

ENGLISH



**Bluebonnet
Learning™**

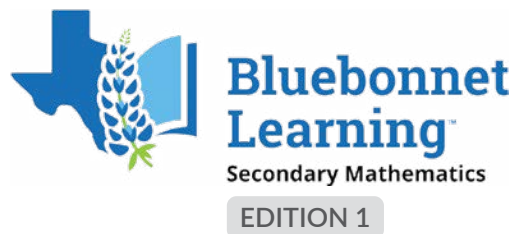
Secondary Mathematics

EDITION 1

Algebra II

Skills Practice

STUDENT EDITION



Algebra II

Skills Practice

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

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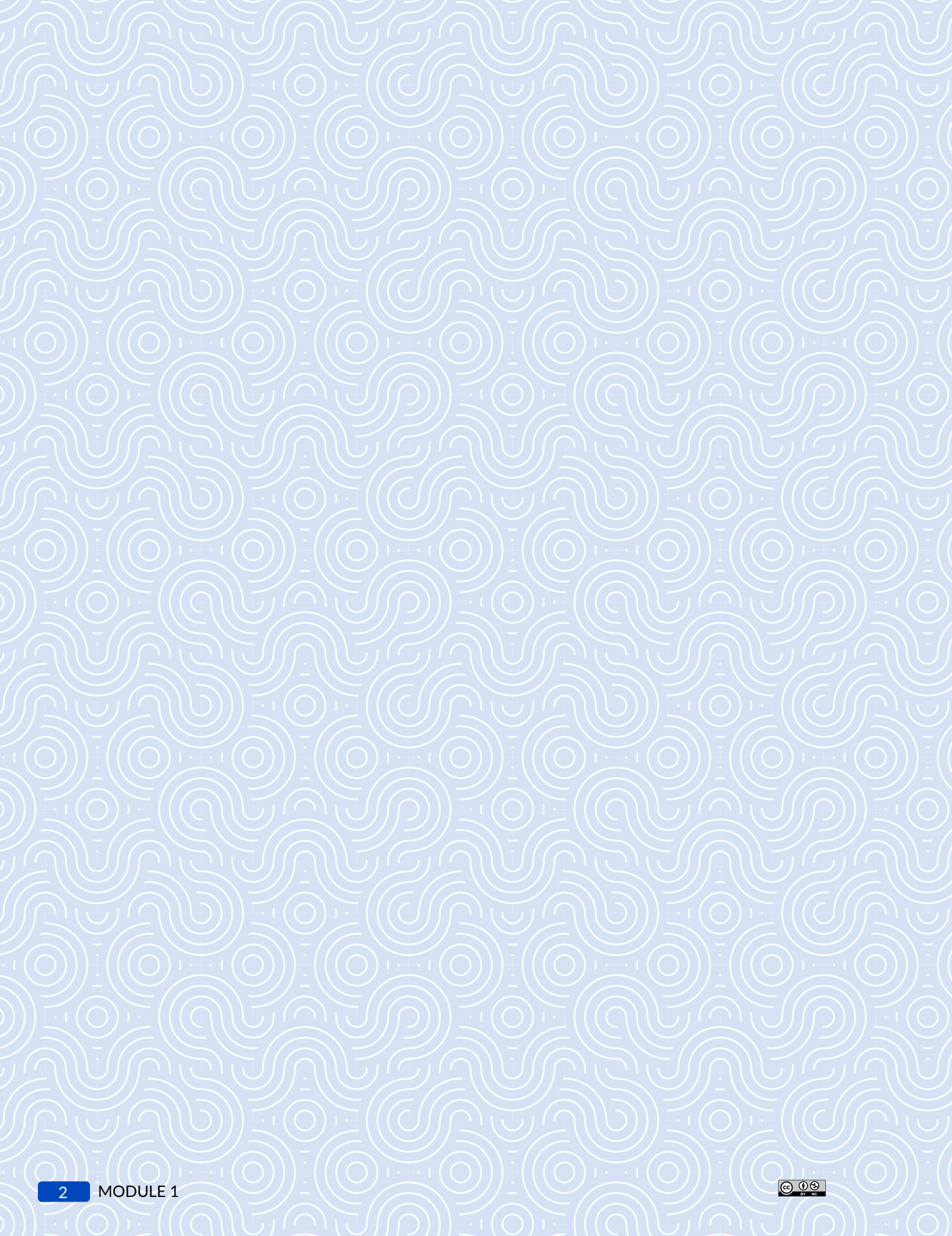
Extending Linear Relationships

TOPIC 1: Absolute Value Functions and Equations

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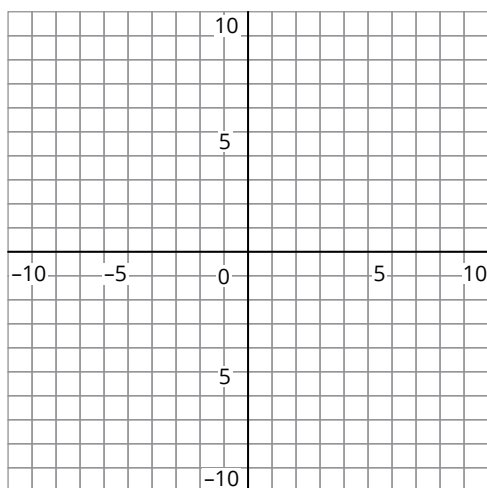
Name _____ Date _____

I. Defining Absolute Value Functions

Topic Practice

- A. Use technology to graph and identify the key attributes of each function. Write the domain and range of the function as inequalities.

1. $f(x) = |x| + 1$



Domain:

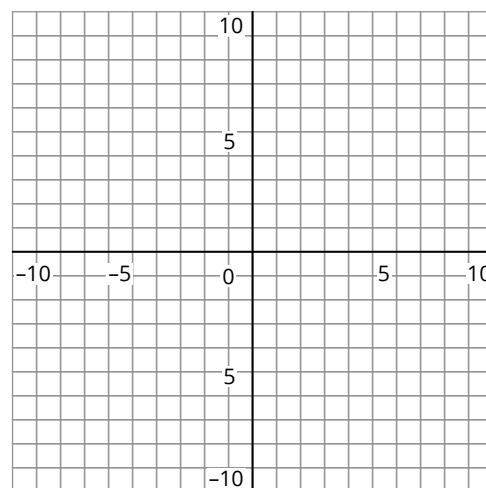
Range:

Vertex:

Intercept(s):

Axis of Symmetry:

2. $f(x) = |x| - 2$



Domain:

Range:

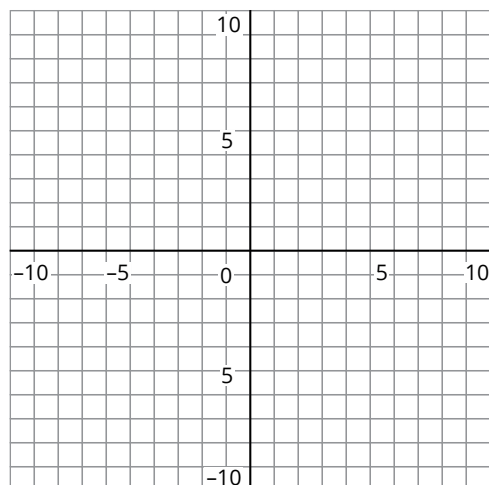
Vertex:

Intercept(s):

Axis of Symmetry:

TOPIC 1 Absolute Value Functions and Equations

3. $f(x) = |x + 2|$



Domain:

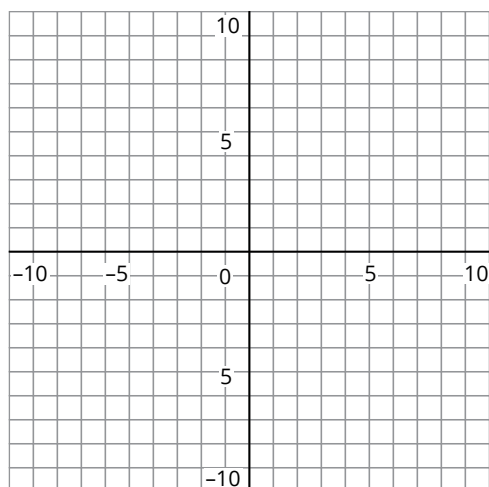
Range:

Vertex:

Intercept(s):

Axis of Symmetry:

5. $f(x) = -|x| + 3$



Domain:

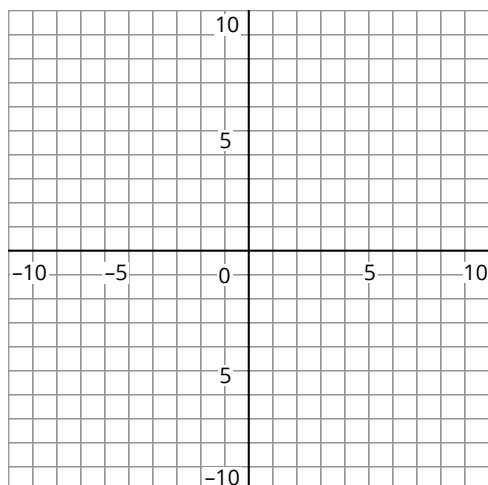
Range:

Vertex:

Intercept(s):

Axis of Symmetry:

4. $f(x) = |x - 5|$



Domain:

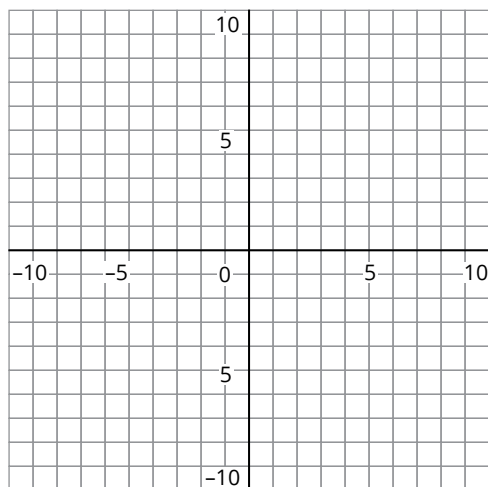
Range:

Vertex:

Intercept(s):

Axis of Symmetry:

6. $f(x) = 2|x| - 1$



Domain:

Range:

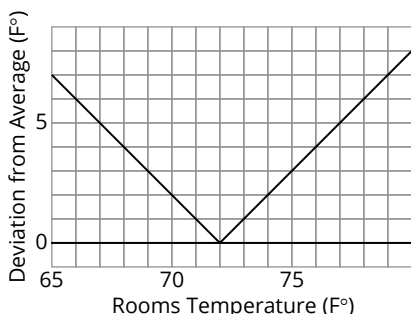
Vertex:

Intercept(s):

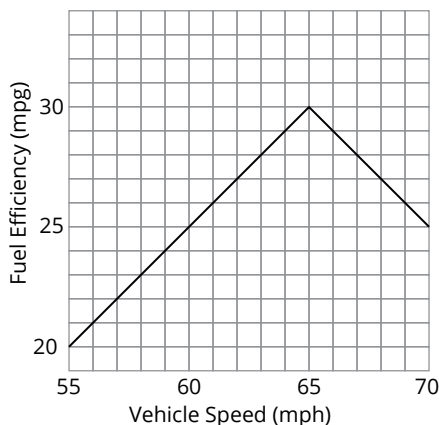
Axis of Symmetry:

B. Use the graph of each problem situation to answer each question.

1. A thermostat is set to maintain an ideal room temperature of 72°F . The deviation of the room temperature x from this ideal value can be represented by $f(x) = |x - 72|$. The room is considered uncomfortable when the temperature is 4.5°F from the ideal temperature. Use the graph to determine when the room is uncomfortable.

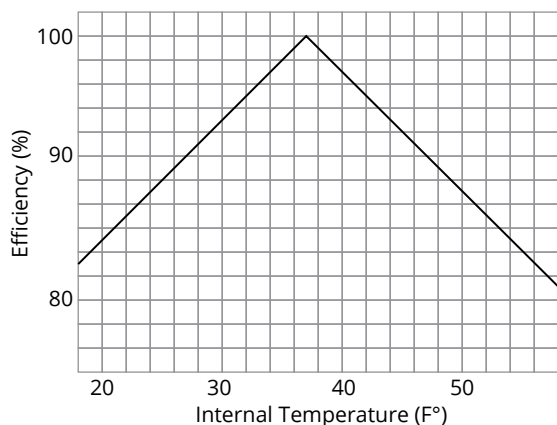


2. The ideal fuel efficiency of a specific car model is 30 miles per gallon (mpg) while traveling 65 miles per hour. When the car deviates from the ideal speed of 65 miles per hour, fuel efficiency is lost. The ideal fuel efficiency can be modeled by $f(x) = -|x - 65| + 30$ where x is the speed of the vehicle. Use the graph to determine the possible vehicle speeds if the car is achieving 24 mpg of fuel efficiency.

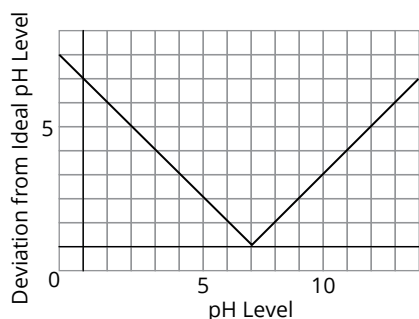


TOPIC 1 Absolute Value Functions and Equations

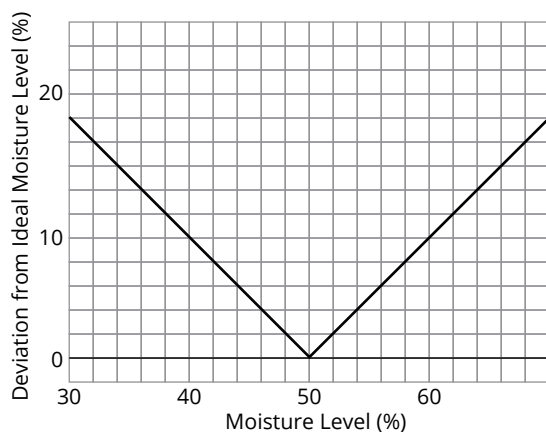
3. A refrigerator operates most efficiently when the internal temperature is maintained at 37°F . When the internal temperature deviates from this ideal temperature, the cooling efficiency of the refrigerator decreases. The cooling efficiency can be modeled by $f(x) = -|x - 37| + 100$ where x represents the internal temperature of the refrigerator. Use the graph of the function to determine the temperature when the refrigerator is working at 85% efficiency.



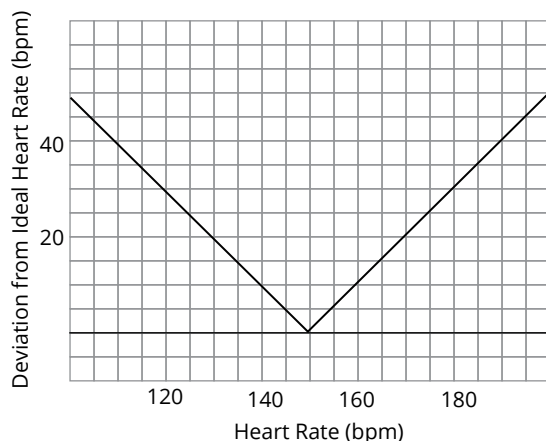
4. A fish tank is designed to maintain an ideal water pH level of 7.0 for the health of the fish. The deviation of the actual pH level x from this ideal value can be represented by $f(x) = |x - 7.0|$. The water is considered unsafe for the fish when the pH level deviates by 0.5 units from the ideal pH level. Use the graph of the function to determine when the water becomes unsafe for the fish.



5. A plant nursery is designed to maintain the soil moisture level at an optimal value of 50% for the healthy growth of plants. The deviation of the actual soil moisture x from this optimal level can be represented by $f(x) = |x - 50|$. The plants begin to show signs of stress if the moisture level is more than 10% away from the ideal value. Use the graph to determine the soil moisture levels at which plants show signs of stress.



6. An athlete's heart rate during exercise should ideally be maintained at 150 beats per minute (bpm) for optimal performance and safety. The deviation of the heart rate x from this ideal rate can be modeled by $f(x) = |x - 150|$. The athlete's performance is considered compromised when their heart rate deviates by more than 35 bpm from the ideal rate. Use the graph to determine the heart rates at which the athlete's performance is compromised.



TOPIC 1 Absolute Value Functions and Equations

Extension

1. A meeting room is designed to maintain an optimal noise level of 30 decibels (dB) for maximum concentration. The deviation of the actual noise level x from this optimal level can be represented by $f(x) = |x - 30|$. The room is considered too noisy or too quiet to maintain concentration when the noise level deviates by more than 15 dB from the optimal level. Write a set of inequalities to represent the undesired noise levels for the room.

Spaced Practice

Solve each system of equations.

1.
$$\begin{cases} 2x + 3y = 12 \\ x - y = 1 \end{cases}$$

2.
$$\begin{cases} 3x + 4y = 27 \\ x = 2y - 1 \end{cases}$$

3.
$$\begin{cases} 2x + 3y = 12 \\ x = 2y - 1 \end{cases}$$

Factor each polynomial to a product of two binomials.

4. $x^2 + 11x + 18$

5. $6x^2 + 11x + 5$

6. $9x^2 - 49$

II. Transformations of Absolute Value Functions

Topic Practice

A. For each transformation, describe the effect on the graph of the parent absolute value function, $f(x) = |x|$.

1. $a(x) = |-3x|$

2. $h(x) = -\left|\frac{1}{2}x\right|$

3. $k(x) = 2|x| - 3$

4. $b(x) = -|x + 2|$

5. $m(x) = \frac{1}{2}|x - 5|$

6. $p(x) = -3|x| - \frac{1}{3}$

TOPIC 1 Absolute Value Functions and Equations

- B. Graph each transformation of the parent absolute value function, $f(x) = |x|$. Then, identify the key attributes, and write the absolute value function. Write the domain and range using inequalities.

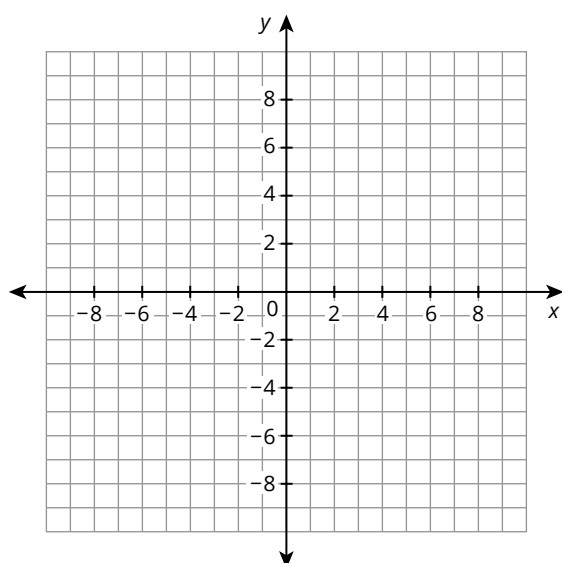
1. $k(x) = -f(2x)$

Vertex:

Domain:

Range:

$k(x) =$



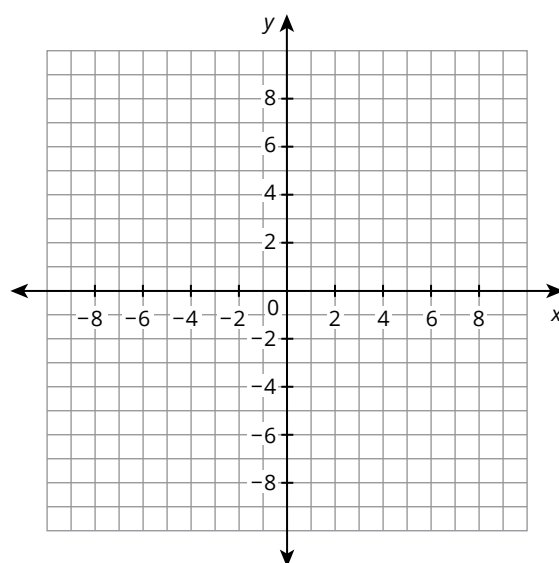
2. $m(x) = f(x) + 5$

Vertex:

Domain:

Range:

$m(x) =$



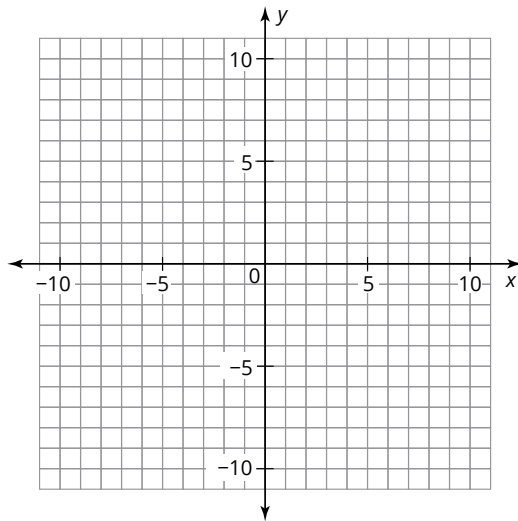
3. $r(x) = f(-2(x + 2))$

Vertex:

Domain:

Range:

$r(x) =$



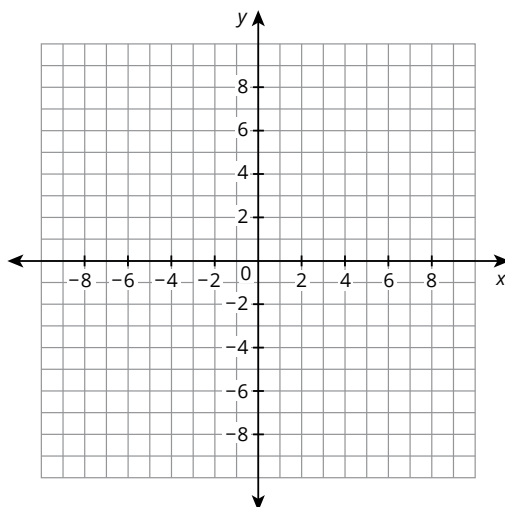
5. $a(x) = -f(x + 1) + 2$

Vertex:

Domain:

Range:

$a(x) =$



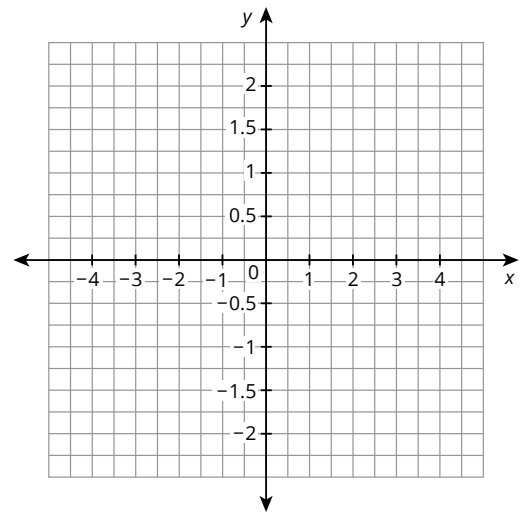
4. $j(x) = \frac{1}{4}f(x) + \frac{1}{2}$

Vertex:

Domain:

Range:

$j(x) =$



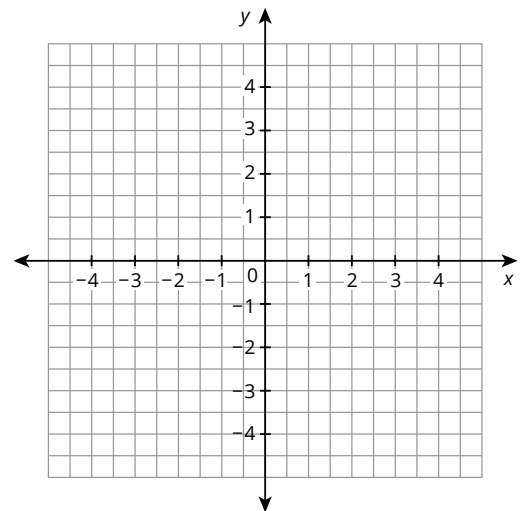
6. $b(x) = -3f(x + 2) - 1$

Vertex:

Domain:

Range:

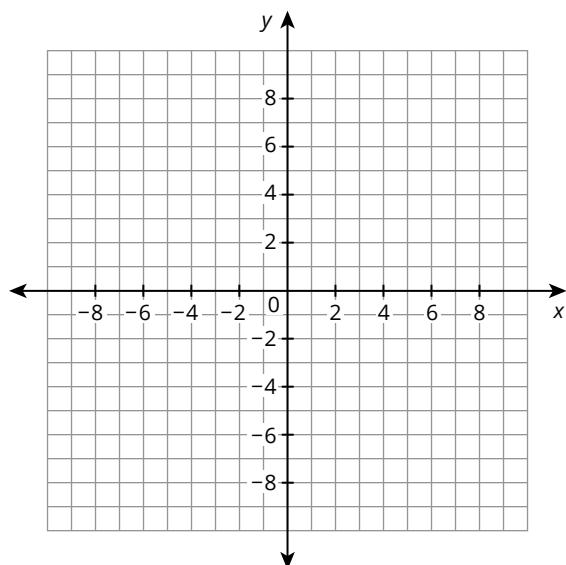
$b(x) =$



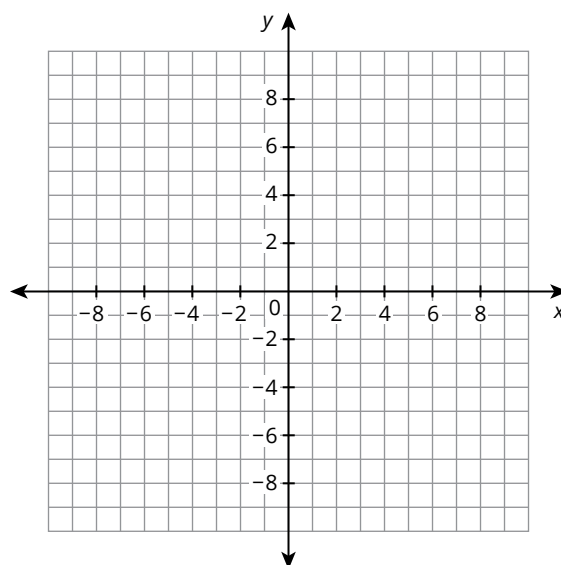
TOPIC 1 Absolute Value Functions and Equations

- C. Write a transformed absolute value function in terms of the parent absolute value function $f(x) = |x|$ for the given characteristics. Then, graph the transformed function.

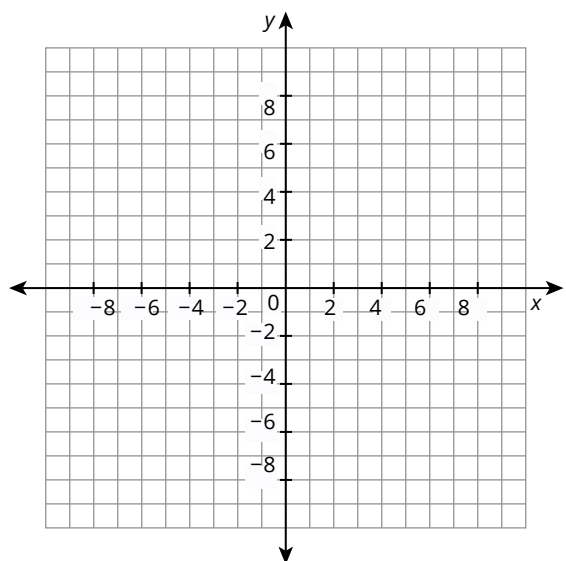
1. Reflected across the line $y = 0$ and translated up 4 units



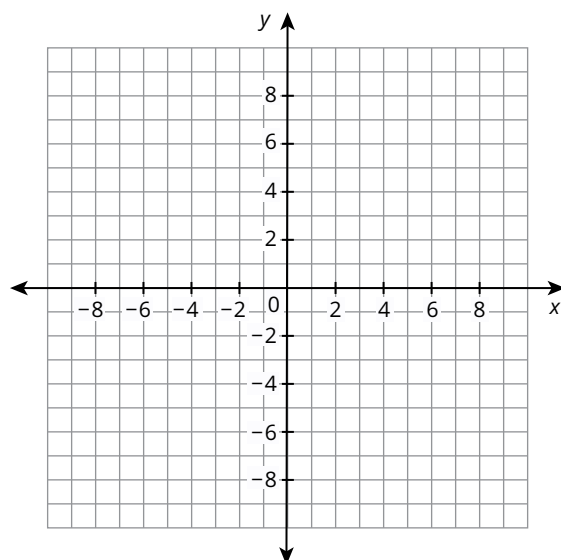
2. Reflected across the line $y = 0$ and horizontally stretched by a factor of $\frac{3}{2}$



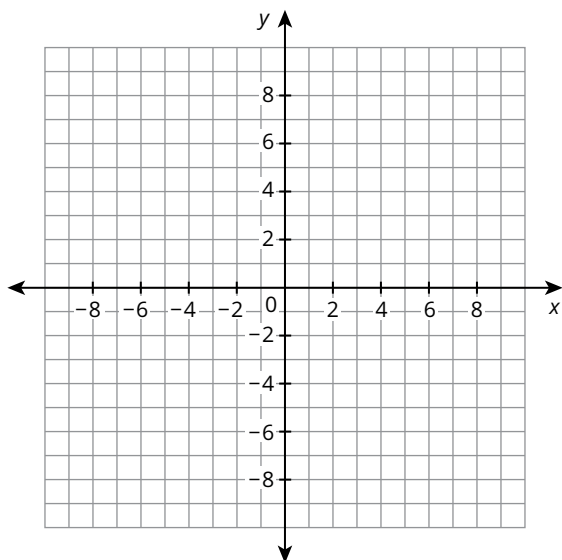
3. Vertex $(0, -2)$ and range $y \leq -2$



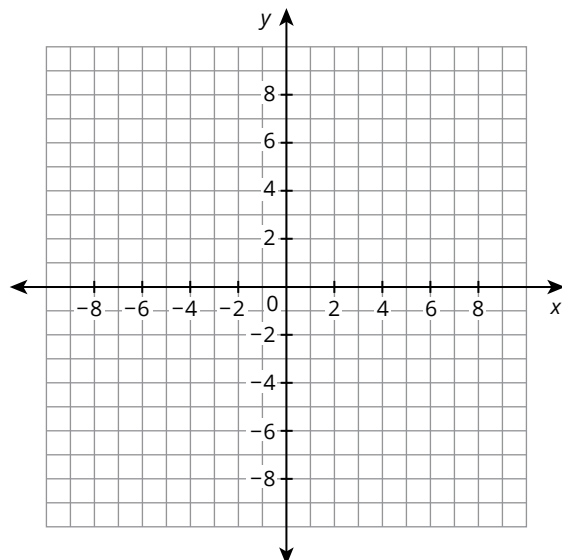
4. Vertex $(4, 0)$ and horizontally stretched by a factor of 2



5. Vertically compressed by a factor of $\frac{1}{3}$ and translated down 1 unit



6. Axis of symmetry $x = 5$ and range $y \geq -3$

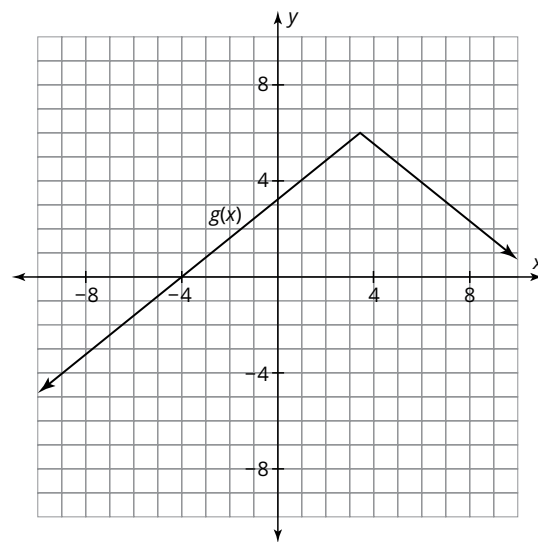


D. Consider the function, $g(x) = |x|$. Complete the table.

Transformation	Transformation Function Form	Equation	Coordinate Notation
1. Vertical stretch by a factor of 3, and horizontal translation of 1 unit right			
2.			$(x, y - 4)$
3.		$y = x + 2 + 1$	
4.	$f(x) = g(-x)$		
5. A horizontal compression of $\frac{1}{2}$ and reflection over the x-axis			
6.			$(x - 3, \frac{1}{3}y - 4)$

Extension

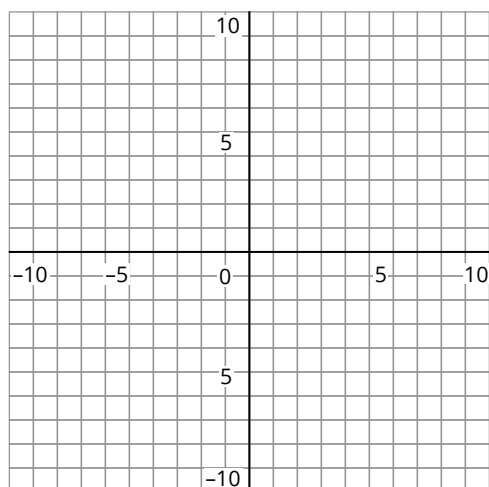
The function $g(x)$ shown is a transformation of $f(x) = |x|$.
Write the function $g(x)$ in terms of $f(x)$.



Spaced Practice

- Graph each absolute value function and identify the key attributes.

a. $f(x) = |x|$



Domain:

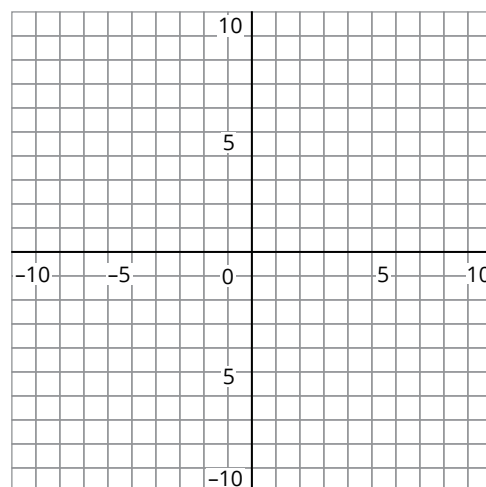
Range:

Vertex:

Intercept(s):

Axis of Symmetry:

b. $f(x) = |x| - 3$



Domain:

Range:

Vertex:

Intercept(s):

Axis of Symmetry:

TOPIC 1 Absolute Value Functions and Equations

2. Factor each polynomial to a product of two binomials.

a. $16x^2 - 25$

b. $4x^2 + 12x + 8$

c. $x^2 + 5x + 6$

III. Absolute Value Equations and Inequalities

Topic Practice

A. Solve each absolute value equation.

1. $|x - 3| = 6$

2. $|\frac{x}{5}| = 4$

3. $|\frac{x+2}{4}| = 7$

4. $3|x - 4| = 27$

5. $|\frac{9-x}{6}| - 2 = 1$

6. $-8|x + 3| = -64$

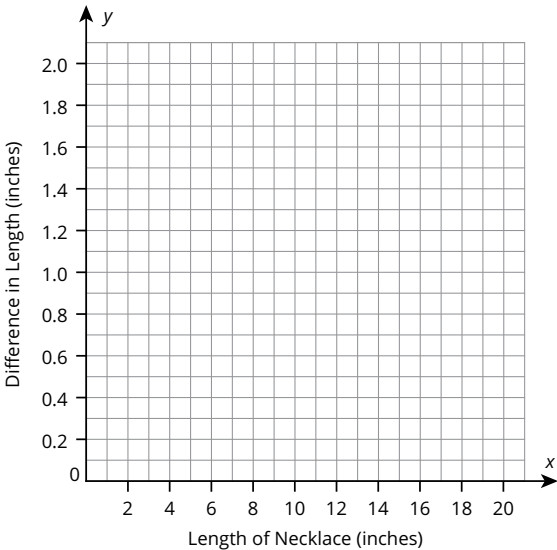
B. Write an equation to represent each problem situation.

- | | |
|---|---|
| <p>1. The recommended tire pressure for a specific tire is 32 psi. The tire is considered underinflated or overinflated if the pressure deviates by more than 3 psi. Write an equation to represent when the tire is underinflated or overinflated.</p> | <p>2. A reservoir is ideally maintained at a water level of 50 feet. If the water level deviates by more than 5 feet from this ideal level, it becomes critical. Write an equation to represent when the water has reached critical levels.</p> |
| <p>3. An electronic circuit operates most efficiently at a voltage of 5 volts. If the voltage deviates by more than 0.1 volts, the circuit's performance is compromised. Write an equation to represent this situation.</p> | <p>4. The ideal operating temperature for an engine is 200°F. If the engine temperature deviates more than 50°F from this optimal level, performance is reduced. Write an equation to determine when performance will be reduced.</p> |
| <p>5. A factory produces a metal rod with an ideal length of 20 cm. The rod is considered defective if its length deviates by more than 0.1 cm from the ideal length. Write an equation to represent when the rod will be considered defective.</p> | <p>6. A protein bar is designed to have an ideal sugar content of 3 grams per serving. If the sugar content deviates by more than 0.25 grams, it affects the taste. Write an equation to represent when the taste will be affected.</p> |

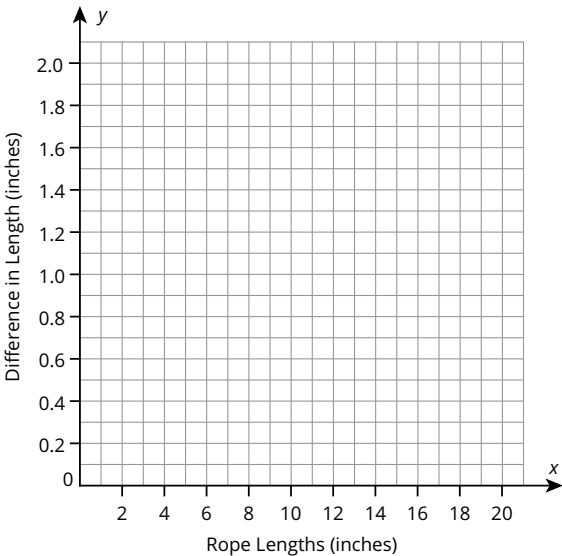
TOPIC 1
 Absolute Value Functions and Equations

C.
 Write and graph the function that represents each problem situation. Verify your answer algebraically.

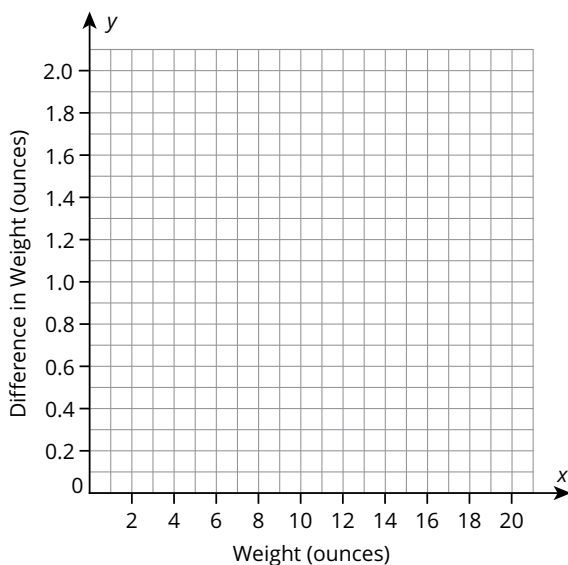
- A jewelry company is making 16-inch bead necklaces. The specifications allow for a difference of 0.5 inch. Write a function to represent the difference between the necklaces manufactured and the specifications. Graph the function. Use your graph to determine the necklace lengths that meet the specifications and verify your answer algebraically.



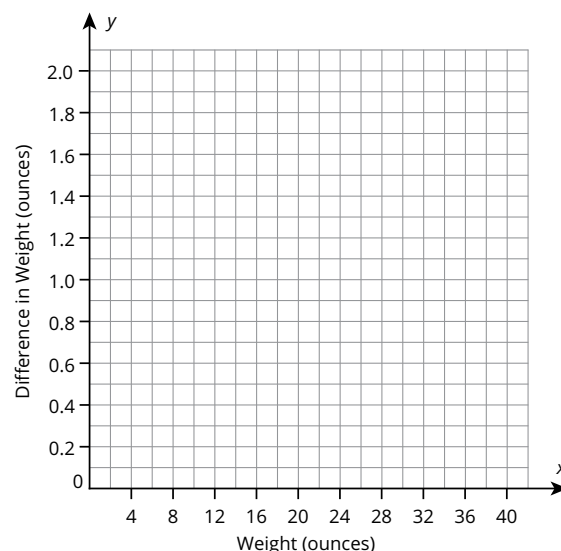
- Julian is cutting lengths of rope for a class project. Each rope length should be 10 inches long. The specifications allow for a difference of 1 inch. Write a function to represent the difference between the rope lengths cut and the specifications. Graph the function. Use your graph to determine the rope lengths that meet the specifications and verify your answer algebraically.



3. A snack company is filling bags with pita chips sold by weight. Each bag should contain 8 ounces of chips. The specifications allow for a difference of 0.25 ounce. Write a function to represent the difference between the weight of a bag of chips and the specifications. Graph the function. Use your graph to determine the weights that meet the specifications and verify your answer algebraically.

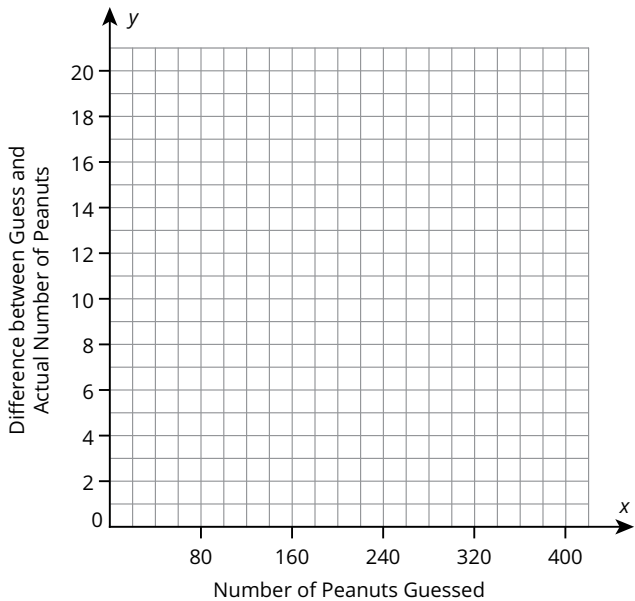


4. A cereal company is filling boxes with cereal sold by weight. Each box should contain 32 ounces of cereal. The specifications allow for a difference of 0.5 ounce. Write a function to represent the difference between the weight of a box of cereal and the specifications. Graph the function. Use your graph to determine the weights that do **NOT** meet the specifications and verify your answer algebraically.

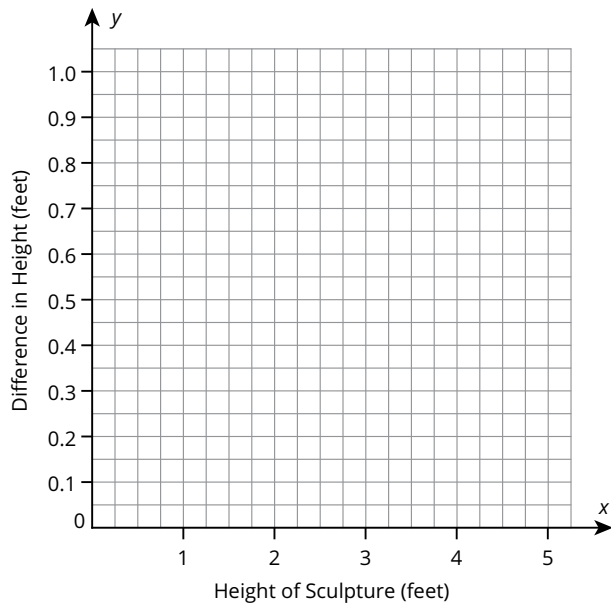


TOPIC 1 Absolute Value Functions and Equations

5. Guests at the school harvest festival are asked to guess how many peanuts are in a jar. The jar contains 260 peanuts. All guests within 10 peanuts of the correct answer win a prize. Write a function to represent the difference between a guess and the actual number of peanuts in the jar. Graph the function. Use your graph to determine the possible guesses that will **NOT** win a prize and verify your answer algebraically.



6. The rules of an art contest state that sculptures submitted should be 3 feet high but allow for a difference of 6 inches. Write a function to represent the difference between a sculpture that is submitted and the specifications. Graph the function. Use your graph to determine the heights that do **NOT** meet the specifications and verify your answer algebraically.



D. Match each compound inequality to its equivalent absolute value inequality.

1. a. $-15 < -3x + 7 < 15$ $|-3x + 7| \leq 15$

b. $-3x + 7 \leq -15$ or $-3x + 7 \geq 15$ $|-3x + 7| > 15$

c. $-15 \leq -3x + 7 \leq 15$ $|-3x + 7| < 15$

d. $-3x + 7 < -15$ or $-3x + 7 > 15$ $|-3x + 7| \geq 15$

2. a. $-2 \leq 6x - 5 \leq 2$ $|6x - 5| > 2$

b. $6x - 5 < -2$ or $6x - 5 > 2$ $|6x - 5| \geq 2$

c. $6x - 5 \leq -2$ or $6x - 5 \geq 2$ $|6x - 5| < 2$

d. $-2 < 6x - 5 < 2$ $|6x - 5| \leq 2$

3. a. $-11 < 2x - 10 < 11$ $|2x - 10| > 11$

b. $2x - 10 \leq -11$ or $2x - 10 \geq 11$ $|2x - 10| \leq 11$

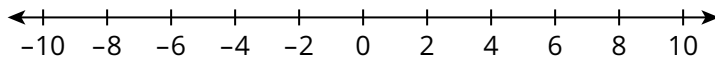
c. $-11 \leq 2x - 10 \leq 11$ $|2x - 10| < 11$

d. $2x - 10 < -11$ or $2x - 10 > 11$ $|2x - 10| \geq 11$

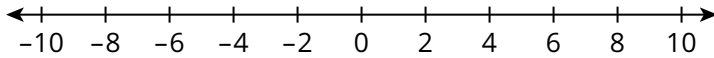
TOPIC 1 Absolute Value Functions and Equations

- E. Solve the linear absolute value inequality by rewriting it as an equivalent compound inequality. Then graph your solution on the number line.

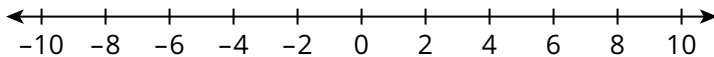
1. $|x - 3| \geq 4$



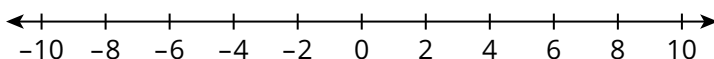
2. $5 > |2x + 7|$



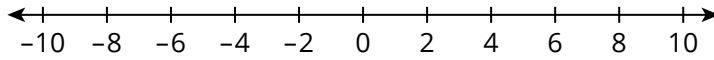
3. $|3x - 1| > 11$



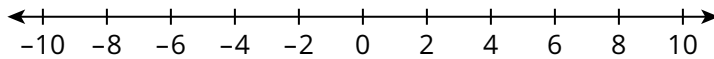
4. $|2x - 8| - 1 \leq 3$



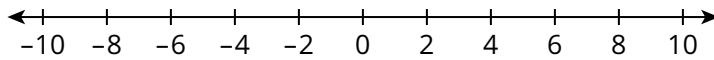
5. $|3x - 2| < -4$



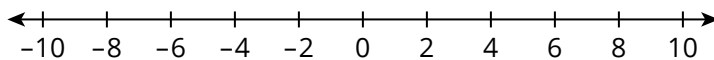
6. $|-4x + 1| \geq 9$



7. $|-2x - 5| < 3$



8. $-12 < |x - 7|$



D. Match each compound inequality to its equivalent absolute value inequality.

1. a. $-15 < -3x + 7 < 15$ $|-3x + 7| \leq 15$

b. $-3x + 7 \leq -15$ or $-3x + 7 \geq 15$ $|-3x + 7| > 15$

c. $-15 \leq -3x + 7 \leq 15$ $|-3x + 7| < 15$

d. $-3x + 7 < -15$ or $-3x + 7 > 15$ $|-3x + 7| \geq 15$

2. a. $-2 \leq 6x - 5 \leq 2$ $|6x - 5| > 2$

b. $6x - 5 < -2$ or $6x - 5 > 2$ $|6x - 5| \geq 2$

c. $6x - 5 \leq -2$ or $6x - 5 \geq 2$ $|6x - 5| < 2$

d. $-2 < 6x - 5 < 2$ $|6x - 5| \leq 2$

3. a. $-11 < 2x - 10 < 11$ $|2x - 10| > 11$

b. $2x - 10 \leq -11$ or $2x - 10 \geq 11$ $|2x - 10| \leq 11$

c. $-11 \leq 2x - 10 \leq 11$ $|2x - 10| < 11$

d. $2x - 10 < -11$ or $2x - 10 > 11$ $|2x - 10| \geq 11$

Extension

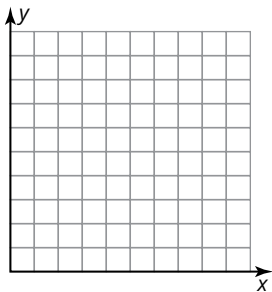
John, Rasheed, and George are different ages. Rasheed is six less than twice John's age. George's age is nine more than half of John's age. The difference between Rasheed and George's ages is no more than nine years.

1. Write an expression that represents the difference between Rasheed and George's ages.
2. Write an absolute value inequality to represent the maximum difference in their ages.
3. Determine whether it is possible for John to be twenty years old. Explain your reasoning.
4. Algebraically determine the greatest and least age John can be so that the difference between Rasheed and George's ages is no more than nine years.

TOPIC 1 Absolute Value Functions and Equations

Spaced Practice

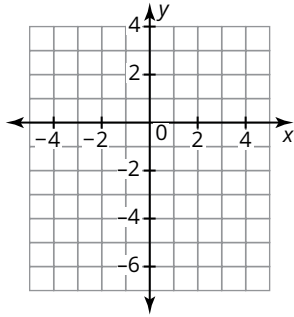
1. The members of a youth football program are required to sell tickets to chicken dinners for the program's fundraiser. They can sell adult tickets for \$10 and child tickets for \$5. Each athlete must bring in at least \$350 from the ticket sales.
 - a. Write an expression to represent the total amount of money an athlete makes from ticket sales.
Let x represent the number of adult tickets sold and let y represent the number of child tickets sold.
 - b. Write an inequality in two variables to represent the amount an athlete must make.
 - c. Graph the inequality from part (b).



