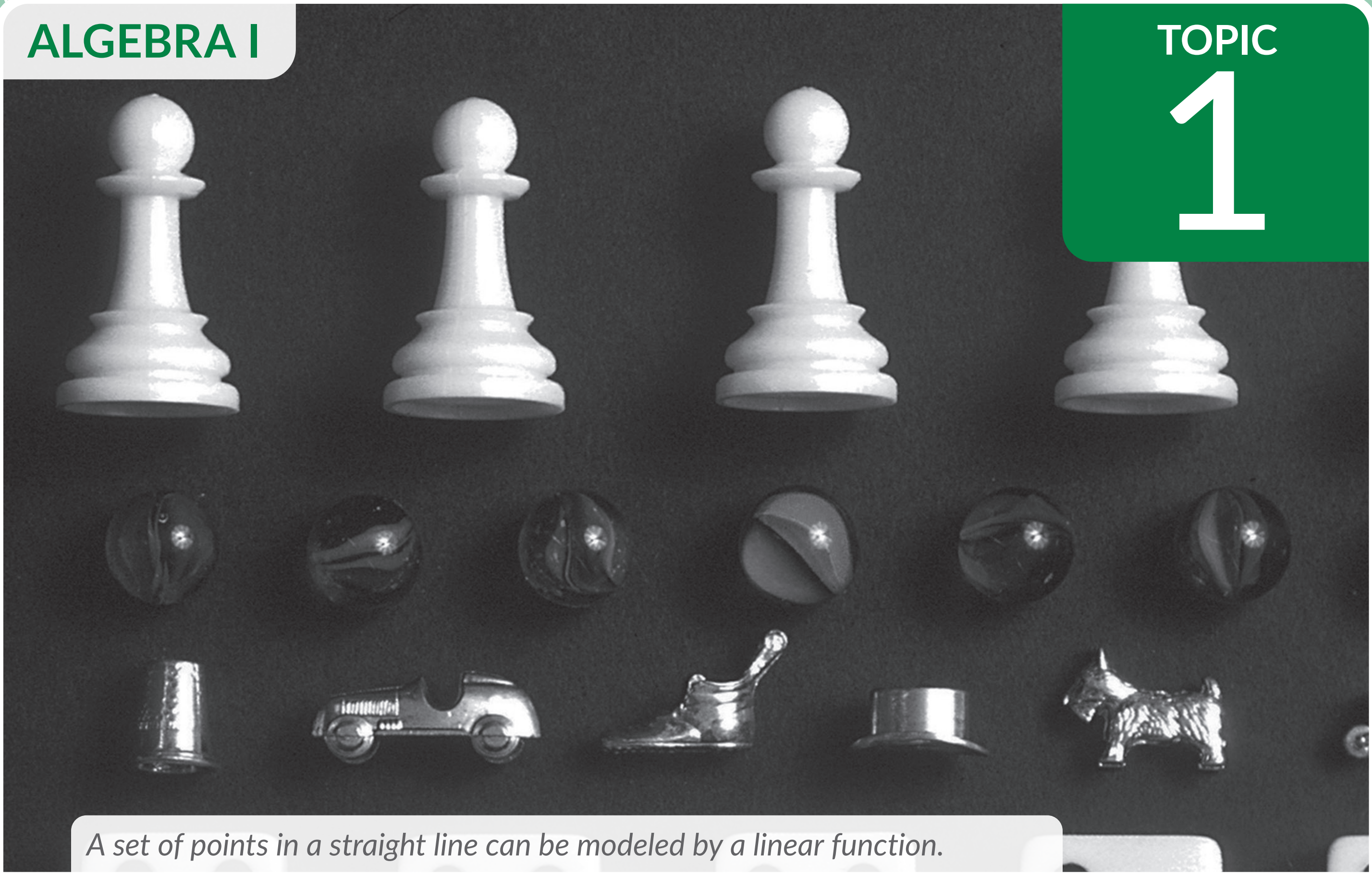
MODULE
2

Exploring Constant Change

TOPIC 1



A set of points in a straight line can be modeled by a linear function.

Linear Functions

1 Least Squares Regression

OBJECTIVES

- Create a graph of data points with and without technology.
- Determine an equation for a line of best fit by visual approximation of a hand-drawn line.
- Determine a linear regression function using technology.
- Make predictions about data using a linear regression function.
- Explain the calculations involved in the Least Squares Method.
- Choose a level of accuracy appropriate when reporting quantities.

NEW KEY TERMS

- Least Squares Method
- centroid
- linear regression function
- interpolation
- extrapolation

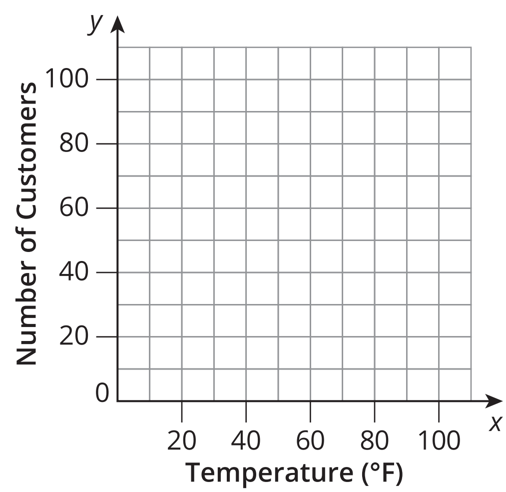
You have searched for patterns in graphs and sequences of numbers.
How can you use what you know to identify patterns in sets of data?

Getting Started

Frozen Yogurt . . . When It's Freezing?

A school's club for aspiring business leaders is helping a local frozen yogurt shop analyze how the business is affected by the weather. The owner is wondering whether there is a relationship between the temperature and the number of customers that buy yogurt during the 2 hours immediately after school. The school's club collected these data.

Temperature (°F)	Number of Customers
45	97
25	55
60	85
15	37
100	100



1. Construct a scatterplot of the collected data.

    a. Plot the first data point. Is there a pattern? Use a piece of spaghetti to approximate a line that models the data.

    b. Add the second data point to the graph. Is there a pattern? Adjust the piece of spaghetti to approximate a line that models the data with the additional point.

c. Add the third data point to the graph. Describe the pattern that you see. Approximate the line using the spaghetti.

d. Continue this process until all five data points are plotted and recorded in the table.

2. Use your linear model to describe the relationship between the temperature outside and the number of customers at the frozen yogurt shop.

ACTIVITY 1.1 A Line Of Best Fit

You have approximated the line that best represents the data with each additional data point.

1. Use the full data set and the line that you approximated to write an equation that you think best represents the data.

2. Based on your equation, predict the number of customers to visit the frozen yogurt shop in the two hours after school for each given temperature.

   a. 85°F

   b. 115°F

   c. 10°F

3. Compare your predictions with your classmates. Did your predictions differ from the other groups? Explain why or why not.

You have noticed that estimating a line of best fit can give different predictions. Fortunately, with technology you can create prediction equations as well as a scatterplots from tables of data. You just need to build a data table that has an independent variable and a dependent variable.

4. Identify the independent and dependent variable. What is the significance of those designations?

5. Use the data table and graphing technology to generate a line of best fit. What is the slope and *y*-intercept of the line and what do they represent?

6. Use the new line of best fit to predict the number of customers at the frozen yogurt shop immediately after school for each given temperature.

    a. 85°F

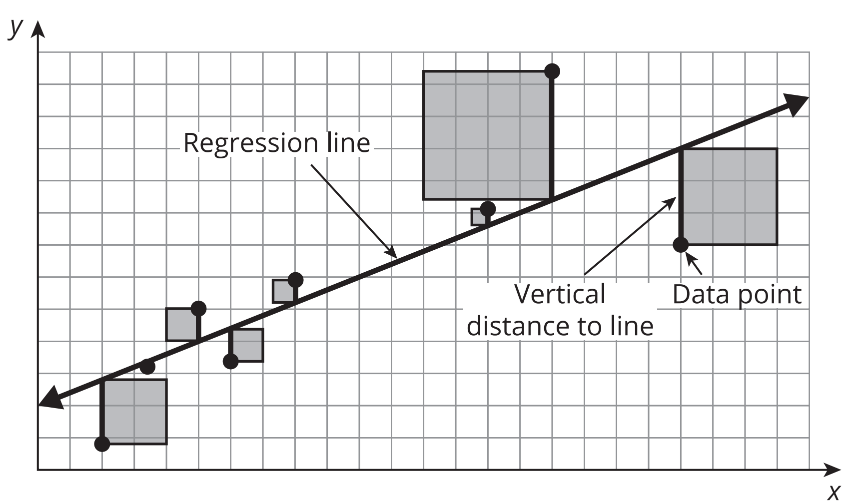
    b. 115°F

    c. 10°F

7. How do your predictions compare to the predictions from the other groups?

The equation that your graphing technology uses to give you the line of best fit is called the **Least Squares Method**. This is a method that creates a line of best fit for a scatterplot that has two basic requirements:

- The line must contain the *centroid* of the data set. The **centroid** is a point whose *x*-value is the mean of all the *x*-values of the points on the scatterplot and its *y*-value is the mean of all the *y*-values of the points on the scatterplot.
- Even though infinitely many lines can pass through the centroid, the **linear regression function** has the smallest possible vertical distances from each given data point to the regression line. The sum of the squares of those distances are at a minimum with this line.



8. Consider the graph of the sample linear regression function.

   a. What do the vertical lines in bold represent?

   b. What do the shaded squares represent? How do they relate to the Least Squares Method?

9. Linh and James each draw a line of best fit to model a set of data. They both record the vertical distances between each point and the line of best fit.

   Linh
   Vertical Distances: 2, 2, 2, 2, 2

   James
   Vertical Distances: 1, 1, 1, 1, 6

   Both students believe they drew the least square regression line. Who's correct? Justify your choice.

10. How does your decision in Question 9 inform you about the placement of a line of best fit using the Least Squares Method?

ACTIVITY
1.2

Making Predictions

The table shows the population of a small town for each year from 2010 to 2017.

Years	2010	2011	2012	2013	2014	2015	2016	2017
Population	21,359	22,906	22,542	23,048	23,562	23,609	24,008	24,716

1. What is the range of the data set?

2. Determine a linear regression model for the data. Round the slope and y-intercept to the nearest whole number.

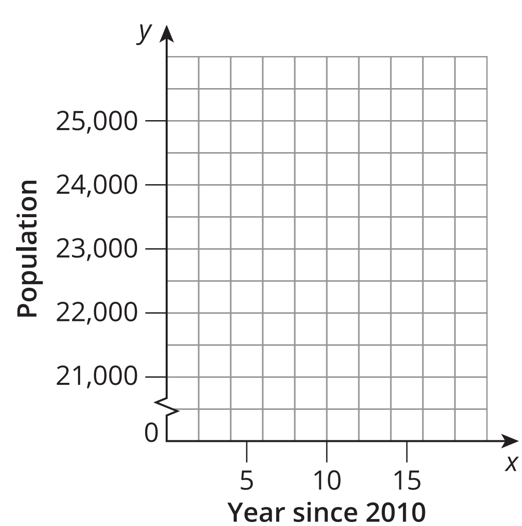
Remember . . .
Data comparing two variables can show a positive association, negative association, or no association.

3. Identify the independent and dependent variables and their units of measure.

4. Does the data represent a function? Does it appear that there is a specific function that could model this data set? When so, describe the function. When not, state why not.

5. Use technology to graph a scatterplot demonstrating the relationship between the year and population. What association do you notice?

6. Between which consecutive years was there a decrease in the population?



7. Use graphing technology to determine the linear regression function for the population. Then, sketch the data points and the line of best fit that you see.

8. What is the relationship between the equation for the line of best fit and any association you notice in the graph? Do you think that this line fits the data well?

9. For each expression from your linear regression function about populations, write an appropriate unit of measure and describe the contextual meaning. Then, choose a term from the word box to describe the mathematical meaning of each part.

Word Box

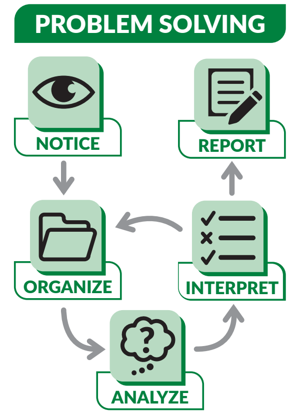
- input value
- output value
- rate of change
- *y*-intercept

		What it Means	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
$f(x)$			
390			
x			
21,855			

10. Use your linear regression function to predict the population for the year 2024.

ACTIVITY 1.3

Making Predictions Within and Outside a Data Set



The music industry is constantly changing how it delivers music to its listeners. The table shows the percent of total U.S. music sales revenues from streaming.

Year	2010	2011	2012	2013	2014	2015
Percent of Total U.S. Music Sales Revenue From Streaming	7	9	15	21	27	34

1. Use graphing technology to determine the linear regression function for the data.

Ask Yourself . . .
What is an appropriate level of accuracy needed throughout this situation?

2. Interpret the equation of the line in terms of this problem situation.

When there is a linear association between the independent and dependent variables of a data set, you can use a linear regression function to make predictions within the data set. Using a linear regression function to make predictions within the data set is called **interpolation**.

3. Use your equation to predict the percent of streaming revenues in 2013. Compare the predicted value percent in 2013 with the actual value.

4. Compute the predicted value percent for 2011 and compare it with the actual value.

5. Do you think a prediction made using interpolation will always be close to the actual value? Explain your reasoning.

To make predictions for values of x that are outside of the data set is called **extrapolation**.

6. Use the equation to predict the percent of streaming revenues:

   a. in 2040.

   b. in 2004.

7. Are these predictions reasonable? Explain your reasoning.

Talk the Talk

Tell Me Ev-ery-thing

You have used technology to determine linear regression functions. You have then used those linear regression functions to predict unknown values within and without a data set.

1. Why is the linear regression function generated using technology more accurate than the line of best fit that you can write using two points?

2. Why are predictions made by extrapolation more likely to be less accurate than predictions made by interpolation?

Lesson 1 Assignment

Write

Complete each sentence with the appropriate vocabulary term.

1. A ____________ can be used to organize and display the values of two variables in a data set.

2. A ________________________ models the relationship between two variables in a data set by producing a line of best fit.

3. A(n) ________________________ is a line that best approximates the linear relationship between two variables in a data set.

4. The ____________________ is used to approximate a line of best fit by minimizing the squares of the distances of the points from the line.

5. ____________ is using a linear regression function to make predictions within the data set.

6. Using a linear regression function to make predictions outside of the data set is ____________.

7. After a scatterplot is created, the ____________ is a point with an x-value that is the mean of all the x-values of the points on the plot and a y-value that is the mean of all the y-values of the points on the plot.

Remember

Patterns in data can be modeled with lines of best fit. The Least Squares Method is one way to create a linear regression function, and it is the method that graphing technology tends to use.

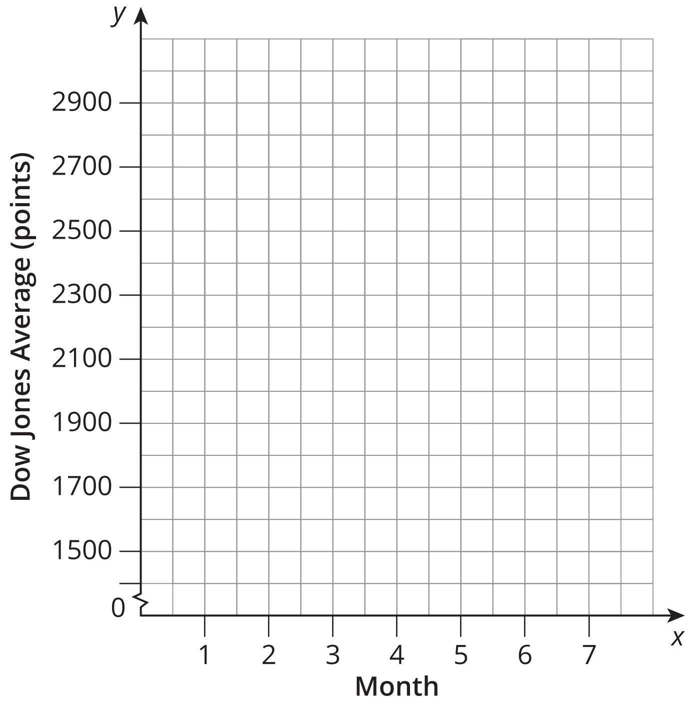
Lesson 1 Assignment

Practice

1. The table shows the Dow Jones Industrial Average on the NYC Stock Exchange over a period of six consecutive months (August through January) in 1987. On Monday, October 19, 1987, stock markets around the world crashed. This decline was the greatest one-day percentage decline in stock market history to date.

Time (months)	Dow Jones Average (points)
1	2700
2	2600
3	1750
4	1950
5	1900
6	2000

   a. Create a scatterplot of the NYC Stock Exchange data. What information can you gather about the stock market from the scatterplot?



   b. Generate a linear regression function that would model the Dow Jones Average using the data for months 1–6.

Lesson 1 Assignment

c. Generate a linear regression function that would model the Dow Jones Average using only the data for months 3–6.

d. When will the model for the data from months 3–6 provide a better prediction?

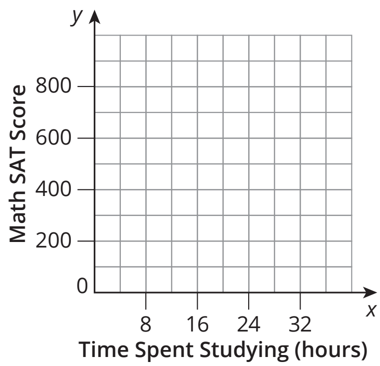
e. Interpret the slope and y-intercept of the linear regression function you wrote in part (c). What do these values represent in terms of the problem situation?

f. Use the linear regression function from part (b) to determine the Dow Jones Average in October 1987. Is this the same as the Dow Jones Average recorded in the table? If not, explain the difference.

2. Mr. Flores is a high school teacher and is preparing his students to take the SAT test. He collected data from 10 students who took the test last year and presented this information to the students in a table. The highest math SAT score a student can achieve is 800. Analyze the data in the table.

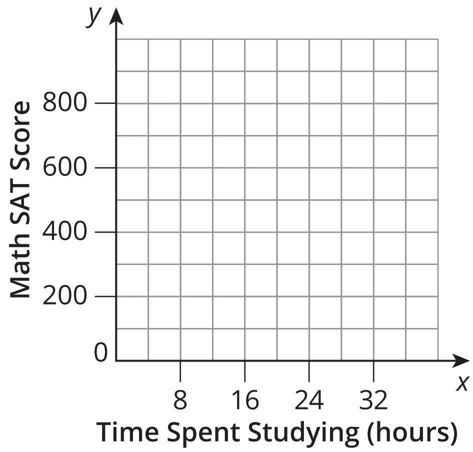
Time Spent Studying (hours)	Math SAT Score
1	350
22	780
12	600
14	700
4	380
10	650
9	580
3	400
7	530
4	410

a. Construct a scatterplot of the data and describe any patterns you see in the data.



Lesson 1 Assignment

b. Determine the equation of a line passing through (14, 700) and (7, 530). Then, determine the equation of a line passing through (22, 780) and (4, 380). Graph both lines on the same graph as the scatterplot.



c. Which line seems to best fit the data? Would you use either one of these lines to make predictions about a student's math SAT score based on the amount of studying they do? Why or why not?

d. Use graphing technology to determine the linear regression function.

e. Interpret the linear regression function in terms of the problem situation.

f. Use the linear regression function to predict the math SAT score for a student who studies for 17 hours. Did you use interpolation or extrapolation to make this prediction? Is this prediction reasonable for this problem situation? Explain your reasoning.

Lesson 1 Assignment

g. Use the linear regression function to predict the math SAT score for a student who studies for 40 hours. Did you use interpolation or extrapolation to make this prediction? Is this prediction reasonable for this problem situation? Explain your reasoning.

h. One of Mr. Flores's students comes back to him the following year and says that he studied for 15 hours for the math SAT and got a score of 610. He argues that the linear regression function predicted that he would have scored a 682. What do you think explains the discrepancy?

Prepare

Describe a possible flaw in the reasoning for each situation.

1. When I wash my hands regularly, I will not get sick.

2. When I practice my guitar every day, I will be a rock star.

3. When I wear my favorite football jersey to support the team, they will win the game.

4. When I am a good driver, I will not have an accident.

2 Correlation

OBJECTIVES

- Determine the correlation coefficient using technology.
- Interpret the correlation coefficient for a set of data.
- Understand the difference between r and r^2.
- Understand the difference between correlation and causation.
- Understand necessary conditions.
- Understand sufficient conditions.
- Choose a level of accuracy appropriate when reporting quantities.

NEW KEY TERMS

- correlation
- correlation coefficient
- coefficient of determination
- causation
- necessary condition
- sufficient condition
- common response
- confounding variable

You have learned how to write a line of best fit relating two variables using the Least Squares Method.

Is there a way to measure the strength of the relationship between the variables?

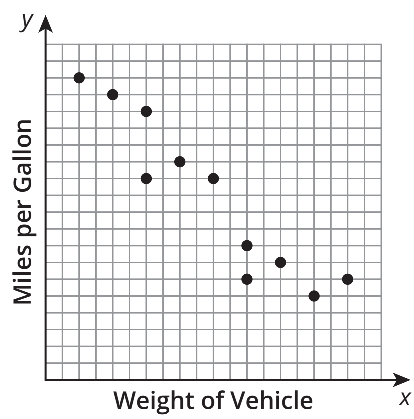
Getting Started

Associate, Formulate, Correlate!

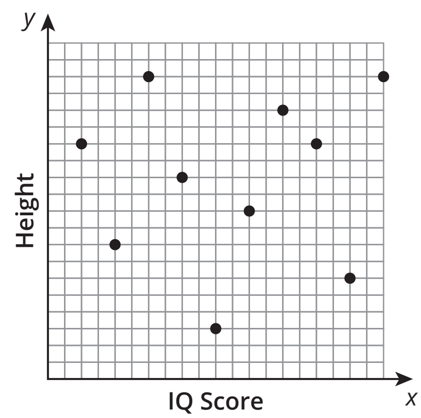
Consider each relationship shown.

1. Describe any associations between the independent and dependent variables, and then draw a line of best fit, if possible.

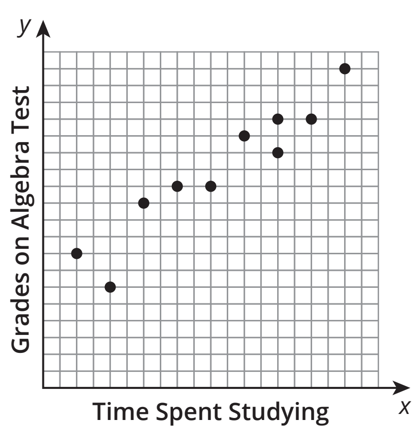
a.



b.



c.



ACTIVITY 2.1 The Correlation Coefficient

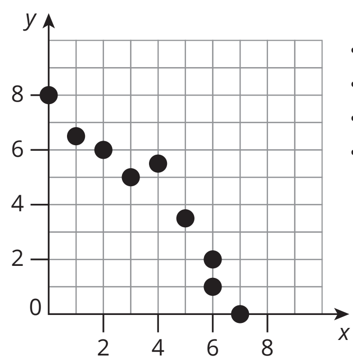
A measure of how well a regression model fits a set of data is called **correlation**. The **correlation coefficient** is a value between −1 and 1, which indicates how close the data are to the graph of the regression function. The closer the correlation coefficient is to 1 or −1, the stronger the relationship is between the two variables. The variable r is used to represent the correlation coefficient.

The correlation coefficient falls between −1 and 0 when the data show a negative association or between 0 and 1 when the data show a positive association.

1. Determine whether the points in each scatterplot have a positive correlation, a negative correlation, or no correlation. Four possible ***r***-values are given. Circle the r-value you think is most appropriate. Explain your reasoning for each.

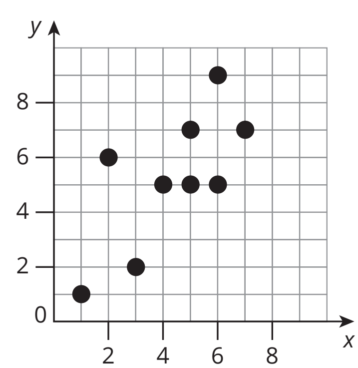
The closer the r-value gets to 0, the less of a linear relationship there is in the data.

a.

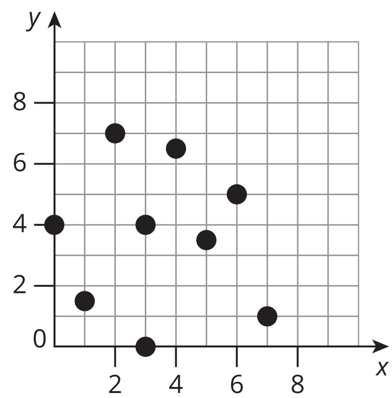


- $r = 0.9$
- $r = -0.9$
- $r = 0.09$
- $r = -0.09$

b.



- $r = 0.7$
- $r = -0.7$
- $r = 0.07$
- $r = -0.07$

c. 

- $r = 1$
- $r = 0.5$
- $r = 0.01$
- $r = -0.5$

You can calculate the correlation coefficient of a data set using the formula:

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

Fortunately, graphing technology can do this arithmetic. Previously you used graphing technology to determine the linear regression function using the Least Squares Method. Along with calculating the equation for the line, graphing technology also calculated the value *r*, the correlation coefficient.

Let's use technology to compute the value of the correlation coefficient.

2. Consider the data set $(-3, -3)$, $(1, 2)$, and $(3, 4)$.

   a. Use technology to compute the correlation coefficient.

   b. Interpret the correlation coefficient of the data set.

ACTIVITY 2.2

Is It Linear?

A group of friends completed a survey about their monthly income and how much they pay for rent each month. The table shows the results.

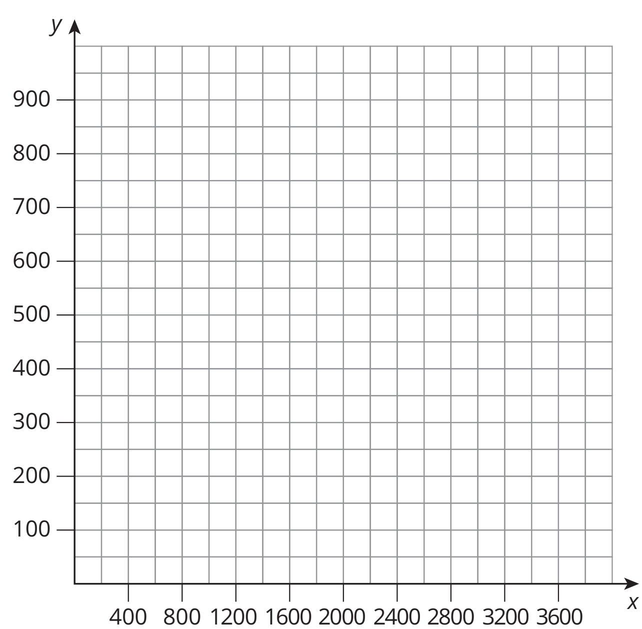
Monthly Net Income (dollars)	Monthly Rent (dollars)
1400	450
1550	505
2000	545
2600	715
3000	930
3400	1000

Ask Yourself . . .
What do you notice as you read through the data?

1. Identify the independent and dependent variables in this problem situation.

2. Construct a scatterplot of the data using technology.

   a. Sketch and label the scatterplot.



   b. Do you think a linear regression function would best describe this situation? Explain your reasoning.

Ask Yourself . . .
What is the appropriate level of accuracy needed for this linear regression function?

3. Use technology to determine whether a line of best fit is appropriate for these data.

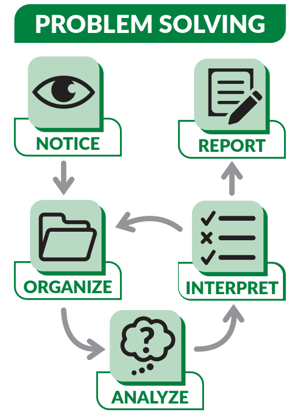
   a. Determine and interpret the linear regression function.

   b. Compute the correlation coefficient.

4. Would a line of best fit be appropriate for this data set? Explain your reasoning.

The correlation coefficient, r, indicates the type (positive or negative) and strength of the relationship that may exist for a given set of data points. The **coefficient of determination**, r^2, measures how well the graph of the regression fits the data. It represents the percentage of variation of the observed values of the data points from their predicted values.

ACTIVITY 2.3 Using the Correlation Coefficient to Assess a Line of Best Fit



The amount of antibiotic that remains in your body over a period of time varies from one drug to the next. The table given shows the amount of Antibiotic X that remains in your body over a period of two days.

Time (hours)	Amount of Antibiotic X in Body (mg)
0	60
6	36
12	22
18	13
24	7.8
30	4.7
36	2.8
42	1.7
48	1

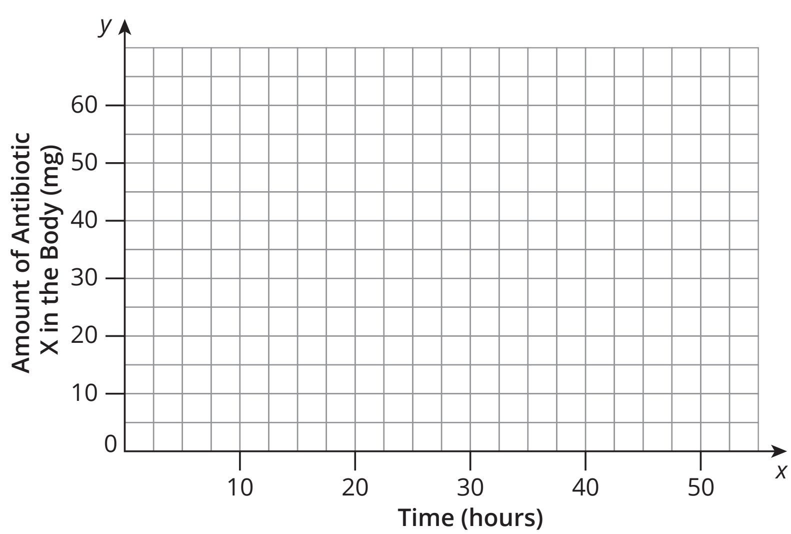
1. Determine and interpret a linear regression function for this data set.

2. Compute and interpret both the correlation coefficient and coefficient of determination of this data set.

3. Does it seem appropriate to use a line of best fit? If no, explain your reasoning.

4. Sketch a scatterplot of the data.

5. Look at the graph of the data. Do you still agree with your answer to Question 3? Explain your reasoning.



ACTIVITY

2.4 Correlation Vs. Causation

Does correlation mean causation? What do you think causation means? That is a question that statisticians are always trying to determine.

Read the three true statements that Jackson and Ricardo are given by their Algebra I teacher. She asks them to decide what conclusions they can draw from the data. Do you agree with them? Why or why not?

1. The number of smartphones sold in the United States has increased every year since 2005. The number of flat screen televisions sold in the United States has also increased during the same period of time.

   Jackson and Ricardo reached the conclusion that owning a cell phone causes a person to buy a flat screen television.

2. Since 2004, the average salary of an NFL football player has increased every year. The average weight of an NFL player has also increased yearly since 2004.

   After much discussion, Jackson and Ricardo reached the conclusion that higher salaries cause the players to gain weight.

3. Worldwide, the number of automobiles sold annually has steadily increased since 1920. Gasoline production has also steadily increased since 1920.

   Jackson and Ricardo concluded that the increase in the number of automobiles sold caused an increase in the amount of gasoline produced.

Proving causation is challenging. The scenarios Jackson and Ricardo analyzed demonstrate that even though two quantities are correlated, this does not mean that one quantity caused the other. This is one of the most misunderstood and misapplied uses of statistics.

Causation is when one event affects the outcome of a second event. A correlation is a **necessary condition** for causation, but a correlation is not a **sufficient condition** for causation. While determining a correlation is straightforward, using statistics to establish causation is very difficult.

4. Many medical studies have tried to prove that not washing your hands can cause you to get sick.

    a. Is not washing your hands a necessary condition for getting sick? Why or why not?

    b. Is not washing your hands a sufficient condition for getting sick?

    c. Is there a correlation between people who don't wash their hands and people who get sick? Explain your reasoning.

    d. Is it true that not washing your hands causes you to get sick? If so, how was it proven?

5. It is often said that being outside for an extended amount of time can cause you to get a sunburn.

    a. Is going outside for a extended amount of time a necessary condition for a sunburn? Why or why not?

    b. Is going outside for an extended amount of time a sufficient condition for a sunburn? Why or why not?

    c. Is there a correlation between being outside for an extended amount of time and getting a sunburn? Explain your reasoning.

    d. Is it true that being outside for an extended amount of time causes sunburns? Explain your reasoning.

Does school absenteeism cause poor performance in school?
A correlation between the independent variable of days absent to the dependent variable of grades makes sense. However, this alone does not prove causation.

6. To prove that the number of days that a student is absent causes the student to get poor grades, we would need to conduct more controlled experiments.

    a. List several ways that you could design experiments to attempt to prove this assertion.

b. Will any of these experiments prove the assertion? Explain your reasoning.

There are two relationships that are often mistaken for causation. A **common response** is when some other reason may cause the same result. A **confounding variable** is when there are other variables that are unknown or unobserved.

7. Consider each relationship. List two or more common responses that could also cause this result.

   a. In North Carolina, the number of shark attacks increases when the temperature increases. Therefore, a temperature increase appears to cause sharks to attack.

   b. A company claims that their weight loss pill caused people to lose 20 pounds when following the accompanying exercise program.

Talk the Talk

Correlations R Us

Consider the given data sets.

Set A

x	y
0	24
2	19
5	12
10	6
20	0

Set B

x	y
8	13
10	4
14	15
15	14
19	73

1. Determine the linear regression function for each set.

2. Compare the correlation coefficient and the coefficient of determination of each data set. Describe which linear regression function is the better fit and why.

