MODULE

3

Modeling Linear Equations and Inequalities

ALGEBRA I

TOPIC 1

Solving linear equations can help determine exact points along a path while solving linear inequalities can help determine points within a region.

Linear Equations and Inequalities

1 Solving Linear Equations

OBJECTIVES

- Write equivalent equations using properties of equality.
- Use properties of equality to solve linear equations and justify a solution method.
- Determine whether an equation has one solution, no solution, or infinite solutions.
- Solve linear equations with variables on both sides.

NEW KEY TERMS

- solution
- no solution
- infinite solutions

You know that equations are one way to represent a linear function, and you have used equations to evaluate linear functions for a given input value.

How can you use equations to solve for unknown input values of a function?

Getting Started

Equation Creation

An *equation* is a mathematical sentence that uses an equals sign to show that two expressions are equivalent. When one of those expressions contains a variable, you can solve the equation.

A **solution** to an equation is a value for the variable that makes the equation a true statement.

Consider the equation $x = 2$. You can substitute the value 2 for x to create the true statement $2 = 2$. Because this is the only value that makes the statement true, 2 is the only solution to the equation $x = 2$.

By performing the same operation on each side of an equation, you can create more complex equations that have the same solution.

1. Consider the equation $x = 2$.

   a. Choose any constant. Add that constant to each side of the equation and simplify.

   b. Choose any constant other than 0. Multiply each side of the equation you created in part (a) by that constant and simplify.

   c. Choose any number other than 0 to represent a in the expression ax. Subtract the term ax from each side of the equation you created in part (b) and simplify.

   d. Choose any constant other than 0. Divide each side of the equation you created in part (c) by that constant and simplify.

Think about . . .
What strategies can you use to verify a solution?

2. Have a partner solve the equation you created in Question 1 to verify that $x = 2$ is the solution.

ACTIVITY 1.1 Using Properties to Justify Solutions

Recall that the properties of equality are rules that allow you to maintain balance and rewrite equations to isolate the variable.

Properties of Equality	For all numbers ***a***, ***b***, and ***c***
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Harper and Sofia both created new equations starting from the solution statement $x = 2$.

Harper

$-3x + 14 = 18 - 5x$

Sofia

$2x = 5 - \frac{1}{2}x$

1. Verify that both equations are equivalent to $x = 2$ using the given strategy.

   a. substitution

   b. properties of equality

There are also basic number properties that you are already familiar with that can be used to justify your steps when solving equations.

Number Properties	For all numbers *a*, *b*, and *c*
commutative property	$a + b = b + a$ $ab = ba$
associative property	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$
distributive property	$a(b + c) = ab + ac$

Justifying your steps with the properties of equality will better prepare you for writing geometrical proofs in future courses.

2. Solve each equation and check your solution. Write the properties that justify each step of your solving strategy.

   a. $24x + 7 = -4x$

   b. $0.6(2x + 1) = 4x + 6.2$

   c. $\frac{1}{2}x - 6 = 2 + (2x + 1)$

3. Compare the solution strategies used by Daniel and Chloe. What do you notice about the properties each student used?

Daniel

$$4(3x + 2) = 8x + 4$$
$$12x + 8 = 8x + 4$$
$$4x = -4$$
$$x = -1$$

Chloe

$$4(3x + 2) = 8x + 4$$
$$\frac{4(3x + 2)}{4} = \frac{8x + 4}{4}$$
$$3x + 2 = 2x + 1$$
$$x = -1$$

ACTIVITY 1.2 Solutions to Linear Equations

In the Getting Started, you built equations from $x = 2$. In this activity, you will build equations from two mathematical sentences and compare your results.

1. Consider the mathematical sentence $2 = 2$.
   Is there any variable or constant that you could add, subtract, multiply, or divide to both sides using the properties of equality to make this true sentence false? Choose variables and constants to create new mathematical sentences to justify your conclusion.

2. Consider the mathematical sentence $2 = 3$.
   Is there any variable or constant that you could add, subtract, multiply, or divide to both sides using the properties of equality to make this false sentence true? Choose variables and constants to create new mathematical sentences to justify your conclusion.

A linear equation can have *one solution*, *no solution*, or *infinite solutions*. The equations you solved in the previous activity are examples of linear equations with one solution. A linear equation with **no solution** means that there is no value for the variable that makes the equation true. A linear equation with **infinite solutions** means that any value for the variable makes the equation true.

3. Consider the equations you created in this activity.

   a. Explain whether the equation(s) you created in Question 1 have one solution, no solution, or infinite solutions.

   b. Explain whether the equation(s) you created in Question 2 have one solution, no solution, or infinite solutions.

4. Consider the equation $2x = 3x$. Does this equation have one solution, no solution, or infinite solutions? Explain your reasoning.

ACTIVITY
1.3

Tic-Tac-Bingo

In this activity, you are going to play a game called Tic-Tac-Bingo. The object of the game is to match two expressions to create an equation with specific solution types. Use the Tic-Tac-Bingo sheet located at the end of the lesson.

Prepare the board.

The board has 9 spaces. Three spaces are already designated. Fill each remaining space with one of the solution types listed. Each option must be used at least once.

Solution Types

- positive rational solution
- negative rational solution
- non-zero integer solution

Play the game.

Your teacher will assign you an expression. When you and a classmate have created an equation with one of the solution types, write your equation in the corresponding box.

Try to be the first person to fill three spaces in a row. Then, try to be the first person to completely fill your board with equations.

1. Reflect on the equations you created.

   a. How can you look at an equation and determine that it has no solution?

   b. How can you look at an equation and determine that it has infinite solutions?

Talk the Talk

One Step at a Time

1. Rectangle A has a width of $(5 + 2x)$ inches and a length of 6 inches. Rectangle B has a width of 8 inches and a length of $(x + 5)$ inches. Write and solve an equation to determine the length of x when both rectangles have the same area. Write the property that justifies each step to solve the equation.

Tic-Tac-Bingo Board

Equation: Solution:	**Solution is neither positive nor negative** Equation: Solution:	Equation: Solution:
No solution Equation: Solution:	**FREE SPACE**	Equation: Solution:
Equation: Solution:	Equation: Solution:	**Infinite solutions** Equation: Solution:

Why is this page blank?

So you can cut out the Bingo Board on the other side

Lesson 1 Assignment

Write

Explain how you know when an equation has no solution and when it has infinite solutions.

Remember

To solve an equation, use the properties of equality to isolate the variable. A linear equation can have one solution, no solution, or infinite solutions.

Practice

1. Solve each equation. Write the properties that justify each step in the solution method.

   a. $3x - 8 = -7x + 18$

   b. $-2(4 - x) = 12x - 3$

   c. $\frac{1}{2}(-10x + 4) = -4(-3 + 2x) + 8$

   d. $\frac{(-2x - 4)}{5} + \frac{8}{5} = 3(x - 1)$

Lesson 1 Assignment

e. $\frac{4}{3}x + 2\left(9 - \frac{1}{3}x\right) = -\frac{7}{3}x + 9$

2. Determine whether each equation has one solution, no solution, or infinite solutions. Explain your reasoning.

a. $-2(x + 5) = -6x + 4(x - 2)$

b. $4(0.2x - 1.2) = -0.5x + 4.3$

c. $\frac{\left(\frac{1}{2}x - 5\right)}{2} = -3x + 4$

d. $2(x - 4) + x = 3(x - 2) - 2$

e. $3 - \frac{2}{5}x - \frac{12}{5} = \frac{10 - 2x}{5}$

f. $6(x - 1) + 21 = 6x + 15$

Prepare

The formula for the circumference of a circle is $C = 2\pi r$. Determine the radius for each circle with the given circumference. Use 3.14 for π and round to the nearest tenth of a unit, if necessary.

1. C = 62.8 in.
2. C = 10 cm
3. C = 15.7 ft
4. $C = 48$ mm

2 Literal Equations

OBJECTIVES

- Rewrite linear equations in different forms.
- Analyze the structure of different forms of linear equations.
- Recognize and use literal equations.
- Rearrange literal equations to highlight quantities of interest.

NEW KEY TERM

- literal equation

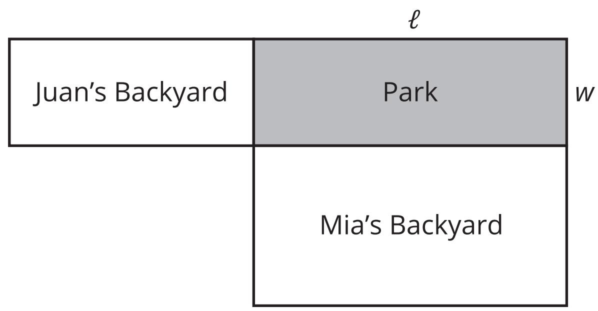
You have used different properties to solve linear equations to determine which value makes the equation true.

How can you use those same properties to solve for one specific variable in an equation that has multiple variables?

Getting Started

Perimeter Perspectives

Juan and Mia have rectangular backyards that each share a side with a neighborhood park. The sides shared with the park are adjacent to each other as shown.



Remember . . .
The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

Juan and Mia are each responsible for constructing their own fence to separate their yard from the park. The city gives them the measure of the perimeter of the park in feet. So that each knows how much fencing to buy, Juan needs to determine the width of the park, and Mia needs to determine the length of the park.

1. How can you write an equation for the perimeter of the park from each person's perspective to help them determine the information they need?

2. Show that the two equations are equivalent.

ACTIVITY 2.1 Equations in Different Forms

The slope, x-intercept, and y-intercept are each important characteristics of linear functions. The structure of equations can reveal these characteristics of a function.

Remember . . .

- You can calculate slope using $\frac{y_2 - y_1}{x_2 - x_1}$.
- You can determine the y-intercept by substituting 0 for x and solving for y.
- You can determine the x-intercept by substituting 0 for y and solving for x.

1. Consider the equation $y = -\frac{2}{3}x + 4$. Determine each characteristic and then explain your work.

   a. Slope

   b. y-intercept

   c. x-intercept

   d. How did you determine each characteristic?

2. Consider the equation $y = 4(x - 2)$. Determine each characteristic and then explain your work.

   a. Slope

   b. y-intercept

   c. x-intercept

   d. How did you determine each characteristic?

3. Consider the equation $3x - 2y = 8$. Determine each characteristic and then explain your work.

   a. Slope

   b. y-intercept

   c. x-intercept

   d. How did you determine each characteristic?

In slope-intercept and point-slope form, the values of m, x_1, and y_1 can be any real numbers. In standard form, however, there are constraints on the variables: A, B, and C must be integers, and A and B cannot both be 0.

Recall that there are three useful forms of linear equations.

Slope-Intercept Form	Point-Slope Form	Standard Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$

4. Identify the form for each equation in Questions 1 through 3.

Consider how the structure of the standard form of a linear function reveals its key characteristics.

5. For the equation $Ax + By = C$, determine the slope, x-intercept, and y-intercept.

6. Which form of a linear equation is more efficient for determining each characteristic?

   a. Slope

   b. a point on the line

   c. y-intercept

7. If you want to graph an equation using technology, which form is more efficient? Explain your reasoning.

8. Rewrite each equation in slope-intercept form.

   a. $y - 2 = 5(x - 7)$

   b. $8x - 2y = 12$

9. Rewrite each equation in standard form.

   a. $y + 7 = 0.5(x - 1)$

   b. $y = -4x + 3$

10. Write the equation for a line in standard form that passes through the given point with the identified slope.

    a. Slope $= \frac{1}{2}$
       Passes through $(-3, 5)$

    b. Slope $= -4$
       Passes through $(2, 1)$

ACTIVITY
2.2 Rewriting Literal Equations

Literal equations are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. These literal equations can be manipulated in order to allow you to solve for one specific variable.

You have used a common literal equation, the formula for converting degrees Fahrenheit to degrees Celsius.

$$C = \frac{5}{9}(F - 32)$$

Jayden is talking on the phone to his friend Mateo who lives in Europe. Mateo is used to describing temperatures in °C, while Jayden is used to describing temperatures in °F. Mateo can use the formula above to quickly convert the temperatures Jayden describes to °C.

1. How can you rewrite the formula so that Jayden can quickly convert the temperatures that Mateo describes to °F? Justify your solution method.

2. Jayden tells Mateo that the temperature where he lives is currently 77°F. Mateo tells Jayden that the temperature where he lives is currently 30°C. Jayden says it is warmer where he lives and Mateo says it is warmer where he lives. Who is correct? Explain your reasoning.

3. Isabella, Ethan, and Minh each convert the given formula to degrees Fahrenheit.

Isabella

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C}{5} + \frac{160}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 32 = F$$

a. Explain Isabella's reasoning.

Ethan

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - 32$$
$$9(C) = 9\left(\frac{5}{9}F - 32\right)$$
$$9C = 5F - 288$$
$$9C + 288 = 5F$$
$$\frac{9C}{5} + \frac{288}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 57.6 = F$$

b. Explain the error in Ethan's work.

Minh

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 45F - 1440$$
$$9C + 1440 = 45F$$
$$\frac{9C}{45} + \frac{1440}{45} = \frac{45F}{45}$$
$$\frac{1}{5}C + 32 = F$$

c. Explain the error in Minh's work.

WORKED EXAMPLE

You can use multiplication to rewrite an equation without fractions. Consider the final equation from Isabella's work.

$\frac{9}{5}C + 32 = F$

$5\left(\frac{9}{5}C + 32\right) = (5)F$	Multiply both sides of the equation by 5 to eliminate the fraction.
$9C + 160 = 5F$	Subtract $9C$ from both sides of the equation.
$160 = 5F - 9C$	

4. Use the strategy from the Worked Example to rewrite Isabella's original equation $C = \frac{5}{9}(F - 32)$ without fractions.

ACTIVITY
2.3

Solving Literal Equations for Specific Variables

Convert each literal equation to solve for the given variable.

1. Think Inside the Box is manufacturing new boxes for Pack-n-Ship. Pack-n-Ship told Think Inside the Box that the boxes must have a specific volume and area of the base. However, Pack-n-Ship did not specify a height for the boxes.

   a. Write a literal equation to calculate the volume of a box. Then, convert the volume formula to solve for height.

   b. Pack-n-Ship specified the volume of the box must be 450 cubic inches and the area of the base must be 75 square inches. Use your formula to determine the height of the new boxes.

Remember . . .
Volume is measured in cubic units since it's calculated using three dimensions. Height measures only one dimension.

2. The volume of an ice cream cone is the measure of how much ice cream fits inside the cone. An ice cream cone company wants to make an ice cream cone with a greater height that still holds the same amount of ice cream.

   a. Write an equation to calculate the volume of a cone. Then, convert the equation to solve for the height.

   b. Explain how your equation determines a linear measurement when the original equation determined a cubic measurement.

The formula for the volume of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2h$, where B is the area of the base of the cone, r is the radius and h is the height.

3. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of each base.

   a. Convert the area formula to solve for the height.

   b. Use your formula to determine the height of a trapezoid with an area of 24 square centimeters and base lengths of 9 cm and 7 cm.

4. For the given literal equation $Z = \frac{A}{B} + \frac{C}{D}$, solve for each variable given. Justify your solution method.

   a. A

   b. D

Talk the Talk

Rearrange It

1. The formula for the lateral surface area of a cylinder is $S = 2\pi rh$, where S is the lateral surface area, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.

2. The lateral surface area of a can of soup is 37.68 square inches, and the length of the radius of the base of the can is 1.5 inches. Use your formula to determine the height of the can of soup. Use 3.14 for π.

3. The formula for the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$, where S is the surface area, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.

4. The surface area of a can of soup is 51.81 square inches, and the length of the radius of the base is 1.5 inches. Use your formula to determine the height of the can of soup. Use 3.14 for π.

Lesson 2 Assignment

Write

Define the term *literal equation* in your own words.

Remember

Literal equations can be rewritten using properties of equality to allow you to solve for one specific variable.

Practice

1. Rewrite each equation in the given form.

   a. $y = \frac{3}{4}x - 8$ in standard form

   b. $y - 12 = -3(x - 1)$ in slope-intercept form

   c. $5x - 2y = -18$ in slope-intercept form

   d. $y + 4 = 0.5(x - 14)$ in standard form

Lesson 2 Assignment

2. Shoe boxes are often used in other capacities once the shopper has bought the shoes. Sometimes the boxes are used to hold other items, so it is helpful to know the volume of the box.

   a. Write the equation to solve for the volume of the shoe box.

   b. If the area of the base of the box is 112 square inches and the height is 3.5 inches, what is the volume of the box?

   c. Rewrite the equation to solve for height.

   d. A box has a volume of 456 cubic inches, with a length of 1 foot and a height of 4 inches. Determine the width of the box.

3. Solve each equation for the specified variable.

   a. $V = \frac{1}{3}Bh$ for B

   b. $I = prt$ for r

   c. $\frac{x + y}{3} = 6$ for y

   d. $A + B + C = 180$ for C

Prepare

Solve each statement for x.

1. $x + 1 = 6$
2. $x - 2 > 5$
3. $\frac{x}{3} \leq -1$
4. $-x > 4$

3 Modeling Linear Inequalities

OBJECTIVES

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

NEW KEY TERM

- linear inequality

You have used horizontal lines on a graph and properties of equality with equations to solve problems that can be modeled with linear equations.

Can you use these same methods to solve problems involving linear inequalities?

Getting Started

Fundraising Function

Ask yourself . . .
How will you represent the $25 credit in your function?

Elijah's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

1. Write a function, $f(b)$, to show Elijah's total sales as a function of the number of boxes of popcorn he sells, b.

2. Analyze the function you wrote.

   a. Identify the independent and dependent quantities and their units.

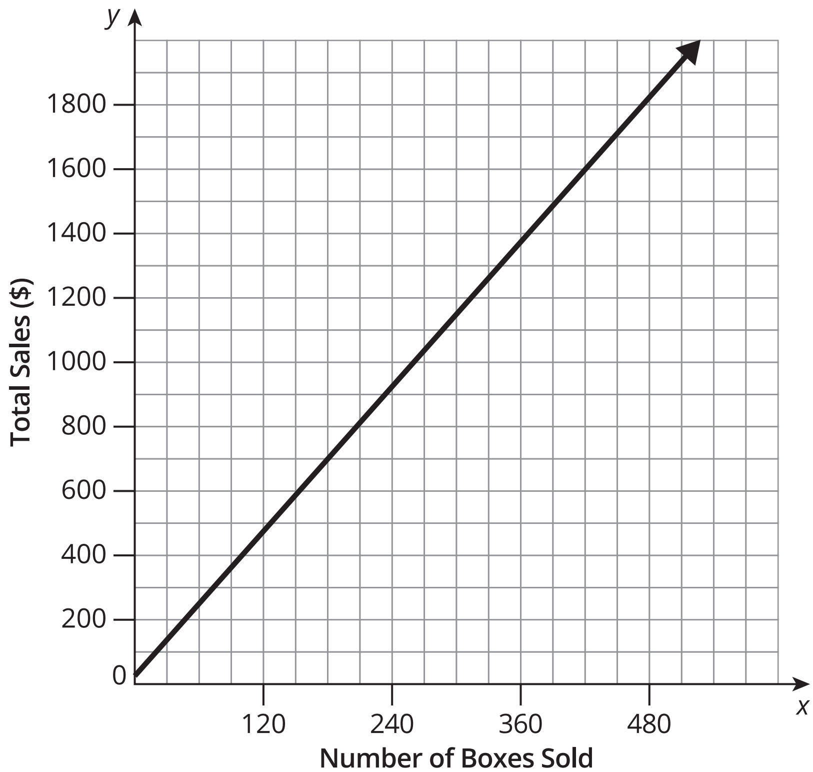
   b. What is the slope of the function? What does it represent in this problem situation?

   c. What is the y-intercept? What does it represent in this problem situation?

ACTIVITY 3.1

Modeling Linear Inequalities

The graph shown represents the function you wrote in the previous activity, $f(b) = 3.75b + 25$.

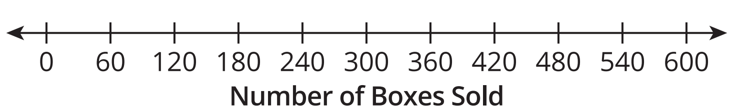


1. Suppose Elijah has a sales goal of $1600.

   a. Draw a horizontal line from the *y*-axis until it intersects with the graphed function to determine the point on the graph that represents $1600 in total sales.

   b. How many boxes would Elijah have to sell to make $1600 in total sales? Explain your reasoning.

   c. Use the number line to represent the number of boxes sold if the total sales is equal to $1600.



Ask yourself . . .
Will the points you graph on the number line be open or closed?

A **linear inequality** is a non-equal comparison of two linear expressions.

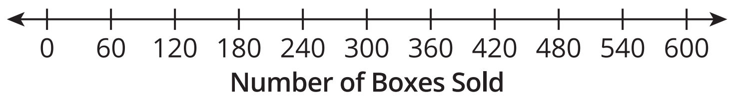
2. Analyze the region of the graph that lies below the horizontal line you drew, up to and including the point intersected by the line.

   a. What does this region of the graph represent?

   b. Write an inequality to represent this region.