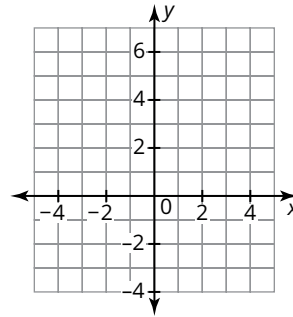
- d. Use the graph to determine whether an athlete will bring in at least \$350 if they sell 10 adult tickets and 30 child tickets. Explain your reasoning.

2. Given the function $f(x) = |x|$. Sketch a graph of each new function.

a. $g(x) = -|x + 1| - 3$



b. $h(x) = \frac{3}{4}|x - 2| + 1$



3. Factor each expression.

a. $9x^2 - 16$

b. $a^2 - 5a - 14$

c. $6y^2 + 11y - 10$

Name _____ Date _____

I. Solving Systems of Equations

Topic Practice

A. Solve each system of linear equations using substitution.

1.
$$\begin{cases} y = 2x + 2 \\ y = -7x - 1 \end{cases}$$

2.
$$\begin{cases} y = 5x - 8 \\ y = x - 20 \end{cases}$$

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3.
$$\begin{cases} x + y = 12 \\ y = -15 + 8x \end{cases}$$

4.
$$\begin{cases} y = -3.5x - 2.4 \\ y = -5.2x + 1 \end{cases}$$

5.
$$\begin{cases} \frac{1}{2}x - y = 10 \\ y = \frac{3}{2}x - 8 \end{cases}$$

6.
$$\begin{cases} x = 4y - 9 \\ 2x - 8y = 10 \end{cases}$$

B. Solve the system of equations using linear combinations.

1.
$$\begin{cases} x - 5y = 8 \\ 4x + 5y = 7 \end{cases}$$

2.
$$\begin{cases} -2.0x + 0.3y = 5.5 \\ 2.0x - 0.8y = -7.5 \end{cases}$$

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3.
$$\begin{cases} y = -7x - 3 \\ y = 4x + 8 \end{cases}$$

4.
$$\begin{cases} 3y = -2x + 4 \\ 4x + 6y = 8 \end{cases}$$

5.
$$\begin{cases} \frac{1}{2}x - \frac{3}{4}y = \frac{5}{8} \\ -2x + \frac{1}{2}y = -\frac{3}{2} \end{cases}$$

6.
$$\begin{cases} -4y = 12x + 10 \\ -y = 3x + 8 \end{cases}$$

- C. Use substitution to solve each system of three linear equations in three variables.

1.
$$\begin{cases} x - 3z = -5 \\ 7x - 3y - 5z = 19 \\ 2x - y + 2z = 16 \end{cases}$$

2.
$$\begin{cases} x + 2y - z = 4 \\ x + 2y + z = 2 \\ x + y + z = -2 \end{cases}$$

$$3. \begin{cases} 5x + y - 2z = 5 \\ 3x + 4y - z = -7 \\ x - 5y + 2z = 19 \end{cases}$$

4.
$$\begin{cases} x + y + z = 500 \\ 2x + 4y + 5z = 2200 \\ y - z = -50 \end{cases}$$

$$5. \begin{cases} 25a + 5b + c = 2 \\ 49a + 7b + c = 6 \\ 9a + 3b + c = 0 \end{cases}$$

6.
$$\begin{cases} x + 2y + 4z = 4 \\ 2x + y + 2z = 7 \\ x + 2y + z = 5 \end{cases}$$

- D. Solve the system of three linear equations in three variables using Gaussian elimination.

$$1. \begin{cases} 2x - 4y + 5z = -33 \\ 4x - y = -5 \\ -2x + 2y - 3z = 19 \end{cases}$$

2.
$$\begin{cases} 3x + 2y + z = 1 \\ x + y + z = 0 \\ 5x + 3y - 2z = -4 \end{cases}$$

$$3. \begin{cases} a - 2b - 3c = 1 \\ 2a - 3b + 3c = 10 \\ 4a - b + c = 10 \end{cases}$$

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4.
$$\begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

$$5. \begin{cases} p - 3q + r = 4 \\ 3p + 2q + r = 3 \\ -6p - 4q - 2r = 1 \end{cases}$$

$$6. \begin{cases} -3x + 2y - 5z = -14 \\ 2x - 3y + 4z = 10 \\ x + y + z = 4 \end{cases}$$

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E. Write and solve the system of linear equations.

1. James is helping raise money for a dog rescue foundation by selling baskets of dog treats.
 - Basket A contains 4 peanut butter treats, 3 bacon treats, and 2 cheese treats and sells for \$12.31.
 - Basket B contains 3 peanut butter treats, 4 bacon treats, and 3 cheese treats and sells for \$13.70.
 - Basket C contains 6 peanut butter treats, 5 bacon treats, and 4 cheese treats and sells for \$20.25.

What is the cost for one of each kind of treat?

2. A local theatre is selling tickets for their upcoming production. They offer 3 different ticket prices for children, adults, and seniors. One receipt shows a customer bought 2 child tickets, 2 adult tickets, and 1 senior ticket for \$110. Another receipt shows a customer bought 3 child tickets, 4 adult tickets, and 2 senior tickets for \$201. A third receipt shows a customer bought 2 child tickets and 2 adult tickets for \$88. How much does each type of ticket cost?

3. A movie theatre sells child tickets for \$7.50, adult tickets for \$9.75, and student tickets for \$8. The movie theatre sold twice as many child tickets as student tickets. If a total of 191 tickets for the film were sold and the total income was \$1593.50, how many of each ticket were sold?
4. Ricardo has 12 bills in his wallet that total \$110. He has a mix of five-dollar bills, ten-dollar bills, and twenty-dollar bills. The number of five-dollar bills is triple the amount of twenty-dollar bills. How many of each denomination does Ricardo have in his wallet?

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- Lauren received an inheritance of \$30,000. She wants to put some of the money into a savings account that earns 1.5% interest annually and invest the rest in certificates of deposit (CDs) and bonds. A broker tells her that CDs pay 4% interest annually and bonds pay 5% interest annually. She wants to earn \$800 interest per year and wants to invest twice as much money in CDs as in bonds. How much should she put in each type of investment?
- Alejandra is helping her grandmother with her flower garden. She decides to add some clematis vines, rose bushes, and peony plants to her garden. The price of a clematis vine is \$3 more than twice a peony plant. The price of a rose bush is \$5 more than a clematis vine. She decides to buy 3 clematis plants, 4 rose bushes, and 2 peony plants for a total of \$233. How much does each plant cost?

Extension

1. Ethan has 15 bills in his wallet that total \$145. He has a mix of five-dollar bills, ten-dollar bills, and twenty-dollar bills. The number of ten-dollar bills is one less than twice the number of twenty-dollar bills.
 - a. Write a system of three linear equations in three variables to represent this situation. Be sure to define your variables.

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- b. How many of each denomination does John have in his wallet?
Solve this system using Gaussian elimination.

Spaced Practice

1. A diver begins at a depth of 4 meters below sea level. He dives to a position that is exactly 10 meters away from the starting depth.
 - a. Formulate an absolute value equation to represent the problem situation.
 - b. Determine the depth of the diver.

2. Solve the absolute value inequality.

$$|3x + 2| \geq 8$$

3. Factor each expression

- a. $16x^2 - 49y^2$

- b. $25a^2 - 1$

II. Optimization

A. Write a system of inequalities to represent the constraints in each problem situation. Be sure to define your variables.

1. A company manufactures two kinds of lawn mowers, gas-powered and electric. It can manufacture at most 100 mowers per day but must manufacture at least 25 gas-powered mowers each day to maintain its inventory.
2. A bike shop sells adult-sized and child-sized bikes. The shop owner believes he will sell at least 20, but no more than 40 bikes per month. He also believes he will sell at least twice as many adult-sized bikes as he will child-sized bikes.

3. A farmer wants to plant corn and oats on her 200-acre farm. She wants to plant at least 50 acres but no more than 100 acres of corn and no more than 125 acres of oats.
4. A university's athletic department has 1200 travel packages to their holiday bowl game. Individuals who buy one of the packages can choose to travel by coach bus or by school bus. Each coach bus can carry 60 passengers and each school bus can carry 48 passengers. The university is only able to locate a total of 20 vehicles.
5. A local runner's club rented an auditorium seating 600 people to hear a presentation by a world-record runner. Tickets to the event cost \$5 for anyone who is 18 years old or younger and \$7 for everyone else. The club already knows that at least fifty \$5 tickets and two hundred \$7 tickets have been sold.
6. A local craft shop sells necklaces and bracelets. The shop owner predicts she will sell more than 30 but less than 60 pieces of jewelry each month. She also believes the number of necklaces she will sell will be more than 3 times as many bracelets she will sell.

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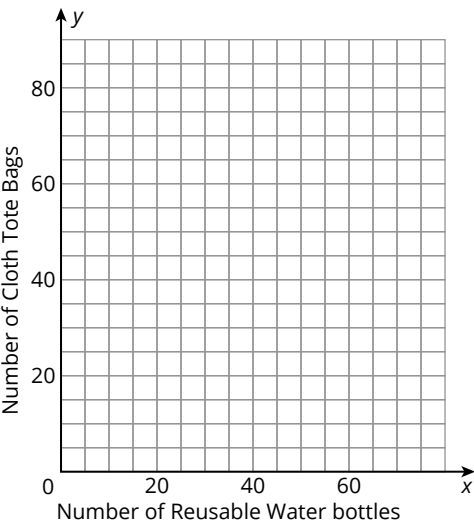
B. For each problem situation, formulate a system of inequalities. Graph the solution set of the system of inequalities. Then, use the graph to answer each question.

1. A school's Green Club sells reusable water bottles and cloth tote bags, which they store in a supply closet. The club purchases each water bottle for \$8, while each tote bag costs \$5.

The club faces the following constraints:

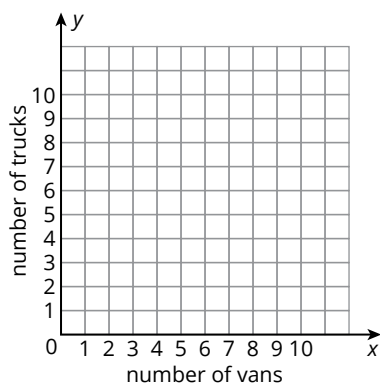
- Their total spending on water bottles and tote bags cannot exceed \$400.
- The supply closet can hold a maximum of 70 items total.
- They must stock at least 3 water bottles and at least 3 tote bags.

What is the maximum number of water bottles the club can purchase?



2. A catering company uses vans and trucks to deliver meals to events. Each truck can deliver up to 20 food trays, while each van can deliver up to 10 food trays. For an upcoming event, the company must deliver at least 100 food trays and only has up to 8 vehicles available. Additionally, the company requires at least 3 food trucks to handle the larger, bulkier food trays.

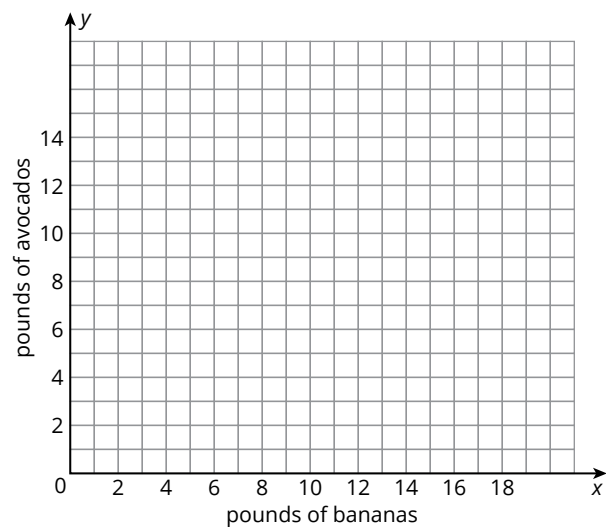
Identify 3 possible combinations of vans and trucks the catering company can use for the delivery.



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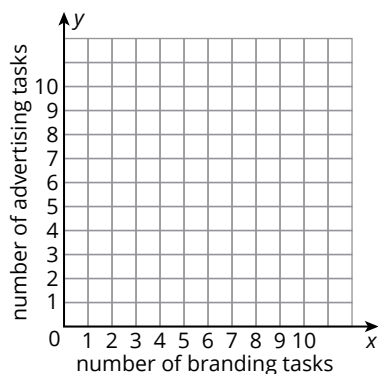
3. Hailey is shopping for bananas and avocados. Bananas cost \$2 per pound, and avocados cost \$3 per pound. She has a budget of \$30 to spend. Hailey wants to buy more than 4 pounds of bananas and at least 2 pounds of avocados. Additionally, the total weight of her purchase cannot exceed 12 pounds.

Determine three different combinations of pounds of bananas and pounds of avocados Hailey can buy.



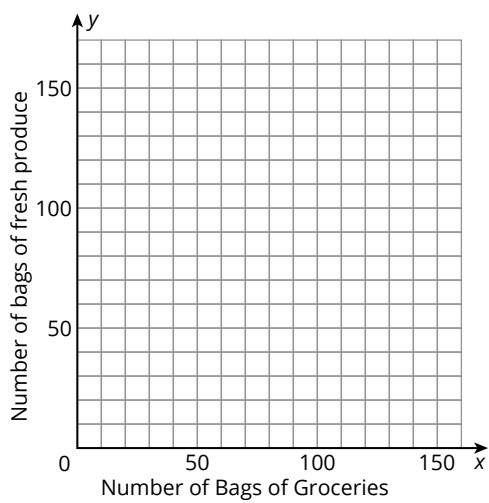
4. A marketing team handles the branding and advertising tasks of a newly launched product. Each branding task requires 5 hours, and each advertising task requires 8 hours. The team is available for less than 40 hours to complete all the branding and advertising tasks. They must complete at least 2 branding and 2 advertising tasks but cannot handle more than 7 total tasks.

Determine whether the marketing team can complete 4 branding and 3 advertising tasks. Explain your reasoning.

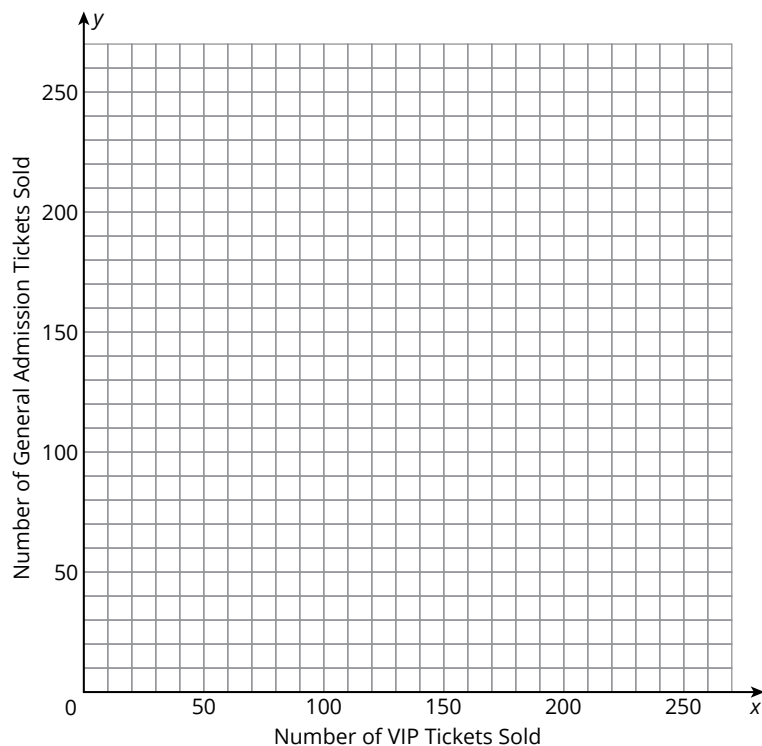


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5. A food bank distributes bags of groceries and packs of fresh produce. Each bag of groceries weighs 13 pounds, and each pack of fresh produce weighs 10 pounds. The distribution truck can only carry up to 1500 pounds and hold a maximum of 150 packages. On a particular day, the food bank distributes more than 30 bags of groceries and more than 25 packs of fresh produce.
- Determine whether the food bank could distribute 40 bags of groceries and 50 packs of fresh produce. Explain your reasoning.



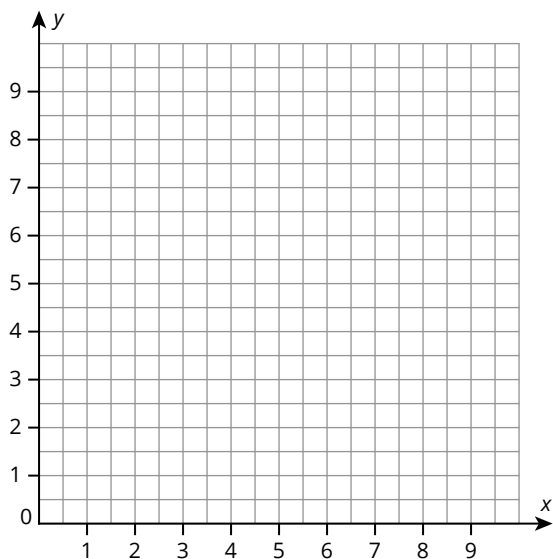
6. Nicky organizes a concert at the music hall, which has a maximum capacity of 300 seats. She sells VIP tickets for \$50 each and general admission tickets for \$30 each. She must sell at least 50 VIP and 100 general admission tickets to cover the cost of renting the hall. She wants to generate at least \$10,000 in ticket revenue. Determine three possible combinations of tickets Nicky can sell to meet her goals.



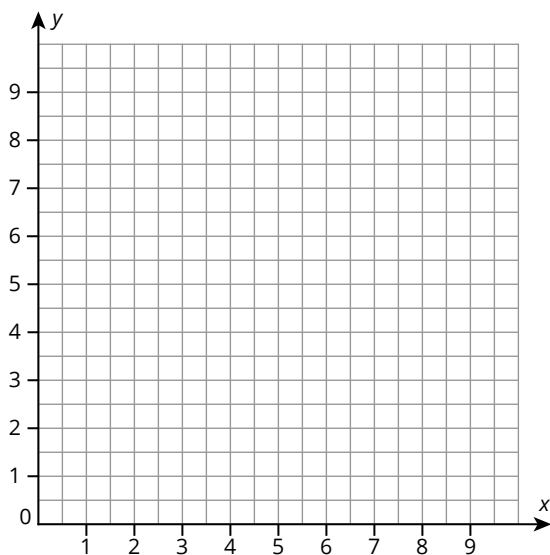
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- C. Graph each system of inequalities represented by the given constraints. Shade the region that represents the solution set. Then, identify three solutions to the systems of inequalities.

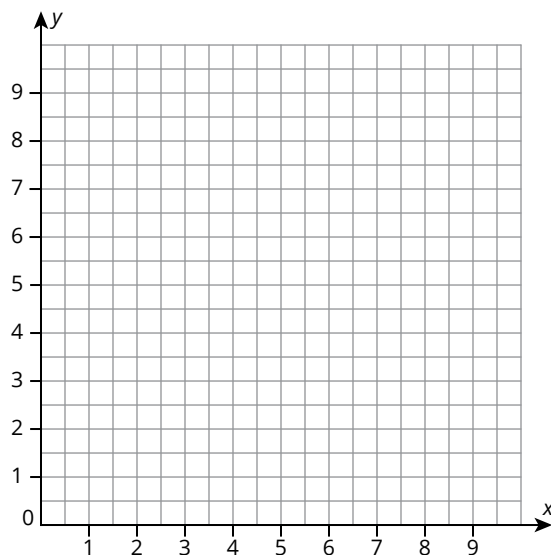
1.
$$\begin{cases} x > 2 \\ x \leq 6 \\ y \geq 0 \\ y < 3 \end{cases}$$



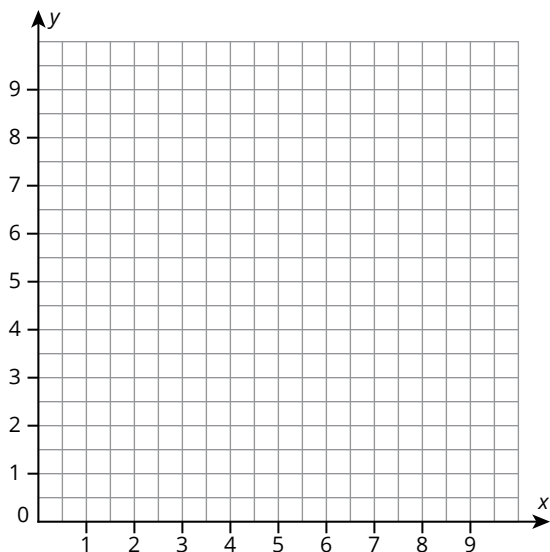
2.
$$\begin{cases} x \geq 1 \\ x \leq 5 \\ y \geq 0 \\ y \leq -\frac{1}{4}x + 5 \end{cases}$$



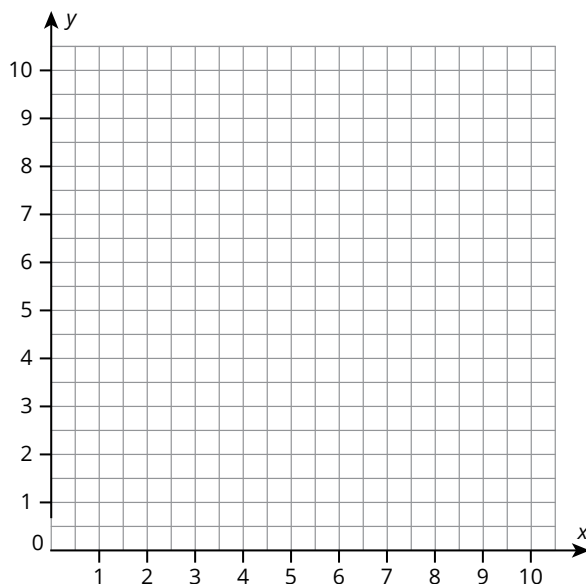
3.
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq -4x + 16 \\ y \leq -\frac{1}{3}x + 5 \end{cases}$$



4.
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 14 \\ 3x + y \leq 12 \end{cases}$$

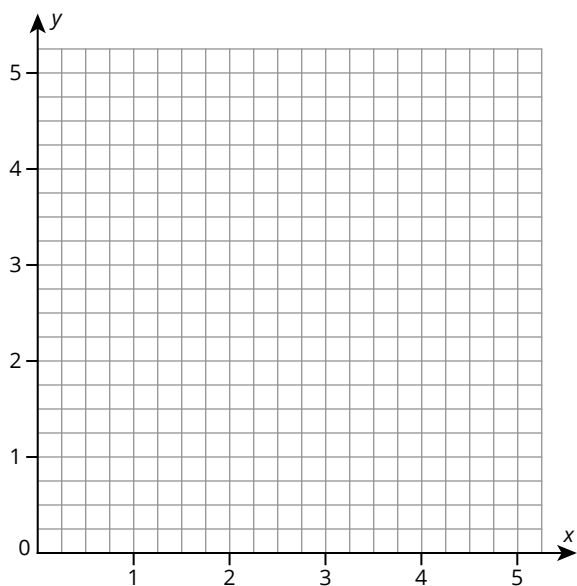


5.
$$\begin{cases} x \geq 0 \\ -x + y > 0 \\ 2x + y > 6 \\ 2x + y \leq 10 \end{cases}$$



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6.
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 3y \geq 3 \\ x + y \geq 2 \end{cases}$$



D. In each problem situation, the coordinates represent the intersection points of a set of inequalities representing the constraints in the problem situation. Using these coordinates, calculate the minimum and maximum value of the function $C(x, y)$.

1. $C(x, y) = x + 3y$
 $\{(0, 5), (2, 0), (7, 0)\}$

2. $C(x, y) = 3x - 2y$
 $\{(6, 7), (10, 3), (12, 10), (20, 6)\}$

3. $C(x, y) = x - 4y$
 $\{(8, 2), (10, 1), (13, 3)\}$

4. $C(x, y) = 2x + 2y$
 $\{(0, 0), (1, 4), (5, 7), (9, 6), (11, 0)\}$

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5. $C(x, y) = x - 3y$

$\{(-2, -1), (-1, 0), (0, 4), (3, 1), (5, -2)\}$

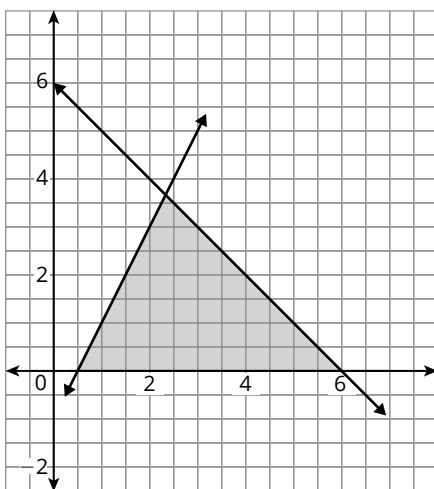
6. $C(x, y) = -x + 2y$

$\{(0, 0), (3, 4), (5, 5), (7, 2), (8, -2)\}$

E. Determine whether each point is a solution to the system of linear inequalities. Explain your reasoning.

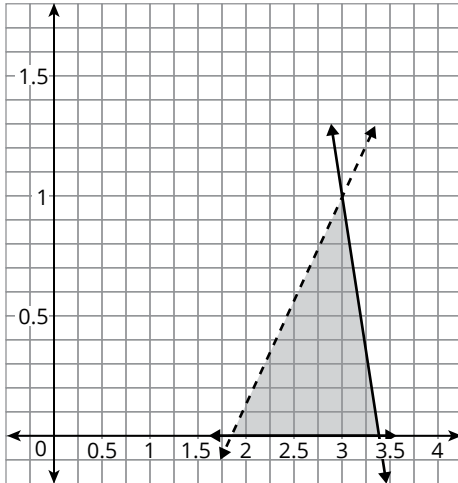
1. Point $(3, 2)$

$$\begin{cases} x + y \leq 6 \\ 2x - y \geq 1 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



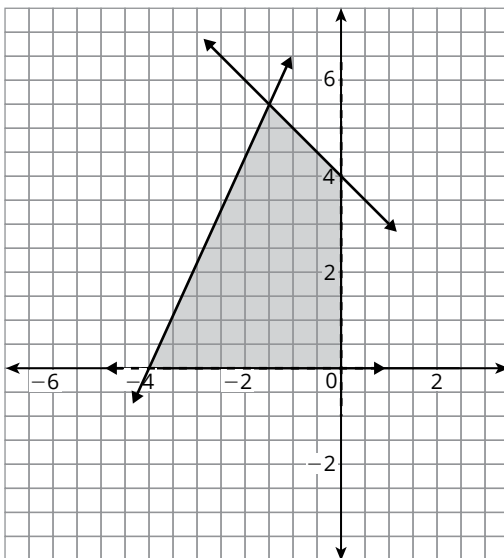
2. Point (4, 1)

$$\begin{cases} 3x + y \leq 10 \\ x - y > 2 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



3. Point (-2, 3)

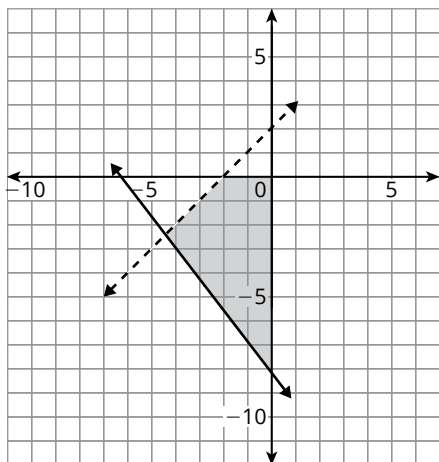
$$\begin{cases} x + y \leq 4 \\ 2x - y \geq -8 \\ x < 0 \\ y > 0 \end{cases}$$



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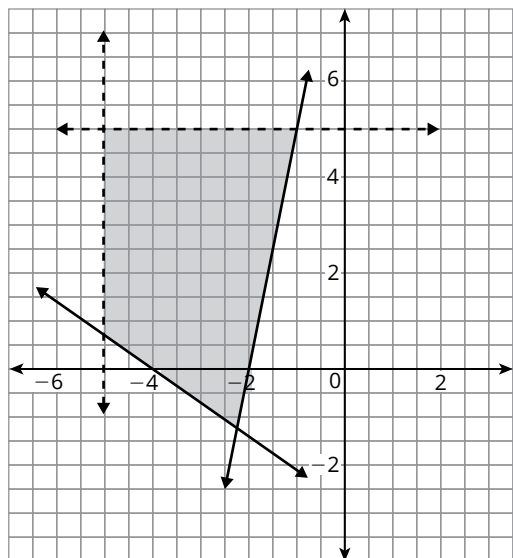
4. Point $(-3, -6)$

$$\begin{cases} 4x + 3y \geq -25 \\ x - y > 2 \\ x \leq 0 \\ y \leq 0 \end{cases}$$



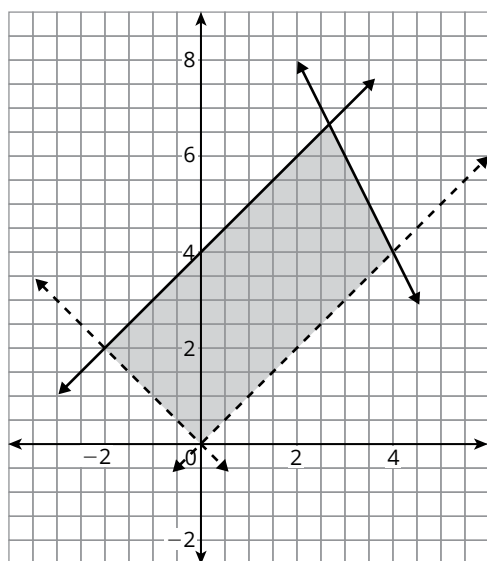
5. Point $(-3, 0)$

$$\begin{cases} 5x - y \leq -10 \\ 2x + 3y \geq -8 \\ x > -5 \\ y < 5 \end{cases}$$



6. Point (0, 0)

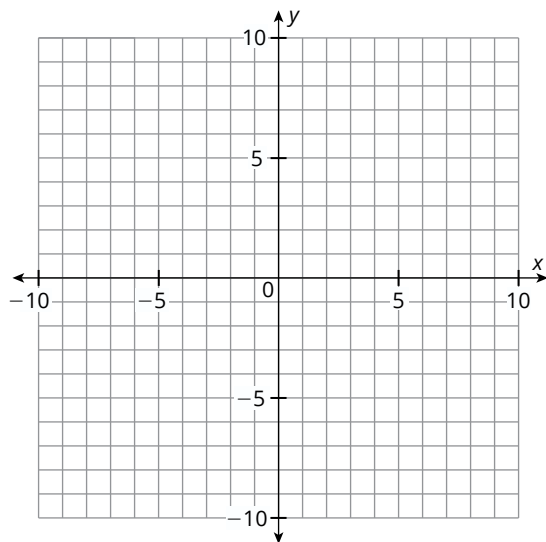
$$\begin{cases} 2x + y \leq 12 \\ x - y \geq -4 \\ y > x \\ y > -x \end{cases}$$



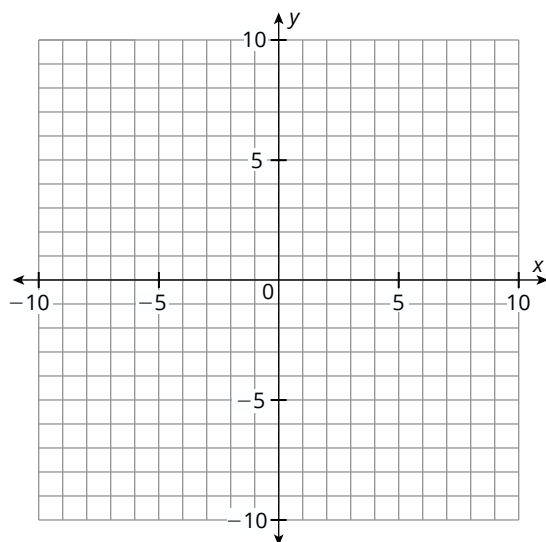
Extension

1. Graph each system of quadratic inequalities.

a.
$$\begin{cases} y > x^2 \\ y \leq -x^2 \end{cases}$$



b.
$$\begin{cases} y \geq (x - 3)^2 \\ y \leq (x - 5)^2 + 3 \end{cases}$$



Spaced Practice

1. Solve the system of equations.

$$\begin{cases} x - y + z = 8 \\ 2x + y - z = -2 \\ 3x - y + 4z = 4 \end{cases}$$

III. Solving Matrix Equations

A. Determine the dimensions of each matrix.

1. $A = \begin{bmatrix} -5 \\ 13 \\ 44 \end{bmatrix}$

2. $D = [7 \ 7 \ 7]$

3. $X = \begin{bmatrix} 23 & 47 \\ -20 & 0 \end{bmatrix}$

4. $C = \begin{bmatrix} 6 & -3 & 14 & 22 & -4 \\ 8 & 7 & -5 & 4 & 1 \end{bmatrix}$

5. $B = \begin{bmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix}$

6. $B = \begin{bmatrix} \frac{3}{4} & 3 & 6 & -2 & 4 & -1 \\ 0 & 2 & -8 & 9 & 5 & 3 \\ -9 & \frac{1}{2} & -6 & -\frac{4}{5} & 22 & 20 \\ -4 & -5 & -\frac{5}{7} & 2 & 44 & 56 \\ -35 & -3 & 0 & 5 & 4 & -10 \end{bmatrix}$

B. Use matrix A to answer each question.

$$A = \begin{bmatrix} 2.1 & 0 & -3.4 & 11.2 & -4.0 & 5.5 \\ -7.3 & 0.8 & 1.9 & -6.7 & -15.2 & 6.9 \\ 4.6 & 9.8 & -3.2 & 3.3 & 0.3 & -0.9 \\ 1.1 & 0.5 & 5.9 & 4.3 & 7.1 & -13.2 \end{bmatrix}$$

1. Determine matrix element a_{35} .

2. Determine the matrix element represented by the value -13.2 .

3. Determine matrix element a_{44} .

4. Determine matrix element a_{54} .

5. Determine the matrix element represented by the value 5.5.

6. Determine the matrix element represented by the value 0.5.

C. Use the matrices to answer each question.

$$A = \begin{bmatrix} 1 & -3 \\ 4 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} -11 & -3 \\ -6 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 8 & 0 & -3 \\ 2 & -1 & 1 & -3 \end{bmatrix}$$

$$D = \begin{bmatrix} 6 & 9 & 3 & 1 \\ 4 & 8 & 1 & 2 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

$$F = \begin{bmatrix} -7 & 3 & -4 \\ 3 & 10 & -1 \\ 14 & -3 & 0 \end{bmatrix}$$

1. Calculate $2A$.

2. Calculate $C - D$.

3. Calculate $D + F$.

4. Calculate $4E + A$.

5. Calculate $-2C + D$.

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6. Calculate $B - 2A$.

7. Calculate $4F$.

8. Calculate $E - B$.

D. Use the matrices to answer each question.

$$A = \begin{bmatrix} -7 & 2 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -9 & 5 \\ 3 & -1 & 0 \\ 10 & -2 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -4 & 11 \\ 3 & -2 \\ 8 & -10 \end{bmatrix}$$

$$E = \begin{bmatrix} -3 & 5 & 0 & -6 & 6 \\ -8 & 7 & 3 & 0 & -1 \end{bmatrix}$$

$$F = \begin{bmatrix} -12 & -4 \\ 4 & 12 \end{bmatrix}$$

1. Calculate AB .

2. Calculate CB .

3. Calculate BC .

4. Calculate DE .

5. Calculate AD .

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6. Calculate $(BA)C$.

E. Use matrix multiplication to determine whether the two matrices are inverses. Explain your reasoning.

1. $R = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ $S = \begin{bmatrix} 2 & -\frac{3}{2} \\ 1 & -\frac{1}{2} \end{bmatrix}$

2. $A = \begin{bmatrix} -9 & 5 \\ -7 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 5 \\ -7 & 9 \end{bmatrix}$

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3. $D = \begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

$$E = \begin{bmatrix} -6 & 8 \\ 3 & -4 \end{bmatrix}$$

4. $M = \begin{bmatrix} -1 & 12 \\ \frac{3}{2} & -15 \end{bmatrix}$

$$N = \begin{bmatrix} 5 & 4 \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

$$5. J = \begin{bmatrix} -5 & 3 & -2 \\ 1 & -1 & 1 \\ 6 & -3 & 2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 & 1 \\ 4 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 1 & 0 & 1 \\ 4 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -1 & 1 \\ -4 & 2 & 3 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

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- F. Use technology to calculate the inverse of each matrix, if it exists.
Use matrix multiplication to verify your answer.

1. $K = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$

2. $C = \begin{bmatrix} 4 & 8 \\ -2 & 6 \end{bmatrix}$

3. $S = \begin{bmatrix} -2 & 2 \\ 4 & -4 \end{bmatrix}$

4. $B = \begin{bmatrix} 4 & -2 & 1 \\ 2 & -3 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

5. $L = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -2 \\ 2 & -3 & -4 \end{bmatrix}$

6. $J = \begin{bmatrix} -25 & 7 & 26 \\ 3 & -1 & -3 \\ 26 & -7 & -27 \end{bmatrix}$

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- G. Write each system of equations as a matrix equation. Then, calculate the solution to each system of linear equations, if it exists, by using technology with matrices.

1.
$$\begin{cases} x + y = 6 \\ -3x + y = 2 \end{cases}$$

2.
$$\begin{cases} -x + 2y = 10 \\ -3x + 6y = 20 \end{cases}$$

$$3. \begin{cases} x + y + 3z = 12 \\ x - y - z = 0 \\ 2x + y - 2z = 3 \end{cases}$$

$$4. \begin{cases} 3x - 5y + 2z = -8 \\ 7x + 5y - 3z = 16 \\ 5x + 3y - 7z = 0 \end{cases}$$

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5.
$$\begin{cases} 2x + 6y + 12z = -6 \\ x + 3y + 6z = -3 \\ 3x - 2y + 3z = 8 \end{cases}$$

6.
$$\begin{cases} x + 4y - 2z = 8 \\ 5x + 7y - 5z = 6 \\ -3x + 2y - 6z = 6 \end{cases}$$

Extension

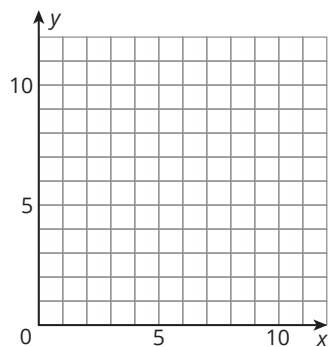
1. The matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ reflects any $2d$ vector across the y -axis.
Explain why.

Spaced Practice

1. Joey, Brianna, and Gracie all work part-time at a local community center, and their pay depends on the type of hours they work: regular hours, overtime hours, and weekend hours. Each of them worked different numbers of hours last week and earned different amounts.
 - Joey worked 5 hours of regular time, 2 hours of overtime, and 3 hours on the weekend, earning \$180.
 - Brianna worked 4 hours of regular time, 4 hours of overtime, and 1 hour on the weekend, earning \$150.
 - Gracie worked 6 hours of regular time, 2 hours of overtime, and 3 hours on the weekend, earning \$190.
 - a. Formulate a system of equations to determine the hourly rates for regular time, overtime, and weekend work.

TOPIC 2 Applications of Linear Relationships

- b. Use technology with matrices to solve the system and determine the hourly pay rate for regular, overtime, and weekend work.
2. Graph the system of inequalities represented by the given constraints. Shade the region that represents the solution set then identify three possible solutions.



IV. Inverses of Linear Functions

Topic Practice

A. Determine the corresponding point on the graph of the inverse of each function.

1. Given that $(2, 5)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?
2. Given that $(-3, 1)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?
3. Given that $(-4, -1)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?
4. Given that $(0, 8)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?
5. Given that $(1, -7)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?
6. Given that $(-6, 0)$ is a point on the graph of $f(x)$, what is the corresponding point on the graph of its inverse?

TOPIC 2 Applications of Linear Relationships

B. Complete each table. Write an equation to represent the relationship. Write an equation for the inverse of the problem situation.

1. One foot is equivalent to 12 inches.

Feet	Inches
1	
2	
3	
4	
5	

2. One meter is equivalent to 100 centimeters.

Meters	Centimeters
1	
2	
3	
4	
5	

3. One pint is equivalent to 2 cups.

Pints	Cups
2	
4	
6	
8	
10	

4. Four quarters is equivalent to 1 dollar.

Quarters	Dollars
4	
16	
32	
64	
128	

5. Three feet is equivalent to 1 yard.

Feet	Inches
3	
9	
12	
18	
24	

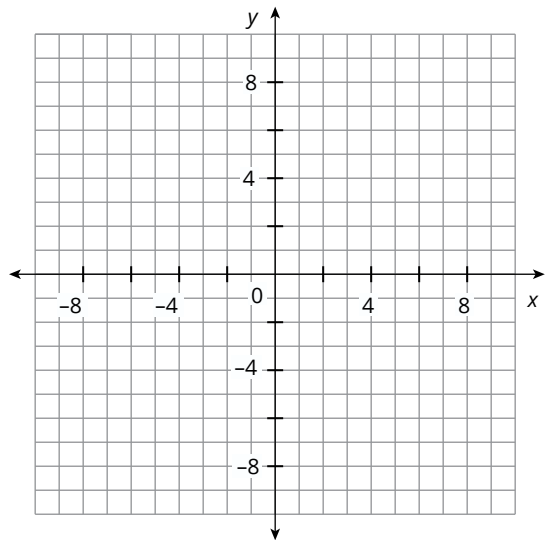
TOPIC 2 Applications of Linear Relationships

6. Twenty nickels is equivalent to 1 dollar.

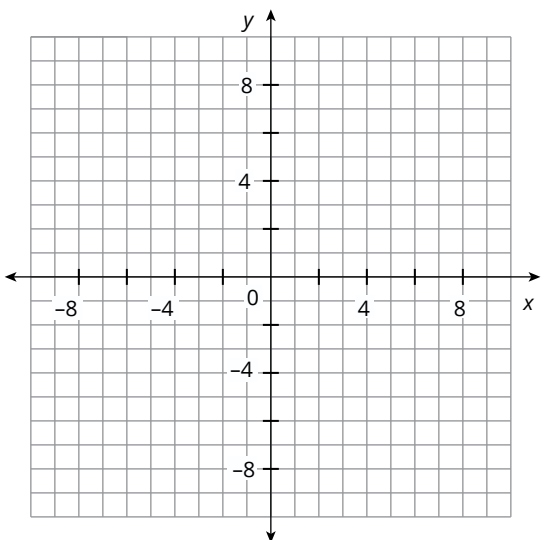
Nickels	Dollars
10	
20	
30	
40	
50	

C. Determine the inverse of each function. Graph the original function and its inverse. Use patty paper to verify your answer.

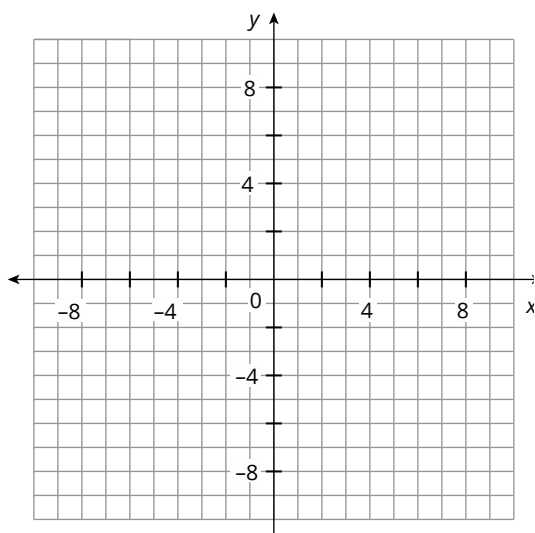
1. $f(x) = 4x$



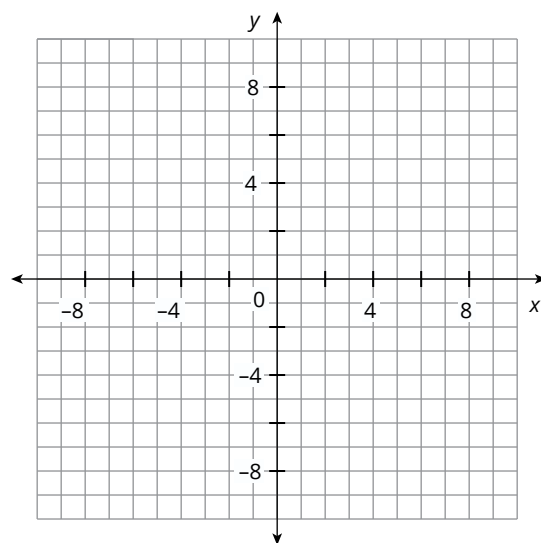
2. $f(x) = \frac{1}{3}x$



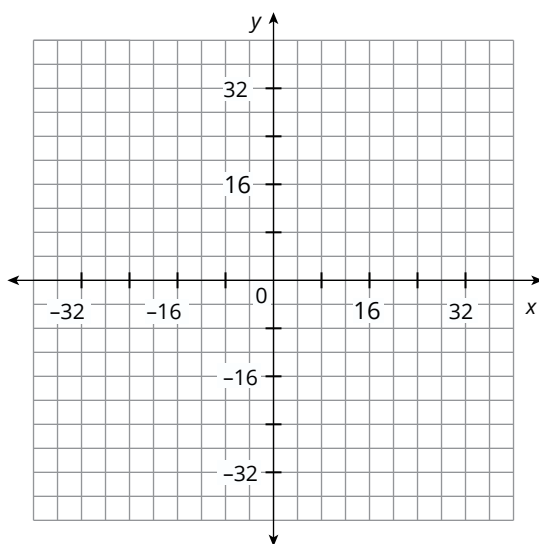
3. $f(x) = 2x + 1$



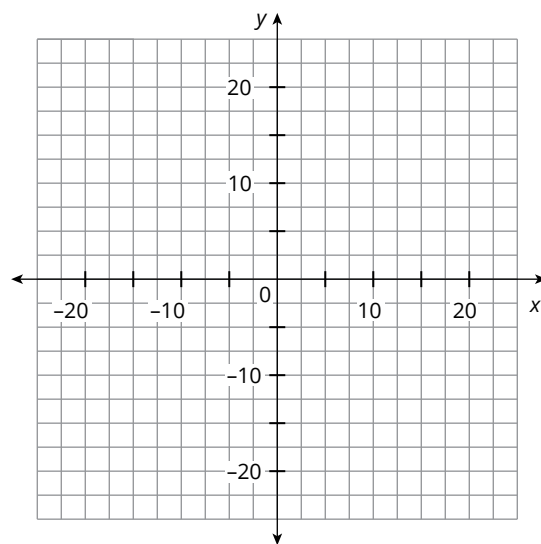
4. $f(x) = -6x - 2$



5. $f(x) = \frac{2}{3}x - 8$



6. $y = 12$



TOPIC 2 Applications of Linear Relationships

Extension

1. For the function $f(x) = x + 1$, the inverse function is $f^{-1}(x) = x - 1$. Evaluate the function $f(x)$ by replacing x with $x - 1$. What do you notice? How does this relate to the process of using a graph to determine an inverse.

Spaced Practice

1. Write the equation of the transformed function from the graph of the parent function $f(x) = |x|$.
 - a. Vertically compressed by a factor of $\frac{1}{2}$.
 - b. Reflected across the x -axis and shifted 5 units down.
 - c. Shifted 2 units to the right and shifted 3 units up.
 - d. Horizontally shifted 4 units to the left, vertically stretched by a factor of 3, and shifted 6 units down.

2. Match each absolute value inequality in Column A to its solution set in Column B.

Column A	Column B
a. $ x - 7 \geq 4$	i. $-\frac{4}{3} < x < 2$
b. $ x + 3 \leq 7$	ii. No solution
c. $ 2x + 5 > 9$	iii. $x \leq 3$ or $x \geq 11$
d. $ 3x - 1 < 5$	iv. $-10 \leq x \leq 4$
e. $ x + 6 \leq -4$	v. $x < -7$ or $x > 2$

3. Factor each quadratic expression.

a. $5x^2 + 17x + 6$

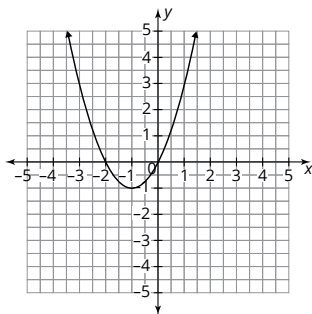
b. $3x^2 - 11x - 4$

c. $9x^2 - 12x + 4$

4. Consider the equations and graphs shown.

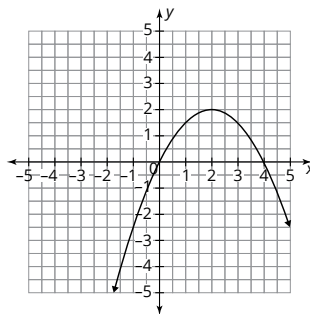
$$f(x) = (x - 2)^2 - 1$$

Graph A



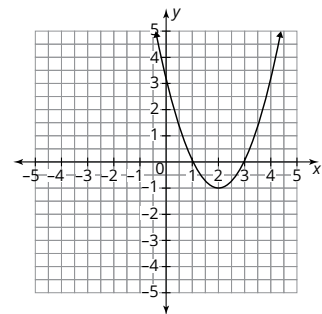
$$g(x) = (x + 1)^2 - 1$$

Graph B



$$h(x) = -\frac{1}{2}(x - 2)^2 + 2$$

Graph C



- a. Match each equation to a graph.

TOPIC 2 Applications of Linear Relationships

- b. Identify the axis of symmetry and zeros of each function.