Lesson 2 Assignment

Write

Complete each sentence.

1. A correlation is a ____________________ for causation, but a correlation is not a ____________________ for causation.
2. A ____________________ is when some other reason may cause the same result.
3. ____________________ is when one event causes a second event.
4. A ____________________ is when there are other variables that are unknown or unobserved.
5. The ______________________ is a value between −1 and 1 that indicates how close the data are to forming a straight line.
6. The percentage of variation of the observed values of the data points from their predicted values is represented by the ____________________________.

Remember

Sets of data can frequently be modeled by using a linear function called a *linear regression function.* A value called the *correlation coefficient* can also be calculated to assist in determining how well the regression model fits the data.

Practice

1. The table shows the percent of the United States population who did not receive needed dental care services.

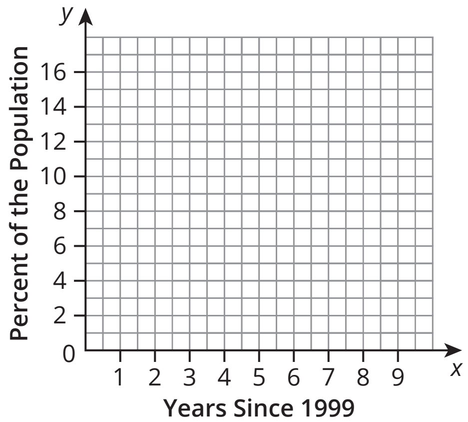
Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Percent	7.9	8.1	8.7	8.6	9.2	10.7	10.7	10.8	10.5	12.6	13.3

a. Do you think a linear regression function would best describe this situation? Why or why not?

b. Determine the linear regression function for these data. Interpret the equation in terms of this problem situation.

c. Compute and interpret the correlation coefficient of this data set. Does it seem appropriate to use a line of best fit? Explain your reasoning.

d. Sketch a scatterplot of the data. Then, plot the equation of the linear regression function on the same grid. Do you still think a linear regression model is appropriate? Explain your answer.



2. A teacher claims that students who study will receive good grades.

   a. Do you think that studying is a necessary condition for a student to receive good grades?

   b. Do you think that studying is a sufficient condition for a student to receive good grades?

   c. Do you think that there is a correlation between students who study and students who receive good grades?

Lesson 2 Assignment

d. Do you think that it is true that studying will cause a student to receive good grades?

e. List two or more confounding variables that could have an effect on this claim.

3. For each situation, decide whether the correlation implies causation. List reasons why or why not.

   a. The number of newspapers sold in a city is highly correlated to the number of runs scored by the city's professional baseball team.

   b. The number of mouse traps found in a person's house is highly correlated to the number of mice found in their house.

Prepare

Use what you know about arithmetic sequences to complete each task.

1. Rewrite the sequence $a_n = 10 - 3(n - 1)$ in simplest terms. Then, write the first 5 terms of the sequences generated by a_n.

2. Given the function $f(x) = -3x + 10$, calculate $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$.

3 Making Connections Between Arithmetic Sequences and Linear Functions

OBJECTIVES

- Use algebraic properties to prove the explicit formula for an arithmetic sequence is equivalent to the equation of a linear function.
- Relate the defining characteristics of an arithmetic sequence, the first term and common difference, and the defining characteristics of a linear function, the y-intercept and slope.
- Connect the slope of a line to the average rate of change of a function.

NEW KEY TERMS

- conjecture
- first differences
- average rate of change

You know that all sequences are functions. What type of function is an arithmetic sequence?

Getting Started

Line Up in Sequential Order

Ask Yourself . . .
What is the difference between an arithmetic and geometric sequence?

Lauren counts the number of new flowers that are blooming in her garden each day in the spring. Sequence A represents the number of new flowers on day 1, day 2, etc.

Sequence A: 3, 6, 12, 24, 48

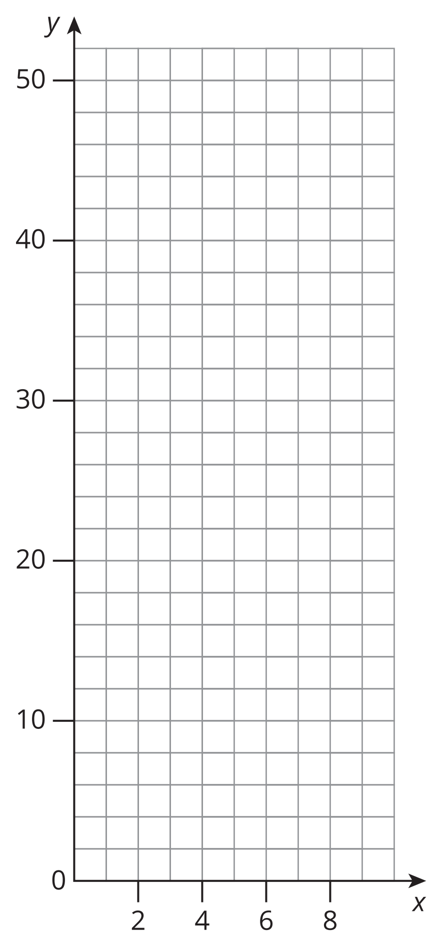
She also measures the height of the first sunflower that has started blooming. Sequence B represents the height in centimeters of the sunflower on day 1, day 2, etc.

Sequence B: 3, 6, 9, 12, 15

1. For each sequence, determine whether it is arithmetic or geometric and write the explicit formula that generates the sequence. Then, graph and label each on the coordinate plane.

   Sequence A:

   Sequence B:



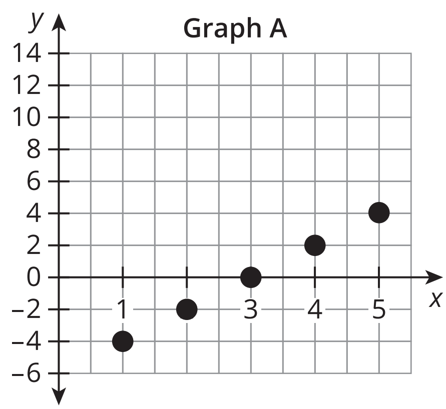
2. List at least three common characteristics of the graphs. How do the two sequences compare?

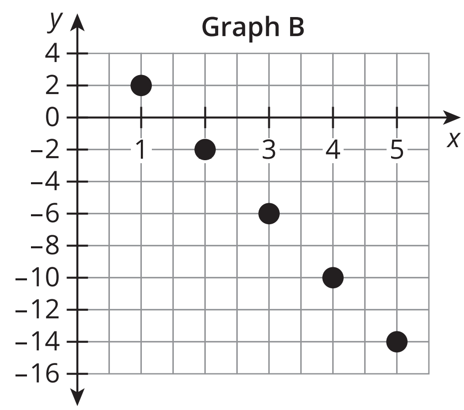
ACTIVITY 3.1 Connecting Forms

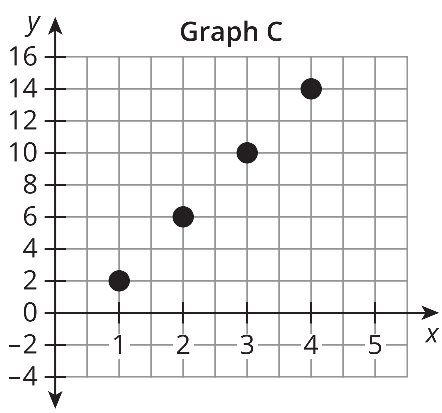
Consider the four explicit formulas, each representing a different arithmetic sequence.

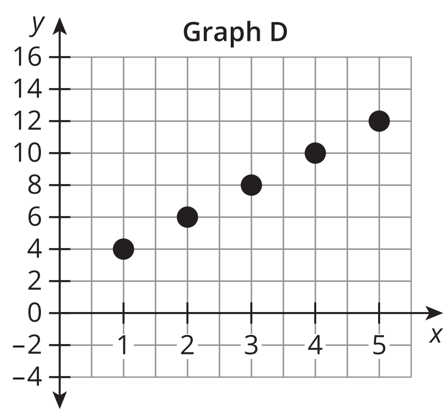
- $a_n = 2 - 4(n - 1)$
- $a_n = 2 + 4(n - 1)$
- $a_n = -4 + 2(n - 1)$
- $a_n = 4 + 2(n - 1)$

1. Match each explicit formula with its graph. Describe the strategies you used.









Ask Yourself . . .
How do you know by the form of the explicit formula that it represents an arithmetic sequence?

2. Consider the set of graphs and identify the function family represented. Based on these formulas and graphs, do you think that all arithmetic sequences belong to this function family? Explain your *conjecture*.

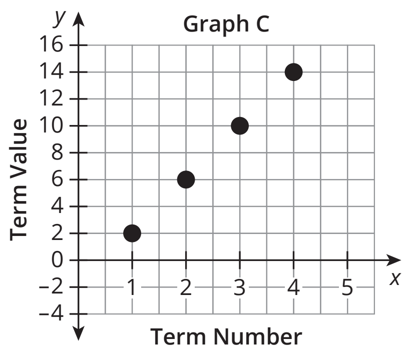
A **conjecture** is a mathematical statement that appears to be true, but has not been formally proven.

Remember . . .

An arithmetic sequence is a sequence of numbers in which the differences between any two consecutive terms is constant. The explicit formula is of the form $a_n = a_1 + d(n - 1)$.

Let's take a closer look at the relationship between arithmetic sequences and the family of linear functions. You know a lot about each relationship.

The explicit formula and the table of values that represents Graph C are shown.



$$a_n = 2 + 4(n - 1)$$

Term Number n	Term Value a_n	
1	a_1	2
2	a_2	6
3	a_3	10
4	a_4	14

3. Describe the domain of the sequence.

4. Identify the common difference in each representation.

A linear function written in general form is $f(x) = ax + b$, where a and b are real numbers. In this form, a represents the slope and b represents the y-intercept.

5. Draw a line to model the linear relationship seen in the graph. Then, write the equation to represent your line. Describe your strategy.

6. Describe the domain of the graph of the linear model.

An explicit formula is one equation that you can use to model an arithmetic sequence. You can rewrite the explicit formula for the arithmetic sequence $a_n = 2 + 4(n - 1)$ in function notation.

WORKED EXAMPLE

You can represent a_n using function notation.

$$a_n = 2 + 4(n - 1)$$
$$f(n) = 2 + 4(n - 1)$$

Next, rewrite the expression $2 + 4(n - 1)$.

$f(n) = 2 + 4n - 4$	Distributive property
$\quad = 4n + 2 - 4$	Commutative property
$\quad = 4n - 2$	Combine like terms

So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$.

7. Compare the equation you wrote to model the graph of the arithmetic sequence to the explicit formula written in function form. What do you notice?

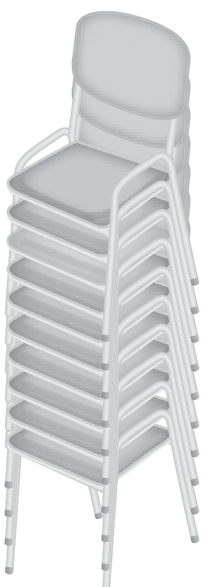
8. Compare the common difference with the slope. What do you notice?

9. Explain why Nakota's reasoning is not correct.

> Nakota
>
> The y-intercept of a linear function is the same as the first term of an arithmetic sequence.

ACTIVITY
3.2

Connecting Constant Difference and Slope



At the end of the last show, stagehands at the community theater stack the audience chairs and place them in storage. The height of one chair is 34 inches, and as each additional chair is stacked, the height increases by 8 inches.

1. Write an equation using the explicit formula to represent this scenario.

2. Rewrite the explicit formula in function form and define your variables.

3. Compare the two algebraic representations of this scenario.
   a. How is the value of d in the explicit formula related to the value of a in function form? How are d and a represented in the scenario? Be sure to include units of measure.

   b. How is the value of a_1 from the explicit formula related to b in function form? How are a_1 and b represented in the scenario?

c. If a_1 represents the first term of a sequence, what does a_0 represent? How can you rewrite the arithmetic sequence using a_0?

$a_n =$ ____________________

In this stacked chair scenario, both d and a represent the additional 8 inches of height per one chair, or the unit rate of change. The common difference, d, of the explicit formula is the same as the slope, a, of a general linear function. They both represent constant change.

In a sequence, the common, or constant, difference is the difference in term values between consecutive terms.	In a linear function, the slope describes the direction and steepness of the line. It is the difference in output values divided by the difference in corresponding input values.

When you see a graph or rewrite an explicit formula for an arithmetic sequence, it is apparent that it represents a linear function. However, the structure of a table requires other strategies to determine whether it represents a linear function.

One strategy is to examine *first differences*. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

The tables that represent the explicit formula and function form of the stacking chairs scenario are shown.

n	a_n
1	34
2	42
3	50
4	58

x	$y = f(x)$
0	26
1	34
2	42
3	50

The expression $y = f(x)$ means that the value of y depends on the value of x. That is, for different values of x, there is a function f which determines the value of y.

4. Determine the first differences in each table to verify they both represent a linear relationship.

5. Joey and Alejandra agree that the table shown represents a linear relationship because there is a constant difference between consecutive points. Joey claims the slope is 5 and Alejandra claims the slope is −5.

Who's correct? Explain your reasoning.

x	y
1	22
2	17
3	12
4	7

$22 - 17 = 5$

$17 - 12 = 5$

$12 - 7 = 5$

Ask Yourself . . .
What does slope describe?

6. Use first differences to determine whether each table represents a linear function. Then, create your own table to represent a linear function. Describe your strategy.

a.

x	y
5	12
6	15
7	21
8	30

b.

x	y
−2	18
−1	14
0	10
1	6

c.

x	y
10	1
11	4
12	9
13	16

d.

x	y

ACTIVITY 3.3

Connecting Constant Difference, Slope, and Average Rate of Change

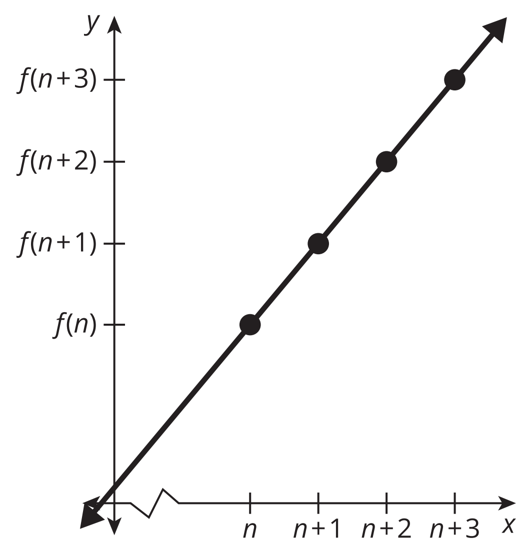
In the previous activity, you determined that the constant difference of the chair heights and the slope of the line are both equal to 8 inches per chair. Is the slope of a linear function always equal to the constant difference of the corresponding arithmetic sequence?

Consider the graph of the arithmetic sequence represented by the general linear function $f(x) = ax + b$.

1. Identify the constant difference of the sequence in the graph.

2. Complete the table for consecutive values of the input.

Term Number	Term Value	
n	$f(n)$	$a(n) + b$
$n + 1$	$f(n + 1)$	$a(n + 1) + b$
$n + 2$		
$n + 3$		



3. Select any two consecutive input values in the sequence. Use the expressions for the term values to determine the constant difference of the sequence.

Recall that the slope of a line is constant between any two points on the line, not just consecutive points.

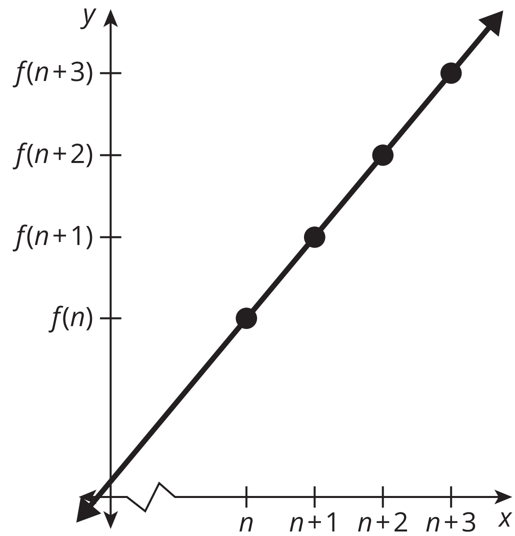
Remember . . .

The slope formula is $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$. Delta ($\Delta$) is a Greek letter used in mathematics to represent "change in". So, $\frac{\Delta y}{\Delta x}$ means "change in y" divided by "change in x".

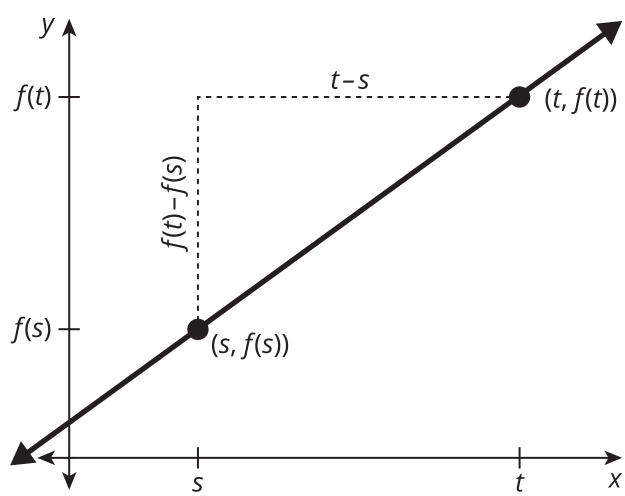
4. Use the table and graph from Question 2 to complete each task.

   a. Identify the slope of the function on the graph.

   b. Select two non-consecutive points in the table and determine the slope between those two points.



The slope, a, is equal to the constant difference. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t)-f(s)}{t-s}$. This represents the change in the output as the input changes from s to t.



5. Show that the slope formula and the average rate of change formula represent the same ratio.

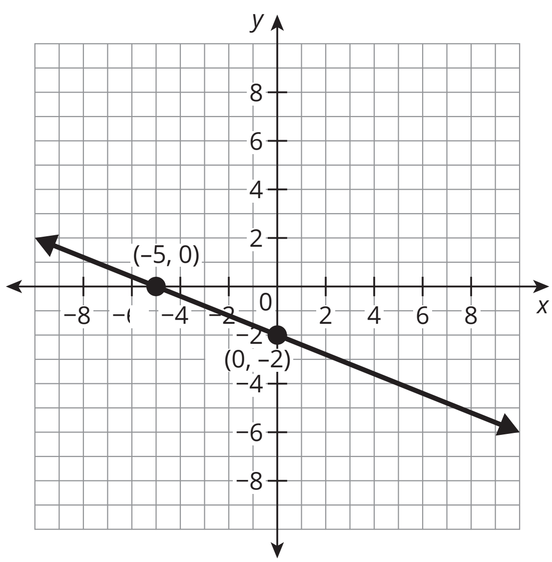
6. Determine the average rate of change for each graph, table, or function.

a. $g(x) = 3x + 2$

b.

x	y
4	1
6	−1
8	−3
10	−5

c.

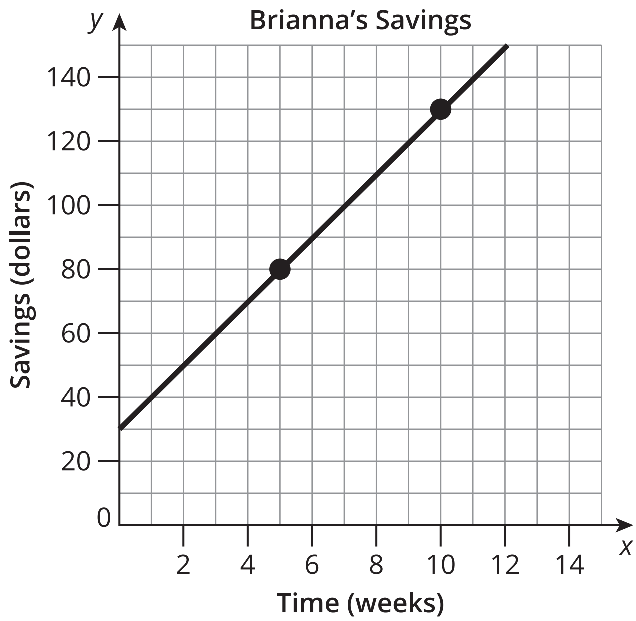


d. The table shows the linear relationships between the distance Hailey runs and the time she spends running.

Average Distance Traveled

Time (minutes)	4	6	8	12
Distance (miles)	0.5	0.75	1	1.5

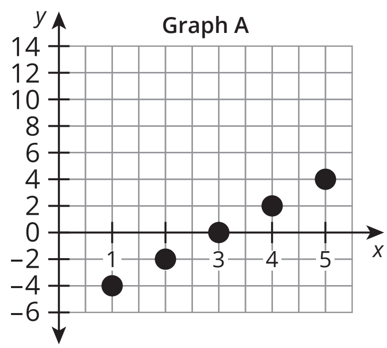
e. The graph shows the relationship between the balance of Brianna's bank account and the number of weeks she has been saving.



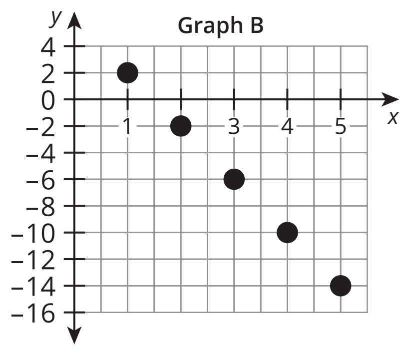
f. The function $y = 2.5x - 15.25$ models the amount of money the student council earns from selling bagels after spending money on supplies for the sale.

ACTIVITY 3.4 Verifying that Common Differences and Slopes are the Same

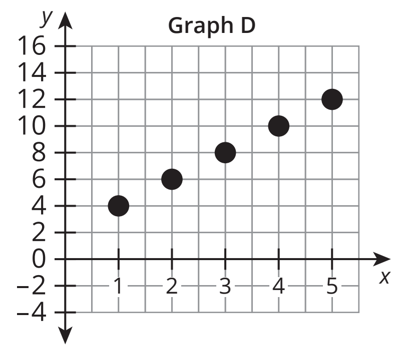
The remaining explicit formulas and graphs from Activity 1 are shown.



$a_n = -4 + 2(n - 1)$



$a_n = 2 - 4(n - 1)$



$a_n = 4 + 2(n - 1)$

1. For each graph and arithmetic sequence, complete each task.

   a. Identify the constant difference in the explicit formula and on the graph.

   b. Rewrite each explicit formula in function notation.

   c. Verify that the constant difference is the same as the slope of the linear function.

   d. Describe how the first term in the explicit formula is related to the *y*-intercept of the function.

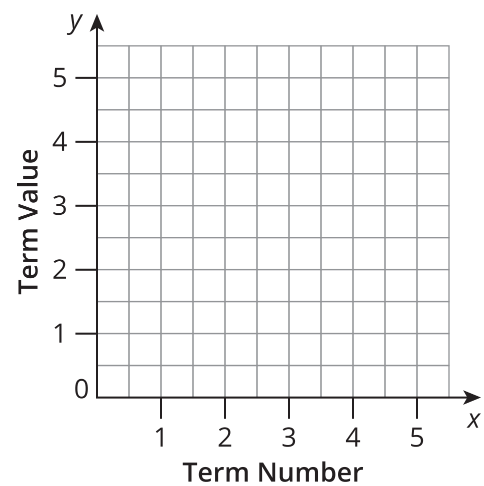
ACTIVITY 3.5 Describing Constant Sequences

Lauren's cat loves to knock over flowerpots. Each morning, she counts the number of flowerpots her cat knocked over during the night before she uprights them again. Sequence C represents the number of flowerpots knocked over on day 1, day, 2, etc.

Sequence C: 3, 3, 3, 3, 3

1. Determine the constant difference and write the explicit formula to represent Sequence C. Then, create a table of values and graph it.

Term Number (n)	Term Value (a_n)



2. Use function notation to write an equation representing the relationship between the number of days, x, and the number of of knocked over flowerpots, C. Interpret the function in terms of the context.

3. Prove that the slope of $C(x)$ is equal to the common difference of Sequence C.

Consider a similar sequence.

Sequence D: $-5, -5, -5, -5, -5$

4. Write the function, $D(x)$, to model this sequence.

5. Prove $D(x)$ is a constant function.

Remember . . .
When the the values of the dependent variable of a function remain constant over the entire domain, then the function is called a constant function.

Talk the Talk

Making It Plain and Clear

You have proven that all arithmetic sequences can be represented by linear functions.

1. Complete the graphic organizer to summarize the connections between arithmetic sequences and linear functions. Then, describe how you can tell a linear relationship exists given a table of values or a graph.

Equation

Arithmetic Sequence	Linear Function	Mathematical Meaning
$a_n = a_1 + d(n - 1)$	$f(x) = ax + b$	
a_n		
d		
n		
$a_1 - d$		

Characteristics and Representations of Linear Functions

Table of Values

Graph

Lesson 3 Assignment

Write

Describe how the terms *constant difference*, *slope*, and *average rate of change* are related.

Remember

The explicit formula of an arithmetic sequence can be rewritten as a linear function in the general form $f(x) = ax + b$, where a and b are real numbers, using algebraic properties. The constant difference of an arithmetic sequence is always equal to the slope of the corresponding linear function.

Practice

1. Gracie claims that the equation $f(n) = 5n - 7$ is the function notation for the sequence that is represented by the explicit formula $a_n = -2 + 5(n - 1)$. Nicky doesn't understand how this can be the case.

   a. Help Nicky by listing the steps to write the explicit formula of the given sequence in function notation. Provide a rationale for each step.

   b. Graph the function. Label the first 5 values of the sequence on the graph.

Lesson 3 Assignment

2. Determine whether each table of values represents a linear function. For those that represent linear functions, write the function. For those that do not, explain why not.

a.

x	$f(x)$
3	14
4	18
5	23
6	29

b.

x	$g(x)$
0	2
1	−1
2	−4
3	−7

c.

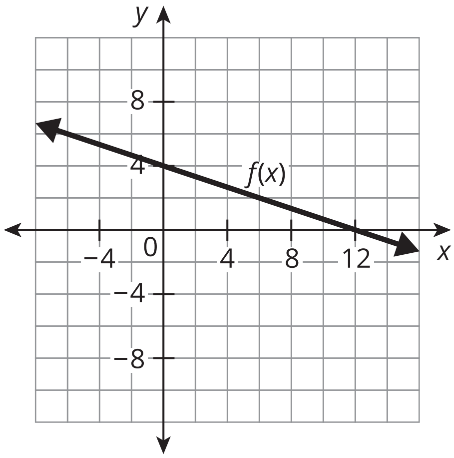
x	$h(x)$
1	11
2	16
3	21
4	26

3. Calculate the average rate of change for each linear function using the formula. Show your work.

a.

x	$f(x)$
3	−4
7	4
9	8
12	14

b.



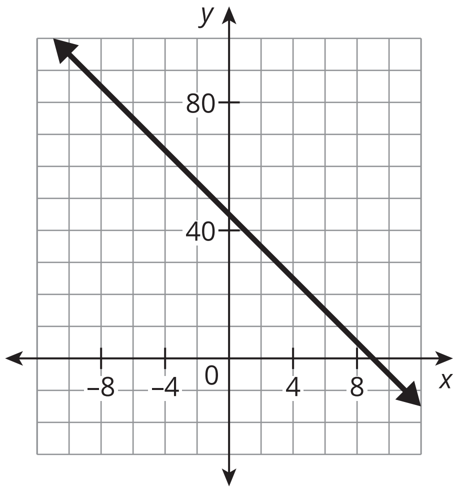
Lesson 3 Assignment

Prepare

Write an equation for each linear relationship.

1. The contestant on a game show has already won a total of \$2750 from a previous episode when the game show continued today. He earns an additional \$250 for each question he answers correctly today.

2.



4 Point-Slope Form of a Line

OBJECTIVES

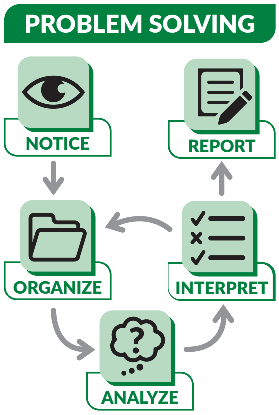
- Use the slope formula to derive the point-slope form of a linear equation.
- Construct an equation in point-slope form to model a linear relationship between two quantities.
- Write equations for vertical and horizontal lines.

NEW KEY TERM

- point-slope form

You have used the slope-intercept form to represent linear relationships. Are there other forms of a linear equation that you can use? How do you write equations for horizontal and vertical lines?

Getting Started



Draining the Pool

Miguel and Nia are pool cleaners who have been hired to drain the community diving pools at the end of the summer. They are comparing the rate at which the three pools drain.

1. For each pool, write an equation in slope-intercept form to represent the linear relationship.

   a. Pool *A* is at a water level of 14 feet and drains at a rate of 3 feet per hour.

   b. Pool *B* is at a water level of 10 feet after draining for 3 hours and drains at a rate of 2 feet per hour.

   c. Pool *C* is at a water level of 15 feet after draining for 2 hours and at 12 feet after draining for 4 hours.

I wonder whether there is a way to make writing the equation of a line more efficient.

2. Compare your process for writing each equation. How are the processes different?

ACTIVITY 4.1

Writing Equations in Point-Slope Form

In the previous lesson, you used the slope, the *y*-intercept, and the slope formula to write a linear equation. You can also determine the equation of a line without knowing the *y*-intercept.

WORKED EXAMPLE

To write an equation of a line from a table of values, you can use the slope formula.

x	y
2	6
4	5
6	4

- First, calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 6}{4 - 2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table.

(2, 6)

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$
$$-\frac{1}{2}(x - 2) = y - 6$$

The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

This linear equation in the Worked Example is written in *point-slope form*. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line.

1. Solve the equation in the Worked Example for y so that the linear equation is in slope-intercept form. What unique information does each form of the linear equation provide? How are they similar?

Write the equation for each linear relationship in point-slope form.

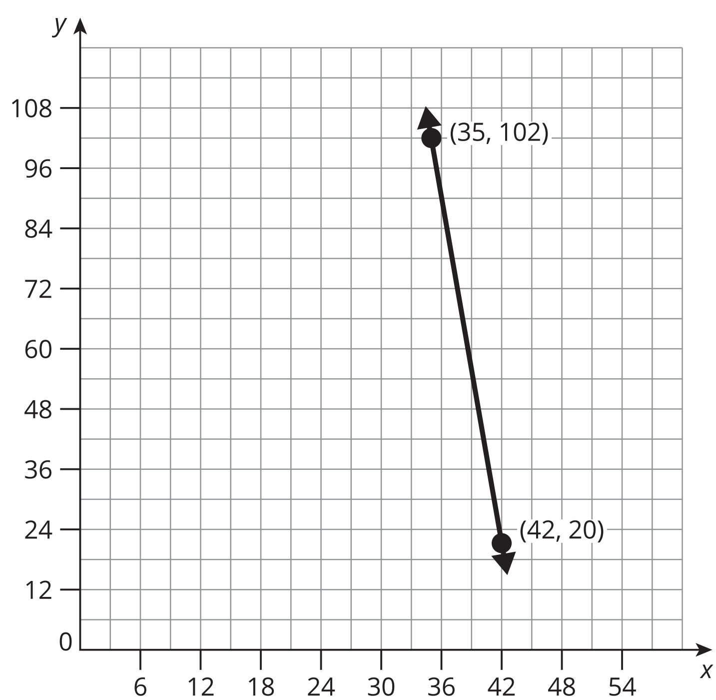
2. The slope is -8. The point (3, 12) lies on the line.

3. The slope is $\frac{2}{3}$. The point $(-4, 5)$ lies on the line.

4. It costs $1.10 per pound plus a basic shipping charge to mail a package. A 10-pound package costs $14 to mail.

5. $m = -\frac{3}{8}$ and point $(50, 7)$

6.



7.

x	y
-5	-6
1	-6
2	-6

8. Rewrite the equation you wrote for Question 7 in slope-intercept form. What do you notice about this equation?

9. Consider the information you may have about a linear relationship. Which form of the equation do you prefer to use in each case? Explain your reasoning.

   a. Given slope and y-intercept

   b. Given two points

   c. Given slope and a point other than the y-intercept

ACTIVITY
4.2 Horizontal and Vertical Lines

Horizontal and vertical lines represent linear relationships, but their equations are different from the equations of lines that are not horizontal or vertical.

1. Consider the equation, $y = -6$, that you wrote for the table shown in the previous activity.

    a. How is this equation different from the other equations? What is its slope?

    b. Describe the graph of the coordinate pairs in this table. Why does the value of its slope make sense?

What is the y-intercept?

x	y
-5	-6
1	-6
2	-6

    c. Explain why the equation makes sense in terms of the graph and the table.

2. Write an equation for each linear relationship. Describe the graph of the linear relationship. State the slope and y-intercept.

    a.

x	y
-7	11
-2	11
0	11

    b. A line that passes through $(-15, -3.75)$ and $(89, -3.75)$

3. Consider a new table of values representing a linear relationship.

x	y
−2	5
−2	14
−2	29

   a. Explain how this table is similar to and different from the tables in Questions 1 and 2.

   b. Write an equation for the linear relationship in the table.

   c. Describe the graph of this linear relationship.

   d. Use the slope formula to calculate the slope between two points in the table. What do you notice?

   e. What is the y-intercept of this linear relationship? Explain why this makes sense.

4. Write an equation for each linear relationship. Describe the graph of the linear relationship.

   a.

x	y
$\frac{17}{2}$	−18
$\frac{17}{2}$	23
$\frac{17}{2}$	267

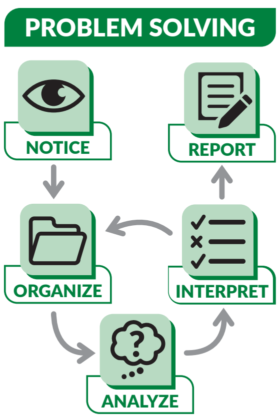
   b. A line that passes through (−7, −973) and (−7, 542)

   c. Create an additional table of values and write the equation for a vertical line.