Remember . . .
There are five inequality signs, less than ($<$), greater than ($>$), less than or equal ($\leq$), greater than or equal ($\geq$) and not equal ($\neq$). The solution set of an inequality is all the values that make the inequality statement true.

   c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



   d. Do all the solutions make sense in context of the problem? Explain your reasoning.

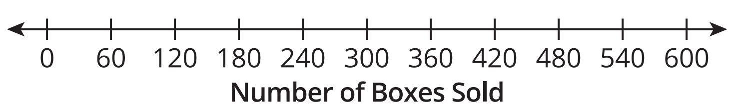
3. Analyze the region of the graph that lies above the horizontal line you drew, not including the point intersected by the line.

   a. What does this region of the graph represent?

   b. Write an inequality to represent this region.

Graphing the solutions to inequalities on number lines will prepare you for graphing linear inequalities in two variables.

   c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



   d. Do all the solutions make sense in context of the problem? Explain your reasoning.

4. Explain the difference between the open and closed circles on your number lines.

Ask Yourself . . .
How does determining the intersection point help you determine your answers?

5. Use the graph to answer each question. Write an equation or inequality statement for each.

   a. How many boxes would Elijah have to sell to earn at least $925?

   b. How many boxes would Elijah have to sell to earn less than $2050?

   c. How many boxes would Elijah have to sell to earn exactly $700?

ACTIVITY 3.2

Solving Two-Step Linear Inequalities

To solve an inequality means to determine the values of the variable that make the inequality true.

Another way to determine the solution set of any inequality is to solve it algebraically. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

To make the first deposit on the trip, Elijah's total sales, $f(b)$, need to be at least \$1100.

WORKED EXAMPLE

You can set up an inequality and solve it to determine the number of boxes Elijah needs to sell.

$$f(b) \geq 1100$$
$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$
$$3.75b + 25 - 25 \geq 1100 - 25$$
$$3.75b \geq 1075$$
$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$
$$b \geq 286.\overline{6}$$

Think about . . .
How accurate does your answer need to be?

1. How many boxes of popcorn does Elijah need to sell to make the first deposit? Explain your reasoning.

2. Write and solve an inequality for each. Show your work.

   a. What is the greatest number of boxes Elijah could sell and still not have made $600 in sales?

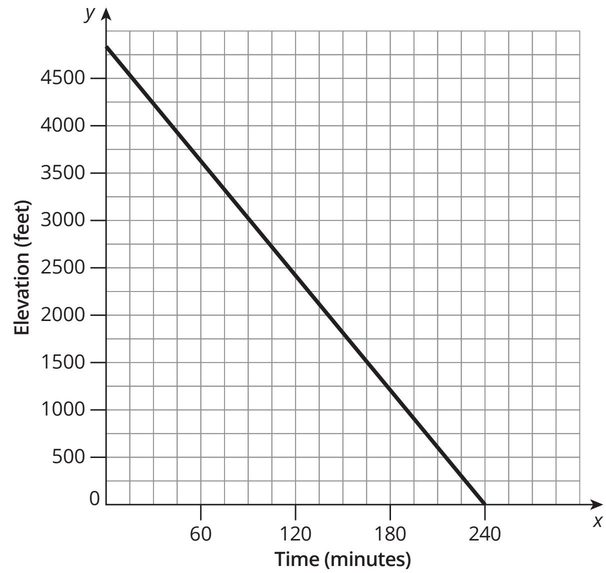
   b. At least how many boxes would Elijah have to sell to make $1500 in sales?

3. The Worked Example showed how to solve an inequality of the form, $y = mx + b$. How does your method change if the inequality is in a different form?

ACTIVITY 3.3 Reversing Inequality Signs

Sea level has an elevation of 0 feet.

Elijah's camping troop hikes down from their campsite at an elevation of 4800 feet to the base of the mountain, which is at sea level. They hike down at a rate of 20 feet per minute.



1. Write a function, $h(m)$, to show the troop's elevation as a function of time in minutes. Label the function on the coordinate plane.

Ask yourself . . .
What are the independent and dependent quantities and their units?

2. Analyze the function. Identify each characteristic and explain what it means in terms of this problem situation.

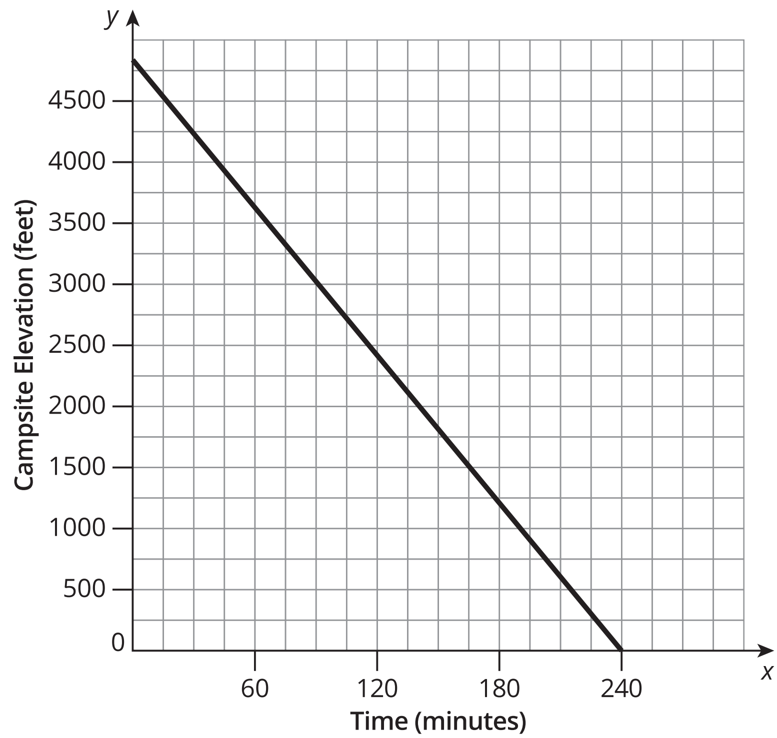
   a. Slope

   b. y-intercept

   c. x-intercept

   d. Domain of the function

3. Use the graph to determine how many minutes passed if the troop is below 3200 feet. Draw an oval on the graph to represent this part of the function and write the corresponding inequality statement.



4. Write and solve an inequality to verify the solution set you interpreted from the graph.

5. Compare and contrast the solution sets you wrote using the graph and the function. What do you notice?

6. Analyze the relationship between the inequality statements representing $h(m)$ and m.

   a. Complete the table by writing the corresponding inequality statement that represents the number of minutes for each height.

   b. Compare each row in the table shown. What do you notice about the inequality signs?

$h(m)$	m
$h(m) > 3200$	
$h(m) \geq 3200$	
$h(m) = 3200$	
$h(m) < 3200$	
$h(m) \leq 3200$	

   c. Explain your answer from part (b). Use what you know about solving inequalities when you have to multiply or divide by a negative number.

ACTIVITY 3.4

Solving Other Linear Inequalities

The inequalities that you have solved in this lesson so far have all been two-step inequalities. Let's consider inequalities in different forms.

Victoria and Kai each solved the inequality $5x + 2 \geq 3x - 10$.

You can rewrite Kai's solution to show it is equivalent to Victoria's. It is sometimes preferred to have the variable on the left side of the inequality. So, $-6 \leq x$ is equivalent to $x \geq -6$.

Victoria

$$5x + 2 \geq 3x - 10$$
$$5x - 3x + 2 \geq 3x - 3x - 10$$
$$2x + 2 \geq -10$$
$$2x + 2 - 2 \geq -10 - 2$$
$$2x \geq -12$$
$$\frac{2x}{2} \geq \frac{-12}{2}$$
$$x \geq -6$$

Kai

$$5x + 2 \geq 3x - 10$$
$$5x - 5x + 2 \geq 3x - 5x - 10$$
$$2 \geq -2x - 10$$
$$2 + 10 \geq -2x - 10 + 10$$
$$12 \geq -2x$$
$$\frac{12}{-2} \leq \frac{-2x}{-2}$$
$$-6 \leq x$$

Remember. . .
When multiplying or dividing by a negative number while solving inequalities, the inequality sign reverses.

1. Describe the process each student used to solve the inequality.
   a. Victoria
   b. Kai

2. How does the process of solving an inequality with variables on both sides compare to the process of solving an equation with variables on both sides?

Aaliyah and Ava each solved the inequality $-4(x - 6) < 22$.

Aaliyah

$$-4(x - 6) < 22$$
$$-4x + 24 < 22$$
$$-4x + 24 - 24 < 22 - 24$$
$$-4x < -2$$
$$\frac{-4x}{-4} > \frac{-2}{-4}$$
$$x > \frac{1}{2}$$

Ava

$$-4(x - 6) < 22$$
$$\frac{-4(x - 6)}{-4} > \frac{22}{-4}$$
$$x - 6 > -5\frac{1}{2}$$
$$x > \frac{1}{2}$$

3. Describe the process each student used to solve the inequality.

   a. Aaliyah

   b. Ava

4. How does the process of solving an inequality using the distributive property compare to the process of solving an equation using the distributive property?

Talk the Talk

It's About Solutions, More or Less

1. Solve each inequality and graph the solution on the number line.

   a. $-\frac{2}{3}x \geq 7$

   b. $32 > 23 - x$

   c. $15 - 4x \leq 3x - 6$

   d. $2(x + 6) < 14 + 4x$

2. If $A < B$, identify the constraints to make each statement true.

   a. $A + C < B + C$

   b. $AC > BC$

   c. $-A < -B$

Lesson 3 Assignment

Write

Describe how to solve an inequality in your own words.

Remember

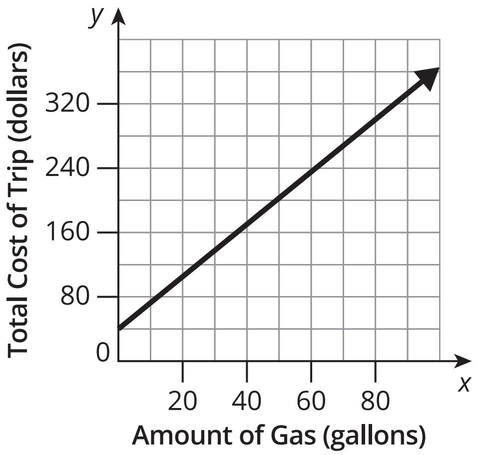
The methods for solving linear inequalities are similar to the methods for solving linear equations. Be sure to reverse the direction of the inequality symbol when multiplying or dividing both sides by a negative number.

Practice

1. Noah is going on a trip to visit some friends from summer camp. He will use \$40 for food and entertainment. He will also need money to cover the cost of gas. The price of gas at the time of his trip is \$3.25 per gallon.
   a. Write a function to represent the total cost of the trip as a function of the number of gallons used.

   b. Identify the rate of change and the y-intercept. Explain their meanings in terms of the problem situation.

   c. Graph the function representing this situation on a coordinate plane.



   d. Use the graph to determine how many gallons of gas Noah can buy if he has \$170 saved for the trip. Draw an oval on the graph to represent the solution. Then, write your answer in words and as an inequality.

Lesson 3 Assignment

e. Verify the solution set you interpreted from the graph.

f. Noah's mom gives him some money for his trip. He now has a total of $220 saved for the trip. What is the greatest number of gallons of gas he can buy before he runs out of money? Show your work and graph your solution on a number line.

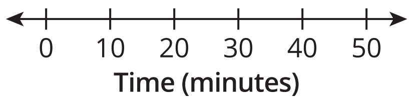
g. If Noah spent more than $92 on his trip, how much gas could he have bought? Show your work and graph your solution on a number line.

2. Noah is on his way to visit his friends at camp. Halfway to his destination, he realizes there is a slow leak in one of the tires. He checks the pressure and it is at 26 psi. It appears to be losing 0.1 psi per minute.

   a. Write a function to represent the tire's pressure as a function of time in minutes.

Lesson 3 Assignment

b. Noah knows that if the pressure in a tire goes below 22 psi, it may cause a tire blowout. What is the greatest amount of time that he can drive before the tire pressure hits 22 psi? Show your work and graph the solution.



3. Solve each inequality for the unknown value.

a. $13 + 4x > 9$

b. $3(4 - 5x) > 8x - 149$

c. $99 - 5d \geq 4d$

d. $3k - 9 \leq -6k - 225$

Prepare

Determine an ordered pair that represents a solution to each equation.

1. $4x + 7y = 24$

2. $5x - 2y = -6$

3. $\frac{1}{2}x + \frac{3}{4}y = 10$

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Linear Equations and Inequalities* topic by:

TOPIC 1: *Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
solving linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.	○	○	○
constructing an argument to justify a solution process for a one-variable equation.	○	○	○
using and interpreting units when solving literal equations for a given variable.	○	○	○
solving literal equations for a specified variable.	○	○	○
recognizing linear relationships from problem situations and writing equations or inequalities to model linear relationships.	○	○	○
solving linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.	○	○	○
interpreting solutions in the context of a problem situations and determining whether they are reasonable.	○	○	○
representing constraints to a problem situation as an inequality.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Linear Equations and Inequalities* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Linear Equations and Inequalities Summary

NEW KEY TERMS

- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

LESSON 1 Solving Linear Equations

A **solution** to an equation is any variable value that makes the equation true. Solving equations requires the use of number properties and the properties of equality. The properties of equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. When the properties of equality are applied to an equation, the transformed equation will have the same solution as the original equation.

A linear equation in the form $y = mx + b$ can have *one solution*, *infinite solutions*, or *no solution*.

Linear equations with **infinite solutions** are created by equating two equivalent expressions.

$$\begin{aligned} 2x + 6 &= 2(x + 3) \\ &= (2 \cdot x) + (2 \cdot 3) \\ &= 2x + 6 \end{aligned}$$

Linear equations with **no solution** are created by equating expressions of the form $mx + b$, with the same value for m and different values for b.

$$\begin{aligned} 4x + 2 &= 4x + 5 \\ 4x - 4x + 2 &= 4x - 4x + 5 \\ 2 &\neq 5 \end{aligned}$$

Linear equations with the solution $x = 0$ are created by equating expressions of the form $mx + b$, with different values for m and the same value for b.

$$\begin{aligned} 2x + 3 &= 5x + 3 \\ 2x - 2x + 3 &= 5x - 2x + 3 \\ 3 &= 3x + 3 \\ 3 - 3 &= 3x + 3 - 3 \\ 0 &= 3x \\ 0 &= x \end{aligned}$$

LESSON 2 Literal Equations

There are three useful forms of linear equations.

Slope-Intercept	Point-Slope Form	Standard Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$

You can use any real numbers for the values of *m*, *b*, x_1, *and* y_1 in slope-intercept and point-slope from. In standard form, however, there are constraints on the variables: *A*, *B*, and *C* must be integers, and *A* and *B* cannot both be 0.

It is often necessary to change between forms, as the structure of each form reveals different key characteristics of the equation, such as the slope and *x*- and *y*-intercepts. Remember, to convert to slope-intercept form from standard form, solve for *y*. To convert to standard form from slope-intercept form, isolate both variables on the same side of the equation and the constant on the other side of the equation.

A **literal equation** is an equation in which the variables represent specific measures. Literal equations are most often seen when studying formulas. These literal equations can be manipulated in order to solve for one specific variable.

For example, a common literal equation is the formula for converting degrees Fahrenheit to degrees Celsius, $C = \frac{5}{9}(F - 32)$. You can use the properties of equality to rewrite the equation to convert degrees Celsius to degrees Fahrenheit, $F = \frac{9}{5}(C + 32)$.

LESSON 3 Modeling Linear Inequalities

To solve a **linear inequality**, first write a function to represent the problem situation. Then, write the function as an inequality based on the independent quantity. To determine the solution, identify the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: isolate the variable on one side of the inequality symbol. Finally, interpret the meaning of the solution.

For example, consider the situation in which Diego has \$25 in his gift fund that he is going to use to buy graduation gifts. Graduation is 9 weeks away. He would like to have at least \$70 to buy gifts, how much should he save each week?

The function is $f(x) = 25 + 9x$, so the inequality is $25 + 9x \geq 70$.

$$\begin{aligned} 25 + 9x &\geq 70 \\ 25 + 9x - 25 &\geq 70 - 25 \\ 9x &\geq 45 \\ \frac{9x}{9} &\geq \frac{45}{9} \\ x &\geq 5 \end{aligned}$$

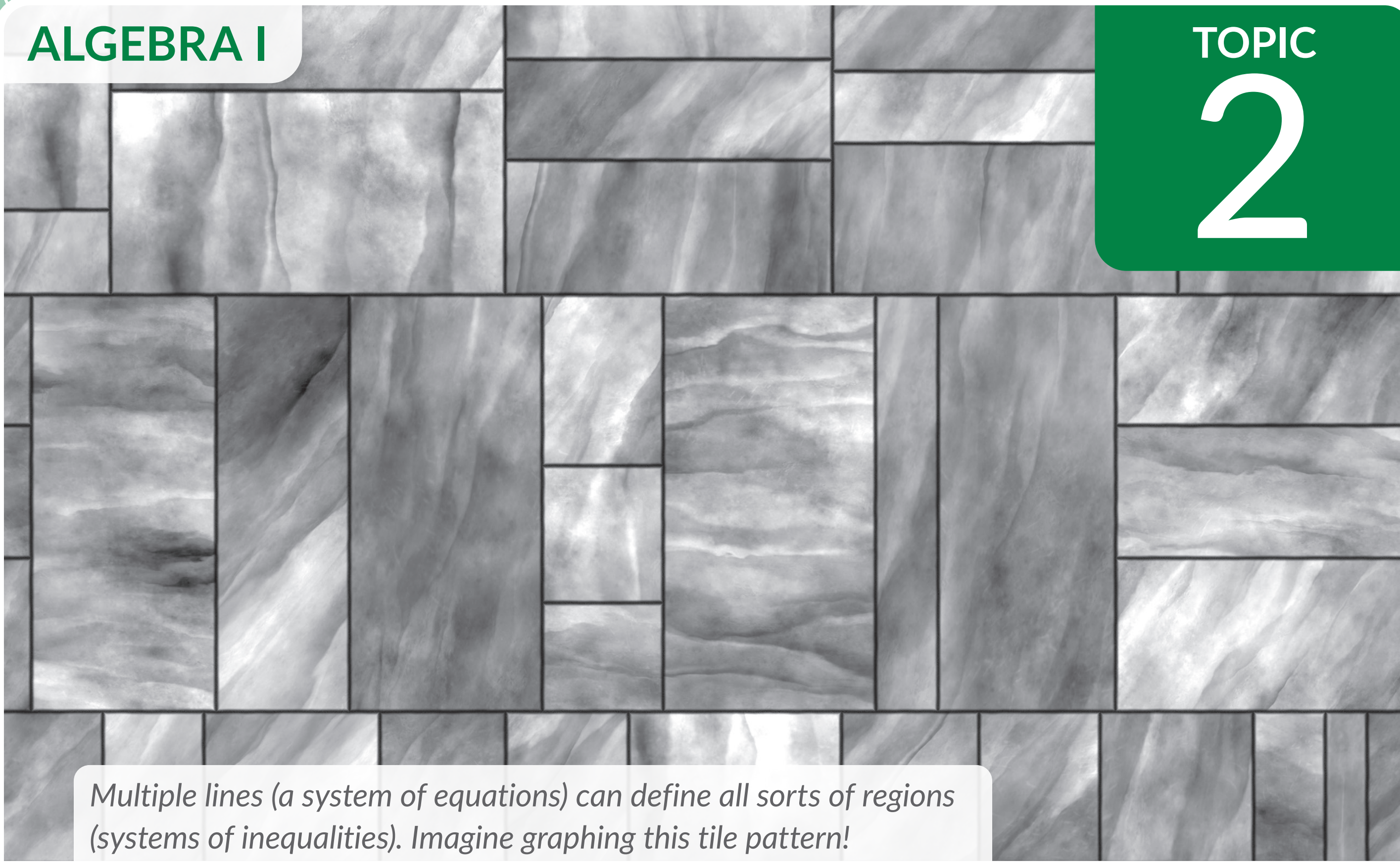
Diego needs to save at least \$5 each week to meet his goal.

When you divide each side of an inequality by a negative number, the inequality sign reverses. For example, consider the inequality $250 - 9.25x < 398$.

$$\begin{aligned} 250 - 9.25x &< 398 \\ 250 - 9.25x - 250 &< 398 - 250 \\ -9.25x &< 148 \\ \frac{-9.25x}{-9.25} &< \frac{148}{-9.25} \\ x &> -16 \end{aligned}$$

ALGEBRA I

TOPIC 2



Multiple lines (a system of equations) can define all sorts of regions (systems of inequalities). Imagine graphing this tile pattern!

Systems of Linear Equations and Inequalities

1

Using Graphing to Solve Systems of Equations

OBJECTIVES

- Write equations in standard form.
- Determine the intercepts of an equation in standard form.
- Use intercepts to graph an equation.
- Write a system of equations to represent a problem context.
- Use a graph to solve a system of linear equations.
- Interpret the solution to a system of equations in terms of a problem situation.
- Use the slope and the *y*-intercept to determine whether the system of two linear equations has one solution, no solution, or infinite solutions.

NEW KEY TERMS

- system of linear equations
- consistent systems
- inconsistent systems

You have examined different linear functions and solved for unknown values.