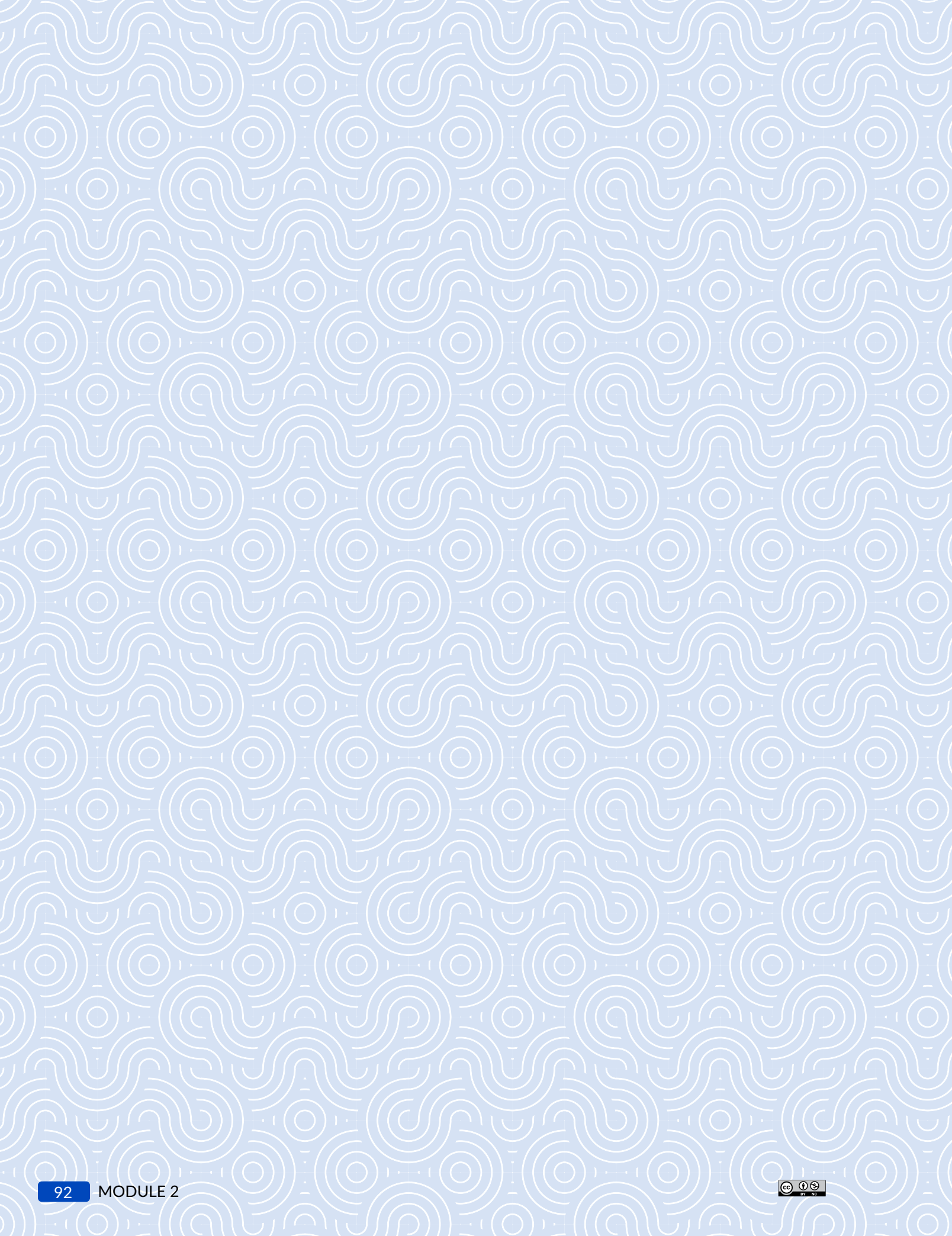
Exploring Quadratic Functions

TOPIC 1: Quadratic Functions and Equations

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Name _____ Date _____

I. Forms of Quadratic Functions

Topic Practice

A. Transform each quadratic function from standard form to vertex form. Then, identify the vertex.

1. $f(x) = x^2 + 2x + 1$

2. $f(x) = -x^2 - 6x + 3$

3. $f(x) = -4x^2 + 8x + 2$

4. $f(x) = -1.5x^2 + 3x + 2$

5. $f(x) = \frac{3}{2}x^2 - 3x + 6$

6. $f(x) = -3x^2 + 6x + 5$

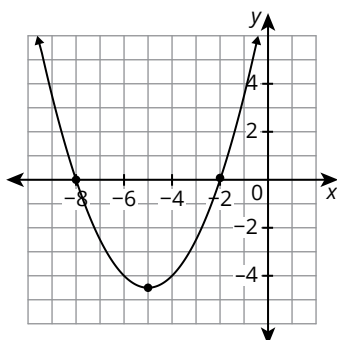
TOPIC 1
 Extending Quadratic Relationships

B. Complete the table by converting the given quadratic function into its other two forms.

	Standard Form	Vertex Form	Factored Form
1		$f(x) = 2(x - 3)^2 - 32$	
2			$f(x) = \frac{1}{2}(x + 2)(x - 4)$
3	$f(x) = 5x^2 - 70x + 225$		
4			$f(x) = -0.25(x - 4)(x + 6)$
5	$f(x) = -3x^2 + 36x - 81$		

C. Select the function that matches each graph. Explain your reasoning.

1.



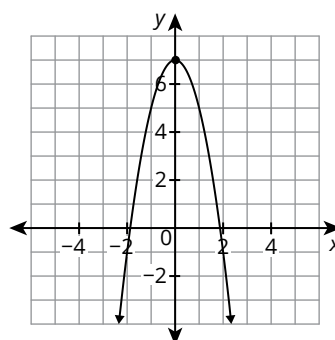
$$f(x) = 6(x - 2)(x - 8)$$

$$f(x) = -\frac{1}{2}(x + 2)(x + 8)$$

$$f(x) = \frac{1}{2}(x + 2)(x + 8)$$

$$f(x) = \frac{1}{2}(x - 2)(x - 8)$$

2.



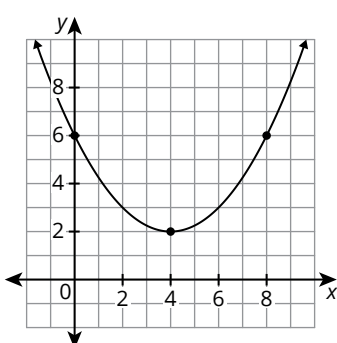
$$f(x) = 2x^2 - x + 7$$

$$f(x) = -2x^2 - x + 7$$

$$f(x) = -x^2 - 2x + 7$$

$$f(x) = -2x^2 - x - 2$$

3.



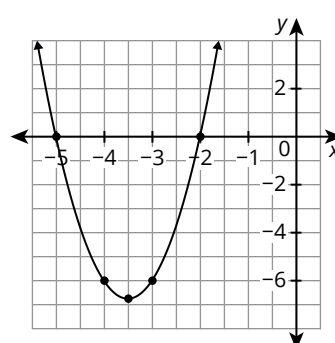
$$f(x) = 0.25(x - 4)^2 + 2$$

$$f(x) = -4(x - 2)^2 - 2$$

$$f(x) = 0.25(x + 4)^2 + 2$$

$$f(x) = 0.25(x - 2)^2 + 4$$

4.



$$f(x) = -3(x + 2)(x - 5)$$

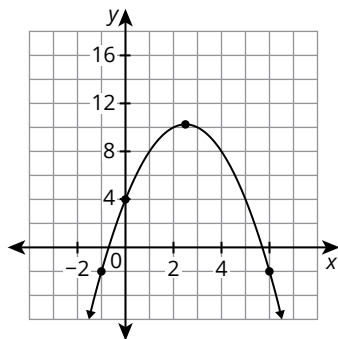
$$f(x) = 3(x + 2)(x + 5)$$

$$f(x) = 3(x - 2)(x - 5)$$

$$f(x) = -3(x - 2)(x - 5)$$

TOPIC 1 Extending Quadratic Relationships

5.



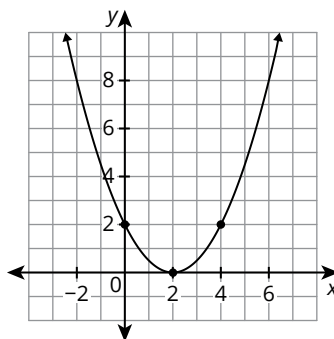
$$f(x) = x^2 + 5x - 4$$

$$f(x) = -x^2 + 5x + 10$$

$$f(x) = -x^2 + 5x + 4$$

$$f(x) = x^2 + 5x + 4$$

6.



$$f(x) = -\frac{1}{2}(x - 2)^2$$

$$f(x) = \frac{1}{2}(x - 2)^2 + 2$$

$$f(x) = \frac{1}{2}(x - 2)^2$$

$$f(x) = \frac{1}{2}(x + 2)^2$$

Extension

1. A scientist is launching a small drone to measure air quality. The height $h(t)$, in meters, of the drone after t seconds, is modeled by the quadratic function $h(t) = -4.9t^2 + 3.5t + 1.2$.
 - a. Rewrite $h(t)$ in vertex form.
 - b. Identify the vertex and explain its meaning in the context of the problem.

- c. Determine the height of the drone at $t = 0.5$ seconds and interpret its meaning in the context of the problem.
2. Given $h(x) = 2x^2 + 8x + 6$, what is the new equation in vertex form when $h(x)$ is reflected over the x -axis and shifted 2 units down.

Spaced Practice

1. The ideal storage temperature for a chemical is 20°C , but the temperature can vary by up to 3°C above or below this value. Write an absolute value equation that represents the possible storage temperatures of the chemical.
2. A delivery drone must drop a package at a delivery point located 50 meters from the starting position. However, the drop may be up to 5 meters off in either direction due to wind. Write an absolute value equation that represents the possible distances the package could be from the starting position.
3. Given the parent function, $f(x) = |x|$. Describe how $f(x)$ was transformed to become the graph of the function $g(x) = 2|x - 5| + 3$.

II. Writing Quadratic Functions

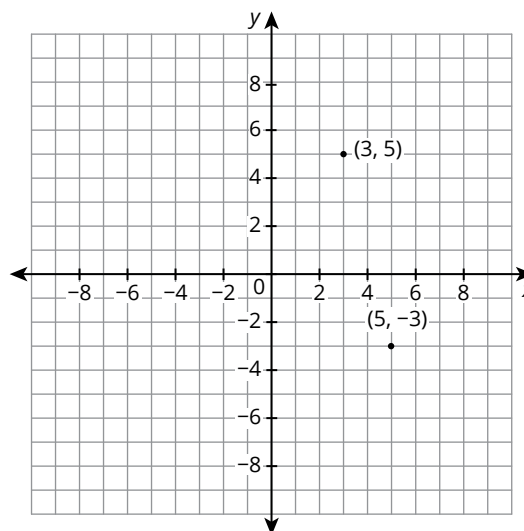
Topic Practice

A. Use the given information to determine the most efficient form you could use to write the quadratic function. Write *standard form*, *factored form*, or *vertex form*.

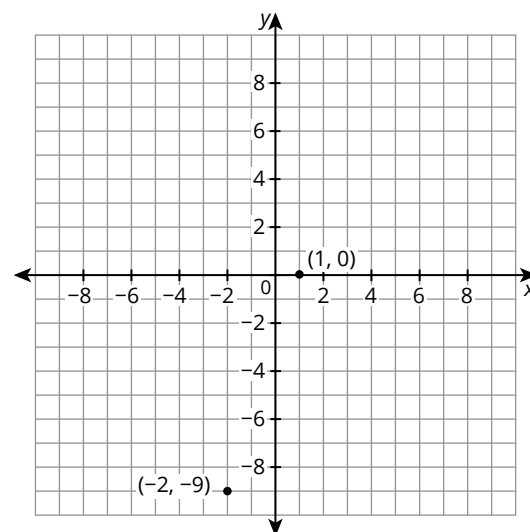
- | | |
|---|---|
| 1. vertex (3, 7) and point (1, 10) | 2. points (1, 0), (4, -3), and (7, 0) |
| 3. y-intercept (0, 3) and axis of symmetry $x = -\frac{3}{8}$ | 4. points (-1, 12), (5, 12), and (-2, -2) |
| 5. roots (-5, 0) and (13, 0) and point (-7, 40) | 6. maximum point (-4, -8) and point (-3, -15) |

B. Use your knowledge of reference points to write an equation for the quadratic function that satisfies the given information. Use the graph to help solve each problem.

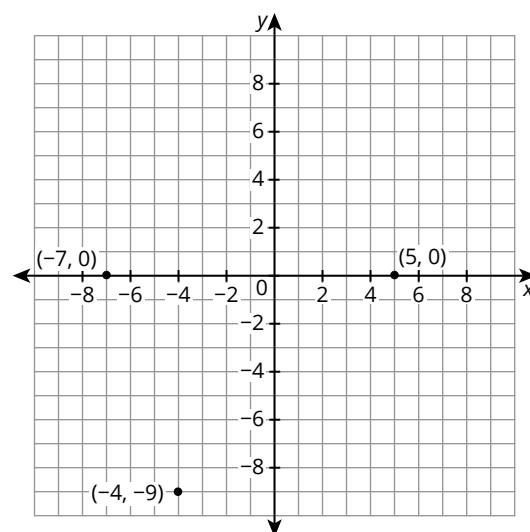
1. Given: vertex (3, 5) and point (5, -3)



2. Given: vertex $(-2, -9)$ and one of two x -intercepts $(1, 0)$

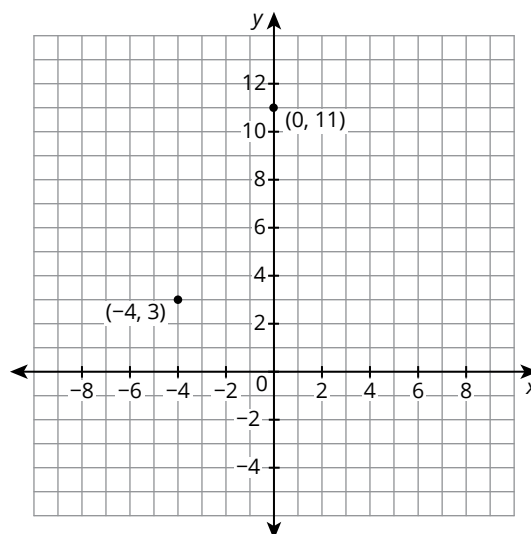


3. Given: two x -intercepts $(-7, 0)$ and $(5, 0)$ and one point $(-4, -9)$

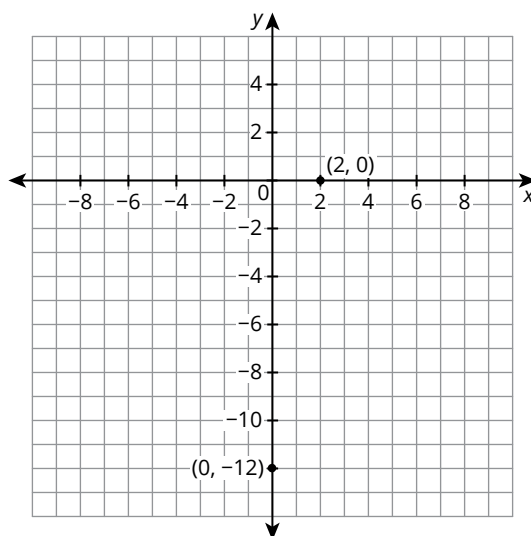


TOPIC 1 Extending Quadratic Relationships

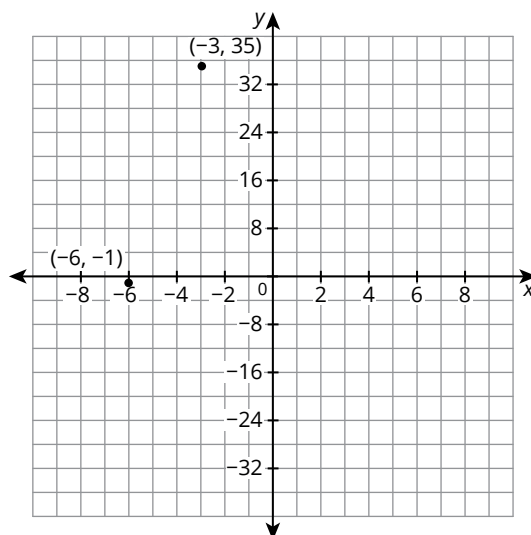
4. Given: vertex $(-4, 3)$ and y -intercept $(0, 11)$



5. Given: exactly one x -intercept $(2, 0)$ and y -intercept $(0, -12)$



6. Given: vertex $(-6, -1)$ and point $(-3, 35)$



C. Create a system of equations and use algebra to write a quadratic equation for each set of three points that lie on a parabola.

1. $(-3, 12), (0, 9), (3, 24)$

TOPIC 1 Extending Quadratic Relationships

2. $(-2, -2), (1, -5), (2, -18)$

3. $(2, 9), (0, -5), (-10, -15)$

TOPIC 1 Extending Quadratic Relationships

4. $(-1, 2), (4, 27), (-3, 20)$

5. $(5, -6), (-2, 8), (3, 4)$

TOPIC 1 Extending Quadratic Relationships

6. $(1, 17), (-1, -9), (2, 105)$

Extension

Use algebra to write a quadratic equation that passes through the points $\left(\frac{1}{2}, \frac{9}{16}\right)$, $\left(1, \frac{9}{2}\right)$, and $\left(-1, \frac{11}{2}\right)$.

Spaced Practice

1. Solve the system of equations using Gaussian elimination.

$$\begin{cases} x + y + z = 6 \\ 2x + 3y + z = 14 \\ x + 2y + 3z = 14 \end{cases}$$

TOPIC 1 Extending Quadratic Relationships

Solve each system of three linear equations in three variables using substitution.

1.
$$\begin{cases} x + 2y - z = 4 \\ 2x - y + z = 3 \\ 3x + y + 2z = 7 \end{cases}$$

2.
$$\begin{cases} 4x - y + z = 8 \\ 2x + 3y - z = 5 \\ x + 2y + z = 6 \end{cases}$$

III. Solving Quadratic Equations

A. Determine the roots of each quadratic equation by factoring.

1. $x^2 + 5x + 6 = 0$

2. $x^2 - 3x - 4 = 0$

3. $m^2 + 2m - 35 = 0$

4. $-x^2 - 4x + 12 = 0$

5. $x^2 + 8x = 0$

6. $w^2 + 50 = -15w$

7. $-t^2 + 12t = 32$

8. $x^2 + 2x + 2 = 0$

9. $2t^2 + t - 3 = 0$

10. $w^2 + 5w - 32 = 2w - 4$

TOPIC 1 Extending Quadratic Relationships

- B. Determine the roots of each quadratic equation by completing the square. Round your answer to the nearest hundredth. Check your answer.**

1. $x^2 + 4x - 6 = 0$

2. $x^2 - 2x - 4 = 0$

3. $x^2 + 10x + 2 = 0$

4. $x^2 - 12x + 25 = 0$

5. $x^2 + 3x - 1 = 0$

6. $x^2 + x - 10 = 0$

C. Determine the zeros or roots of each function or equation.

1. $f(x) = x^2 + 3x - 5$

2. $f(x) = -3x^2 - x + 7$

3. $2x^2 + 6x - 7 = 2$

4. $4x^2 - x - 1 = 5$

5. $f(x) = -8x^2 + 2x + 1$

6. $3x^2 + x + 3 = 5$

TOPIC 1 Extending Quadratic Relationships

7. $f(x) = -2x^2 - 8x + 1$

8. $5x^2 + 8x - 3 = 1$

9. $-3x^2 + 6x + 2 = -5$

10. $f(x) = x^2 + 6x + 5$

11. $f(x) = -2x^2 + 5x - 1$

12. $-3x^2 + 8x - 2 = -6$

13. $f(x) = -x^2 + 6x + 7$

14. $2x^2 + 8x + 3 = -5$

15. $f(x) = 9x^2 + 5x + 1$

16. $6x^2 + 3x - 5 = 2$

17. $f(x) = 5x^2 + 10x + 5$

18. $f(x) = 7x^2 + 9x + 5$

TOPIC 1 Extending Quadratic Relationships

Extension

The function g is defined by $g(x) = x^2 - 3x - 10$. If $g(x + 3) = x^2 + bx - c$, what are the values of b and c ? Show your work and justify your answer.

Spaced Practice

1. Consider the function $f(x) = \left(x + \frac{1}{2}\right)\left(x - \frac{3}{4}\right)$.
 - a. Identify the form of the function as factored, standard, or vertex.
 - b. Identify the zeros and axis of symmetry of the function.
2. Write a quadratic equation for the parabola that passes through the point $(-2, 12)$ with roots $(-5, 0)$ and $(-3, 0)$.
3. Solve each equation for the unknown value.
 - a. $|2x + 3| = 25$
 - b. $9 = |-3x - 1| - 7$

IV. Imaginary and Complex Numbers

A. Rewrite each expression using i .

1. $\sqrt{-72}$

2. $\sqrt{-49} + \sqrt{-23}$

3. $38 - \sqrt{-200} + \sqrt{121}$

4. $\sqrt{-45} + 21$

5. $\frac{\sqrt{-48} - 12}{4}$

6. $\frac{1 + \sqrt{4} - \sqrt{-15}}{3}$

7. $-\sqrt{-28} + \frac{\sqrt{21}}{3} - \frac{\sqrt{12}}{6}$

8. $\frac{\sqrt{-75} + \sqrt{80}}{10}$

TOPIC 1 Extending Quadratic Relationships

B. Simplify each expression.

1. $(2 + 5i) - (7 - 9i)$

2. $(3 - 4i) + (-6 - 9i)$

3. $-6 + 8i - 1 - 11i + 13$

4. $-3(5i - 4 + 10i) - 4(9 + 3i)$

5. $-(4i - 1 + 3i) + (6i - 10 + 17)$

6. $22i + 13 - (7i + 3 + 12i) + 16i - 25$

7. $9 + 3i(7 - 2i)$

8. $(4 - 5i)(8 + i)$

9. $-0.5(14i - 6) - 4i(0.75 - 3i)$

10. $\left(\frac{1}{2}i - \frac{3}{4}\right)\left(\frac{1}{8} - \frac{3}{4}i\right)$

C. Determine each product.

1. $(3 + i)(3 - i)$

2. $(4i - 5)(4i + 5)$

3. $(7 - 2i)(7 + 2i)$

4. $\left(\frac{1}{3} + 3i\right)\left(\frac{1}{3} - 3i\right)$

5. $(0.1 + 0.6i)(0.1 - 0.6i)$

6. $-2[(-i - 8)(-i + 8)]$

D. Use the discriminant to determine whether each function has real or imaginary zeros.

1. $f(x) = x^2 + 12x + 35$

2. $f(x) = -3x^2 + x - 9$

TOPIC 1 Extending Quadratic Relationships

3. $f(x) = x^2 - 4x + 7$

4. $f(x) = 9x^2 - 12x + 4$

5. $f(x) = -\frac{1}{4}x^2 + 3x - 8$

6. $f(x) = x^2 + 6x + 9$

**E. Factor each function over the set of real or imaginary numbers.
Then, identify the type of zeros.**

1. $k(x) = x^2 - 25$

2. $n(x) = x^2 - 5x - 14$

3. $p(x) = x^2 - 8x + 17$

4. $g(x) = x^2 + 6x + 10$

5. $h(x) = -x^2 + 8x - 7$

6. $m(x) = \frac{1}{2}x^2 + 8$

F. Determine the zeros of each given function.

1. $f(x) = 4x^2 + 1$

2. $f(x) = x^2 + 9$

3. $f(x) = x^2 + 2x + 5$

4. $f(x) = -x^2 + 4x - 6$

5. $f(x) = x^2 + 2x + 2$

6. $f(x) = -x^2 + 6x - 25$

7. $f(x) = x^2 - 4x + 9$

8. $f(x) = 2x^2 + 8x + 10$

TOPIC 1 Extending Quadratic Relationships

G. Solve each quadratic equation over the set of complex numbers.

1. $x^2 + 4 = 0$

2. $x^2 + 2x + 6 = 0$

3. $2x^2 + 8x + 12 = 0$

4. $x^2 - 6x + 13 = 0$

5. $3x^2 - 12x + 15 = 0$

6. $x^2 + 4x + 8 = 0$

Extension

How could you use your knowledge of quadratic functions to solve for a quadratic inequality by graphing?

Spaced Practice

1. Rewrite the quadratic function, $f(x) = 2x^2 - 8x + 5$, in vertex form. Then, identify the key attributes of the function.

Vertex:

Axis of symmetry:

Minimum or maximum:

Solve each quadratic equation with the indicated method.

2. Solve by factoring: $x^2 - 8x + 15 = 0$

TOPIC 1 Extending Quadratic Relationships

3. Solve using the quadratic formula: $-2x^2 + 8x - 10 = 20$

4. Solve using completing the square: $3x^2 - 5x + 2 = 0$

Name _____ Date _____

I. Solving Quadratic Inequalities

Topic Practice

A. Determine the roots of each quadratic inequality. Use the interval method to determine the solution set of the inequality. Round your answer to the nearest thousandth if necessary.

1. $x^2 - 7x + 16 \geq 10$

2. $x^2 + 7x - 2 < -12$

3. $x^2 + x - 15 < 4$

4. $x^2 + 11x - 21 \leq 2$

5. $-x^2 + 4x - 5 \leq -2$

6. $-x^2 - 3x + 14 > -3$

TOPIC 2 Applications of Quadratics

B. A water balloon is thrown upward from a height of 5 feet with an initial velocity of 35 feet per second. The quadratic function $h(t) = -16t^2 + 35t + 5$ represents the height of the balloon, h , in feet t seconds after it is thrown. Use this information to answer each question. Round your answer to the nearest thousandth.

1. How long does it take for the balloon to reach the ground?
2. Determine when the balloon is less than 10 feet above the ground.
3. Determine when the balloon is more than 10 feet above the ground.



4. Determine when the balloon is 10 feet above the ground.

5. Determine when the balloon is less than 20 feet above the ground.

6. Determine when the balloon is 20 feet above the ground.

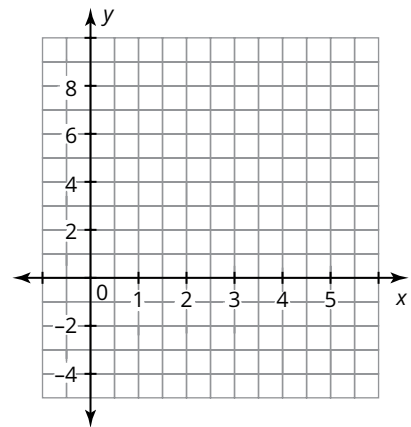
7. Determine when the balloon is more than 20 feet above the ground.

TOPIC 2 Applications of Quadratics

8. Determine when the balloon is less than 30 feet above the ground.

Extension

Ava and Emma both started their own companies with \$3000. Ava's profits can be represented as $g(x) \geq x^2 - 5x + 3$. Emma's profits can be represented by $h(x) = -x^2 + 5x + 3$. Graph the solutions to the quadratic inequalities and state what the shaded region means in regards to Ava and Emma's profits.



Spaced Practice

1. Determine each product.

a. $(3x - 9)^2$

b. $(6x^2 + 5x + 4)(-x - 3)$

2. Solve each equation.

a. $x^2 = 5x - 4$

b. $x^2 + 9x - 23 = 0$

3. Determine the roots of each function.

a. $f(x) = (x + 5)^2 + 9$

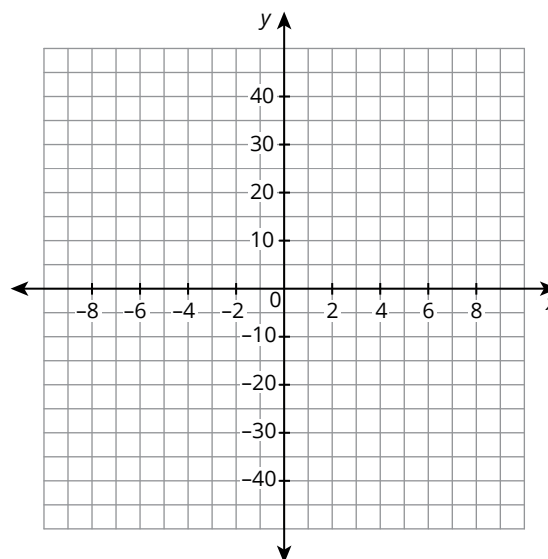
b. $g(x) = x^2 - 3x + 5$

II. Systems of Linear and Quadratic Equations

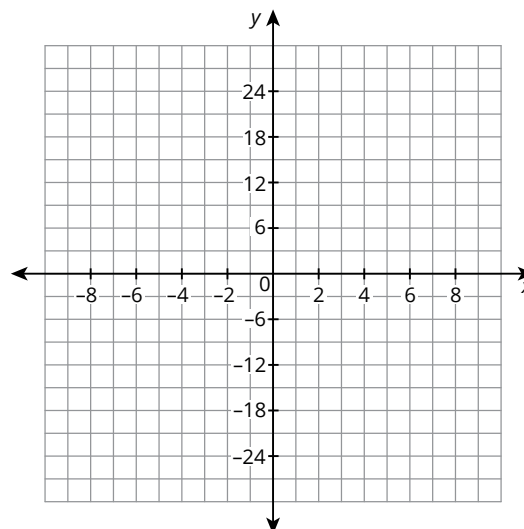
Topic Practice

- A. Solve each system of two equations in two variables graphically.
Then, verify the solution algebraically.

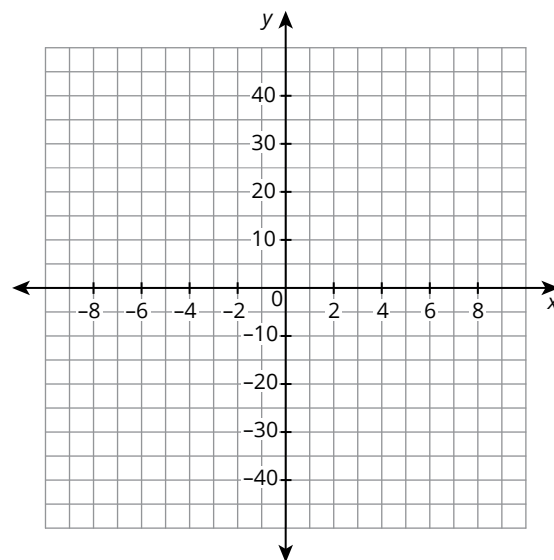
1.
$$\begin{cases} y = x^2 + 2x + 5 \\ y = 5x + 15 \end{cases}$$



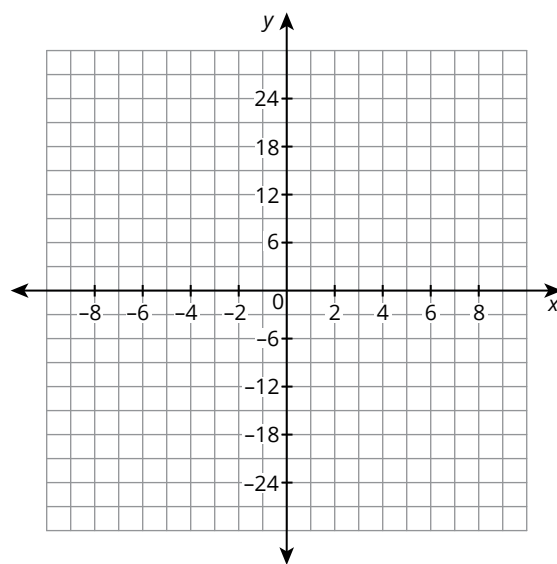
2.
$$\begin{cases} y = -x^2 + 7x - 16 \\ y = 4x - 20 \end{cases}$$



3.
$$\begin{cases} y = x^2 + 8x + 4 \\ y = x - 9 \end{cases}$$

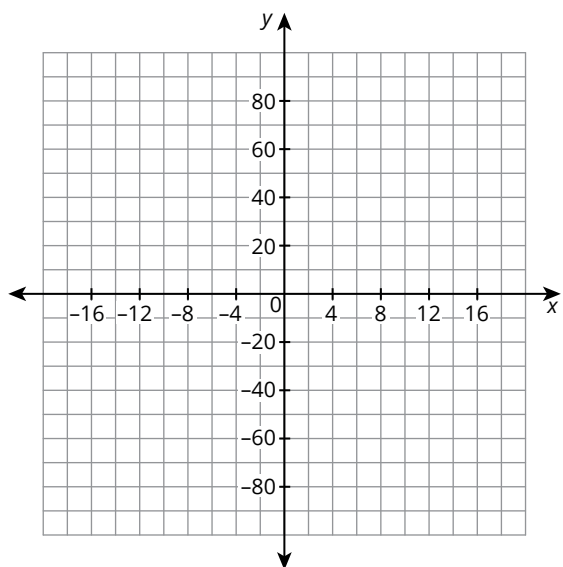


4.
$$\begin{cases} y = -2x^2 + 8x + 1 \\ y = -4x + 19 \end{cases}$$

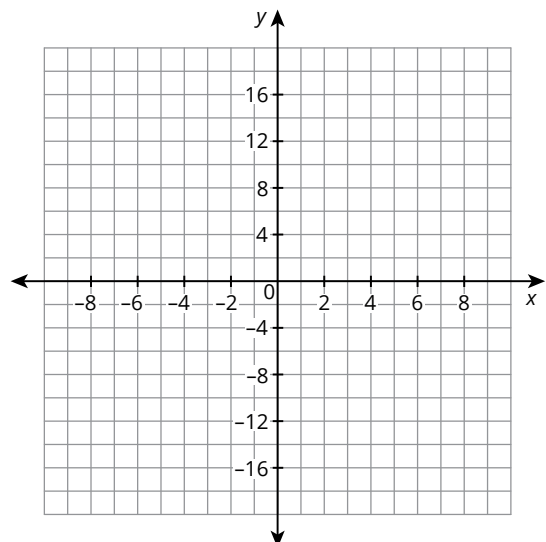


TOPIC 2 Applications of Quadratics

5.
$$\begin{cases} 5x - 3 = y \\ x^2 - 2y = 30 \end{cases}$$

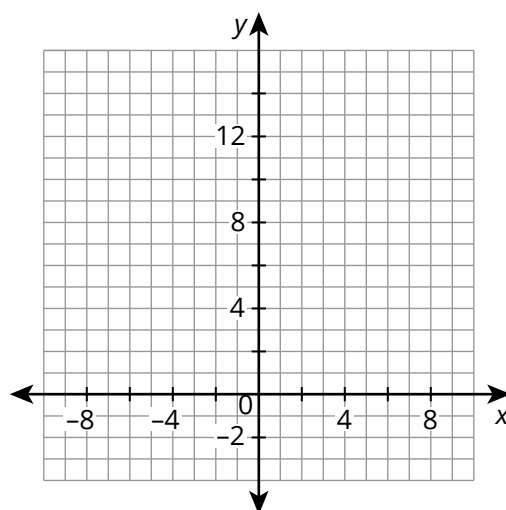


6.
$$\begin{cases} y = x^2 + 10x + 11 \\ y = x - 7 \end{cases}$$

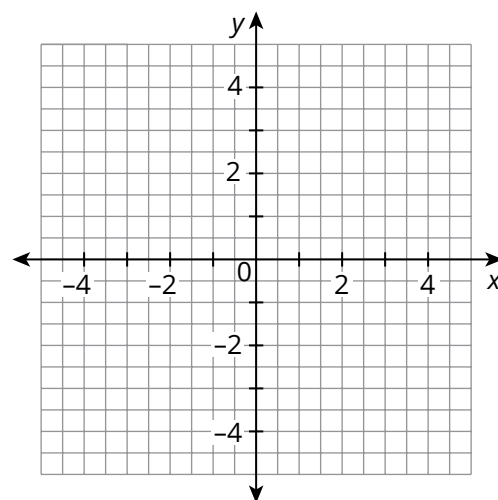


B. Solve each system of equations algebraically. Then, verify each solution graphically.

1.
$$\begin{cases} y = x^2 - 6x + 7 \\ y = 2x \end{cases}$$

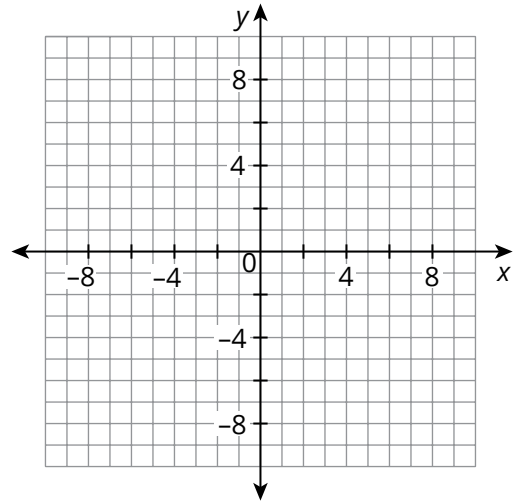


2.
$$\begin{cases} y = x^2 - 3x + 1 \\ y = x - 3 \end{cases}$$

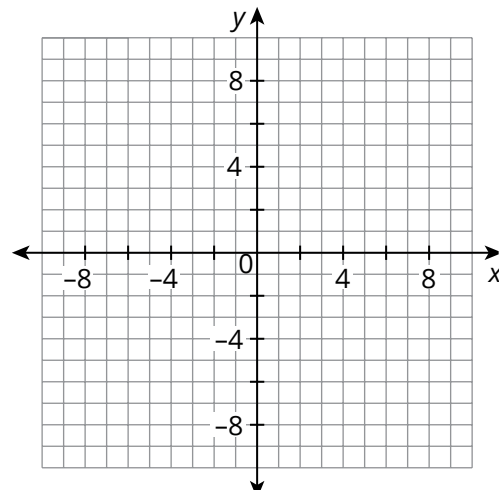


TOPIC 2 Applications of Quadratics

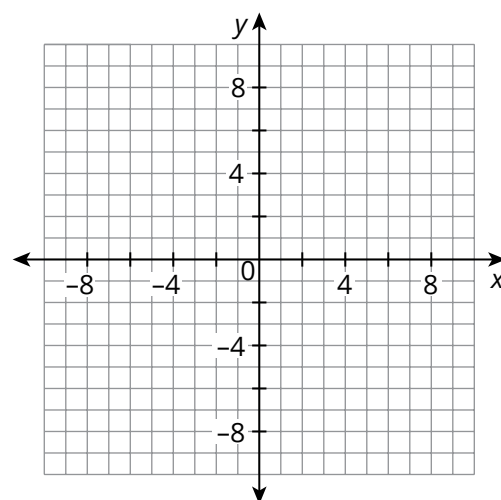
3.
$$\begin{cases} y = 2x^2 + 16x + 24 \\ y = -x - 2 \end{cases}$$



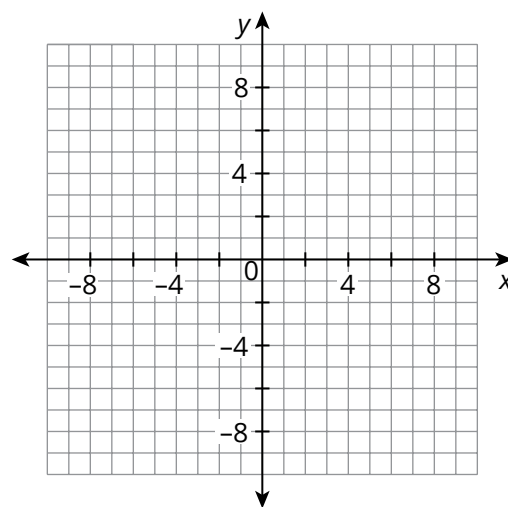
4.
$$\begin{cases} y = -x^2 + 6x - 6 \\ y = 3x + 1 \end{cases}$$



5.
$$\begin{cases} y = 4x^2 + 6x + 3 \\ y = -6x - 6 \end{cases}$$



6.
$$\begin{cases} y = 3x^2 + 24x + 50 \\ y = 4x + 1 \end{cases}$$



TOPIC 2 Applications of Quadratics

- C. Write and solve a system of equations for each situation. Then, explain whether the solution(s) to the system make sense in the problem situation.
1. A student throws a ball straight upward from a height of 5 meters with an initial velocity of 40 feet per second. The student is in the gymnasium, where the height of the ceiling is 30 feet. Will the ball hit the ceiling? Recall that vertical motion can be modeled by the equation $h(t) = -16t^2 + v_0 t + h_0$, where h is the height, in feet, of the object after t seconds, v_0 is the initial velocity, and h_0 is the initial height.

- Each month, a company that specializes in making skateboards must keep track of their costs and revenue. The company's costs consist of fixed costs that include rent, utilities, and workers' salaries, as well as the variable cost to make the skateboards. Its costs can be represented by the function $C(x) = 20x + 700$. The company's revenue for every skateboard sold can be represented by the function $R(x) = 150x - 0.6x^2$. Determine the break-even point(s) for the month.
- Another company decides they want to compete with the company from question 2. This new company determines its monthly costs will be represented by the function $C(x) = 17x + 800$. They predict revenue will be represented by the function $R(x) = 60x - 0.7x^2$. Determine how many skateboards the new competitor must make to break even each month.

TOPIC 2 Applications of Quadratics

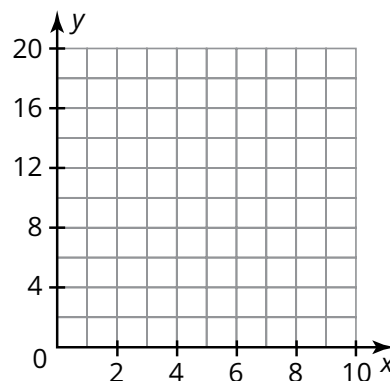
4. A professional base jumper jumps off the Pecos River High Bridge near Comstock, Texas. The Pecos River High Bridge is the highest highway bridge in Texas. When he jumps, he free falls for a few seconds and then opens his parachute so that he can land safely. Before he opens the parachute, the equation $h(t) = -4.9t^2 + t + 325$ models his height in meters t seconds after he jumps. The equation $h(t) = -5t + 275$ represents his height after he opens the parachute. How long after jumping did he open the parachute?
5. A punter kicks a football. The height of the football, in meters, is modeled by the function $h(t) = -4.9t^2 + 8.5t + 0.25$, where t represents time, in seconds. A blocker can only attempt to knock down the football as it travels upward from the punter's foot. The height in meters of the approaching blocker's hands is modeled by the function $h(t) = -0.4t + 3$, where t represents the same time. Will the blocker knock down the football? If so, when will the blocker knock it down?

6. Samuel and his sister are playing a guessing game. Samuel tells his sister that he is thinking of two negative integers. The first number minus the second number is -4 . The square of the first number minus 4 times the second number is equal to 61.

Extension

1. Graph the inequalities given and describe what the double shaded region means in your own words.

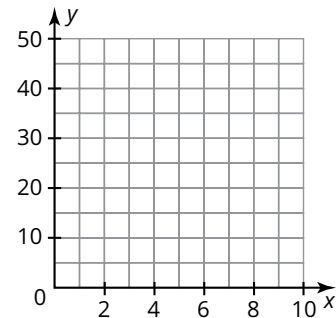
$$\begin{cases} y > 2x + 5 \\ y \leq -3x^2 + 15x \end{cases}$$



TOPIC 2 Applications of Quadratics

Spaced Practice

1. Solve $x^2 - 12 = 5$.
2. Perform each operation.
 - a. $(7x^3 + 5x^2 - 8x) + (3x^3 - 4x^2 + 11)$
 - b. $(6x - 2y) - (3x - 5y)$
3. A soccer ball is kicked up off a 5-meter-high platform with an initial velocity of 27 meters per second.
 - a. Write an inequality to represent when the soccer ball will be above 40 meters.
 - b. Graph the inequality and state when the soccer ball will be above 40 meters.



III. Modeling with Linear and Quadratic Functions

- A. Calculate the first and second differences for each table of values.
Describe the type of function represented by the table.

1.

x	y
-2	-10
-1	-7
0	-4
1	-1
2	2

First Differences

Second Differences

2.

x	y
-2	12
-1	3
0	0
1	3
2	12

First Differences

Second Differences

3.

x	y
-3	35
-2	31
-1	27
0	23
1	19

First Differences

Second Differences

4.

x	y
-1	16
1	15
1	18
2	25
3	36

First Differences

Second Differences

TOPIC 2 Applications of Quadratics

5.

x	y
-4	-58
-3	-37
-2	-22
-1	-13
0	-10

First Differences

Second Differences

6.

x	y
-1	24
0	30
1	36
2	42
3	48

First Differences

Second Differences

B. Use graphing technology to determine whether to use a linear or quadratic function to represent each table. Then, write the function to represent the data.

1.

x	y
-2	-3
-1	-1
0	1
1	3
2	5

2.

x	y
-2	1
-1	0
0	1
1	4
2	9

3.

x	y
1	0
2	-1
3	0
4	3
5	8

4.

x	y
1	5
2	9
3	13
4	17
5	21

5.

x	y
0	8
1	11
2	14
3	17
4	20

6.

x	y
-1	0
0	-3
1	-4
2	-3
3	0

TOPIC 2 Applications of Quadratics

- C. Use technology to determine the key attributes of the graph of each given regression function. Then, determine what these values mean in terms of the problem situation and tell whether the values make sense.**
1. A household collects data on the monthly temperature, in °F, and the cost of their corresponding electric bill. The cost is modeled by the linear regression function $f(t) = -5t + 400$, where t represents the monthly temperature, in °F, and $f(t)$ represents the monthly bill in dollars.
 2. An athlete throws a disc upward at an angle. The height in feet of the disc can be modeled by the quadratic regression function $h(x) = -0.002x^2 + 0.440x + 5.621$, where x represents the distance in feet that the disc has traveled horizontally.

3. Elijah and his friend are playing catch with a baseball. Elijah tosses the ball to his friend, but he overthrows it, and it hits the ground. The height, in feet, of the baseball can be modeled by the quadratic regression function $h(x) = -0.005x^2 + 0.752x + 5.220$, where x represents the distance in feet that the baseball has traveled horizontally.

TOPIC 2 Applications of Quadratics

4. A company's profit in dollars, p , can be modeled by the quadratic regression function $p(x) = -116.938x^2 + 4010.166x - 10,590.863$, where x represents the number of years since the company was started.

D. Use the data given to create a regression function to answer each question.

1. A real estate agent collected data about houses in a neighborhood, measuring their size (in square feet) and their selling price (in dollars). If a house is sold for \$290,000, estimate its size using the line of best fit.

Size (ft ²)	Cost (\$)
1400	225,000
1800	275,000
2000	300,000
2125	315,625
2633	379125

2. A quadratic regression function that models the relationship between the selling price of a necklace, x , in dollars and the profit earned each month, y , in dollars. To reach a profit of \$300 per month from the necklaces, what should the selling price be?

Necklace Price (\$)	Profit (\$)
1	2830
6	5130
22	5770
29	2830
34	−470

3. A company manufactures bicycles. The relationship between the number of bicycles made and the cost to produce those bicycles can be modeled by a quadratic regression function. What would be the cost to manufacture 50 bicycles.

Number of bicycles	Cost (\$)
30	6757
41	7941.70
45	8432.50
70	12,225
75	13,133.50

4. A car dealer tracks the value of cars based on how many miles they've been driven. The car value can be modeled by the linear regression. Predict the value of a car that has driven 100,000 miles.

Miles Driven	Car Value (\$)
5000	27,250
11,111	26333.35
60,000	19,000
92,000	14,200
115,500	10,675

TOPIC 2 Applications of Quadratics

5. A company tracks their advertising budget and their monthly sales. The monthly sales can be modeled by the linear regression function. If the company wants sales of \$130,000, how much should they spend on advertising?

Advertising Budget (\$)	Sales (\$)
3300	9901
3721	11164
3967	11902
4500	13501
6225	18676

6. A farmer finds that his crop yield per acre can be modeled by a quadratic regression function. What is the approximate yield when 20 pounds of fertilizer are applied per hundred square feet?

Fertilizer (pounds per hundred ft ²)	Crop Yield (bushels)
1	3.89
5	8.79
15	15.59
21	17.75
28	18.45



- E. Use technology to determine the quadratic regression function for each data set. Round decimals to the nearest thousandth.

1.

x	y
1	22
6	6
20	20
42	64
80	99

2.

x	y
12	38
24	12
30	3
40	16
54	54

3.

x	y
25	60
50	80
75	140
100	210
125	250
150	322
175	400

4.

x	y
10	14
20	36
30	70
40	120
60	240
70	350

TOPIC 2 Applications of Quadratics

5.

x	y
0.1	8
0.2	12
0.3	14
0.4	16
0.5	18
0.6	10
0.7	2

6.

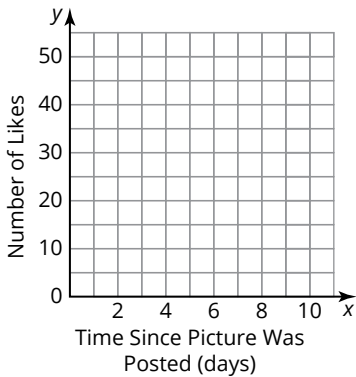
x	y
0.1	2.2
0.2	3.1
0.3	4
0.4	4.8
0.5	5.4
0.6	6
0.7	4.2

Extension

- Sophia painted a picture and posted it on a social media site. The table shows the number of people who responded that they liked the picture since Sophia posted it.

Time Since Picture Was Posted (days)	1	3	5	7	9
Number of Likes	1	5	11	21	38

- Graph the data.



- Based on the graph, is a linear, a quadratic, or an exponential regression model best for the data?
- Use graphing technology to write a regression function that best models the data.

Spaced Practice

1. Solve each inequality.

a. $4x^2 - 12 > 3x^2 + x$

b. $4x^2 - 6x - 9 \geq x^2$

2. Solve by factoring. $x^2 - 11x + 19 = -5$

TOPIC 2 Applications of Quadratics

3. Solve by completing the square. $x^2 - 10x + 26 = 8$

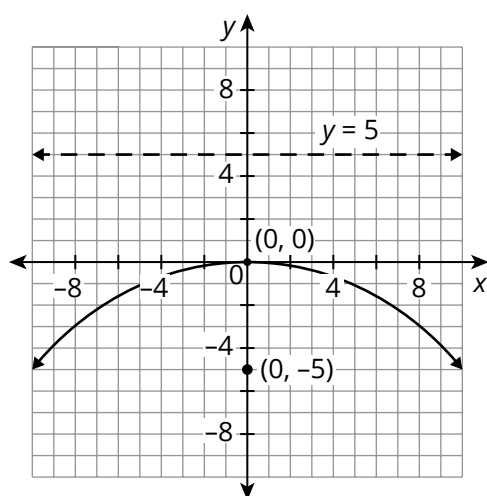
4. Solve by quadratic formula. $2x^2 - 3x - 5 = 0$

IV. Equation of a Parabola

Topic Practice

- A. For the graph of each parabola centered at the origin, write its equation in conic form. Show your work for deriving the equation. Then, identify the axis of symmetry and concavity of the parabola.

1.

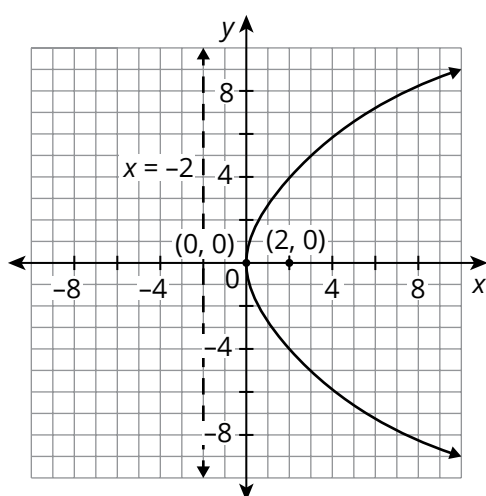


Equation:

Axis of Symmetry:

Concavity:

2.