In a horizontal line there is no change in the y-values as the x-values change. Therefore, the slope is 0. A horizontal line has zero steepness. In a vertical line there is no change in the x-values as the y-values change. Therefore, the slope is undefined. A vertical line has an undefined steepness.

ACTIVITY 4.3 Matching Representations

Take out your scissors. It's time to cut and sort!



1. Carefully cut out the graphs, tables, contexts, and equations located the end of the lesson. Match each equation with its correct graph, table, or context. Explain how you matched the equations with the representations.

2. Compare the graphs.

   a. How are they different? How are these differences reflected in the slope-intercept form of their equation?

   b. Identify the *y*-intercept for each graph. How can you determine this point in the slope-intercept form of the equation for each graph?

   c. Identify the slope for each graph. How is the slope represented in the slope-intercept form of each equation?

3. Analyze the equation for each table.

    a. Determine the coefficient of x for each linear relationship using the slope formula.

Can you remember the ways to determine the rate of change from a table?

    b. How can the number that is added in each equation written in slope-intercept form be determined from the table?

4. Analyze the equation for each context. Explain what each term of the equation means in each context.

Talk the Talk

Say What?

You have learned about two forms of a linear equation: the slope-intercept form, $y = mx + b$, and the point-slope form, $y - y_1 = m(x - x_1)$.

1. What information can you determine about each line by looking at the structure of the equation?

   a. $y = \frac{3}{5}x - 4$

   b. $y - 6 = 2(x + 1)$

   c. $y + 4 = 2(x - 0)$

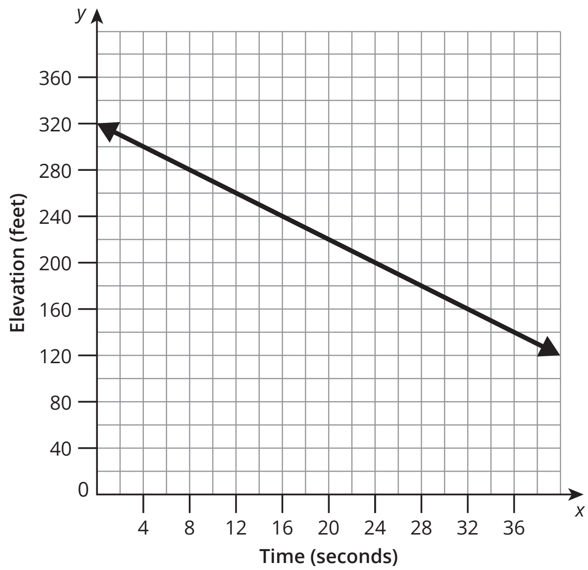
   d. $y = -\frac{2}{7}x$

   e. $y + 5 = -(x - 4)$

   f. $y = 19$

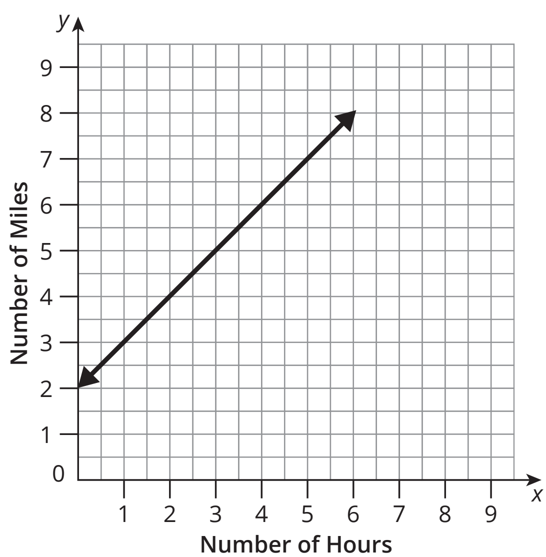
2. Create a context that represents a linear relationship that passes through the point (2, 56) and has a positive slope. Then, write the equation of the line in point-slope form and slope-intercept form.

a.



b. Juliana read the first 40 pages of a mystery novel before she fell asleep. The next day, she read one page every two minutes until she finished the book, which was a total of 325 pages.

c.



d.

Time (hours)	Water Level (feet)
x	***y***
2	15
4	13.5
8	10.5
10	9

e.

Number of Games Ben Won Today	Number of Credits on Ben's Player's Card
x	***y***
12	216
18	264
25	320
40	440

f. A pizza shop charges $4.50 for a small pizza, $7.00 for a medium pizza, and $9.00 for a large pizza. Additional toppings cost extra depending on the size of the pizza ordered. Antonio ordered a large pizza with three toppings that cost a total of $12.60.

Why is this page blank?

So you can cut out the cards on the other side.

$y = 1.2x + 9$	$y = -\frac{3}{4}x + \frac{33}{2}$
$y = \frac{1}{2}x + 40$	$y - x = 2$
$y - 200 = -5(x - 24)$	$y = 8x + 120$

Why is this page blank?

So you can cut out the cards on the other side.

Lesson 4 Assignment

Write

Compare the slope-intercept and point-slope forms of a linear equation.

Remember

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

Practice

Write an equation in point-slope form.

1. $m = 2$; (5, 6)

2. $m = -9.2$; (−17, 10)

3. (−2, −3) and (8, −8)

4. (79, 52) and (−87, 550)

5. A photography studio charges $50 for a sitting fee and 6 prints. Luigi increased his order to 11 prints and paid $65.

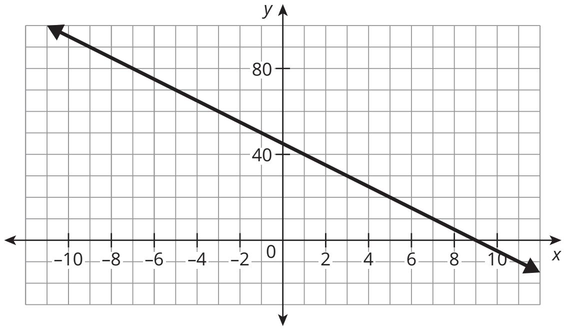
6. Lucia is taking the stairs in her building from her floor to the top of the building. After 2 minutes, she was 100 steps from the bottom floor. After 5 minutes, she was 196 steps from the bottom floor.

Write an equation in any form.

7. A newspaper charges a flat fee plus a charge per day to place a classified ad.

Number of Days	Total Charge ($)
2	8.00
4	13.00
6	18.00

8.

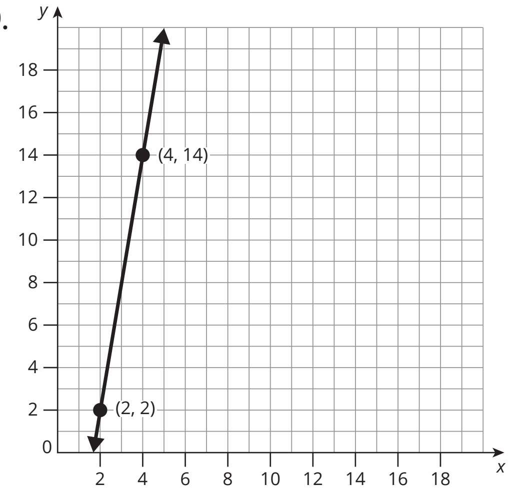


Lesson 4 Assignment

9.

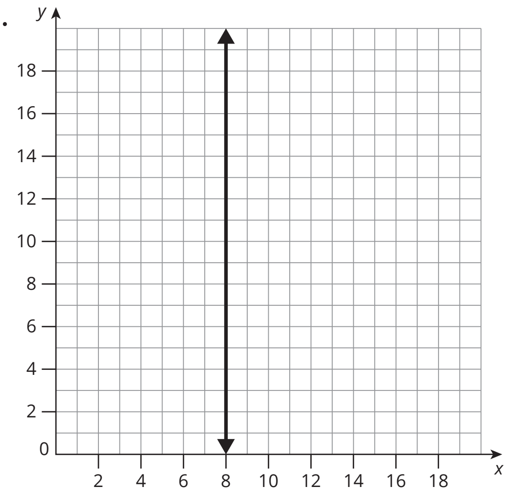
x	y
−10	50
−2	10
4	−20
14	−70

10.



11. Xavier is traveling on a toll road. He plans to exit the road 5 miles ahead and pay \$1.75. He changes his plans and travels 9 miles and pays \$2.75.

12.



Prepare

Solve each equation for y.

1. $-2y = -x + 7$

2. $\frac{3}{4}y = x - 6$

3. $2x + 3y = 6$

4. $\frac{1}{2}x - 4y = 8$

5 Using Linear Equations

OBJECTIVES

- Construct linear equations to model relationships between two quantities.
- Graph lines using the slope-intercept form of a linear equation.
- Graph lines using the point-slope form of a linear equation.
- Graph lines using the standard form of a linear equation.
- Convert equations from point-slope or standard form to slope-intercept form.
- Discuss the advantages and disadvantages of slope-intercept, point-slope, and standard form.

NEW KEY TERMS

- standard form
- *x*-intercept

You have graphed equations using tables of values.

Is there a more efficient method to graphing a linear relationship? Can you use the equation of a linear relationship to create a graphical representation?

Getting Started

Jump In the Line

Describe what you know about the graph of each relationship by analyzing each equation. Then, explain how you might graph each line given its equation.

1. $y = \frac{2}{3}x + 7$

2. $y - 3 = 5(x + 1)$

3. $x = -4$

4. $-3x + 8y = 10$

ACTIVITY 5.1 Using Slope-Intercept Form to Graph a Line

As you learned previously, the slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. You can use the equation to graph the relationship without first creating a table of values using the y-intercept and the slope.

Douglas is giving away tickets to a concert that he won from a radio station contest. Currently, he has 10 tickets remaining. He gives a pair of tickets to each person who asks for them.

This situation can be modeled by the equation $y = -2x + 10$, where x represents the number of people who request tickets and y represents the number of tickets available.

To graph the equation $y = -2x + 10$, you will first plot the y-intercept, (0, 10), and then use the slope, -2, to plot two more points. Remember, slope describes the steepness and direction of a line. Slope is the ratio of the change in y-values to the change in x-values, commonly referred to as rise over run. In this equation, you can think of $m = -2$ as two different ratios: $\frac{-2}{1}$ or $\frac{2}{-1}$. The sign of the number tells you the direction to go to plot a new point. The ratio $\frac{-2}{1}$ has a negative rise and a positive run. It is interpreted as down 2 units and to the right 1 unit. The ratio $\frac{2}{-1}$ has a positive rise and a negative run. It is interpreted as up 2 units and to the left 1 unit.

A rule of thumb when graphing a line is to plot at least three points.

WORKED EXAMPLE

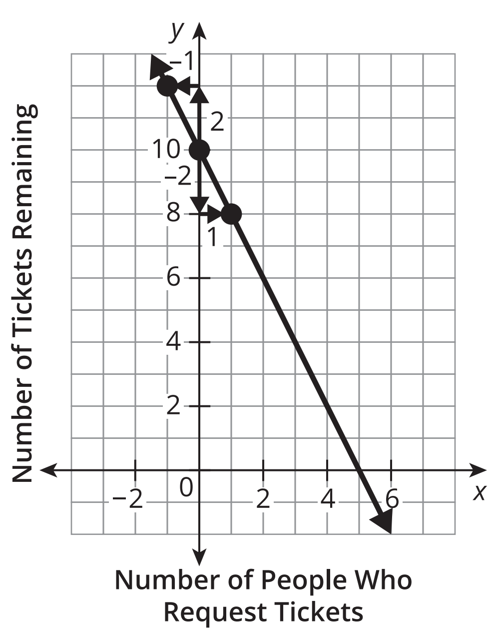
Graph $y = -2x + 10$.

Begin by plotting the y-intercept, (0, 10).

Use the slope and count from the y-intercept to graph two more points on the line.

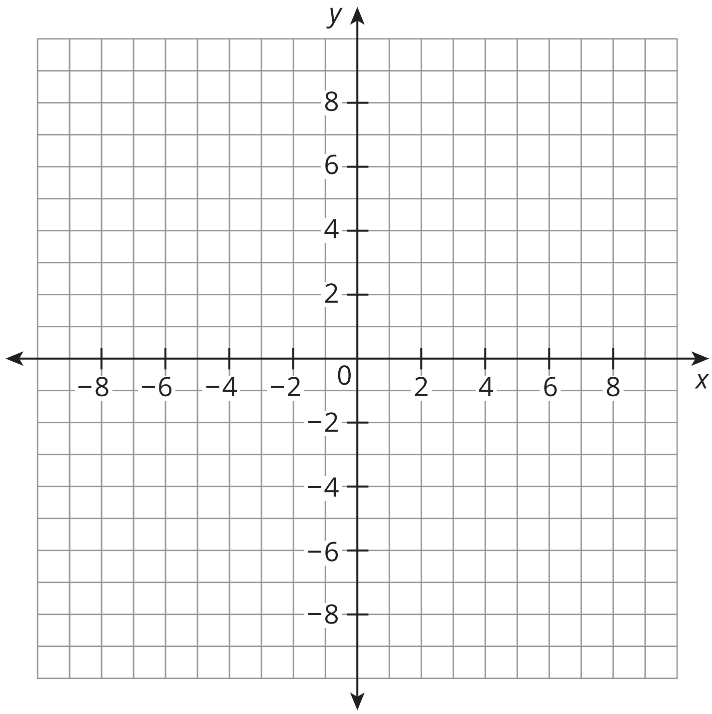
- For $m = \frac{-2}{1}$, go down 2 units and to the right 1 unit.
- For $m = \frac{2}{-1}$, go up 2 units and to the left 1 unit.

Connect the points to form a straight line.



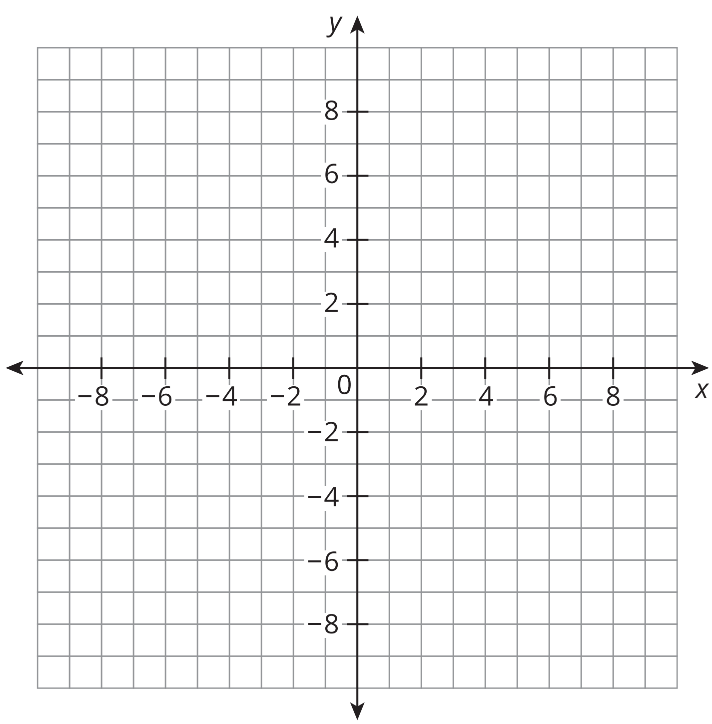
Use the equations to graph each line.

1. $y = \frac{3}{2}x - 1$

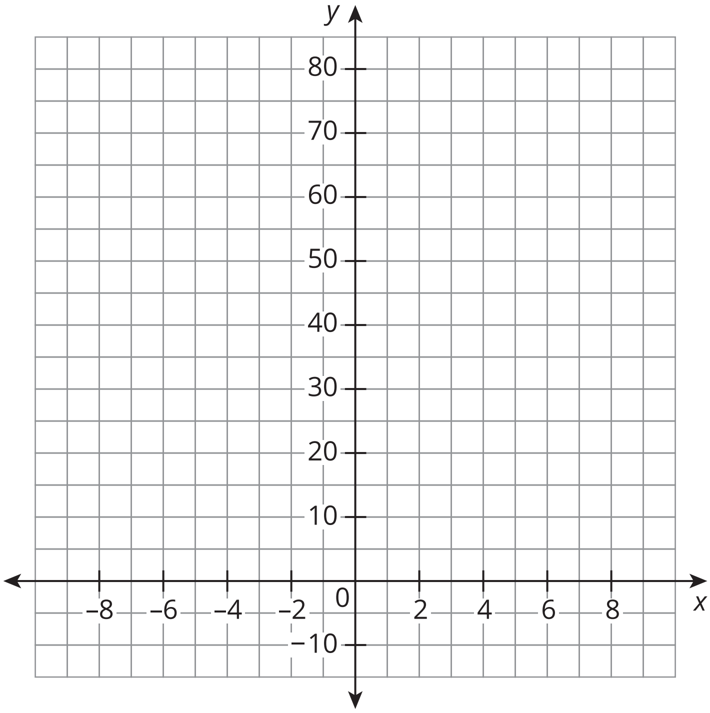


Think about the direction of the line before you start graphing.

2. $y = -\frac{5}{2}x + 3$



3. $y = 10x + 25$

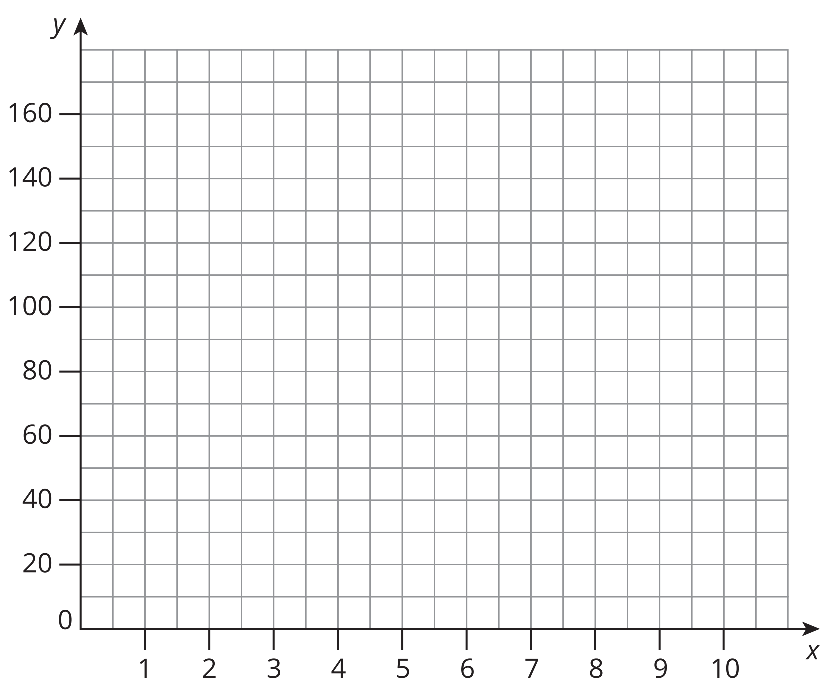


Use a straightedge to draw your lines.

Ask Yourself . . .
Should each graph have arrows or points on each end? When using a point, how do you know whether to use an open circle or a closed circle?

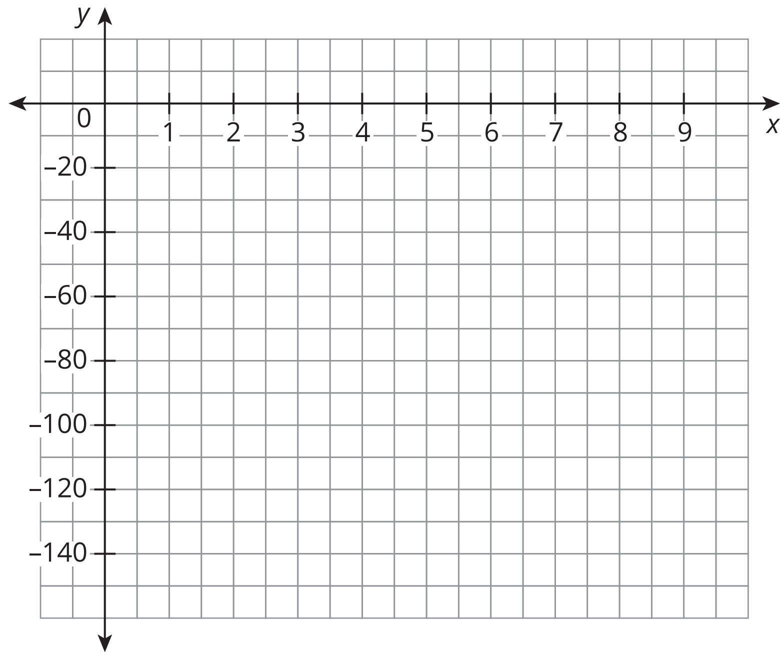
Write an equation for each problem situation. Then, graph each equation.

4. Avery wants to buy a virtual reality headset. He already has $30 saved and plans to mow lawns for $15 each.



Remember to label the axes with the appropriate variable quantities.

5. A scuba diver is 130 feet below sea level. The diver ascends at a constant rate of 20 feet per minute.



ACTIVITY 5.2 Using Point-Slope Form to Graph a Line

In the previous lesson, you learned that the point-slope form of a linear equation is $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is a point on the line. Use this form of a linear equation for the next problem situation.

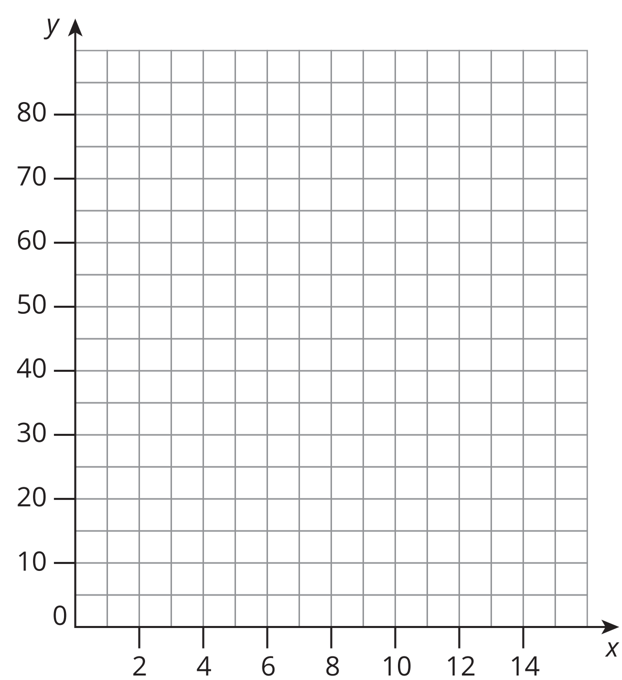
The jazz band is selling tickets to raise money for new music stands. They already had some money in their account when they started selling tickets at $5.00 each. After selling 3 tickets, the band had a total of $50 in their account.

1. Let $x =$ the number of tickets sold, and let $y =$ the total amount of money in the jazz band's account.

   a. Write an equation in point-slope form.

   b. Use the point-slope form to graph the equation.

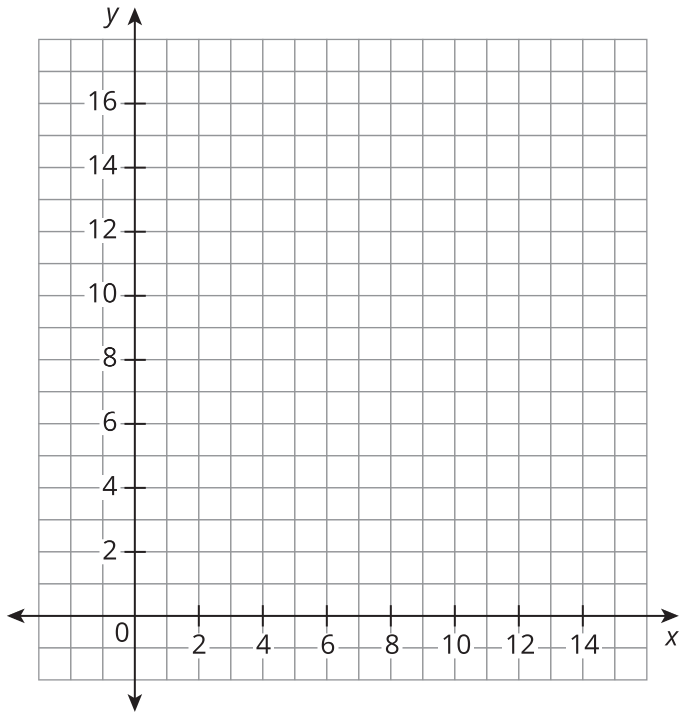
      - Write the coordinates for the known point. Plot the point on the coordinate plane.
      - Write the slope as a ratio. Then, use the slope and count from the point. To identify another point on the graph, start at the point and count either down (negative) or up (positive) for the rise. Then, count either left (negative) or right (positive) for the run.
      - Continue the counting process to plot at least two more points.
      - Connect the points to form a straight line.



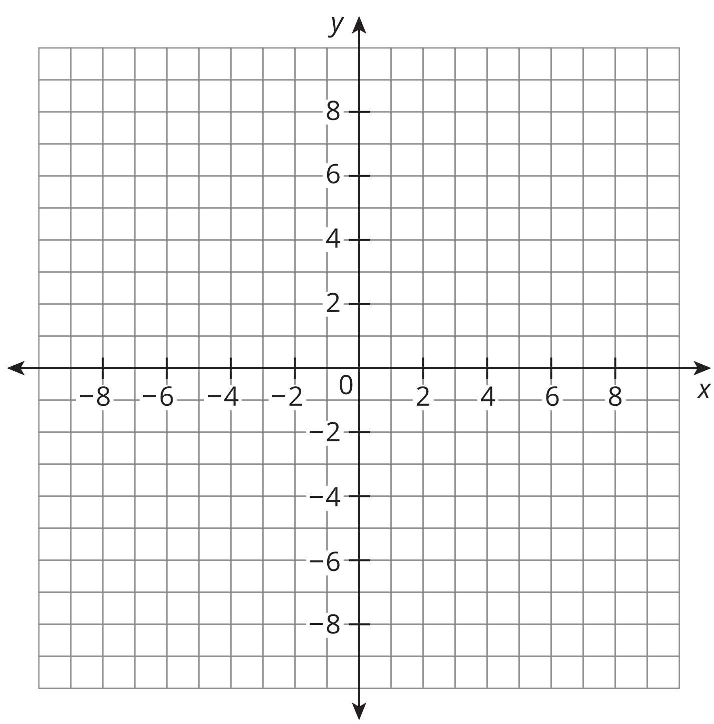
Remember, start with the given point and then use the slope to plot two more points.

2. Identify the slope and a point on the line from the given equation. Then, graph each line. Be careful to take into account the scales on the axes and the signs of the points given in the equation.

a. $y - 5 = -\frac{3}{4}(x - 14)$

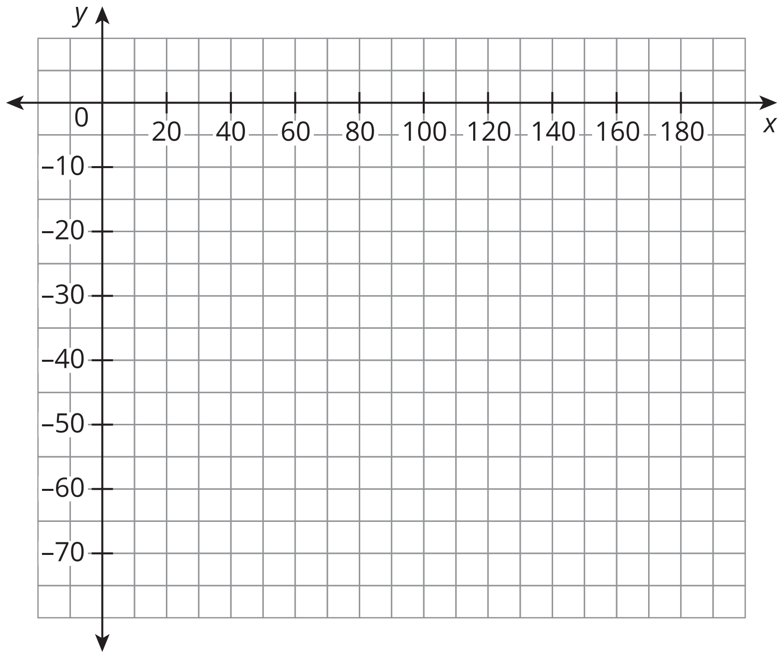


b. $y + 8 = \frac{1}{2}(x + 6)$



3. An underwater remote operated vehicle (ROV) is lowered from a boat at a constant rate of 0.3 meters per second. After 90 seconds, the ROV is at a depth of -25.8 meters.

Define your variables and units. Then, write and graph an equation for the depth of the submarine over time.



ACTIVITY 5.3

Standard Form of a Linear Equation

Tickets for a school play cost \$5.00 for students and \$8.00 for adults. On opening night, \$1600 is collected in ticket sales.

This situation can be modeled by the equation $5x + 8y = 1600$. You can define the variables as shown.

x = number of student tickets sold
y = number of adult tickets sold

This equation is not written in slope-intercept form or in point-slope form. It is written in *standard form*. The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

1. Explain what each term of the equation represents in the problem situation.

   a. $5x$

   b. $8y$

   c. 1600

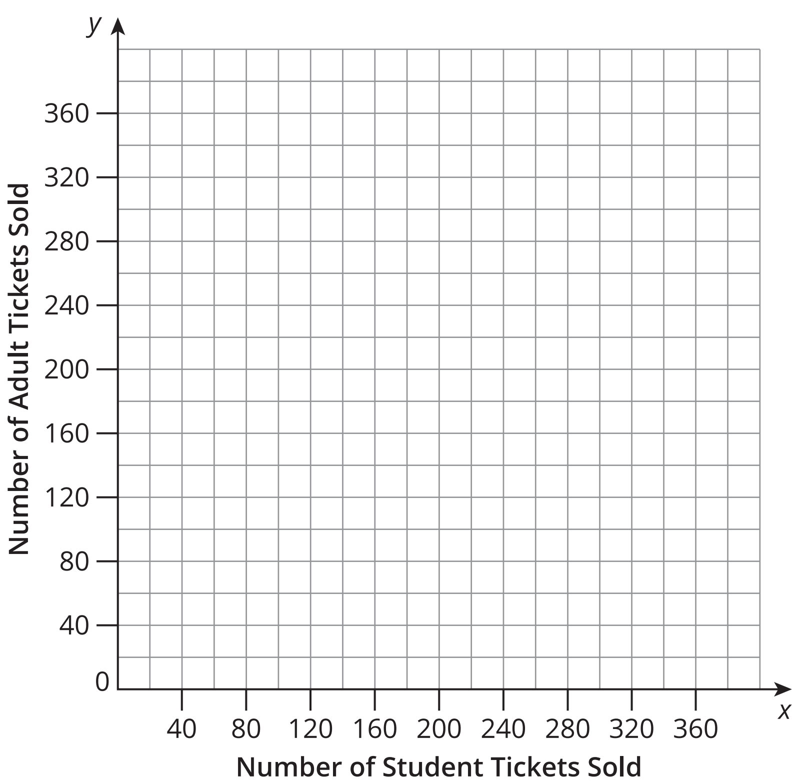
2. What is the independent variable? What is the dependent variable? Explain your reasoning.

Remember, the y-intercept, $(0, b)$ is where a line crosses the y-axis, so the value of x is 0. To calculate a y-intercept, substitute 0 for x and solve the equation for b.

The *x-intercept*, $(x, 0)$, is where the line crosses the x-axis, so the value of y is 0. To calculate an x-intercept, substitute 0 for y and solve the equation for x.

3. Calculate and interpret the meanings of the x-intercept and y-intercept for this equation.

4. Use the x-intercept and y-intercept to graph the equation of the line.



5. Determine the slope of this line. Interpret the meaning of the slope in this problem situation.

6. Complete the table.

Standard Form	x-Intercept	y-Intercept	Slope
$5x + 2y = 6$			
	$\left(\frac{7}{3}, 0\right)$		$m = -\frac{3}{4}$
		$(0, -3)$	$m = \frac{2}{3}$
$-5x + 7y = 11$			

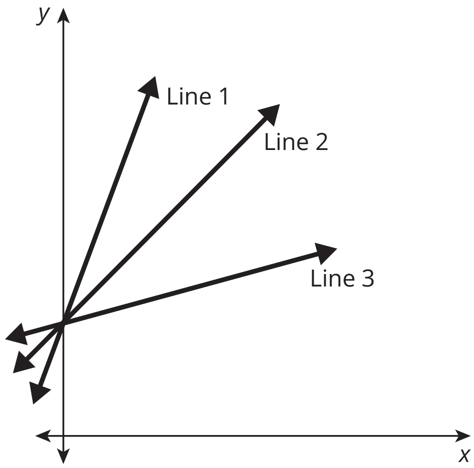
7. What do you notice about the relationship between the integers *A*, *B*, and *C* from the standard form and

   a. the x-intercepts?

   b. the y-intercepts?

   c. the slope?

8. Match each graph with the correct equation written in standard form. Explain your reasoning.



a. $3x - 12y = -60$

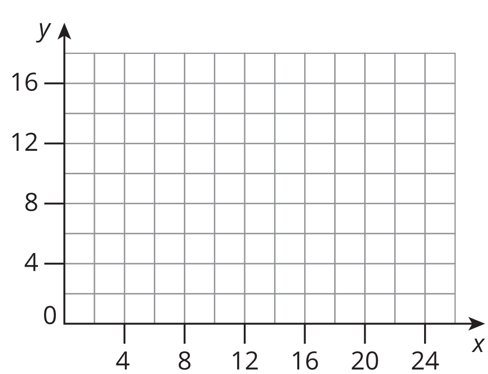
b. $6x - 2y = -10$

c. $9x - 9y = -45$

Notice that there are no values on the *x*- and *y*-axis. What strategies can you use to determine which graph goes with which equation?