How can you solve problems that require two linear functions? How many solutions exist when you consider two functions at the same time?

Getting Started

Ticket Tabulation

A high school sells tickets for its basketball games. Students pay $5, and adults pay $10 for a ticket. The high school needs to raise $3000 selling tickets to send the team to an invitational tournament.

1. Write an equation to represent this situation.

2. What combination of student and adult ticket sales would achieve the high school's goal?

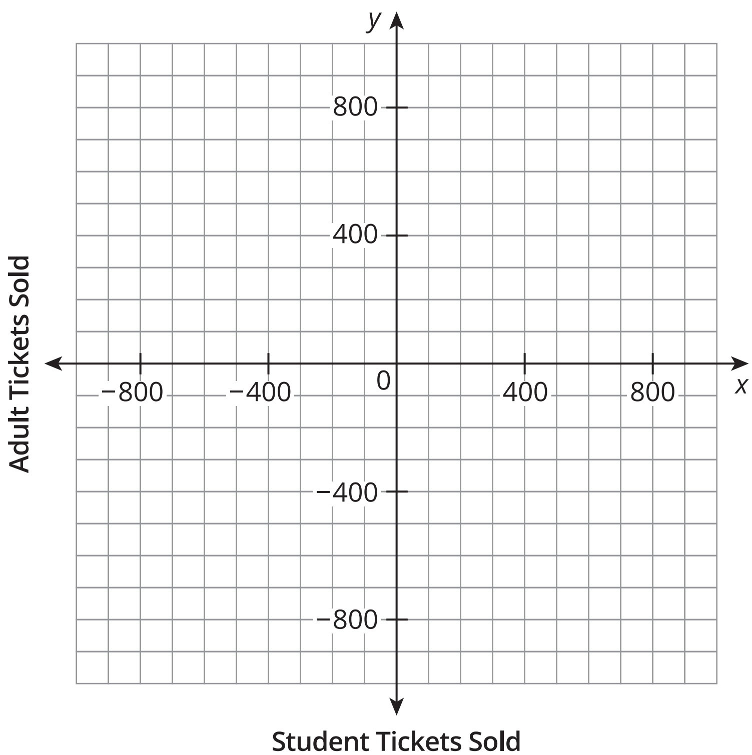
3. Compare your combination of ticket sales with your classmates'. Did you all get the same answer? Explain why or why not.

ACTIVITY 1.1 Analyzing the Graph of an Equation in Standard Form

Consider the goal of the athletic association described in the previous activity. Let s represent the number of student tickets sold, and let a represent the number of adult tickets sold. Written in standard form, the equation that represents the situation is $5s + 10a = 3000$.

One efficient way to graph a linear function in standard form is to use x- and y-intercepts. You can calculate the x-intercept by substituting $y = 0$ and solving for x. You can calculate the y-intercept by substituting $x = 0$ and solving for y.

1. Use the x- and y-intercepts to graph the equation.



Ask Yourself . . .
Which quantity is represented on each axis?

2. Determine the domain and range of each.

   a. The mathematical function

   b. The function modeling the real-world situation

3. Explain what each intercept means in terms of the problem situation.

4. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

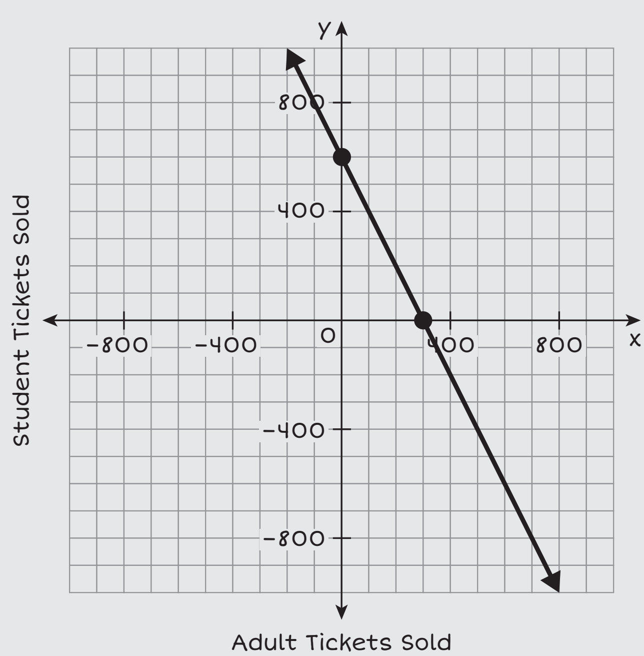
Ask Yourself . . .
What does each point on the graph of an equation represent?

5. How can you use the graph to determine a combination of ticket sales to meet the goal of $3000?

6. Ethan graphed the equation $5s + 10a = 3000$ in a different way. Explain why Ethan's graph is correct.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

Ethan



7. Use Ethan's graph to describe the domain and range of each.

   a. The mathematical function

   b. The function modeling the real-world situation

8. Explain what each intercept means in terms of the problem situation.

9. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

10. Compare the domain and range of the two functions that model the real-world situation. What do you notice?

11. Compare the *x*-intercepts and the *y*-intercepts of the two graphs. What do you notice?

12. Is there a way to determine the total amount of money collected from either graph? Explain why or why not.

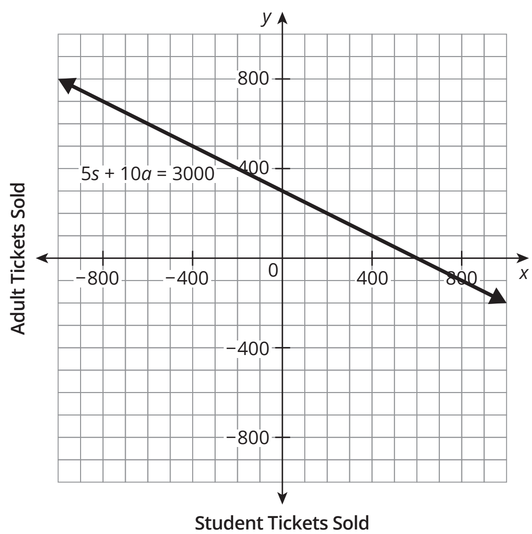
ACTIVITY 1.2 Determining the Solution to a System of Linear Equations

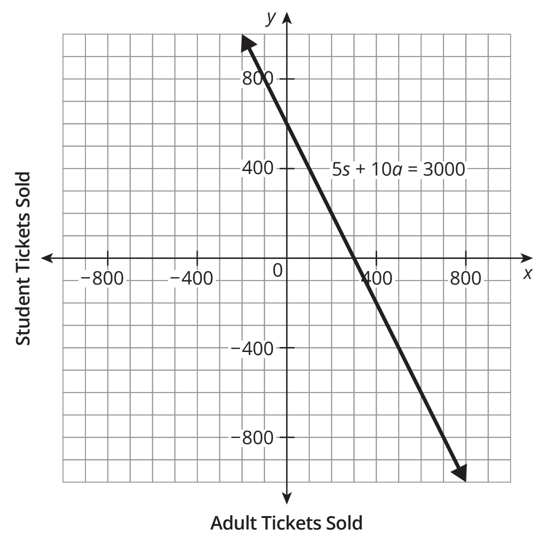
When two or more linear equations define a relationship between quantities, they form a **system of linear equations**.

The athletic director of the high school says that 450 total tickets were sold to the home game.

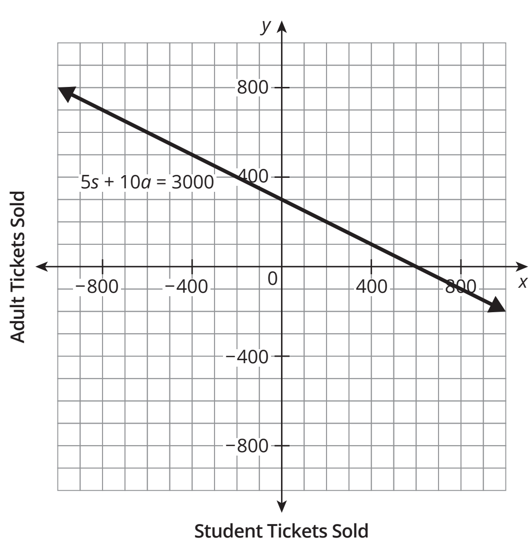
1. Write an equation that represents this situation. Let s represent the number of student tickets sold, and let a represent the number of adult tickets sold.

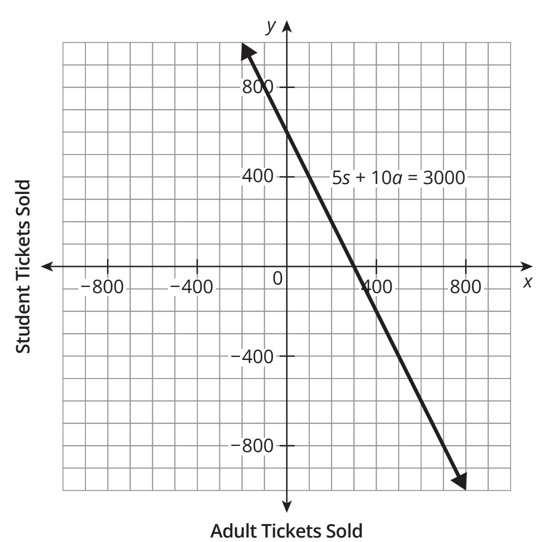
The coordinate planes shown already contain the function that models the earnings from ticket sales.





2. Use x- and y-intercepts to graph the function modeling the total number of tickets sold on each coordinate plane.





3. When the high school reached its goal of $3000 in ticket sales, how many of each type of ticket was sold? Is there more than one solution?

Remember . . .
According to the situation, 450 tickets were sold to the game.

4. Use technology to locate the exact point of intersection. Explain the process you used.

5. Algebraically justify that your solution is correct.

ACTIVITY
1.3

Systems with No Solution, One Solution, or an Infinite Number of Solutions

Samuel and Diego are in the Robotics Club. They are both saving money to buy materials to build a new robot.

Samuel opens a new bank account. He deposits $25 that he won at a robotics competition. He also plans on depositing $10 a week that he earns from tutoring. Diego decides he wants to save money as well. He already has $40 saved from mowing lawns over the summer. He plans to also save $10 a week from his allowance.

1. Write equations to represent the amount of money Samuel saves and the amount of money Diego saves.

2. Use your equations to predict when Samuel and Diego will have the same amount of money saved.

You can prove your prediction by solving and graphing a system of linear equations.

3. Analyze the equations in your system.

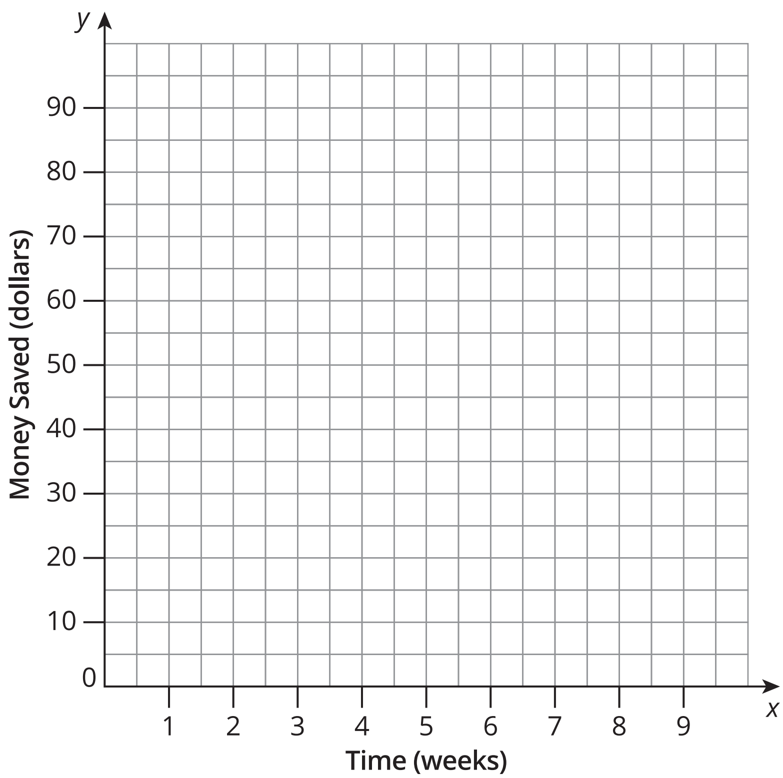
   a. How do the slopes compare? Describe what this means in terms of this problem situation.

   b. How do the y-intercepts compare? Describe what this means in terms of this problem situation.

4. Determine the solution of the system of linear equations graphically.

   a. Predict what the graph of this system will look like. Explain your reasoning.

   b. Graph both equations on the coordinate plane.



5. Analyze the graph you created.

   a. Describe the relationship between the graphs.

   b. Does this linear system have a solution? Explain your reasoning.

6. Was your prediction in Question 2 correct? Explain how you algebraically and graphically proved your prediction.

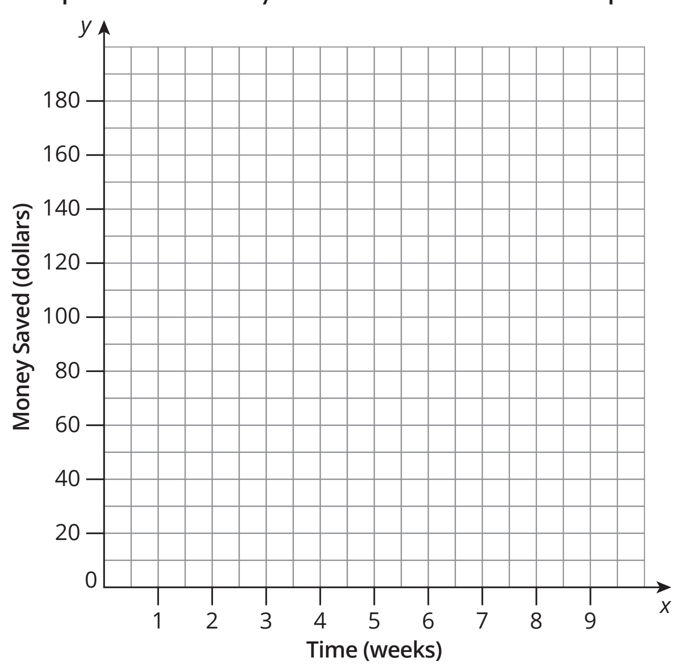
Isabella is also in the Robotics Club and has heard about Samuel's and Diego's savings plans. She wants to be able to buy her new materials before Diego, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week, she also deposits $4 from her tips.

Remember . . .
Don't forget to define your variables!

7. Use equations and graphs to determine when Isabella and Diego have saved the same amount of money.

   a. Write a linear system to represent the total amount of money Isabella and Diego have saved after a certain amount of time.

   b. Graph the linear system on the coordinate plane.



   c. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

Diego and Isabella went on a shopping spree this weekend and spent all their savings except for $40 each. Diego is still saving $10 a week from his allowance. Isabella now deposits her tips twice a week. On Tuesdays, she deposits $4, and on Saturdays, she deposits $6.

8. Diego claims he is still saving more each week than Isabella.

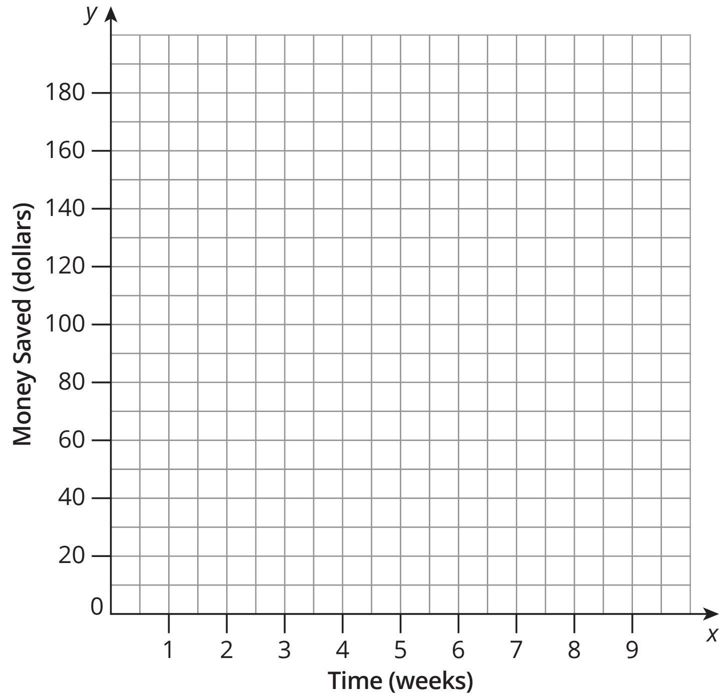
   a. Do you think Diego's claim is true? Explain your reasoning.

   b. How can you prove your prediction?

9. Prove your prediction graphically.

   a. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

b. Graph the linear system on the coordinate plane.



10. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

11. Was Diego's claim that he is still saving more than Isabella a true statement? Explain why or why not.

Talk the Talk

Beating the System

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one solution or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

1. Complete the table.

System of Two Linear Equations	Consistent Systems		Inconsistent Systems
Description of *y*-Intercepts	Same or different *y*-intercepts	Same *y*-intercepts	Different *y*-intercepts
Number of Solutions			
Description of Graph			

2. Explain why the *x*- and *y*-coordinates of the points where the graphs of a system intersect are solutions.

Ask Yourself . . .
What patterns do you notice?

Lesson 1 Assignment

Write

Define each term in your own words.

1. Consistent systems
2. Inconsistent systems

Remember

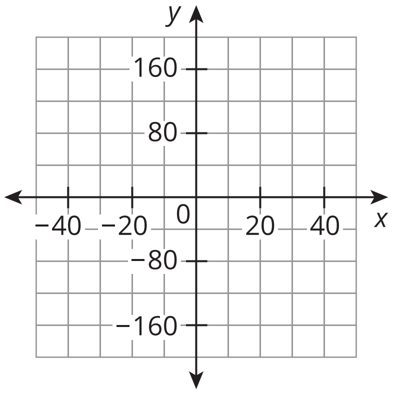
When two or more equations define a relationship between quantities, they form a system of linear equations. The point of intersection of a graphed system of linear equations is the solution to both equations. A system of linear equations can have one solution, no solution, or infinite solutions.

Practice

1. Mr. Nguyen gives his class 50-question multiple choice tests. Each correct answer is worth 2 points, while a half point is deducted for each incorrect answer. If the student does not answer a question, that question does not get any points.

   a. A student needs to earn 80 points on the test in order to keep an A grade for the semester. Write an equation in standard form that represents the situation. Identify three combinations of correct and incorrect answers that satisfy the equation.

   b. Determine the *x*- and *y*-intercepts of the equation and use them to graph the equation. Explain what each intercept means in terms of the problem situation.



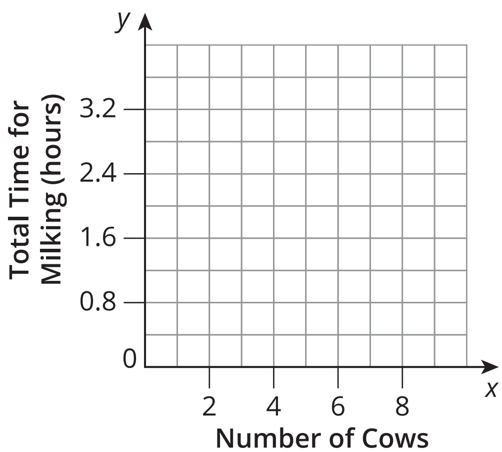
Lesson 1 Assignment

2. Harper owns a dairy farm. In the morning, it takes her 0.3 hour to set up for milking the cows. Once she has set up, it takes Harper 0.2 hour to milk each cow by hand. She is contemplating purchasing a milking machine in hopes that it will speed up the milking process. The milking machine she is considering will take 0.4 hour to set up each morning and takes 0.05 hour to milk each cow.

   a. Write a system of linear equations that represents the total amount of time Harper will spend milking the cows using the two different methods.

   b. Graph both equations on a coordinate plane.

   c. Estimate the point of intersection. Explain how you determined your answer.



   d. What does the point of intersection represent in this problem situation?

   e. Does the solution make sense in terms of this problem situation? Explain your reasoning.

   f. Is this system of equations consistent or inconsistent? Explain your reasoning.

Lesson 1 Assignment

3. Identify whether each system is consistent or inconsistent. Explain your reasoning.

   a. $\begin{cases} -3x = 4y = 3 \\ -12x + 16y = 8 \end{cases}$

   b. $\begin{cases} 7x + 3y = 0 \\ -14x + 6y = 0 \end{cases}$

   c. $\begin{cases} 6x + y = 1 \\ -6x - 4y = -4 \end{cases}$

Prepare

Analyze each system of equations. What can you conclude about the value of y in each?