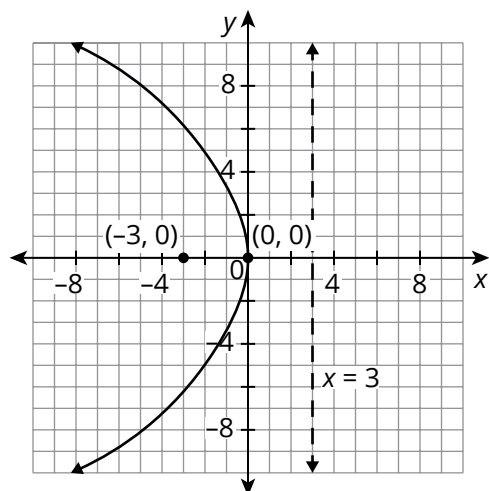
Equation:

Axis of Symmetry:

Concavity:

TOPIC 2 Applications of Quadratics

3.

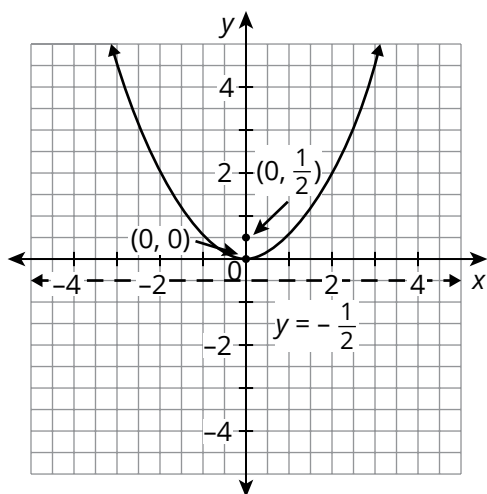


Equation:

Axis of Symmetry:

Concavity:

4.

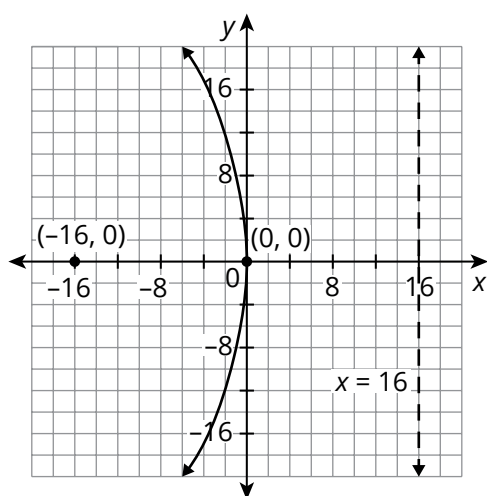


Equation:

Axis of Symmetry:

Concavity:

5.

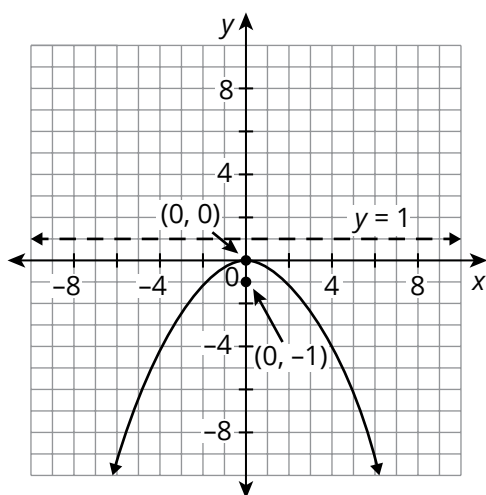


Equation:

Axis of Symmetry:

Concavity:

6.



Equation:

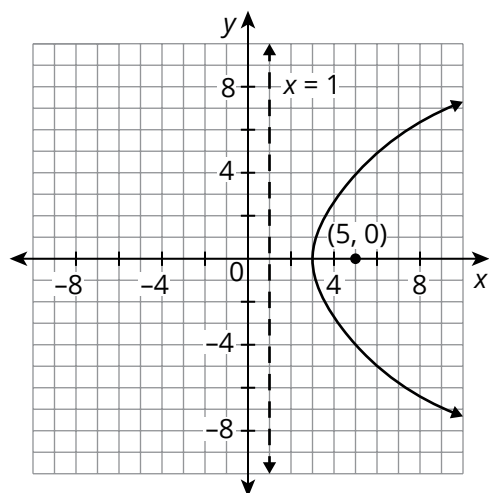
Axis of Symmetry:

Concavity:

TOPIC 2 Applications of Quadratics

- B. Write the conic form equation of each parabola. Identify the vertex and axis of symmetry. Show your work for deriving the equation.

1.



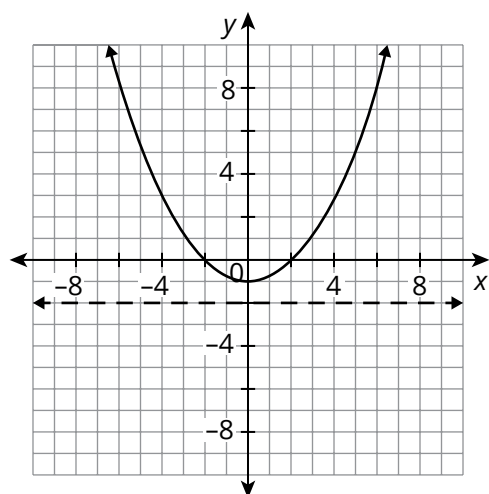
Vertex:

Axis of Symmetry:

p -value:

Equation:

2.



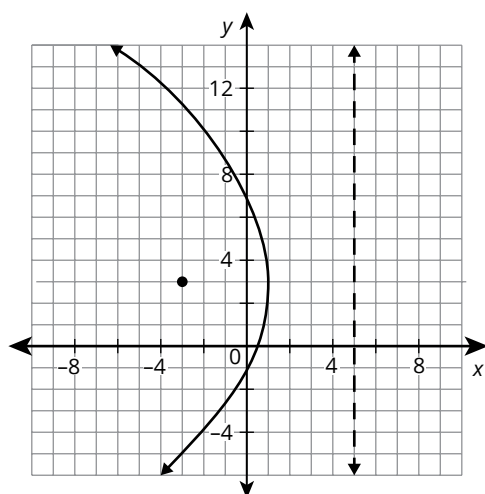
Vertex:

Axis of Symmetry:

p -value:

Equation:

3.



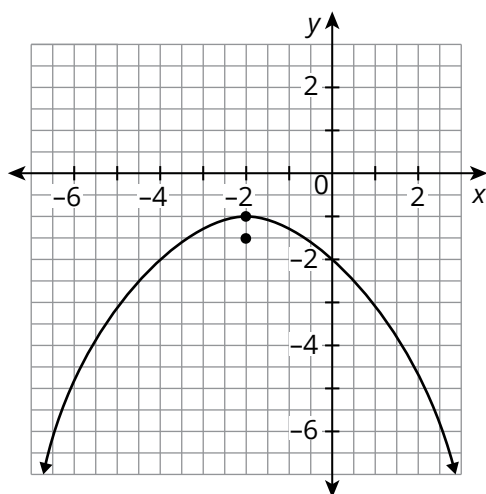
Vertex:

Axis of Symmetry:

p -value:

Equation:

4.



Vertex:

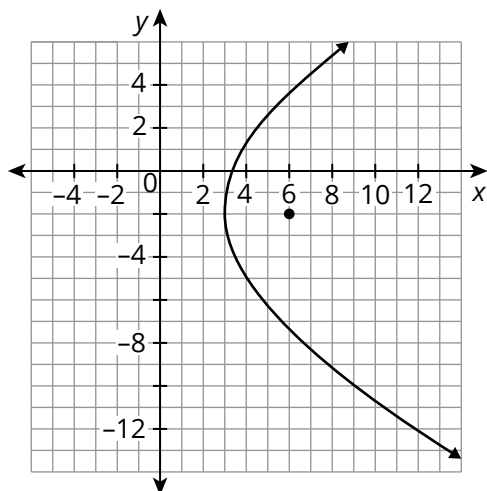
Axis of Symmetry:

p -value:

Equation:

TOPIC 2 Applications of Quadratics

5.



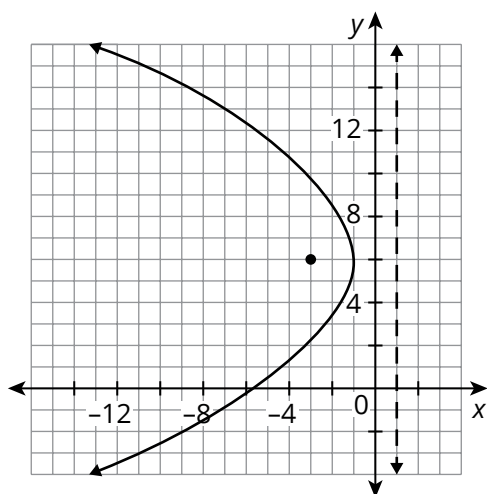
Vertex:

Axis of Symmetry:

p -value:

Equation:

6.



Vertex:

Axis of Symmetry:

p -value:

Equation:

C. Determine the equation of a parabola with the given vertex and focus.

1. A parabola has a vertex at $(0, 0)$ and a focus at $(0, -2)$.

2. A parabola has a vertex at $(3, 0)$ and a focus at $(5, 0)$.



- | | |
|--|---|
| <p>3. A parabola has a vertex at $(-2, -3)$ and a focus at $(-2, -6)$.</p> | <p>4. A parabola has a vertex at $(4, -2)$ and a focus at $(-1, -2)$.</p> |
| <p>5. A parabola has a vertex at $(10, 7)$ and a focus at $(10, 18)$.</p> | <p>6. A parabola has a vertex at $(-5, 8)$ and a focus at $(2, 8)$.</p> |

D. Determine the equation of a parabola with the given focus and directrix. Explain your method for determining the equation.

- | | |
|---|---|
| <p>1. A parabola has a focus at $(-1, -5)$ and a directrix at $y = 3$.</p> | <p>2. A parabola has a focus at $(5, 0)$ and a directrix at $x = -1$.</p> |
| <p>3. A parabola has a focus at $(-3, -8)$ and a directrix at $y = -6$.</p> | <p>4. A parabola has a focus at $(-7, -5)$ and a directrix at $x = -3$.</p> |

TOPIC 2 Applications of Quadratics

5. A parabola has a focus at $(3, 3)$ and a directrix at $y = 5$.
6. A parabola has a focus at $(-10, 1)$ and a directrix at $y = -1$.

E. Determine the equation of a parabola with the given attributes.

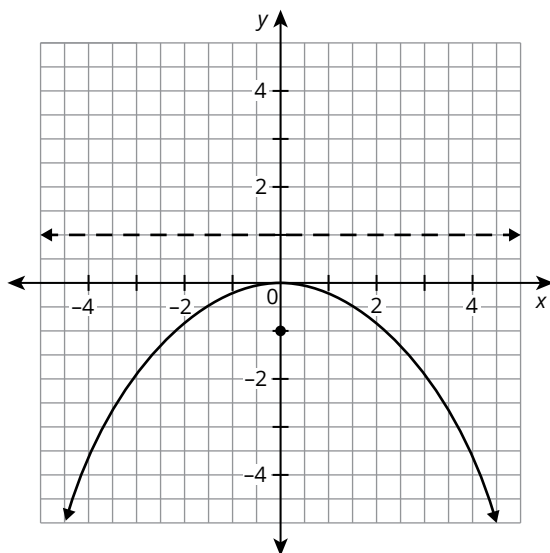
1. The parabola has an axis of symmetry at $x = 2$, vertex at $(2, 3)$, and passes through the point $(3, 4)$.
2. The parabola has a vertex at $(0, 0)$ and a focus at $(0, 3)$.
3. The parabola has a vertex at $(2, 1)$ and directrix at $x = -2$.
4. The parabola opens downward, has a vertex at $(1, 3)$, and passes through the point $(2, 2)$.

5. The parabola has a vertex at $(-3, 2)$, opens to the right, and passes through the point $(1, 4)$.

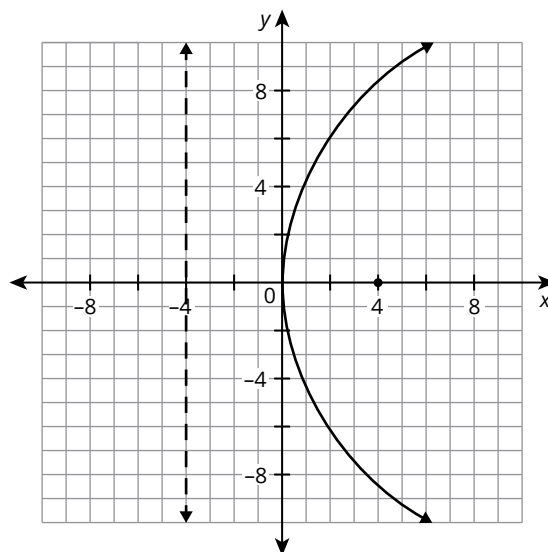
6. The parabola has a focus at $(0, 5)$, a directrix at $y = 1$, and an axis of symmetry at $x = 0$.

- F. Use the graph to write an equation for the parabola in geometric form.

1.

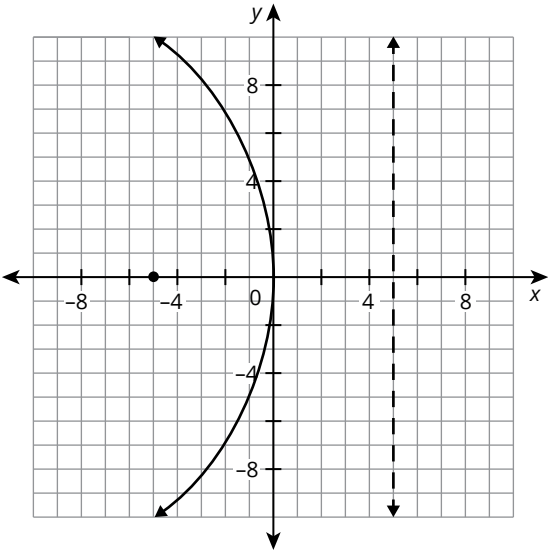


2.

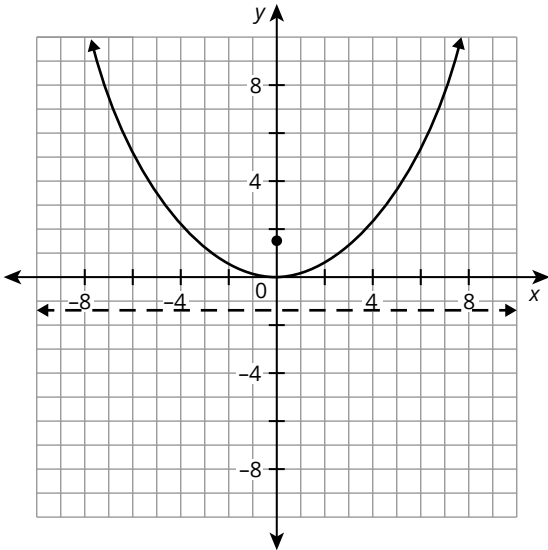


TOPIC 2 Applications of Quadratics

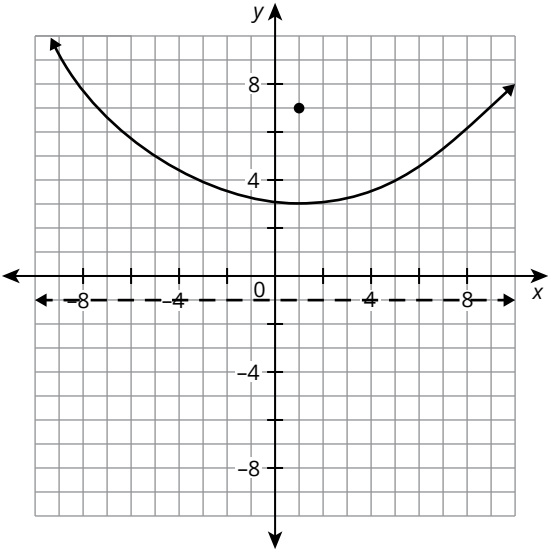
3.



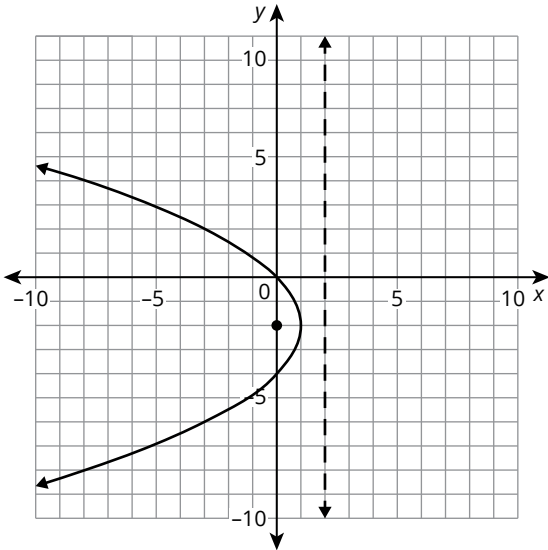
4.



5.

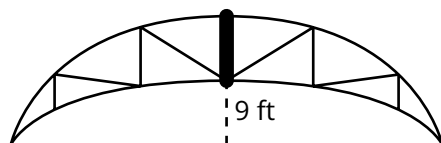


6.



Extension

The finish line of a 5K race is an archway of balloons. The archway is formed by two parabolas, one representing the top of the archway and one representing the bottom of the archway as shown.



The width of the archway on the ground is 60 feet. The height of the top of the archway is 18 feet. The height of the bottom of the archway is 9 feet. The framework of the archway consists of vertical posts 10 feet apart with posts connecting the tops and bottoms of adjacent vertical posts. Calculate the sum of the lengths of the posts.

Spaced Practice

1. Solve each system of equations.

a.
$$\begin{cases} y = x^2 + 3x - 7 \\ x + y = -2 \end{cases}$$

b.
$$\begin{cases} y = x^2 + 3x + 2 \\ y = 2x + 4 \end{cases}$$

Analyzing Structure

TOPIC 1: Composing and Decomposing Functions

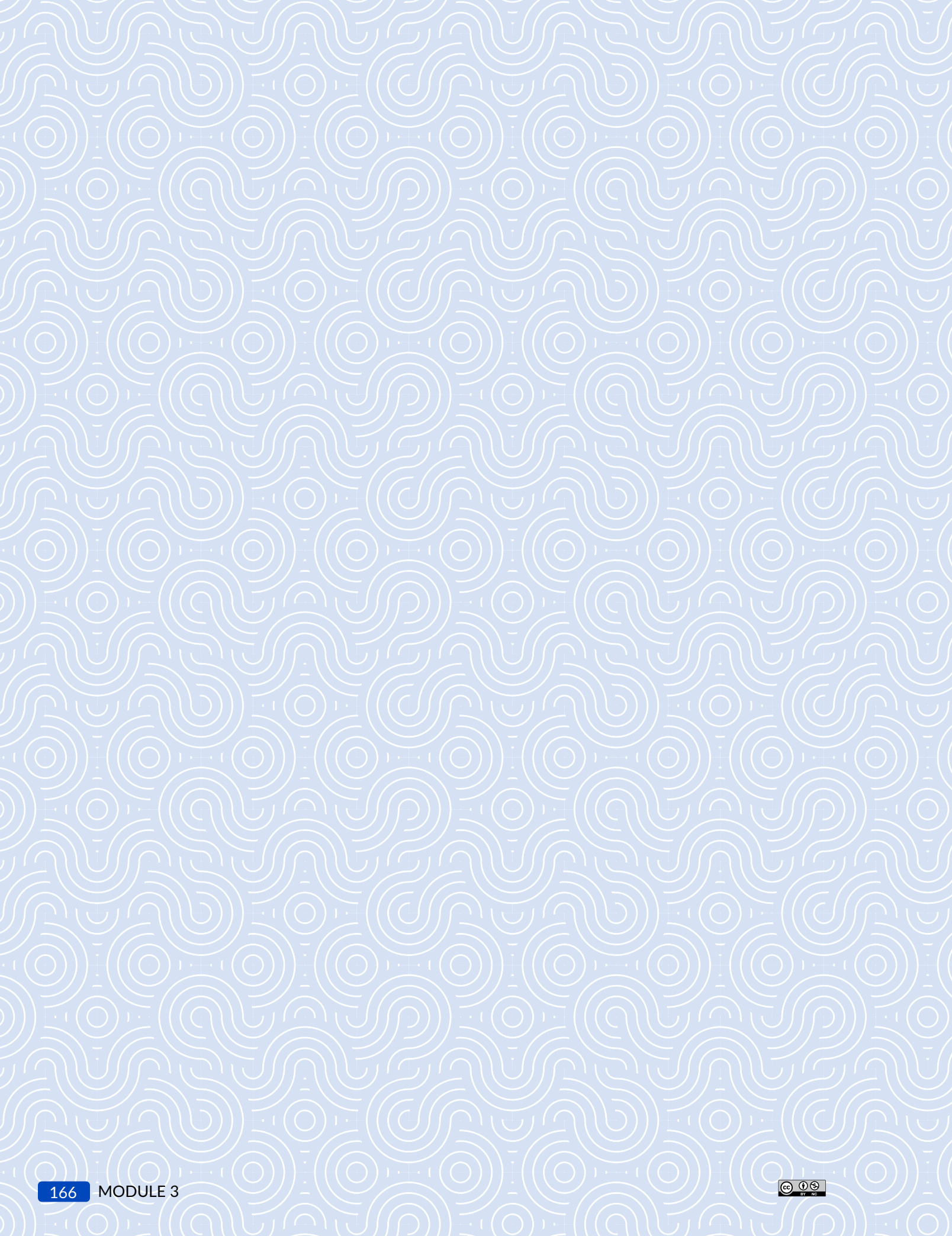
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Name _____ Date _____

I. Modeling with Functions

Topic Practice

A. Analyze the data in each table. Then, determine whether each data set is modeled by a linear or quadratic function. Explain your reasoning.

1.

x	y
1	3
2	5
3	7
4	9

2.

x	y
1	2
2	6
3	12
4	20

3.

x	y
0	3
1	2
2	3
4	11

4.

x	y
0.5	2.5
1.5	7.5
2.5	12.5
3.5	17.5

5.

x	y
-2	0
0	22
2	44
4	66

6.

x	y
-4	0
-2	4
0	4
2	0

TOPIC 1 Composing and Decomposing Functions

B. Write a quadratic function in standard form that represents each area as a function of the width. Remember to define your variables.

1. A builder is designing a rectangular parking lot. She has 300 feet of fencing to enclose the parking lot around three sides.
2. Aiko is enclosing a new rectangular flower garden with a rabbit garden fence. She has 40 feet of fencing.
3. Pedro is building a rectangular sandbox for the community park. The materials available limit the perimeter of the sandbox to at most 100 feet.
4. Lea is designing a rectangular quilt. She has 16 feet of piping to finish the quilt around three sides.
5. Kiana is making a rectangular vegetable garden alongside her home. She has 24 feet of fencing to enclose the garden around the three open sides.
6. Nelson is building a rectangular ice rink for the community park. The materials available limit the perimeter of the ice rink to at most 250 feet.

C. Use the quadratic formula to answer each question.

1. A ball is thrown upwards with an initial velocity of 25 meters per second from a height of 5 meters. The height, $h(x)$ in meters at time x seconds is modeled by $h(x) = -4.9x^2 + 25x + 5$. Determine when the ball reaches the ground. Round your answer to the nearest tenth.
2. The length of a rectangle is 5 meters longer than its width. If the area of the rectangle is 60 square meters, determine the dimensions of the rectangle. Round your answer to the nearest hundredth.

TOPIC 1 Composing and Decomposing Functions

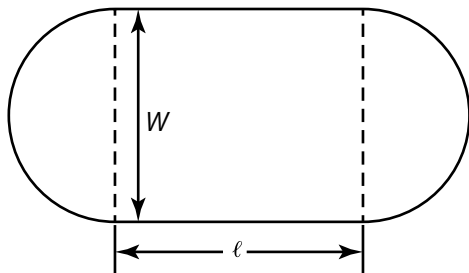
- A soccer player kicks a ball toward a goal, and its height in meters after t seconds is given by $h(t) = -4.9t^2 + 14t + 1$. Determine when the ball is 5 meters above the ground. Round your answer to the nearest tenth.
- A new amusement park ride launches riders into the air from a platform. The height h of a rider in meters after t seconds is given by the equation $h(t) = -5t^2 + 30t + 20$. The ride will launch the rider, and they will travel upward before falling back toward the ground. Determine when the rider is 65 meters off the ground.

- A company produces custom bicycles and their profit, $p(x)$, in thousands of dollars is given by the equation $p(x) = -0.01x^2 + 2x - 80$ when x represents the number of bicycles sold. Determine the number of bicycles that must be produced and sold for the profit to be zero (when the company breaks even).
- A landscaper is designing a rectangular flower garden. The area A of the garden is 200 square meters. The length of the garden is 5 meters longer than its width. Determine the dimensions of the garden. Round your answer to the nearest whole number.

TOPIC 1 Composing and Decomposing Functions

Extension

1. A team of engineers is going to build a recreational field with a path around it. The field will be rectangular in shape with a semicircle at each end. The path will be 1320 feet long. Determine the dimensions for the length and width that will give the greatest possible area for the rectangular section of the field. Round your answers to the nearest tenth.



Spaced Practice

1. Use properties to rewrite each expression.
 - a. $(8 - 3i) - (8 + 3i)$
 - b. $(8 - 3i)(8 + 3i)$
2. A baseball is hit from an initial height of 3 feet. The baseball reaches its maximum height of 81 feet when it is 156 feet from home plate.
 - a. Write a quadratic function to represent the height of the baseball as a function of its distance from home plate.
 - b. Determine the height of the baseball when it is 180 feet from home plate. Round your answer to the nearest tenth.
3. Solve the equation $7x^2 - 11x + 3 = 0$. Round your answer(s) to the nearest hundredth, if necessary.

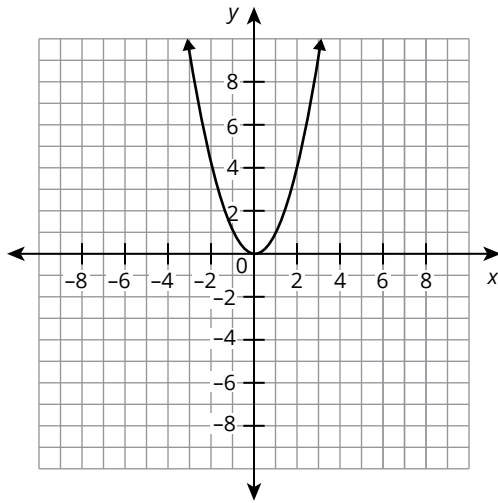
II. Transforming Function Shapes

Topic Practice

A. Dilate each given function to create a new function of a higher degree. Sketch the graph and then identify the zeros of each new function.

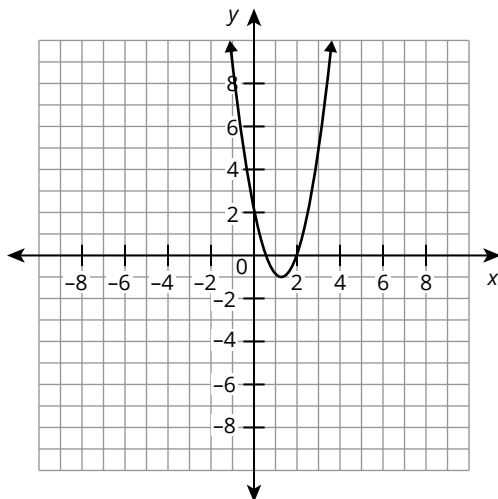
1. $f(x) = x^2$

Dilate the function by $x + 4$.



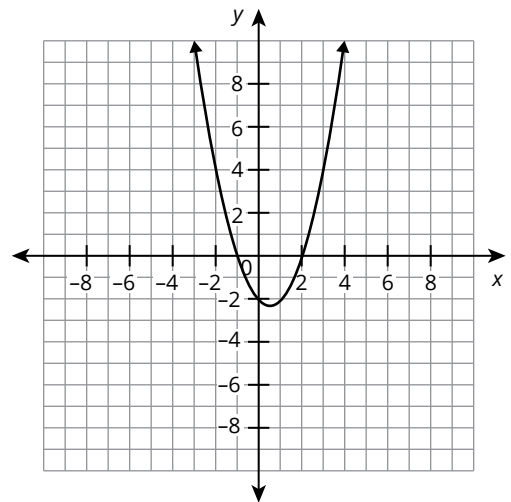
3. $h(x) = (2x - 1)(x - 2)$

Dilate the function by x .



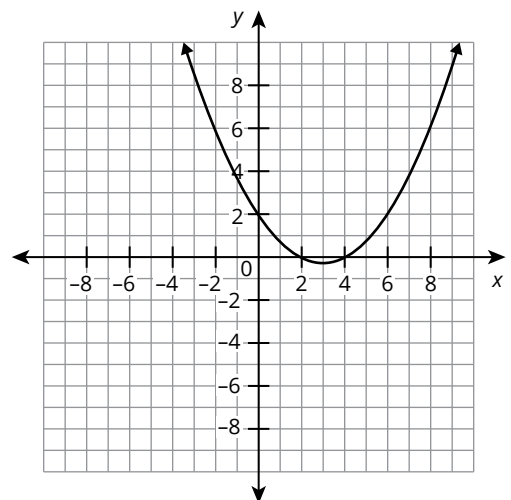
2. $g(x) = (x - 2)(x + 1)$

Dilate the function by $x + 2$.



4. $m(x) = (x - 4)\left(\frac{1}{4}x - \frac{1}{2}\right)$

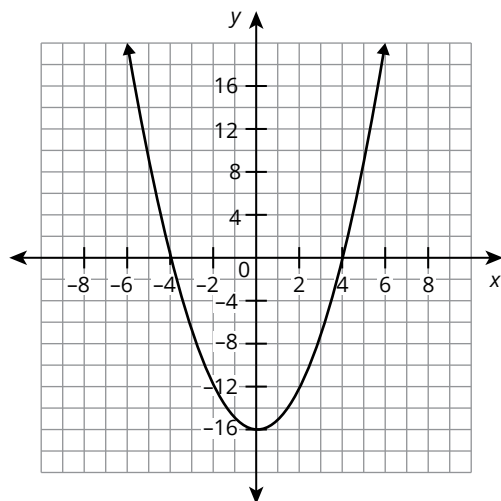
Dilate the function by $\frac{1}{2}x + 2$.



TOPIC 1 Composing and Decomposing Functions

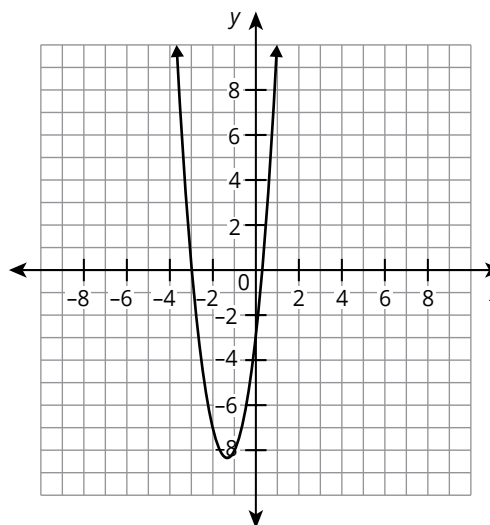
5. $n(x) = (x + 4)(x - 4)$

Dilate the function by $\frac{1}{3}x$.



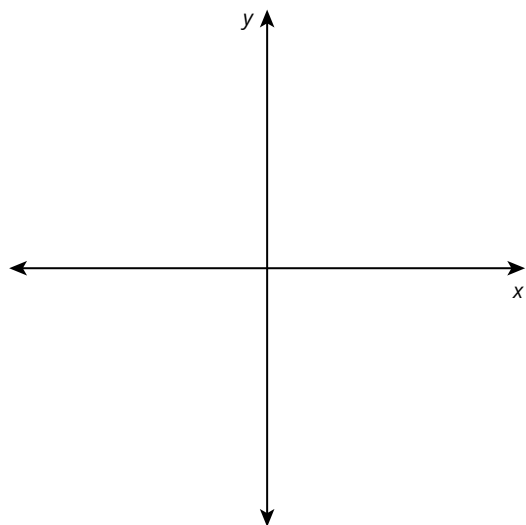
6. $p(x) = (x + 3)(3x - 1)$

Dilate the function by $x + 1$.

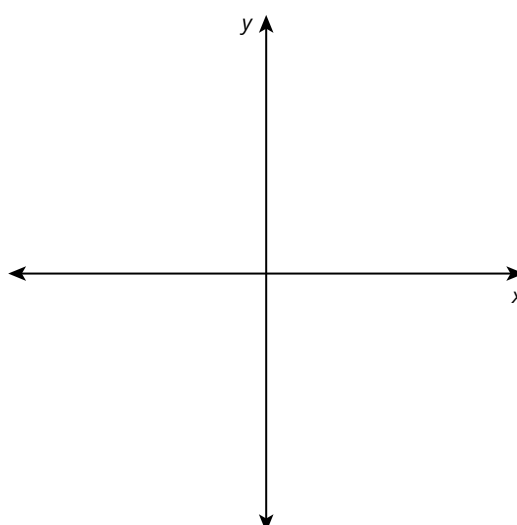


B. Predict the shape of each function. Sketch it on the coordinate plane and label the zeros. Then, use graphing technology to test each prediction.

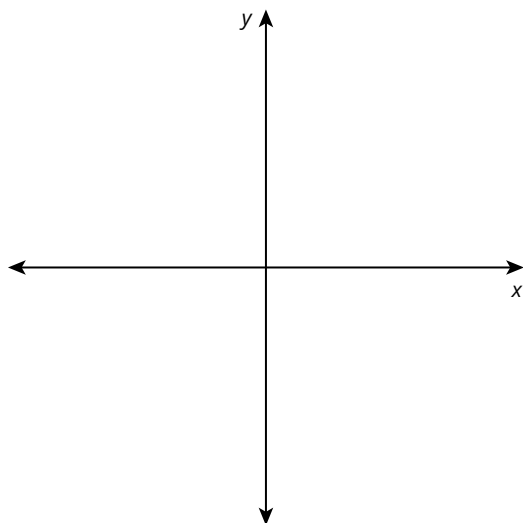
1. $f(x) = x(x - 2)(x + 4)$



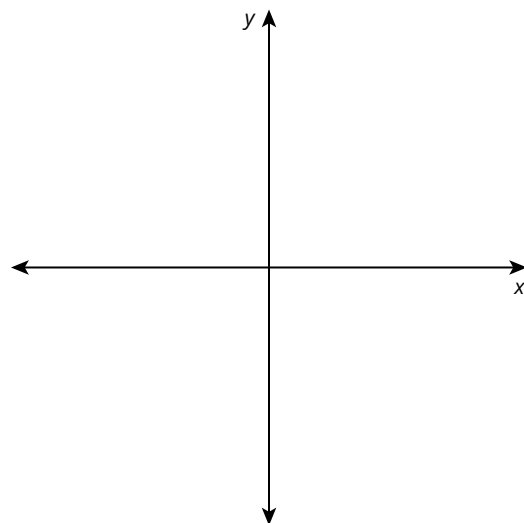
2. $g(x) = (x - 5)(x + 5)(x - 5)$



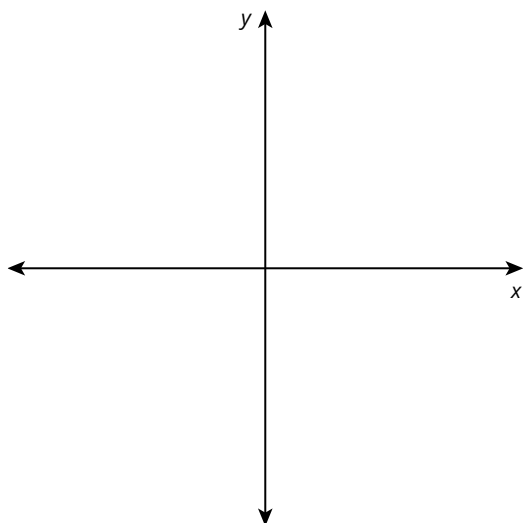
3. $h(x) = (x^2)(x + 6)$



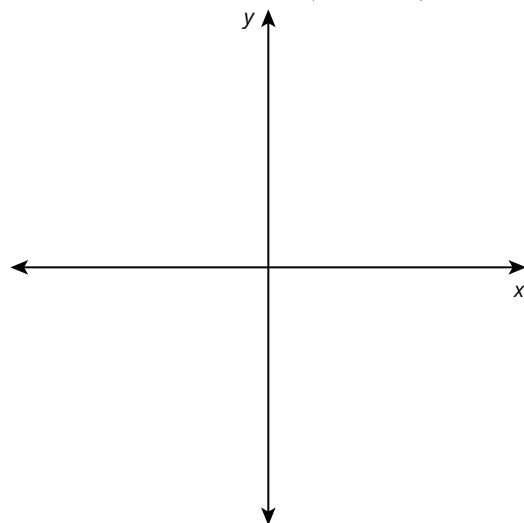
4. $m(x) = (2x + 1)(x - 8)(x - 4)$



5. $n(x) = (x + 7)(x + 7)(x + 7)$



6. $p(x) = (x + 10)(x - 3)\left(\frac{1}{2}x + 3\right)$

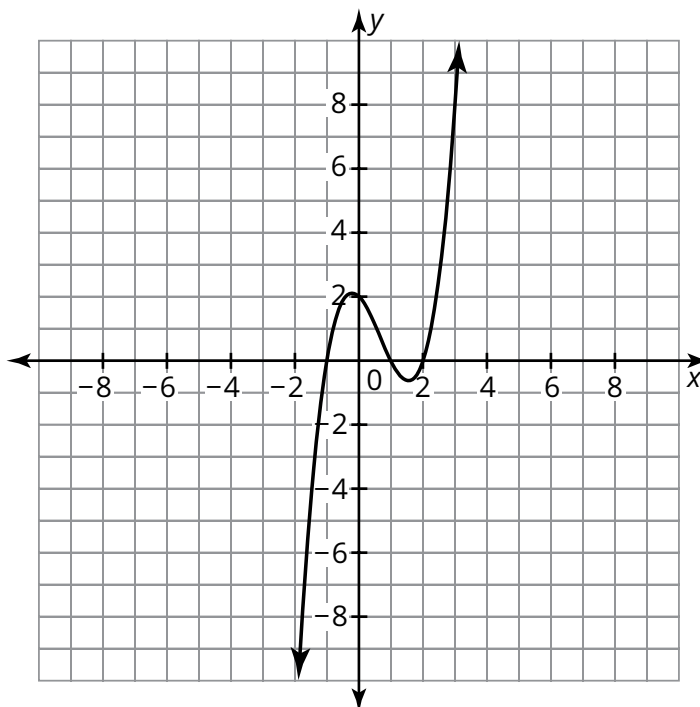


TOPIC 1 Composing and Decomposing Functions

Extension

1. Consider the graph of a cubic function.

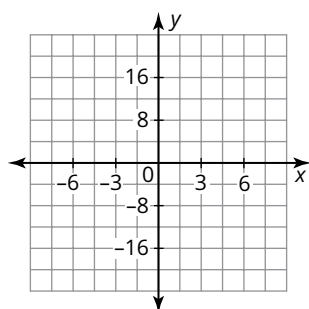
- a. Determine a quadratic function that could have been dilated to produce the cubic function. State the function used to dilate the quadratic function.



- b. Determine a linear function that could have been dilated to produce the cubic function. State the function used to dilate the linear function.

2. Consider the function $f(x) = x^3 + 2x^2 - 5x - 6$.

- a. Use technology to graph the function, and then sketch it on the grid.



- b. Determine the domain and range of the function.
- c. Determine the intervals over which the function is increasing and decreasing.
- d. Determine the x - and y -intercepts of the graph of $f(x)$.
- e. Determine the values of x for which $f(x) = -4$.

TOPIC 1 Composing and Decomposing Functions

Spaced Practice

1. A baseball is hit straight up into the air. The height of the baseball, in meters, is modeled by the function $h(t) = -4.9t^2 + 15t + 1.2$ where t represents time, in seconds, after the ball is hit. A fielder, attempting to catch the ball, raises his glove into the air as it travels upward. The height of the glove, in meters, is modeled by the function $g(t) = -0.4t + 2.8$ where t also represents time, in seconds.
 - a. Write a system of equations to model the problem situation.
 - b. Solve the system of equations you wrote in part (a).
 - c. Can the fielder's glove reach high enough to catch the ball while it is on its upward path? Explain your reasoning.

III. Exploring Cubic Functions

Topic Practice

- A. Complete the table. Include an expression for the volume.
Circle the relative maximum or minimum, if there is one.

1.

Height of Box (in.)	Width of Box (in.)	Length of Box (in.)	Volume of Box (cu. in.)
0	8	10	0
1			
1.5			
2			
3			
4			
h	$8 - 2h$	$10 - 2h$	

2.

Radius of Cylinder (in.)	Height of Cylinder (in.)	Base Area of Cylinder (sq. in.)	Volume of Cylinder (cu. in.)
0	0	0	0
-1			
1			
2			
3			
4			
r	$3r$	$3.14r^2$	

TOPIC 1 Composing and Decomposing Functions

3.

Height of Cube (cm)	Width of Cube (cm)	Length of Cube (cm)	Volume of Cube (cm)
-2			
0			
1			
3			
5			
			1000
s			

4.

Width of Tank (m)	Height of Tank (m)	Length of Tank (m)	Volume of Tank (cu. m)
10			
20			
30			
			58,682
40			
50			
w	$100 - 2w$	$3w - 50$	

5.

Height of Square Pyramid (ft)	Side of Base Length (ft)	Area of Base (sq. ft)	Volume of Square Pyramid (cu. ft)
-4			
0			
3			
6			
9			
12			
p	$\frac{1}{2}p$	$\frac{1}{2}p^2$	

6.

Length of Base (dm)	Height of Base (dm)	Length of triangular Prism (dm)	Volume of Triangular Prism (cu. dm)
-0.5			
0			
0.4			
0.5			
			5
2	4		
b		$10b - 5$	

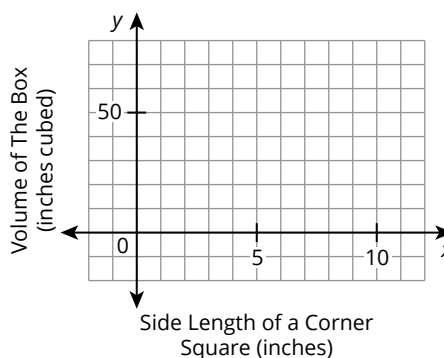
TOPIC 1 Composing and Decomposing Functions

- B. Jasmine is making open nets of rectangular prisms out of construction paper as part of an art project. She cuts squares from each of the four corners of the construction paper, and then folds up the remaining sides to form a box without a lid. Each sheet of construction paper is 12 inches long by 8 inches wide.**

1. Let x represent the side length of a corner square in inches. Write a function $V(x)$ to represent the volume of a box in terms of x .

3. For what value of x is the volume of the box a maximum? State the volume and dimensions of the box. Round your answers to the nearest hundredth.

2. Graph the function $V(x)$ using technology and sketch the graph below.



4. Determine the domain and range of the function and the domain and range of the problem situation using inequalities. Explain your reasoning.

TOPIC 1 Composing and Decomposing Functions

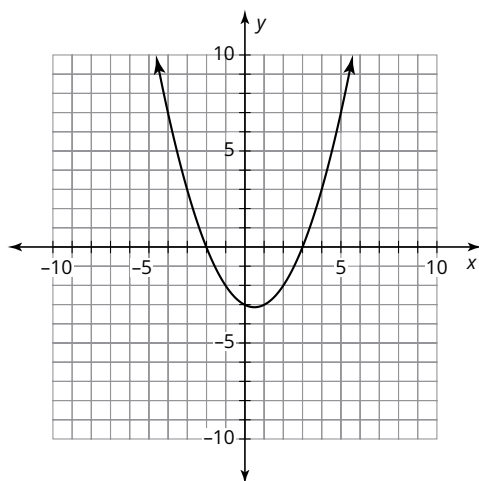
- b. Graph the function $V(x)$ using technology. Determine the domain and range of the function. Determine the domain and range of the function as they relate to this problem situation. Explain your reasoning.
- c. Determine the maximum volume of a bin. What are the dimensions of a bin with the maximum volume?
- d. Determine any relative maximums or relative minimums of $V(x)$. Then, determine the intervals over which the function is increasing and decreasing.
- e. Determine the x - and y -intercepts of the graph of $V(x)$. What do they represent in this problem situation?
- f. Daniela's boss asks her to make several bins with volumes of exactly 40 cubic feet. Determine the bin dimensions that will work.

Spaced Practice

- Dilate each function by the given factor to create a new function of higher degree. Sketch the graph and then identify the zeros of the new function.

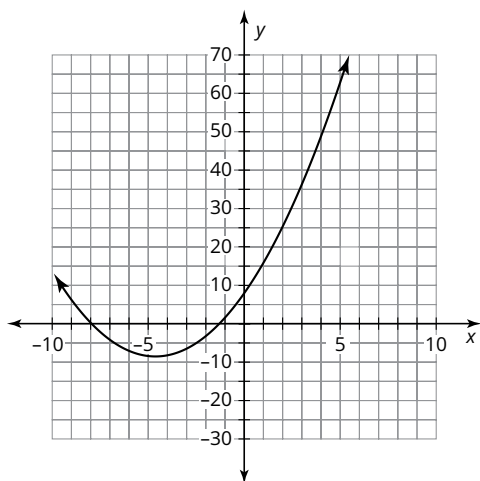
a. $f(x) = \left(\frac{1}{2}x + 1\right)(x - 3)$

Sketch $(x + 1) \cdot f(x)$.



b. $g(x) = (3x + 4)\left(\frac{1}{4}x + 2\right)$

Sketch $(x - 2) \cdot f(x)$.



- Solve the equation $x^2 - 6x + 35 = 10$.

IV. Decomposing Cubic Functions

Topic Practice

A. Use multiplication tables to determine the product of the three linear factors.

1. $4x(x + 2)(x - 4)$

2. $(2x - 3)(4x + 5)(x + 6)$

3. $(5x - 8)^3$

4. $(7 - 2x)(5 + x)(7 + 3x)$

5. $\left(\frac{3}{4}x\right)\left(\frac{2}{5}x\right)\left(\frac{2}{3}x - 1\right)$

6. $(0.75x)(4x - 5)(-2x + 5)$

B. Determine the product of three linear factors. Use graphing technology to verify that the expressions are equivalent.

1. $3x(x + 3)(x - 2)$

2. $(2x - 1)(2x + 1)(x + 4)$

3. $(4x - 7)^3$

4. $(10 - 3x)(7 + x)(8 + 6x)$

TOPIC 1 Composing and Decomposing Functions

5. $\left(\frac{1}{2}x\right)\left(\frac{2}{3}x\right)\left(\frac{1}{4}x - 1\right)$

6. $0.25x(12x - 1)(8 - 3x)$

C. Determine the product of linear and quadratic factors. Use graphing technology to verify that the expressions are equivalent.

1. $x(x^2 + 3x - 4)$

2. $(2x - 9)(4x^2 - 5x - 12)$

3. $7x(x + 5)^2$

4. $(x^2 + 1)(8 - x)$

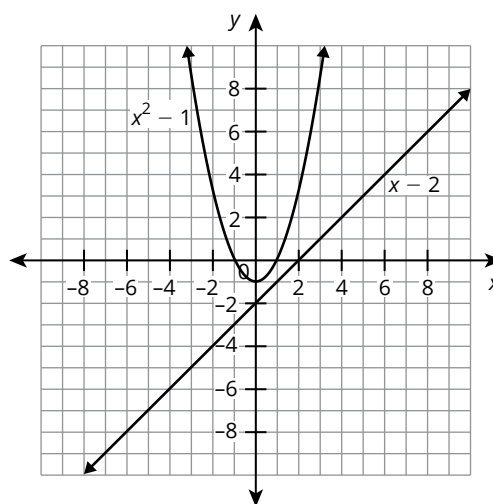
5. $(-2.3 + 1.1x + 0.9x^2)(4.5x - 3.8)$

6. $\left(-\frac{3}{4}x^2 + \frac{1}{8}\right)\left(\frac{1}{4} - \frac{7}{8}x\right)$

7. How do problems 1–6 demonstrate the Fundamental Theorem of Algebra?

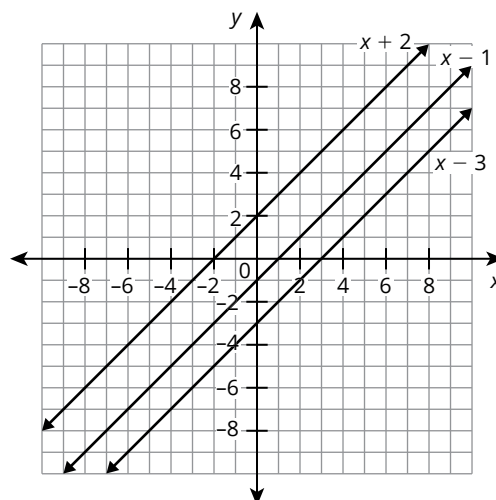
- D. Sketch the graph of the cubic function that is the product of the functions shown. Then, determine the product of the functions algebraically. Verify your sketch by graphing the product on a graphing calculator.