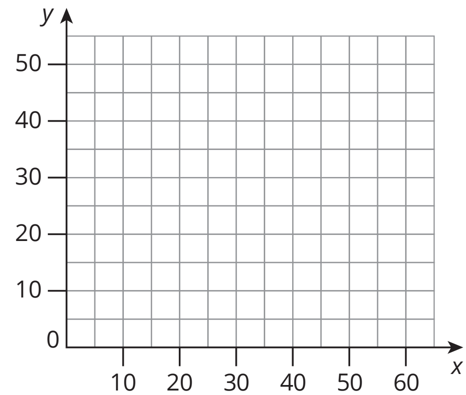
Define variables for each problem situation. Then, write an equation in standard form and use the intercepts to graph the linear relationship.

9. Nia burns 20 calories for every 5 minutes she jumps rope and 50 calories for every 5 minutes she runs. On Tuesday, Nia burned a total of 500 calories.



Ask Yourself . . .
Why is the standard form of a linear equation appropriate to represent this situation?

10. For the show choir's holiday performance, they are selling tickets for \$5 per student and \$6.00 per adult. On the night of their final performance, they collect \$270 in ticket sales.



WORKED EXAMPLE

Write a linear equation in standard form, $Ax + By = C$, that goes through the points (−1, 3) and (−4, −3).

Begin by determining the slope of the line.

$$m = \frac{(-3 - 3)}{(-4 - (-1))} = \frac{(-6)}{(-3)} = 2$$

Pick one of the points and write an equation in point-slope form for the line.

$y - 3 = 2(x - (-1))$

$y - 3 = 2(x + 1)$

Rewrite the equation in standard form.

$$\begin{aligned} y - 3 &= 2x + 2 \\ y &= 2x + 5 \\ -2x + y &= 5 \end{aligned}$$

In standard form, A, B, and C are integers.

11. Write a linear equation in standard form given two points on the line.

a. (0, −4) and (2, 2)

b. (1, −5) and (−2, 7)

c. (12, 10) and (16, 12)

ACTIVITY 5.4

Identifying Slope and y-Intercept

Each equation represents a linear relationship. Examine each and determine the slope and the y-intercept. Write the y-intercept as an ordered pair.

1. $y = 11x - 9$

2. $4x + 6y = 270$

3. $y = 5(2x - 9)$

4. $8y = -6x + 24$

5. $y + 9 = 6(x - 3)$

6. $y = 9$

7. $4x - 12y = 48$

8. $x = 10$

9. $y - 4 = 3(x + 1)$

10. $y = 9 - \frac{1}{2}x$

Talk the Talk

Choose Your Medium

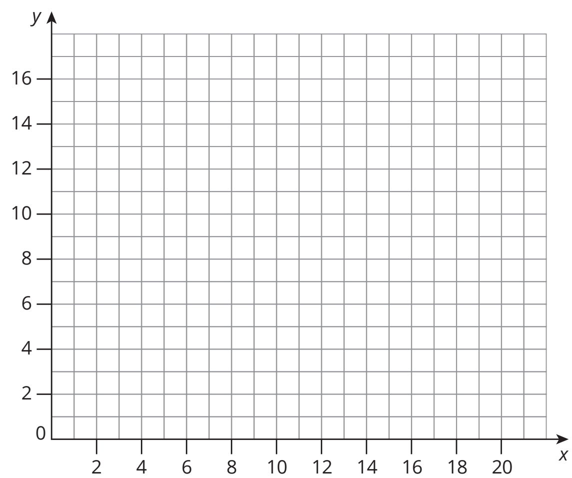
Ask Yourself . . .
What tools or strategies can you use to solve this problem?

For each context, complete each task:

- Write an equation in slope-intercept, point-slope, or standard form.
- State the form of the equation you used and your reason for using it.
- Graph the line using any method.
- Explain the graphing method you used and your reason for using it.
- Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. On a math quiz, students earned 2 points for every correct multiple-choice question and 3 points for every correct short answer question. Joey earned a total of 36 points on the quiz.

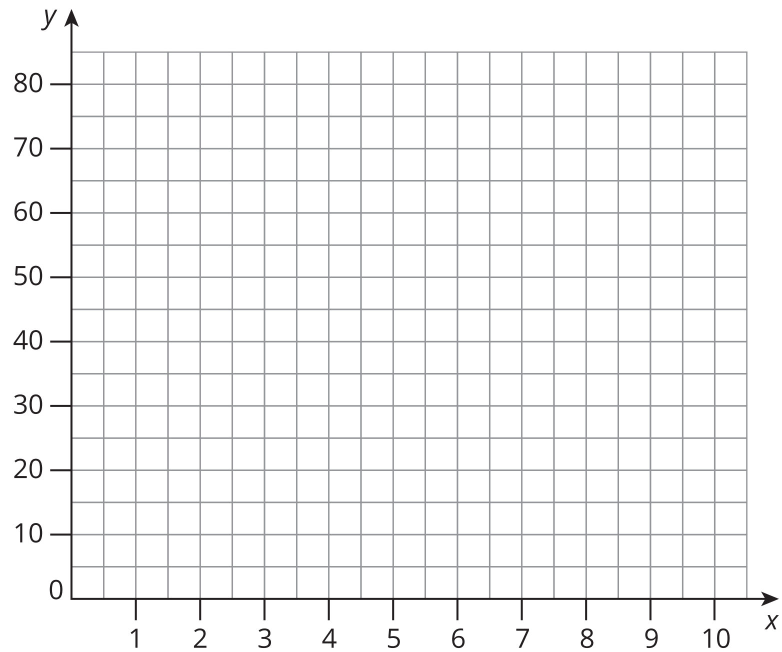
Let x = number of correct multiple-choice questions
Let y = number of correct short answer questions
Equation: Reasoning:



Reasoning:

2. Nicky has $20, and he plans to save an additional $10 every two weeks.

Let x = number of weeks
Let y = Nicky's total savings
Equation: Reasoning:



Reasoning:

3. What are the advantages and disadvantages of using each form of a linear equation?

   a. Slope-intercept form

b. Point-slope form

c. Standard form

Lesson 5 Assignment

Write

Explain how to graph a line when the equation is written in slope-intercept form, point-slope form, or standard form.

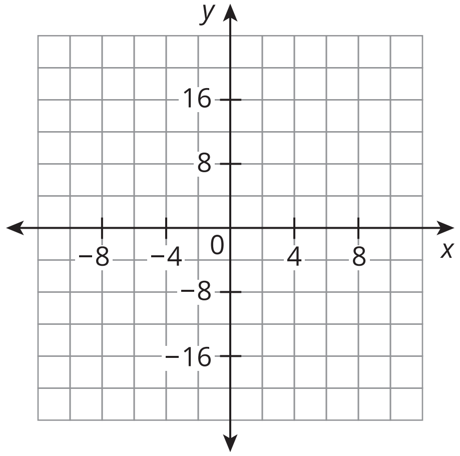
Remember

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

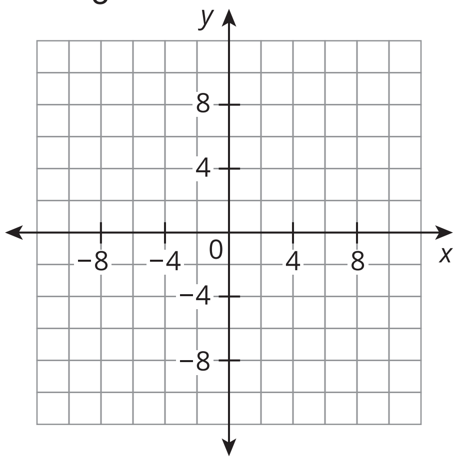
Practice

1. Graph each equation using its given form.

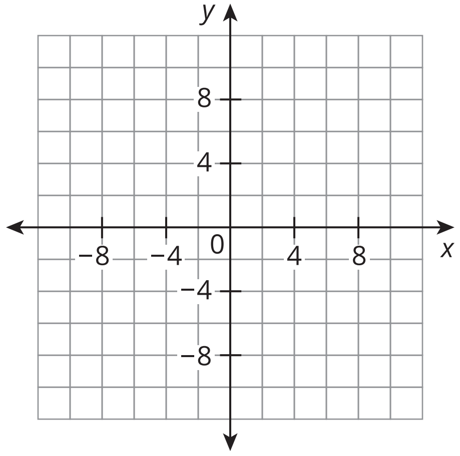
a. $y = 4x + 2$



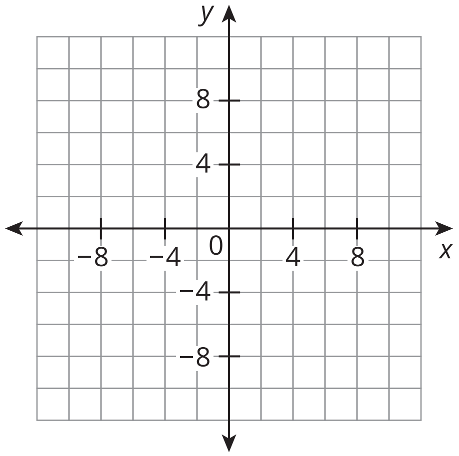
b. $y = -\frac{1}{3}x - 5$



c. $y + 1 = \frac{3}{4}(x - 8)$



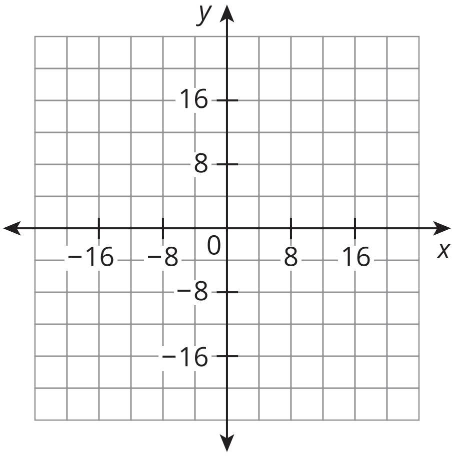
d. $y - 4 = -\frac{2}{3}(x - 6)$



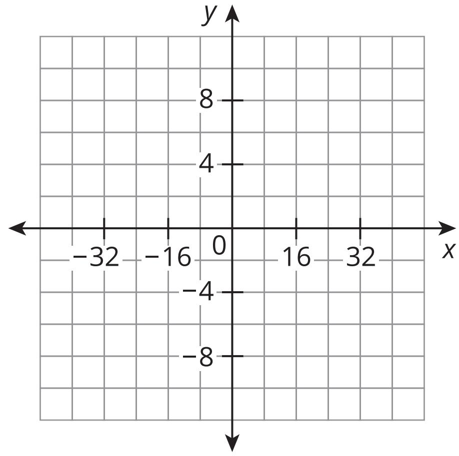
Lesson 5 Assignment

2. Graph each equation using its intercepts.

   a. $4x + 6y = 48$



   b. $-2x + 8y = 56$



3. Write a linear equation in standard form given two points on the line.

   a. $(0, -9)$ and $(-3, 6)$

   b. $(-4, 2)$ and $(2, 5)$

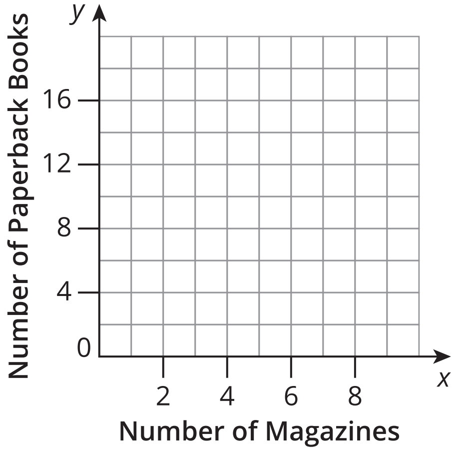
4. Brianna bought magazines for \$6 each and paperback books for \$3 each for a total of \$54.

   a. Define your variables and write an equation in standard form to represent the situation.

   b. Calculate and interpret the x-intercept and the y-intercept for this equation.

Lesson 5 Assignment

c. Graph the equation of the line using the intercepts.



d. Calculate and interpret the slope of this line.

5. Each equation represents a linear relation. State the slope and y-intercept for each.

a. $-9x + 2y = -36$

b. $y + 5 = -7(x + 3)$

c. $y = 2$

d. $y = \frac{5}{2}x - 9$

Prepare

Determine the slope of the line between each pair of points.

1. (0, 10) and (3, 12)

2. (−1, 4.5) and (1, −4.5)

3. (21, 0) and (0, 12)

6 Making Sense of Different Representations of a Linear Function

LEARNING OBJECTIVES

- Determine whether a scenario, equation, table, or graph represents a linear relationship.
- Calculate the average rate of change from a table.
- Write functions given a table of values.
- Interpret expressions that represent different quantities in terms of a context and a graph.
- Compare different equation representations of linear functions.

NEW KEY TERMS

- polynomial
- degree
- leading coefficient
- zero of a function

You know how to determine whether a relationship represents a linear function, and you know how to write an equation for the function. How can you use the structure of the equation to identify characteristics of the function?

Getting Started

Well, Are Ya or Aren't Ya?

1. Determine whether each representation models a linear or nonlinear function. Explain your reasoning.

Scenario A

A tree grows 3.5 inches each year.

Scenario B

The strength of a medication decreases by 50% each hour it is in the patient's system.

Scenario C

The area of a square depends on its side length.

Equation A

$y = 14 - 9x$

Equation B

$y = 2^x + 1$

Equation C

$y = \frac{1}{4}(x + 7) - 1$

Table A

x	y
1	3
1	4
1	5

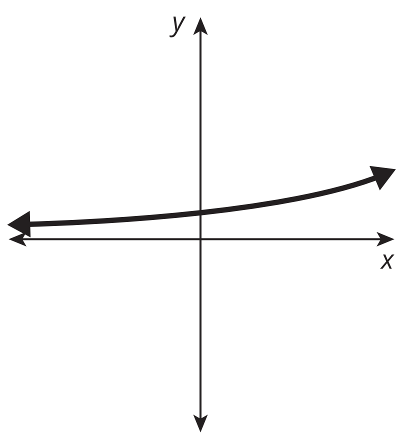
Table B

x	y
3	1
4	1
5	1

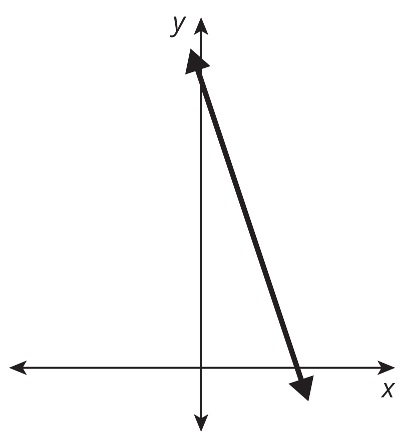
Table C

x	y
−9	45
−8	30
−7	15

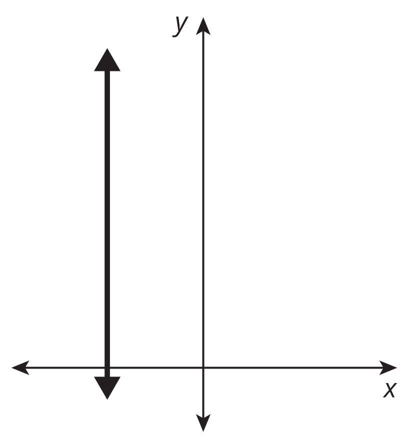
Graph A



Graph B



Graph C



ACTIVITY 6.1

Interpreting Linear Functions

Each table of values in the previous activity had consecutive input values. Tables in this format allow you to use differences to determine whether the representation is linear. Often, input values are in intervals other than 1, and sometimes the input values are in random order. To determine whether these tables represent linear functions, you need to make sure the slope, or average rate of change, is constant between all given points.

Ask Yourself . . .
Is there a pattern in the input values?

Remember . . .
In a linear relationship, when the value of the dependent variable is directly proportional to the value of the independent variable, the relationship is a direct variation. Another way to say this is that in a direct variation the value of the dependent variable varies directly with the independent value. You can use these phrases interchangeably.

1. Determine whether each table represents a linear function. If so, write the function.

a.

x	y
–2	5.5
1	4.75
4	4
7	3.25

b.

x	y
0	5
2	13
4	21
8	29

Analyze each situation represented as a table of values.

2. Nakota sells pretzels at festivals on weekends. The table shown is a record of past sales.

Number of Pretzels Sold	Amount of Money Earned (dollars)
15	37.5
42	105
58	145
29	72.5

   a. What does this table tell you about his sales?

   b. Determine whether the amount of money Nakota earns is directly proportional to the number of pretzels sold. Justify your reasoning.

   c. Write an equation to represent the situation.

   d. How much money does Nakota earn when he sells 75 pretzels?

3. Alejandra and her friends recently went to the community fair. They had to pay an entrance fee and then purchase 1 ticket for each ride. Lauren is going to the fair tomorrow and wants to know the cost of each ride ticket. Alejandra and her friends help Lauren by writing down how much money they spent and the number of tickets they purchased.

Number of Ride Tickets	Amount of Money Spent (dollars)
2	7.5
4	9
6	10.5
11	14.25

a. What does this table tell you about the cost to go to the fair and ride the rides?

b. Determine whether the amount of money Alejandra and and her friends spent varies directly with the number of ride tickets they purchased. Justify your reasoning.

c. Write an equation to represent the situation.

d. If Lauren has $20 to spend, how many ride tickets can she buy?

4. The local pet store has a fish tank on display at the community fair. Darren is responsible for draining the tank at the end of the fair. The pet store manager provides him with this information from when they drained the tank at the end of the fair last year.

Time (hours)	Amount of Water Remaining (gallons)
$\frac{1}{4}$	169
$\frac{1}{2}$	163
$\frac{3}{4}$	157
1	151

a. Determine whether the amount of water remaining is directly proportional to the time the drain spent draining the tank. Justify your reasoning.

b. How many gallons did the fish tank hold?

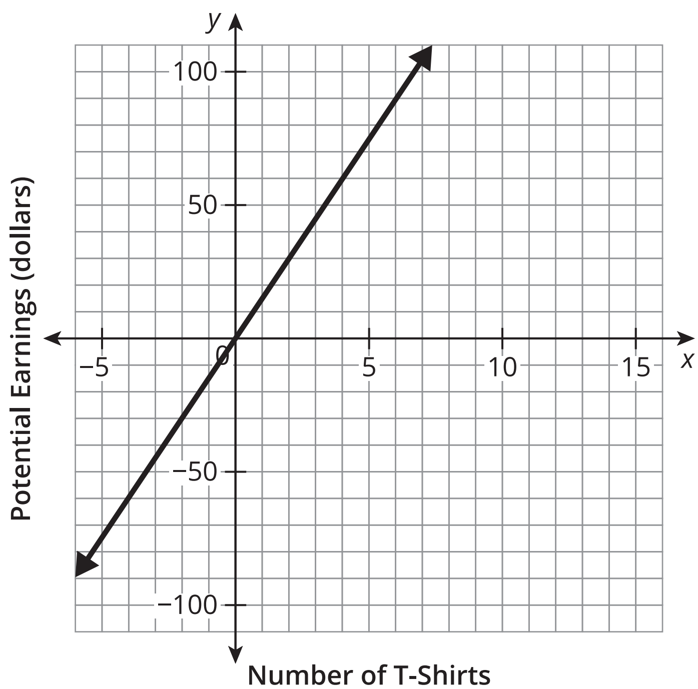
c. Write an equation to represent the situation.

d. When will the tank be empty?

ACTIVITY 6.2

Interpreting Graphs of Linear Functions

Gracie sells silk screened T-shirts for her mom at local festivals. After each festival, she returns whatever money she earns to her mom. The graph shown represents her potential earnings based on the number of T-shirts she sells.



Ask Yourself . . .
What is the meaning of the slope, *x*- and *y*-intercepts, domain, and range in terms of this situation?

1. Analyze and interpret the graph. List as many facts as you can about the scenario based on what you see in the graph and describe how they relate to the scenario.

Ask Yourself . . .
Does the function $E(t)$ represent a direct variation? Explain your reasoning.

2. Interpret the meaning of the x- and y-intercept.

3. Write a function, $E(t)$, to model Gracie's potential earnings given the number of T-shirts she sells.

4. What does $(t, E(t))$ represent in terms of the function and the graph?

5. Evaluate each and interpret the meaning in terms of the equation, the graph, and the scenario.

   a. $E(2)$

   b. $E(5)$

   c. $E(2.75)$

Gracie has a goal to earn \$100 at the festival. Let's consider how to determine the number of T-shirts she needs to sell to meet her goal.

WORKED EXAMPLE

To determine the number of T-shirt sales it takes to earn \$100 using the function, $E(t) = 15t$, substitute 100 for $E(t)$ and solve.

$$E(t) = 15t$$
$$100 = 15t$$
$$\frac{100}{15} = t$$
$$6.67 = t$$

6. Consider the Worked Example.

   a. Interpret the meaning of $t = 6.67$.

b. Why can you substitute 100 for $E(t)$?

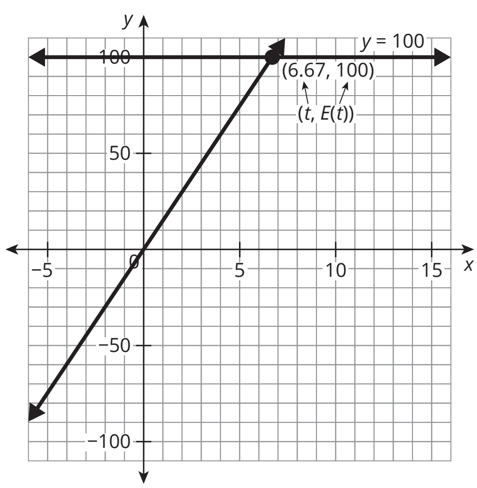
Remember . . .
The graph of an equation plotted on the coordinate plane represents the set of all its solutions.

You can also use the graph to determine the number of T-shirts Gracie needs to sell to earn $100.

WORKED EXAMPLE

To determine the number of T-shirts sales it takes to earn $100 using your graph, you need to determine the intersection of the two lines represented by the equation $100 = 15t$.

First, graph the function defined by each side of the equation, and then determine the intersection point of the two graphs.



$$h(t) = 15t$$
$$100 = 15t$$
$$y = 100 \qquad y = 15x$$

Solution: (6.67, 100)

In terms of the graph, Gracie needs to sell 6.67 T-shirts to earn $100. In terms of the context, she needs to sell 7 T-shirts.

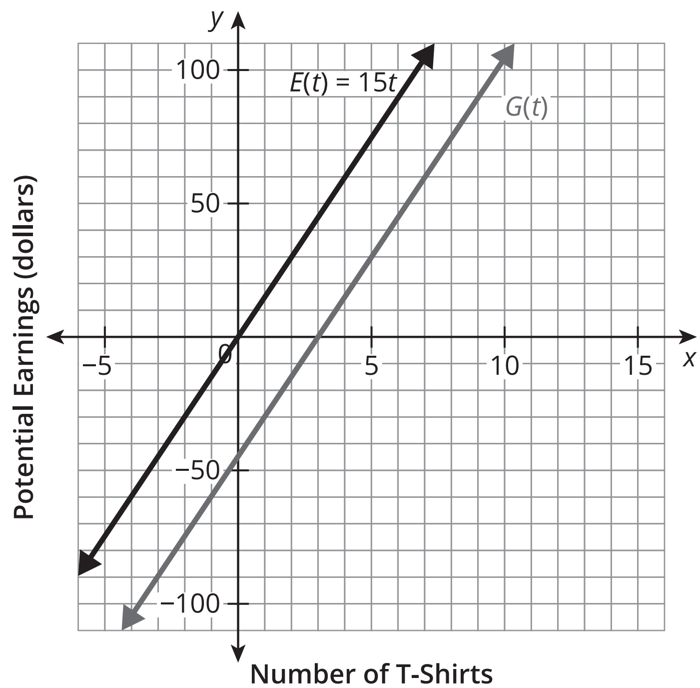
7. Consider the equation and graphical representations. What are the limitations of using each to answer questions about the number of T-shirts sold or the amount of money earned?

a. Equation

b. Graph

ACTIVITY 6.3 Interpreting Changes to the Graph of a Linear Function

For the next festival, Gracie's mom suggests that she still sells each T-shirt for \$15, but should give away 3 T-shirts in a raffle. This new relationship, $G(t)$, is shown on the graph.



1. Compare the two graphs. What do you notice?

   a. How do the graphs show the selling price per T-shirt remains the same?

   b. Determine and interpret the meaning of $y = G(0)$ in terms of the graph and this scenario. Label the point on the graph.

   c. Determine and interpret the meaning of $G(t) = 0$ in terms of the graph and this scenario. Label the point on the graph.

Ask Yourself . . .
Does $G(t)$ vary directly with t?

Brianna and Hailey each wrote an equation to describe the effect of giving away three T-shirts.

Brianna

Gracie is giving away 3 T-shirts, so she has 3 fewer shirts to sell.

$$G(t) = 15(t - 3)$$

Hailey

The cost of giving away three shirts is $45.

$$G(t) = 15t - 45$$

2. Verify the two equation representations are equivalent.

3. How many T-shirts will Gracie need to sell to earn $100? Use the graph and an equation.

4. Consider the expressions in the first two rows that define the quantities of the function and then the parts of each equation written by Brianna and Hailey to complete the table. First, determine the unit of measure for each expression. Then, describe the contextual meaning and the mathematical meaning of each part of the function.

		What It Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
t			
$G(t)$			
15			
$(t - 3)$			
$15t$			
-45			

The linear functions that Brianna and Hailey each wrote are equivalent; however, they are written in different forms. The linear function $G(t) = 15(t - 3)$ is written in factored form and $G(t) = 15t - 45$ is written in general form.

Polynomial comes from *poly*- meaning "many" and -*nomial* meaning "term," so it means "many terms."

A linear function can also be referred to as a *polynomial* function. A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. The **degree** of a polynomial is the greatest variable exponent in the expression. The **leading coefficient** of a polynomial is the numeric coefficient of the term with the greatest power.

WORKED EXAMPLE

A few examples of polynomial functions.

Polynomial Functions		Degree
Constant	$P(x) = 7$	0
Linear	$P(x) = 2x - 5$	1
Quadratic	$P(x) = 3x^2 - 2x + 4$	2
Cubic	$P(x) = 4x^3 - 2$	3

When you graph a polynomial the degree tells you the maximum number of times the graph can cross the x-axis.

The structure of each linear function tells you important information about the graph. Let's consider the general form of a linear function, $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept.

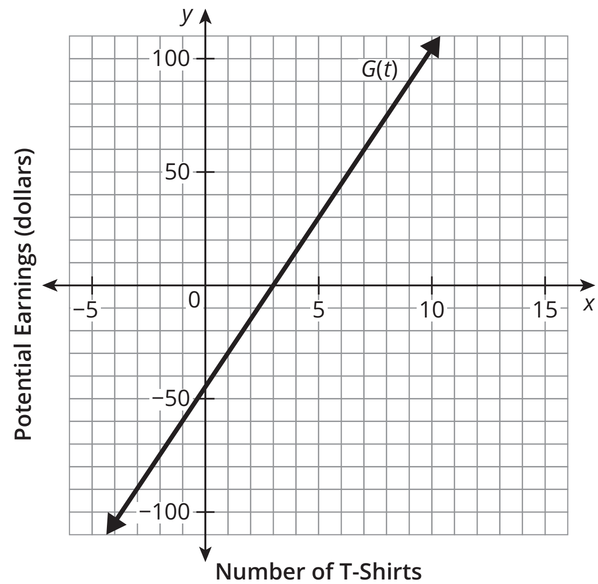
The variables used to represent any real number in the general linear form are irrelevant. Think about the position of the number as either the leading coefficient or a constant and the potential effect on the function.

You know the form $y = mx + b$ as slope-intercept form, where m represents the slope and b represents the y-intercept. Notice that the general form has the same structure. The general form shows that a linear equation is a polynomial of degree 1. You will learn more about polynomials as you progress through future mathematical courses.

5. Consider the general form of the linear function $G(t)$.

   a. Label a on the graph.

   b. Label b on the graph.



Next, consider the factored form of a linear function, $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a polynomial is in factored form, the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept. This value is called the *zero of the function*. A **zero of a function** is a real number that makes the value of the function equal to zero, or $f(x) = 0$.

You can set $(x - c)$ equal to zero and determine the point where the graph crosses the x-axis.

6. Consider the factored form of the linear function $G(t)$.

   a. Label a on the graph.

   b. Label c on the graph.

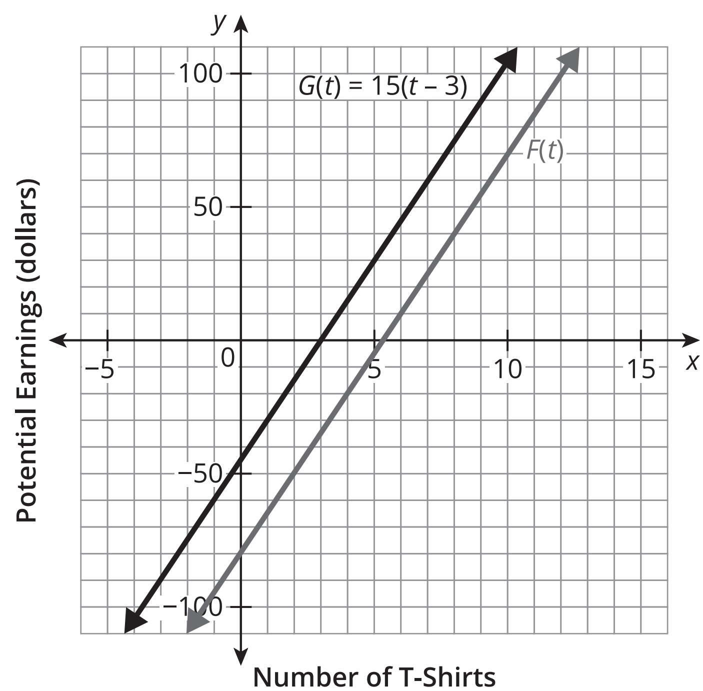
7. What is the zero of $G(t)$? Explain your reasoning.

ACTIVITY
6.4

Interpreting More Changes to the Graph of a Linear Function

The next festival that Gracie is attending charges a \$35 fee to rent a booth. She is still selling her mom's T-shirts for \$15 each and giving 3 away in a raffle. The graph shows this new relationship, $F(t)$.

Ask Yourself . . .
Is $F(t)$ directly proportional to t?



1. Consider the relationship between graphs of $G(t)$ and $F(t)$.

   a. How do the graphs show that the selling price per T-shirt remains the same?

   b. How did the new booth fee of \$35 change the graph?

c. How many T-shirts will Gracie need to sell before she will have any money to return to her mom? Explain your reasoning.

d. How many T-shirts will Gracie need to sell to earn $100?

2. Consider the relationship between the equations of $G(t)$ and $F(t)$.

a. Write the function $F(t)$ in terms of $G(t)$.

b. Rewrite $F(t)$ in general form. Then describe how the a- and b-values are represented on the graph.

The general form of a linear function is $f(x) = ax + b$.

The factored form of a linear function is $f(x) = a(x - c)$.

The values a, b, and c are real numbers and $a \neq 0$.

c. Rewrite $F(t)$ in factored form. Use a fraction to represent the c-value. Then describe how the a- and c-values are represented on the graph.

Talk the Talk

Reading Between the Lines

Complete each "I can" sentence using *always*, *sometimes*, or *never.*

1. Suppose you are given a dependent value and need to calculate an independent value of a linear function.

   a. I can ______________________ use a table to determine an *approximate* value.

   b. I can ______________________ use a table to calculate an *exact* value.

   c. I can ______________________ use a graph to determine an *approximate* value.

   d. I can ______________________ use a graph to calculate an *exact* value.

   e. I can ______________________ use an equation to determine an *approximate* value.

   f. I can ______________________ use an equation to calculate an *exact* value.

2. Write the function that models each table of values. Then, evaluate the function for each independent and dependent value.

   a.

x	1	2	3	4	5
$f(x)$	−20	5	30	55	80

$f(x) =$

$f(12) =$

$f(x) = -145$

b.

x	1	3	5	7	9
$g(x)$	18	6	-6	-18	-30

$g(x) =$

$g(-9) =$

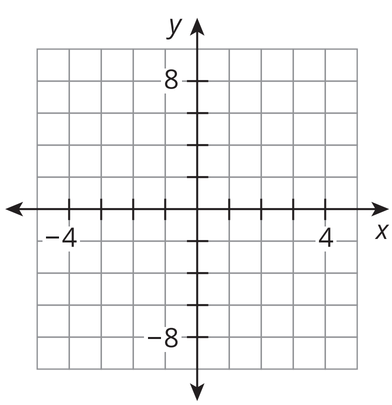
$g(x) = -54$

3. Complete the graphic organizer located at the end of this lesson for the linear function $f(x) = 2x - 8$.

   a. Write $f(x)$ in general form. Then, describe the information given in this form.

   b. Write $f(x)$ in factored form. Then, describe the information given in this form.

   c. Graph $f(x)$. Describe how you know this graph can cross the x-axis only one time.



   d. Create a table of values for $f(x)$.

x	y

Graphic Organizer

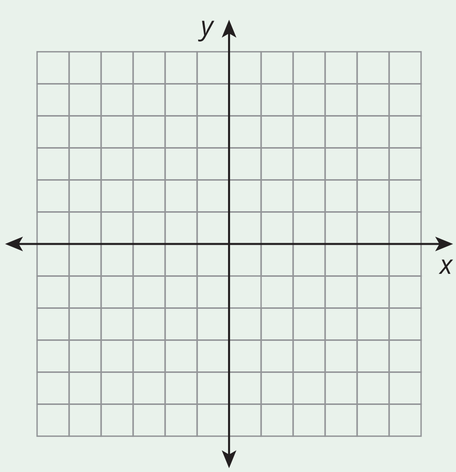
General Form

$f(x) = ax + b$, where a and b are real numbers and $a \neq 0$

Factored Form

$f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$

$f(x) = 2x - 8$



Graph

x	y

Table

Lesson 6 Assignment

Write

Describe a zero of a function in your own words.