1. $\begin{cases} x = 12 \\ y = x + 22 \end{cases}$

2. $\begin{cases} x = 0 \\ y = x - 45 \end{cases}$

3. $\begin{cases} x = y \\ y = 2x - 10 \end{cases}$

4. $\begin{cases} x = y + 3 \\ y = 2x - 10 \end{cases}$

2 Using Substitution to Solve Linear Systems

OBJECTIVES

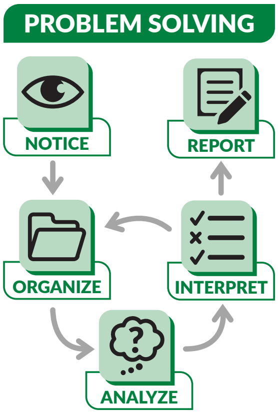
- Write a system of equations to represent a problem context.
- Algebraically solve a system of equations using substitution.
- Interpret a solution to a system of linear equations in terms of the problem situation.
- Solve real-world and mathematical problems with two linear equations in two variables.

NEW KEY TERM

- substitution method

You have graphed systems of equations.

Suppose you graph a system of equations, but the point of intersection is not clear from the graph. How can you determine the solution to the system?



Getting Started

Goats, Chickens, and Pigs

At the county fair, farmers bring some of their animals to trade with other farmers. To make all trades fair, a master of trade oversees all trades. Assume all chickens are of equal value, all goats are of equal value, and all pigs are of equal value.

- In the first trade of the day, 4 goats were traded for 5 chickens.
- In the second trade, 1 pig was traded for 2 chickens and 1 goat.
- In the third trade, Farmer Lyndi put up 3 chickens and 1 pig against Farmer Simpson's 4 goats.

1. Is this a fair trade? If not, whose animals are worth more? How could this be made into a fair trade?

ACTIVITY 2.1 Introduction to Substitution

Sofia is helping her mother make potato salad for the county fair and is asked to go to the market to buy fresh potatoes and onions. Sweet onions cost \$1.25 per pound, and potatoes cost \$1.05 per pound. Her mother told her to use the \$30 she gave her to buy these two items.

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

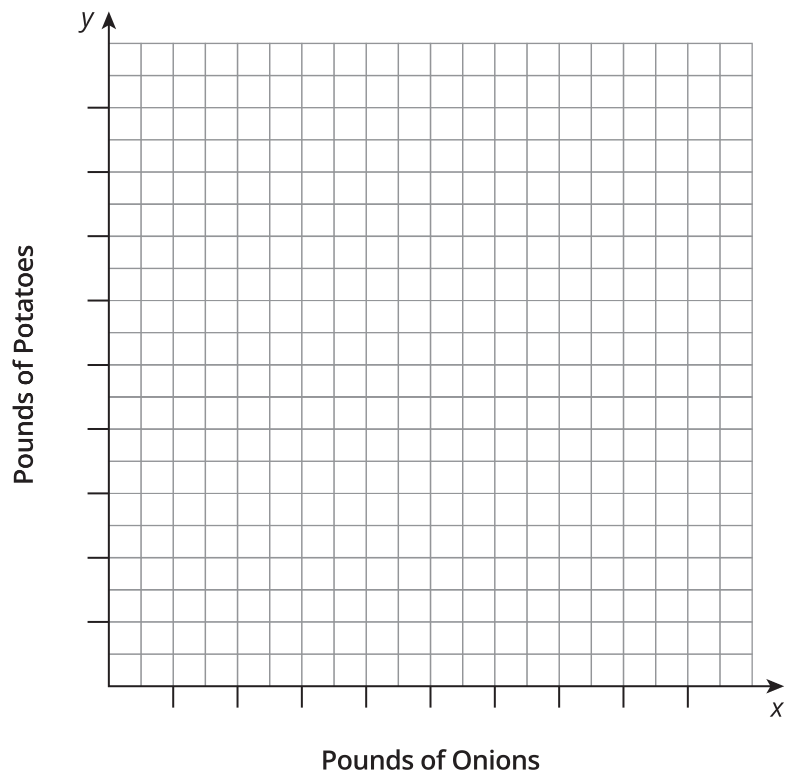
1. Write an equation that relates the number of pounds of potatoes and the number of pounds of onions that Sofia can buy for \$30. Use x to represent the number of pounds of onions and y to represent the number of pounds of potatoes that Sofia can buy. Then, rewrite your equation in standard form.

2. Sofia's mother told her that the number of pounds of potatoes should be 8 times greater than the number of pounds of onions in the salad. Write an equation in x and y that represents this situation.

3. Will 1 pound of onions and 8 pounds of potatoes satisfy both equations? Explain your reasoning.

4. Create graphs of both equations. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



5. Can you determine the exact solution of this linear system from your graph? Explain your reasoning.

6. Estimate the point of intersection from your graph.

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

WORKED EXAMPLE

Let's consider the system you wrote.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$.

Write the new equation.

$$1.25x + 1.05y = 30$$
$$1.25x + 1.05(8x) = 30$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$1.25x + 8.40x = 30$$
$$9.65x = 30$$
$$x \approx 3.1$$

Therefore, Sofia should buy approximately 3.1 pounds of onions.

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

Therefore, Sofia should buy approximately 24.8 pounds of potatoes.

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept of the line.

Ask Yourself . . .
Keep in mind what the value represents.

7. Check that the solution is correct. Show your work.

8. What is the solution to the system? What does it represent in terms of the problem situation?

9. Compare your solution using the substitution method to the solution on your graph. What do you notice?

ACTIVITY 2.2

Substitution with Special Systems

Daniel and Chloe are helping to set up the booths at the fair. They are each paid $7 per hour to carry the wood that is needed to build the various booths. Daniel arrives at 7:00 A.M. and begins working immediately. Chloe arrives 90 minutes later and starts working.

1. Write an equation that gives the amount of money that Daniel will earn, *y*, in terms of the number of hours he works, *x*.

2. How much money will Daniel earn after 90 minutes of work?

3. Write an equation that gives the amount of money Chloe will earn, *y*, in terms of the number of hours since Daniel started working, *x*.

4. How much money will each student earn by noon?

5. Will Chloe ever earn as much money as Daniel? Explain your reasoning.

6. Write a system of linear equations for this problem situation.

Think About . . .
How is this similar to solving linear equations with no solution or with infinite solutions?

7. Analyze the system of linear equations. What do you know about the solution of the system by observing the equations? Explain your reasoning.

Let's see what happens when we solve the system algebraically.

8. Since both equations are written in slope-intercept form as expressions for y in terms of x, substitute the expression from the first equation into the second equation.

   a. Write the new equation.

   b. Solve the equation for x.

   c. Does your result for x make sense? Explain your reasoning.

9. What is the result when you algebraically solve a linear system that contains parallel lines?

On Monday night, the fair is running a special for the local schools: if tickets are purchased from the school, you can buy student tickets for $4 and adult tickets for $4. You buy 5 tickets and spend $20.

10. Write an equation that relates the number of student tickets, x, and the number of adult tickets, y, to the total amount spent.

11. Write an equation that relates the number of student tickets, x, and the number of adult tickets, y, to the total number of tickets purchased.

12. Write both equations in slope-intercept form.

13. Analyze the system of linear equations. What do you know about the solution of the system by looking at the equations?

Let's see what happens when you solve the system algebraically.

14. Since both equations are now written in slope-intercept form as expressions for y in terms of x, substitute the expression from the first equation into the second equation.

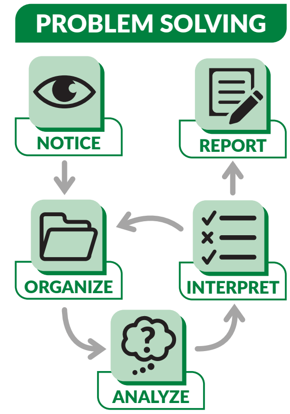
    a. Write the new equation and solve the equation for x.

    b. Does your result for x make sense? Explain your reasoning.

15. How many student tickets and adult tickets did you purchase?

16. What is the result when you algebraically solve a linear system that contains two lines that are actually the same line?

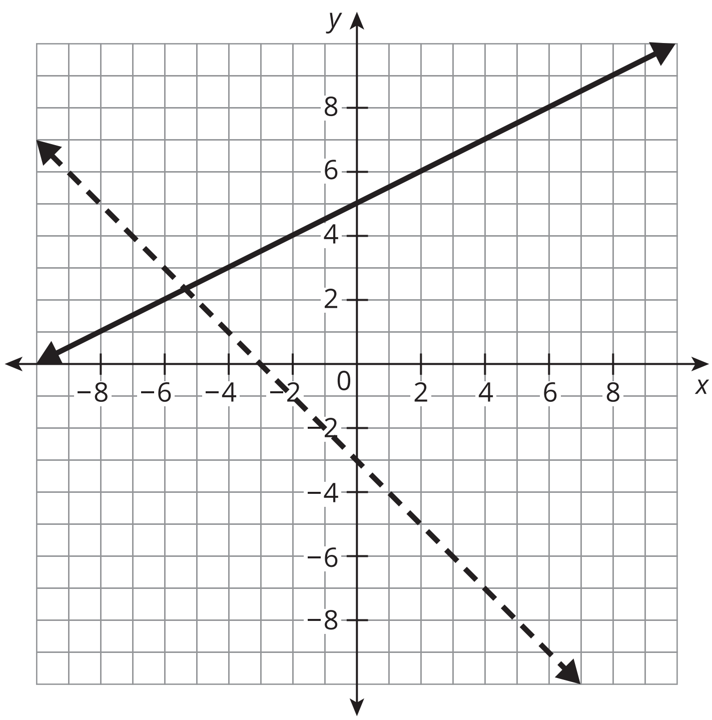
ACTIVITY
2.3 Writing and Solving Systems



Write a system of equations for each graph, table, or description.

1. The admission fee for a fair is $7 for an adult and $5 for children under the age of 5. There are 449 people at the fair. They collect $2768 in admission fees. How many adults, x, and children, y, are at the fair?

2. Write a system of equations from the graph.



3. Write a system of linear equations from the tables provided.

Line *A*	
x	*y*
−2	−20
0	−15
2	−10
4	−5

Line *B*	
x	*y*
−4	16
−1	10
2	4
5	−2

Write and solve a system of equations to solve each problem.

4. The business manager for a band must make $236,000 from ticket sales to cover costs and make a reasonable profit. The auditorium where the band will play has 4000 seats, with 2800 seats on the main level and 1200 on the upper level. Attendees will pay $20 more for main-level seats.

   a. Write a system of equations with *x* representing the cost of the main-level seating and *y* representing the cost of the upper-level seating.

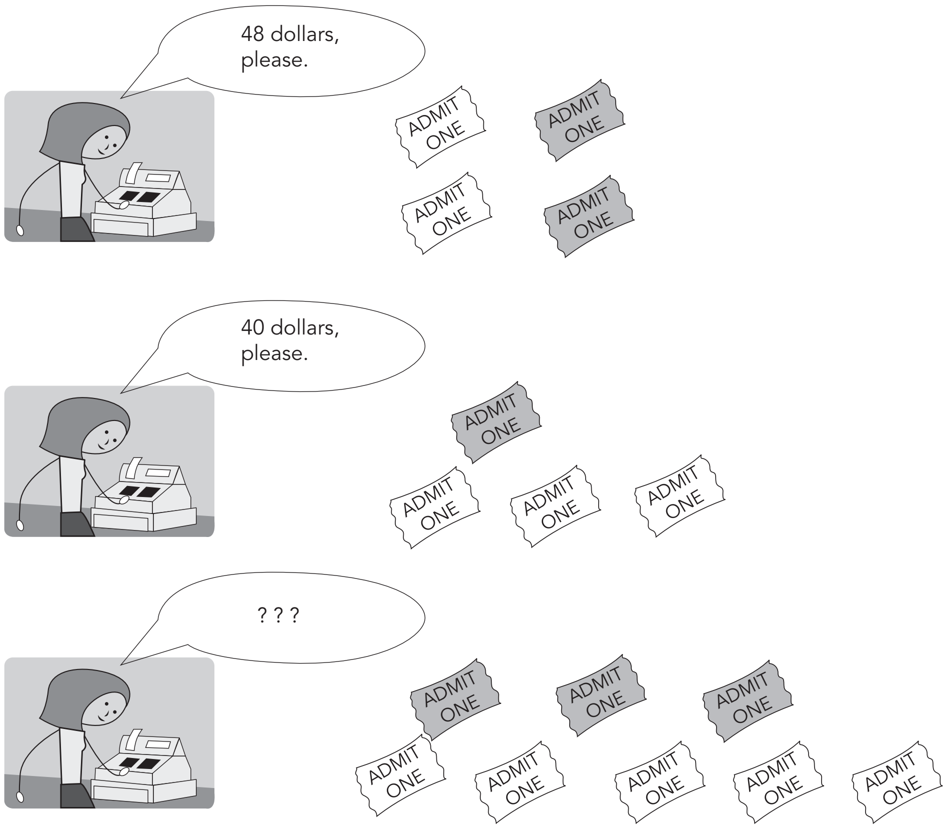
   b. Without solving the system of linear equations, interpret the solution.

   c. Solve the system of equations using the substitution method. Then, interpret the solution of the system in terms of the problem situation.

5. Mia is working as a cashier at the sports arena. What should she tell the next person in line?


Student ticket


Adult ticket



Write and solve a system of equations that represents the problem situation. Define the variables. Then, determine the cost of each type of ticket. Finally, state the amount Mia charges the third person.

Talk the Talk

The Substitution Train

1. Determine the solution to each linear system by using the substitution method. Check your answers algebraically.

a. $\begin{cases} 2x + 3y = 34 \\ y = 5x \end{cases}$

b. $\begin{cases} y = 4x + 2 \\ y = 3x - 2 \end{cases}$

c. $\begin{cases} 3x + 2y = 4 \\ 2x - y = 5 \end{cases}$

d. $\begin{cases} 3x + y = 8 \\ 6x + 2y = 10 \end{cases}$

Lesson 2 Assignment

Write

Explain how to use the substitution method to solve systems of linear equations.

Remember

When a system has no solution, the equation resulting from the substitution step has no solution.

When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

Practice

1. Victoria is trying to become more environmentally conscious by making her own cleaning products. She researches different cleaners and decides to make furniture polish using olive oil and lemon juice. She wants to make enough to fill two 24-fluid-ounce bottles.

   a. Write an equation in standard form that relates the amount of olive oil and lemon juice to the total amount of mixture Victoria wants to make. Use x to represent the amount of lemon juice and y to represent the amount of olive oil.

   b. The recommendation for the mixture is that the amount of olive oil be twice the amount of lemon juice. Write an equation in terms of x and y as defined in part (a) that represents this situation.

   c. Use substitution to solve the system of equations. Check your answer.

   d. What does the solution of the system represent in terms of the mixture?

Lesson 2 Assignment

e. The best price Victoria can find for lemon juice is \$0.25 per fluid ounce. The best price she can find for olive oil is \$0.39 per fluid ounce. She buys a total of 84 fluid ounces of lemon juice and olive oil and spends \$29.40. Write equations in standard form for this situation. Use x to represent the amount of lemon juice she buys, and use y to represent the amount of olive oil she buys.

f. Solve the system of equations you wrote using the substitution method. Check your answer. Describe the solution in terms of the problem situation.

2. In an effort to eat healthier, Juan is tracking his food intake by using an application on his phone. He records what he eats, then the application indicates how many calories he has consumed.

   One day, Juan eats 10 medium strawberries and 8 vanilla wafer cookies as an after-school snack. The caloric intake from these items is 192 calories. The next day, she eats 20 medium strawberries and 1 vanilla wafer cookie as an after-school snack. The caloric intake from these items is 99 calories.

   a. Write a system of equations for this problem situation. Define your variables.

   b. Without solving the system of linear equations, interpret the solution.

c. Solve the system of equations using the substitution method. Check your work.

d. Interpret the solution of the system in terms of the problem situation.

e. Juan's friend Jayden also has a calorie counting application on his phone. The two friends decide to compare the two programs. Juan eats 1 banana and 5 pretzel rods, and his application tells him he consumed 657 calories. Jayden eats 1 banana and 5 pretzel rods, and his application tells him he consumed 656 calories. The boys want to know how many calories are in each food. Write a system of equations for this problem. Define your variables.

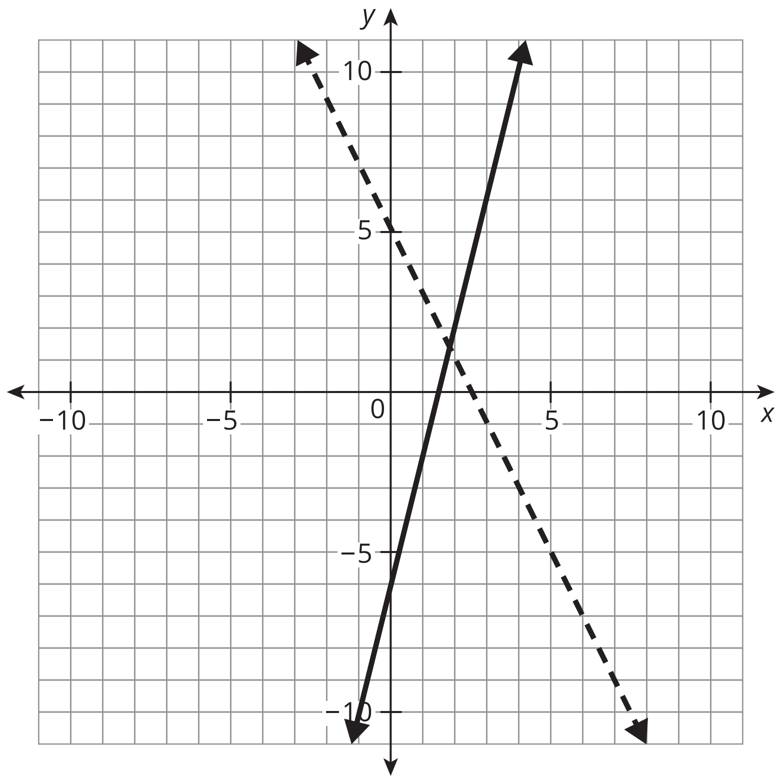
f. Solve the system of equations using the substitution method. Interpret your answer in terms of the problem.

3. Write a system of linear equations for each graph, table, or description.

   a. Mateo has 13 coins. The coins are nickels and quarters. The coins have a total value of $2.05. Let n represent the number of nickels, and let q represent the number of quarters.

Lesson 2 Assignment

b.



c.

Line *A*	
x	y
−3	−15
0	−3
3	9
6	15

Line *B*	
x	y
−4	12
−2	5
2	−9
6	−23

Lesson 2 Assignment

4. Use the substitution method to determine the solution of each system of linear equations. Check your solutions.

   a. $\begin{cases} 9x + y = 16 \\ y = 7x \end{cases}$

   b. $\begin{cases} 3x + \frac{1}{2}y = -3.5 \\ y = -6x + 11 \end{cases}$

   c. $y = -5x$
   $21x - 7y = 28$

   d. $2x + 4y = -32$
   $y = -\frac{1}{2}x - 8$

Prepare

Determine the additive inverse for each expression.

1. 4
2. x
3. $20x$
4. $-9x$
5. $78.5x$

3 Using Linear Combinations to Solve a System of Linear Equations

OBJECTIVES

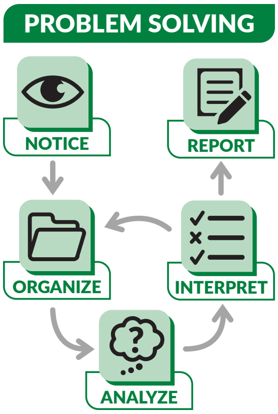
- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations.
- Interpret the solution to a system of equations in terms of a problem situation.

NEW KEY TERM

- linear combinations method

You have solved a system of linear equations graphically and algebraically, using the substitution method.

What are other algebraic strategies for solving systems of equations?



Getting Started

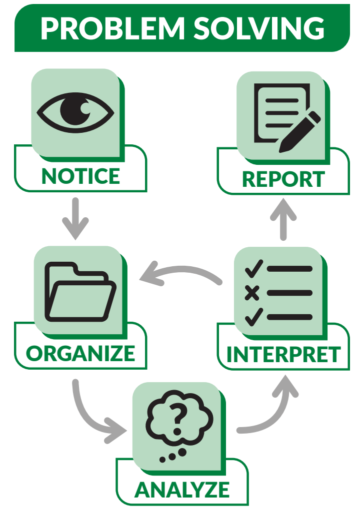
Coding Class

There are a total of 324 students taking a coding class. The number of 9th-grade students outnumber the number of 10th-grade students by 34.

1. Use reasoning to determine the number of 9th- and 10th-grade students who joined the coding class.

2. How can you check that your solution is correct?

ACTIVITY 3.1 Combining Linear Systems



Consider the scenario from the Getting Started. Let's explore an algebraic strategy to determine a solution.

1. Write the system that represents the problem situation. Use *x* to represent the 9th-grade students in the class, and use *y* to represent the 10th-grade students in the class. Write the equations in standard form.

Remember . . .
As long as you maintain equality you can rewrite equations any way you want.

Mateo and Minh used different strategies to solve the system of equations in a similar way. Analyze each student's reasoning.