1. $(x - 2)(x^2 - 1)$

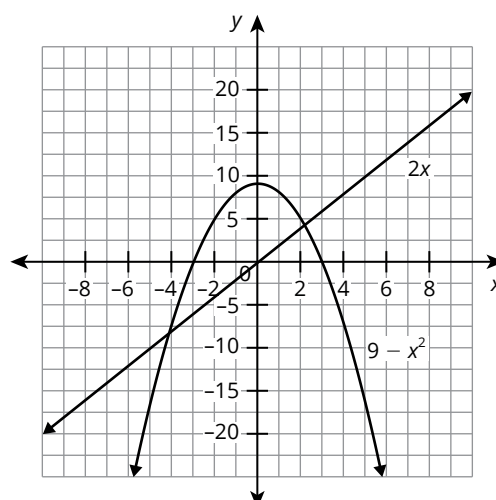


TOPIC 1 Composing and Decomposing Functions

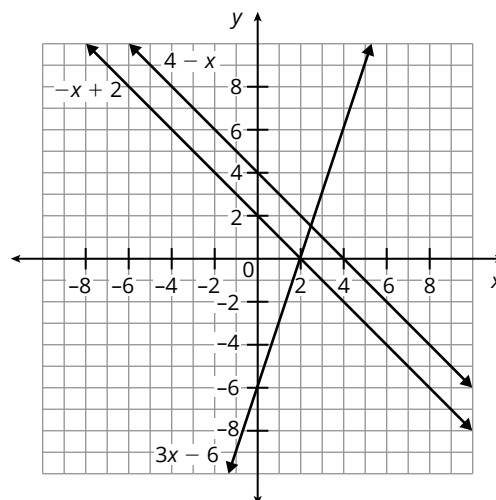
2. $(x + 2)(x - 1)(x - 3)$



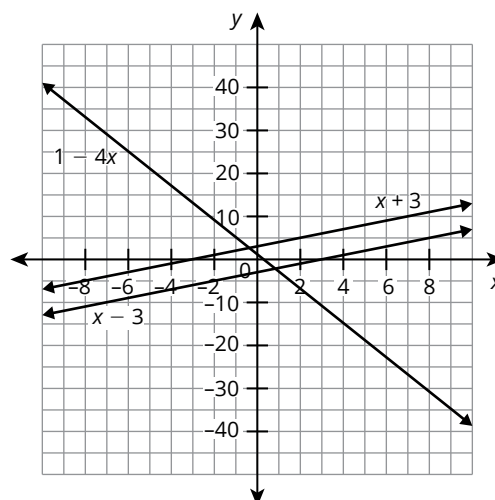
3. $2x(9 - x^2)$



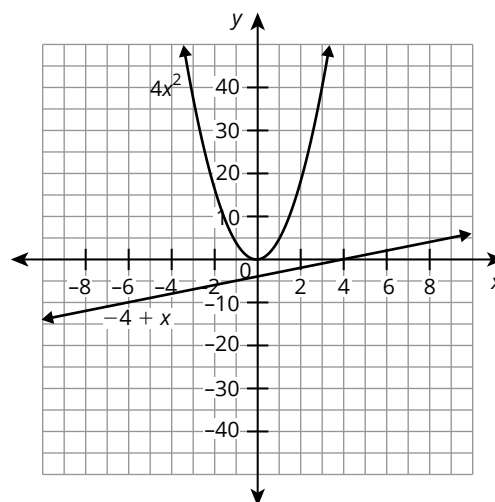
4. $(4 - x)(-x + 2)(3x - 6)$



5. $(x + 3)(1 - 4x)(x - 3)$



6. $(-4 + x)(4x^2)$

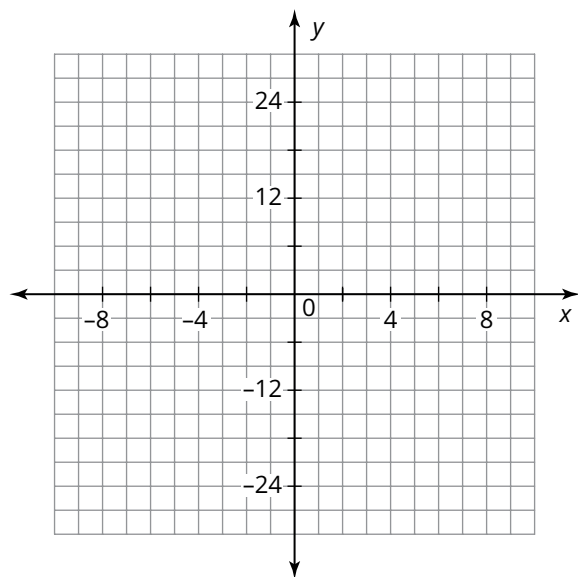


TOPIC 1 Composing and Decomposing Functions

Extension

1. Consider the function $f(x) = 3x^3 + 5x^2 - 16x - 12$.

a. Graph the function.



b. Determine the zeros of the function from the graph.

c. Rewrite the function into factored form.

d. Determine the intervals over which the value of $f(x)$ is positive.
Determine the intervals over which the value of $f(x)$ is negative.
Explain your reasoning.

Spaced Practice

1. An open box is to be made from a rectangular piece of cardboard, 30 inches by 24 inches, by cutting equal squares from the corners and turning up the sides.
 - a. Write a function $V(x)$ to represent the volume of a box in terms of the side length, x , of the removed squares. Explain your reasoning.
 - b. Represent the function $V(x)$ using graphing technology. Determine the domain and range of the function and of this problem situation using inequalities. Explain your reasoning.

2. Write a quadratic equation that includes the given points.
 - a. vertex $(4, -3)$; point $(6, -1)$
 - b. points $(-2, 8)$, $(1, 14)$, and $(0, 10)$

3. Solve the equation $2m^2 - 6m + 14 = 0$.

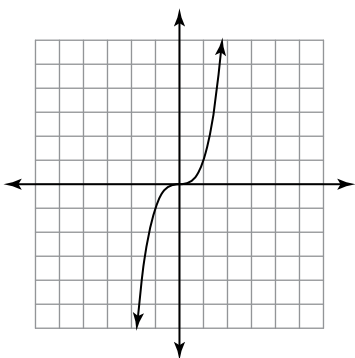
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I. Symmetric Functions

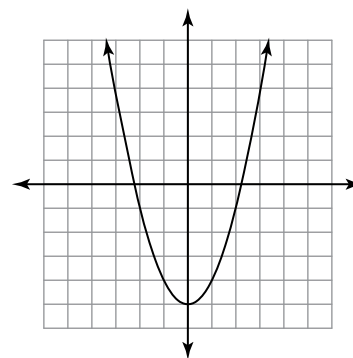
Topic Practice

A. Determine whether the function represented by each graph is symmetric about a line, a point, or neither.

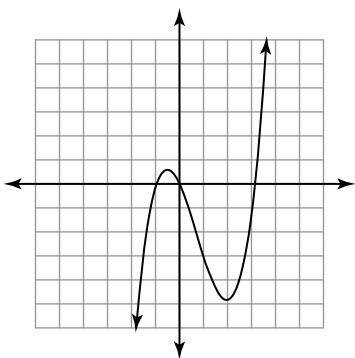
1.



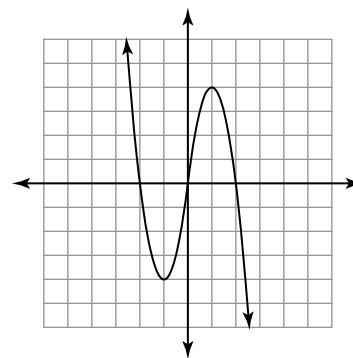
2.



3.

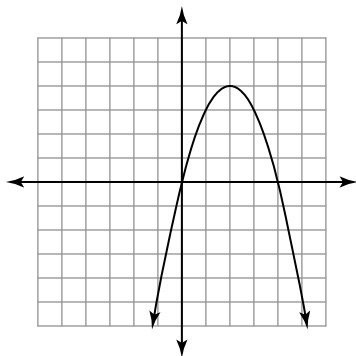


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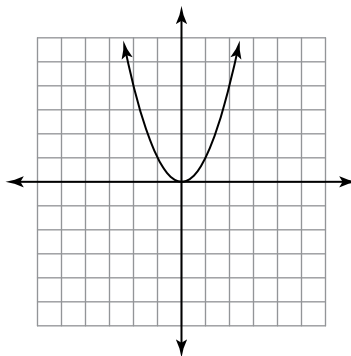


TOPIC 2 Attributes of Cubic Functions

5.

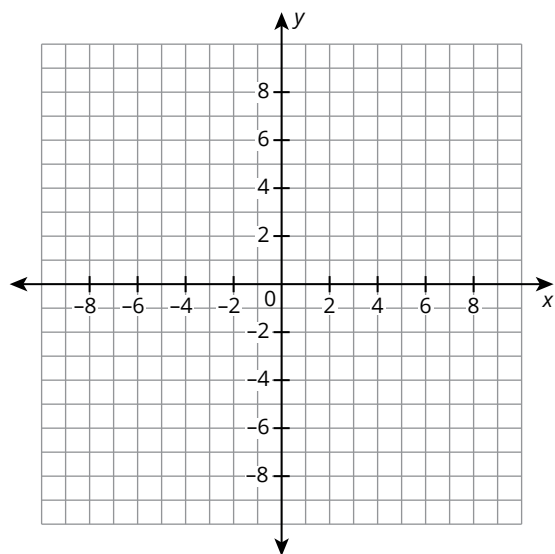


6.

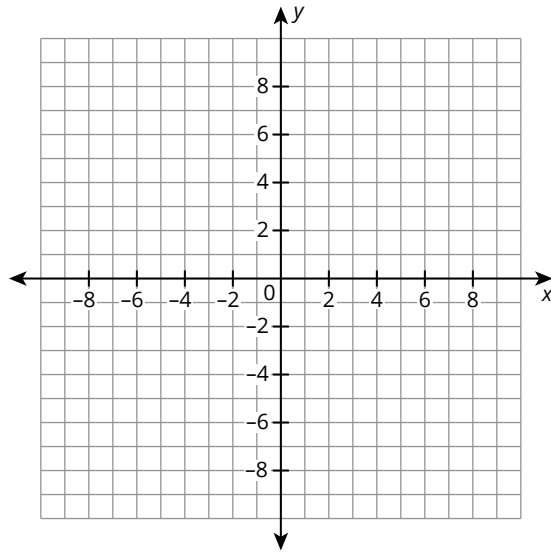


B. Graph and describe the key attributes of each function. Be sure to include whether the graph is symmetric about a point, symmetric about a line, or neither.

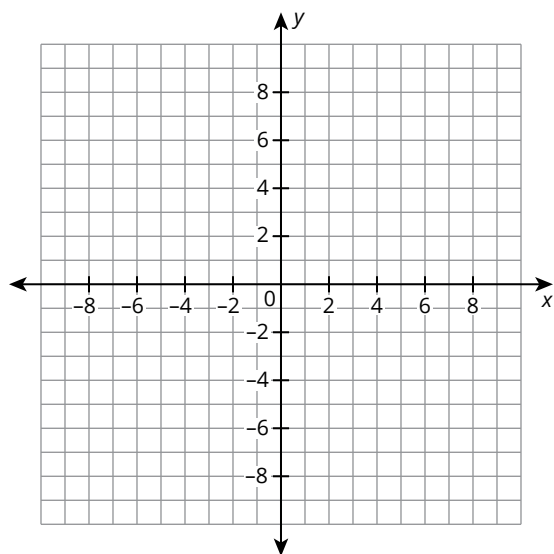
1. $f(x) = x^3 - 3x$



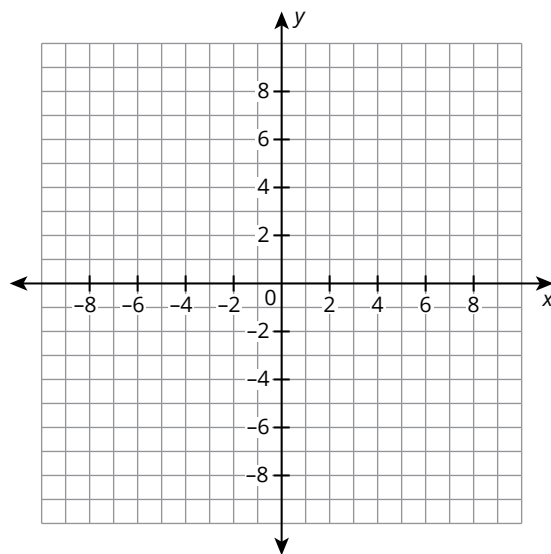
2. $f(x) = x^3 + 2x^2 - 3x$



3. $f(x) = -x^3 + x^2 + 2x$

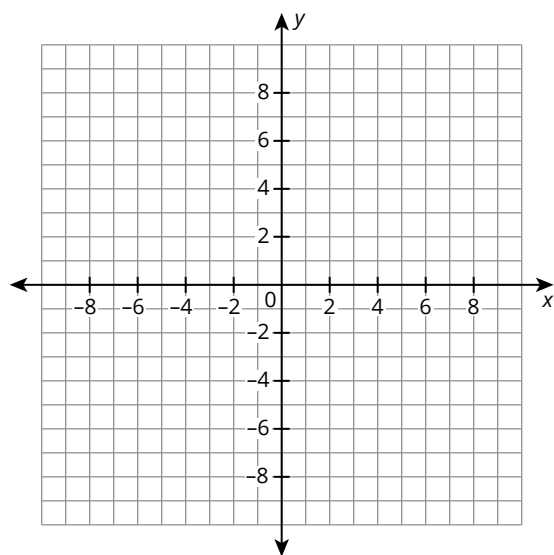


4. $f(x) = x^3 + 4x^2 + 4x$

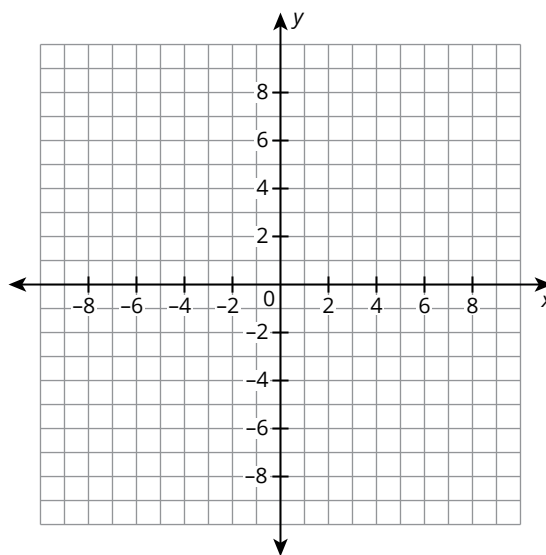


TOPIC 2 Attributes of Cubic Functions

5. $f(x) = x^3 + 3(x^2) + 2x$



6. $f(x) = -x^3 + 4x^2 - 2x$



Extension

1. Use technology to graph the function $f(x) = 3x^6 - 5x^4 + 2x^2 - 100$. Then describe the symmetry of the graph.

Spaced Practice

1. Consider the functions $k(x) = x + 1$, $m(x) = x - 4$, $n(x) = x + 5$, and $f(x) = k(x) \cdot m(x) \cdot n(x)$.
 - a. Determine the degree of the function $f(x)$. Explain your reasoning.
 - b. Determine the zeros of $f(x)$. Explain your reasoning.
 - c. Determine the intervals over which the value of $f(x)$ is positive. Determine the intervals over which the value of $f(x)$ is negative. Explain your reasoning.

2. Add or subtract each expression.
 - a. $(5 + 3i) + (8 - 9i)$
 - b. $(-10 - 7i) - (4 - 15i)$

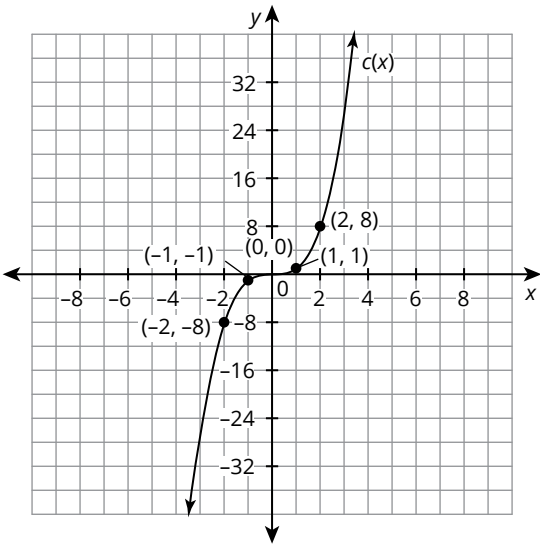
II. Transformations of Cubic Functions

Topic Practice

A. Use reference points and symmetry to complete the table of values for each function. Then, graph the function on the coordinate plane.

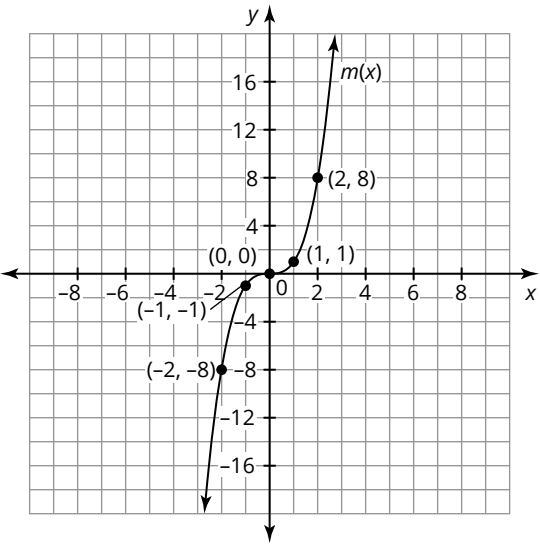
1. $c(x) = x^3$; $h(x) = -3c(x)$

Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



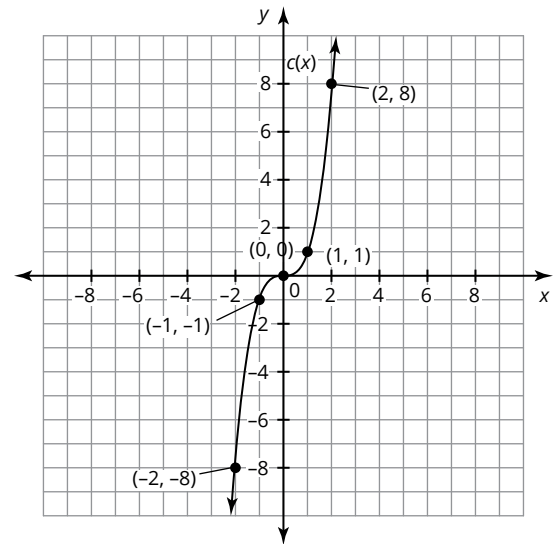
2. $m(x) = x^3$; $n(x) = m(x) + 4$

Reference Points on $m(x)$	→	Corresponding Points on $n(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



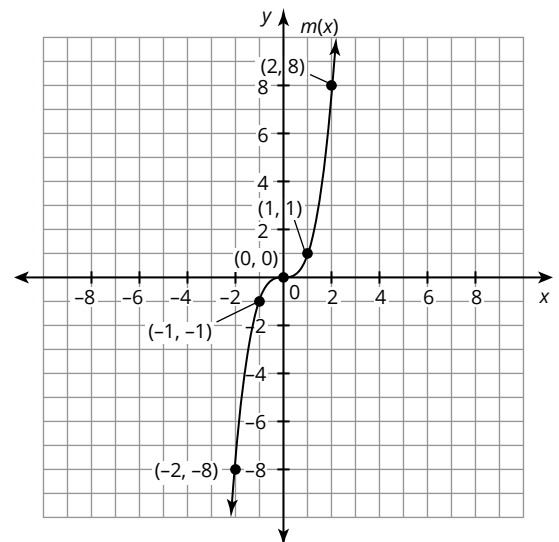
3. $c(x) = x^3$; $g(x) = \frac{1}{4}c(x)$

Reference Points on $c(x)$	→	Corresponding Points on $g(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



4. $m(x) = x^3$; $p(x) = m(x - 1)$

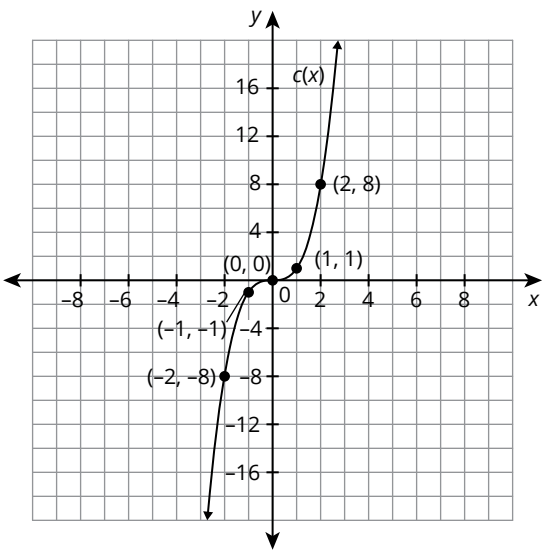
Reference Points on $m(x)$	→	Corresponding Points on $p(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



TOPIC 2
 Attributes of Cubic Functions

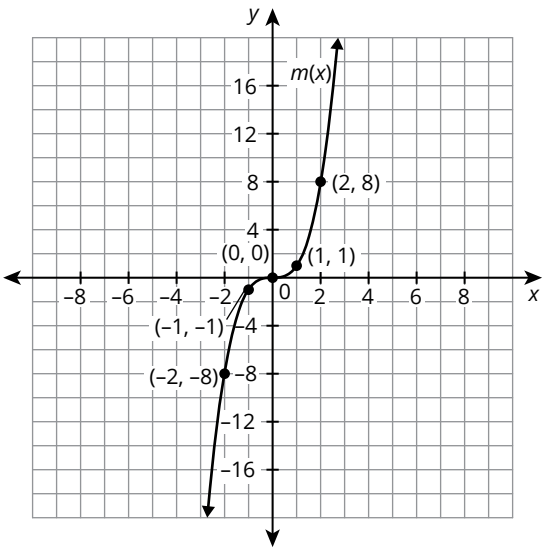
5. $c(x) = x^3$; $d(x) = c(x) - 3$

Reference Points on $c(x)$	→	Corresponding Points on $d(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



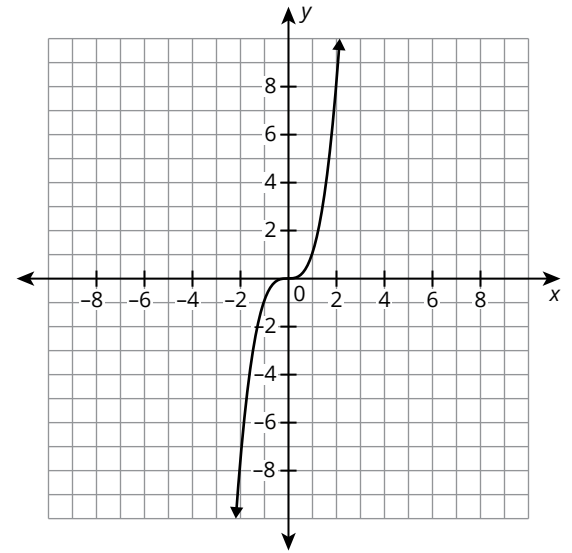
6. $m(x) = x^3$; $t(x) = m(-2x)$

Reference Points on $m(x)$	→	Corresponding Points on $t(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



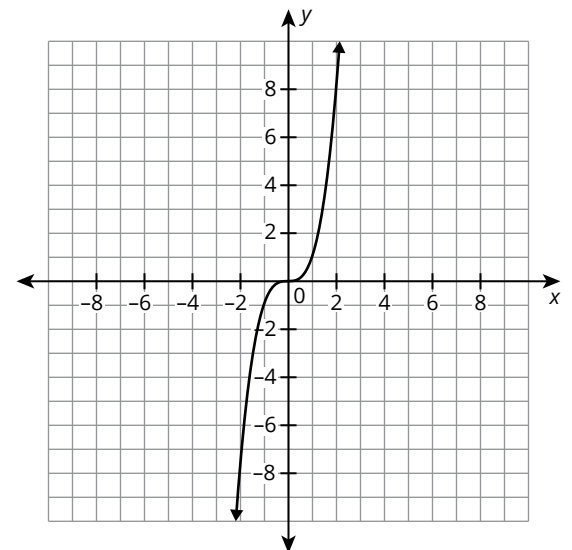
7. $c(x) = x^3$; $h(x) = 3c(x)$

Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



8. $c(x) = x^3$; $h(x) = c(x + 3)$

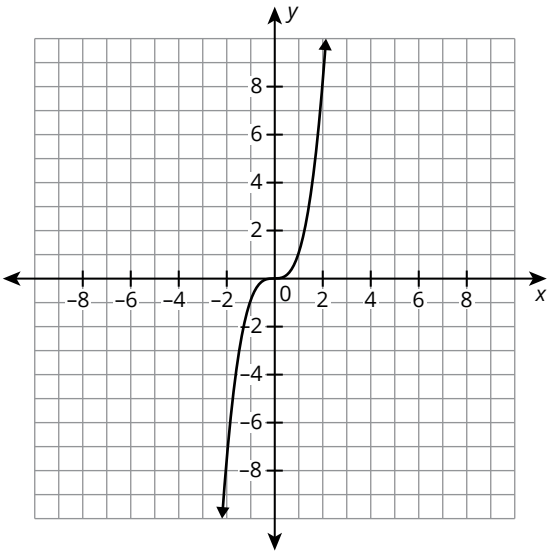
Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	
(1, 1)	→	
(2, 8)	→	



TOPIC 2
 Attributes of Cubic Functions

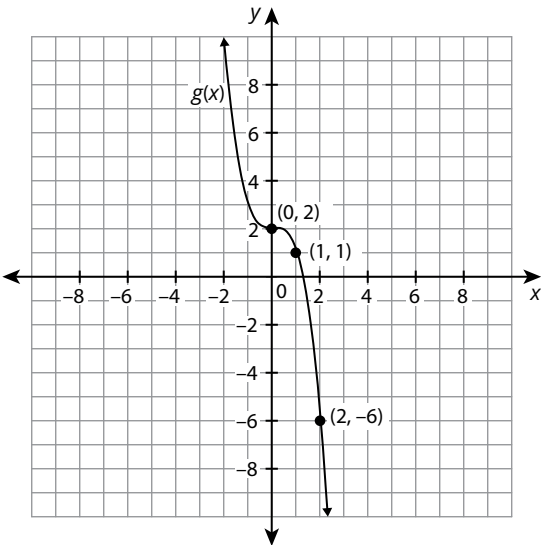
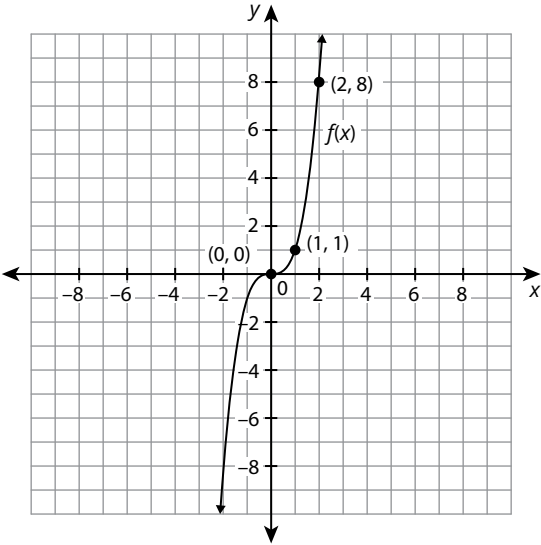
9. $c(x) = x^3$; $h(x) = c(2x)$

Reference Points on $c(x)$	$\rightarrow$	Corresponding Points on $h(x)$
(0, 0)	$\rightarrow$	
(1, 1)	$\rightarrow$	
(2, 8)	$\rightarrow$	

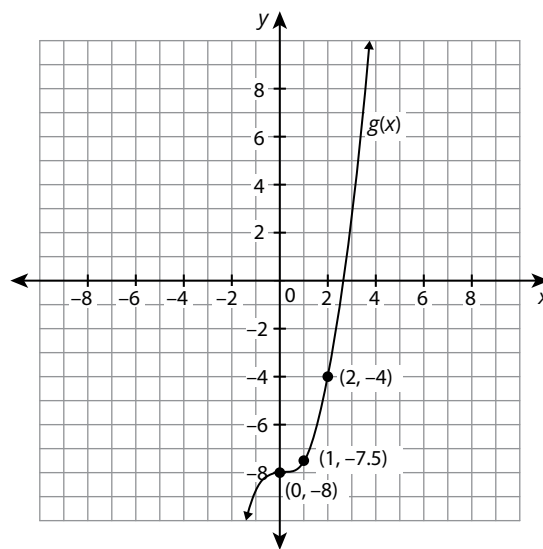
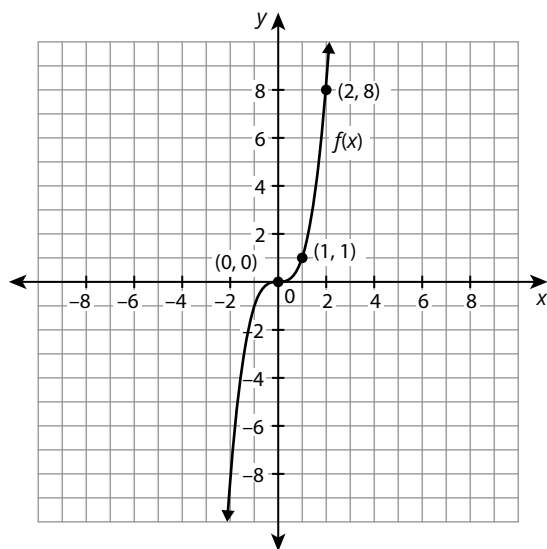


B. Analyze the graphs of $f(x)$ and $g(x)$. Write an equation for $g(x)$ in terms of $f(x)$.

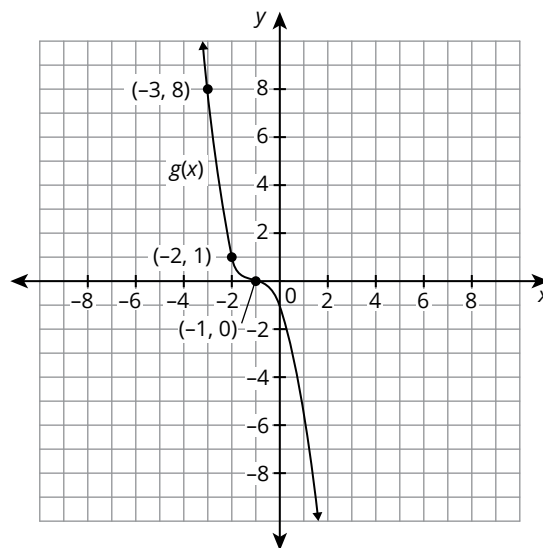
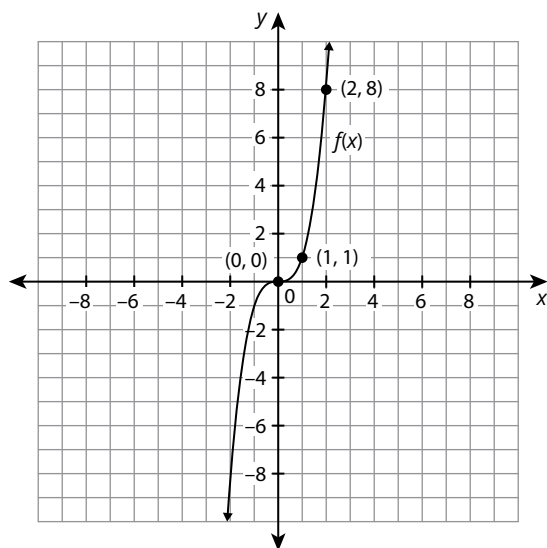
1. $g(x) =$



2. $g(x) =$

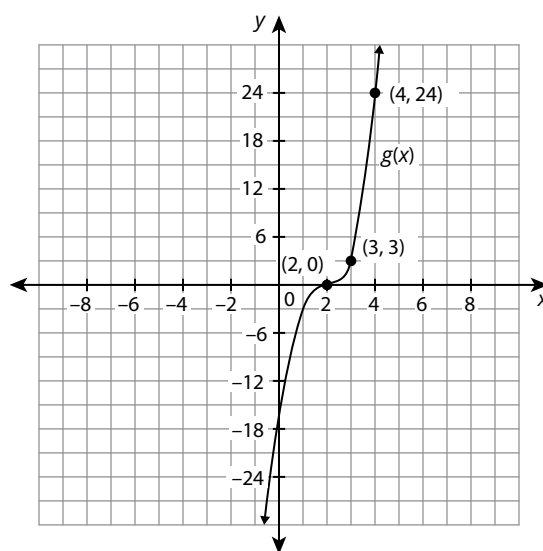
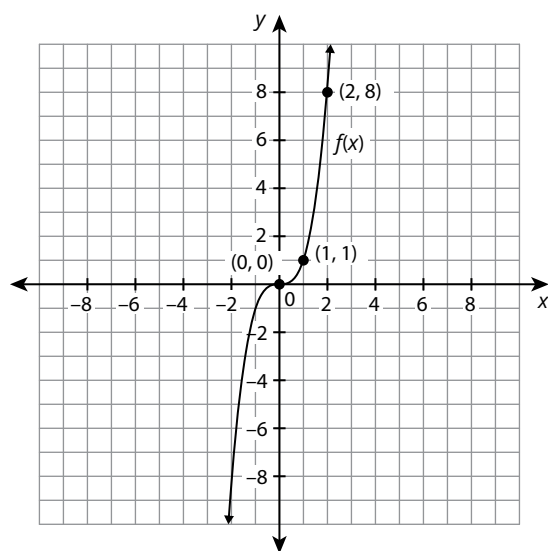


3. $g(x) =$

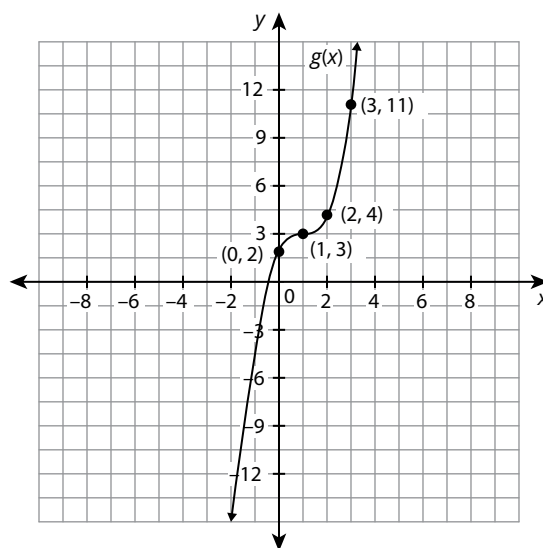
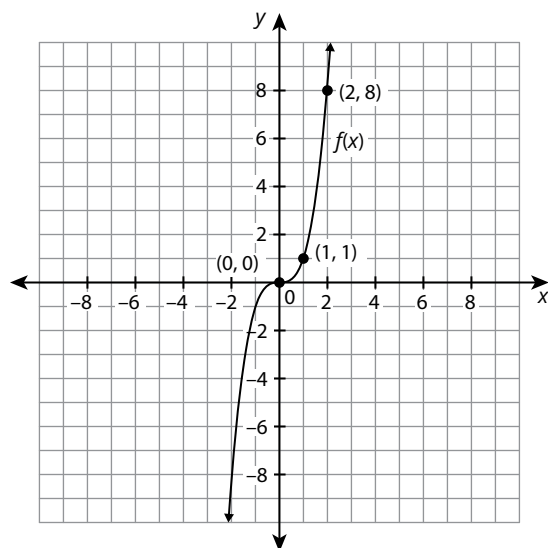


TOPIC 2 Attributes of Cubic Functions

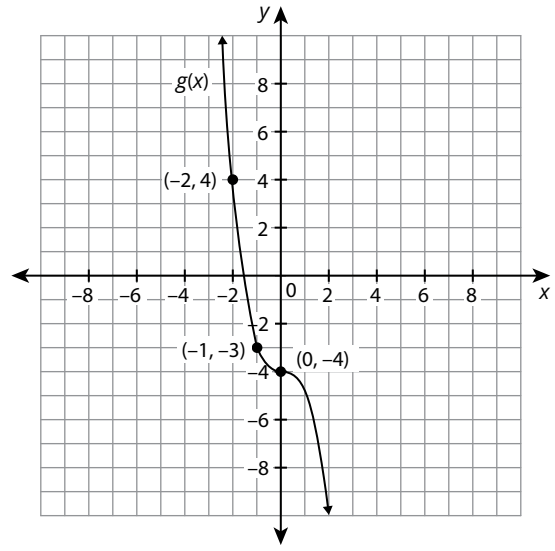
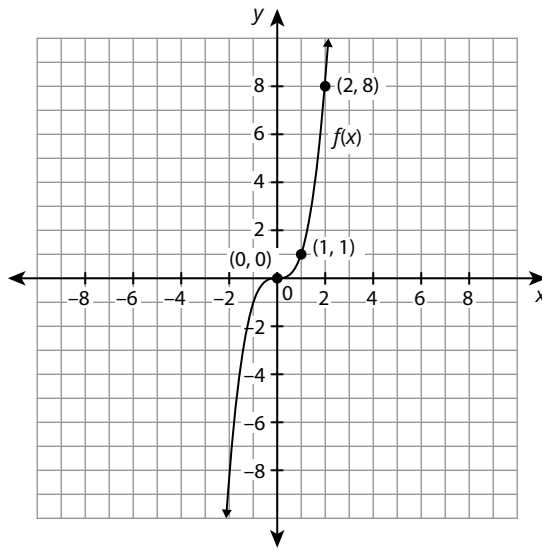
4. $g(x) =$



5. $g(x) =$



6. $g(x) =$



C. The equation for a cubic function $c(x)$ is given. The equation for the transformed function $m(x)$ in terms of $c(x)$ is also given. Describe the transformation(s) performed on $c(x)$ that produced $m(x)$. Then, write a specific equation for $m(x)$.

1. $c(x) = x^3$; $m(x) = -c(0.5x) + 2$

2. $c(x) = x^3$; $m(x) = 4c(x - 3) - 5$

TOPIC 2 Attributes of Cubic Functions

3. $c(x) = x^3$; $m(x) = -4p(x) + 4$

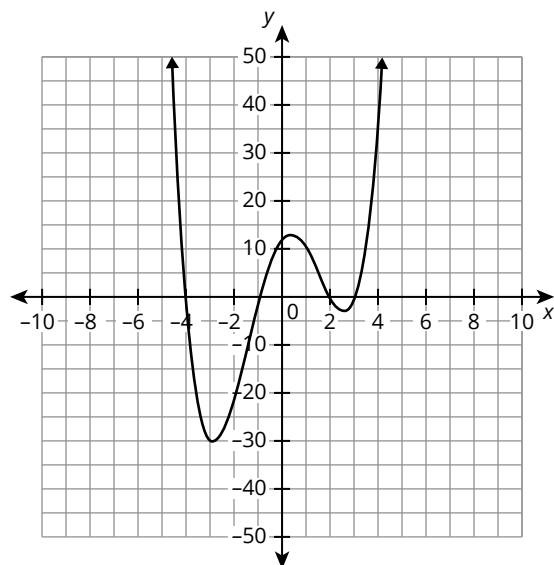
4. $c(x) = x^3$; $m(x) = -c(x + 5)$

5. $c(x) = x^3$; $m(x) = 2c(-x - 2)$

6. $c(x) = x^3$; $m(x) = c(x + 4) - 1$

Extension

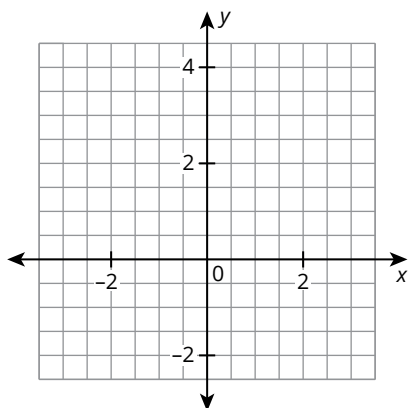
1. Consider the graphed function $f(x)$ and sketch $g(x)$ when $g(x) = -f(x - 5)$.



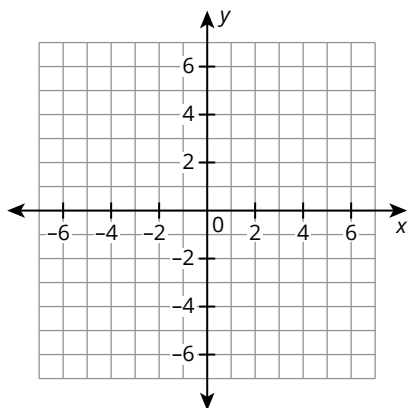
Spaced Practice

1. Determine the symmetry of each graph.

a. $f(x) = x^3 - 3x$



b. $f(x) = x^3 - 2x^2 - 3x$

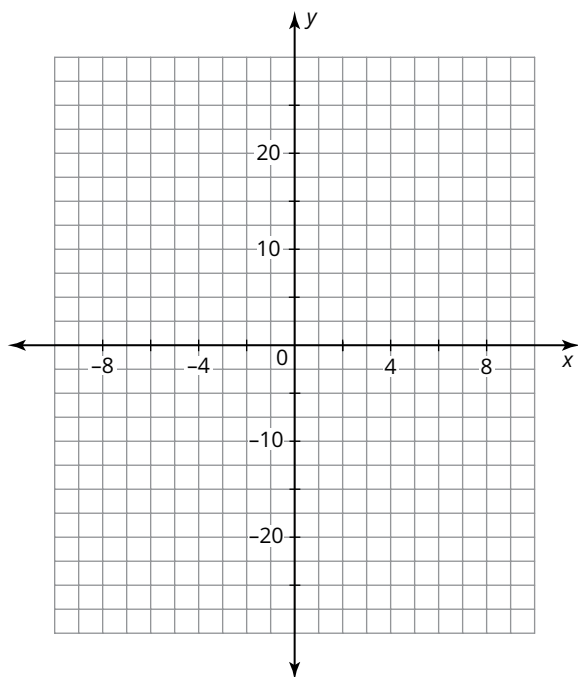


TOPIC 2 Attributes of Cubic Functions

2. Dilate each function by the given factor to create a new function of higher degree. Sketch the graph and then identify the zeros of the new function.

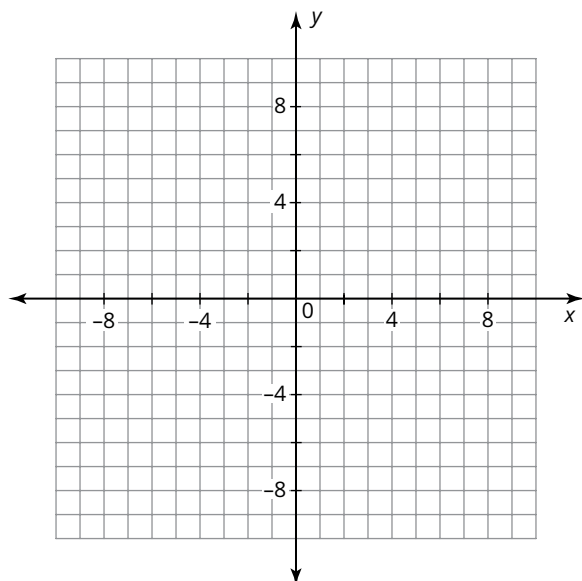
a. $f(x) = (x + 3)(4x - 1)$

Dilate by $(x - 1)$.



b. $f(x) = (x + 2)(x - 2)$

Dilate by x .



3. Rewrite each expression using multiplication.

a. $(3 - 2i)(4 + 5i)$

b. $(7 - 6i)^2$

III. Key Attributes of Cubic Functions

Topic Practice

- A. Lucas has been tracking the stock price of a tech company over the 40 days since it started trading on the New York Stock Exchange. He models the stock price over the 40 days with the function $p(x) = 0.00025x^3 - 0.0075x^2 - 0.1x + 12$, where $p(x)$ represents the stock price, in dollars, on the x th day since the stock started trading.
1. Graph the function $p(x)$ using graphing technology. Is each of the extrema for the graph of $p(x)$ visible with these settings? Explain how you know.
 2. Write the domain and range of $p(x)$ using interval notation.
 3. Explain what the domain and range mean in the context of the problem. Then, write the domain and range of $p(x)$ in terms of the problem situation using interval notation.
 4. Determine the y -intercept of the graph of $p(x)$. Then, describe the meaning of the y -intercept in the context of the problem.

5. Determine the x -intercepts of the graph of $p(x)$. Include only the x -intercepts that are in the domain of the problem situation. Then, describe the meaning of the x -intercepts in the context of the problem.

6. Determine the intervals over which the function is increasing and decreasing. Include only the intervals within the context of the problem. Explain how these intervals relate to the problem situation.

7. At what point during the 40 days since the stock started trading did the price of the stock reach a maximum? What was the maximum price at that point? Explain your reasoning.

8. On the 40th day since the stock started trading, Lucas analyzes his model and tries to decide whether to buy stock in the tech company. Based on his model, should Lucas buy the stock?

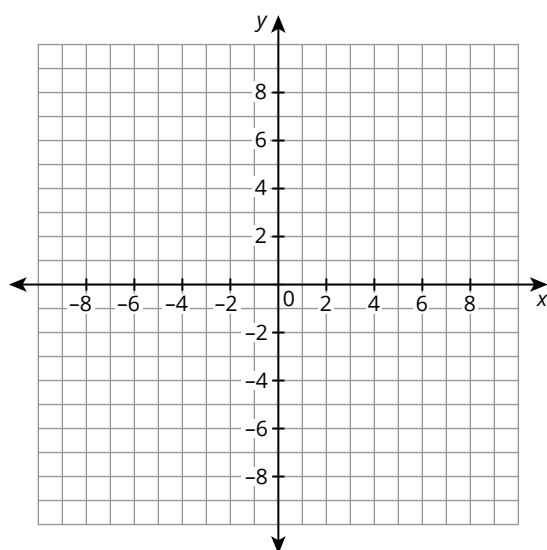
9. Do you think the function $p(x)$ will effectively model the stock price beyond the 40th day?

TOPIC 2 Attributes of Cubic Functions

B. Use the coordinate plane to sketch a cubic function with the given key attributes. If the graph is not possible to sketch, explain why.

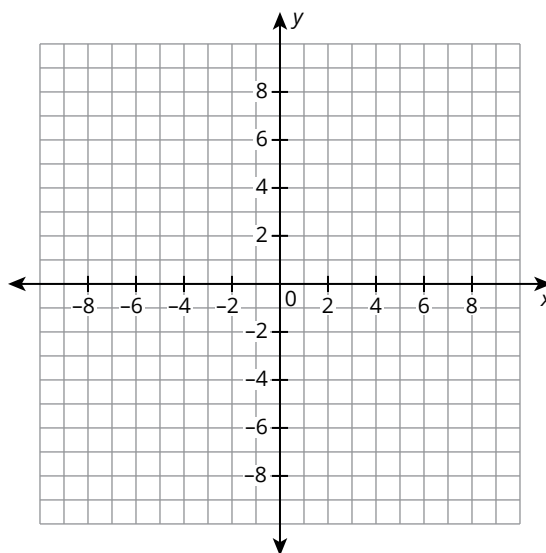
1. Key Attributes:

- degree 3
- starts in Quadrant II
- ends in Quadrant IV
- relative maximum at $x = 3$
- relative minimum at $x = -3$



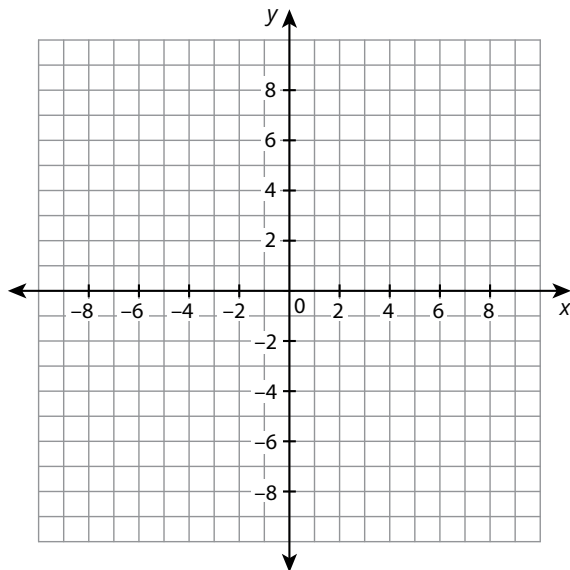
2. Key Attributes:

- odd degree
- starts in Quadrant II
- does not enter Quadrant I
- relative minimum at $x = -2$



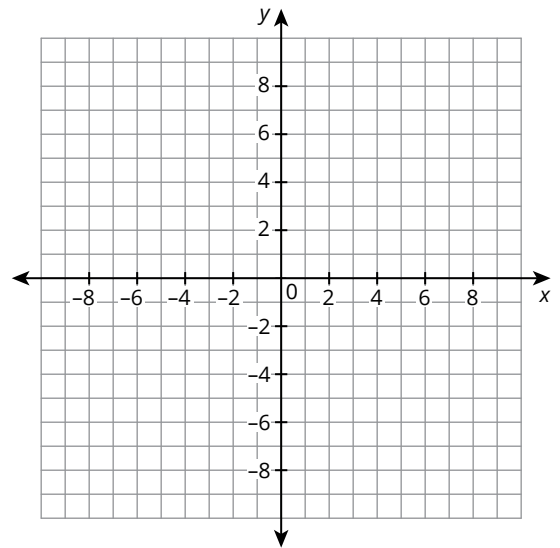
3. Key Attributes:

- degree 3
- negative a -value
- y -intercept at $y = -4$
- x -intercepts at $x = -5$, $x = -2$, $x = 1$, and $x = 3$



4. Key Attributes:

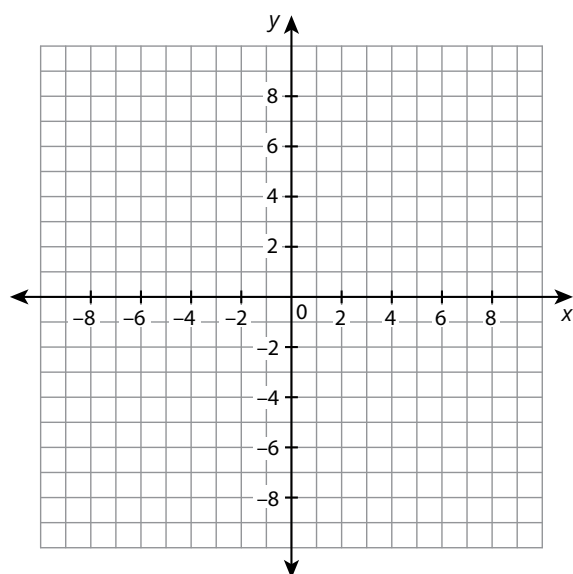
- positive a -value
- y -intercept at $y = -2$
- three x -intercepts
- two relative extrema



TOPIC 2 Attributes of Cubic Functions

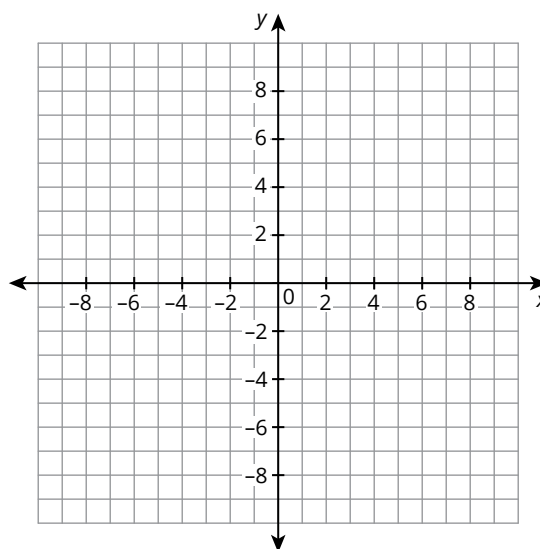
5. Key Attributes:

- degree 3
- positive a -value
- two x -intercepts
- absolute maximum at $y = 1$



6. Key Attributes:

- always decreasing
- y -intercept at $y = -2.5$
- x -intercept at $x = -3$

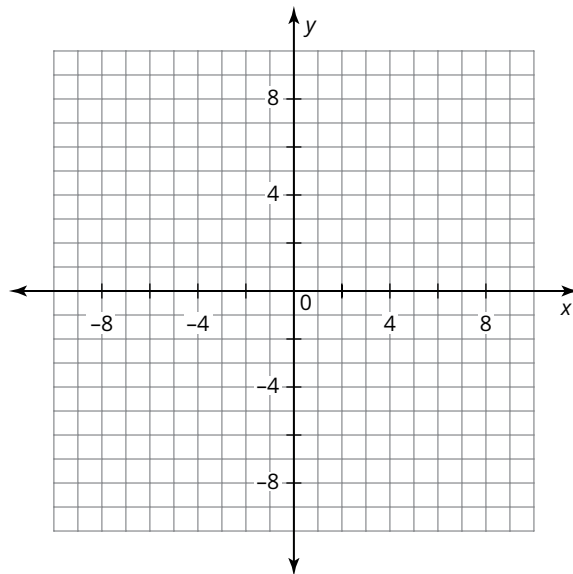


Extension

1. Sketch a graph of a polynomial function with the attributes given. If the graph is not possible to sketch, explain why.