Remember

The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient which describes the steepness and direction of the line. The b-value describes the y-intercept.

The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. In this form, the a-value is the slope and the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept.

Practice

Determine whether the table of values represents a linear function. If so, write the function.

1.

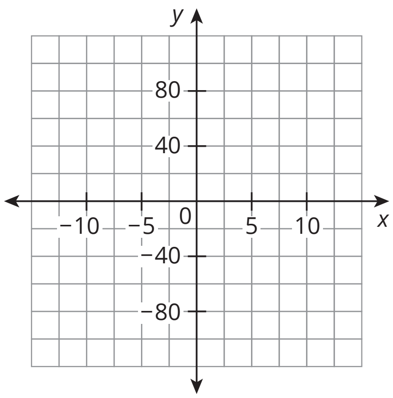
x	y
−2	$5\frac{2}{3}$
0	5
2	$4\frac{1}{3}$
4	$3\frac{2}{3}$

2.

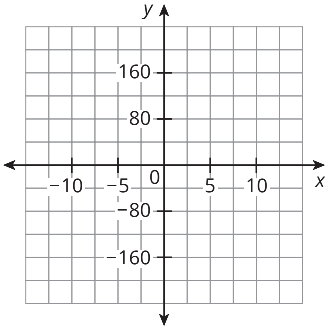
x	y
−5	−27
0	−2
5	20
10	48

For each scenario, write a linear function in factored form and in general form. Then, sketch a graph and label the x- and y-intercepts. Finally, answer each question.

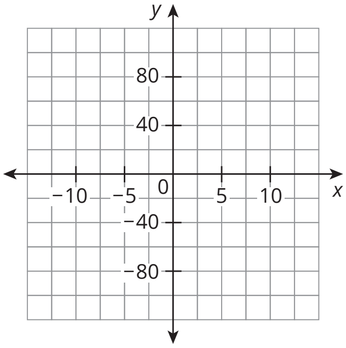
3. Omar prints and sells T-shirts for $14.99 each. Each month 5 T-shirts are misprinted and cannot be sold. How much money will he earn when he prints 22 T-shirts? How many T-shirts will he need to sell to earn $200?



4. Juliana paints and sells ceramic vases for \$35 each. Each month, she typically breaks 3 vases in the kiln. How much money will she earn if she sells 17 ceramic vases? How many ceramic vases will she need to sell to earn \$600?



5. Antonio builds and sells homemade wooden toys for \$12 each. The festival he is attending charges \$50 to set up his booth. How much money will he earn if sells 35 wooden toys? How many wooden toys will he need to sell to earn \$250?



Prepare

Determine each value, given the function $h(x) = -2x - 5$.

1. $h(3)$

2. $2 \cdot h(-2)$

3 $-1 \cdot h(1) + 5$

4. $5 \cdot h(0) + 5$

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Linear Functions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Linear Functions* topic by:

TOPIC 1: *Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
determining when a data set should be modeled by a linear function.	○	○	○
using technology to determine the linear regression function for a data set.	○	○	○
making predictions using the linear regression function.	○	○	○
interpreting the meaning of the slope and *y*-intercept of a linear regression function in terms of the problem context.	○	○	○
explaining the correlation coefficient as a measure of how well a function fits a data set.	○	○	○
understanding that the sign of a correlation coefficient indicates the direction of the association and that the magnitude indicates the strength of the fit.	○	○	○
analyzing a function on a scatterplot and its correlation coefficient to determine whether a function is an appropriate fit for a data set.	○	○	○
recognizing that correlation does not imply causation.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
recognizing that all arithmetic sequences are linear functions and rewriting an arithmetic sequence as a linear function using function notation.			
writing equations of lines in the appropriate form: slope-intercept, point-slope, or standard form.			
graphing lines from an equation written in slope-intercept, point-slope, or standard form.			
writing and solving equations involving direct variation.			
connecting average rate of change of a linear function to the slope of a line.			
constructing a linear function from a scenario, table of values, graph, or arithmetic sequence.			
interpreting the real-world meaning of the coefficients and constants in a linear function.			
recognizing a linear relationship and writing an equation to represent that situation.			
identifying the variables and quantities of a linear function.			
determining key features of graphs of linear functions including domain, range, x-intercept, y-intercept, zeros, and slope.			

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Linear Functions* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Linear Functions Summary

LESSON 1 Least Squares Regression

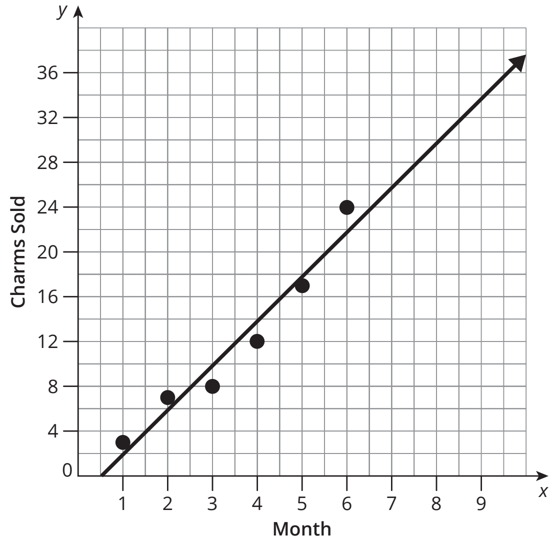
Technology calculates the line of best fit of data on a scatterplot using the **Least Squares Method**. The line includes the **centroid**, or the point whose *x* value is the mean of all *x* values and whose *y* value is the mean of all *y* values. The **linear regression function** has the smallest possible vertical distance from each given data point to the line. The sum of the squares of these distances is at a minimum with the linear regression function.

When there is a linear association between the independent and dependent variables, you can use a linear regression function to make predictions within the data set. Using a linear regression function to make predictions within the data set is called **interpolation**. To make predictions outside the data set is called **extrapolation**.

For example, consider the situation of Lucia selling charms to her classmates. The table records the sales of her charms over the months since she began selling them.

Month	Charms Sold
1	3
2	7
3	8
4	12
5	17
6	24

The linear regression function modeling the situation is graphed on the scatterplot shown.



The linear regression function is $y = 3.97x - 2.07$.

Using the equation to interpolate, Lucia should sell about 14 charms in the fourth month.

$$\begin{aligned} y &= 3.97(4) - 2.07 \\ &= 13.81 \end{aligned}$$

Using the equation to extrapolate, Lucia should sell about 30 charms in the eighth month.

$$\begin{aligned} y &= 3.97(8) - 2.07 \\ &= 29.69 \end{aligned}$$

NEW KEY TERMS

- Least Squares Method
- centroid [centroide]
- linear regression function [función de regresión lineal]
- interpolation [interpolación]
- extrapolation [extrapolación]
- correlation [correlación]
- correlation coefficient [coeficiente de correlación]
- coefficient of determination [coeficiente de determinación]
- causation [causalidad]
- necessary condition [condición necesaria]
- sufficient condition [condición suficiente]
- common response [respuesta común]
- confounding variable [variable de confusión]
- conjecture [conjetura]
- first differences

NEW KEY TERMS

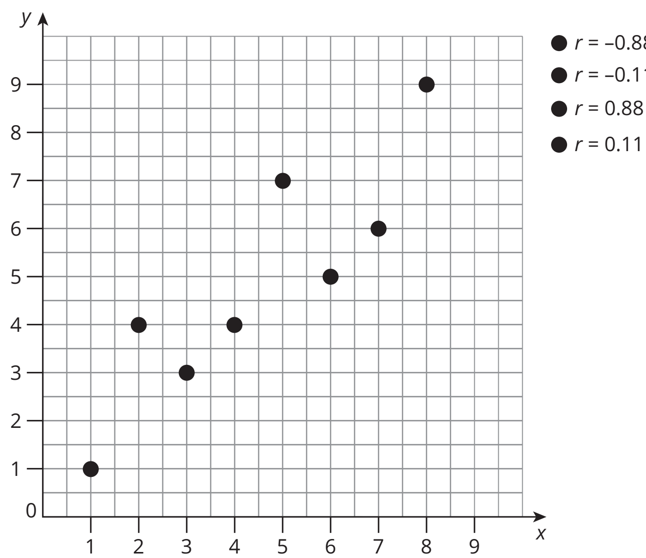
- average rate of change
- point-slope form
- standard form [forma estándar/general]
- polynomial [polinomio]
- degree
- leading coefficient
- zero of a function [cero de una función]

LESSON 2

Correlation

The measure of how well a regression fits a set of data is called **correlation**. When dealing with regression functions, the variable r is used to represent a value called the **correlation coefficient**. The correlation coefficient indicates how close the data are to the graph of the regression function. The correlation coefficient falls between -1 and 0 when the data show a negative association or between 0 and 1 when the data show a positive association. The closer the r-value is to 1 or -1, the stronger the relationship is between the two. The **coefficient of determination**, r^2, measures how well the regression line fits the data. It represents the percentage of variation of the observed values of the data points from their predicted values.

For example, consider the possible r-values for a linear regression function given for the data graphed in the scatterplot.



The data has a positive correlation. Because of this, r-value must be positive. Also, the data are fairly close to forming a straight line, so of the choices, $r = 0.88$ would be the most accurate. Technology can be used to verify the correlation coefficient. The coefficient of determination for this data set is 0.7744.

When interpreting the correlation between two variables, you are looking at the association between the variables. While an association may exist, that does not mean there is causation between the variables. **Causation** is when one event causes a second event. A correlation is a **necessary condition** for causation, but a correlation is not a **sufficient condition** for causation. Correlation may be due to a **common response**, which is when another reason may cause the same result, or a **confounding variable**, which is when other variables are either unknown or unobserved.

For example, consider an experiment conducted by a group of college students that found that more class absences correlated to rainy days. The group concluded that rain causes students to be sick. However, this correlation does not imply causation. Rain is neither a necessary condition (because students can get sick on days it does not rain) nor a sufficient condition (because not every student who is absent is necessarily sick) for students being sick.

LESSON 3

Making Connections Between Arithmetic Sequences and Linear Functions

You can make a conjecture that the graphs of arithmetic sequences represent linear functions. A **conjecture** is a mathematical statement that appears to be true, but has not been formally proven. An explicit formula for an arithmetic sequence can be rewritten in function notation to prove your conjecture.

For example, to write the explicit formula $a_n = 2 + 4(n - 1)$ as a linear function in general form, first use function notation to represent a_n.

$a_n = 2 + 4(n - 1)$
$f(n) = 2 + 4(n - 1)$

Next, rewrite the expression $2 + 4(n - 1)$.

$f(n) = 2 + 4n - 4$	distributive property
$= 4n + 2 - 4$	commutative property
$= 4n - 2$	combine like terms

So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$.

One strategy to determine if a table of values represents a linear function is to examine first differences. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

x	*y*
1	22
2	17
3	12
4	7

$17 - 22 = -5$
$12 - 17 = -5$
$7 - 12 = -5$

For example, the first differences of the table shown are constant, so the relationship is linear.

The slope, a, of a linear function is equal to the constant difference of an arithmetic sequence. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$. This represents the change in the output as the input changes from s to t.

You can determine slope of a line when given two points using the formula $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$. Delta ($\Delta$) is a greek letter used in mathematics to represent "change in". So $\frac{\Delta y}{\Delta x}$ means "change in y" divided by "change in x".

LESSON 4 Point-Slope Form of a Line

You can determine the equation of a line from a table of values without knowing the y-intercept using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For example, to write an equation of a line from a table of values, you can use the slope formula.

x	y
2	6
4	5
6	4

- First, calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 6}{4 - 2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table. Let's use (2, 6).

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$
$$-\frac{1}{2}(x - 2) = y - 6$$

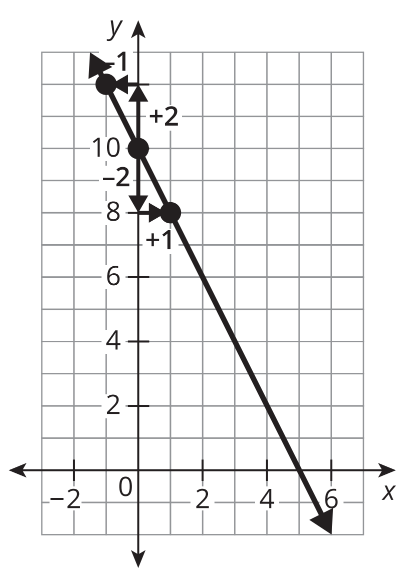
The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

The linear equation you wrote is written in point-slope form. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line. For example, given a slope of 3 and the point (1, 4), you can write the equation for the line in point-slope form as $y - 4 = 3(x - 1)$.

The slope of a horizontal line is 0. The slope of a vertical line is undefined.

LESSON 5 Using Linear Equations

You can use an equation in slope-intercept form to graph a relationship without first creating a table of values using the y-intercept and the slope.



For example, graph the equation $y = -2x + 10$. Begin by plotting the y-intercept, (0, 10). Use the slope and count from the y-intercept to graph two more points on the line.

- For $m = \frac{-2}{1}$, go down 2 units and to the right 1 unit.
- For $m = \frac{-2}{1}$, go up 2 units and to the left 1 unit.

Connect the points to form a straight line.

Another way to write a linear equation is in standard form. The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are integers, and A and B are not both zero.

When an equation is in standard form, you can calculate a y-intercept, where a line crosses the y-axis, by substituting 0 for x and solving the equation for y. To calculate an x-intercept, where the line crosses the x-axis, substitute 0 for y and solve the equation for x.

To write a linear equation in standard form when given two points, first write the equation in point-slope form. Then, rewrite the equation in standard form.

Write a linear equation in standard form, $Ax + By = C$, that goes through the points $(-1, 3)$ and $(-4, -3)$.

Begin by determining the slope of the line.

$$m = \frac{(-5 - 3)}{(-4 - (-1))} = \frac{(-6)}{(-3)} = 2$$

Pick one of the points and write an equation in point-slope form for the line.

$y - 3 = 2(x - (-1))$
$y - 3 = 2(x + 1)$

Rewrite the equation in standard form.

$$\begin{aligned} y - 3 &= 2x + 2 \\ y &= 2x + 5 \\ -2x + y &= 5 \end{aligned}$$

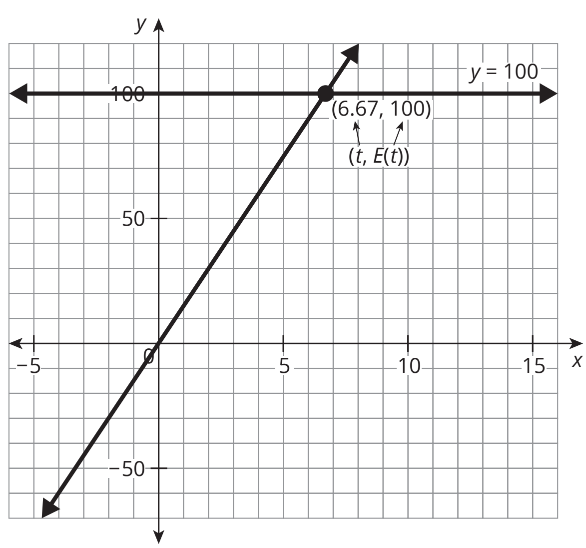
LESSON 6 Making Sense of Different Representations of a Linear Function

To determine whether a table of values represents a linear function, the slope, or average rate of change, needs to be constant between all given points.

You can use substitution or a graph to determine the output for a given input of a function.

For example, consider the function $E(t) = 15t$ which models the amount of money Gracie earns for selling t T-shirts. To determine the number of shirts she needs to sell to earn \$100, substitute \$100 for $E(t)$ and solve.

$$\begin{aligned} E(t) &= 15t \\ 100 &= 15t \\ 6.67 &= t \end{aligned}$$



Or you can determine the intersection of the graphs of the two lines represented by the equation $100 = 15x$.

Gracie needs to sell 6.67 T-shirts to earn \$100. In terms of the context, she needs to sell 7 T-shirts.

A linear function can also be referred to as a polynomial function. A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. The **degree** of a polynomial is the greatest variable exponent in the expression. The **leading coefficient** of a polynomial is the numeric coefficient of the term with the greatest power.

The chart shows a few examples of polynomial functions.

Polynomial Function		Degree
Constant	$P(x) = 7$	0
Linear	$P(x) = 2x - 5$	1
Quadratic	$P(x) = 3x^2 - 2x + 4$	2
Cubic	$P(x) = 4x^3 - 2$	3

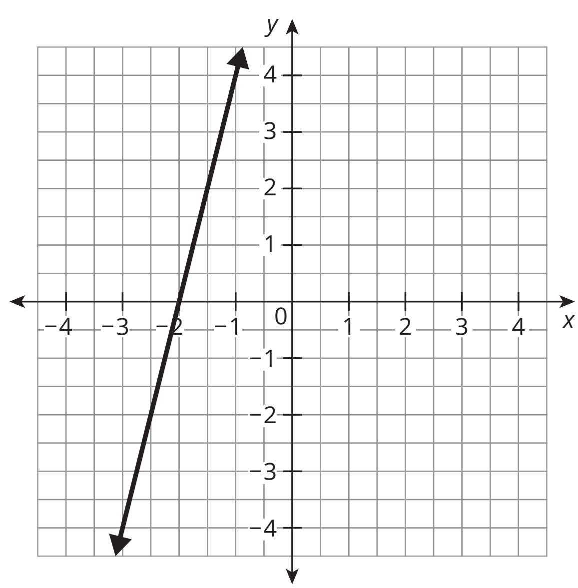
The structure of each linear function provides important information about the graph. The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept.

When you graph a polynomial, the degree indicates the maximum number of times the graph can cross the x-axis. A linear function has a degree of 1, so it crosses the x-axis at most one time.

The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a linear function is in factored form, the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept. This value is called the zero of the function. A **zero of a function** is a real number that makes the value of the function equal to zero, $f(x) = 0$.

You can set the factor $(x - c)$ equal to zero to determine the point where the graph crosses the x-axis.

For example, the linear function $f(x) = 4x + 8$ in factored form is $f(x) = 4(x + 2)$. Set the factor $x + 2$ equal to zero and then solve for x to determine the zero of the function, which is the point at which the graph of the function will cross the x-axis.



$$\begin{aligned} x + 2 &= 0 \\ x &= -2 \end{aligned}$$

ALGEBRA I

TOPIC 2

A region can be described as above, below, to the left, or to the right of a line.

Transforming and Comparing Linear Functions

1 Transforming Linear Functions

OBJECTIVES

- Determine the effects on the graph of a linear function when $f(x)$ is replaced by $f(x) + d$ or $a \cdot f(x)$.
- Graph linear function transformations expressed symbolically and show intercepts.
- Identify key characteristics of the graphs of linear functions, such as slope and y-intercept, in terms of quantities from a verbal description.
- Prove that a translated line and its pre-image have the same slope and are, therefore, parallel.
- Write equations of parallel lines.

NEW KEY TERM

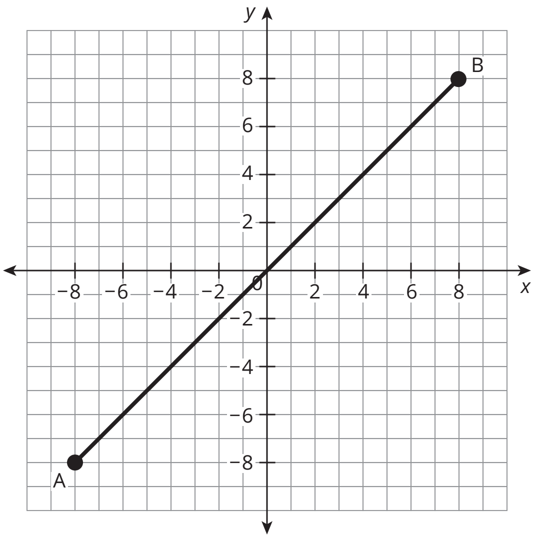
- parent function

You have learned about linear functions and their characteristics, including slope and y-intercept.

How can you transform a function? What effects do different transformations have on the characteristics of linear functions?

Getting Started

Returning to Transformation Station



Consider $\overline{AB}$ with coordinates $A(-8, -8)$ and $B(8, 8)$.

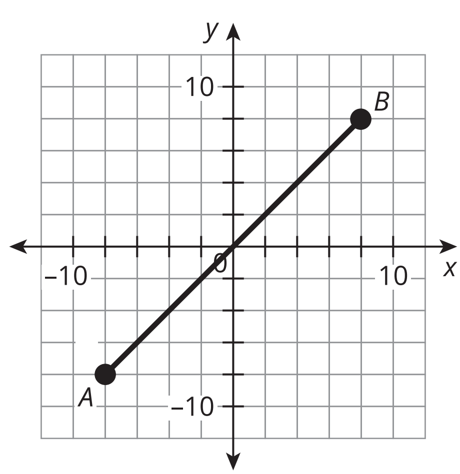
Follow your teacher's instructions to copy this line segment on a coordinate plane on the floor of your classroom, with different students standing at different points of the line segment.

1. Move on the coordinate plane to translate the entire line segment up 4 units and then down 4 units. Describe how the student "points" move.

2. How do the translations affect the coordinates of the figure?

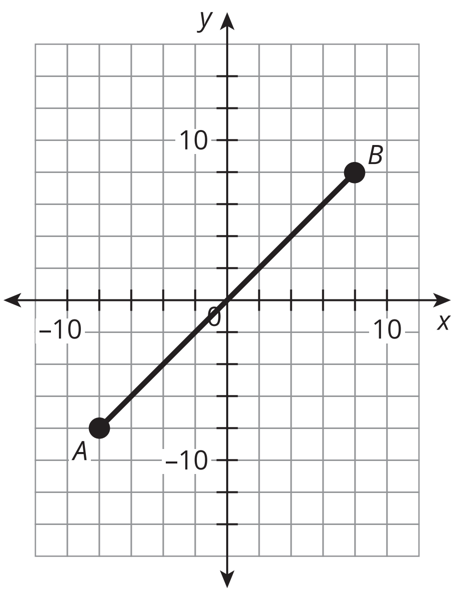
Original Point	4 Units Up	4 Units Down
(x, y)		

3. Draw $\overline{CD}$ on the coordinate plane so that it is a vertical translation of $\overline{AB}$ up 4 units. Compare the two line segments. Describe how they are related.

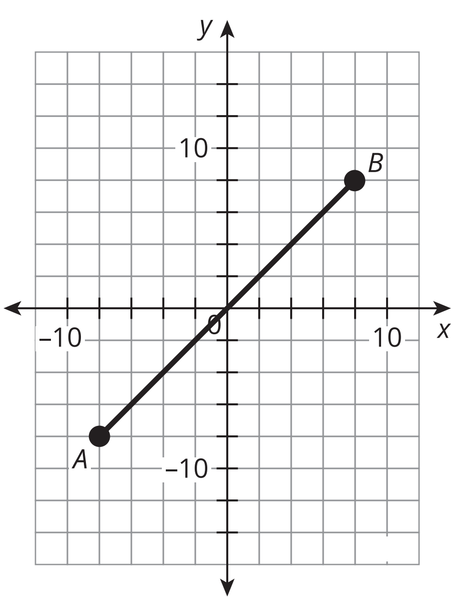


4. Move back to where you started. Then, multiply all the y-coordinates by 2 and then by -2. Describe how the student "points" move.

5. Draw $\overline{EF}$ on the coordinate plane by multiplying all the y-coordinates of $\overline{AB}$ by 2. Compare the two line segments. Describe how they are related.



6. Draw $\overline{GH}$ on the coordinate plane by multiplying all the y-coordinates of $\overline{AB}$ by -2. Compare the two line segments. Describe how they are related.



ACTIVITY 1.1 Vertical Translations of Functions

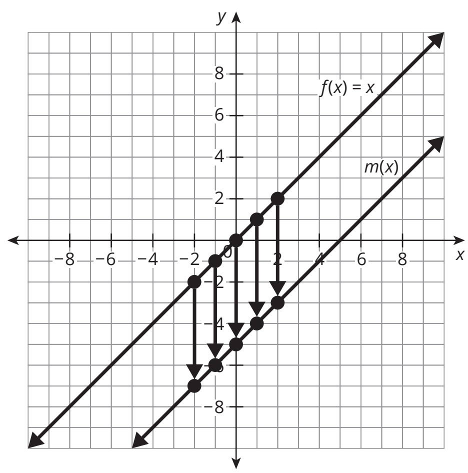
Let's determine how translations impact the graph of the linear function $f(x) = x$. The function $f(x) = x$ is the *parent function* for the linear function family. A **parent function** is the simplest function of its type.

For the family of linear functions having the general form $f(x) = ax + b$, where a and b are any real numbers, the function $f(x) = x$, where $a = 1$ and $b = 0$, is the parent function. It is the simplest linear function.

WORKED EXAMPLE

You can translate the graph of $f(x)$ down 5 units by moving each point 5 units down. The transformed graph is labeled as $m(x)$.

To translate the point $(-2, -2)$ on $f(x)$, subtract 5 units from the output value, or y-value. The input value, or x-value, remains unchanged. The coordinates of the translated point on $m(x)$ are $(-2, -7)$. The coordinates of four additional points on $f(x)$ are translated for you.



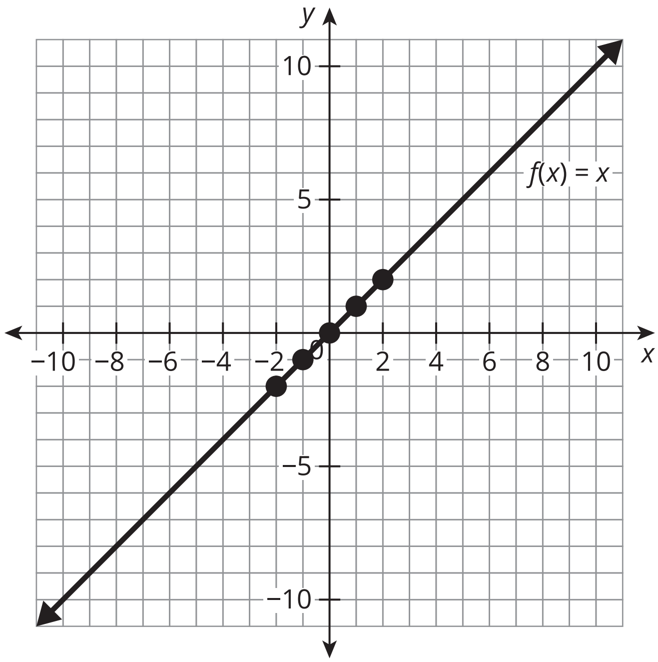
Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$m(x)$
−2	−7
−1	−6
0	−5
1	−4
2	−3

1. Consider the translated function, $m(x)$. Identify the slope and y-intercept of the graph of the function. Then, write the equation for the function in general form.

2. Translate $f(x)$ again to create a new function, $p(x)$.

   a. Translate the graph of $f(x)$ up 5 units. Label your graph as $p(x)$. Complete the table of corresponding points on $p(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

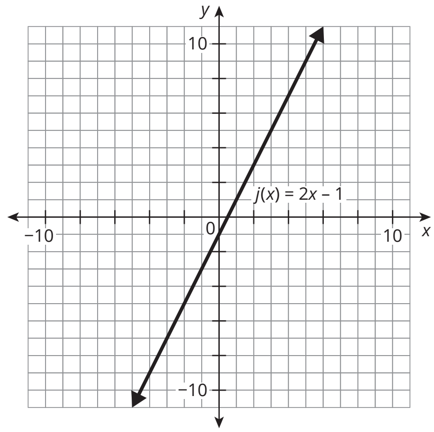
Transformed Graph	
x	$p(x)$

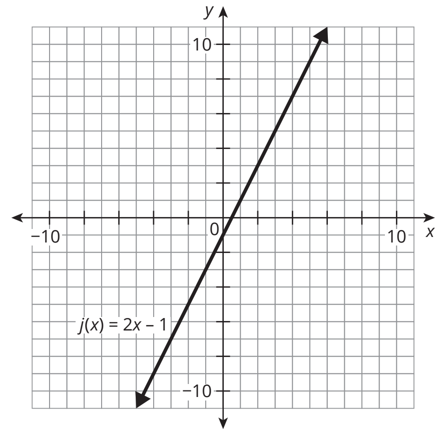
   b. Identify the slope and y-intercept of the graph of the function. Then, write the equation for the function in general form.

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. This translation affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up. For $d < 0$, the resulting graph vertically shifts down. The distance the graph is shifted is the absolute value of d, or $|d|$.

3. Compare the values of $f(2)$ and $p(2)$. How did the transformation of the function affect the value of the function at $x = 2$?

4. Consider the graph of $j(x) = 2x - 1$.





a. Translate the graph of $j(x)$ up 4 units. Label the graph as $q(x)$. Then, write an equation for $q(x)$ in terms of $j(x)$.

b. Translate the graph of $j(x)$ down 4 units. Label the graph as $r(x)$. Then, write an equation for $r(x)$ in terms of $j(x)$.

c. Rewrite $q(x)$ and $r(x)$ in general form.

Ask Yourself . . .
Will the graphs of $j(x)$, $q(x)$, and $r(x)$ ever intersect?

d. Compare the equations and graphs of $j(x)$, $q(x)$, and $r(x)$. What do you notice?

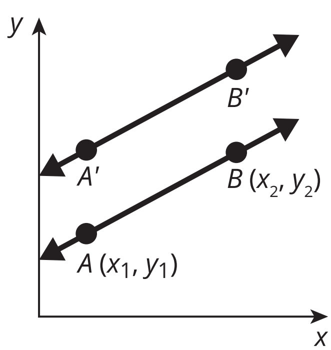
ACTIVITY
1.2 Slopes of Parallel Lines

In the previous activity you translated the function $j(x) = 2x - 1$ up 4 units to create $q(x) = 2x + 3$.

1. Will the graphs of $j(x)$ and $q(x)$ ever intersect? Explain your reasoning.

You can algebraically prove that a line and its translation are parallel to each other.

2. Line *AB* was translated a units up to create line *A′B′*.



   a. Identify the *x*- and *y*-coordinates of each corresponding point on the image.

   b. Use the slope formula to calculate the slope of the pre-image.

   c. Use the slope formula to calculate the slope of the image.

Ask Yourself . . .
The original figure is the pre-image and the transformation of the figure is the image.

d. How does the slope of the image compare to the slope of the pre-image?

e. How would you describe the relationship between the graph of the image and the graph of the pre-image?

Catalina and James each write the equation of a line that is parallel to $y = \frac{1}{2}x + 3$ and passes through the point (2, 9).

Catalina

I know that since the lines are parallel, the new line is a translation of $y = \frac{1}{2}x + 3$. I can substitute the x-value from (2, 9) into $y = \frac{1}{2}x + 3$ to determine the corresponding point on the pre-image.

$$y = \frac{1}{2}(2) + 3 = 4$$

The point that corresponds to the point (2, 9) on $y = \frac{1}{2}x + 3$ is (2, 4). Going from (2, 4) to (2, 9) is a translation up five units.

Since

$y = \left(\frac{1}{2}x + 3\right) + 5 = \frac{1}{2}x + 8$, the equation of the parallel line is $y = \frac{1}{2}x + 8$.

James

To write the equation of the line, all I need to know is the slope formula,

$m = \frac{y_2 - y_1}{x_2 - x_1}$.

I used $m = \frac{1}{2}$ and (2, 9) for (x, y).

$\frac{1}{2} = \frac{y - 9}{x - 2}$.

Then, I rearranged the equation to write it in general form.

$$\begin{aligned} 2(y - 9) &= x - 2 \\ y - 9 &= \frac{1}{2}(x - 2) \\ y &= \frac{1}{2}(x - 2) + 9 \\ y &= \frac{1}{2}x - 1 + 9 \\ y &= \frac{1}{2}x + 8 \end{aligned}$$

3. Which student's method do you prefer and why?

4. Write the equation of a line that is parallel to $y = -3x - 1$ and passes through the point $(-1, 5)$.

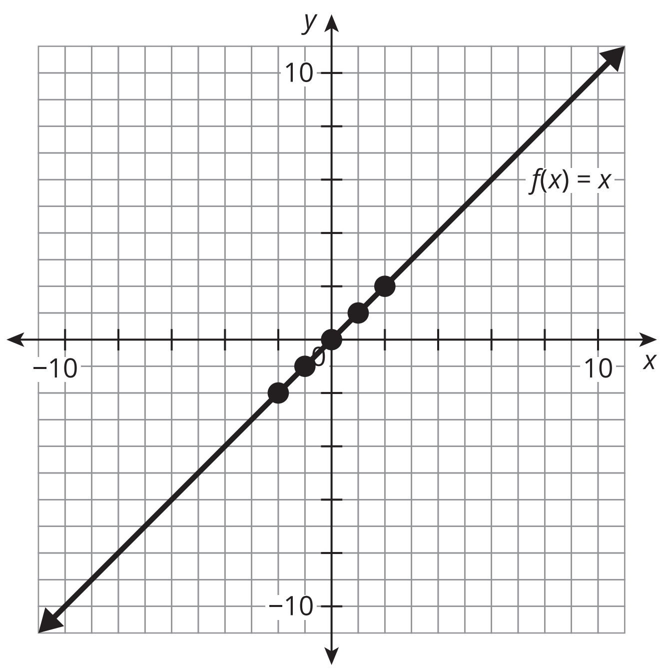
ACTIVITY 1.3

Vertical Dilations of Functions

In this activity, let's consider how dilations impact the graph of the linear function $f(x) = x$.

1. Suppose the output values of $f(x)$ are changed by a factor of 4 to create $a(x)$.

   a. Sketch the graph of $a(x)$ and complete the table of values.