Mateo

I can use the substitution method to solve this system.

$$x + y = 324$$
$$x - y = 34 \rightarrow x = y + 34$$
$$(y + 34) + y = 324$$
$$2y + 34 = 324$$
$$2y = 290$$
$$y = 145$$

$$x = 145 + 34$$
$$x = 179$$

Minh

You can eliminate one of the quantities by adding the two equations together.

$$\begin{array}{r} x + y = 324 \\ + \; x - y = 34 \\ \hline 2x = 358 \\ x = 179 \end{array}$$

$$179 + y = 324$$
$$y = 145$$

2. Explain why each student is correct in their reasoning.

3. Examine the structure of the system. What characteristic of the system made Minh's strategy efficient?

4. Identify the solution to the linear system as an ordered pair. Then, interpret the solution in terms of this problem situation.

The algebraic method used by Minh to solve the linear system is called the *linear combinations method*. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

5. Solve each system of equations using the linear combinations method.

   a. $\begin{cases} -4x + y = 10 \\ -2x - y = -1 \end{cases}$

   b. $\begin{cases} -\frac{1}{2}x + 5y = -6 \\ \frac{1}{2}x + y = -6 \end{cases}$

ACTIVITY 3.2 Using Additive Inverses to Combine Linear Systems

In the system of equations from the previous activity, one of the variables in both equations has coefficients that are additive inverses. What if a system doesn't have variables that are additive inverses? Let's use the strategy of linear combinations to solve other systems.

WORKED EXAMPLE

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}.$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{array}{r} 7x + 2y = 24 \\ +\ \ -8x - 2y = -30 \\ \hline -x = -6 \\ x = 6 \end{array}$$

Now that the y-values are additive inverses, you can solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

Remember . . .
Two numbers with a sum of zero are called *additive inverses*.

1. In the Worked Example, only one equation needs to be rewritten to solve using the linear combinations method. Why?

Ask Yourself . . .
How can you solve the system of equations by transforming the first equation instead of the second?

Now, let's consider a system where both equations need to be rewritten.

Ask Yourself . . .
If you multiply both sides of an equation by the same number, is the equation still true?

WORKED EXAMPLE

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$3(4x + 2y) = 3(3)$$
$$-2(5x + 3y) = -2(4)$$

Multiply each equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$12x + 6y = 9$$
$$-10x - 6y = -8$$

2. Determine the solution for the linear system shown in the second Worked Example.

3. How could you have solved this system by creating *x*-values that are additive inverses?

4. Describe the first step needed to solve each system using the linear combinations method. Identify the variable that will be solved for when you add the equations.

   a. $\begin{cases} 5x + 2y = 10 \\ 3x + 2y = 6 \end{cases}$

   b. $\begin{cases} x + 3y = 15 \\ 5x + 2y = 7 \end{cases}$

c. $\begin{cases} 4x + 3y = 12 \\ 3x + 2y = 4 \end{cases}$

5. Analyze each system. How would you rewrite the system to solve for one variable? Explain your reasoning.

a. $\begin{cases} \frac{1}{2}x - 5y = -45 \\ -\frac{1}{2}x + 10y = -20 \end{cases}$

b. $\begin{cases} 4x + 3y = 24 \\ 3x + y = -2 \end{cases}$

c. $\begin{cases} 3x + 5y = 17 \\ 2x + 3y = 11 \end{cases}$

d. $\begin{cases} 6x + 3y = 5 \\ 2x + y = 1 \end{cases}$

e. $\begin{cases} x + 2y = -6 \\ 2x + 4y = -12 \end{cases}$

ACTIVITY 3.3

Applying the Linear Combinations Method

A snow resort offers two winter specials: the Get-Away Special and the Extended Stay Special. The Get-Away Special offers two nights of lodging and four meals for $270. The Extended Stay Special offers three nights of lodging and eight meals for $435. Determine what Let It Snow charges per night of lodging and per meal.

1. Write the system of linear questions that represents the problem situation. Let *n* represent the cost for one night of lodging at the resort and *m* represent the cost for each meal. Write the equations in standard form.

2. How are these equations the same? How are these equations different?

3. Solve the system comparing the two winter specials.

4. Interpret the solution of the linear system in the problem situation.

5. Check your solution algebraically.

6. Is the Extended Stay Special the better deal? Explain why or why not.

ACTIVITY
3.4 Fractions and Linear Combinations

The School Spirit Club is making beaded friendship bracelets with the school colors to sell in the school store. The bracelets are black and orange and come in two lengths: 5 inches and 7 inches. The club has enough beads to make a total of 84 bracelets. So far, they have made 49 bracelets, which represents $\frac{1}{2}$ the number of 5-inch bracelets plus $\frac{3}{4}$ the number of 7-inch bracelets they plan to make and sell. Determine how many 5-inch and 7-inch bracelets the club plans to make.

1. Let x represent the number of 5-inch bracelets, and let y represent the number of 7-inch bracelets. Write a system of equations in standard form to represent this problem situation.

2. Aaliyah says that the first step to solve this system is to multiply the second equation by the least common denominator (LCD) of the fractions. Liam says that the first step is to multiply the first equation by $-\frac{1}{2}$. Who is correct? Explain your reasoning.

3. Rewrite the equation containing fractions as an equivalent equation without fractions.

4. Determine the solution to the system of equations by using linear combinations and check your answer.

5. Interpret the solution of the linear system in terms of this problem situation.

Talk the Talk

There's a Method in My Madness

You have used three different methods for solving systems of equations: graphing, substitution, and linear combinations.

1. Describe how to use each method and the characteristics of the system that makes this method most appropriate.

   a. Graphing method:

   b. Substitution method:

   c. Linear combinations method:

Lesson 3 Assignment

Write

Explain how you would combine the two equations to solve for x and y. Use the following terms in your explanation: *linear combination* and *additive inverses.*

$$3x + 2y = -25$$
$$x - 4y = 5$$

Remember

You can use the linear combinations method to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

Practice

1. Two high schools are taking field trips to the state capital. A total of 408 students from East High will be going in 3 vans and 6 buses. A total of 516 students from West High will be going in 6 vans and 7 buses. Each van has the same number of passengers, and each bus has the same number of passengers.

   a. Write a system of equations that represents this problem situation. Let x represent the number of students in each van, and let y represent the number of students in each bus.

   b. How are the equations in the system the same? How are they different?

   c. Describe the first step needed to solve the system using the linear combinations method. Identify the variable that will be eliminated as well as the variable that will be solved for when you add the equations.

Lesson 3 Assignment

d. Solve the system of equations using the linear combinations method. Show your work.

e. Interpret the solution of the linear system in terms of the problem situation.

f. Check your solution algebraically.

2. Solve each system of linear equations.

a. $\begin{cases} 3x + y = 9 \\ 7x + y = 32 \end{cases}$

b. $\begin{cases} \frac{2}{3}x + \frac{1}{4}y = 18 \\ \frac{1}{6}x - \frac{3}{8}y = -6 \end{cases}$

Lesson 3 Assignment

c. $\begin{cases} 5x - 8y = 25 \\ -x + 4y = -8 \end{cases}$

d. $\begin{cases} 5x + 4y = -14 \\ 3x + 6y = 6 \end{cases}$

Prepare

Determine if each point is a solution to $y > x$, $y < x$, or $y = x$.

1. $(8, -2)$

2. $(0, 7)$

3. $(-1, -1)$

4. $(-4, -3)$

5. $(9, 9)$

6. $(-3, -10)$

4 Graphing Inequalities in Two Variables

OBJECTIVES

- Write an inequality in two variables.
- Graph an inequality in two variables on a coordinate plane.
- Determine whether a solid or dashed boundary line is used to graph an inequality on a coordinate plane.
- Interpret the solutions of inequalities mathematically and in the context of real-world problems.

NEW KEY TERMS

- half-plane
- boundary line

You have graphed linear inequalities in one variable.

What does the graph of a linear inequality in two variables look like? How does it compare to the graph of a linear equation?

Getting Started

Making a Statement

Consider each solution statement.

$$x = 2$$

$$x < 2 \qquad\qquad x \leq 2$$

$$x > 2 \qquad\qquad x \geq 2$$

1. Compare the solution statements. What does each one mean?

2. Choose a solution statement and write a scenario to represent it. Then, modify the scenario so the resulting interpretation is one of the other four solution statements.

ACTIVITY

4.1 Linear Inequalities in Two Variables

1. Coach Garcia is analyzing the scoring patterns of players on his basketball team. Kai is averaging 20 points per game from scoring on two-point and three-point shots.

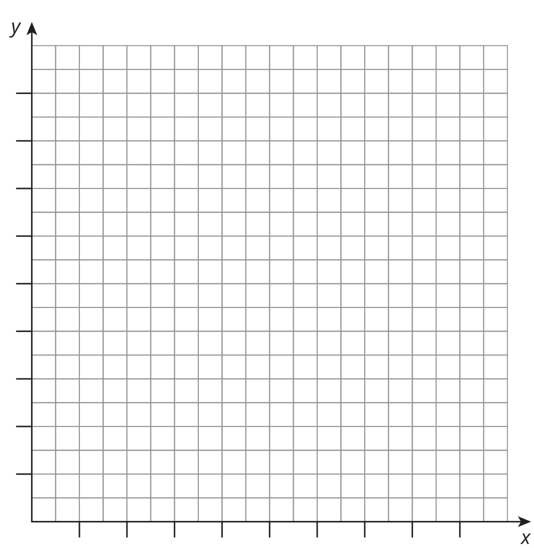
   a. If she scores 6 two-point shots and 2 three-point shots, will Kai meet her points-per-game average?

   b. If she scores 7 two-point shots and 2 three-point shots, will Kai meet her points-per-game average?

   c. If she scores 7 two-point shots and 4 three-point shots, will Kai meet her points-per-game average?

2. Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.

3. Graph the equation you wrote on the coordinate plane.



Ask Yourself . . .
How should you label the graph?

4. Coach Garcia believes that the team can win the district playoffs if Kai scores at least 20 points per game.

   a. How can you rewrite the equation you wrote in Question 2 to represent that Kai must score at least 20 points per game?

   b. Write an inequality in two variables that represents this problem situation.

Remember. . .
An inequality is a statement formed by placing an inequality symbol ($>$, $\geq$, $<$, $\leq$,) between two expressions.

5. Complete the table of values. Then, add the ordered pairs in the table to the graph in Question 3. If the number of total points scored does not meet or exceed Kai's points-per-game average, use an "x" to plot the point. If the number of total points scored meets or exceeds Kai's points-per-game average, use a dot to plot the point.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total-Points Scored
4	1	
6	1	
7	1	
8	2	
6	4	
9	5	

6. What do you notice about your graph?

7. What can you interpret about the solutions of the inequality from the graph?

8. Choose a different ordered pair located above the line and a different ordered pair that is located below the line. How do these points confirm your interpretation of the situation? Explain your reasoning.

9. Shade the side of the graph that contains the combinations of shots that are greater than or equal to Kai's points-per-game average.

10. How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?

11. Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not.

Like linear equations, linear inequalities take different forms. Each of the linear inequalities in two variables shown represents a different relationship between the variables.

$$ax + by < c \qquad ax + by > c$$
$$ax + by \leq c \qquad ax + by \geq c$$

ACTIVITY 4.2

Determining the Graphs of Linear Inequalities

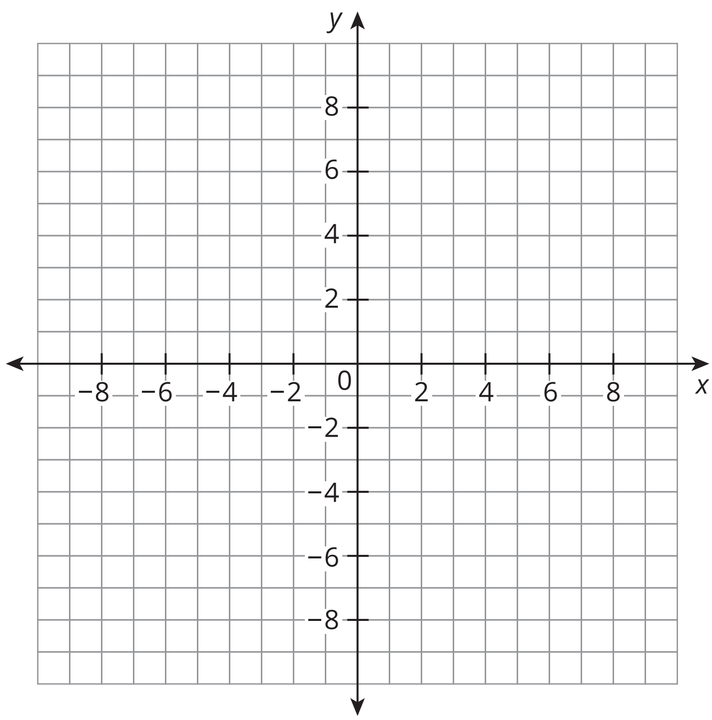
If the inequality symbol is $\geq$ or $\leq$, the boundary line is a solid line because all points on the line are part of the solution set. If the symbol is $>$ or $<$, the boundary line is a dashed line because no point on that line is a solution.

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes, and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane.

Consider the linear inequality $y > 4x - 6$. The boundary line that divides the plane is determined by the equation $y = 4x - 6$.

1. Should the boundary line in this graph be a solid line or a dashed line? Explain your reasoning.

2. Graph the boundary line on the coordinate plane shown.



After you graph the inequality with either a solid or a dashed boundary line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains the solutions and should be shaded.

3. Decide which half-plane to shade.

    a. Is (0, 0) a solution? Explain your reasoning.

    b. Shade the correct half-plane on the coordinate plane.

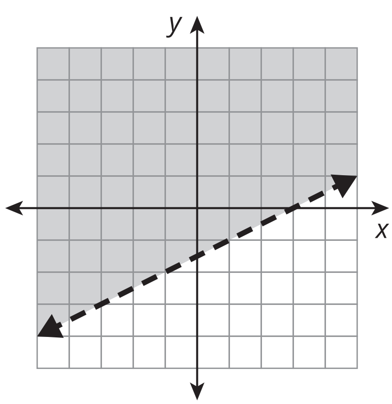
Think About . . .
It's a good idea to check points in both half-planes to verify your solution.

4. Match each graph to one of the inequalities given. In part (d), graph the inequality that was not graphed in parts (a) through (c).

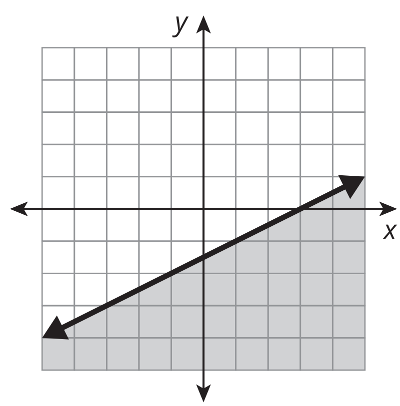
$$y \geq \frac{1}{2}x - 3 \qquad y \leq \frac{1}{2}x - 3$$

$$y > \frac{1}{2}x - 3 \qquad y < \frac{1}{2}x - 3$$

a.



b.



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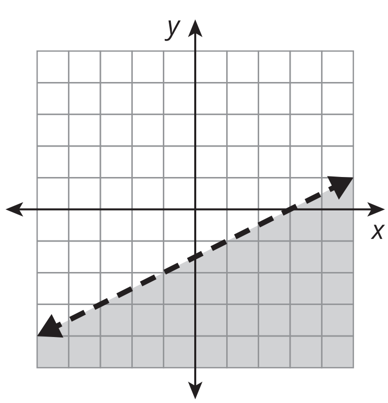
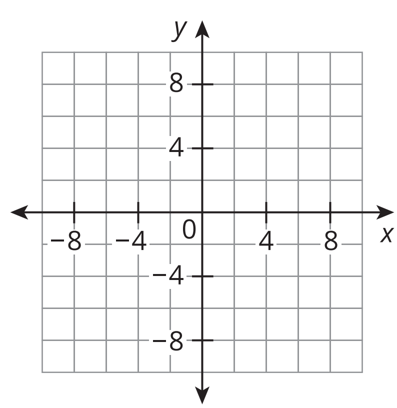


d.

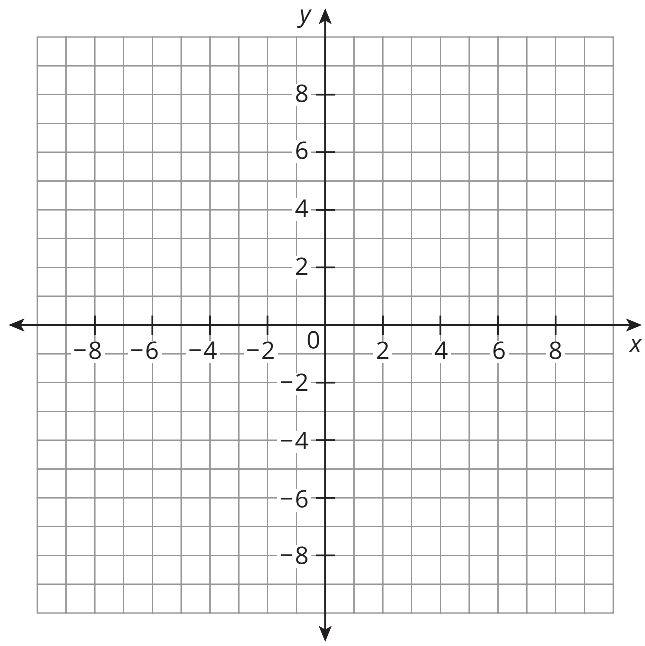


Think About . . .

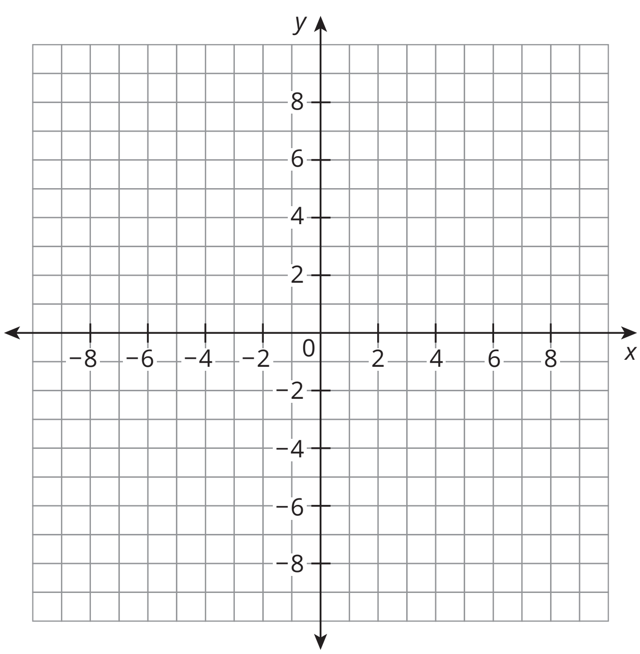
Consider the inequality symbol and which half-plane will be shaded before you test any points.

5. Graph each linear inequality.

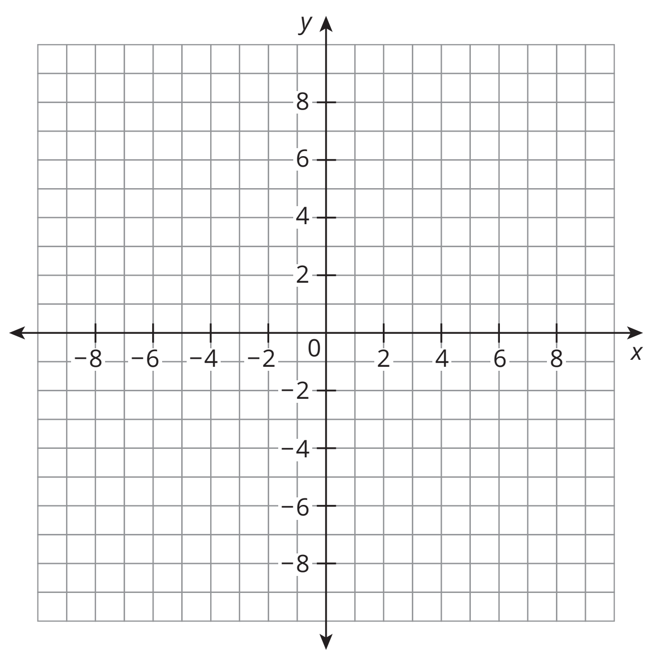
a. $y > x + 3$



b. $y \leq -\frac{1}{3}x + 4$



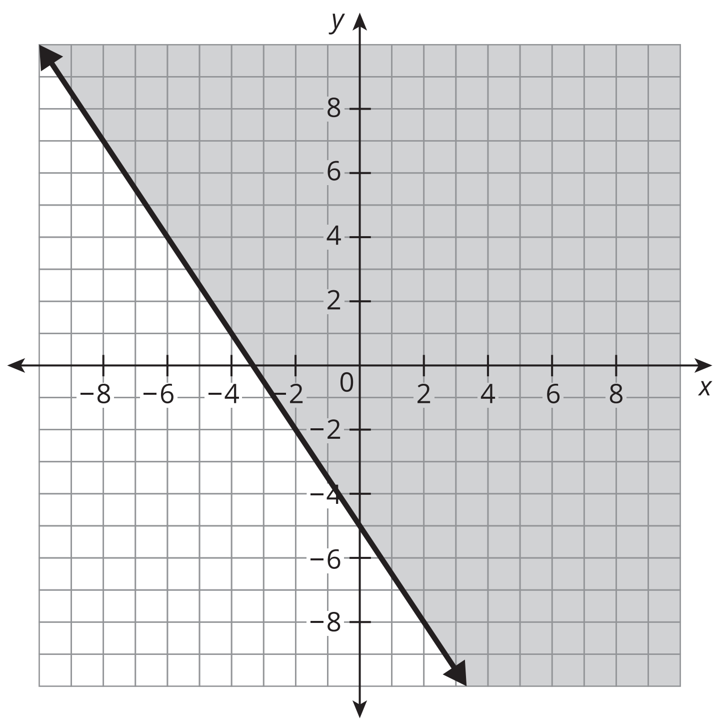
c. $2x - y < 4$



Previously, you have written a linear equation given various representations, including two points, one point and the slope, a table of values, or a graph. You can use a similar approach when writing a linear inequality.