Key Attributes:

- degree 6
- two real roots
- relative maximum at $y = 1$
- absolute minimum at $x = -4$

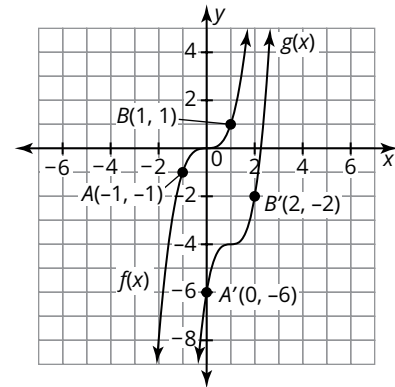


TOPIC 2 Attributes of Cubic Functions

Spaced Practice

1. Analyze the graphs of the functions $f(x)$ and $g(x)$.

a. Write the equation for $f(x)$.



b. The function $g(x)$ is a transformation of the function $f(x)$. Describe the transformations performed on $f(x)$ that result in the function $g(x)$. Explain your reasoning.

c. Write the equation for $g(x)$.

2. Solve the equation $b(b + 3) = -12$.

Name _____ Date _____

I. Adding, Subtracting, and Multiplying Polynomials

Topic Practice

A. Determine each sum or difference.

1. $(3x^2 + 5x - 7) + (2x^2 - 3x + 4)$

2. $(6x^2 - 4x + 3) - (2x^2 + 3x - 5)$

3. $(4x^3 + 2x^2 - 3x + 5) + (x^3 - 3x^2 + 7x - 2)$

4. $(7x^3 - 5x^2 + 4x - 6) - (3x^3 + 2x^2 - x + 4)$

5. $(3x^2 + 4x - 1) - (2x^2 - 3x + 6) + (x^2 + x - 4)$

6. $(5x^3 + 3x^2 - 2x + 1) + (x^3 - x^2 + 4x - 3) - (2x^3 + x^2 - 3x + 2)$

B. Multiply the polynomials using a multiplication table.

1. $(x^3 - 2x^2 + 3x - 4)(x^2 + x - 1)$

2. $(x^4 + 3x^3 + 3x - 4)(x^3 - 4x + 5)$

TOPIC 3 Relating Factors and Zeros

3. $(2x^3 - x^2 + 4x - 3)(x^3 + 2x^2 - 5x + 1)$

4. $(x^4 + 2x^3 - x^2 - 4)(x^3 - 5x + 2)$

5. $(x^3 + 4x^2 - 2x + 5)(x^3 - 3x^2 + x - 4)$

6. $(x^4 + 2x^3 - 3x^2 + 4x)(2x^3 - x^2 + 3x - 2)$

C. Multiply the polynomials using the distributive property.

1. $3x(x^2 + 2x + 4)$

2. $-2x(x^2 - 3x + 5)$

3. $4(x^3 - 2x^2 + x - 7)$

4. $-3x^2(x^2 + 4x - 2)$

5. $5x(2x^2 - x + 3)$

6. $-x(x^3 + 2x^2 - x + 4)$

D. Determine each sum, difference, or product.

1. $(3x^3 + 4x^2 - 2x + 5) + (2x^3 - x^2 + 6x - 3)$

2. $(4x^4 - 3x^3 + 2x^2 - x + 7) - (x^4 - 5x^3 + 3x^2 + 2x - 1)$

3. $(2x^2 + 3x - 4)(x^2 - x + 2)$

4. $(5x^5 y^3 - 3x^3 y + 7x^3 - 6y) + (2x^5 y^3 + 4x^4 y^3 - x^3 + 8y)$

5. $(6x^6 - 5x^4 y^2 + 3x^2 - 2y + 1) - (3x^6 + 2x^4 y^2 - 4x^2 + y - 3)$

6. $(7x^5 y^3 - 2x^4 y^2)(x^2 y^3 + xy^2 - 1)$

Extension

1. Consider the polynomials $(2x^2 + 2x - 1)$, $(2x - 1)$, and $(x^2 - 3x + 4)$.
 - a. Without multiplying, make a conjecture about the degree of the product of these polynomials. Explain how you determined your answer.

TOPIC 3 Relating Factors and Zeros

- b. Without multiplying, make a conjecture about the number of terms in the product of these polynomials.
- c. Determine the product of the three polynomials.

Spaced Practice

1. Write a quadratic equation in geometric form for a parabola with a focus at $(3, 2)$ and directrix $y = 0$.
2. Write a quadratic equation in geometric form for a parabola with a vertex at $(1, 1)$ and directrix $y = -1$.
3. Write a quadratic equation in geometric form for a parabola with an axis of symmetry $x = -2$ and vertex at $(-2, 4)$, that goes through the point $(-1, 6)$.

II. Factoring Polynomials to Identify Zeros

Topic Practice

A. Factor each expression completely.

1. $x^2 + 12x - 13$

3. $x^2 - 12x - 28$

5. $x^2 - 5x - 14$

2. $x^2 + 6x + 8$

4. $x^2 + 30x + 81$

6. $x^2 - 16x - 36$

B. Factor each expression by factoring out the greatest common factor.

1. $2x^5 - 8x^4 + 10x^3$

2. $-9x^4 + 45x^3 - 9x^2$

3. $105x^3 - 147x$

4. $-\frac{3}{5}x^4 + \frac{3}{5}x^3 - \frac{27}{5}x^2$

5. $\frac{1}{3}x^4 - \frac{8}{3}x^3 + \frac{1}{3}x^2 - \frac{11}{3}x$

6. $8x^4 - 16x^3 + 56x^2 - 24x$

TOPIC 3 Relating Factors and Zeros

C. Factor each expression completely using the chunking method.

1. $4x^2 + 8x + 3$

2. $25x^2 - 35x + 12$

3. $121x^2 - 44x - 12$

4. $49x^2 + 63x + 18$

5. $9x^2 + 30x - 11$

6. $169x^2 - 130x + 24$

D. Factor each perfect square trinomial.

1. $4x^2 + 12x + 9$

3. $16x^4 + 104x^2 + 169$

5. $9x^4 + 42x^2y + 49y^2$

2. $x^2 - 12xy + 36y^2$

4. $25x^4 + 80x^2 + 64$

6. $64x^2 + 16xy^2 + y^4$

E. Factor each expression completely using the factor by grouping method.

1. $x^3 - 2x^2 + 3x - 6$

3. $x^3 - 6x^2 - 9x + 54$

2. $x^3 + x^2 - 4x - 4$

4. $x^4 - 3x^3 - x^2 + 3x$

TOPIC 3 Relating Factors and Zeros

5. $-x^3 + 5x^2 + 16x - 80$

6. $x^3 - 3x^2 - 4x + 12$

7. $x^3 - 3x^2 + 5x - 15$

8. $2x^3 + x^2 + 14x + 7$

F. Factor each expression over the set of real numbers.

1. $x^3 + 2x^2 - 5x - 10$

2. $3x^3 + 7x^2 + 12x + 28$

3. $x^4 - 6x^3 + x^2 - 6x$

4. $2x^4 + 4x^3 - 14x^2 - 28x$

5. $6x^4 - 12x^3 + 9x^2 - 18x$

6. $x^4 + 4x^3 - 8x^2 - 32x$

G. Factor each quartic expression completely using the quadratic form method.

1. $x^4 - 13x^2 + 36$

3. $x^4 - 29x^2 + 100$

5. $x^4 - 164x^2 + 6400$

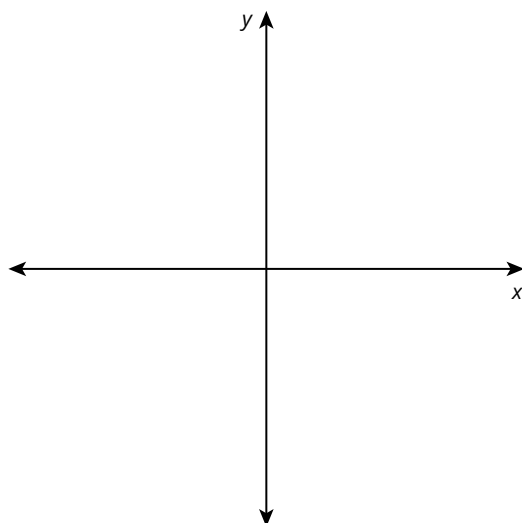
2. $x^4 - 50x^2 + 49$

4. $x^4 - 25x^2 + 144$

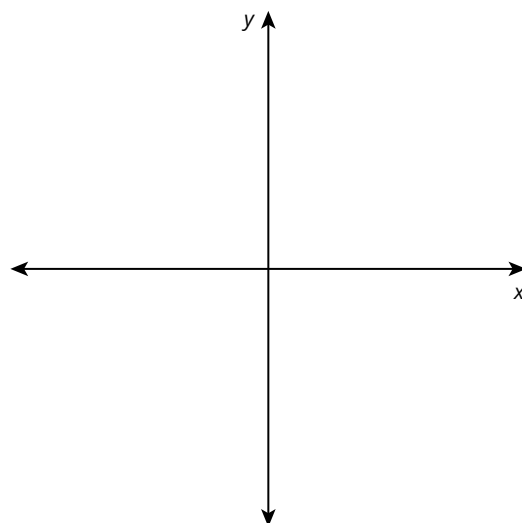
6. $x^4 - 61x^2 + 900$

H. Determine the linear factors of each polynomial function. Then, use the factors to sketch a graph of the function.

1. $f(x) = x^3 - 6x^2 + 11x - 6$

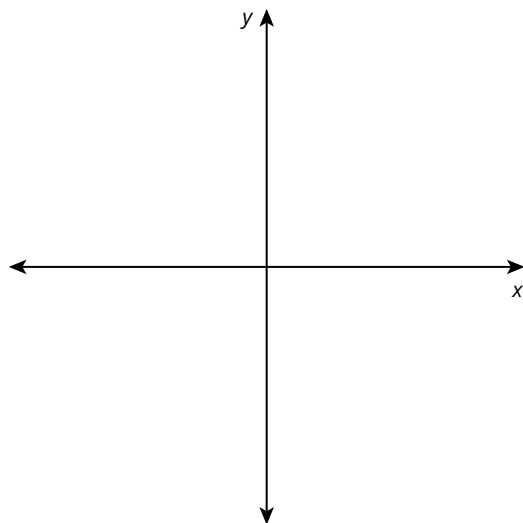


2. $f(x) = x^3 - 4x^2 - 5x + 20$

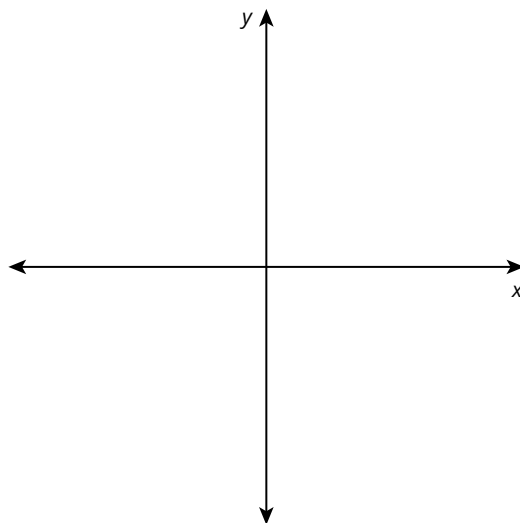


TOPIC 3 Relating Factors and Zeros

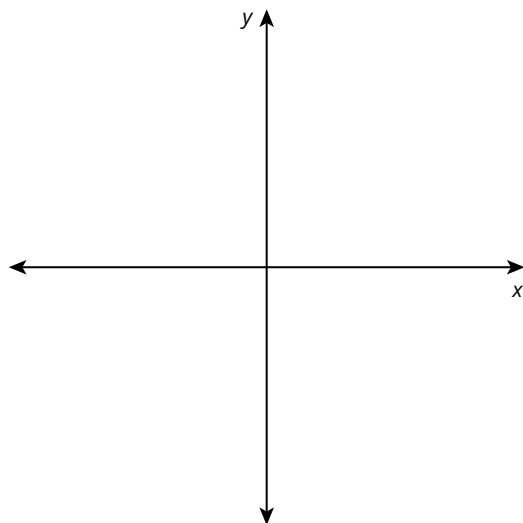
3. $f(x) = x^3 + 4x^2 - 5x - 20$



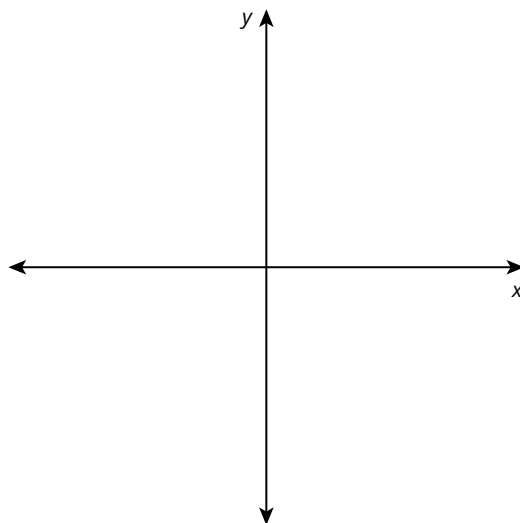
4. $f(x) = x^4 - 5x^2 + 4$



5. $f(x) = x^4 - 6x^2$



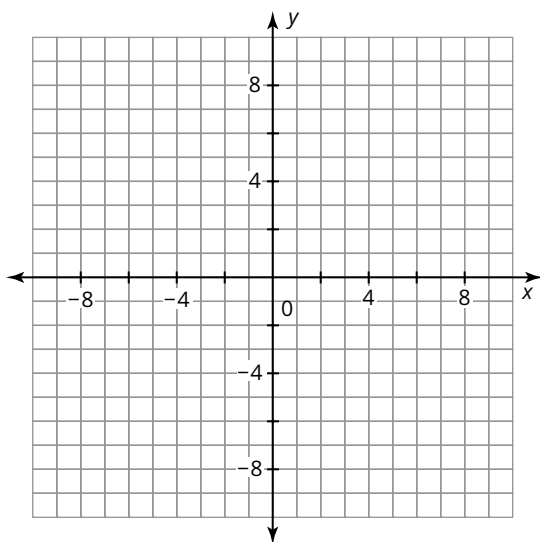
6. $f(x) = x^4 - 2x^3 - 3x^2 + 6x$



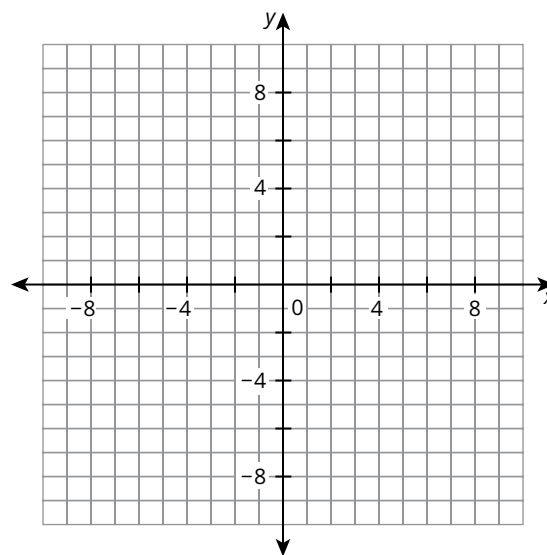
Extension

1. Sketch each piecewise function.

a. $g(x) = \begin{cases} -x + 1, & x < 0 \\ 2x^2 - 8x & x \geq 0 \end{cases}$



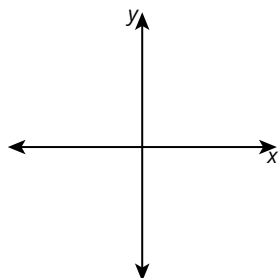
b. $f(x) = \begin{cases} x, & x < -1 \\ x^3 + x^2 - x - 1, & -1 \leq x \leq 1 \\ 4 & x > 1 \end{cases}$



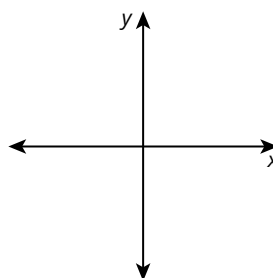
Spaced Practice

1. Use the factors to sketch each cubic function and label the zeros.

a. $f(x) = x(x - 5)(x + 2)$



b. $g(x) = (x + 4)(x - 6)(x + 1)$



TOPIC 3 Relating Factors and Zeros

2. Determine the linear or quadratic factors of each equation.

a. $f(x) = 8x^3 + 216$

b. $h(x) = x^4 + 3x^3 - 4x^2 - 12x$

III. Polynomial Division

A. Write the zero that corresponds to each factor.

1. $x + 5$

2. $x - 12$

3. $2x + 1$

4. $10x - 9$

5. $x - 13$

6. $3x + 4$

B. Write the factor that corresponds to each zero.

1. $x = 2$

2. $x = -7$

3. $x = -75$

4. $x = \frac{2}{3}$

5. $x = -\frac{3}{8}$

6. $x = \frac{5}{4}$

TOPIC 3 Relating Factors and Zeros

C. Determine each function value. When the output is 0, write the associated factor for the function.

- | | |
|--|---|
| 1. Determine $p(-3)$ when
$p(x) = 2x^3 - 2x^2 - 36x - 36$. | 2. Determine $p(-2)$ when
$p(x) = x^4 - 10x^3 + 8x^2 + 106x - 105$. |
| 3. Determine $p(-3)$ when
$p(x) = 2x^4 + 5x^3 + 8x^2 + 15x + 6$. | 4. Determine $p(1)$ when
$p(x) = x^4 + 3x^3 - 6x^2 - 8x$. |
| 5. Determine $p(0.5)$ when
$p(x) = 6x^3 + 11x^2 - 3x - 2$. | 6. Determine $p(-8)$ when
$p(x) = x^4 + 9x^3 + 7x^2 - 9x - 8$. |

D. Use the factor theorem to determine whether the given expression is a factor of each polynomial.

- | | |
|--|---|
| 1. Is $x - 2$ a factor
of $f(x) = x^3 + 8x^2 - 31x + 22$? | 2. Is $x - 3$ a factor
of $f(x) = 4x^4 - x^3 - 52x^2 - 35x + 12$? |
| 3. Is $x - 12$ a factor
of $f(x) = x^4 - 12x^3 + x^2 - 12x$? | 4. Is $x - 8$ a factor
of $f(x) = x^3 - 7x^2 - 14x + 48$? |
| 5. Is $2x - 5$ a factor
of $f(x) = x^3 + 5x^2 - x - 5$? | 6. Is $3x + 4$ a factor
of $f(x) = 3x^3 + 13x^2 + 18x + 8$? |

E. Use the factor theorem to determine the unknown coefficient so that the given linear expression is a factor of the function.

1. Determine a if $x + 3$ is a factor of $f(x) = x^3 + 9x^2 + ax + 15$.

3. Determine a if $x - 1$ is a factor of $f(x) = ax^3 - 10x^2 - 13x + 20$.

5. Determine a if $x + 2$ is a factor of $f(x) = x^3 - x^2 + ax - 36$.

2. Determine a if $x - 4$ is a factor of $f(x) = x^3 + ax^2 - 20x - 48$.

4. Determine a if $x - 7$ is a factor of $f(x) = x^4 - 4x^3 + ax^2 - 8x - 42$.

6. Determine a if $x - 8$ is a factor of $f(x) = x^4 + ax^3 - 5x^2 - 21x - 24$.

F. Use polynomial long division to determine if the given factor is a factor of each polynomial.

1. Is $(x - 1)$ a factor of $x^4 - 3x^3 + 6x^2 - 12x + 8$?

2. Is $(x - 1)$ a factor of $x^4 + 6x^3 - 12x^2 - 38x - 21$?

TOPIC 3 Relating Factors and Zeros

3. Is $(x^2 - x + 6)$ a factor
of $x^4 - 3x^3 - 4x^2 + 12x$?

4. Is $(x - 3)$ a factor
of $x^3 + 12x^2 + 17x - 30$?

5. Is $(2x^2 - x - 6)$ a factor
of $2x^3 + 7x^2 - 10x - 24$?

6. Is $(x + 2)$ a factor of $x^4 - 2x^3 - 4x - 6$?

G. Determine each quotient using polynomial long division. Write the dividend as the product of the divisor and the quotient, plus the remainder.

1. $x - 4 \overline{) 2x^3 - 7x^2 - 19x + 60}$

2. $x - 2 \overline{) 2x^3 - x^2 - 13x - 6}$

3. $x + 3 \overline{) x^3 + 8x^2 + 7x + 5}$

4. $x^2 + 2x - 20 \overline{) 4x^4 + 9x^3 - 82x^2 - 28x + 18}$

TOPIC 3 Relating Factors and Zeros

5. $x + 2 \overline{) 3x^3 + 5x^2 - 2x}$

6. $2x^2 + 11x + 15 \overline{) -10x^4 - 49x^3 - 40x^2 + 56x + 15}$

7. $x^2 + 2x + 1 \overline{) 3x^3 + 7x^2 + 5x + 2}$

8. $x - 2 \overline{) 2x^4 + 6x^3 - 8x^2 + 4x - 56}$

H. Factor each binomial completely over the set of real numbers using the difference of squares method.

1. $x^2 - 100$

2. $49x^2 - 4y^2$

3. $x^4 - 81$

4. $x^4 - 36$

5. $9x^4 - 121y^2$

6. $4x^4 - 9y^2$

I. Factor each binomial using the sum or difference of two cubes.

1. $x^3 + 27$

2. $x^3 - 8y^3$

3. $8x^3 - 125$

4. $x^3 + 64y^3$

5. $343x^3 - 1$

6. $216x^3 + 125y^3$

TOPIC 3 Relating Factors and Zeros

- J. Determine each quotient using synthetic division. Write the dividend as the product of the divisor and the quotient, plus the remainder.

1. $(x^4 + 8x^3 - 3x^2 - 24x) \div (x - 3)$

2. $(x^4 - 3x^3 + 6x^2 - 12x + 8) \div (x - 1)$

3. $(2x^3 + 21x^2 + 22x - 45) \div (2x + 5)$

4. $(x^3 + x^2 - 16x - 16) \div (x + 2)$

5. $(x^4 - 6x^3 - 19x^2 + 24x) \div (x + 3)$

6. $(x^4 + 5x^3 - 33x^2 + 27x) \div (x - 9)$

Extension

- Determine each quotient using synthetic division. Write the dividend as the product of the divisor and the quotient, plus the remainder.

$$x^3 - 2x^2 + x - 1 \overline{) 2x^5 - 3x^4 + 5x^3 - 7x^2 + 4x - 6}$$

$$2x^5 - 4x^4 + 2x^3 - 2x^2$$

$$x^4 + 3x^3 + 5x^2 - 4x$$

$$x^4 - 2x^3 + x^2 - x$$

$$5x^3 - 6x^2 + 5x - 6$$

$$5x^3 - 10x^2 + 5x - 5$$

$$4x^2 - 1$$

Spaced Practice

- Determine the zeros of each function.

a. $f(x) = 3(x - 1)^4 - 9$

b. $f(x) = -2(x + 2)^3 - 1$

c. $f(x) = 5(x + 3)^3$

TOPIC 3 Relating Factors and Zeros

2. Add or subtract the polynomial expressions.

a. $(4x^2 + 7x - 5) + (3x^2 - 2x + 9)$

b. $(5x^3 - 4x^2 + 6x - 8) - (2x^3 + 3x^2 - x + 4)$

c. $(6x^3 + 2x^2 - x + 7) + (3x^3 - 5x^2 + 4x - 2) - (x^3 + 2x^2 - 3x + 1)$

d. $(7x^4 - 3x^3 + 5x^2 - 6x + 2) - (2x^4 - x^3 + 4x^2 - x - 3)$

IV. Rational Root Theorem

Topic Practice

A. Determine the possible rational roots of each polynomial using the rational root theorem.

1. $x^3 - 4x^2 + 6x - 8 = 0$

2. $2x^4 - 4x^2 + 15 = 0$

3. $-2x^3 + 5x + 18 = 0$

4. $x^3 + 12x^2 - 21x + 32 = 0$

5. $5x^4 - 7x^3 + 5x - 30 = 0$

6. $12x^4 - 15x^3 + 24x^2 + 11x - 2 = 0$

TOPIC 3 Relating Factors and Zeros

- B. Use the rational root theorem to determine the possible rational roots for each polynomial equation. Then, solve completely. Use the graph, when given, to identify possible zeros.

1. $x^3 + 3x^2 - 18x - 40 = 0$

- Possible rational roots:

- Solve completely:

2. $6x^3 + 35x^2 - 52x - 21 = 0$

- Possible rational roots:

- Solve completely:

3. $x^4 + 4x^3 - 21x^2 - 36x + 108 = 0$

- Possible rational roots:

TOPIC 3 Relating Factors and Zeros

- Solve completely:

4. $x^2 - 8x + 17 = 0$

- Possible rational roots:

- Solve completely:

5. $x^4 - 4x^3 + 5x^2 - 4x + 4 = 0$

- Possible rational roots:

- Solve completely:

TOPIC 3 Relating Factors and Zeros

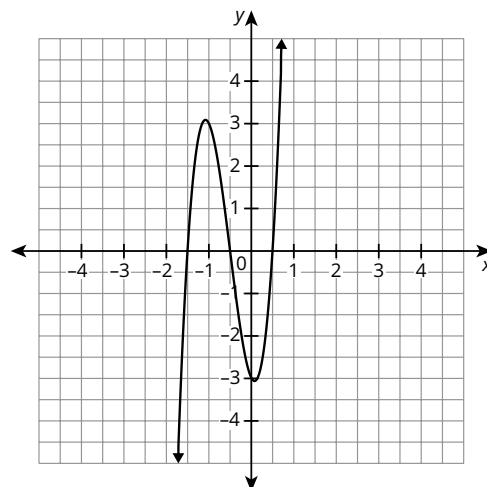
6. $x^3 - 1.5x^2 - 1.5x + 1 = 0$

- Possible rational roots:

- Solve completely:

7. $8x^3 + 12x^2 - 2x - 3 = 0$

- Possible rational roots:

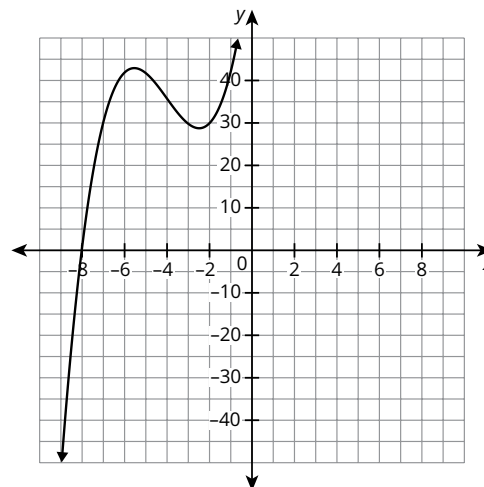


- Solve completely:

TOPIC 3 Relating Factors and Zeros

8. $x^3 + 12x^2 + 41x + 72 = 0$

- Possible rational roots:



- Solve completely:

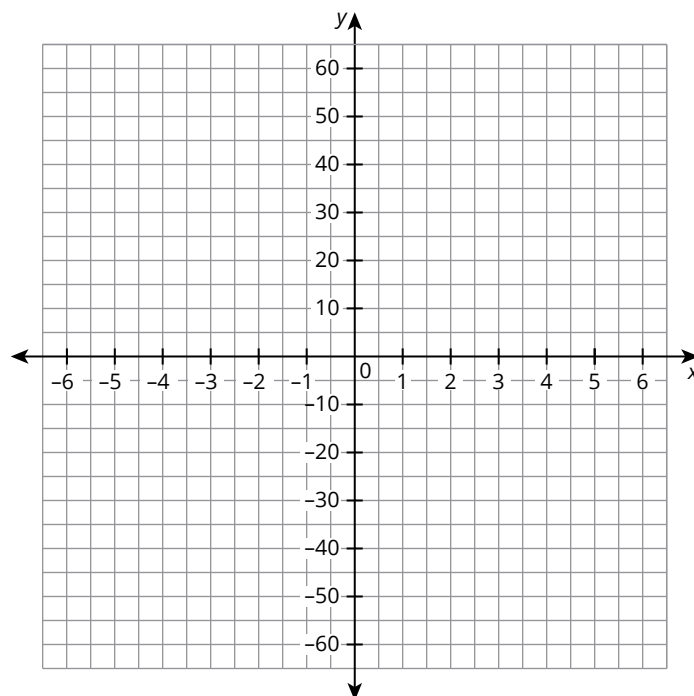
Extension

- Using the graph identify one of the roots, then factor completely.

$$x^5 - 12x^4 + 46x^3 - 60x^2 + 25x$$

- Possible rational roots:

- Solve completely:



Spaced Practice

Solve a system of equations using technology with matrices.

1. The community college theater department sold three kinds of tickets to its latest play production. The adult tickets sold for \$15, the student tickets for \$10 and the child tickets for \$8. The theater department was thrilled to have sold 250 tickets and brought in \$2,825 in one night. The number of student tickets sold is twice the number of adult tickets sold.
 - a. Formulate a system of equations to represent the scenario.
 - b. Use technology to solve the system of linear equations using matrices. How many of each type of ticket did the department sell?

2. Factor each polynomial over the set of real numbers.

a. $16x^2 - 3x - 3$

b. $x^4 - 10x^2 + 25$

Extending Beyond Polynomials

TOPIC 1: Rational Functions

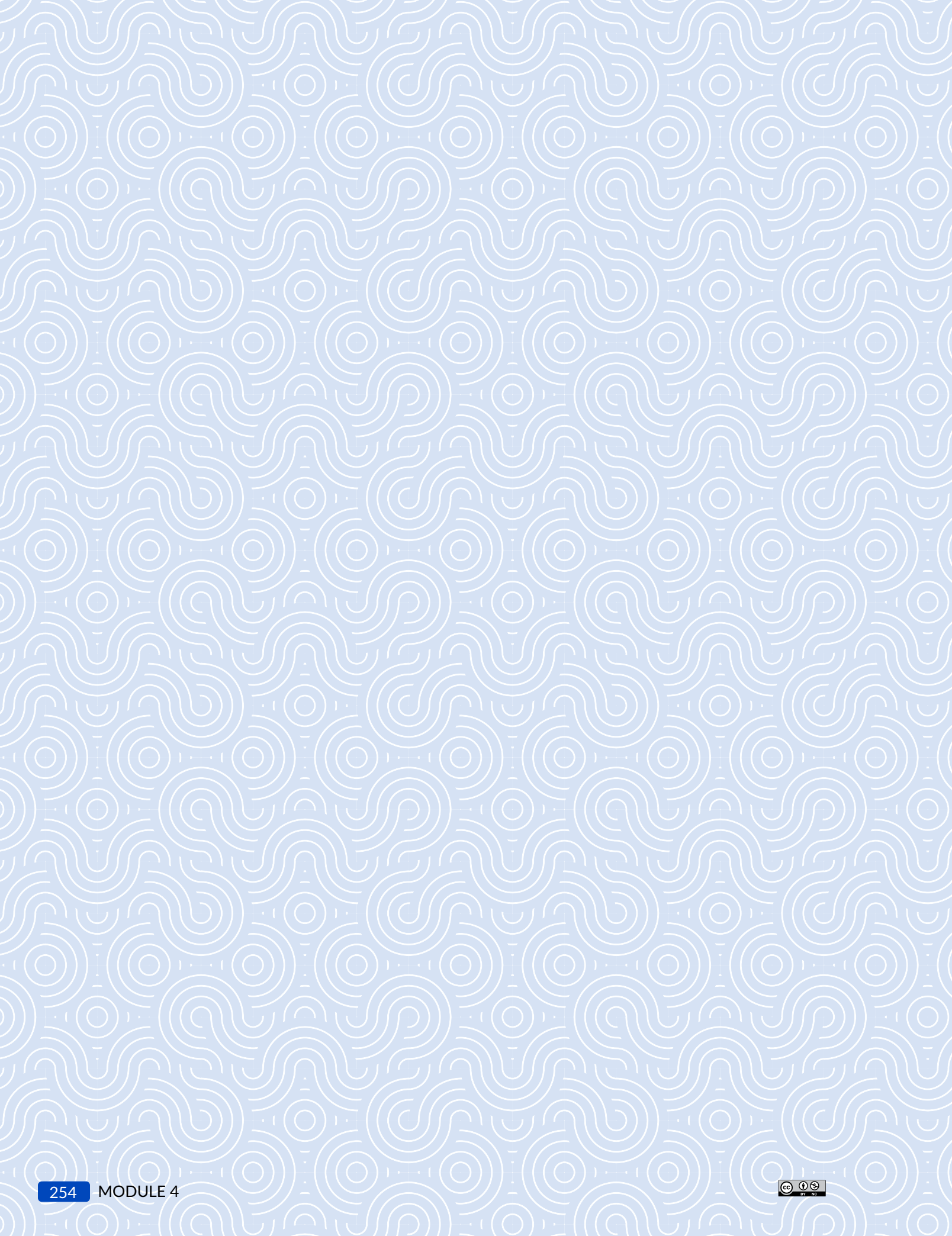
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Name _____ Date _____

I. Introduction to Rational Functions

Topic Practice

- A. Use graphing technology to graph each function. Then, use the graph to determine the domain and range. Write the domain and range in interval notation, set notation, and using inequalities.

1. $f(x) = x^3 - 3x^3 + 2x - 1$

2. $f(x) = |x - 4| + 2$

3. $f(x) = \frac{1}{x-1}$

TOPIC 1 Rational Functions

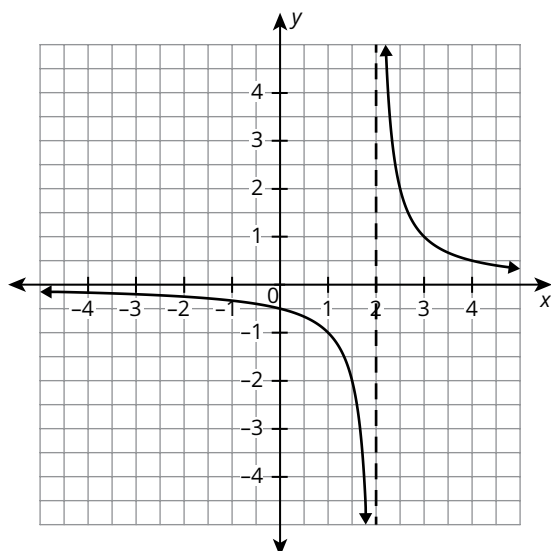
4. $f(x) = |x - 3|, 0 \leq x \leq 6$

5. $f(x) = x^3 - 4x, -2 \leq x \leq 2$

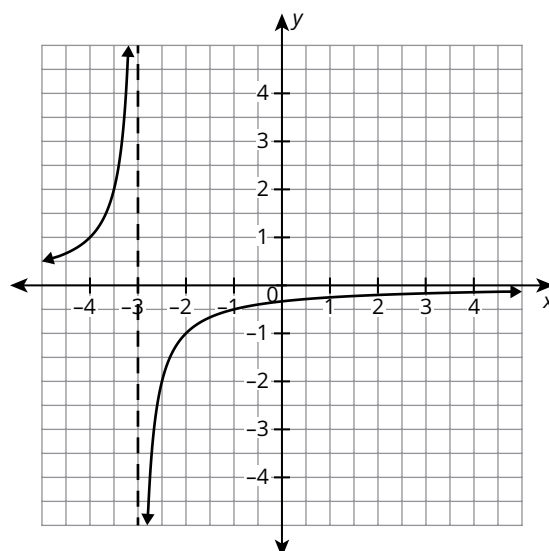
6. $f(x) = x^2, -2 < x < 2$

B. Each graph represents a rational function. Describe the vertical and horizontal asymptotes for each graph.

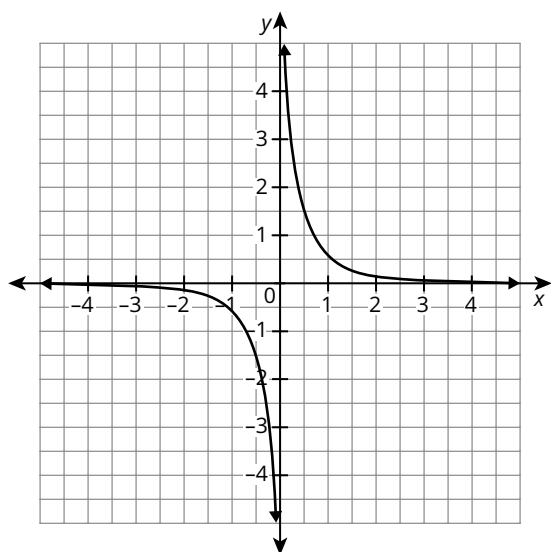
1.



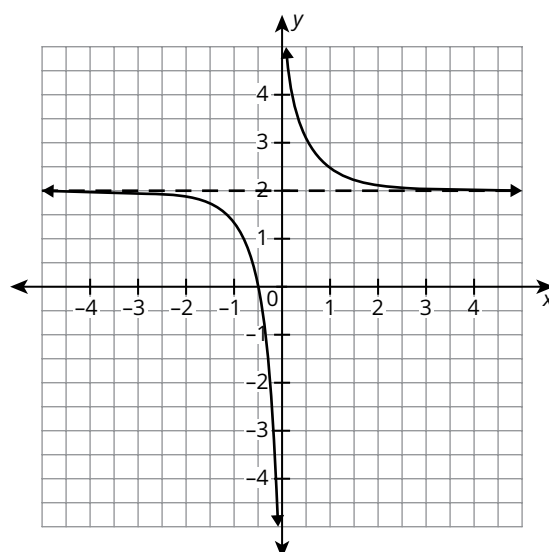
2.



3.

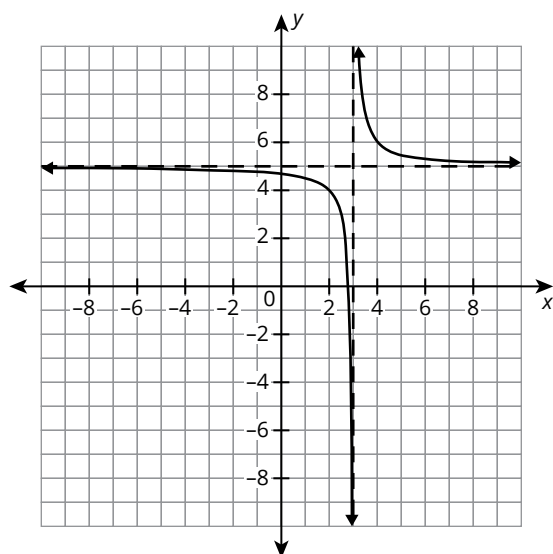


4.

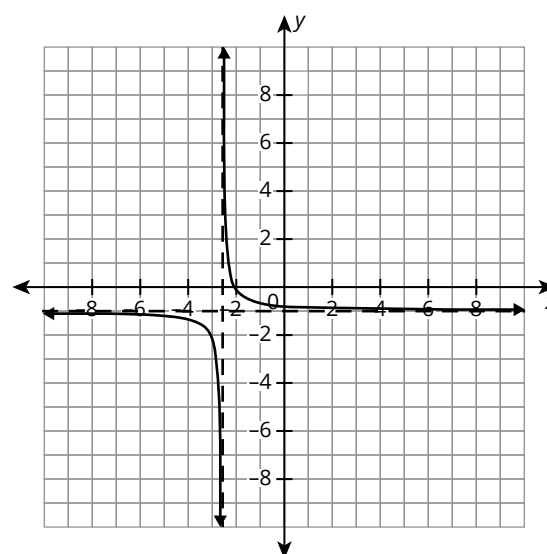


TOPIC 1 Rational Functions

5.



6.



C. Determine whether each function is a rational function or not a rational function. If the function is not rational, explain why.

1. $f(x) = x^2 - 6x + 2$

2. $g(x) = x^{\frac{2}{3}} - 1$

3. $q(x) = \frac{x^3 - 2x + 1}{\sqrt{x}}$

4. $r(x) = \left(\frac{1}{3}\right)^x$

5. $h(x) = \frac{x}{x-3}$

6. $t(x) = \frac{5}{x} - x$

7. $s(x) = 1 - 4^x$

8. $p(x) = \frac{1}{(x-1)(x+1)}$

D. Use an inverse variation function to determine the constant of variation for each situation. Then, write the inverse variation equation that represents the situation.

1. The number of vibrations per second of a guitar string varies inversely with its length. A guitar string that is 0.75 meter long vibrates 3.7 times per second.
2. The force needed to break a board varies inversely with the length of the board. It takes 1.25 pounds of pressure to break a board that is 8 feet long.
3. The pressure exerted by the sole of the foot of a person walking varies inversely with the area of the sole of the shoe. Each of Darlene's boots has a sole with an area of 23.2 square inches and the pressure exerted by the sole is 5 pounds per square inch.
4. At depths greater than 900 meters, the water temperature in degrees Celsius of the Pacific Ocean varies inversely with the depth. At a depth of 3000 meters, the temperature is 1.48°C .

TOPIC 1 Rational Functions

5. The amount of carpeting that John can buy varies inversely with the cost per square foot. John can afford 100 square feet of carpeting when the cost is \$4 per square foot.

6. The volume of a gas varies inversely with the pressure exerted on it. The pressure of a 56.25 cubic inch sample of gas was measured at 40 pounds per cubic inch.

E. Formulate an inverse variation function to solve each question.

1. The number of workers needed to clean a stadium after an event varies inversely with the amount of time that is available, in hours, for the workers to clean before the next event. If the constant of variation is 150, how much time is available for 25 workers to clean the stadium?



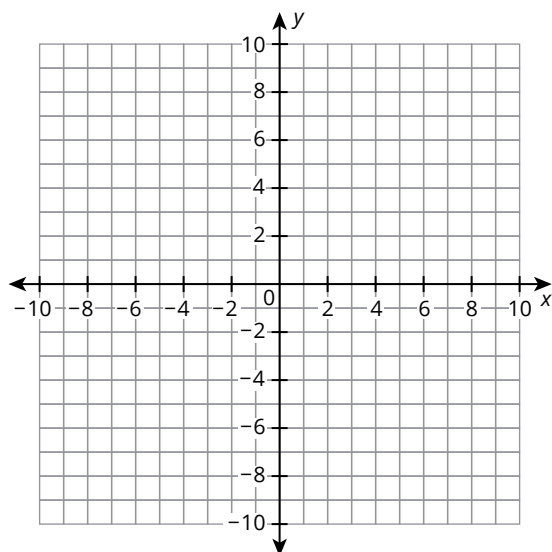
3. A property owner rents houses to college students. The amount of rent each student pays is inversely proportional to the number of students sharing the house. If the constant of variation is 1150, how many students are renting the house if they are each paying \$230 in rent?

TOPIC 1 Rational Functions

4. The daily demand for a company's newest television varies inversely with its price. If the constant of variation is 175,000, what is the daily demand for the television if the price is \$500?
5. The force, in pounds, required to loosen a bolt with a wrench varies inversely with the length of the wrench, in inches. If the constant of variation is 1500, what is the length of the wrench if 250 pounds of force are needed to loosen a bolt?
6. The current, in amperes, in a battery varies inversely with the resistance of the circuit, in ohms, to which the battery is connected. If the constant of variation is 24, what is the resistance of the circuit if the current in a battery is 1.2 amperes?

- F. Graph each rational function and identify the key attributes.
Write the domain and range of the function using set notation.

1. $f(x) = \frac{1}{x}$



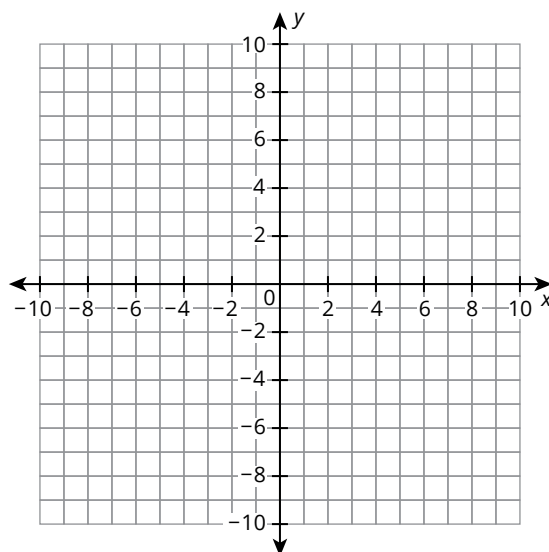
Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

2. $f(x) = \frac{1}{x-2}$



Domain:

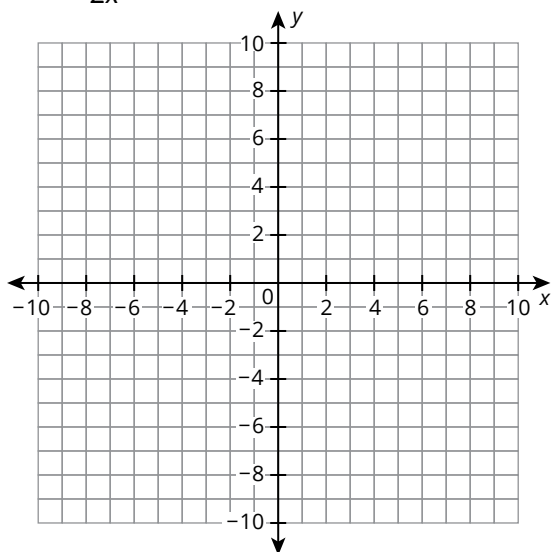
Range:

Vertical asymptote:

Horizontal asymptote:

TOPIC 1 Rational Functions

3. $f(x) = \frac{1}{2x}$



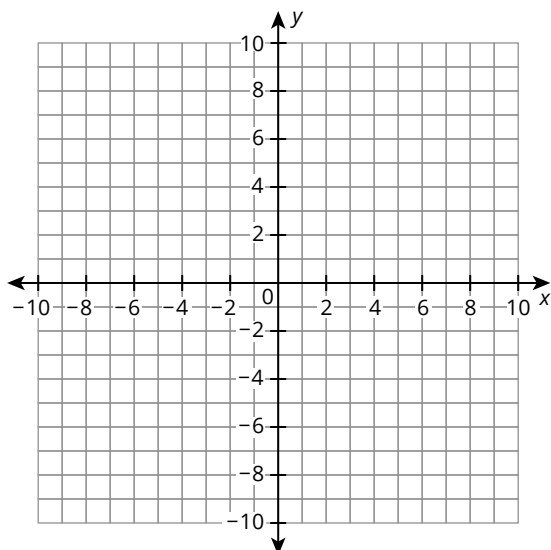
Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

5. $f(x) = \frac{-1}{x}$



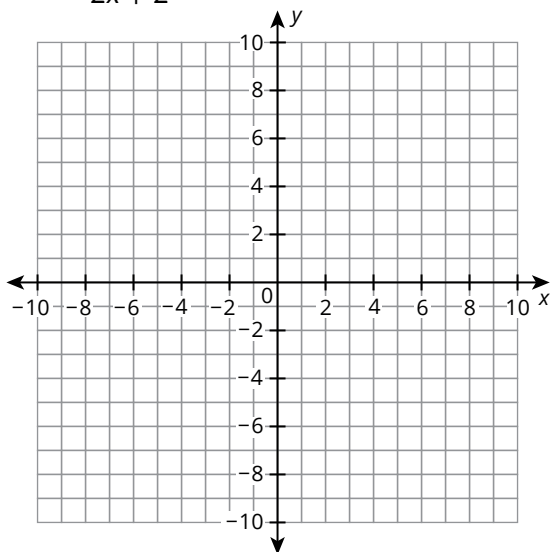
Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

4. $f(x) = \frac{1}{2x+2}$



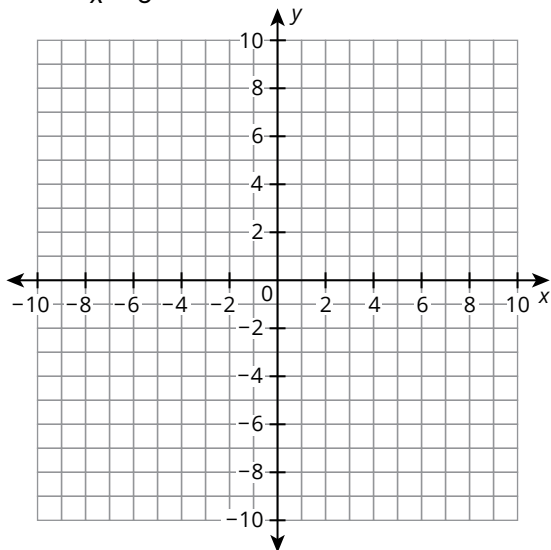
Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

6. $f(x) = \frac{-3}{x-6}$



Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

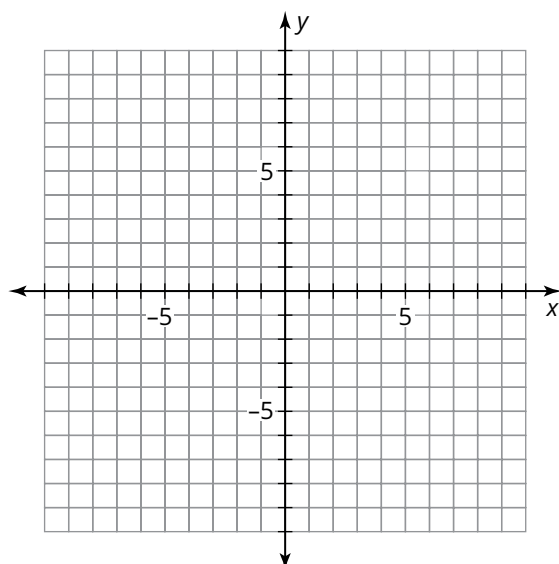
Extension

1. Consider the function $f(x) = \frac{1}{x-2}$

a. Complete the table.

x	-2	0	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4	6
$f(x)$									

b. Use the table to graph the function.