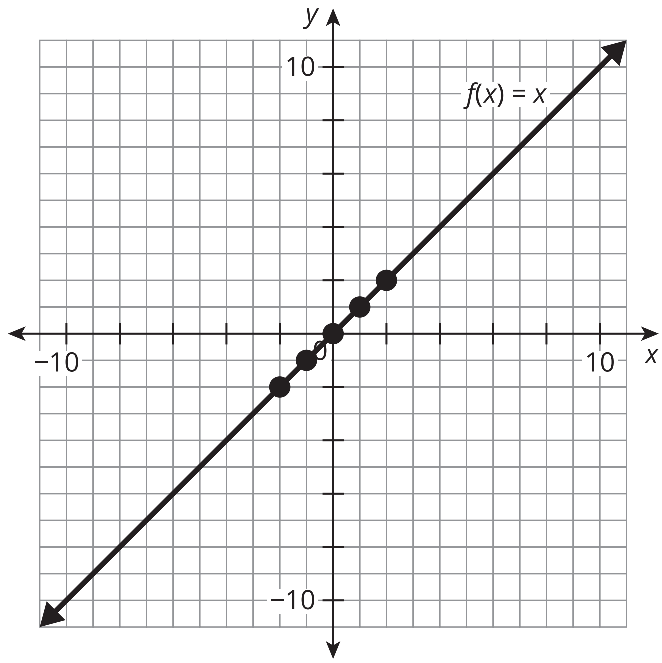
Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$a(x)$
−2	
−1	
0	
1	
2	

   b. Identify the slope and y-intercept of the function $a(x)$. Then, write the equation for the function $a(x)$ in general form.

2. Suppose the output values of $f(x)$ are changed by a factor of $\frac{2}{3}$ to create $b(x)$.

a. Sketch the graph of $b(x)$ and complete the table of values.



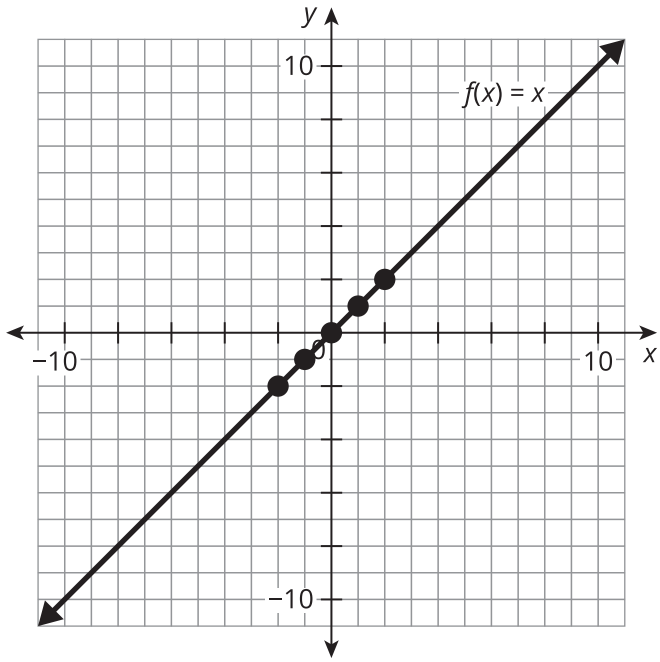
Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$b(x)$
−2	
−1	
0	
1	
2	

b. Identify the slope and y-intercept of the function $b(x)$. Then, write the equation for the function $b(x)$ in general form.

3. Suppose the output values of $f(x)$ are changed by a factor of -4 to create $c(x)$.

   a. Sketch the graph of $c(x)$ and complete the table of values.



Original Graph	
x	$f(x)$
–2	–2
–1	–1
0	0
1	1
2	2

Transformed Graph	
x	$c(x)$
–2	
–1	
0	
1	
2	

   b. Identify the slope and y-intercept of the function $c(x)$. Then, write the equation for the function $c(x)$ in general form.

For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ shows a vertical dilation of the function. This dilation affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph is vertically stretched or compressed and is reflected across the x-axis.

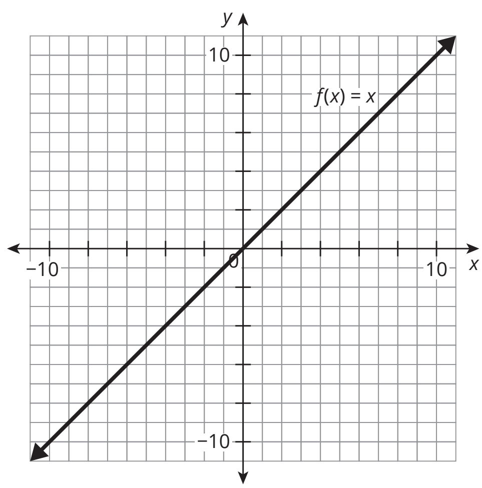
4. Compare the values $f(-1)$ and $c(-1)$. How did the transformation of the function affect the value of the function at $x = -1$?

ACTIVITY 1.4 Vertical Dilations and Vertical Translations of Functions

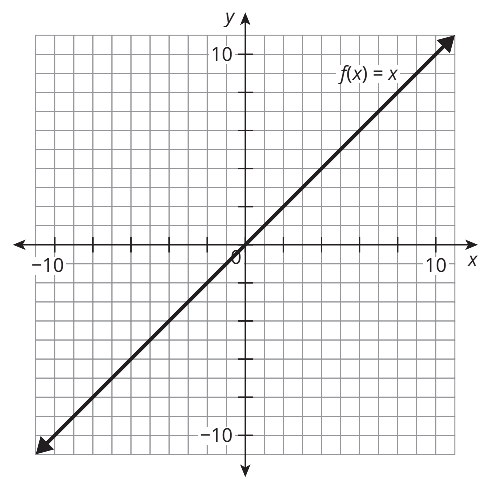
Let's consider more transformations of the parent function $f(x) = x$.

1. Describe the transformations performed on $f(x)$ to produce $g(x)$. Then, graph $g(x)$. Write the function equation in general form.

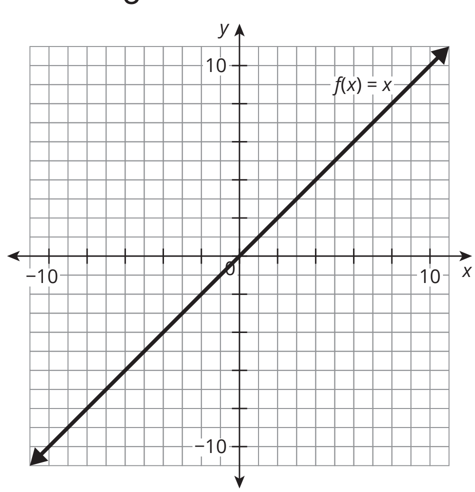
a. $g(x) = 2 \cdot f(x) + 7$



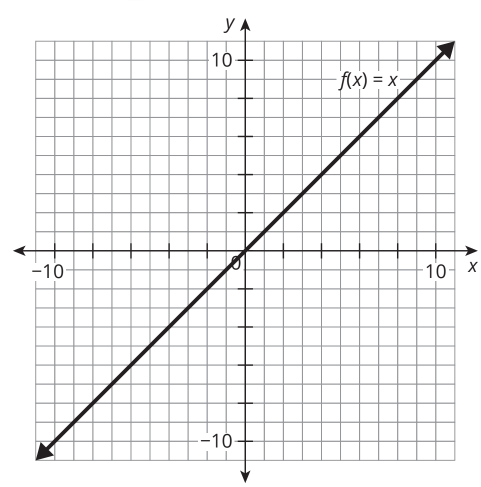
b. $g(x) = 3 \cdot f(x) - 1$



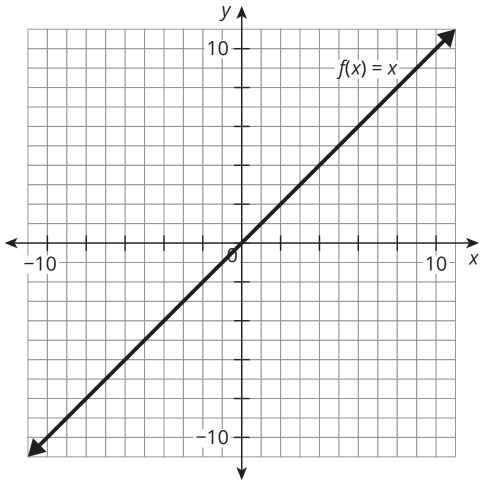
c. $g(x) = \frac{1}{3} \cdot f(x) + 2$



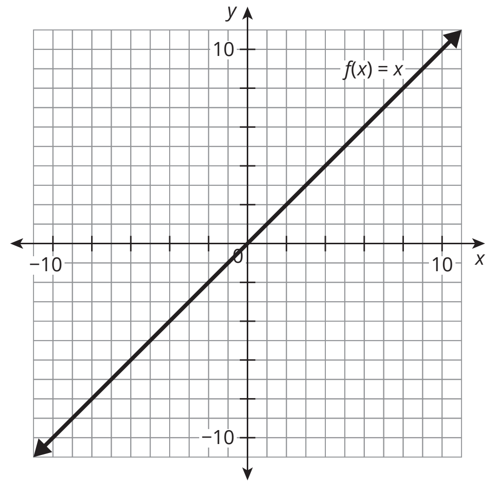
d. $g(x) = \frac{1}{2} \cdot f(x) - 3$



e. $g(x) = -1 \cdot f(x) - 4$



f. $g(x) = -\frac{2}{3} \cdot f(x) + 5$

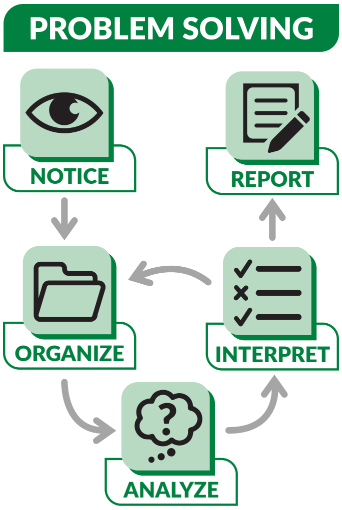


When a function is both translated and stretched vertically, the resulting function can be written in the form $a \cdot f(x) + d$, where d represents the vertical translation of $f(x)$, and a represents the vertical dilation of $f(x)$.

2. Consider the function $g(x) = a \cdot f(x) + d$.

   a. How does changing the a-value affect the slope of the function? The y-intercept of the function?

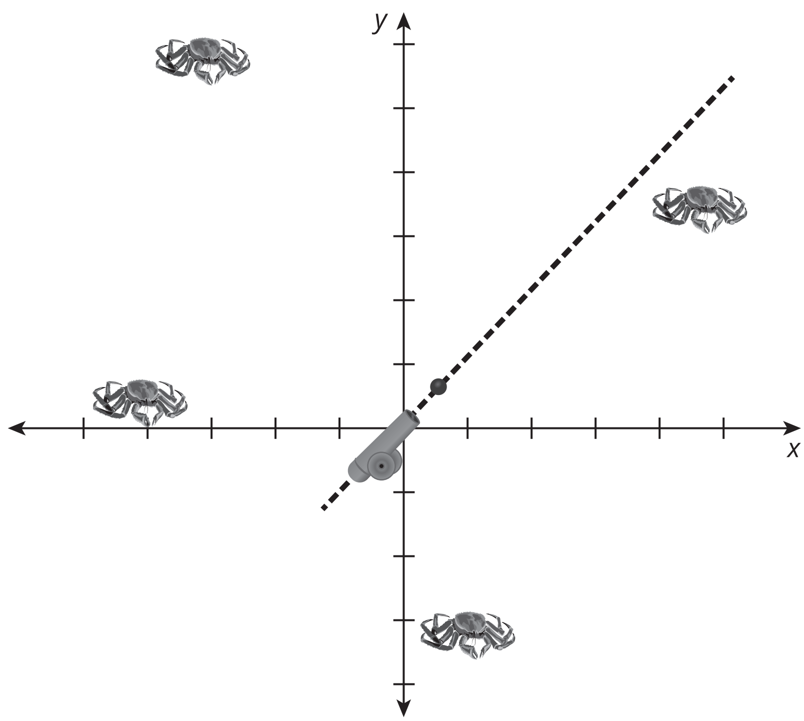
   b. How does changing the d-value affect the slope of the function? The y-intercept of the function?



ACTIVITY 1.5 Applying Linear Function Transformations

Your company is developing a new video game for kids, which involves scoring points by shooting targets with a cannon. The cannon starts at the origin of a coordinate system as shown.

By default it shoots along the line $f(x) = x$. Players can move the cannon up and down the y-axis and also change the angle of the cannon. When a player shoots the cannon, the game program determines the values of a and d for the linear function $a \cdot f(x) + \text{d}$. If the target is on that line, the program will show an animation of the cannonball hitting the target.



Ask Yourself . . .
The cannon can shoot in only one direction. Does that affect the domain or range of the linear function?

1. Test the program. Determine values for a and d for four linear functions that should hit the targets shown.

2. Identify the domain on which each graph hits the target.

The game developer is also testing two other versions of the game, with different abilities for the cannon.

Version B
The cannon can only shoot along a line parallel to $f(x) = x$ or $f(x) = -x$. It can move up and down the y-axis.

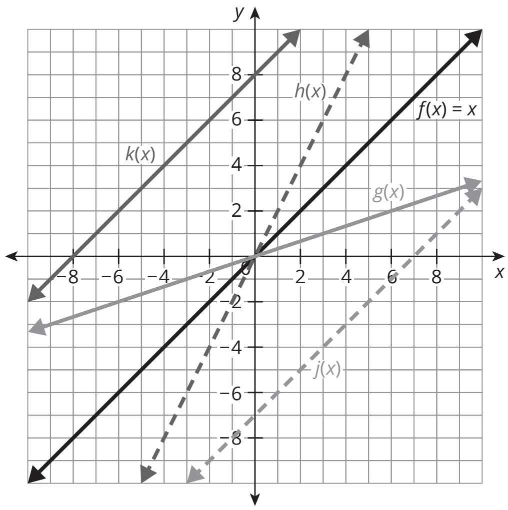
Version C
The cannon can change angles but remains fixed at the origin.

3. Determine values for a and d, along with their corresponding domains, for four linear functions that can hit the targets in each version of the game. Show your work.

Talk the Talk

Function Matching

The graph shows the linear function $f(x) = x$ and four transformations of $f(x)$.



Transformations
$\frac{1}{3} \cdot f(x)$
$2 \cdot f(x)$
$f(x) - 7$
$f(x) + 8$

Ask Yourself . . .
Did you justify your mathematical reasoning?

Match each transformed graph to one of the transformations in the table. Explain your reasoning.

1. $k(x) =$ ____________________

2. $h(x) =$ ____________________

3. $g(x) =$ ____________________

4. $j(x) =$ ____________________

Lesson 1 Assignment

Write

Describe the term *parent function* in the context of transformations using your own words.

Remember

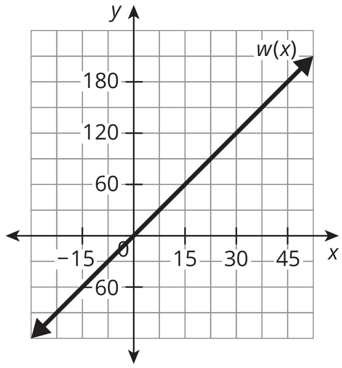
For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. For $d > 0$, the resulting graph vertically shifts up. For $d < 0$, the resulting graph vertically shifts down. The parent function and the resulting graph are parallel because they have the same slope but different *y*-intercepts.

The transformed function $y = af(x)$ shows a vertical dilation of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph is vertically stretched or compressed and is reflected across the *x*-axis.

Practice

1. Given $w(x) = 4x$.

   a. Graph $r(x) = \frac{1}{2} \cdot w(x)$. Then, complete the table of corresponding points on $r(x)$.



x	$w(x)$	$r(x)$
0		
15		
30		
45		

   b. Describe the transformation performed on $w(x)$ to produce $r(x)$.

   c. Write the equation for the function $r(x)$ in general form.

2. Write the equation of a line parallel to the line $y = 2x$ that passes through the given points.

   a. (0, 4)

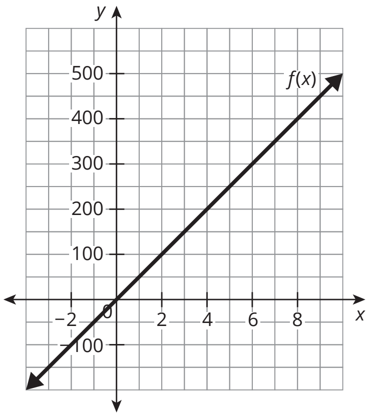
   b. (–2, –1)

   c. (2, 0)

Lesson 1 Assignment

3. Given $f(x) = 50x$.

   a. Graph $b(x) = f(x) - 150$. Then, complete the table of corresponding points on $b(x)$.



x	f(x)	b(x)
2		
4		
6		
8		

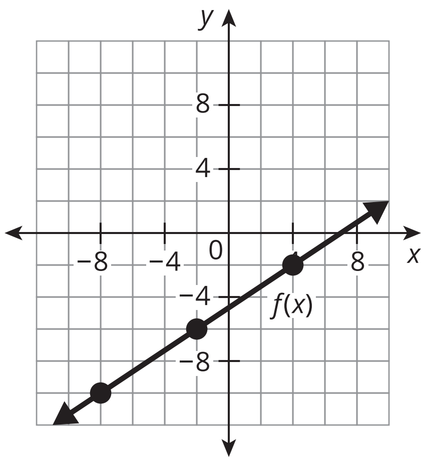
   b. Write the equation for the function $b(x)$ in general form.

   c. Describe the transformation performed on $f(x)$ to produce $b(x)$.

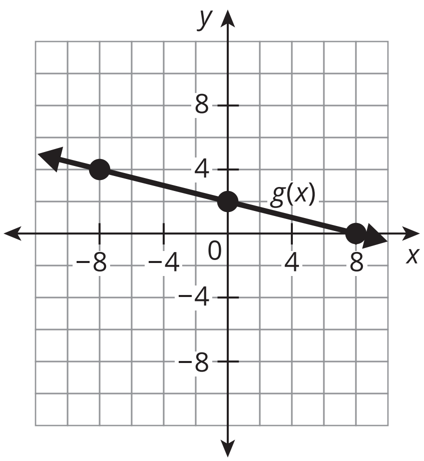
Prepare

Write a linear function for each graph.

1.



2.



2 Vertical and Horizontal Transformations of Linear Functions

OBJECTIVES

- Identify key characteristics of linear functions.
- Determine the effects on the graph of a linear function when $f(x)$ is replaced by $f(x) + d$, $f(x - c)$, $af(x)$, and $f(bx)$.

You have learned about the effects of vertical transformations on linear functions.

What effect do horizontal transformations have on linear functions? How do they compare to vertical transformations?

Getting Started

Hey, I Remember This!

Graph each linear function and describe the key characteristics of the graph.

1. $g(x) = 3x - 8$

   Domain: ____________

   Range: ____________

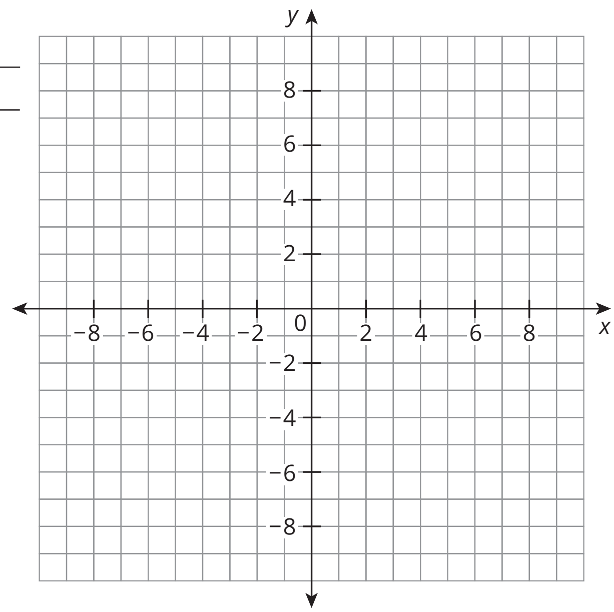
   x-intercept: ________

   y-intercept: ________

   Slope: ____

   Increasing/Decreasing:

   ____



2. $h(x) = -\frac{2}{3}x + 5$

   Domain: ____________

   Range: ____________

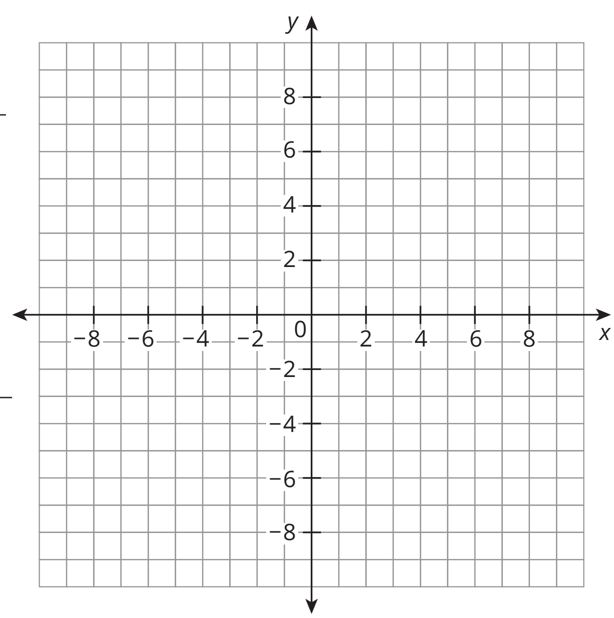
   x-intercept: ________

   y-intercept: ____________

   Slope: __________

   Increasing/Decreasing:

   ____



ACTIVITY **2.1**

Comparing Vertical and Horizontal Translations

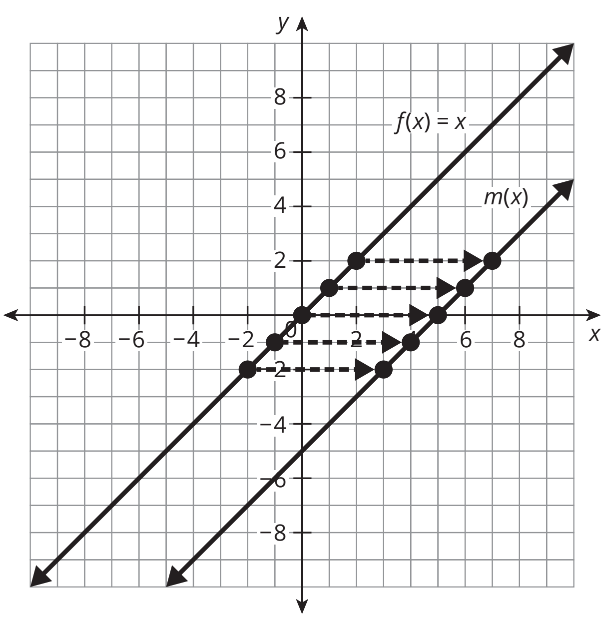
Let's determine how translations impact the graph of the linear function $f(x) = x$.

WORKED EXAMPLE

You can translate the graph of $f(x)$ to the right 5 units by moving each point 5 units to the right. The transformed graph is labeled as $m(x)$.

To translate the point $(-2, -2)$ on $f(x)$, add 5 units to the input value, or x-value. The output value, or y-value, remains unchanged. The coordinates of the translated point on $m(x)$ are $(3, -2)$.

The coordinates of four additional points on $f(x)$ are translated for you.

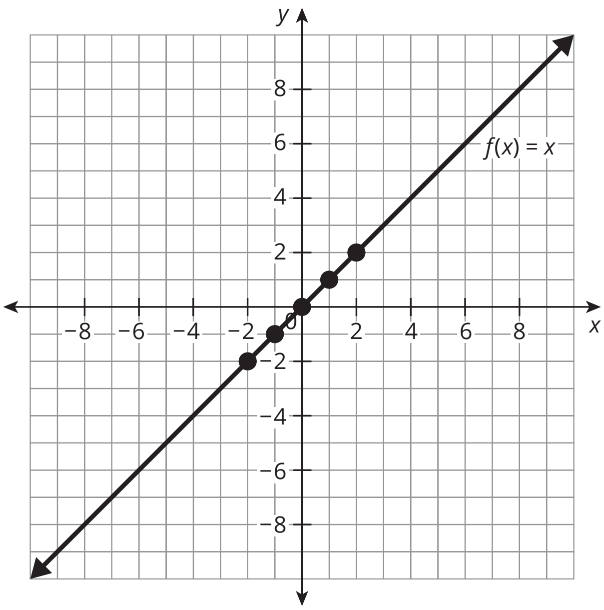


Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$m(x)$
3	−2
4	−1
5	0
6	1
7	2

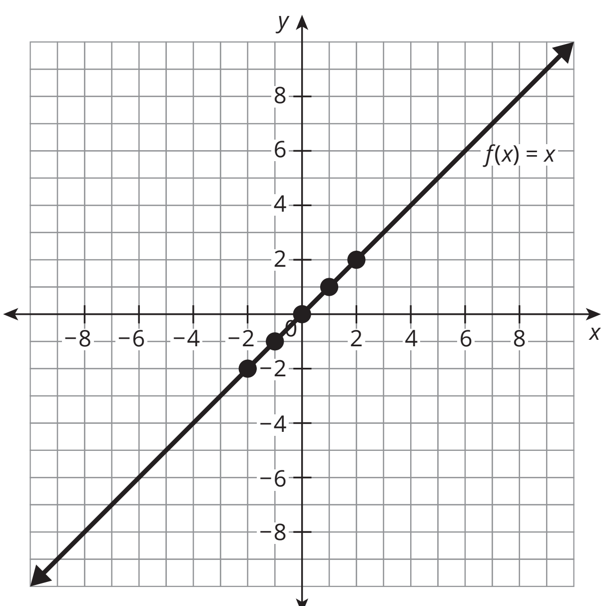
1. Given the graph and a table of values for $f(x) = x$, complete the following transformations.

   a. Translate the graph of $f(x)$ left 3 units. Label your graph as $t(x)$. Complete the table of corresponding points on $t(x)$.



Original Graph		Transformed Graph	
x	$f(x)$	x	$t(x)$
–2	–2		
–1	–1		
0	0		
1	1		
2	2		

   b. Translate the graph of $f(x)$ up 3 units. Label your graph as $w(x)$. Complete the table of corresponding points on $w(x)$.



Original Graph		Transformed Graph	
x	$f(x)$	x	$w(x)$
–2	–2		
–1	–1		
0	0		
1	1		
2	2		

Ask Yourself . . .
What observations can you make?

c. Compare the table of values for $f(x)$ and $t(x)$. Are the input values or output values affected by this horizontal translation?

d. Compare the table of values for $f(x)$ and $w(x)$. Are the input values or output values affected by this vertical translation?

e. Compare the graphs and tables of values for $t(x)$ and $w(x)$. What do you notice?

2. Which transformation(s) of the parent function $f(x) = x$ affect the input values of the corresponding points on the transformed function?

3. Which transformation(s) of the parent function $f(x) = x$ affect the output values of the corresponding points on the transformed function?

4. Based on your observations, horizontally translating the parent function $f(x) = x$ right n units is equivalent to what other transformation?

5. Based on your observations, vertically translating the parent function $f(x) = x$ up n units is equivalent to what other transformation?

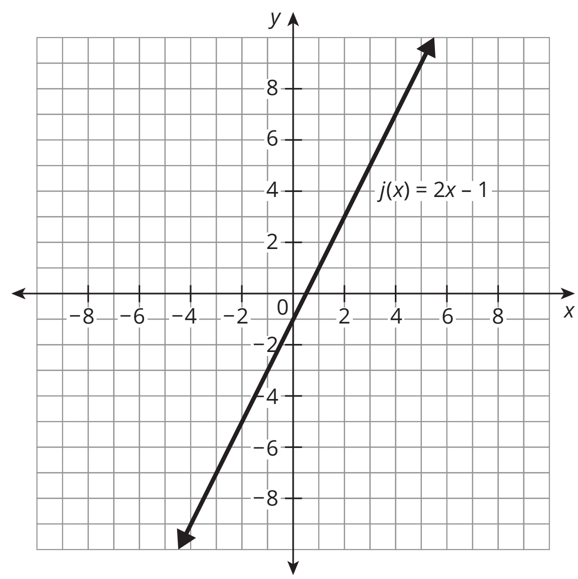
For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values, or x-values, of the function. For $c > 0$, the resulting graph horizontally shifts right c units. For $c < 0$, the resulting graph horizontally shifts left c units.

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up d units. For $d < 0$, the resulting graph vertically shifts down d units.

You have determined that for the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units.

Let's see whether these translations have similar effects on the graph of any linear function.

6. Consider the graph of $j(x) = 2x - 1$.



   a. Sketch the graph of $j(x)$ shifted up 4 units. Label the graph as $q(x)$.

   b. Write an equation for $q(x)$ in terms of $j(x)$.

   c. Sketch the graph of $j(x)$ shifted left 4 units. Label the graph as $r(x)$.

   d. Write an equation for $r(x)$ in terms of $j(x)$.

e. Compare the graphs of $q(x)$ and $r(x)$. What do you notice?

f. The equation for $q(x)$ is simplified for you. Simplify the equation for $r(x)$. What do you notice?

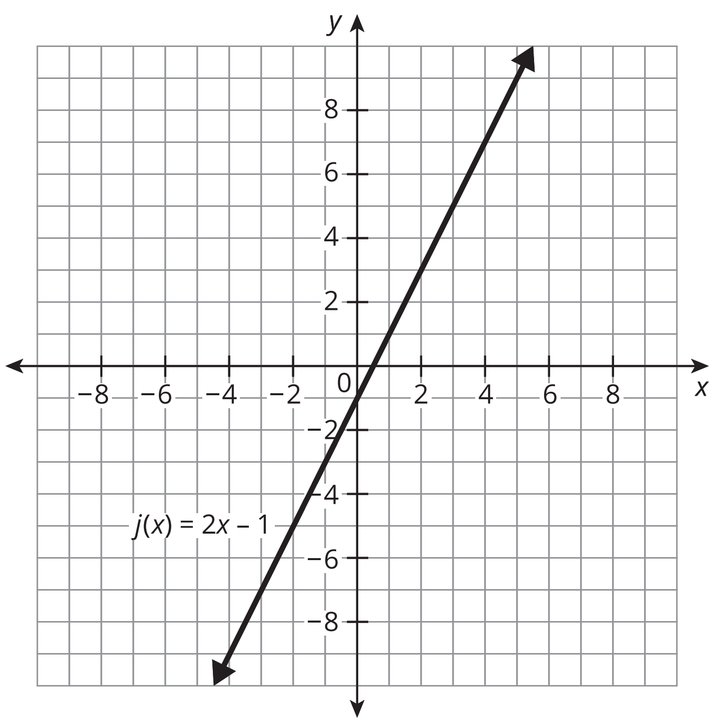
$q(x) = j(x) + 4$ $\qquad\qquad\qquad$ $r(x) = j(x + 4)$

$\quad = 2x - 1 + 4$

$q(x) = 2x + 3$

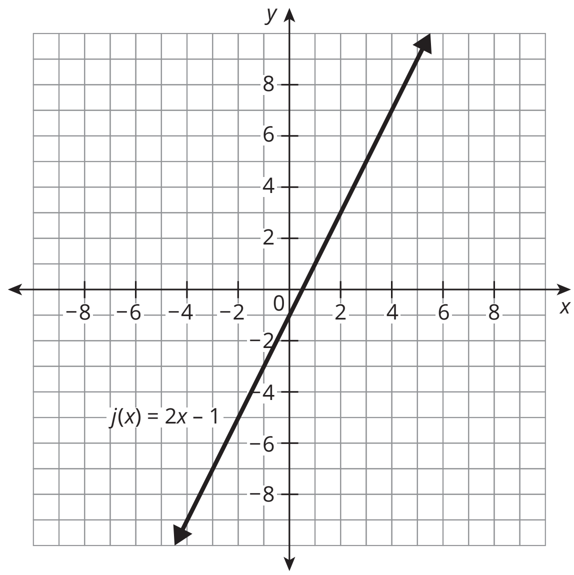
7. Again, consider the graph of $j(x) = 2x - 1$.

a. Sketch the graph of $j(x)$ shifted right 2 units. Label the graph as $b(x)$.



b. Write an equation for $b(x)$ in terms of $j(x)$.

c. Sketch the graph of $j(x)$ shifted down 2 units. Label the graph as $v(x)$.



d. Write an equation for $v(x)$ in terms of $j(x)$.

e. Compare the graphs of $b(x)$ and $v(x)$. What do you notice?

f. Simplify the equations for $b(x)$ and $v(x)$. What do you notice?

$b(x) =$ $v(x) =$

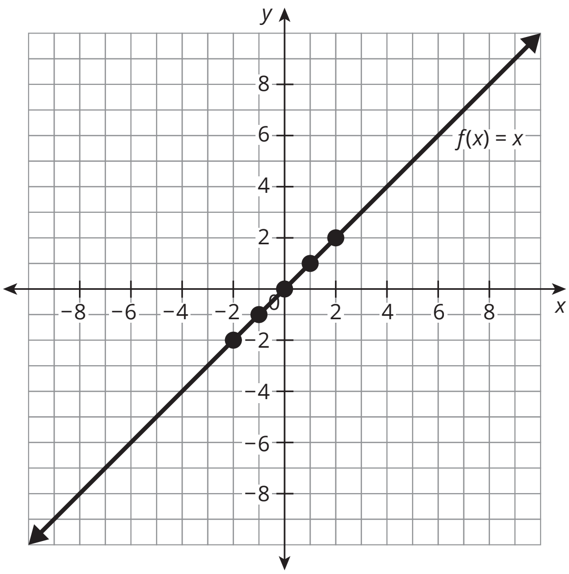
8. For any linear function, does a horizontal translation right n units produce the same graph as a vertical translation down n units?

9. For any linear function, does a horizontal translation left n units produce the same graph as a vertical translation up n units?