WORKED EXAMPLE

Write a linear equality for the graph.



You can use what you have previously learned about the graphs of linear equations to determine that the boundary line is represented by the equation $y = -\frac{3}{2}x - 5$. Now, you must decide which inequality symbol should replace the equals sign in the equation.

Since the graph shows a solid boundary line and the half-plane above the line is shaded, use the symbol $\geq$.

$y \geq -\frac{3}{2}x - 5$

Test a point in the solution set to check the linear inequality.

Test the point (0, 0):

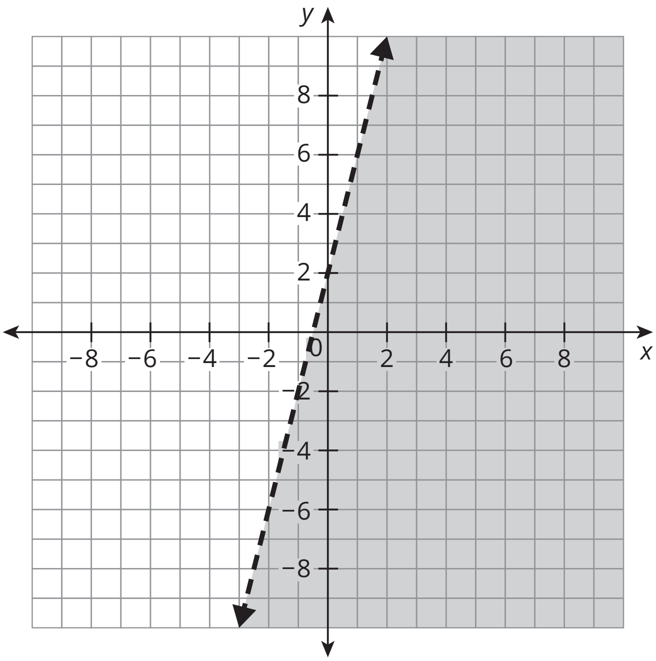
$0 \overset{?}{\geq} -\frac{3}{2}(0) - 5$

$0 \geq -5$ ✓

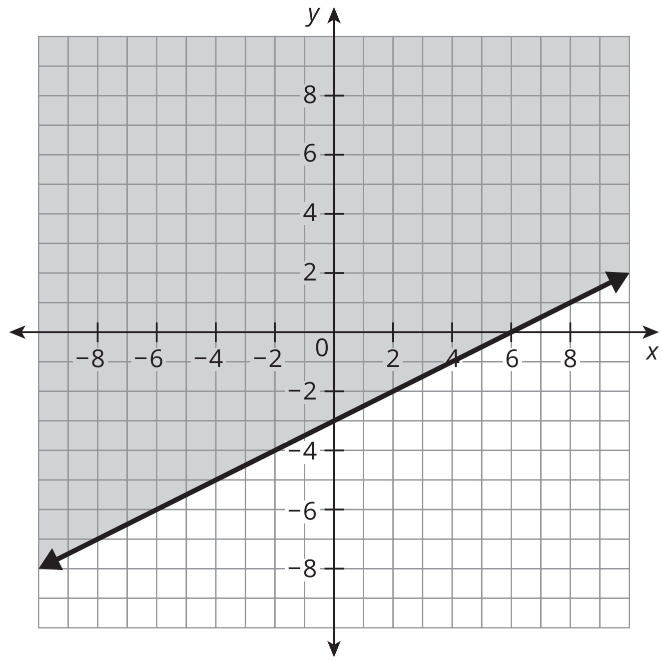
Remember. . .
The point (0, 0) can be used as a test point, unless the boundary line passes through (0, 0).

6. Write a linear inequality for each graph.

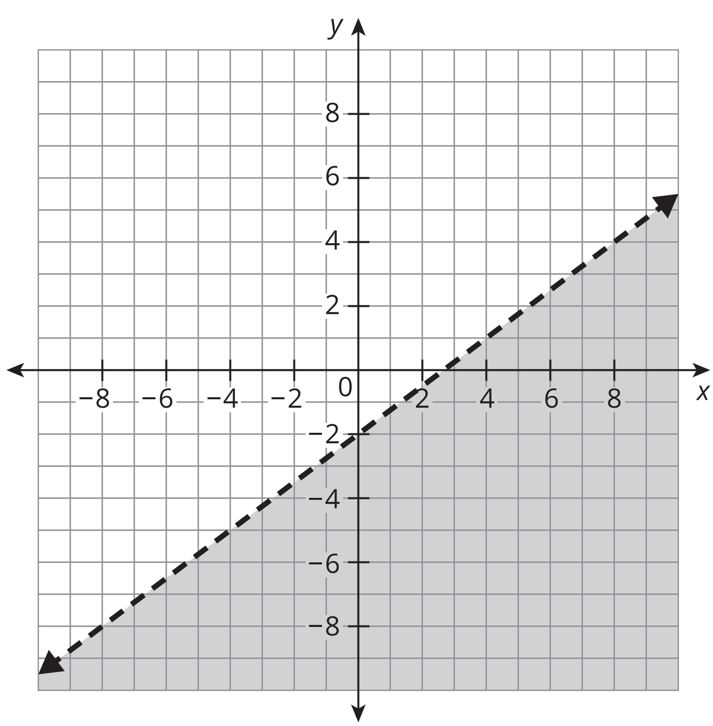
a.



b.



c.



ACTIVITY 4.3

Interpreting the Graph of a Linear Inequality

César has relatives living in both the United States and Mexico. He is given a prepaid phone card worth $50 for his birthday. The table of values shows combinations of minutes for calls within the United States, x, and calls to Mexico, y, that expend his $50 prepaid phone card.

Length of Calls Within United States (minutes)	Length of Calls to Mexico (minutes)
0	100
50	80
140	44
200	20
240	4

1. Write an inequality modeling the number of minutes César can use for calls within the United States and for calls to Mexico.

2. Graph your inequality on the given coordinate grid. Be sure to label your axes.

3. If César speaks with his aunt in Guadalajara, Mexico, for 70 minutes using his phone card, how long can he speak with his cousin in New York using the same card?

4. Can César call his uncle in San Antonio for 100 minutes and also call his grandmother in Juárez, Mexico, for 80 minutes using his phone card? Explain your reasoning.

Ask Yourself . . .
How can you use linear inequalities in everyday life?

5. Can César call his brother in Mexico City, Mexico, for 55 minutes and also call his sister in Denver for 90 minutes using his phone card? Explain your reasoning.

6. Interpret the meaning of each.

   a. Points on the line

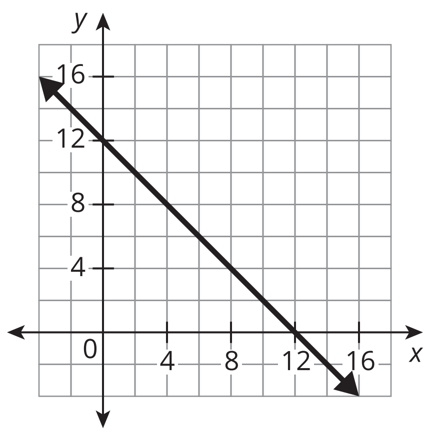
   b. Points above the line

   c. Points below the line

Talk the Talk

There's a Fine Line

Consider the graph of the linear equation $x + y = 12$.



Use the graph to answer each question.

1. Describe how to graph $x + y < 12$ and choose a point to test this region.

2. Describe how to graph $x + y \leq 12$ and choose a point to test this region.

3. Describe how to graph $x + y > 12$ and choose a point to test this region.

4. Complete the table.

Equation or Inequality	Description of the Solution Set
$x + y = 0$	
$x + y \geq 0$	
$x + y \leq 0$	
$x + y > 0$	
$x + y < 0$	

Lesson 4 Assignment

Write

Describe a half-plane in your own words.

Remember

The graph of a linear inequality in two variables is the half-plane that contains all the solutions. If the inequality symbol is $\leq$ or $\geq$, the graph shows a solid boundary line because the line is part of the solution set. If the symbol is $<$ or $>$, the boundary line is a dashed line because no point on the line is a solution.

Practice

1. Elijah is working two jobs to save money for his college education. He makes \$8 per hour working for his uncle at a pizzeria bussing tables and \$10 per hour tutoring peers after school in math. His goal is to make \$160 per week.

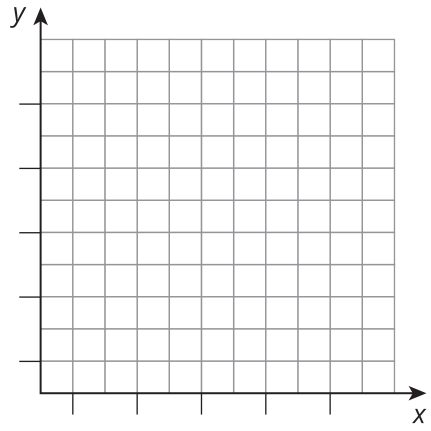
   a. If Elijah works 8 hours at the pizzeria and tutors 11 hours during the week, does he reach his goal?

   b. Write an expression to represent the total amount of money Elijah makes in a week from working both jobs. Let x represent the number of hours he works at the pizzeria and y represent the number of hours he tutors.

   c. After researching the costs of colleges, Elijah decides he needs to make more than \$160 each week. Write an inequality with two variables to represent the amount of money Elijah needs to make.

Lesson 4 Assignment

d. Graph the inequality from part (c).



e. Is the point (0, 0) in the shaded region of the graph? Explain why or why not.

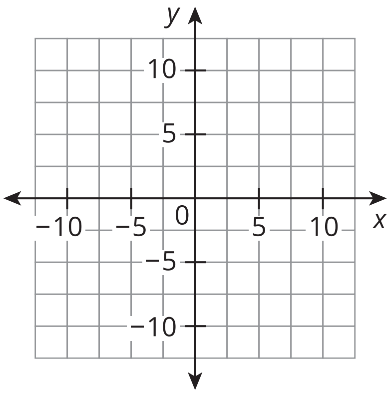
f. According to the graph, if Elijah works 5 hours at the pizzeria and tutors for 10 hours, will he make more than $160? Explain why or why not.

g. Due to days off from school, Elijah will only be tutoring for 6 hours this week. Use the graph to determine the least amount of full hours he must work at the pizzeria to still reach his goal. Then, show that your result satisfies the inequality.

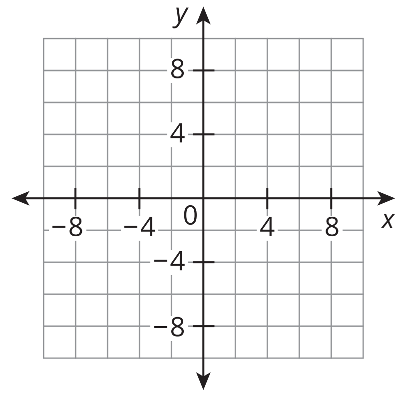
Lesson 4 Assignment

2. Graph each inequality on a coordinate plane.

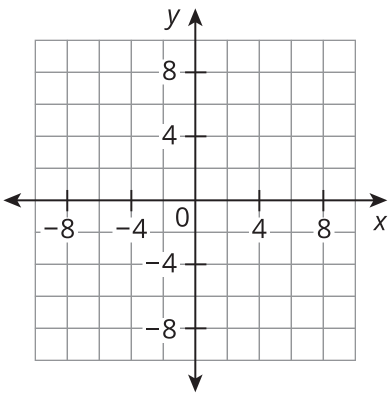
   a. $x + 3y > 9$



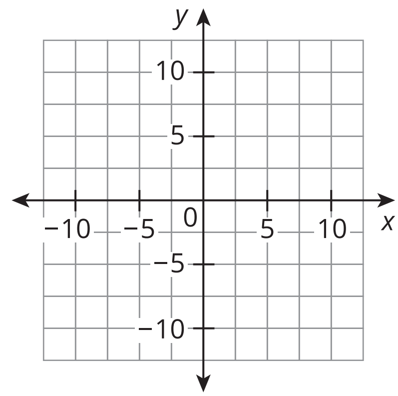
   b. $2x - 6y \leq 15$



   c. $2x + y < 6$



   d. $3x - y \geq 1$



Prepare

Determine an ordered pair (x, y) that satisfies each inequality.

1. $x + y < 18$

2. $x - y > -7$

3. $2x + 3y \leq -5$

4. $-5x - 2y \geq 10$

5 Systems of Linear Inequalities

OBJECTIVES

- Represent constraints in a problem situation with systems of inequalities.
- Write and graph systems of linear inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- Verify solutions to systems of linear inequalities algebraically.

NEW KEY TERMS

- constraints
- solution set of a system of linear inequalities

You have graphed a linear inequality in two variables and interpreted the solutions.

What does the graph of a system of linear inequalities look like, and how can you describe the solution set?

Getting Started

A River Runs Through It

Noah is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds can cause the raft to sink, hit more rocks, and/or maneuver more slowly.

Think About . . .
Does Chase count himself when determining the weight and the cost?

Noah estimates the weight of each adult as approximately 200 pounds and the weight of each child as approximately 100 pounds. Noah charges adults $75 and children $50 to ride down the river with him. His goal is to earn at least $150 each rafting trip.

1. Write an inequality to represent the most weight Noah can carry in terms of rafters. Define your variables.

2. Write an inequality to represent the minimum amount of money Noah wants to collect for each rafting trip.

3. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Noah wants to earn per trip.

ACTIVITY
5.1

Determining Solutions to Systems of Linear Inequalities

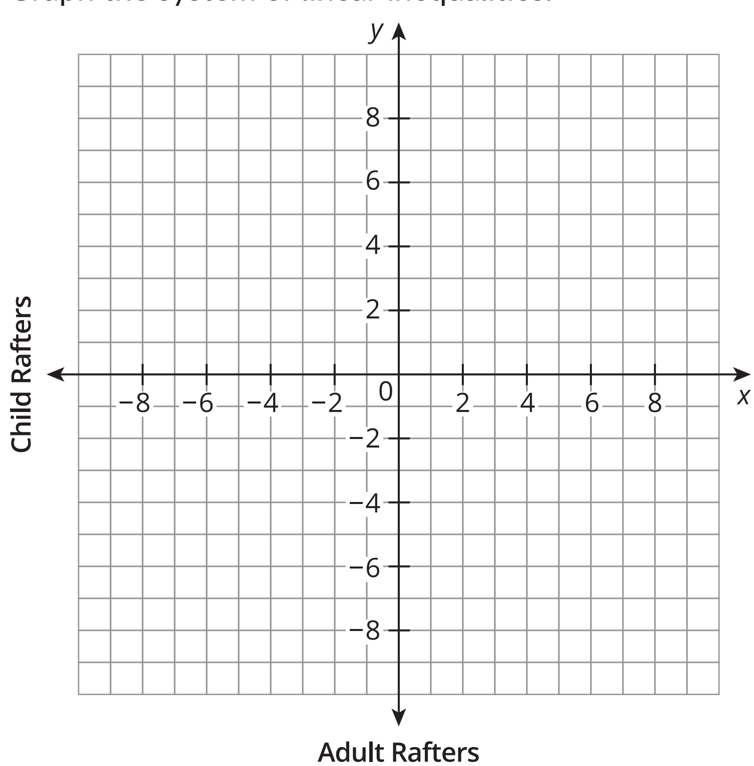
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

1. Let's consider two trips that Noah guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then, describe the meaning of the solution in terms of this problem situation.

   a. First Trip: Noah guides 2 adults and 2 children.

   b. Second Trip: Noah guides 5 adults.

2. Graph the system of linear inequalities.



Shade the half-plane of each inequality differently. You can use colored pencils or simply vertical and horizontal lines.

The **solution set of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

3. Analyze your graph.

    a. Describe the possible number of solutions for a system of linear inequalities.

    b. Is the intersection point a solution to this system of inequalities? Why or why not?

    c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?

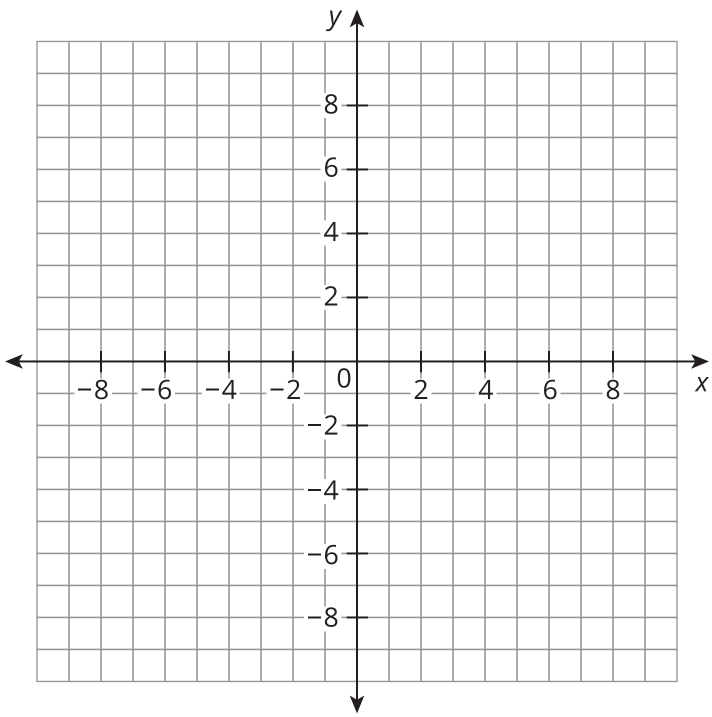
    d. Determine one combination of adults and children that is not a solution for this system of linear inequalities. Explain your reasoning.

ACTIVITY 5.2 Analyzing Graphs of Systems of Linear Inequalities

Determine the solution set of the given system of linear inequalities.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

1. Graph the system of linear inequalities.



Think About . . .
Notice the inequality symbols. How does this affect your graph?

2. Choose a point in each shaded region of the graph. Determine whether each point is a solution of the system. Then, describe how the shaded region represents the solution.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
$(-8, 2)$	$-8 + 2 > 1$ $-6 > 1$ ✗	$-(-8) + 2 \leq 3$ $10 \leq 3$ ✗	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.

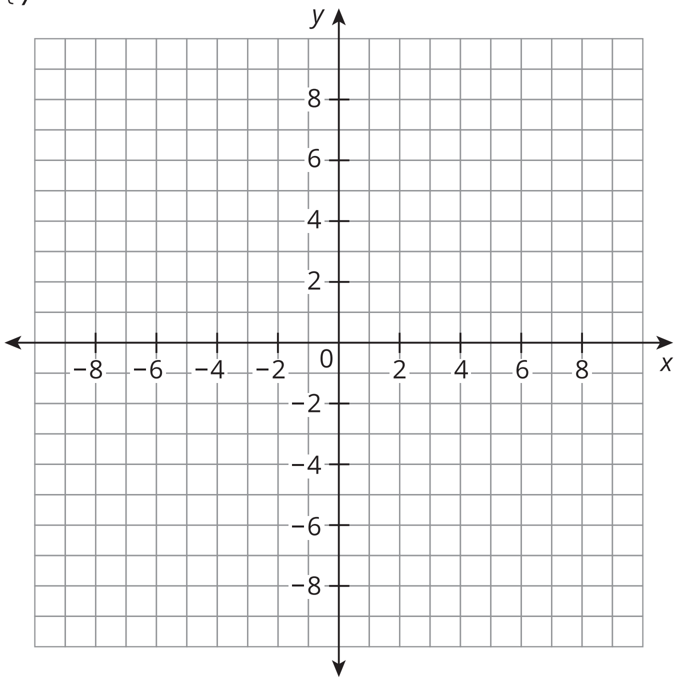
3. Ava makes the statement about the intersection point of a system of inequalities. Explain why Ava's statement is incorrect.