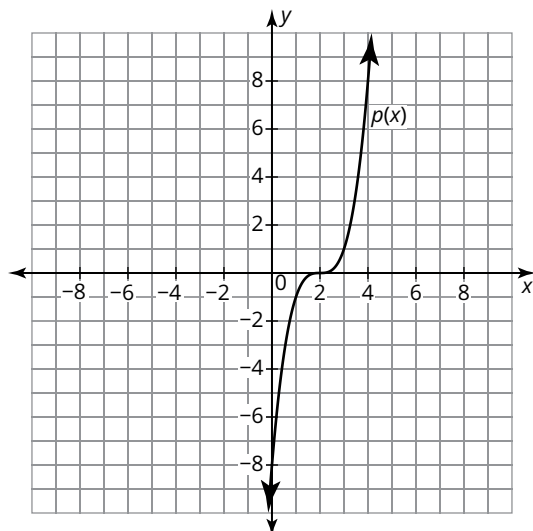


c. Analyze the function and the corresponding table and graph. Describe the domain and range of the function. Determine all of the asymptotes of the function. Explain your reasoning.

TOPIC 1 Rational Functions

Spaced Practice

1. Identify the extrema, zeros, and intercepts of the graph of the cubic function $p(x)$.



2. Determine the inverse of each linear equation

a. $y = 3x + 8$

b. $y = \frac{1}{2}x + 8$

$$x = \frac{1}{2}y - 4$$

$$x + 4 = \frac{1}{2}y$$

$$2(x + 4) = y$$

II. Transformations of Rational Functions

Topic Practice

A. Use technology to graph each rational function and complete the table. Use set notation for the domain and range.

	c-value	$g(x) = \frac{1}{x - c}$	Vertical Asymptote(s)	Horizontal Asymptote(s)	Domain	Range
1	3					
2	-4					
3	$\frac{1}{2}$					
4	-2.7					
5	5					
6	$-\frac{7}{8}$					

TOPIC 1 Rational Functions

- B. Determine the domain, range, and vertical and horizontal asymptotes of each rational function without using a graphing calculator. Write the domain and range using interval notation.**

1. $f(x) = \frac{3}{x}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

3. $f(x) = \frac{1}{x+4}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

5. $f(x) = \frac{-7}{2x+3}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

7. $f(x) = \frac{8}{-2(x-1)}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

2. $f(x) = \frac{1}{x-9}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

4. $f(x) = \frac{1}{2\left(x + \frac{1}{2}\right)}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

6. $f(x) = \frac{x}{7}$

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

8. $f(x) = \frac{1}{-2x}$

Domain:

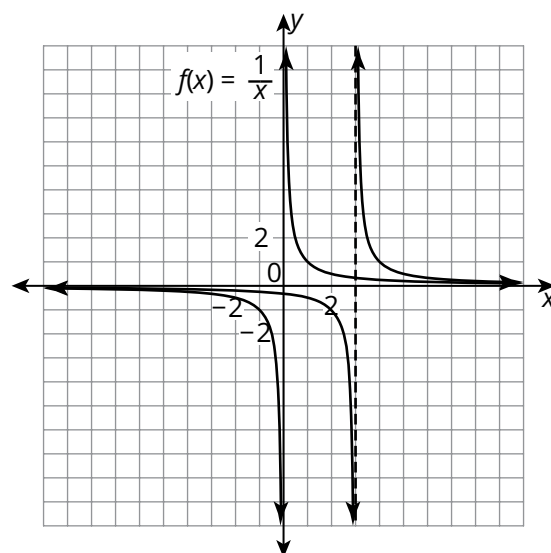
Range:

Vertical asymptote:

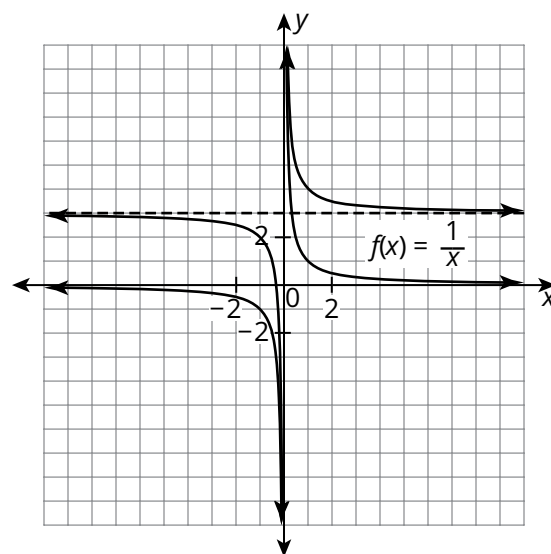
Horizontal asymptote:

- C. The function $f(x) = \frac{1}{x}$ is shown on each coordinate plane. Determine whether the other function shown is the graph of $g(x)$, $p(x)$, or $q(x)$. Explain your reasoning.

1. $g(x) = \frac{1}{x-3}$
 $p(x) = \frac{1}{x+3}$
 $q(x) = \frac{1}{x} + 3$

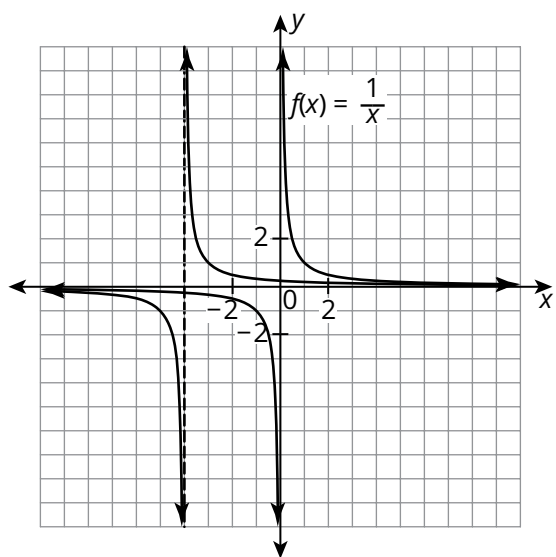


2. $g(x) = \frac{1}{x-3}$
 $p(x) = \frac{1}{x+3}$
 $q(x) = \frac{1}{x} + 3$

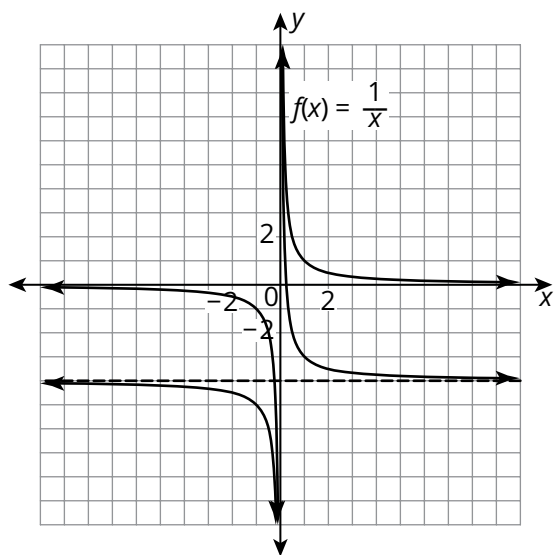


TOPIC 1 Rational Functions

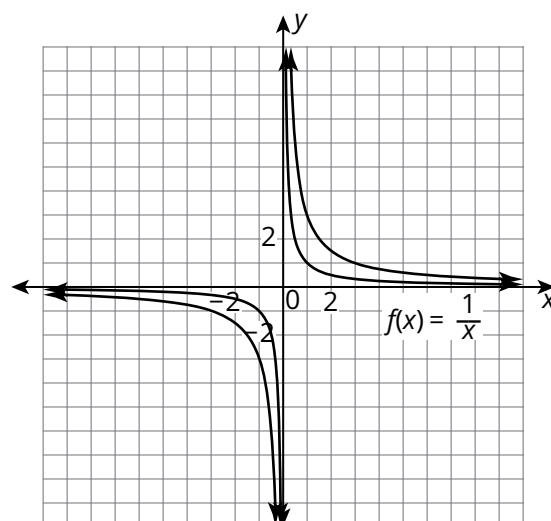
3. $g(x) = \frac{1}{x-4}$
 $p(x) = \frac{1}{x+4}$
 $q(x) = \frac{1}{x} - 4$



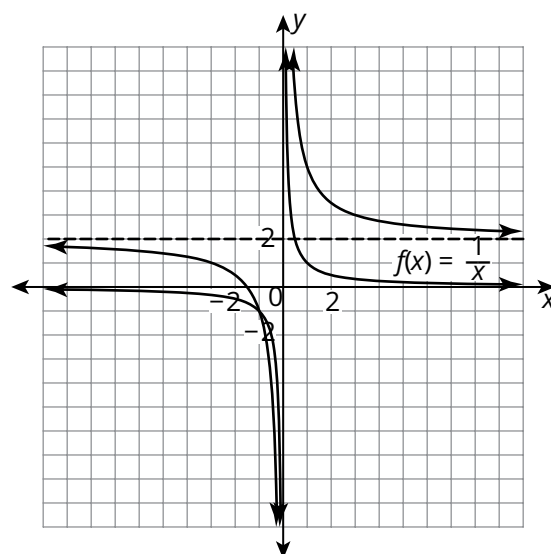
4. $g(x) = \frac{1}{x-4}$
 $p(x) = \frac{1}{x+4}$
 $q(x) = \frac{1}{x} - 4$



5. $g(x) = \frac{3}{x}$
 $p(x) = \frac{-3}{x}$
 $q(x) = \frac{3}{x} + 2$



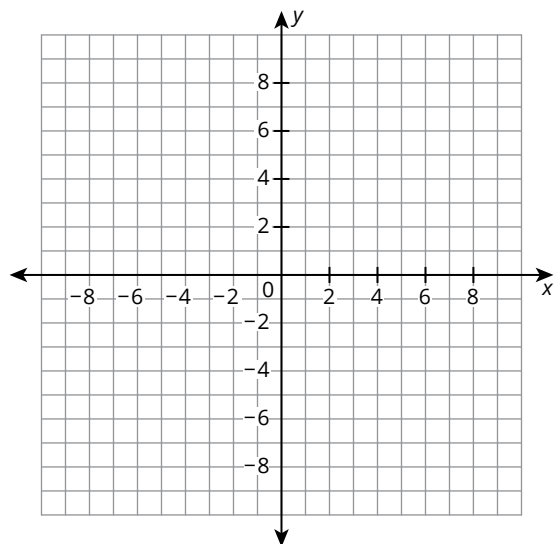
6. $g(x) = \frac{3}{x}$
 $p(x) = \frac{-3}{x}$
 $q(x) = \frac{3}{x} + 2$



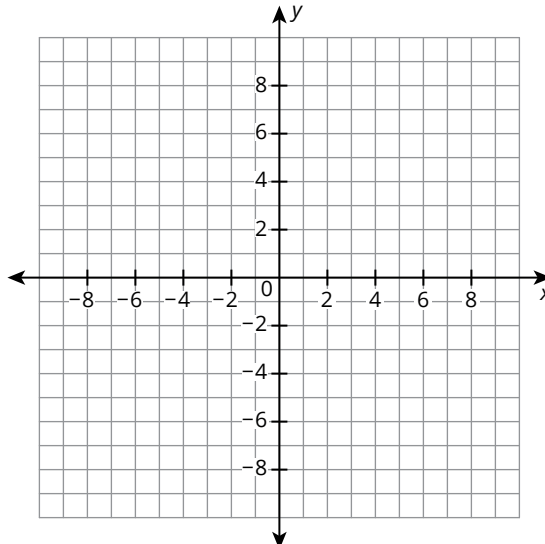
TOPIC 1 Rational Functions

D. Sketch $g(x)$ on each coordinate plane, given $f(x) = \frac{1}{x}$

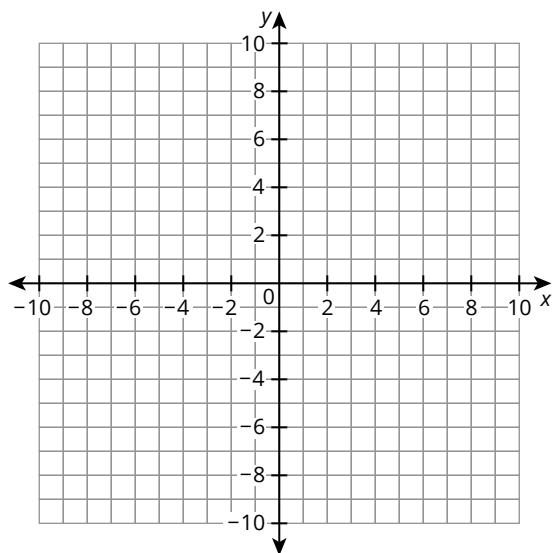
1. $g(x) = f(x - 2)$



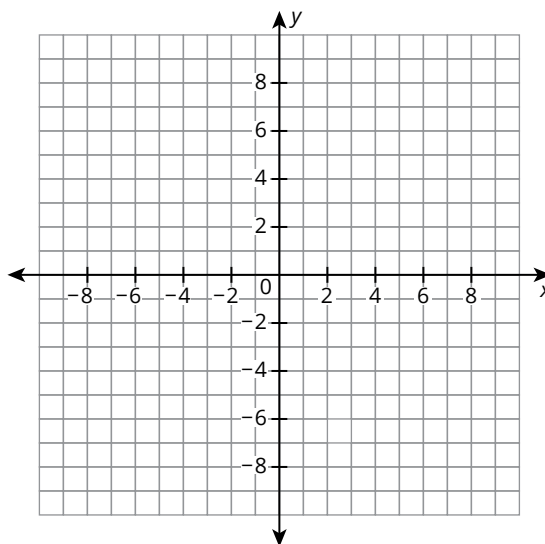
2. $g(x) = f(x) - 4$



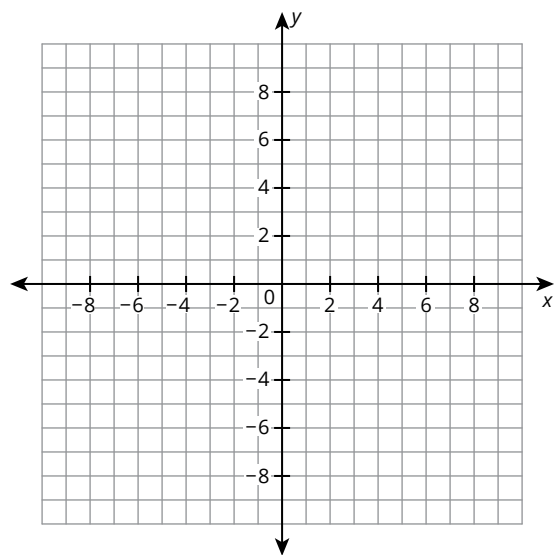
3. $g(x) = -4f(x + 5)$



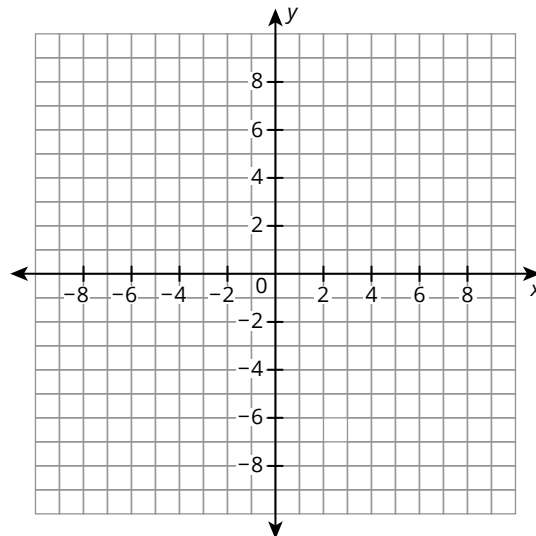
4. $g(x) = 2f(-x)$



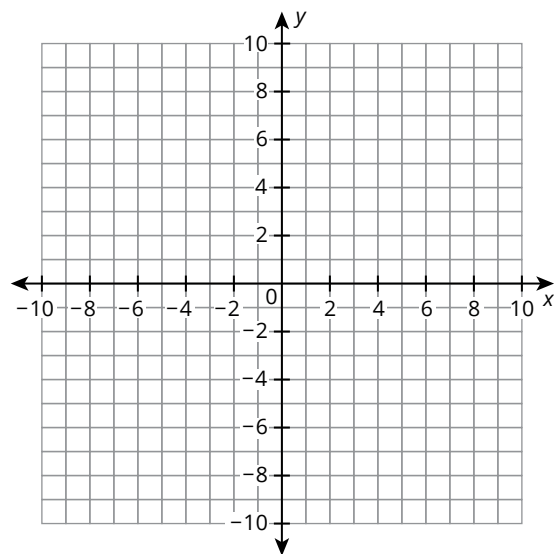
5. $g(x) = f(x + 2) - 1$



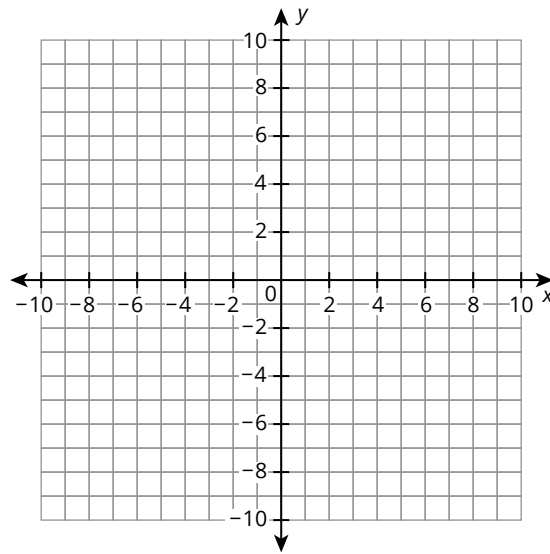
6. $g(x) = -f(x - 1) + 3$



7. $g(x) = f(-(x - 4)) + 3$



8. $g(x) = f(2(x + 5)) - 1$

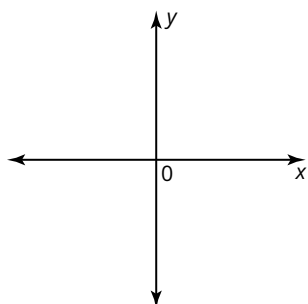


Extension

1. Consider the functions $f(x) = x^2 - x - 12$ and $g(x) = x - 4$.
 - a. Use the rational function $h(x) = \frac{f(x)}{g(x)}$. Create a table of values for the function $h(x)$ given the following x -values: $-5, -4, -3, -2, 4, 6$.
 - b. Sketch the function, $h(x)$.
 - c. Does the sketch include any vertical asymptotes? Why or why not? Use algebra to explain.

- d. Create the rational function $m(x) = \frac{g(x)}{f(x)}$. Complete the table of values for the function $m(x)$ and then sketch the function.

x	-5	-4	-3	-2	4	6
$m(x)$						



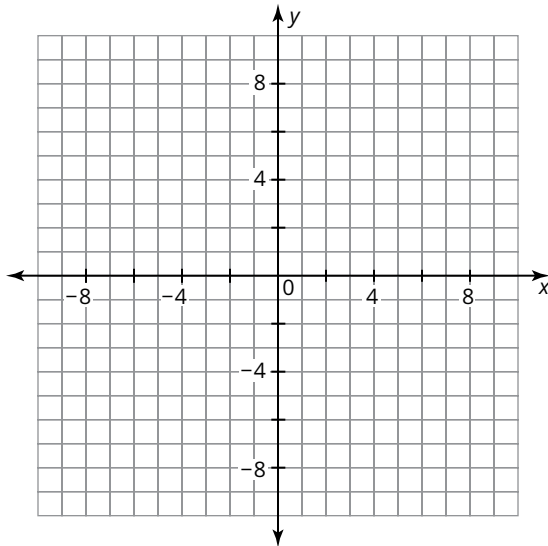
- e. Does the sketch include any vertical asymptotes? Why or why not? Use algebra to explain.
- f. If $f(x)$ and $g(x)$ are polynomials, are the vertical asymptotes of the rational function $R(x) = \frac{f(x)}{g(x)}$ always going to be the zeros of the function $g(x)$? Explain.

TOPIC 1 Rational Functions

Spaced Practice

1. Consider the function $f(x) = \frac{2}{x}$.

a. Graph the function.



b. Analyze the function and the corresponding graph. Describe the domain, and range of the function. Determine all of the asymptotes of the function. Explain your reasoning.

2. Rewrite each expression.

a. $(9 - 4i) - (6 + 3i)$

b. $(12i + 4) + (-3i - 1)$

c. $(4 - 2i)(4 + 2i)$

III. Exploring Asymptotes

Topic Practice

A. Determine the domain, range, and vertical and horizontal asymptotes of each rational function without using graphing technology. Write the domain and range using the indicated notation.

1. $f(x) = \frac{3}{x}$; Set notation

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

2. $f(x) = \frac{1}{x-9}$; interval notation

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

3. $f(x) = \frac{1}{x+9}$; Inequalities

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

4. $f(x) = \frac{1}{2x+1}$; Set notation

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

5. $f(x) = \frac{-7}{2x+3}$; Interval notation

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

6. $f(x) = \frac{x}{7}$; Inequalities

Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

TOPIC 1 Rational Functions

- B. Write a rational function for each table, graph, or description provided. Explain your reasoning include any transformations of $y = \frac{1}{x}$ necessary to create the associated function.

1. Vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 0$

2. Vertical asymptote at $x = -5$ and a horizontal asymptote at $y = 0$

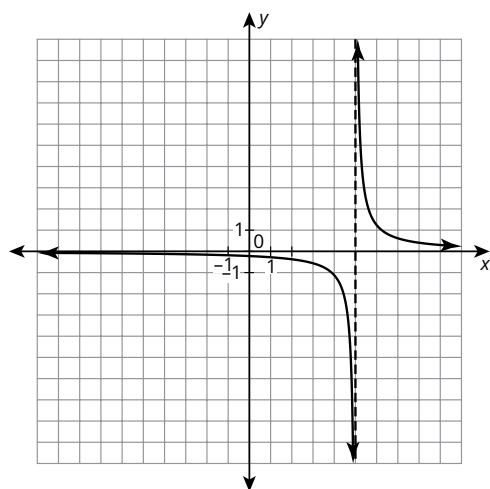
3.

x	$f(x)$
-4	-1
-3	$-\frac{3}{2}$
-2	-3
-1	undefined
0	3
1	$\frac{3}{2}$
2	1

4. The domain is all real numbers except $x = 6$. The range is all real numbers except $y = 0$.

5. Vertical asymptote is at $x = -3$. The range is all real numbers except $y = 0$.

6.



C. Analyze each rational function. Use algebra to determine the vertical asymptotes. Then, write the domain and range of the function using inequalities, interval notation and set notation.

1. $f(x) = \frac{2}{3x - 15}$

2. $f(x) = \frac{1}{(x + 2)(x - 3)}$

3. $f(x) = \frac{12}{x^2 + 4x - 5}$

4. $f(x) = \frac{-3}{x^2 - x}$

5. $f(x) = \frac{7}{x^2 + 1}$

6. $f(x) = \frac{-2}{(x - 3)(x - 2)}$

TOPIC 1
 Rational Functions

D. Write a rational function $g(x)$ that matches the given characteristic(s).

1. Vertical asymptote at $x = 5$

2. Vertical asymptote at $x = -2$
3. Vertical asymptote at $x = 4$
Horizontal asymptote at $y = -\frac{1}{2}$

4. Vertical asymptote at $x = -\frac{3}{2}$
Horizontal asymptote at $y = 1$
5. For $f(x) = \frac{1}{x}$, $g(x) = f(x + 7) - 2$.

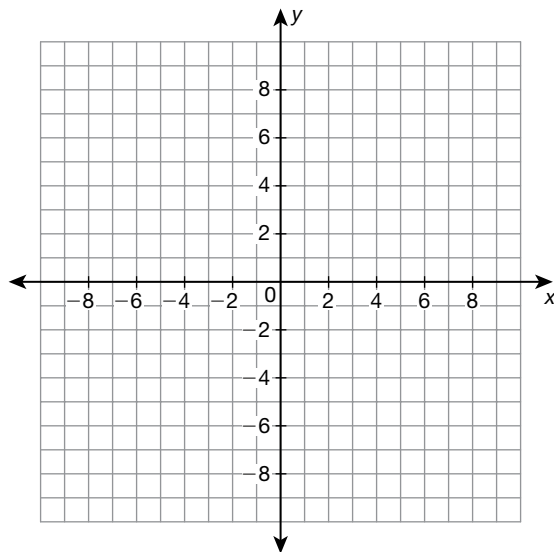
6. For $f(x) = \frac{1}{x}$, $g(x)$ shifts $f(x)$ left 1 unit and down 2 units.

Extension

1. Consider the function $h(x) = \frac{8}{x^2}$.
- a. Complete the table.

x	-8	-4	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4	8
$h(x)$											

- b. Use the table to graph the function.



- c. Analyze the function and the corresponding table and graph.
Describe the domain and range of the function using set notation.
Determine all of the asymptotes of the function.
Explain your reasoning.

Spaced Practice

1. Formulate an inverse variation function and solve each problem.
 - a. The number of workers x and the time y needed to complete a task vary inversely. When there are 5 workers, the task is completed in 30 hours. How many hours would it take to complete the task with 10 workers?

TOPIC 1 Rational Functions

- b. The speed of a car y and the time it takes to travel a fixed distance x are related by an inverse variation. When the car is traveling at 60 miles per hour, it takes 3 hours to reach the destination. What is the inverse variation function, and how long will it take to reach the destination if the car is traveling at 80 miles per hour?

2. Solve each absolute value equation.

a. $2|x - 4| = 18$

b. $\left|\frac{x}{3}\right| + 7 = -4$

IV. Operations with Rational Expressions

Topic Practice

A. Calculate each sum and difference. Simplify the answer when possible.

1. $\frac{x}{2} + \frac{7x}{6}$

2. $\frac{x+2}{4} - \frac{z}{10}$

3. $\frac{2x^2-3}{4} + \frac{x^2+5}{10}$

4. $\frac{x^2+3x-4}{6} - \frac{2x^2+3x+4}{12} - \frac{x^2+9}{24}$

5. $\frac{-2x}{5} - \frac{y}{10} + z$

6. $\frac{x-3}{15} - \frac{x-3}{10}$

7. $\frac{2x}{7} - \frac{y}{2} + \frac{x+1}{3}$

8. $\frac{x-1}{4} + \frac{x+2}{2} - \frac{x^2}{8}$

9. $\frac{2x}{5} + \frac{2x^2-1}{10} - \frac{4x+1}{6}$

10. $\frac{x+4}{2} - \frac{x+5}{3} + \frac{x+6}{21}$

B. Calculate the least common denominator (LCD) for each sum and difference. Describe any restriction(s) for the value of x .

1. $\frac{x}{3} + \frac{x+1}{15}$

2. $\frac{7x}{12} + \frac{x-2}{6} - \frac{x^2}{3}$

TOPIC 1 Rational Functions

3. $\frac{x+1}{x} - \frac{x-1}{(x^2+x)}$

5. $\frac{3x+4}{x} - \frac{5}{6x} + \frac{9}{2x}$

7. $\frac{3x}{x-2} + \frac{x}{2x+4} + \frac{5}{x}$

4. $\frac{3}{2x} + \frac{x^2+1}{4x^2+8}$

6. $\frac{x}{x^2-1} - \frac{x-3}{x-1}$

8. $\frac{x-3}{2x^2+7x+6} - \frac{x}{2x+3}$

C. Calculate each sum and difference. Describe any restriction(s) for the value of x and simplify the answer when possible.

1. $\frac{3}{x} + \frac{1}{x+1}$

3. $\frac{x}{2x-1} + \frac{x+2}{x}$

5. $\frac{1}{x^2-4} - \frac{1}{x-2}$

2. $\frac{2}{x-2} - \frac{5}{x+3}$

4. $\frac{1}{x+3} - \frac{1}{x-3}$

6. $\frac{x+3}{x-1} + \frac{x-4}{x+2}$

7. $\frac{x+1}{x^2-16} - \frac{x}{x^2+7x+12}$

9. $\frac{1}{x-4} - \frac{x}{x+2} + \frac{x^2}{x-1}$

11. $\frac{x+1}{x^2-3x-4} + \frac{x-3}{x-2}$

8. $\frac{x^2}{x^2-8x+7} - \frac{x^2+2}{2x^2-9x-35}$

10. $\frac{3x^2}{2x^2-11x+5} + \frac{7x^2}{x^2-2x-15}$

12. $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$

D. Multiply each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

1. $\frac{3x}{8} \cdot \frac{7}{2x}$

3. $\frac{2ab^2}{5c^2} \cdot \frac{15c}{4a}$

5. $\frac{3mn^2}{10} \cdot \frac{m^2}{8n} \cdot \frac{20}{3n^2}$

7. $\frac{x^2-4}{x+5} \cdot \frac{x+5}{x-2}$

2. $\frac{6x}{5} \cdot \frac{x-5}{x}$

4. $\frac{5x^2}{7} \cdot \frac{14}{3x}$

6. $\frac{x+1}{x} \cdot \frac{x^2}{2x+2}$

8. $\frac{x^2+2x-3}{x^2} \cdot \frac{x^2+x}{x^2+4x+3}$

TOPIC 1 Rational Functions

9. $\frac{x^2 - 4x}{x - 2} \cdot \frac{2 - x}{x}$

11. $\frac{x + 3}{x - 5} \cdot \frac{1}{x^2 + 6x + 9} \cdot (x^2 - 25)$

10. $\frac{1}{2x^2 + 3x - 2} \cdot \frac{x^2 - 2x - 8}{x - 4}$

12. $\frac{5x^2}{x + 4} \cdot \frac{3x^2 + 12x}{7x - 7} \cdot \frac{x^2 - 2x + 1}{3}$

E. Determine the quotient of each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

1. $\frac{3c^2}{5ab} \div \frac{9}{2a}$

3. $\frac{x^2 + 1}{x} \div \frac{x^2 - 1}{2x}$

5. $\frac{x^2 + 6x + 8}{3x + 2} \div \frac{-x - 4}{3x^2 - x - 2}$

7. $\frac{2x^2 - 2x}{x^2 + 2x + 1} \div \frac{3x - 3}{2x + 2}$

9. $\frac{x^2 - 121}{x^2 + x - 20} \div \frac{x^2 - 10x - 11}{x^2 - 25}$

2. $\frac{4x^2 y}{5z^4} \div \frac{2x}{z} \div \frac{1}{2z}$

4. $\frac{x^2 + 6x - 27}{x^2} \div \frac{x^2 - 3x}{9}$

6. $\frac{x^2 - 9}{x + 3} \div (x - 3)$

8. $\frac{x^2 + 4x + 3}{2x^2 - 11x + 5} \div \frac{x^2 + 3x}{2x - 1}$

10. $\frac{(x - 5)^3}{(x + 2)^2(2x - 3)^4} \div \frac{(x - 5)^5}{(x + 2)(2x - 3)^2}$

Extension

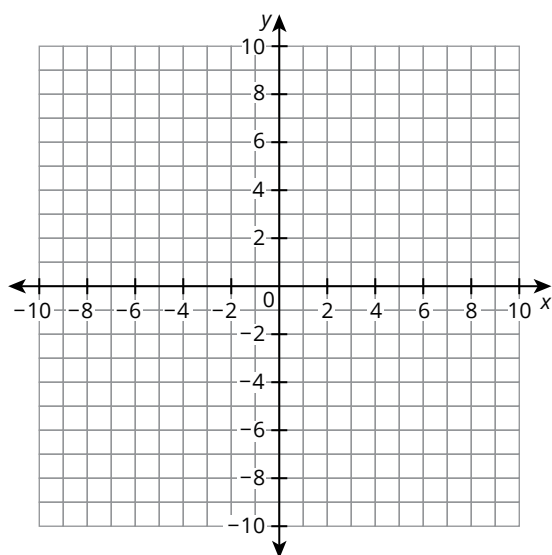
1. Perform the operations.

$$\frac{12x}{3x^2 - 10x + 3} - \frac{x-2}{x^2 - 9} + 2$$

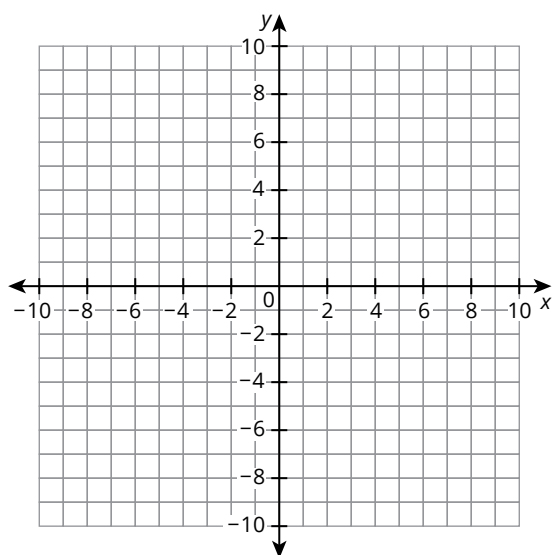
Spaced Practice

Graph each function.

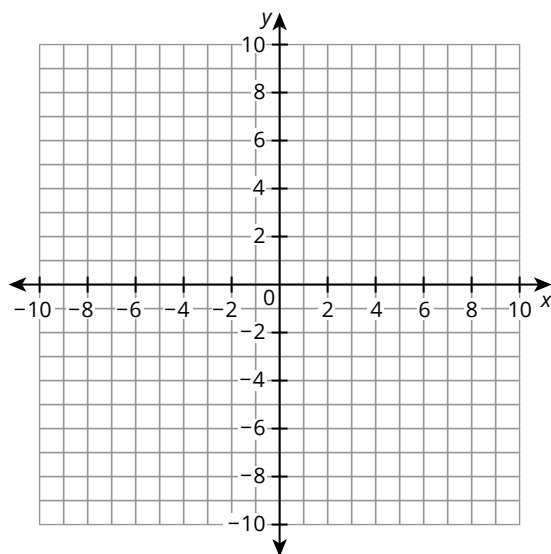
1. $f(x) = 2|x - 1|$



3. $h(x) = \frac{1}{x-7}$



2. $g(x) = -x^3 - 4$



V. Solving Problems with Rational Equations

Topic Practice

A. Solve each rational equation. Describe any restrictions for the value of x . Check your answer(s) and identify any extraneous roots should they occur.

1. $\frac{x-1}{x+3} = \frac{x-2}{x+1}$

2. $\frac{x+2}{x-7} = \frac{x}{x-3}$

3. $\frac{2x-1}{x+1} = \frac{2x-2}{x}$

4. $\frac{x-3}{x^2} = \frac{x-3}{x^2-1}$

TOPIC 1 Rational Functions

5. $\frac{x^2 - 1}{x - 1} = \frac{x^2 + 1}{x + 1}$

6. $\frac{x + 5}{x - 4} = \frac{x + 4}{x - 5}$

B. Solve each rational equation. Describe any restrictions for the value of x . Check your answer(s).

1. $\frac{2}{x} - \frac{3}{2x} = \frac{1}{x^2}$

2. $\frac{1}{x} + \frac{1}{x^2} = 2$

3. $\frac{5}{2x-4} = \frac{15}{x^2-4}$

4. $\frac{2}{x+3} + \frac{6}{x^2+3x} = \frac{1}{x}$

5. $\frac{2}{x^2-x} - \frac{1}{x-1} = 0$

6. $\frac{x}{x+2} + \frac{4x+6}{2x^2+5x+3} = \frac{x-1}{2x+4}$

Extension

1. Blaise is running in a ten-mile race. Blaise runs the first 4 miles of the race at a pace that averages 0.5 miles an hour faster than the last 6 miles of the race. It takes Blaise one hour and 45 minutes to run the 10-mile race. Write and solve an equation to determine Blaise's average speed for the first 4 miles and the last 6 miles of the race. Show your work.

Spaced Practice

1. Perform each operation. List any restrictions on the variables.

a. $\frac{3}{3x^2 - 11x - 4} + \frac{2x}{x^2 - 3x - 4}$

b. $\frac{2x - 4}{3x^3} \cdot \frac{9x}{x - 2}$

VI. Solving Work, Mixture, Distance, and Cost Problems

Topic Practice

A. Write an equation to model each work scenario. Then, solve each equation.

1. Michael can paint a room in 8 hours. Olivia can paint the same room in 6 hours. If they paint the room together, how long will it take them to paint the room?

TOPIC 1 Rational Functions

2. Koda owns a lawn service company. Currently it takes her 50 hours a week to service all of her customers. To reduce the number of hours a week she needs to work, Koda hires Madison to help her. While Koda was on vacation, Madison was able to complete all of the work in 60 hours. If Madison and Koda work together, after Koda returns from vacation, how long will it take them to service all their customers?
3. Using a forklift, Jacob can unload a box car in 90 minutes. Ana takes twice as long to complete the same task. If Jacob and Ana work together, how long will it take them to unload a box car?

4. Fernando can complete a 4 foot by 6 foot quilt in 16 days. Alexandar can complete the same task in 12 days. If they solicit Destiny's help, who can complete the task in 14 days by herself, how long will it take the three of them to complete a 4 foot by 6 foot quilt?
5. Luis can wash 24 golf carts in a 4-hour shift. If Malik helps him, they can get the job done in 2 hours. How long will it take Malik to do the job by himself?

TOPIC 1 Rational Functions

6. Hannah and William own a pastry shop. Working alone, Hannah can decorate 8 dozen cookies in 90 minutes. William, on the other hand, needs 120 minutes to decorate 8 dozen cookies. If they work together, how long does it take them to decorate 16 dozen cookies?

B. Write an equation to model each mixture scenario. Then, solve each equation.

1. Abby knows that if she needs to add antifreeze to her car's radiator the mixture used must contain 50% antifreeze and 50% water. How many gallons of a mixture containing 80% antifreeze must be added to a 3-gallon mixture containing 40% antifreeze to obtain the mixture Kaitlin needs?

2. The directions on the back of a 2-quart bottle of a 60% orange concentrate says it needs to be mixed with water to obtain a 20% orange drink. How much water should Huyen add to the concentrate to obtain a drink that is 20% orange concentrate?

TOPIC 1 Rational Functions

3. A research scientist has 10 liters of a 40% acid solution. She needs to create a solution containing 35% acid by mixing the 10 liters with a second solution containing 20% acid. How much of this second solution should she use?
4. Maria combines 2 gallons of 2% milk and 6 gallons of 4% milk. How many additional gallons of 6% milk must she add to obtain a 5% milk mixture?

- A jeweler has 30 ounces of an alloy consisting of 60% gold and 40% silver. How much of a second alloy containing 80% gold and 20% silver must be mixed with the first alloy to obtain an alloy containing 75% gold and 25% silver?
- Javier has 3 quarts of a 5% sugar solution. He wants to mix this with 2 quarts of an 8% sugar solution and x quarts of a 12% sugar solution to make a 10% sugar solution. How many quarts of the 12% sugar solution should Keyon use?

TOPIC 1 Rational Functions

- C. Write an equation to model each distance scenario. Then, solve each equation.**
1. Olivia walked to the grocery store, which was 2 miles away. Her walking rate on the way back was 0.75 of her walking rate on the way to the store because she was carrying a bag of groceries. If it took Olivia 1 hour to make the round trip, what was her walking rate on the way to the store?
 2. Jacob can paddle his canoe at a rate of 6 miles per hour in still water. Last weekend, he went to the Guadalupe River, a river in Texas popular for canoeing. Jacob canoeing the Guadalupe River going downstream with the current for 9 miles and then returning upstream against the current. If the trip took him 4 hours to complete, what was the speed of the current?

3. Suppose flying in calm air, a robin can reach a speed of 25 kilometers per hour. Each day this robin flies from its nest to the nearest body of water 1 kilometer away. On one particular day, it flew into a headwind and on its return trip the wind was at its back. If the total trip took $\frac{1}{10}$ of an hour, what was the speed of the wind?

TOPIC 1 Rational Functions

- Natalia walked a half a mile to her sister's house to pick up her little brother and then walked back. The round trip took 60 minutes. If the rate at which she walked to her sister's house was 25% faster than the rate she walked while returning home, how fast did she walk on the way home?
- An 8-man rowing crew rows at a speed of 10 miles per hour in still water. Every morning they practice by rowing 4 miles upstream and then 4 miles downstream. If it takes them $\frac{5}{6}$ of an hour to complete the trip, what is the speed of the current?

6. Trung runs the 100-meter dash in 15 seconds with no wind. Yesterday she ran the 100-meter dash twice. The first time she ran it in 18.75 seconds against the wind and the second time she ran it in 12.5 seconds with the wind. What was the speed of the wind?

TOPIC 1 Rational Functions

- D. Write an equation or inequality to model each cost scenario. Then, solve each equation or inequality. Explain whether you think the solution is reasonable for the problem situation.**
1. Avery purchased a power boat for \$15,000 and was told by the salesman that the yearly average cost to operate the boat was approximately \$900. In what year of ownership will Avery's average annual cost of owning the boat be \$3900?

2. Mason purchased a new cell phone for \$350 and added an annual warranty plan that cost him \$35 dollars per year. In what year will Mason's average annual cost of owning the phone be \$122.50?

TOPIC 1 Rational Functions

3. Avery always wanted a designer purse, but they were too expensive to purchase. Recently she was introduced to a company that allowed individuals to rent one. The purse that Avery rented initially cost her \$80 with a \$15 monthly rental fee. She plans on renting a new purse until the average monthly cost of renting the purse is less than \$19. When will the average monthly cost of renting the purse drop to less than \$19?

4. Shopping around for a freezer, Diego finally settled on one with a purchase price of \$850. The annual cost of operating the freezer is \$40 per year. When Diego's average cost of owning the freezer is less than \$117, he plans to shop for a new freezer. When can Diego shop for a new freezer?

TOPIC 1 Rational Functions

5. Minh purchased a flat screen TV for \$700 and added an annual warranty plan that cost her \$23 per year. Liam purchased a similar TV for \$650 and added an annual warranty plan that costs him \$28 per year. In how many years will Minh's average annual cost of owning her TV be less than Liam's annual cost of owning his TV?

6. Emma is considering purchasing one of two microwave ovens. The first microwave costs \$300 and comes with an annual warranty plan that costs \$20 per year. The second microwave costs \$350 and comes with an annual warranty plan that costs \$10 per year. When will the average annual cost of owning the second microwave be less than the average annual cost of owning the first microwave?

TOPIC 1 Rational Functions

Extension

Three friends own a landscape service. One of their biggest clients is a museum. When Kayden, Jaylen, and Zion work together they can mow the museum lawn in 3 hours. When Kayden works alone, he can mow the lawn in 6 hours. It takes Jaylen an hour more than Zion to mow the lawn. Write and solve an equation to determine how long it would take Jaylen and Zion to mow the lawn on their own. Show your work.

Spaced Practice

1. Krish plays chess online. So far, Krish has won 325 out of 464 games. In order to move up to the next skill level his winning percent must be 75%. Estimate the number of consecutive games Krish will need to win to move up to the next skill level. Does your solution seem reasonable? Explain your reasoning.

2. Claudina is going to join a yoga studio. The Peaceful Zen studio costs \$480 for the year and \$5 for each class that she takes. The Ohm Tree studio cost \$375 for the year and \$7 for each class that she takes.
 - a. Write a function to represent the average cost of each class per year for the two studios.
 - b. Which studio will have a lower average cost per class over the next year if she takes two classes a week? Show all of your work and explain your reasoning.

3. DeMarcus has 340 out of 400 points possible in his biology class. He needs an 80% in the class in order to be on the Dean's List for the semester. The only remaining graded items will be class participation points worth 1 point each. Estimate the number of consecutive class participation points DeMarcus can miss and have his grade be an 80%. Show all of your work and explain your reasoning.

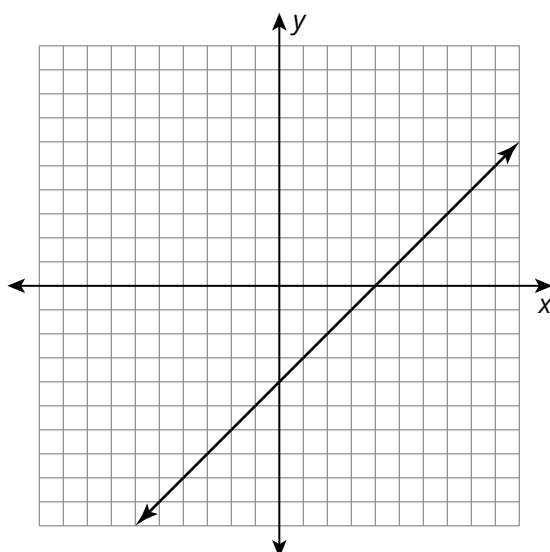
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I. Inverse of Power Functions

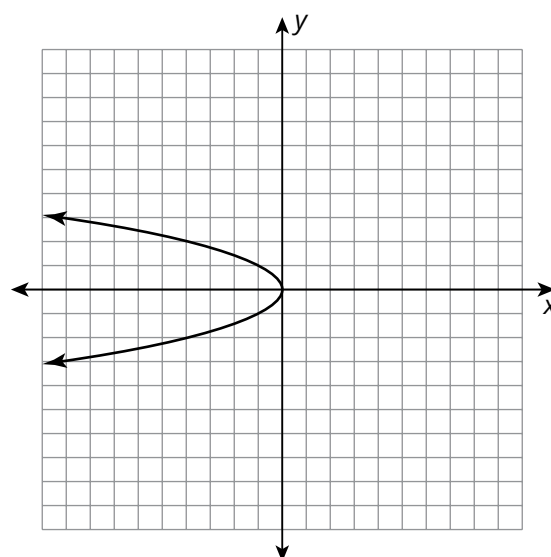
Topic Practice

A. Determine whether or not each relation is a function. Use the Vertical Line Test.

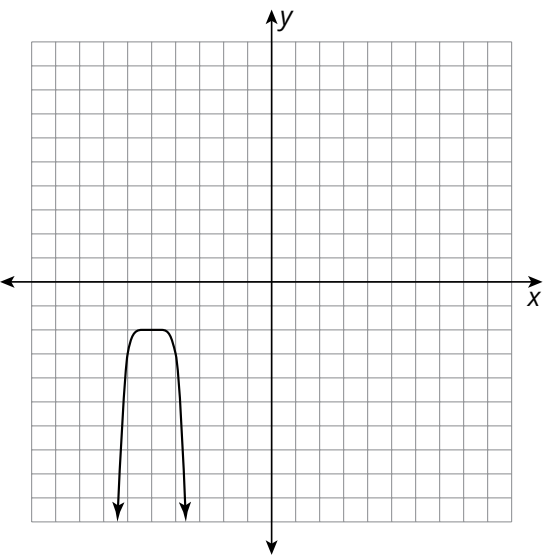
1.



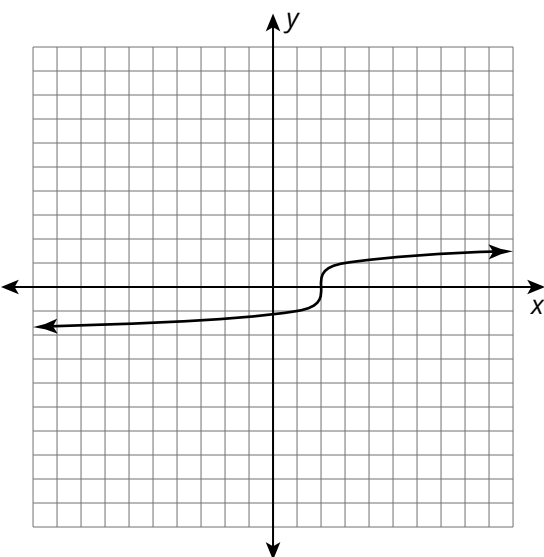
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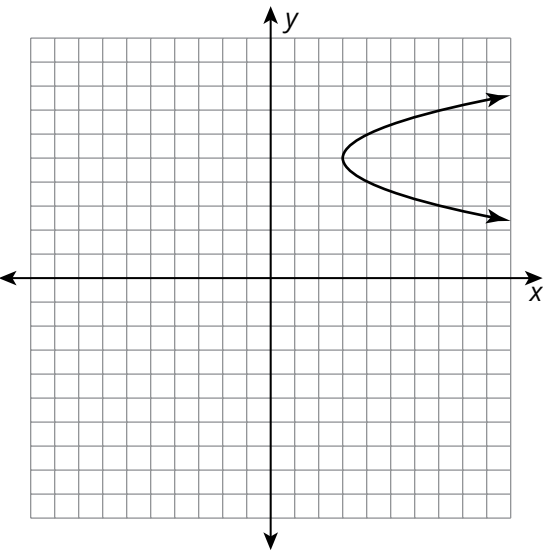
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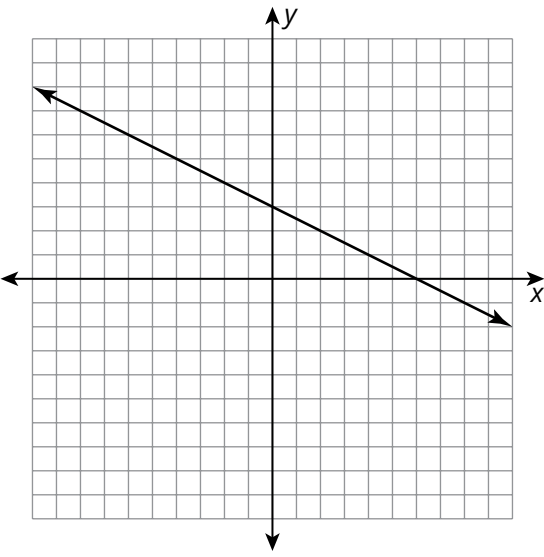
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5.