ACTIVITY 2.2 Comparing Vertical and Horizontal Dilations

1. The graph and a table of values for $f(x) = x$ are provided for you.

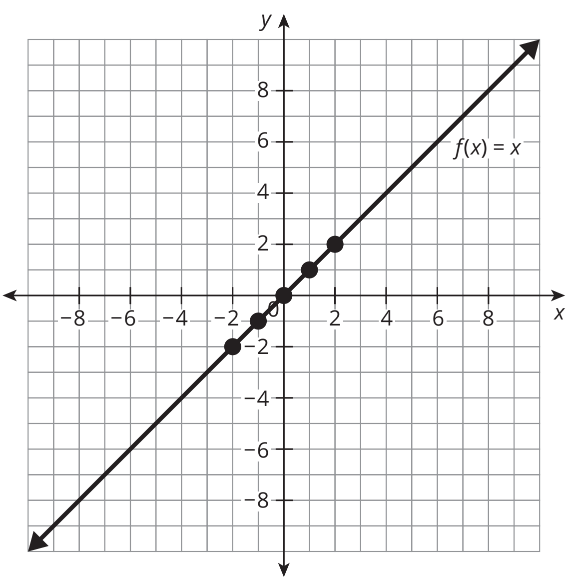
   a. Sketch the graph of $a(x)$, if the output values of $a(x)$ are changed by a factor of 2. Complete the table of corresponding points on $a(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$a(x)$
−2	
−1	
0	
1	
2	

   b. Sketch the graph of $b(x)$, if the input values of $b(x)$ are changed by a factor of $\frac{1}{2}$. Complete the table of corresponding points on $b(x)$.



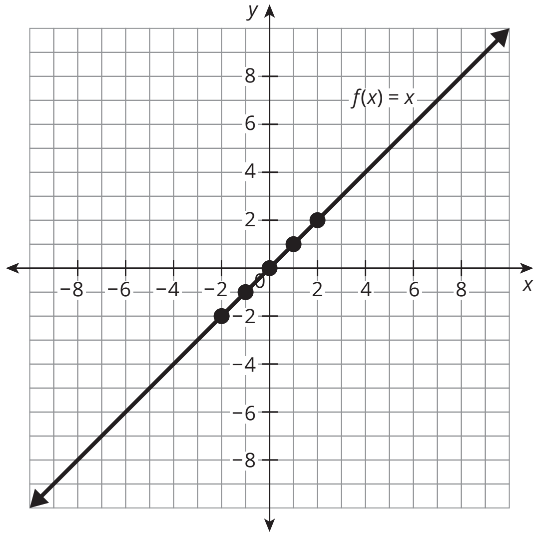
Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$b(x)$
	−2
	−1
	0
	1
	2

   c. Compare the graphs and tables of values for $a(x)$ and $b(x)$. What do you notice?

2. Again, the graph and a table of values for $f(x) = x$ are provided for you.

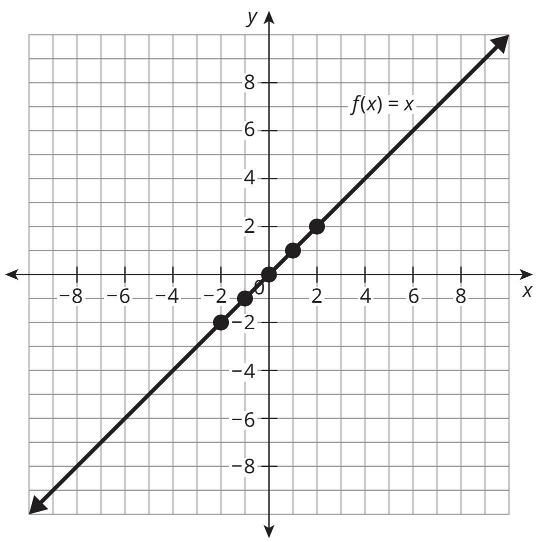
a. Sketch the graph of $c(x)$, if the output values of $c(x)$ are changed by a factor of $\frac{4}{5}$. Complete the table of corresponding points on $c(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$c(x)$
−2	
−1	
0	
1	
2	

b. Sketch the graph of $d(x)$, if the input values of $d(x)$ are changed by a factor of $\frac{5}{4}$. Complete the table of corresponding points on $d(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$d(x)$
	−2
	−1
	0
	1
	2

c. Compare the graphs and tables of values for $c(x)$ and $d(x)$. What do you notice?

d. Based on your observations, changing the output values of the parent function $f(x) = x$ by a factor of n units is equivalent to changing the input values by what factor?

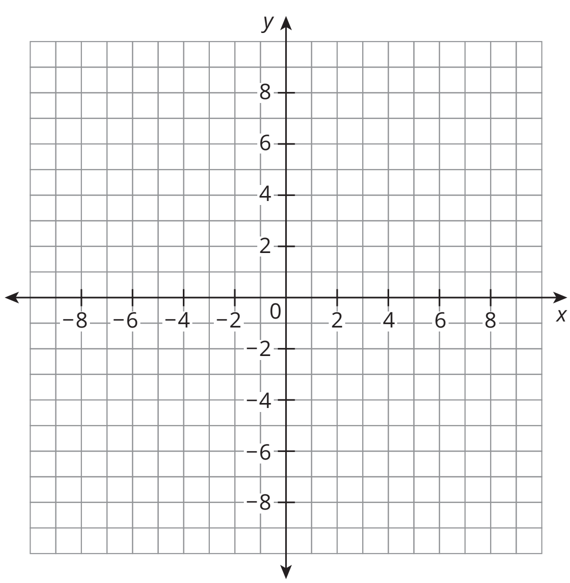
e. Based on your observations, changing the input values of the parent function $f(x) = x$ by a factor of n units is equivalent to changing the output values by what factor?

For the parent function $f(x) = x$, the transformed function $y = af(x)$ affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph reflects across the x-axis.

For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values, or x-values, of the function. For $|b| > 1$, the resulting graph horizontally compresses by a factor of $\frac{1}{|b|}$ units. For $0 < |b| < 1$, the resulting graph horizontally stretches by a factor of $\frac{1}{|b|}$ units. For $b < 0$, the resulting graph reflects across the y-axis.

You have determined that for the parent function $f(x) = x$, a vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

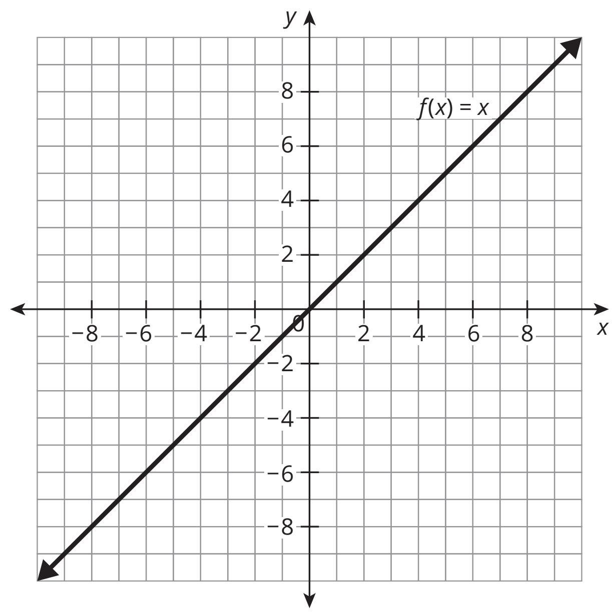
3. Does this relationship between horizontal and vertical dilations hold true for any linear function? Explain your reasoning. Provide a counterexample if necessary.



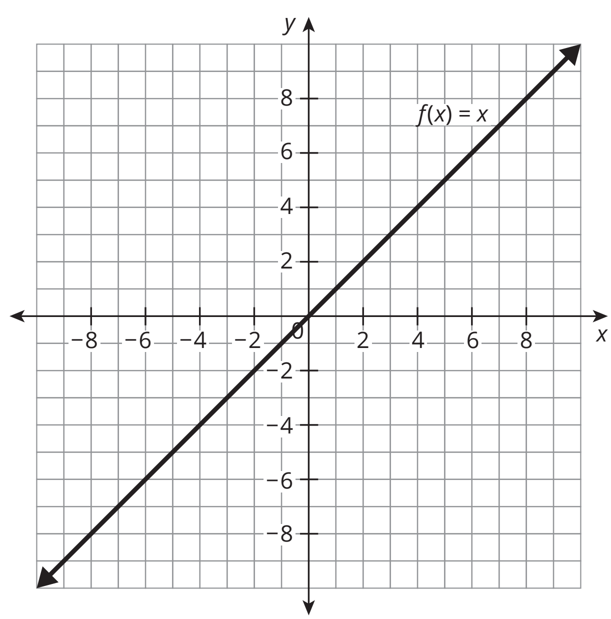
ACTIVITY 2.3 Graphing Linear Transformations

The equation of the parent linear function $f(x)$ is given. The equation for the transformed function $g(x)$ is also given. Describe the transformation(s) performed on $f(x)$ to produce $g(x)$. Then, graph $g(x)$.

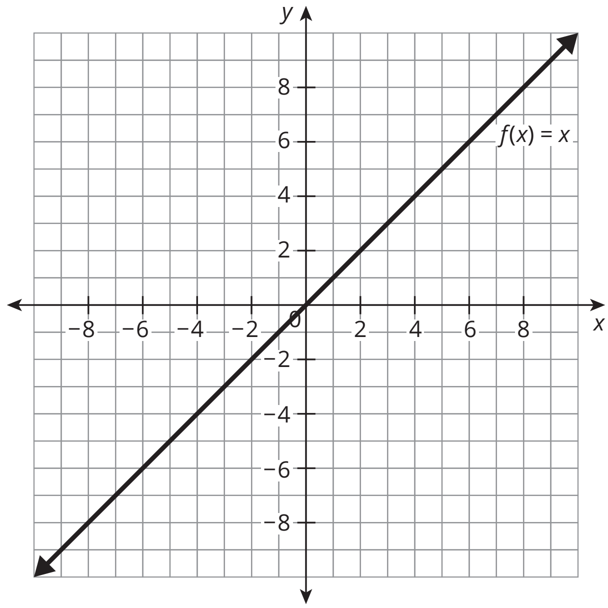
1. $f(x) = x$
   $g(x) = f(x - 7)$



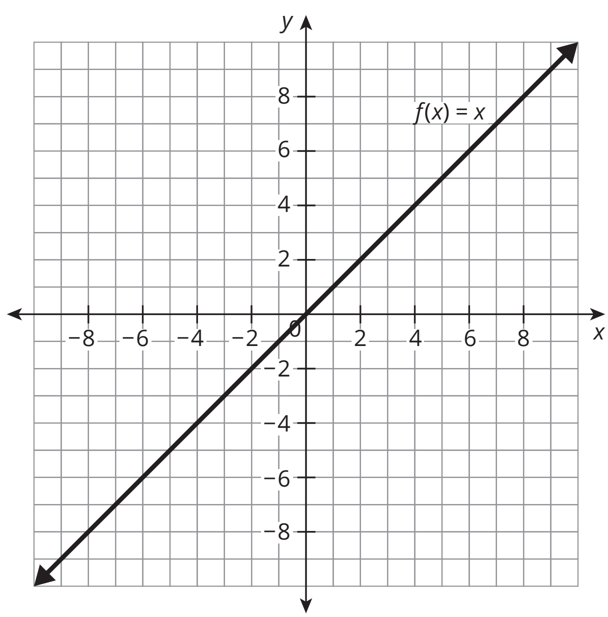
2. $f(x) = x$
   $g(x) = 3f(x)$



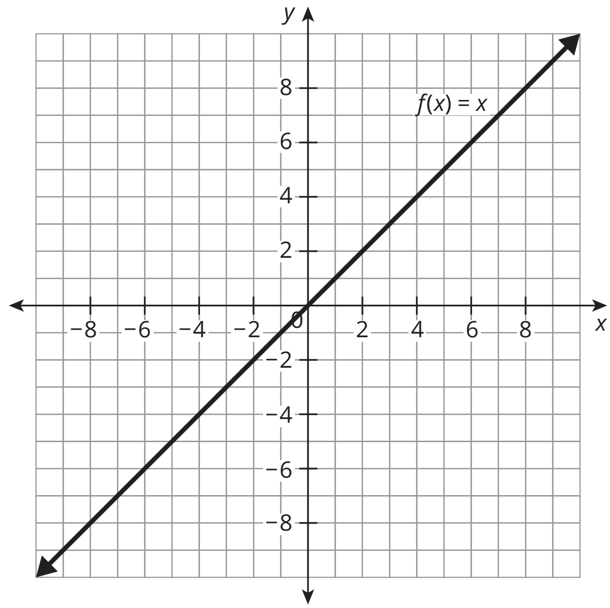
3. $f(x) = x$
$g(x) = f(x) + 2$



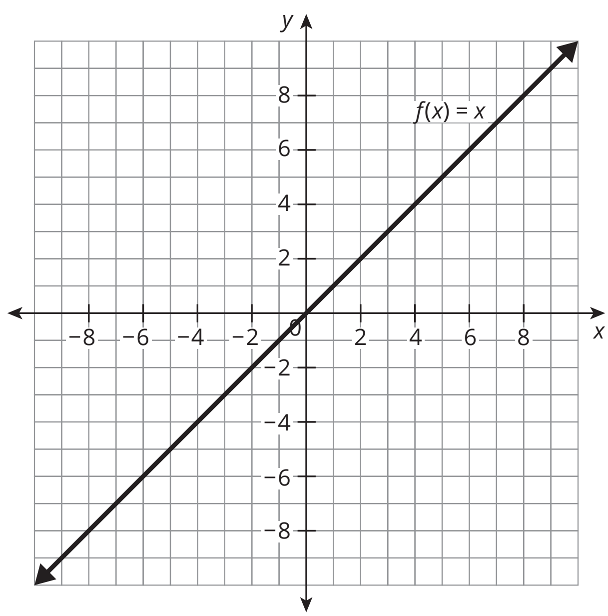
4. $f(x) = x$
$g(x) = f(\frac{1}{2}x)$



5. $f(x) = x$
   $g(x) = f(x) - 4$



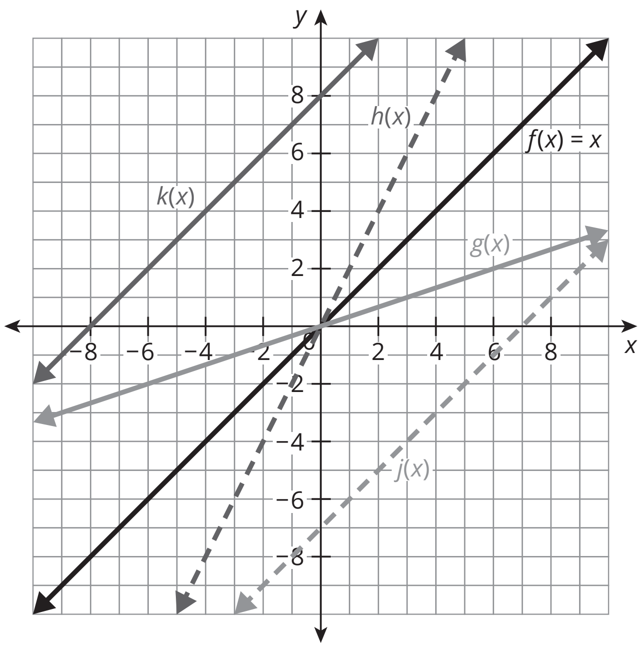
6. $f(x) = x$
   $g(x) = f(x + 5)$



Talk the Talk

Show Me What You've Got!

The graph shows the linear function $f(x) = x$ and four transformations of $f(x)$.



Equation
$f(\frac{1}{3}x)$
$2f(x)$
$f(x - 7)$
$f(x) + 8$

Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. $k(x) =$ ____________________

2. $h(x) =$ ____________________

3. $g(x) =$ ____________________

4. $j(x) =$ ____________________

Lesson 2 Assignment

Write

For the parent function, $f(x) = x$, how do horizontal transformations compare to vertical transformations?

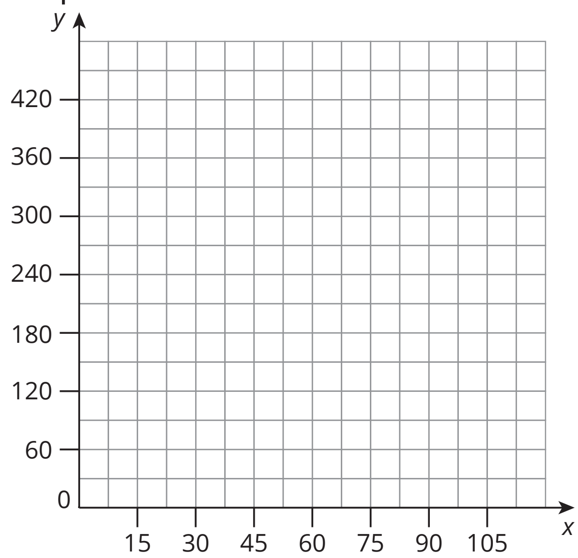
Remember

When $f(x)$ is replaced by $af(b(x - c)) + d$, b and c affect the input values of the function.

Practice

1. Ricardo started an exercise program. He researched the number of calories burned by walking and calisthenics. He learned that walking burns 4 calories per minute. The number of calories Ricardo can burn by walking is modeled by the function $w(x) = 4x$, where x represents the number of minutes he walks.

   a. Graph the linear function.



   b. Describe the key characteristics of the graph.
   Domain:
   Range:
   x-intercept/zero:

   c. Do the domain and range make sense in terms of the problem situation? Explain your reasoning. If the domain and range do not make sense, how could they be changed to better model the situation?

Lesson 2 Assignment

d. Ricardo burned 330 calories doing calisthenics before going for a walk. Let $c(x)$ be the function that represents Ricardo's calories burned from doing calisthenics and walking. Write an equation for $c(x)$ in terms of $w(x)$.

e. Simplify the equation for $c(x)$.

f. Describe the transformation performed on $w(x)$ to produce $c(x)$.

Prepare

Determine the reciprocal of each value.

1. 3

2. -10

3. $\frac{1}{5}$

4. $-\frac{5}{4}$

5. $-c$

6. $\frac{a}{b}$

3 Determining Slopes of Perpendicular Lines

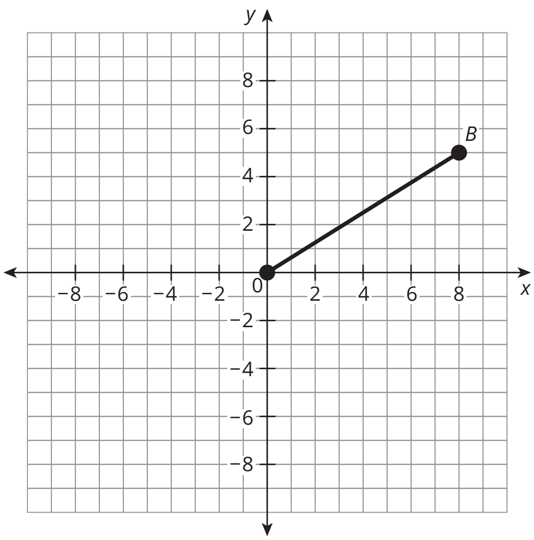
OBJECTIVES

- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of horizontal and vertical lines.

You have translated the graphs of linear equations to determine the slopes of parallel lines. How can you rotate the graphs of linear equations to determine the slopes of perpendicular lines?

Getting Started

Coordinate Rotation



Consider $\overline{AB}$ with coordinates A (0, 0) and B (8, 5).

1. Suppose the line segment is rotated through an angle with the origin as the center of rotation. Complete the table with the coordinates of the rotated figure.

Original Line Segment	90° Counterclockwise	180°	270° Counterclockwise
$\overline{AB}$	$\overline{A'B'}$	$\overline{A''B''}$	$\overline{A'''B'''}$
A (0, 0)			
B (8, 5)			

2. What do you notice about the coordinates of the endpoints of each rotated figure?

3. Determine the slope of each line segment.

   a. $\overline{AB}$

   b. $\overline{A'B'}$

   c. $\overline{A''B''}$

   d. $\overline{A'''B'''}$

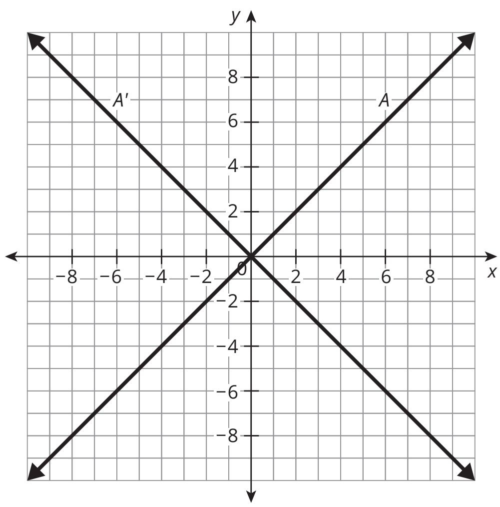
4. **Describe** any patterns you notice in the slopes of the figures.

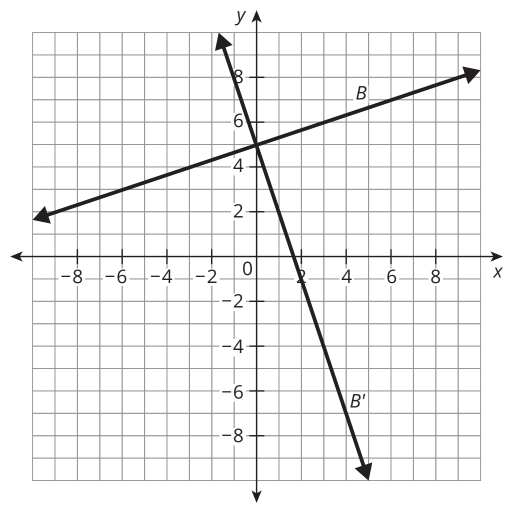
Review the definition of **describe** in the Academic Glossary.

ACTIVITY 3.1 Slopes of Perpendicular Lines

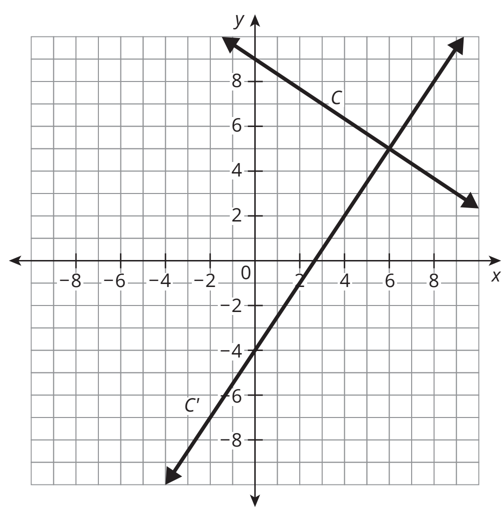
Recall that perpendicular lines or line segments form a right angle at the point of intersection.

Consider the three graphs shown. Each shows a line and its rotation 90° about a point, which is also the point of intersection.





1. Are the lines in each graph perpendicular? Explain your reasoning.



Remember . . .

The reciprocal of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. Because the product of a number and its reciprocal is one, reciprocal numbers are also known as multiplicative inverses.

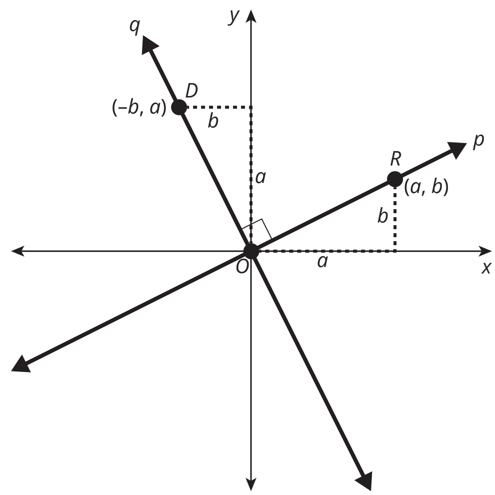
2. Write the equation for each line and its transformation. What do you notice?

Consider the theorem.

WORKED EXAMPLE

Theorem: If two lines are perpendicular, their slopes are negative reciprocals.

Use the graph and the proof shown to analyze the validity of the theorem.



Given: $p \perp q$

Let m_1 = slope of line *p* and let m_2 = slope of line *q*.

Point *R* lies on line *p*.

Prove: $m_1 = -\frac{1}{m_2}$

Rotate point *R* 90° counterclockwise using point *O* as the center of rotation. Since *p* and *q* are perpendicular, the image (point *D*) will lie on line *q* under this 90° rotation.

Since this rotation maps the positive *x*-axis to the positive *y*-axis and the positive *y*-axis to the negative *x*-axis, then the coordinates of *R*(*a*, *b*) are transformed into the coordinates of *D*(−*b*, *a*). Graphically, you can see the movement of lengths *a* and *b* under the rotation.

The graph shows the slope of line *p*, $m_1 = \frac{b}{a}$, and the slope of line *q*, $m_2 = \frac{a}{-b}$.

Using these slopes, you can demonstrate that $m_1 = -\frac{1}{m_2}$.

$$\frac{b}{a} = -\frac{1}{\frac{a}{-b}}$$

$$= -1 \cdot \frac{-b}{a}$$

$$= \frac{b}{a}$$

The slope of line *q* is the negative reciprocal of the slope of line *p*.

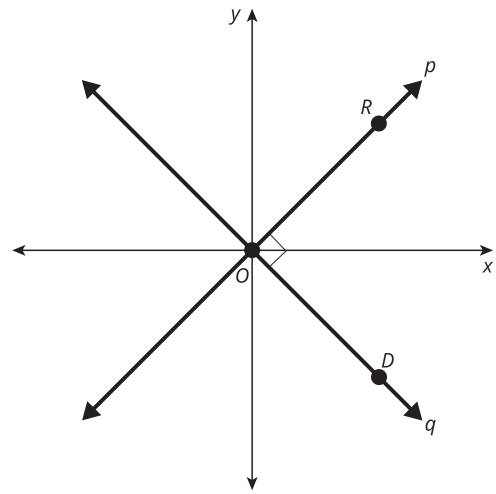
Remember . . .
The symbol $\perp$ means *is perpendicular* to.

The product of the slopes of perpendicular lines is −1.

The proof shown is a paragraph proof. You will learn different formats of proof as you investigate more properties of lines and figures.

There is often more than one way to prove a theorem. Suppose that point R is rotated 90° clockwise using point O as the center of rotation.

3. Rewrite the proof using the clockwise rotation of point R.



4. Line j and line k are perpendicular. Given each slope of line j, determine the slope of line k.

   a. $m = \frac{2}{3}$

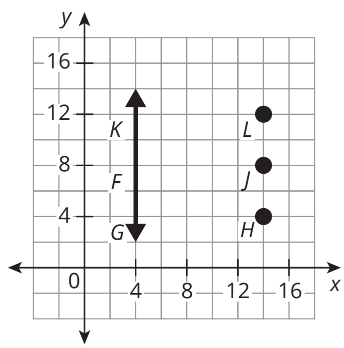
   b. $m = -\frac{4}{5}$

   c. $m = -3$

ACTIVITY
3.2 Horizontal and Vertical Lines

Consider the graph shown.

1. Use a straightedge to extend $\overline{GK}$ to create line p, extend $\overline{GH}$ to create line q, extend $\overline{FJ}$ to create line r, and extend $\overline{KL}$ to create line s.



2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if x increases by one unit, by how many units does y change?

3. Describe the slope of any horizontal line. Explain your reasoning.

4. Consider the vertical line you drew in Question 1. Suppose that y increases by one unit. By how many units does x change?

5. Describe the slope of any vertical line. Explain your reasoning.

6. Determine whether each of the given statements is always, sometimes, or never true. Explain your reasoning.

   a. All vertical lines are parallel.

   b. All horizontal lines are parallel.

7. Describe the relationship between any vertical line and any horizontal line.

8. Write an equation for a horizontal line and an equation for a vertical line that pass through the point $(2, -1)$.

9. Write an equation for a line that is perpendicular to the line given by $x = 5$ and passes through the point $(1, 0)$.

10. Write an equation for a line that is perpendicular to the line given by $y = -2$ and passes through the point $(5, 6)$.

ACTIVITY 3.3 Writing Equations of Perpendicular Lines

Remember . . .
You can use point-slope form to write an equation for any line when you know its slope and one point on that line.

In a previous lesson you wrote the equation of a line parallel to a given line that passes through a given point. You can write the equation of a perpendicular line using what you know about the slope of that line and any point on that line.

1. Write the equation of the line perpendicular to $y = 2x + 1$ that passes through the point (6, 2).

2. Write the equation of the line perpendicular to $y = -\frac{3}{4}x$ that passes through the point (3, −8).

3. Write the equation of the line that passes through the point (6, 2) and is perpendicular to a line that passes through the points (−5, 3) and (−1, −9).

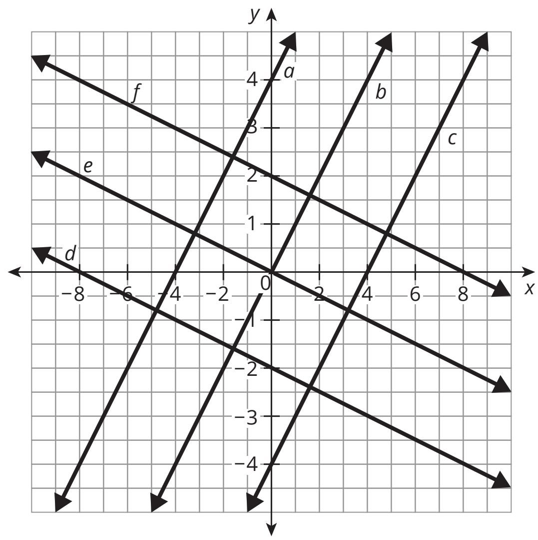
4. Write the equation of a line that passes through the point $(-2, 7)$ and is perpendicular to a line that passes through the points $(-6, 1)$ and $(0, 4)$.

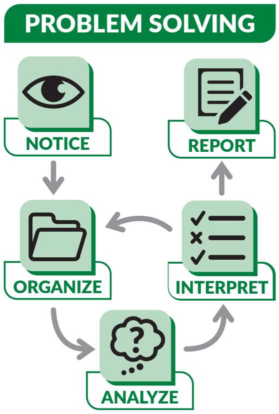
5. A pair of perpendicular lines intersect at the point $(5, 9)$. Write the equation of the line that is perpendicular to the line that also passes through point $(-4, 4)$.

Talk the Talk

Things Aren't Always What They Seem

Consider the graphs of six linear equations shown.





1. Linh says lines *a*, *b*, and *c* are parallel to each other and perpendicular to lines *d*, *e*, and *f*. Jackson agrees that lines *a*, *b*, and *c* are parallel to each other but says they are not perpendicular to lines *d*, *e*, and *f*. Who is correct? Justify your reasoning.

Lesson 3 Assignment

Write

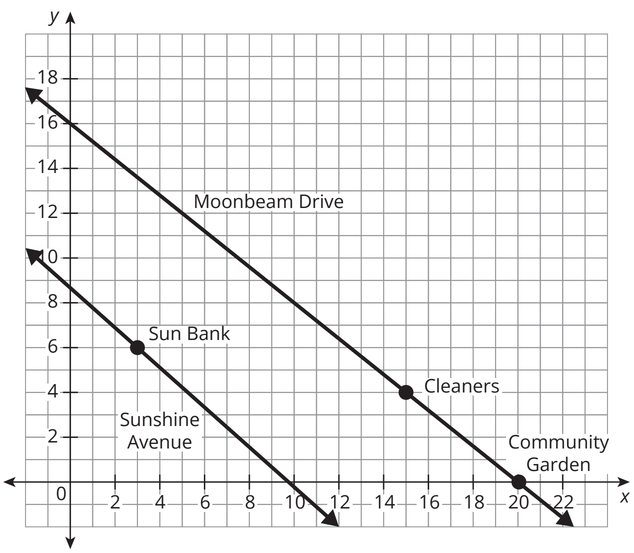
Explain in your own words why the slope of a vertical line is undefined.

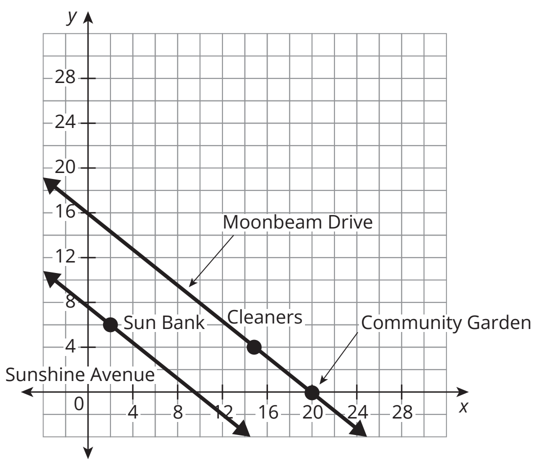
Remember

The slopes of perpendicular lines are negative reciprocals. Any vertical line is perpendicular to any horizontal line.

Practice

Nakota is a developer and plans to build a new development. Use the grid to help Nakota create a map for his development. Each gridline represents one block.





1. There are two main roads that pass through the development, Moonbeam Drive and Sunshine Avenue. Are these two roads parallel to each other? Explain your reasoning.

Lesson 3 Assignment

2. Nakota wants to build a road named Stargazer Boulevard that will be parallel to Moonbeam Drive. On this road, he will build a diner located seven blocks north of the community garden. Determine the equation of the line that represents Stargazer Boulevard. Show your work. Then, draw and label Stargazer Boulevard on the coordinate plane.

3. Nakota wants to build a road named Rocket Drive that connects Sun Bank to Moonbeam Drive. He wants this road to be as short as possible. Determine the equation of the line that represents Rocket Drive. Show your work. Then, draw and label Rocket Drive on the coordinate plane.

4. Two office buildings are to be located at the points (8, 4) and (12, 10). Would the shortest road between the two office buildings be a line that is perpendicular to Moonbeam Drive? Explain your reasoning.