Ava

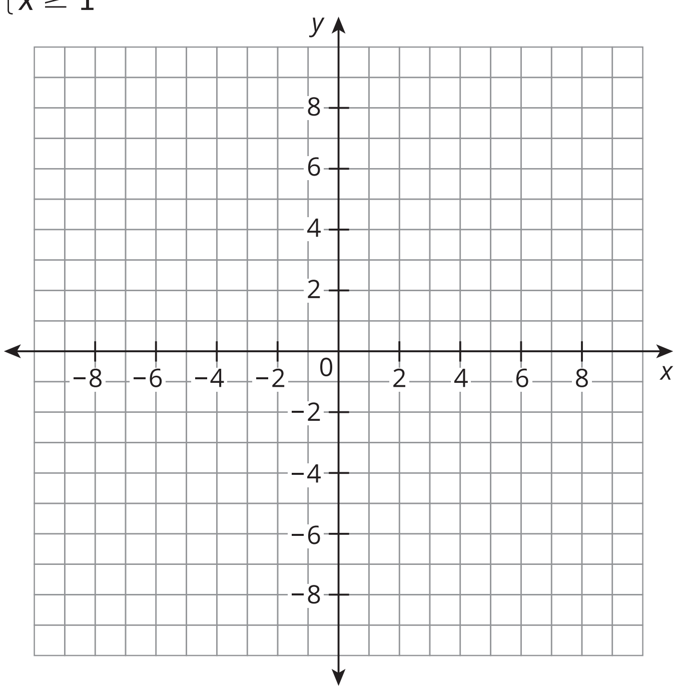
The intersection point is always a solution to a system of inequalities because that is where the two lines meet.

4. Solve each system of linear inequalities by graphing the solution set. Then, identify two points that are solutions of the system.

a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$



b. $\begin{cases} x \geq -5 \\ x \geq 1 \end{cases}$



Ask Yourself . . .
Did you share your solution(s) with others?

ACTIVITY 5.3

Applying Systems of Linear Inequalities

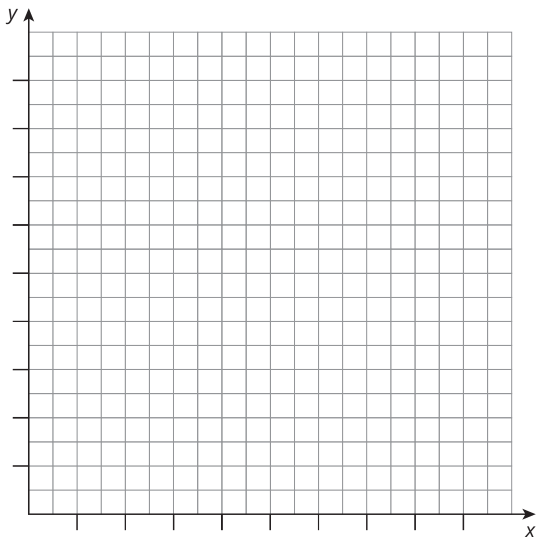
Emma and a group of friends decide to use the fitness room after school. A sign on the wall provides the information shown.

Exercise	Calories Burned per Minute
Treadmill—light effort	7.6
Treadmill—vigorous effort	12.4
Stair Stepper—light effort	6.9
Stair Stepper—vigorous effort	10.4
Stationary Bike—light effort	5.5
Stationary Bike—vigorous effort	11.1

Emma decides to use the stair stepper. She has, at most, 45 minutes to exercise and she wants to burn at least 400 calories.

1. Write a system of linear inequalities to represent Emma's workout. Define your variables.

2. Graph the system of inequalities from Question 1 on the coordinate plane. Be sure to label your axes.



Use technology to graph your inequalities and check your answer.

3. Analyze your graph.

   a. Identify two different solutions of the system of inequalities.

   b. Interpret your solutions in terms of Emma's workout.

   c. Algebraically prove that your solutions satisfy the system of linear inequalities.

ACTIVITY 5.4

Identifying Systems of Linear Inequalities

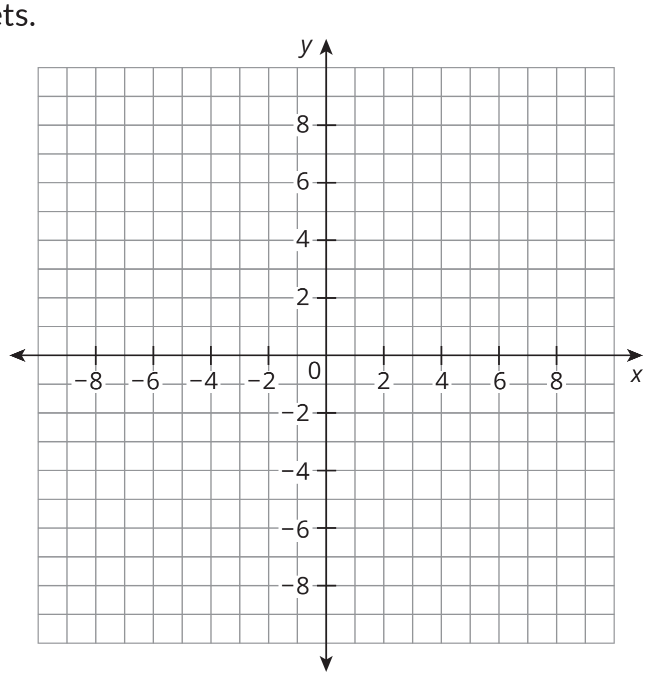
Consider the four systems shown.

System A	System B	System C	System D
$\begin{cases} y < \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y < \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$

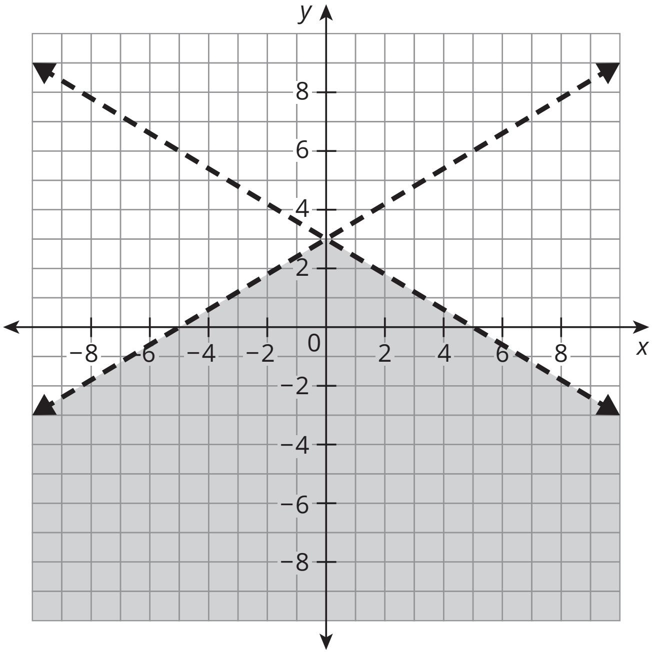
1. Analyze Isabella's statement and explain why it is incorrect.

> **Isabella**
> Since the equations in each system are the same, the graphs and solutions should all be identical.

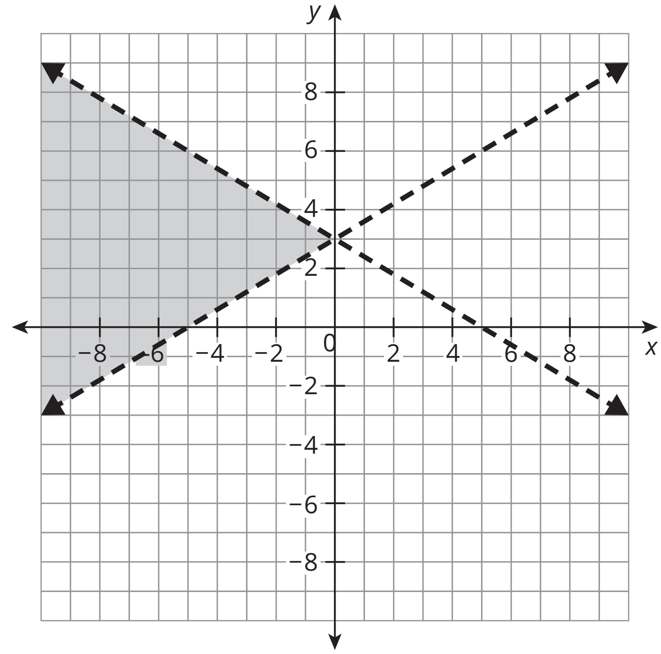
2. Match a graph and possible solution to each given system of linear inequalities. Complete the blank graph and partial solution set to make four complete sets.



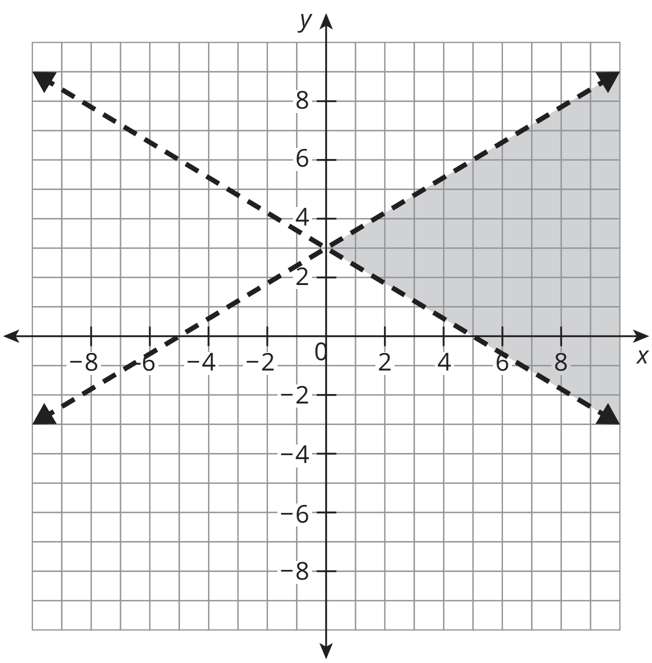
Graph A



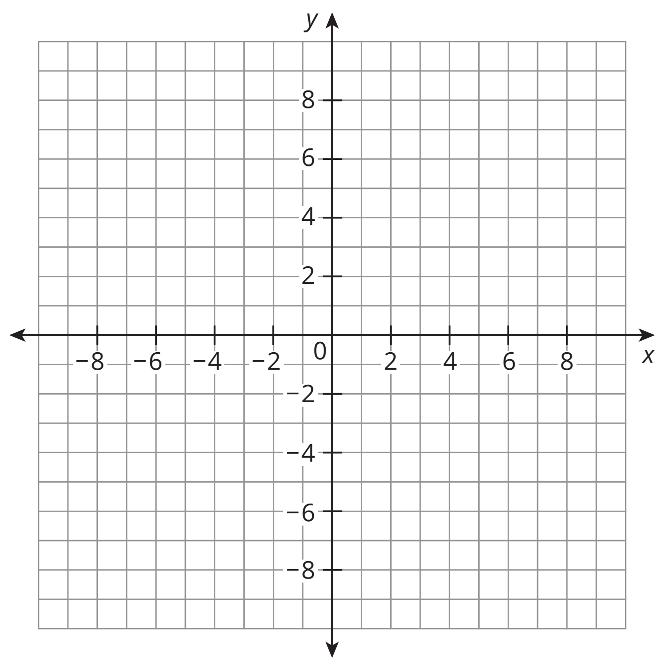
Graph B



Graph C



Graph D



Possible Solutions A	Possible Solutions B	Possible Solutions C	Possible Solutions D
(0, −6) and (4, −4)	(0, 7.5) and (−4, 10)	(−6, 4) and (−10, 8)	_____and_____

Talk the Talk

Get to Know the Region

The solution set to a system of inequalities can be any of four regions on the coordinate plane.

1. Consider each system of linear inequalities and decide which region represents the solution set. Explain your reasoning.
   - A region above both lines.
   - A region below both lines.
   - A region between the lines.
   - No solution.

   a. $y < 8 + 2x$
   $3 + 2x > y$

   b. $5 + x < y$
   $y > 7 + x$

   c. $y > 12 - 3x$
   $10 - 3x > y$

   d. $6 - x < y$
   $y < 9 - x$

2. How is the solution to a system of linear inequalities the same as or different from the solution to a system of linear equations?

Lesson 5 Assignment

Write

Describe how you know which region, if any, represents the solution to a system of linear inequalities.

Remember

The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

Practice

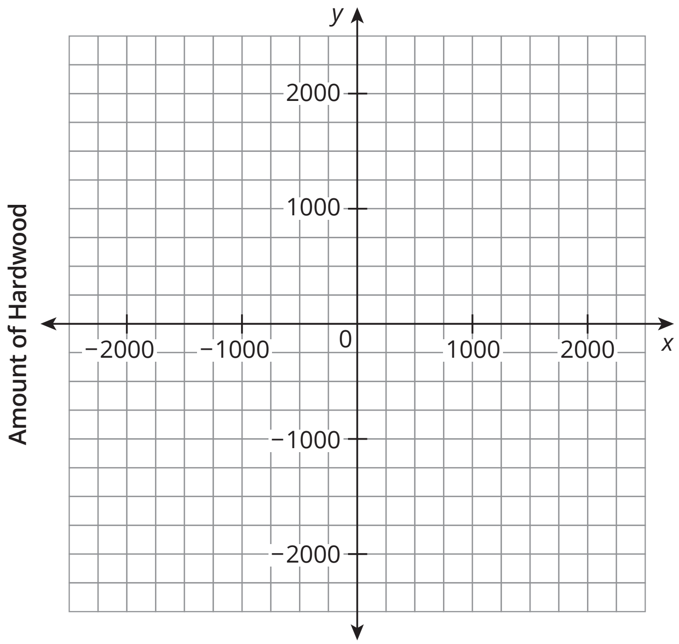
1. Samuel is remodeling his basement. One part of the planning involves the flooring. He knows that he would like both carpet and hardwood, but he isn't sure how much of each he will use. The most amount of flooring area he can cover is 2000 square feet. The carpet is \$4.50 per square foot, and the hardwood is \$8.25 per square foot. Both prices include labor costs. Samuel has budgeted \$10,000 for the flooring.

   a. Write a system of inequalities to represent the maximum amount of flooring needed and the maximum amount of money Samuel wants to spend.

   b. One idea Samuel has is to make two rooms—one having 400 square feet of carpeting and the other having 1200 square feet of hardwood. Determine whether this amount of carpeting and hardwood are solutions to the system of inequalities. Explain your reasoning in terms of the problem situation.

Lesson 5 Assignment

c. Graph this system of inequalities.



d. Determine the intersection point of the two lines. Is this a solution to this system of inequalities in terms of the problem situation?

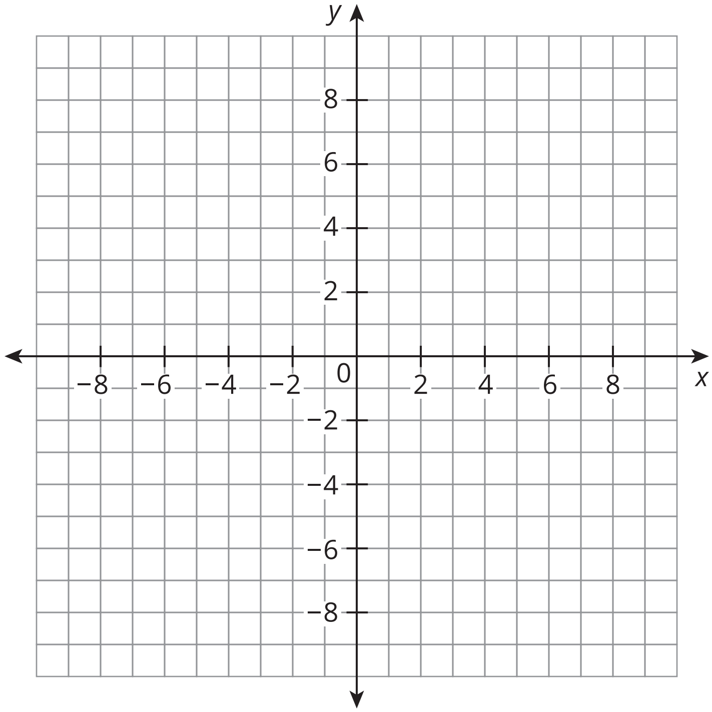
e. Identify two different solutions to the system of inequalities. Explain what the solutions represent in terms of the problem situation.

Lesson 5 Assignment

f. Determine one combination of amounts of carpet and hardwood that is not a solution for the system of inequalities. Explain your reasoning.

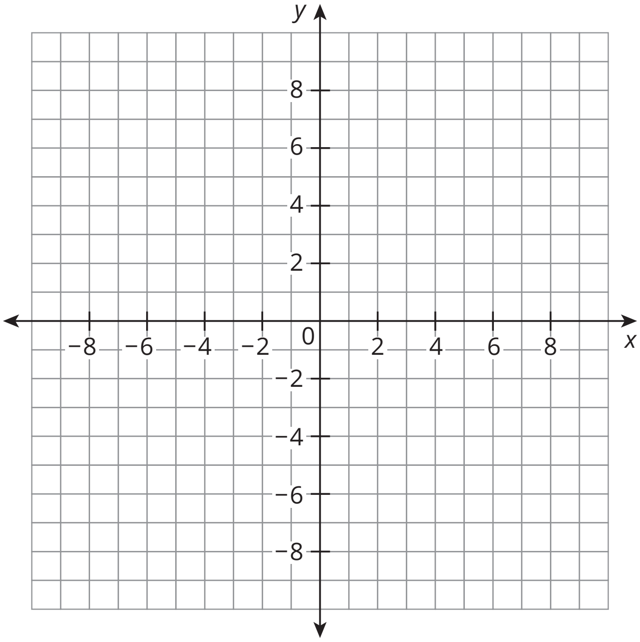
2. Solve each system of linear inequalities.

a. $\begin{cases} -x + 3y \leq -6 \\ -5x + 3y \geq 6 \end{cases}$

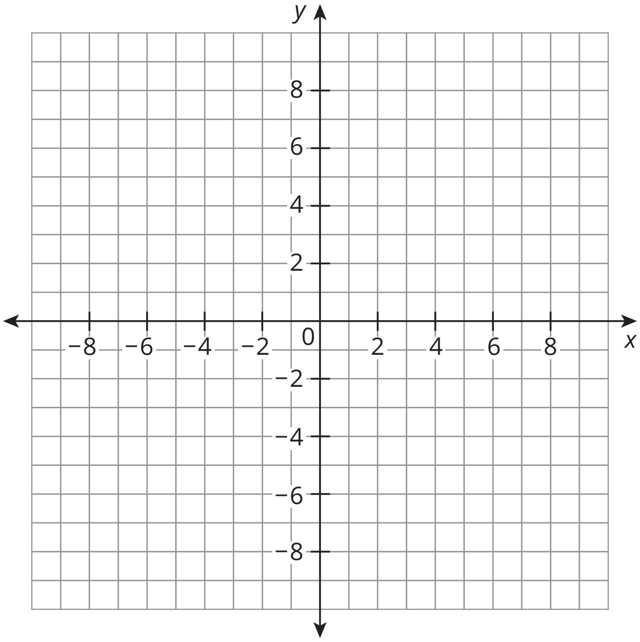


Lesson 5 Assignment

b. $\begin{cases} -x + 2y < 6 \\ 3x + 2y \leq 2 \end{cases}$



c. $\begin{cases} -x + 3y \leq 18 \\ x \leq 3 \end{cases}$



Lesson 5 Assignment

Prepare

Determine whether the point (1, 7) is a solution to each system.

1. $\begin{cases} 4x - y = -3 \\ -2x + y = 5 \end{cases}$

2. $\begin{cases} x + y > 4 \\ 4x - y < -4 \end{cases}$

3. $\begin{cases} y = -3.5x - 2 \\ y = 4.5x - 10 \end{cases}$

4. $\begin{cases} -2x + y < 8 \\ x - y > -8 \end{cases}$

6 Solving Systems of Equations and Inequalities

OBJECTIVES

- Use various methods of solving systems of linear equations to determine the better buy or the better job offer.
- Solve systems of linear inequalities with more than two inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

You have solved systems of linear equations by graphing, by using substitution, and by using linear combinations. You have also graphed systems of linear inequalities to determine possible solutions.

How can you use these various methods to reason about real-world problems?

Getting Started

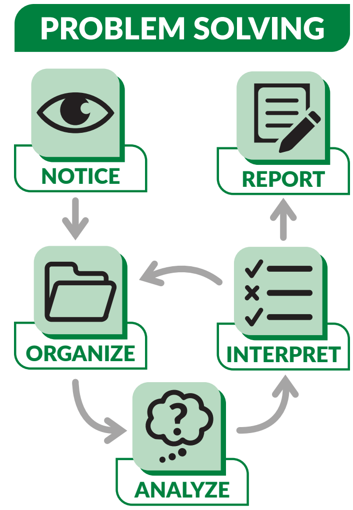
Systems for Summer Savings

A neighborhood pool club offers two membership plans. Plan A includes a seasonal sign-up fee of \$100 and charges \$2 each time you use the pool. Plan B has no sign-up fee but charges \$6 each time you use the pool. Harper chooses Plan A. She has a budget of \$200 to spend on pool fees during the months of July and August.

Harper wants to be sure she uses the pool enough times so that the plan she chooses works out to be the better deal between the two plans, but she does not want to go over her budget.

1. Use a system of linear equations and a system of linear inequalities to make a recommendation to Harper as to how often she should use the pool in July and August.

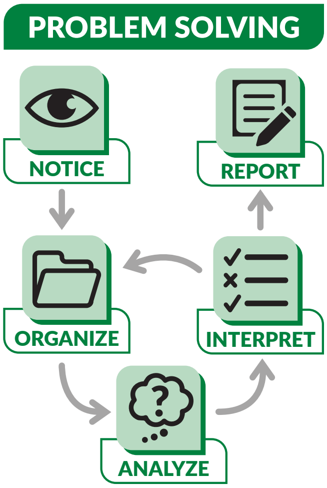
ACTIVITY 6.1 Determining the Better Deal



A bicycle company is planning to make a low price ultra-light bicycle. There are two different plans being considered for building this bicycle. The first plan includes a cost of $125,000 to design and build a prototype bicycle. The combined materials and labor costs for each bike made under the first plan will be $225. The second plan includes a cost of $100,000 to design and build the prototype. The combined materials and labor costs for each bike made under the second plan will be $275.

1. You recently got a job at the bicycle company as a financial analyst. Analyze the costs for each proposed bicycle prototype and determine which plan they should follow. Provide evidence for your proposal.

Ask Yourself . . .
Can others follow and understand your process and reasoning?



ACTIVITY 6.2

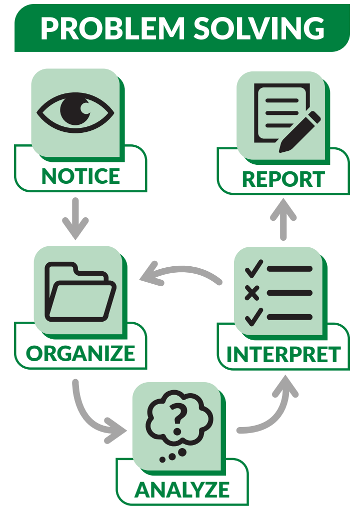
Determining the Better Buy

Diego is in search of a new cell phone plan. He is considering two different cell phone services from two different providers.

One plan is for a phone with GPS capabilities that has a monthly fee of \$99.99 and 2000 MB of data per month. Once a user exceeds the free monthly data allowance, each additional MB of data used is \$0.05. The other plan is for a phone without GPS capabilities for a monthly fee of \$79.99 and 1500 MB of data per month. Once a user exceeds the free monthly data allowance, each additional MB of data used is \$0.08. Diego is unsure which plan to choose. He wasn't very careful with his last contract and paid a lot of extra money in charges for data.

1. Write an email to advise Diego which plan to choose. Provide evidence in your response.

ACTIVITY 6.3 Solving a System of Linear Inequalities with Four Constraints



Daniel's eye doctor informed him that he needs glasses. Luckily, the local vision store is having a sale on all eyeglass frames. The advertisement in the window is as shown.

Previously, you solved a system containing two linear inequalities. However, systems can consist of more than two linear inequalities.

Save 60% to 75% on All Frames
Regularly Priced at
\$120—\$360

1. Use the advertisement to write two inequalities that represent the regular price of eyeglass frames. Let *r* represent the regular price of the frames.

2. Use the same advertisement to write two inequalities that represent the reduced price of the eyeglass frames. Let *s* represent the sales price of the frames in terms of *r*.

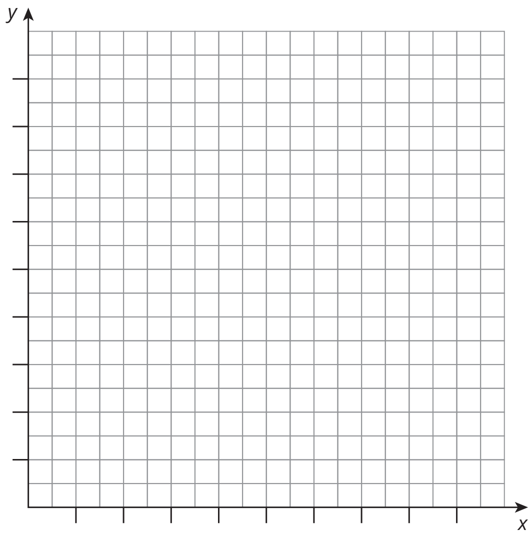
Remember . . .
When an item is 20% off the regular price, you can think of that item costing 80% of the regular price.

3. Sofia wrote this system of linear inequalities for the problem situation. Explain why it is incorrect.

Sofia

$$\begin{cases} r \geq 120 \\ r \leq 360 \\ s \leq 0.6r \\ s \geq 0.75r \end{cases}$$

4. Graph each inequality on the grid.



5. Shade the portion of the graph that satisfies the system of linear inequalities. What shape does the solution region resemble?

6. About how much would Daniel expect to spend if he purchases eyeglasses that are regularly priced at $320?

Remember . . .
When graphing a system of linear inequalities, you must determine the portion of the graph that satisfies all the inequalities in the system.

7. Daniel is definitely going to purchase a pair of eyeglasses that are on sale. What is the least amount of money Daniel can expect to spend? What is the greatest amount he can expect to spend?

8. Daniel decides on a pair of eyeglasses that are regularly priced at $240.

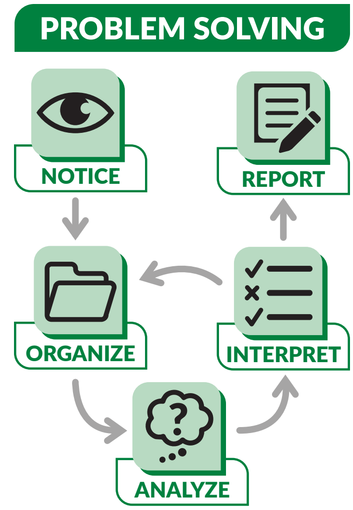
   a. Can Daniel expect to save more or less than $140 off the purchase price of this pair of eyeglasses? Use your graph to determine an approximate answer.

The graph shows you the sale price of the eyeglasses, but how can you determine how much he will save?

   b. Use algebra to determine the greatest amount of money Daniel can save by purchasing eyeglasses that are regularly priced at $240.

ACTIVITY 6.4

Determining the Better Job Offer



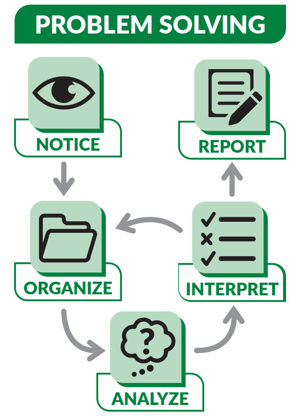
Juan interviewed for two different sales positions at competing companies. Company A has offered Juan a salary of \$31,200 per year, plus a 9% commission on his total sales. Company B will offer him \$26,000 per year, plus a 15% commission on his total sales.

Juan isn't sure which offer to accept. He's great at making a sale, but he's just not sure which job will be better in terms of his pay. He is confident that he can make at least \$2000 worth of sales each week.

1. Write an email to Juan with your recommendation of which job offers better compensation. Provide evidence in your response.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Talk the Talk



Which Cab Is More Fab?

Cab Company A charges \$3.50 upon entry and an additional \$1.25 per mile driven. Cab Company B charges \$5 upon entry and an additional \$1.15 per mile driven.

1. Emma needs to take a cab to the airport. Which company should she use if she wants to minimize the cost? Use any method to solve.

Lesson 6 Assignment

Write

You have used three methods to solve systems: graphing, substitution, and linear combinations. Describe the characteristics you would look for when determining which method to use.

Remember

The solution set to a system of inequalities with more than two constraints can be described as the region where all the graphs overlap.

Practice

1. Jayden wants to subscribe to a service that will allow him to rent DVDs and stream movies online. Company A offers a subscription for \$14.25 a month. With this subscription, Antonio can check out as many DVDs as he wants each month and must pay \$1.40 for each movie he streams online. Company B offers a subscription for \$8.50 a month. With this subscription, Jayden can checkout as many DVDs as he wants each month and must pay \$3.25 for each movie he streams online.

   a. Write a system of linear equations to represent this problem situation.

   b. Analyze the two subscription plans and determine which one is the better deal. Use any or all of the methods you have learned to determine your answer.

   c. Write a short paragraph recommending which subscription Antonio should choose.

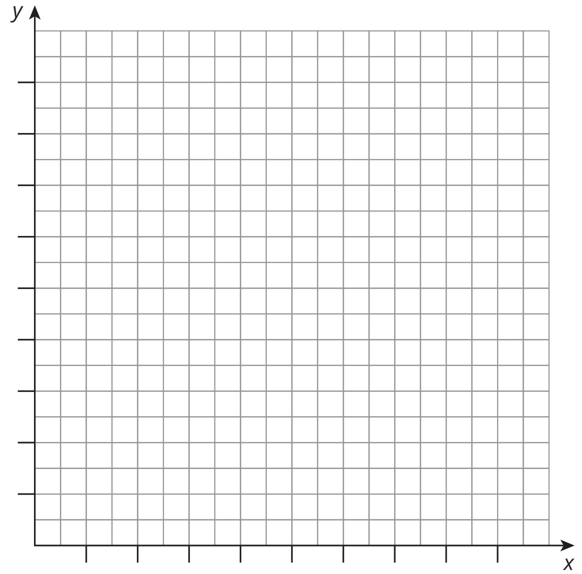
Lesson 6 Assignment

d. Which method do you think provides the quickest way to analyze a system of equations to determine which one is the better deal? Explain your reasoning.

2. A ballet company needs to rent a venue for their production. There are a number of arenas they are considering. The arenas have seating capacities that range from 800 to 1776 seats. The management of the company knows the ticket sales may not be good this year, but their goal is to sell between 65% and 90% of the available seats. Whichever arena they choose, 100 seats must be set aside for the company's donors.

   a. Write a system of inequalities that represents the problem situation. Define your variables.

   b. Graph each inequality on a coordinate plane.



Lesson 6 Assignment

c. One of the arenas they are considering has 1200 available seats. Determine the minimum and maximum number of seats they would need to sell for management to reach their goal.

d. If the company sells 900 seats, what is the range of seating capacities for the arenas they may rent?

e. If they rent an arena that has a 1300-seat capacity and sell 800 tickets, would management reach their goal? Explain your reasoning.

Prepare

Simplify each expression.

1. $(-10)(-10)(-10)$

2. $(-10)(-10)(-10)(-10)$

3. $(-1)(2)(-3)(4)(-5)$

4. $(-2)(-3)(-4)(-5)$

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Systems of Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Systems of Linear Equations and Inequalities* topic by:

TOPIC 2: *Systems of Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
understanding that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.	○	○	○
writing an equation, an inequality, or a system of linear equations or inequalities that best models a problem.	○	○	○
graphing systems of equations on a coordinate grid with appropriate labels and scales.	○	○	○
solving a system of two linear equations in two variables approximately (graphically) and exactly (algebraically).	○	○	○
interpreting the meaning of the solution to a system of linear equations in terms of a problem situation.	○	○	○
identifying linear systems that have no solution and explain why they have no solution.	○	○	○
identifying linear systems that have infinite solutions and explain why they have infinite solutions.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Systems of Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
writing and graphing an inequality and a system of inequalities in two variables on a coordinate plane.	◯	◯	◯
understanding that the solution of a system of linear inequalities is the intersection of the shaded regions of both inequalities.	◯	◯	◯
interpreting the solution to an inequality in two variables graphed on a coordinate plane.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Systems of Linear Equations and Inequalities* topic.

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2. What mathematical understandings from the topic do you feel you are making the most progress with?

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continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

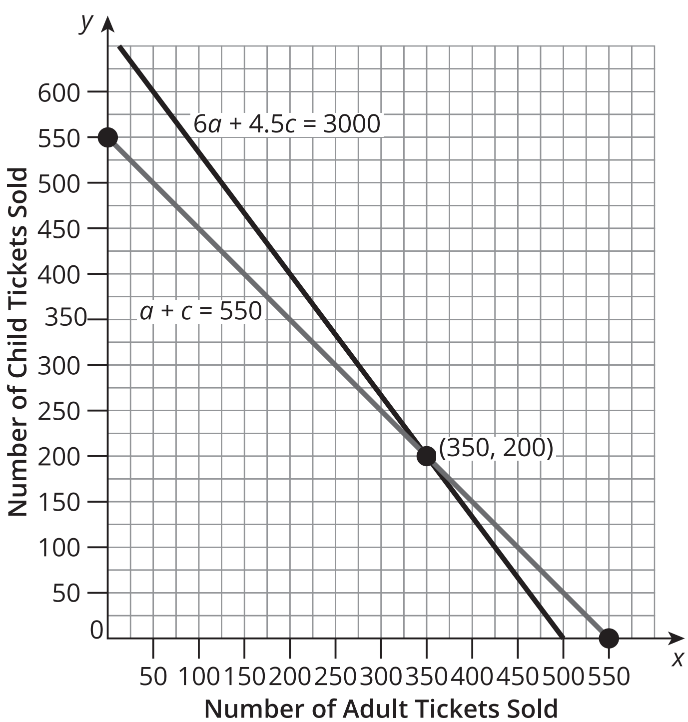
Systems of Equations and Inequalities Summary

LESSON 1 Using Graphing to Solve Systems of Equations

When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system of equations is to graph both equations and identify the point at which the two graphs intersect.

For example, suppose a fundraiser sells 550 tickets for a spaghetti dinner to raise $3000 for charity. An adult ticket costs $6, and a child ticket costs $4.50. To determine how many adult tickets and child tickets were sold, let *a* represent the number of adult tickets purchased, and let *c* represent the number of child tickets purchased.

$$\begin{cases} a + c = 550 \\ 6a + 4.5c = 3000 \end{cases}$$



NEW KEY TERMS

- system of linear equations [sistema de ecuaciones lineales]
- consistent systems [sistemas consistentes]
- inconsistent systems [sistemas inconsistentes]
- standard form of a linear equation [forma estándar/genera de una ecuación lineal]
- substitution method [método de sustitución]
- linear combinations method [método de combinaciones lineales]
- half-plane
- boundary line
- constraints
- solution of a system of linear inequalities [solución de un sistema de desigualdades lineales]

You can use x- and y-intercepts to graph each of the two equations to determine how many of each type of tickets were sold.

The intersection point appears to be (350, 200). There were 350 adult tickets and 200 child tickets sold.

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

LESSON 2 Using Substitution to Solve Linear Systems

The **standard form of a linear equation** can be written as $Ax + By = C$, where A, B, and C are integers, and A and B are not both zero.

Sometimes, a system of equations may not be accurately solved using graphs. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

For example, consider this system of equations.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating the variable. This will be considered the first equation. Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation. Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$. Write the new equation: $1.25x + 1.05(8x) = 30$.

Step 3: Solve the new equation.

$$\begin{aligned} 1.25x + 8.40x &= 30 \\ 9.65x &= 30 \\ x &\approx 3.1 \end{aligned}$$

Now, substitute the value for x into $y = 8x$ to determine the value for y.

$y \approx 8(3.1) \approx 24.8$

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

When a system has no solution, the equation resulting from the substitution step has no solution. When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

LESSON 3 Using Linear Combinations to Solve a System of Linear Equations

The **linear combinations method** is a process used to solve a system of equations by adding two equations so they result in an equation with one variable. Once the value of this variable is determined, it can be used to calculate the value of the other variable. In many cases, one or both of the equations may need to be multiplied by a constant so that the coefficients of the term containing either *x* or *y* are opposites. Then, when the equations are added, the result is an equation in one variable.

For example, consider this system of linear equations.

$$\begin{cases} 7x + 2y = 24 \\ x + y = 15 \end{cases}$$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the bottom equation by a constant that results in opposite coefficients for one of the variables.

$$\begin{array}{r} 7x + 2y = 24 \\ + \;\; -8x - 2y = -30 \\ \hline -x = -6 \\ x = 6 \end{array}$$

Now that the *y*-values are opposite, you can add the equations and solve this linear system.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ -2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for *x* into one of the equations to calculate the value for *y*.

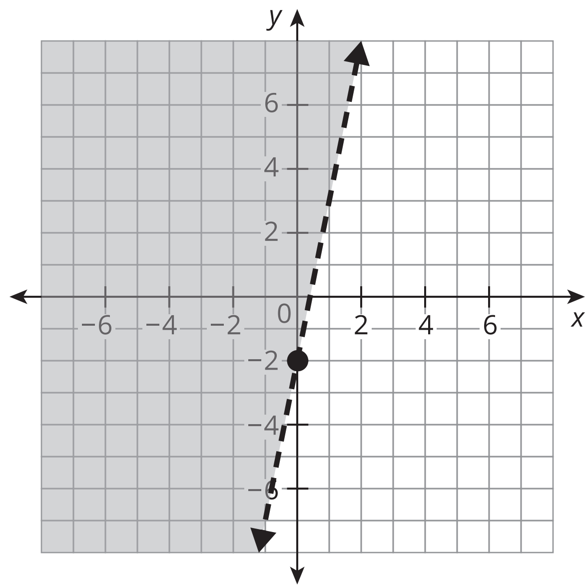
The solution to the system of linear equation is $(6, -9)$.

LESSON 4 Graphing Inequalities in Two Variables

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes. The inequality symbol identifies which half-plane contains the solutions. When the symbol is $\leq$ or $\geq$, the boundary line is part of the solution and is solid. When the symbol is $<$ or $>$, the boundary line is not part of the solution and is represented with a dashed line. Use (0, 0) as a test point to determine which half of the plane is the solution of the inequality and should, therefore, be shaded.

For example, the graph shows the solution to the inequality $y < 5x - 2$. Since the inequality symbol in the solution is $<$, the shaded half-plane does not include points on the line. Since (0, 0) is not a solution, then the region to the right of the dashed line is the solution set.

$$y < 5x - 2$$
$$0 \overset{?}{<} 5(0) - 2$$
$$0 \overset{?}{<} -2$$



LESSON 5 Systems of Linear Inequalities

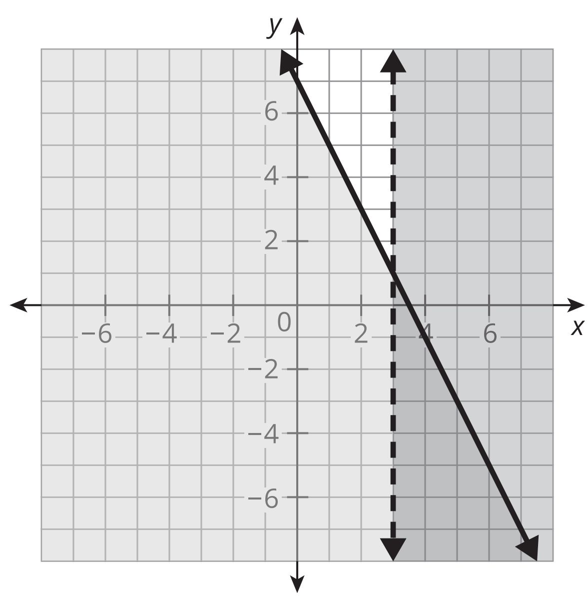
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are “constrained” to lie within a certain region.

The **solution of a system of linear inequalities** is the intersection of the solutions of each inequality. Every point in the intersection region satisfies the system. To determine the solution set of the system, graph each inequality on the same coordinate plane. The region that overlaps is the solution to the system.

For example, the overlapping region of the graphs of the inequalities $2x + y \le 7$ and $x > 3$ is the solution to the system.

$$\begin{cases} 2x + y \le 7 \\ x > 3 \end{cases}$$

Two points that are solutions of the system are (4, −5) and (5, −8).



LESSON 6 Solving Systems of Equations and Inequalities

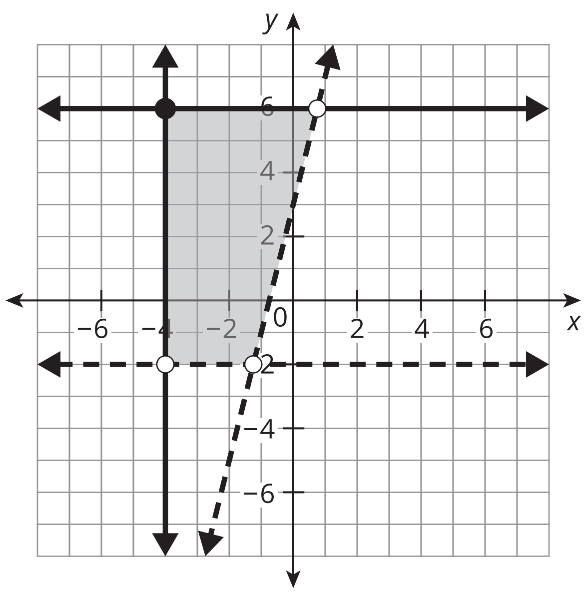
Substitution, linear combinations, and graphing are three methods for determining the solution of a linear system of equations. Use substitution when a variable in one equation can easily be isolated or when the equations are in slope-intercept form. Use the linear combinations method when the coefficients of like terms are opposites or can be easily made into opposites by multiplication. Use the graphing method when the numbers are convenient to graph or when an exact solution is not needed.

To solve a system of linear inequalities with more than two inequalities, graph each linear inequality and shade the correct region that contains the solution sets that satisfy all the inequalities in the system. Also, determine all the points of intersection for the boundary lines that make the vertices of the solution region. Use a closed point when the point is part of the solution region, and use an open dot when the point is not part of the solution region but is a point of intersection.

The graphs show the solution to a system of four inequalities.

$$\begin{cases} x \geq -4 \\ y > -2 \\ y \leq 6 \\ y > 4x + 3 \end{cases}$$

One solution of the system of inequalities is (−2, 2).



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