Lesson 3 Assignment

5. A straight road named Planet Drive is planned that will connect the diner and the community garden. What is the equation of the line that represents Planet Drive? Show your work. Draw and label Planet Drive on the coordinate plane.

6. Nakota decides that another road to be named Saturn Avenue is needed that will go past the cleaners and be perpendicular to Planet Drive. Determine the equation of the line that represents Saturn Avenue. Show your work. Draw and label Saturn Avenue on the coordinate plane.

Prepare

Determine whether each set of ordered pairs represents a function. Explain your reasoning.

1. $\{(-1, -1), (0, 0)\ (1, 1), (2, 2)\}$

2. $\{(-1, -2), (0, 0)\ (1, 2), (2, 4)\}$

3. $\{(-1, -1), (0, -1)\ (1, -1), (2, -1)\}$

4. $\{(-1, -1), (-1, 0)\ (-1, 1), (-1, 2)\}$

4 Comparing Linear Functions in Different Forms

OBJECTIVES

- Compare linear functions represented algebraically, graphically, in tables, or with verbal descriptions.
- Choose and interpret appropriate units to represent independent and dependent quantities in situations modeled by linear functions.
- Choose and interpret appropriate scales and origins for graphs of linear functions.

You have represented linear functions in a variety of different ways. How does each linear function representation compare with the others?

Getting Started

Odd One Out

1. Choose the function that does not belong with the others and justify your choice.

Function A

x	y
−5	10
−1.5	3
0	0
2.5	−5

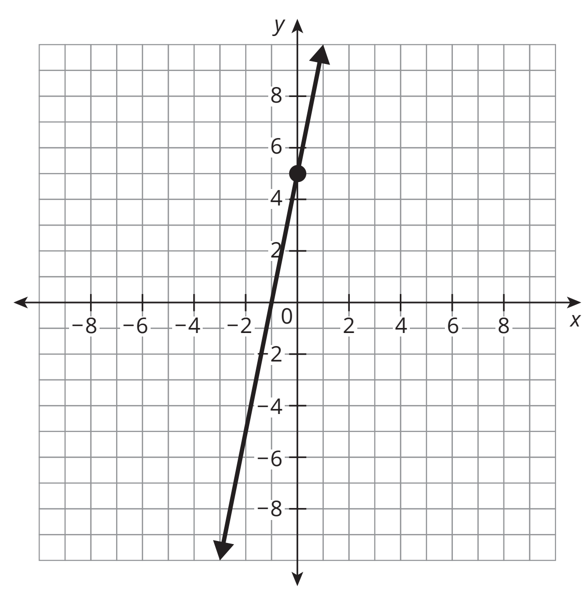
Function B

During one year at a high school, the ratio of 9th-graders to 10th-graders was 1 : 1.

Function C

$c(x) = \frac{1}{5}x$

Function D



ACTIVITY
4.1

Slopes of Linear Relationships

Consider the line that represents the equation $y = 4x - 3$.

1. Determine whether the graph of each linear relationship is parallel, perpendicular, or neither parallel nor perpendicular to the graph of $y = 4x - 3$.

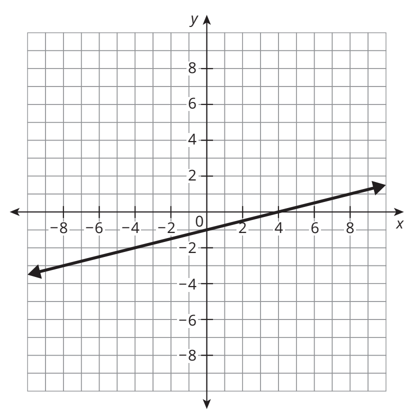
a.

x	y
−2.5	−6
0	4
0.5	6
3	16

b. An elevator descends 4 feet every second.

c. $y = \frac{1}{4}x + 5$

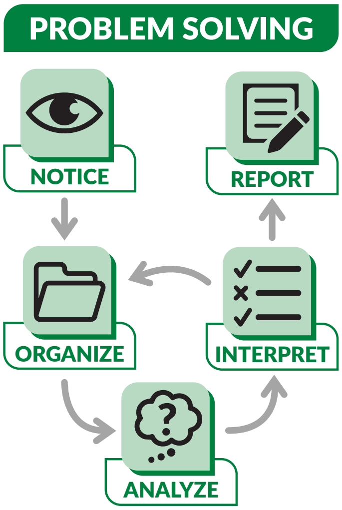
d.



e.

x	y
−4	3
0	2
2	1.5
8	0

f. For every hour Chiletso bakes, she makes 4 dozen cookies.



ACTIVITY 4.2

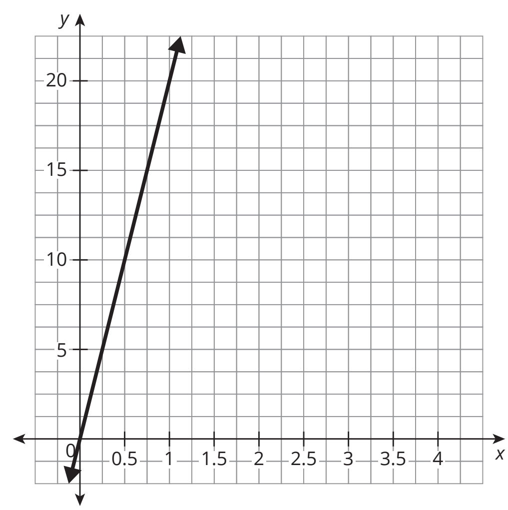
Comparing Tables and Graphs

Large recycling centers pay out different amounts per ton of cardboard recycled. Travis County Recycling Center lists its payout amounts in a table, and Hays County uses a graph to advertise its payout amounts. The two centers' payout amounts are close but not equal.

Travis County Recycling

0.5	\$7.50
1	\$15
1.5	\$22.50
2	\$30

Hays County Recycling



1. Each representation shows a functional relationship between quantities. Label the quantities and their units in the table and on the graph.

2. Let $w(x)$ represent the function for Travis County, and let $j(x)$ represent the function for Hays County. Use $>$, $<$, or $=$ to complete each statement.

   a. $w(0.5)$ _______ $j(0.5)$ b. $w(1)$ _______ $j(1)$ c. $w(3)$ _______ $j(3)$

3. Compare the y-intercepts of each relationship. What does each y-intercept mean in terms of the relationship between the quantities?

4. Which recycling center would you choose? Provide evidence to justify your choice.

Ask Yourself . . .
How else can you represent this information?

Ask Yourself . . .
Did you complete all the steps in the problem-solving model?

ACTIVITY

4.3 Comparing Equations and Tables

Xavier and Joey are friends who live in different countries. When they chat online and talk about the weather, one of them always uses temperature in degrees Celsius while the other talks about temperature using degrees Fahrenheit.

Xavier uses an equation to convert Joey's temperatures, and Joey looks at a table to convert Xavier's temperatures. The equation and part of the table are shown.

Remember . . .
0°C = 32°F

Joey's Table

10	−12.2
30	−1.1
50	10
75	23.9

Xavier's Equation

$J(x) = \frac{9}{5}x + 32$

1. Identify quantities in the table and equation. Who is converting from °C to °F, and who is converting from °F to °C? Explain your reasoning.

2. Compare the given characteristics for each function in terms of the quantities.

   a. slope

   b. y-intercept

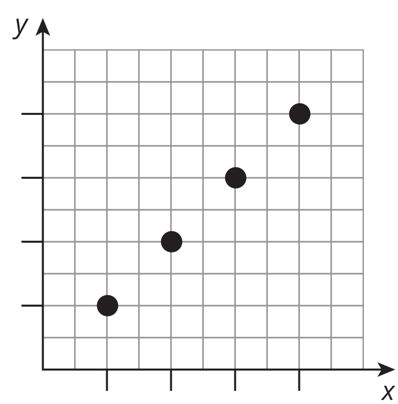
Think About . . .
The units of the quantities are very important.

ACTIVITY 4.4

Comparing Descriptions and Graphs

Lauren and Miguel both opened bank accounts at the same time. Miguel's account balance is shown in the graph. He started with $0 and deposited the same amount each month. In the 4th month, he had $80 saved. Lauren opened her bank account on September 1st with $25 and continues to deposit $25 each month.

Miguel



Lauren

Lauren opened her bank account on September 1st with $25 and continues to deposit $25 each month.

1. Use what you know about Miguel's account to determine the scale of each axis and interpret the origin on the graph. Explain your reasoning.

2. Compare the given characteristics for each function in terms of the quantities.

   a. slope

   b. *y*-intercept

Ask Yourself . . .
How can you use comparing graphs, tables, and descriptions in everyday life?

Talk the Talk

Function Maker Space

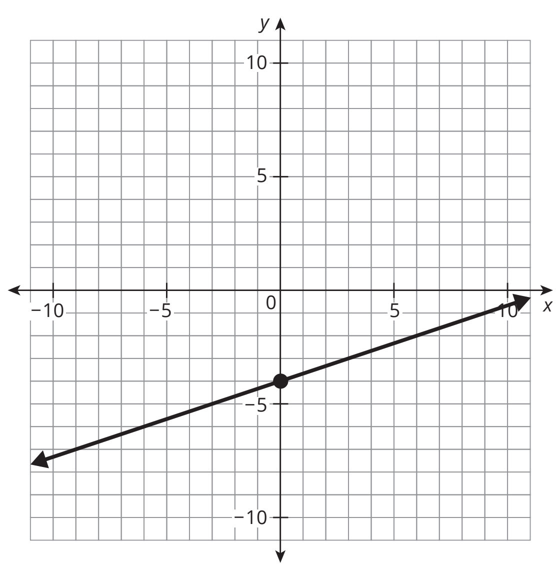
Consider the table of values.

x	−10	10	20	25
y	0	5	7.5	8.75

1. Create a situation to represent the table of values shown.

2. Write an equation that has a slope that is less steep than the relationship in the table.

3. How does the slope of the graph shown compare to the slope from the table of values and your equation?



4. What strategies did you use to create your linear functions and to compare the slopes?

Lesson 4 Assignment

Write

Describe how to compare the slopes and y-intercepts of two linear functions if one is represented as a graph and one is represented as a table.

Remember

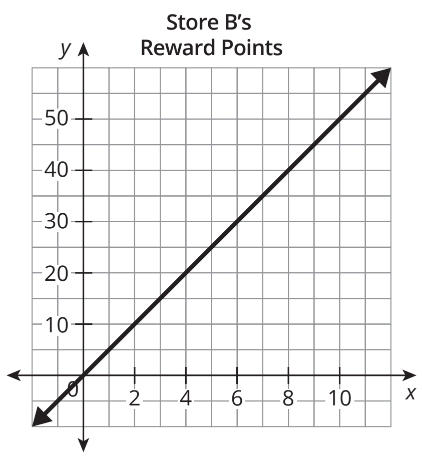
A linear function can be represented using an equation, a table, a graph, or with a verbal description. Characteristics of linear functions, such as slope, y-intercepts, and independent and dependent quantities, can be understood from different representations of functions.

Practice

1. Bookstores specializing in selling used books award different amounts of points to customers who supply them with used books. The points are used toward the purchase of other books in the store. Store A lists its point values in a table, and Store B uses a graph to post its point values.

Store A's Reward Points

2	12
4	24
6	36
8	48



   a. Label each column of values in the table and label the x-axis and y-axis on the graph with the appropriate variable quantities.

   b. Compare the slope of each function and explain what each represents in context.

Lesson 4 Assignment

c. Compare the y-intercepts of each function and explain what each represents in context.

2. Hailey and Ben live in different cities. They are planning to meet in Nashville, but they will both need to drive several days to get there. They have each calculated the distance to Nashville from their homes, but one calculated the distance in miles and the other calculated the distance in kilometers.

Hailey uses an equation to convert the distances Ben plans to drive each day, and Ben uses a table to convert the distances Sherry plans to drive each day. The equation and part of the table are shown.

Ben's Table

300	482.80
382	614.77
426	685.58
475	764.44

Hailey's Equation

$y = 0.6214x$

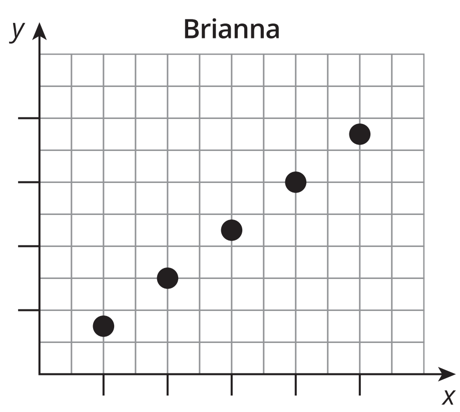
a. Label each column of quantities in Ben's table and identify the meaning of x and y in Hailey's equation. Who is converting from miles to kilometers and who is converting from kilometers to miles? Explain your reasoning.

Lesson 4 Assignment

b. Compare the slope for each function. Explain what each slope represents in context.

c. Compare the y-intercepts of each function and explain what each y-intercept represents in context.

3. Antonio and Brianna collect movies. Brianna's movie collection is shown in the graph. She started with 0 movies and added the same number of movies to her collection each month. In the 5th month, she had 15 movies. Antonio's movie collection is given by a description.



Antonio

Antonio started his collection with 27 movies he inherited from his uncle and continues to buy 2 movies each month.

a. Use what you know about Brianna's movie collection to determine the scale of her graph. Label the x- and y-axis, the origin, and the intervals on both axes. Explain your reasoning.

Lesson 4 Assignment

b. Compare the slope for each function. Explain what each represents in context.

c. Compare the y-intercepts of each function and explain what each represents in context.

Prepare

Solve each equation for x.

1. $\frac{1}{3}x = 8$

2. $5 + x = 12.7$

3. $2x - 9 = 6$

4. $12 + 2x = 3x - 1$

Name: ______________________________

Transforming and Comparing Linear Functions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Transforming and Comparing Linear Functions* topic by:

TOPIC 2: *Transforming and Comparing Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
recognizing that the parent function for linear functions is $f(x) = x$.	◯	◯	◯
determining the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, $f(x - c)$ and $f(bx)$.	◯	◯	◯
describing the transformations performed on a parent function to create a new function in transformation form $a \cdot f(b(x - c)) + d$.	◯	◯	◯
writing the equation of a line that contains a given point and is parallel or perpendicular to a given line.	◯	◯	◯

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Transforming and Comparing Linear Functions* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Transforming and Comparing Linear Functions Summary

NEW KEY TERM
- parent function

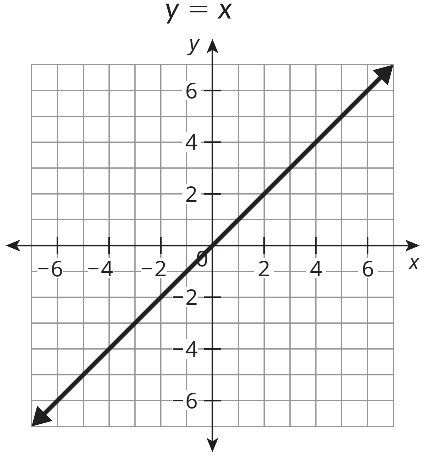
LESSON 1 Transforming Linear Functions

A **parent function** is the simplest function of its type. For example, $f(x) = x$ is the simplest linear function. It is in the form $f(x) = ax + b$, where $a = 1$ and $b = 0$.

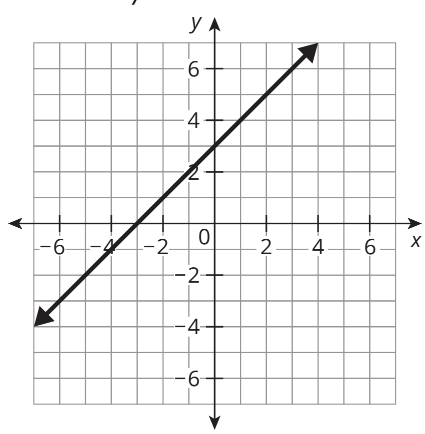
For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up. For $d < 0$, the graph vertically shifts down. The amount of shift is given by $|d|$.

For example, the function $y = x + 3$ translates the graph of $y = x$ vertically up 3 units.

$y = x$



$y = x + 3$



You can algebraically prove that a line and its translation are parallel to each other.

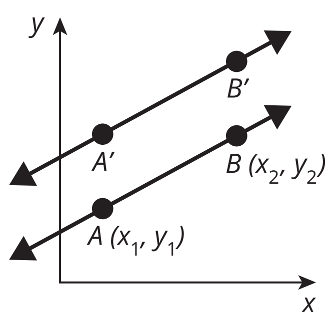
Line AB was translated a units up to create line $A'B'$.

The coordinates of point A' are $(x_1, y_1 + a)$ and the coordinates of point B' are $(x_2, y_2 + a)$.

slope of line $AB = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

slope of line $A'B' = \frac{[(y_2 + a) - (y_1 + a)]}{(x_2 - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

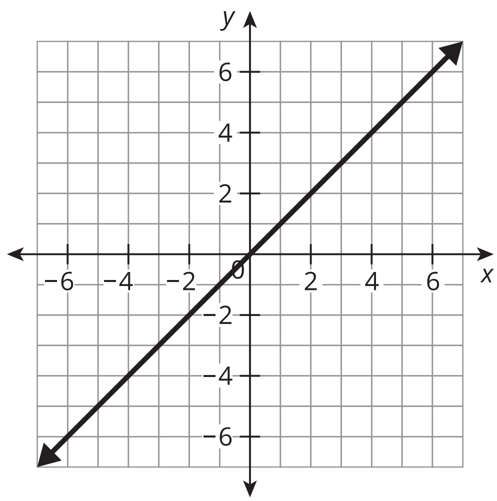
Since the slope of line AB is equal to the slope of line $A'B'$, the lines are parallel.



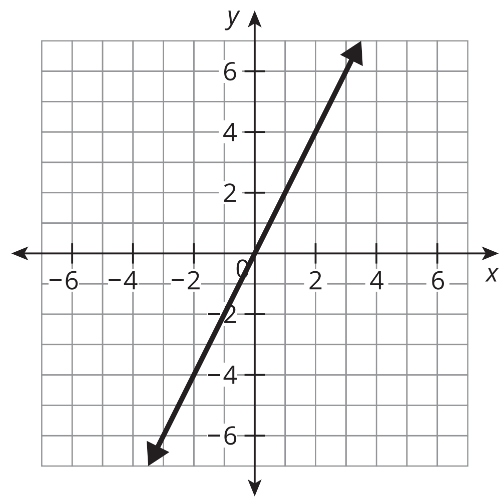
For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values of the function. For $|a| > 1$, the graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the graph vertically compresses by a factor of a units. For $a < 0$, the graph reflects across the x-axis.

For example, the function $y = 2x$ dilates the graph of $y = x$ by a factor of 2.

$y = x$



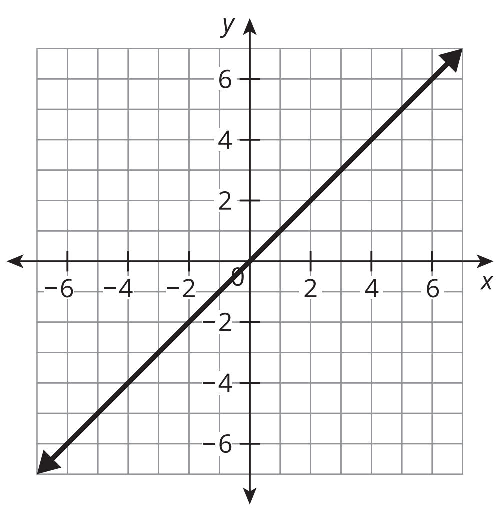
$y = 2x$



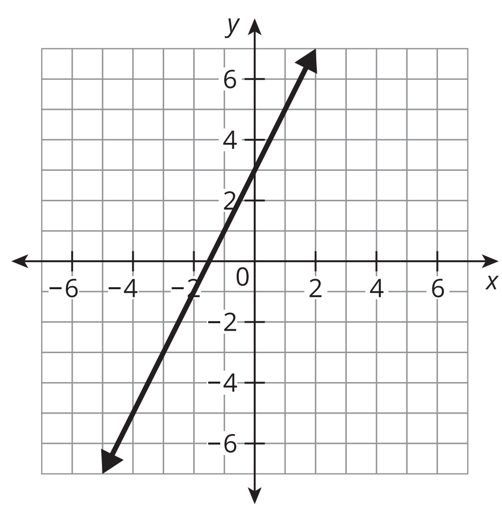
When a function is both translated and vertically dilated, the resulting function can be written in the form $a \cdot f(x) + d$, where d represents the vertical translation of $f(x)$ and a represents the vertical dilation of $f(x)$.

For example, the graph of $y = 2x + 3$ represents both a vertical translation of 3 units and vertical dilation by a factor of 2.

$y = x$



$y = 2x + 3$



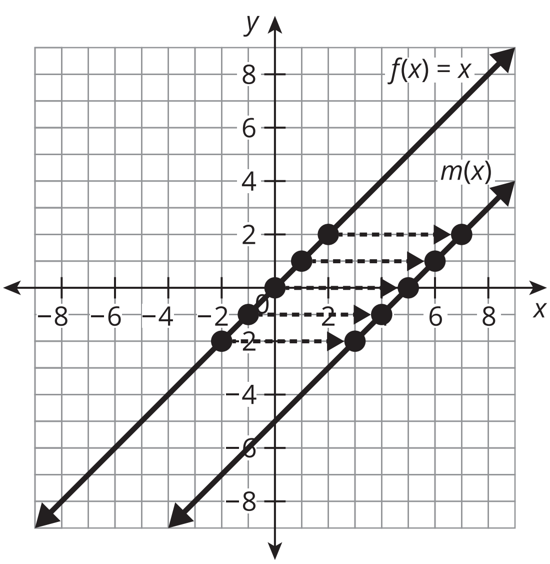
LESSON 2 Vertical and Horizontal Transformations of Linear Functions

For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values, or x-values, of the function. For $c > 0$, the resulting graph horizontally shifts right c units. For $c < 0$, the resulting graph horizontally shifts left c units.

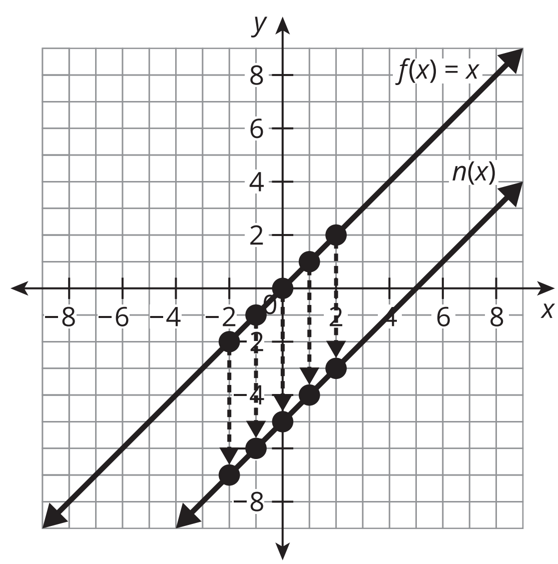
For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up d units. For $d < 0$, the resulting graph vertically shifts down d units.

You have determined that for the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units.

For example, you can translate the graph of $f(x) = x$ to the right 5 units by moving each point 5 units to the right. The transformed graph is labeled as $m(x)$. $m(x) = f(x - 5)$ which is $m(x) = x - 5$. You can translate the graph of $f(x) = x$ down 5 units by moving each point 5 units down. The transformed graph is labeled as $n(x)$. $n(x) = f(x) + (-5)$ which is $n(x) = x - 5$, thus producing the same equation and graph as $m(x)$.



Original Graph	
x	**f(x)**
−2	−2
−1	−1
0	0
1	1
2	2



Transformed Graph	
x	**m(x)**
3	−2
4	−1
5	0
6	1
7	2

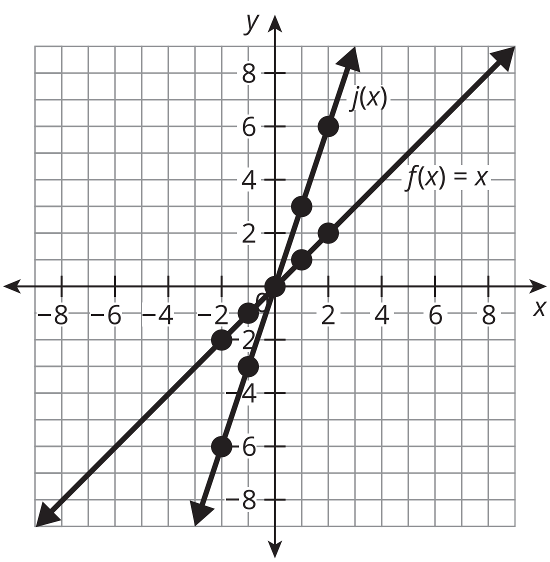
Transformed Graph	
x	**n(x)**
−2	−7
−1	−6
0	−5
1	−4
2	−3

For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph reflects across the x-axis.

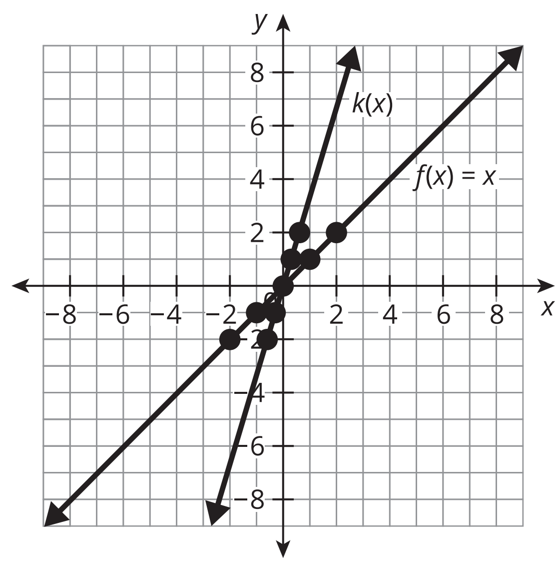
For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values, or x-values, of the function. For $|b| > 1$, the resulting graph horizontally compresses by a factor of $\frac{1}{|b|}$ units. For $0 < |b| < 1$, the resulting graph horizontally stretches by a factor of $\frac{1}{|b|}$ units. For $b < 0$, the resulting graph reflects across the y-axis.

You have determined that for the parent function $f(x) = x$, a vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

For example, you can vertically dilate the graph of $f(x) = x$ by a scale factor of 3. The transformed graph, $j(x)$, is a result of vertically stretching the graph of $f(x)$ by a factor of 3 units; $j(x) = 3f(x)$ which is $j(x) = 3x$. You can horizontally dilate the graph of $f(x) = x$ by a scale factor of $\frac{1}{3}$. The transformed graph, $k(x)$, is a result of horizontally compressing the graph $f(x)$ by a factor of $\frac{1}{|b|}$ units or 3 units in this case. $k(x) = f\left(\frac{1}{3}x\right)$,which is $k(x) = 3x$.



Original Graph	
x	**f(x)**
−2	−2
−1	−1
0	0
1	1
2	2



Transformed Graph	
x	**j(x)**
−2	−6
−1	−3
0	0
1	3
2	6

Transformed Graph	
x	**k(x)**
$-\frac{2}{3}$	−2
$-\frac{1}{3}$	−1
0	0
$\frac{1}{3}$	1
$\frac{2}{3}$	2

LESSON 3 Determining Slopes of Perpendicular Lines

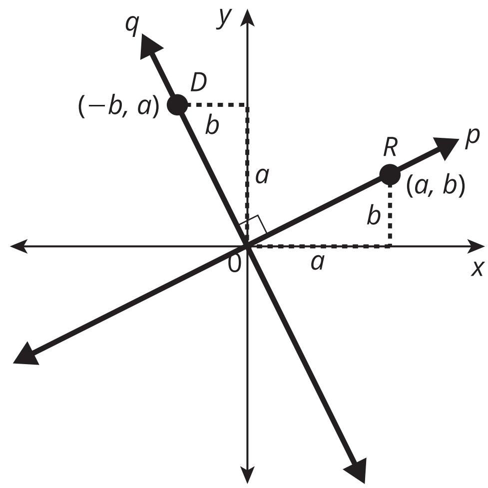
Perpendicular lines or line segments form a right angle at the point of intersection. You can think of perpendicular lines as a line and its rotation 90° about a point, which is also the point of intersection.

The product of the slopes of perpendicular lines is -1.

For example, consider the theorem.

Theorem: If two lines are perpendicular, their slopes are negative reciprocals.

Use the graph and the proof shown to analyze the validity of the theorem.



Given: $p \perp q$

Let m_1 = slope of line p and let m_2 = slope of line q.

Point R lies on line p.

Prove: $m_1 = -\frac{1}{m_2}$

Rotate point R 90° counterclockwise using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q under this 90° rotation.

Since this rotation maps the positive x-axis to the positive y-axis and the positive y-axis to the negative x-axis, then the coordinates of $R(a, b)$ are transformed into the coordinates of $D(-b, a)$. Graphically, you can see the movement of lengths a and b under the rotation.

The graph shows the slope of line p, $m_1 = \frac{b}{a}$, and the slope of line q, $m_2 = \frac{a}{-b}$.

Using these slopes, we can demonstrate that $m_1 = \frac{-1}{m_2}$.

$$\begin{aligned}\frac{b}{a} &= \frac{-1}{\frac{a}{-b}}\\ &= -1 \cdot \frac{-b}{a}\\ &= \frac{b}{a}\end{aligned}$$

The slope of line q is the negative reciprocal of the slope of line p.

The slope of any horizontal line is 0 since no matter the change for x, there is 0 change for y.

$$\frac{0}{x_2 - x_1} = 0$$

The slope of any vertical line is undefined since no matter the change for y, there is 0 change for x.

$\frac{y_2 - y_1}{0}$ is undefined since a value cannot be divided by zero.

All horizontal lines are parallel to each other since their slopes are equal and all vertical lines are parallel since their slopes are equal. A horizontal and a vertical line are always perpendicular to each other.

For example, to write an equation for a line that passes through the point $(-4, -2)$ and is perpendicular to the line $y = 3$, first determine that the line given by $y = 3$ is a horizontal line. Therefore, a line that is perpendicular to $y = 3$ is a vertical line. A vertical line that passes through the point $(-4, -2)$ has the equation $x = -4$.

You can use what you know about the slopes of perpendicular lines and slope-intercept form to write the equation of a perpendicular line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line perpendicular to $y = 4x - 1$ must have a slope of $-\frac{1}{4}$, since $4 \cdot -\frac{1}{4} = -1$.

Using the given point and slope-intercept form, you can set up an equation to solve for b, the y-intercept.

$$2 = -\frac{1}{4}(-4) + b$$

$$2 = 1 + b$$

$$b = 1$$

Therefore, the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$ is $y = -\frac{1}{4}x + 1$.

LESSON 4 Comparing Linear Functions in Different Forms

Functions can be represented using tables, equations, graphs, and with verbal descriptions. Features of linear functions, such as *y*-intercepts, slopes, and independent and dependent quantities, can be determined from different representations of functions.

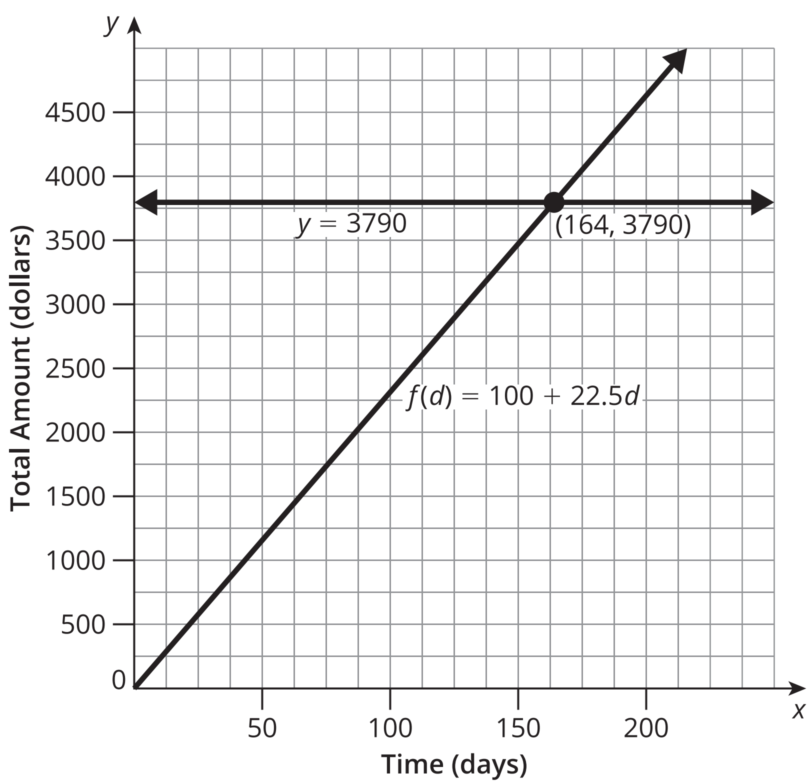
A table can help you calculate solutions given a few specific input values. A graph can help you determine exact solutions if the graph of the function crosses the grid lines exactly. A function can be solved for any value, so any and all solutions can be determined. Technology can allow for more accuracy when using a graph to determine a solution.

For example, suppose Omar had \$100 in his car fund. He earns \$7.50 per hour at his after-school job. He works 3 hours each day, including weekends. Omar puts all of his earned money in his car fund. How many days will it take him to have enough money to buy a car that costs \$3790?

A table can be used to estimate that it will take between 100 and 175 days to buy the car. A graph can be used to estimate that it will take about 160 days to buy the car. A function will give an exact solution. It will take exactly 164 days to buy a car that costs \$3790.

d	$100 + 22.50d$
0	100
10	325
20	550
50	1225
100	2350
175	4037.5

$$f(d) = 100 + 22.50d$$
$$3790 = 100 + 22.50d$$
$$3690 = 22.50d$$
$$\frac{3690}{22.50} = \frac{22.50d}{22.50}$$
$$164 = d$$



ISBN: 978-1-970197-81-5

Printed in the USA