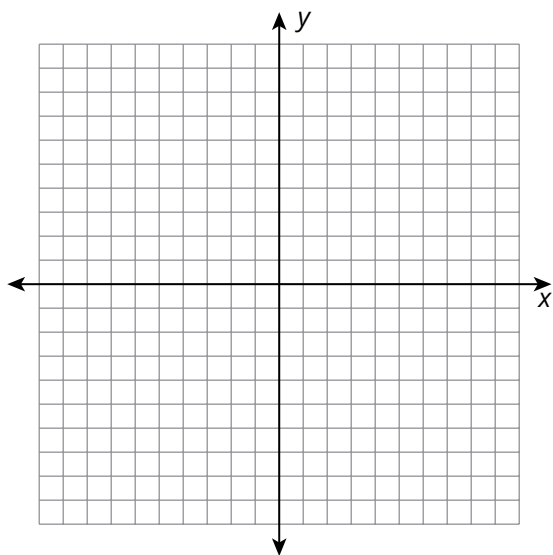


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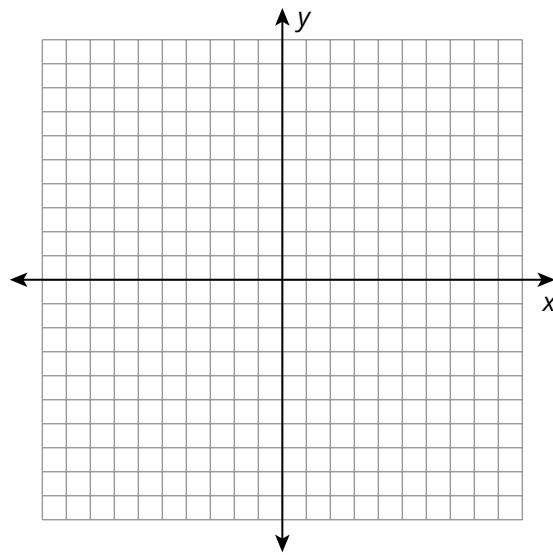


B. Sketch the graph of the inverse of each equation.

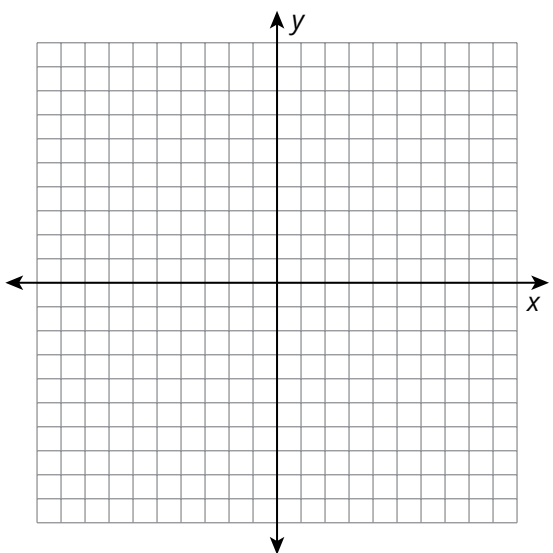
1. $y = -(x^3)$



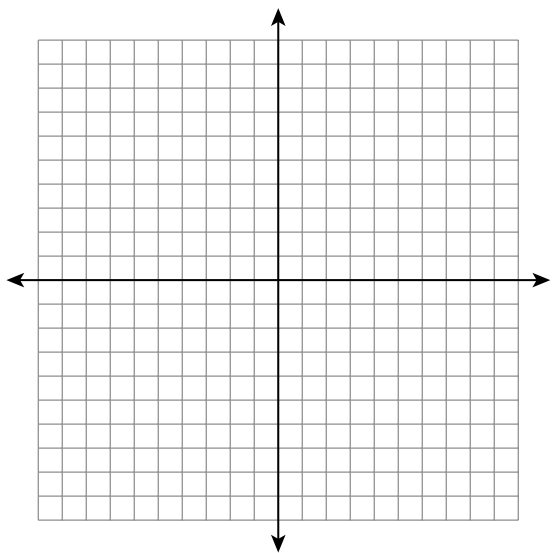
2. $y = -(x^2)$



3. $y = x^2 + 3$

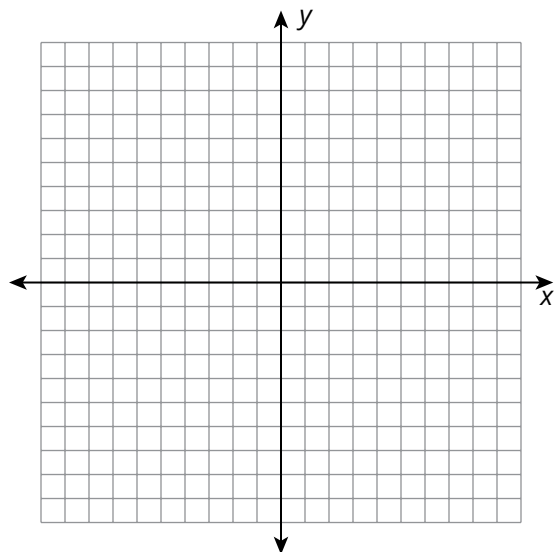


4. $y = x^3 - 2$

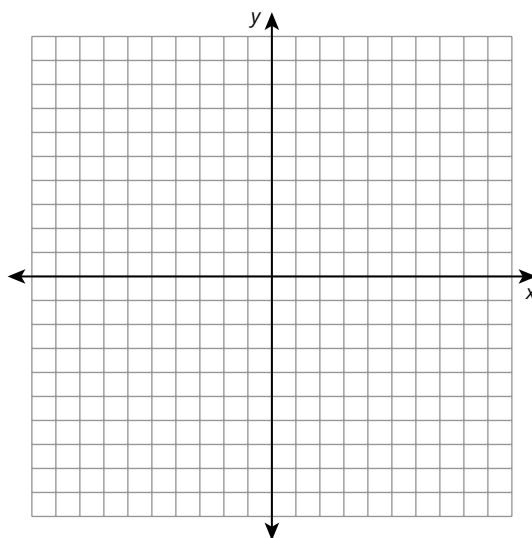


TOPIC 2 Radical Functions

5. $y = -\frac{1}{4}x^3$

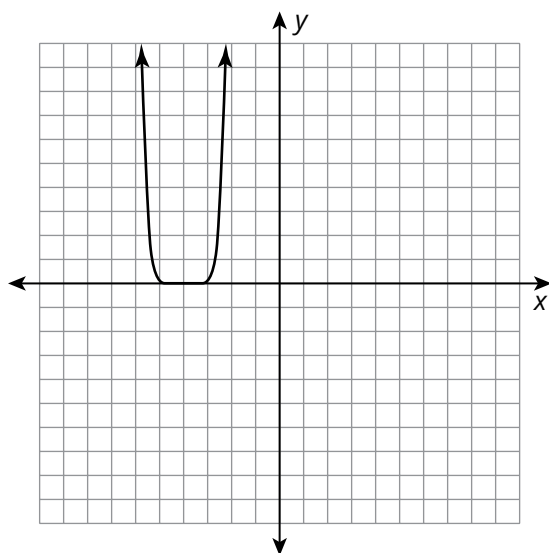


6. $y = \frac{1}{2}x^2$

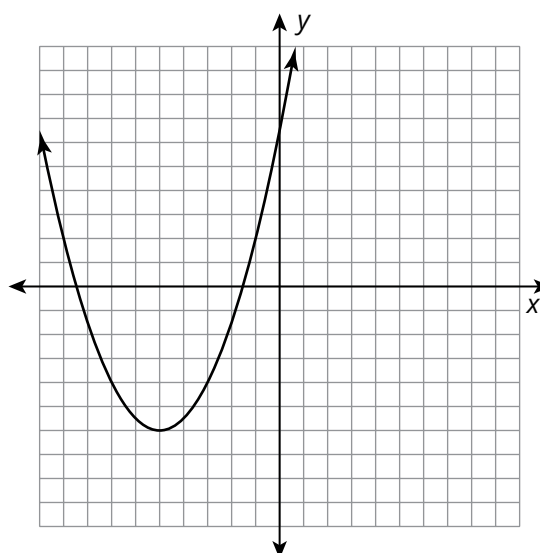


C. Determine whether each function is invertible. Use the Horizontal Line Test.

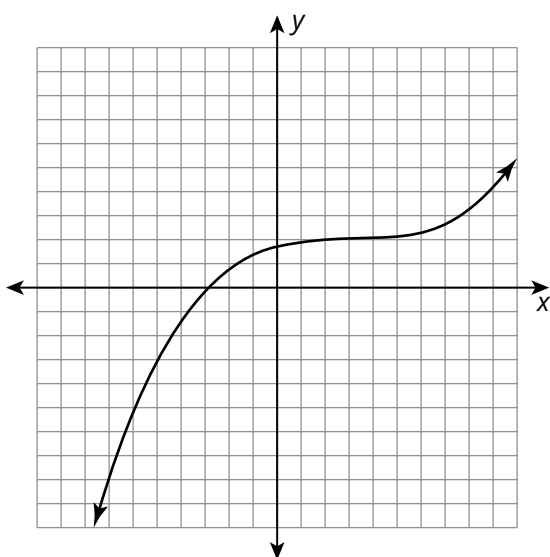
1.



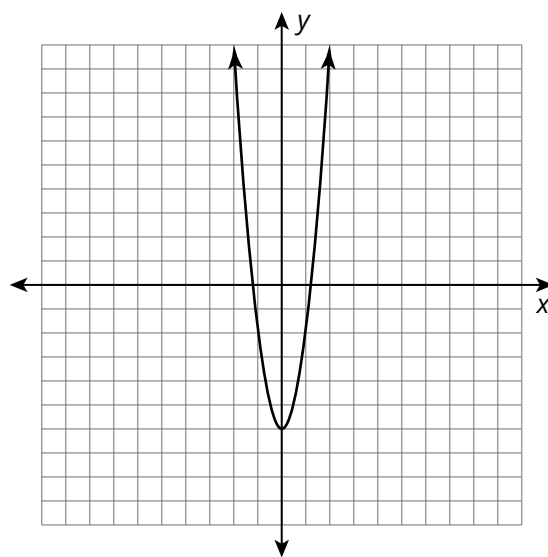
2.



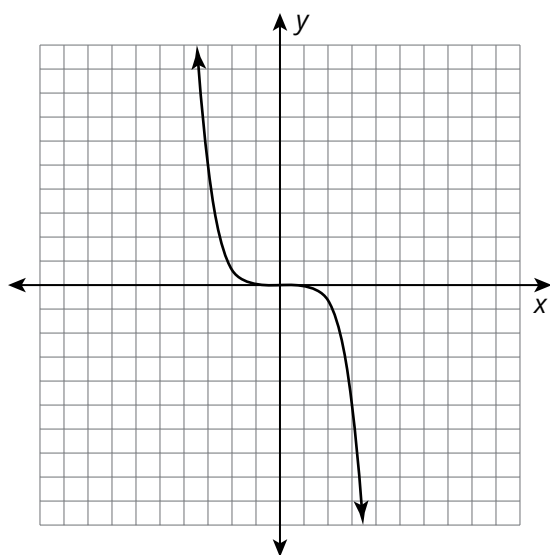
3.



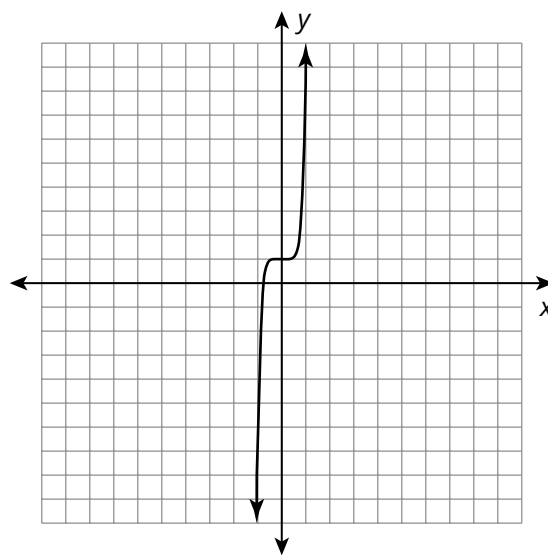
4.



5.



6.



TOPIC 2 Radical Functions

D. Without graphing, determine whether or not each function is invertible.

1. $f(x) = 3x^2$

2. $f(x) = x^2 - 4$

3. $f(x) = -x^3$

4. $f(x) = 1.257x^2$

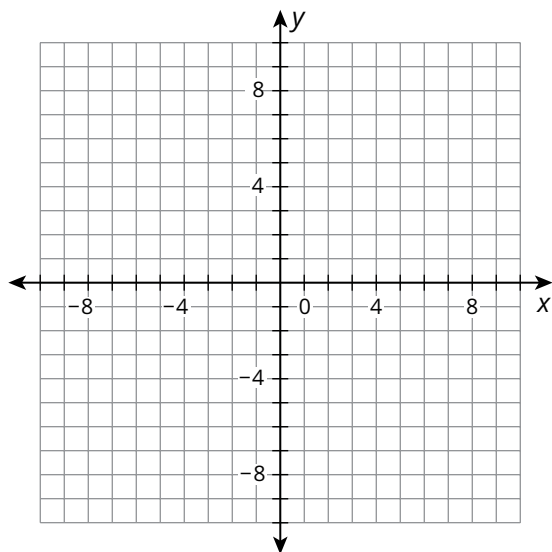
5. $f(x) = 4x^3 - 5x^2 + 1$

6. $f(x) = -\frac{3}{5}x$

Extension

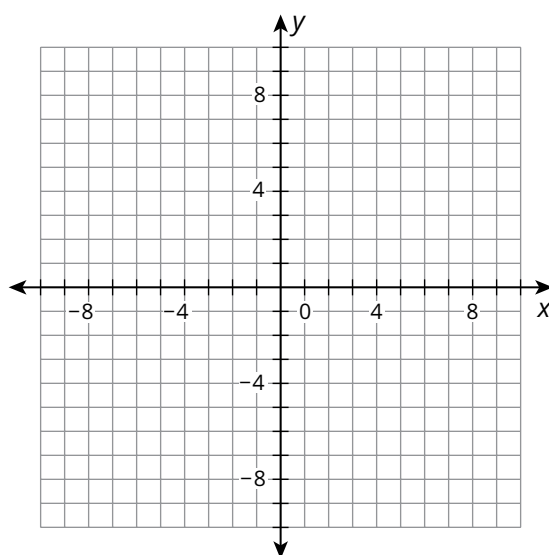
1. Consider the function $f(x) = (x - 1)^5 + 2$.

a. Sketch the graph of $f(x)$.



b. Is $f(x)$ invertible? Explain your reasoning.

c. If $f(x)$ is invertible, sketch the graph of $f^{-1}(x)$.



Spaced Practice

1. Koda and Michael are refinishing the hardwood floors in an old estate property. Michael can refinish the floors in 100 hours by himself. Working together, it takes Koda and Michael 60 hours to refinish all the floors. Write and solve an equation to determine how long it would take Koda to refinish the floors by herself. Show your work.

TOPIC 2
 Radical Functions

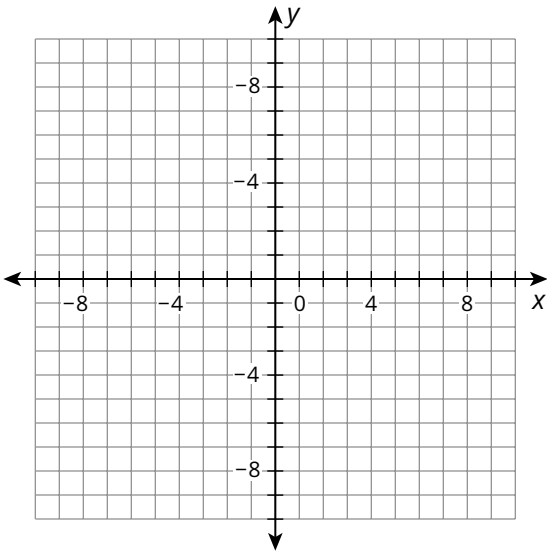
2. Write a quadratic equation that has vertex (5, 2) and includes the point (4, 6).

3. Consider the function $f(x) = -\frac{3}{x}$.

a. Complete the table.

x	−9	−6	−3	−1	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	3	6	9
f(x)											

b. Use the table to graph the function.



- c. Describe the domain and range of the graph using interval notation. Determine all of the asymptotes of the function. Explain your reasoning.

4. Use long division to determine the quotient of $(x^3 - 4x^2 + 2x + 5) \div (x - 2)$.

II. Radical Functions

Topic Practice

A. Determine the equation of the inverse for each quadratic function.

1. $f(x) = 7x^2$

2. $f(x) = -x^2$

3. $f(x) = 6x^2 + 11$

4. $f(x) = 2x^2 - 12$

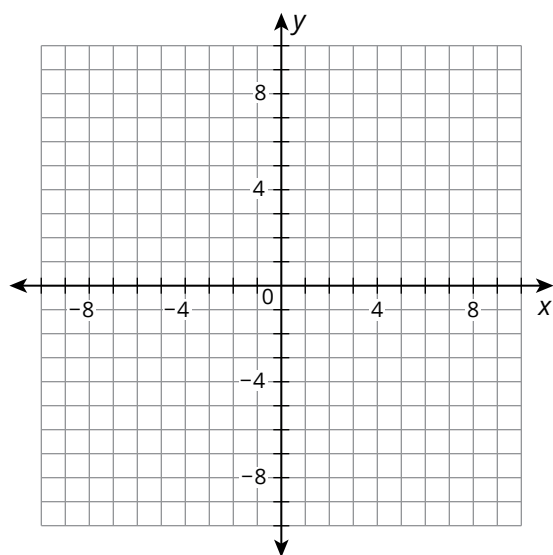
5. $f(x) = -4x^2 - 6$

6. $f(x) = -3x^2 + 20$

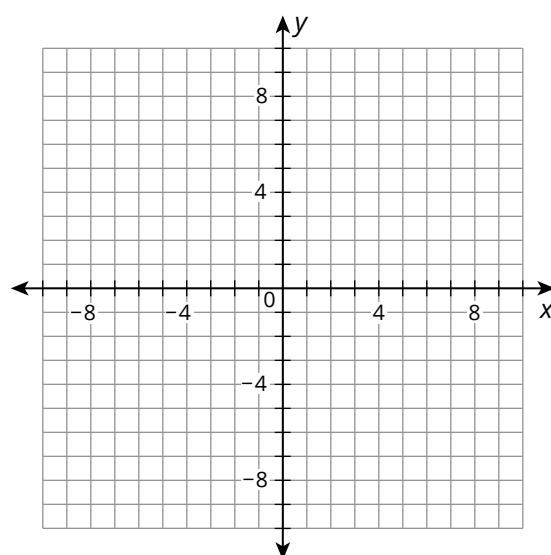
B. Determine the equation of the inverse for each given function.

Graph the function and its inverse. Restrict the domain of the original function and the inverse so that the inverse is also a function. Write the domain and range of each function using inequalities.

1. $f(x) = 2x^2$

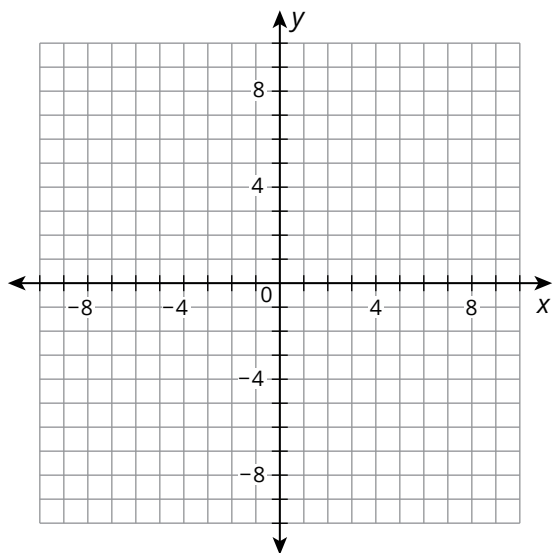


2. $f(x) = x^2 + 3$

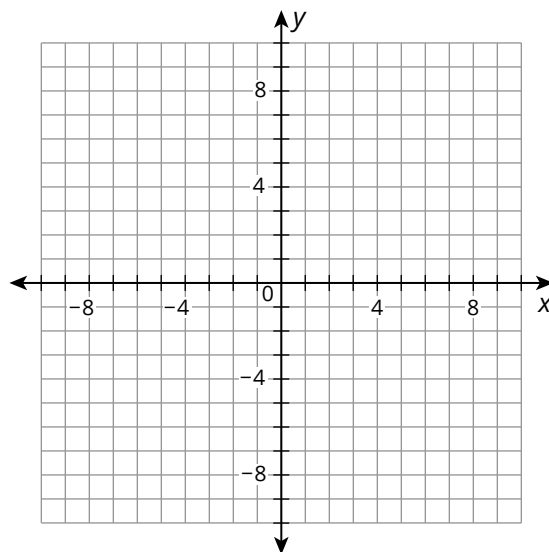


TOPIC 2 Radical Functions

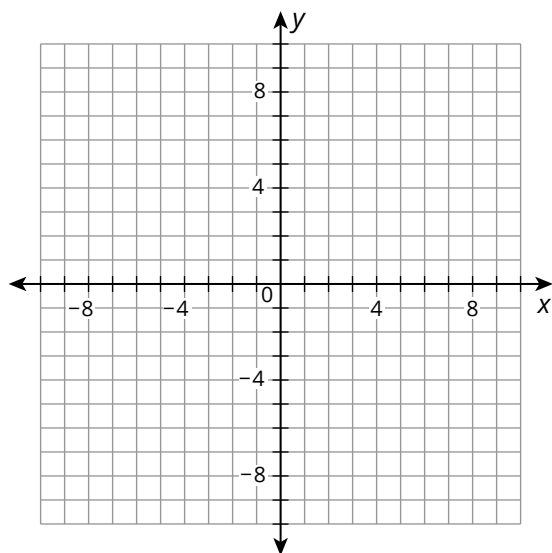
3. $f(x) = -4x^2 - 2$



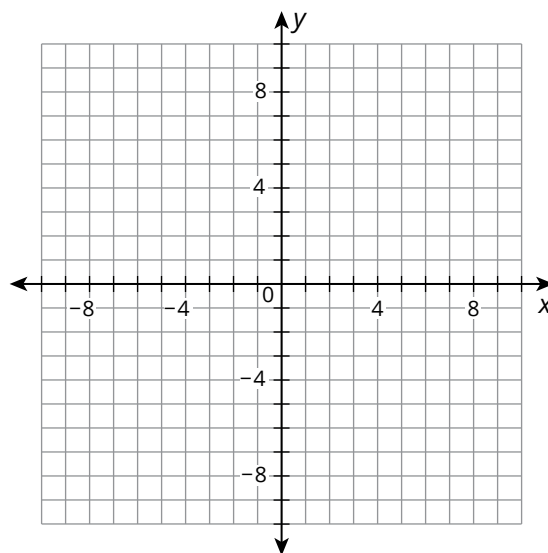
4. $f(x) = 3x^2 - 4$



5. $f(x) = -\frac{1}{2}x^2$



6. $f(x) = -x^2 + 5$

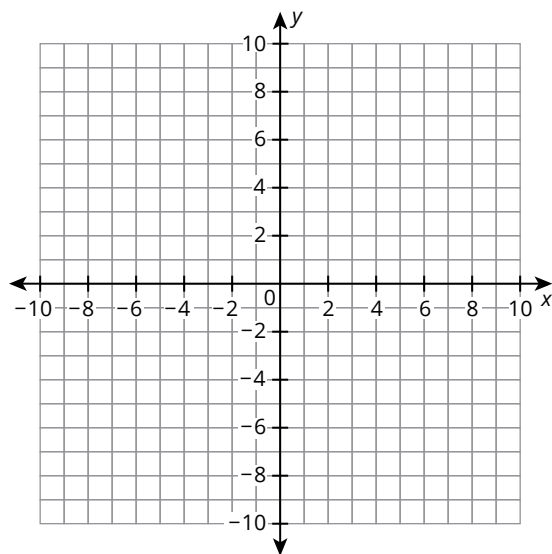


7. Analyze the domain and range of each function and its inverse. Describe the relationship between the restricted domain and range of the function and the domain and range of the inverse function.

TOPIC 2 Radical Functions

- C. Graph each function and identify the key attributes (domain, range, maximum or minimum, and the x - and y -intercepts) of each function. Write the domain and range using set notation.

1. $f(x) = \sqrt{x}$



Domain:

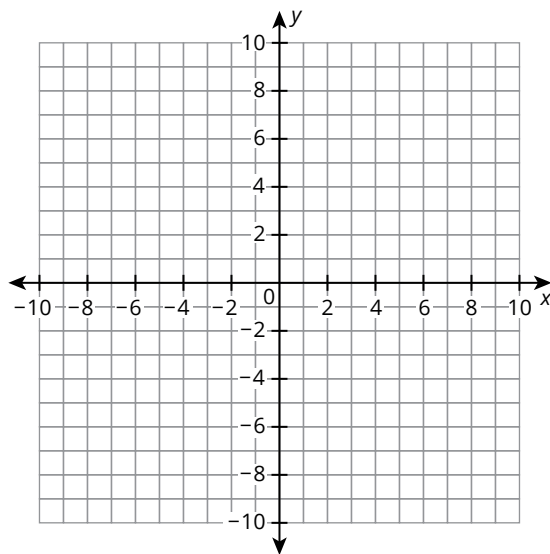
Range:

x -intercept:

y -intercept:

Minimum/Maximum:

2. $f(x) = \sqrt{x + 4}$



Domain:

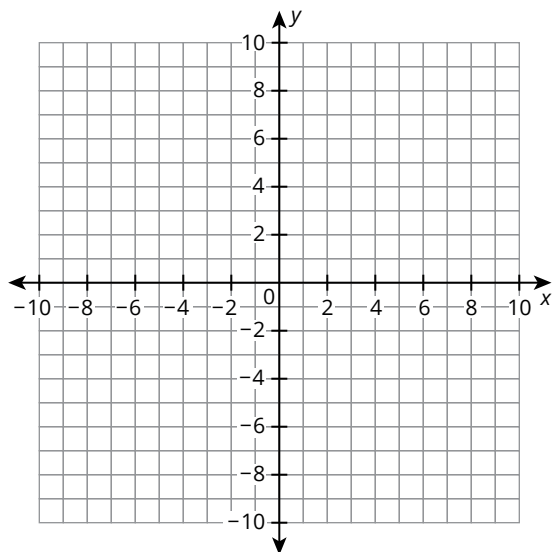
Range:

x -intercept:

y -intercept:

Minimum/Maximum:

3. $f(x) = \sqrt{x} + 1$



Domain:

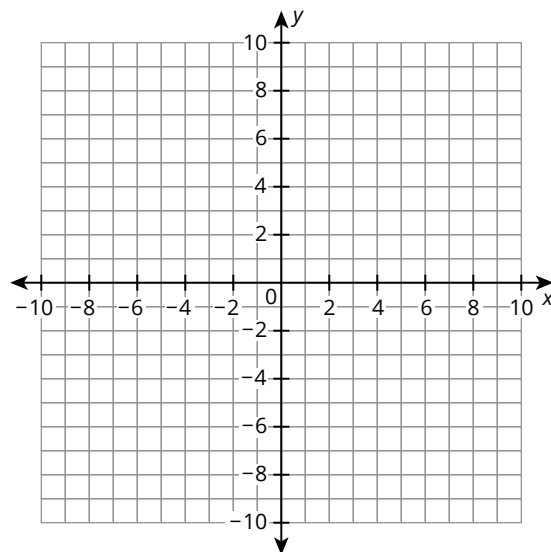
Range:

x-intercept:

y-intercept:

Minimum/Maximum:

4. $f(x) = \frac{\sqrt{x}}{2}$



Domain:

Range:

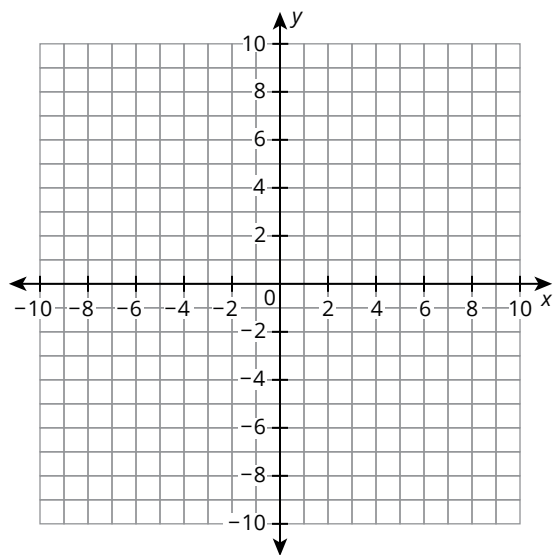
x-intercept:

y-intercept:

Minimum/Maximum:

TOPIC 2 Radical Functions

5. $f(x) = \sqrt{-5x}$



Domain:

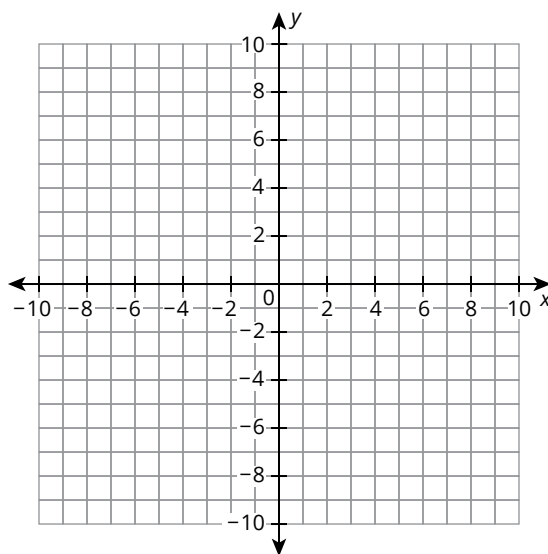
Range:

x-intercept:

y-intercept:

Minimum/Maximum:

6. $f(x) = \sqrt{3 - x}$



Domain:

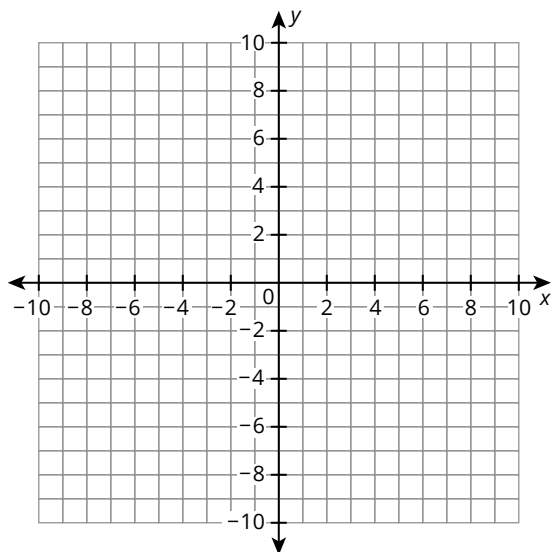
Range:

x-intercept:

y-intercept:

Minimum/Maximum:

7. $f(x) = \sqrt[3]{x}$



Domain:

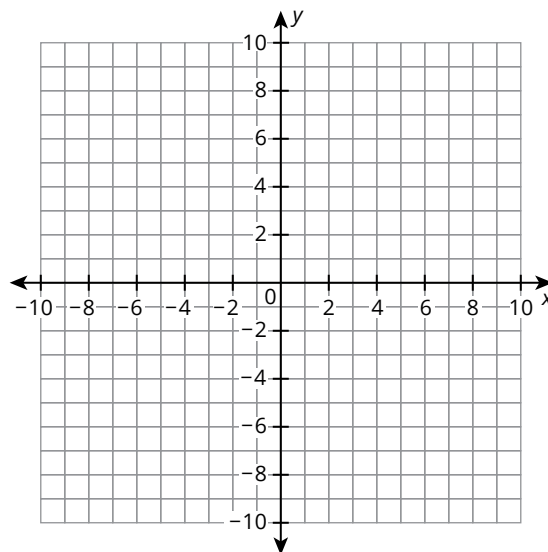
Range:

x-intercept:

y-intercept:

Maximum/Minimum:

8. $f(x) = \sqrt[3]{x - 2}$



Domain:

Range:

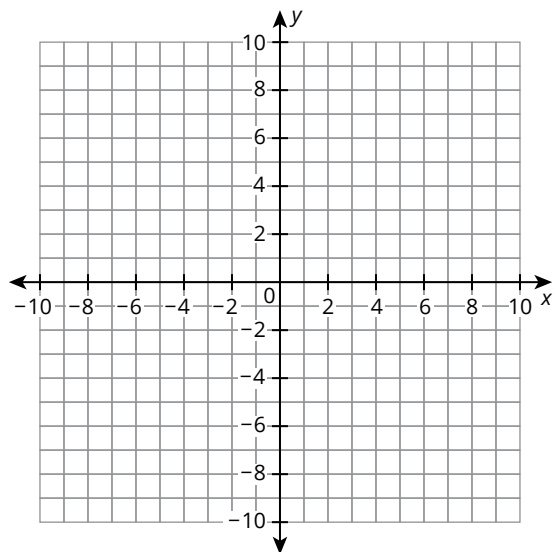
x-intercept:

y-intercept:

Maximum/Minimum:

TOPIC 2 Radical Functions

9. $f(x) = \sqrt[3]{x} - 5$



Domain:

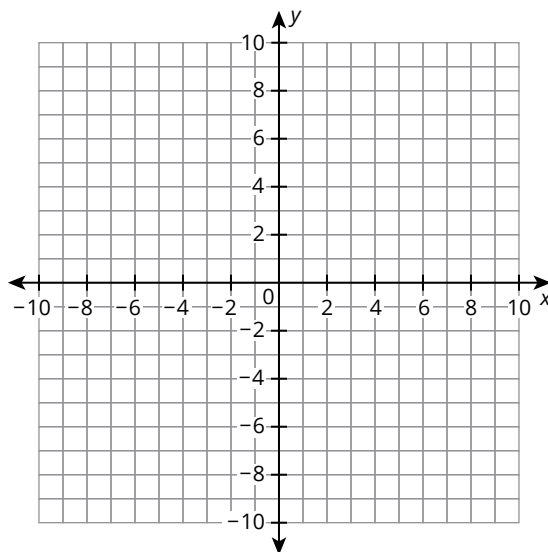
Range:

x-intercept:

y-intercept:

Maximum/Minimum:

10. $f(x) = \frac{\sqrt[3]{x}}{4}$



Domain:

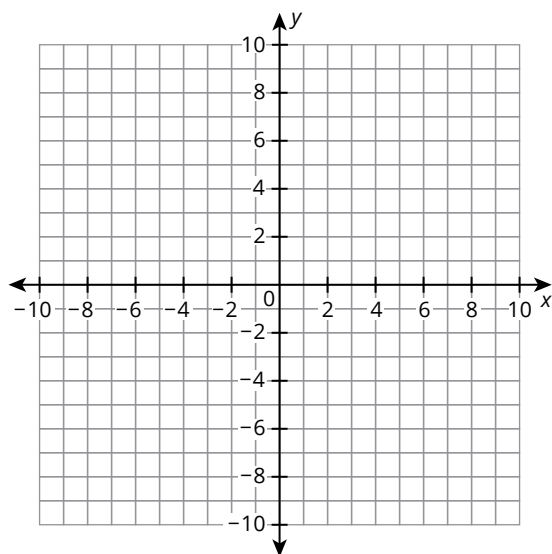
Range:

x-intercept:

y-intercept:

Maximum/Minimum:

11. $f(x) = \sqrt[3]{-2x}$



Domain:

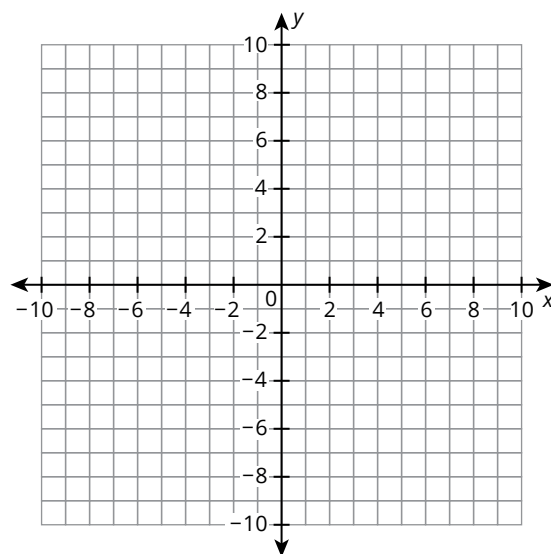
Range:

x-intercept:

y-intercept:

Maximum/Minimum:

12. $f(x) = \sqrt[3]{1-x}$



Domain:

Range:

x-intercept:

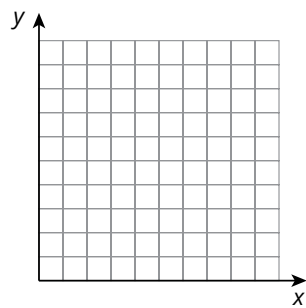
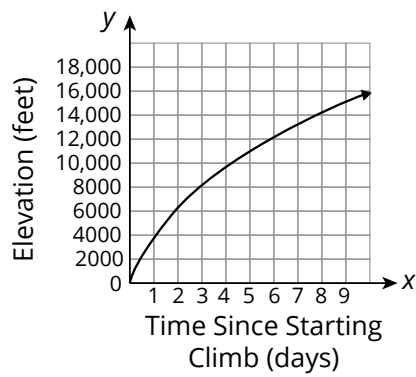
y-intercept:

Maximum/Minimum:

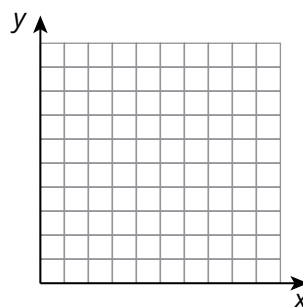
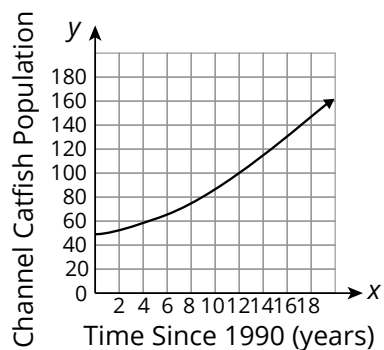
TOPIC 2 Radical Functions

D. The function modeling each problem situation is given. Sketch the inverse of each function on the grid provided and label the axes. Then, describe the domain and range of both functions as they relate to the problem situation.

1. The function models a mountain climber's elevation over his 10-day mountain climb.

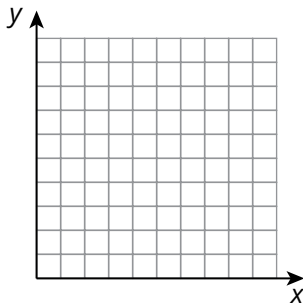
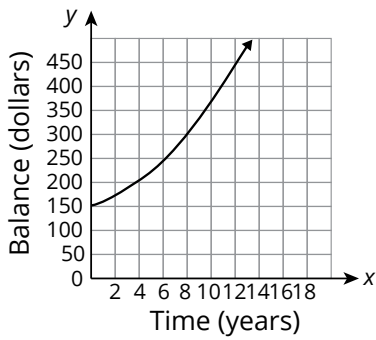


2. In the year 1990, park rangers stocked an unpopulated lake with 50 channel catfish. The function models the channel catfish population over a period of time.

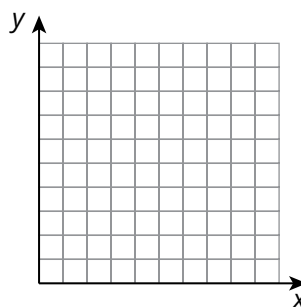
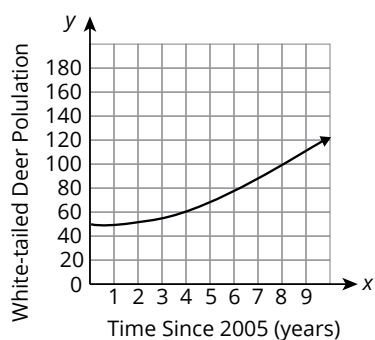


TOPIC 2 Radical Functions

3. Daniel deposited \$150.00 in a savings account. The function models the balance in the savings account in dollars.

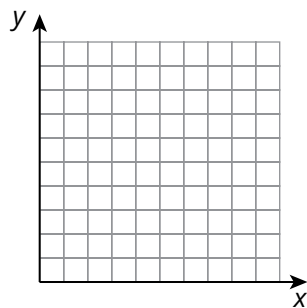
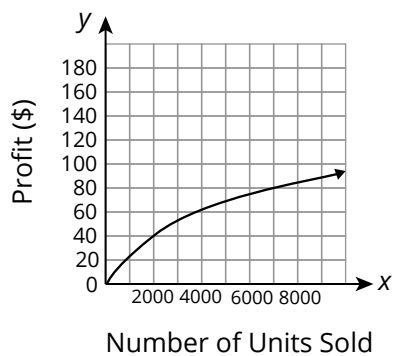


4. In the year 2005, a park had a population of 47 white-tailed deer. The function models the population of white-tailed deer in a park over a period of time.

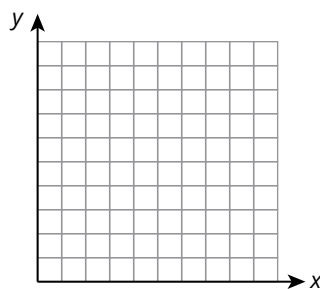
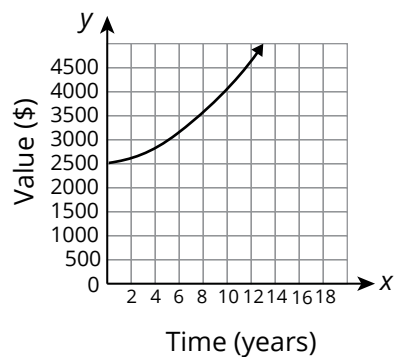


TOPIC 2 Radical Functions

5. A company's profit over a period of a year is modeled on the graph below.



6. Lance bought a rare baseball memorabilia item 15 years ago for \$2500. The function models the value of the item over time.



TOPIC 2 Radical Functions

E. Use compositions to determine whether $f(x)$ and $g(x)$ are inverse functions. Show your work.

1. $f(x) = \frac{-8+x}{2}$

$$g(x) = 2x + 8$$

2. $f(x) = -4x + 9$

$$g(x) = \frac{x-4}{-9}$$

3. $f(x) = (x-2)^2$

$$g(x) = \sqrt{x} - 2$$

4. $f(x) = 5x^2$, for $x > 0$

$$g(x) = \frac{\sqrt{x}}{5}$$

5. $f(x) = 2\sqrt[3]{x+3}$

$g(x) = \frac{x^3}{8} - 3$

6. $f(x) = 3(x+1)^3$

$g(x) = \sqrt[3]{\frac{x}{3}} - 1$

Extension

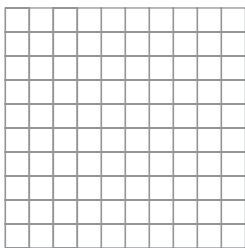
1. The number of ants in an ant population after x days can be modeled by the function $a(x) = 20x^3$ for $1 \leq x \leq 45$.
 - a. Determine the corresponding range of $a(x)$ for the given domain. Describe what the domain and range represent in this problem.

TOPIC 2 Radical Functions

- b. Use the function $a(x)$ to predict the ant population after 15 days.
- c. Write the inverse function $a^{-1}(x)$.
- d. Use compositions to verify that $a(x)$ and $a^{-1}(x)$ are inverse functions. Show your work.
- e. Determine the domain and range of $a^{-1}(x)$. Describe what the domain and range represent in this problem.
- f. Use the inverse function to determine when the ant population will be 895,000.

Spaced Practice

1. Write a rational function with vertical asymptote $x = -3$ and a horizontal asymptote at $y = -5$. Sketch a graph of the function.



2. Use synthetic division to divide the given polynomials.

a. $(x^3 + 13x^2 + 40x + 26) \div (x + 9)$

b. $(x^4 - 8x^3 + 10x^2 + 2x + 4) \div (x - 2)$

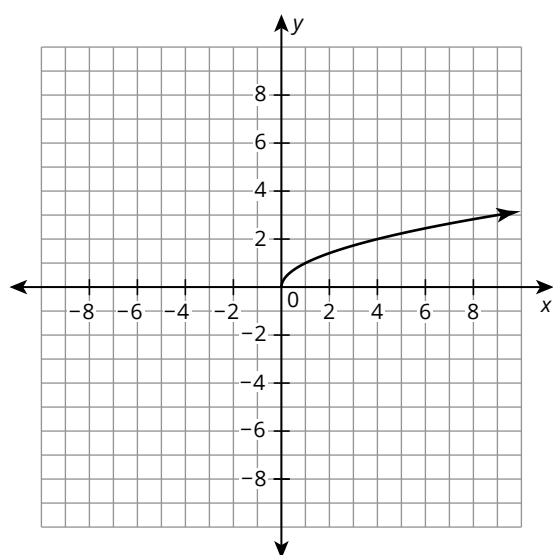
3. Solve $x^2 + 6x + 12 = 4$.

III. Transformations of Radical Functions

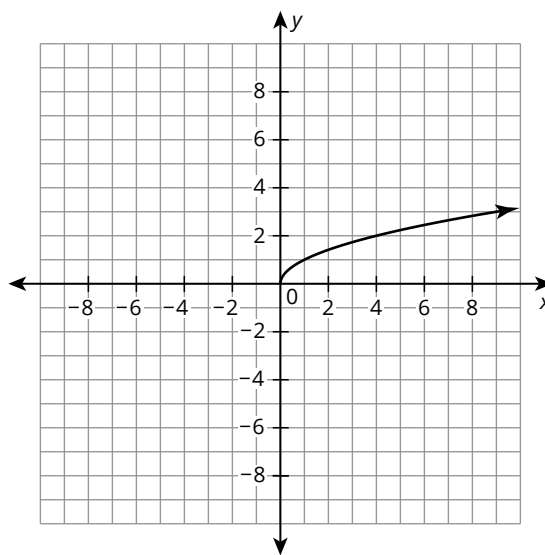
Topic Practice

- A. Sketch the graph of the transformation of $f(x) = \sqrt{x}$ as described in each exercise. Write the equation to describe each new function. The graph of $f(x) = \sqrt{x}$ is shown on each grid.

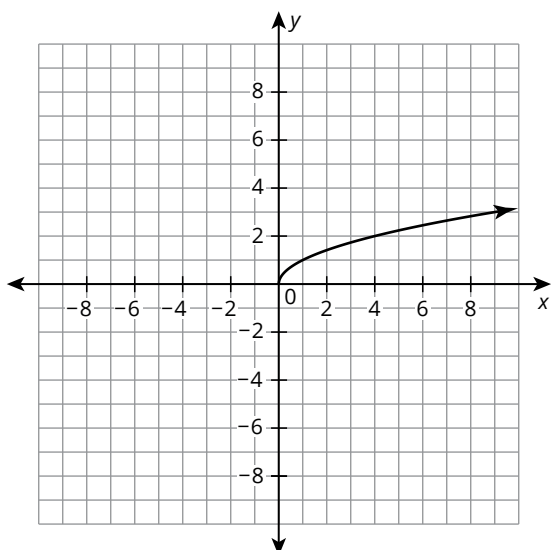
1. Translate the graph up 2 units.



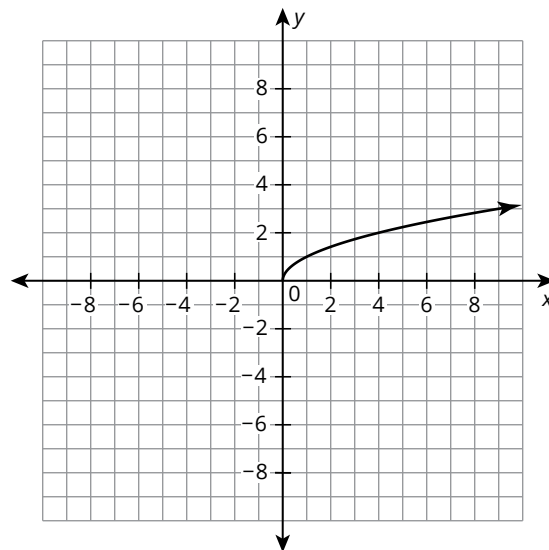
2. Translate the graph down 5 units.



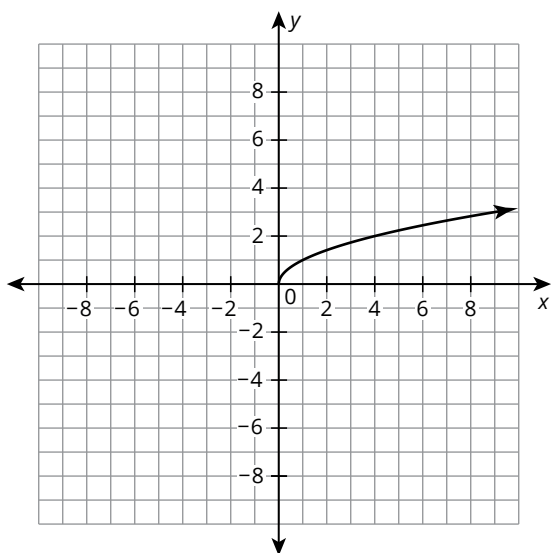
3. Translate the graph to the left 4 units.



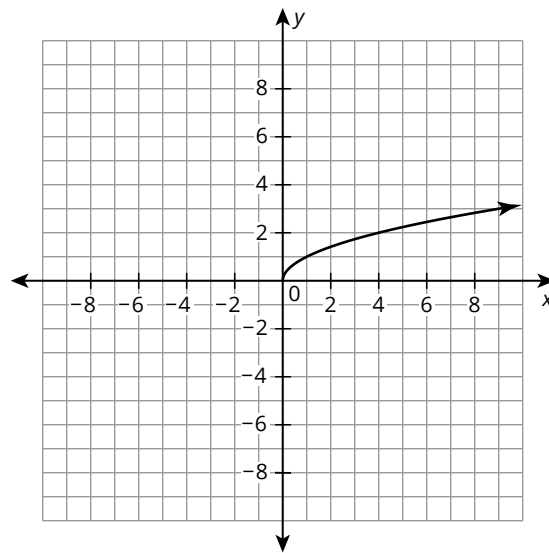
4. Translate the graph to the right 7 units.



5. Stretch the graph vertically by a factor of 2.

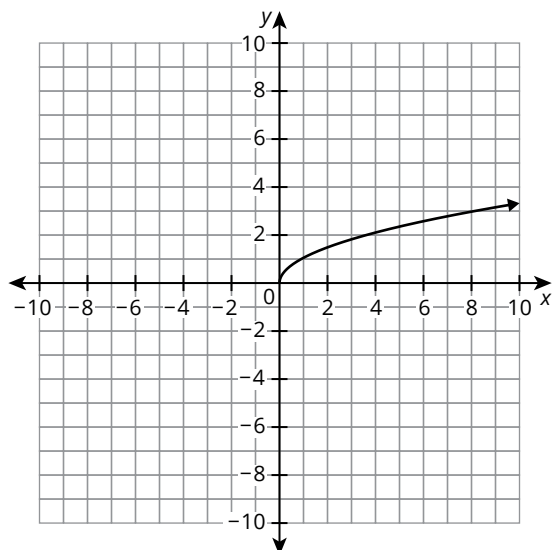


6. Reflect the graph over the x-axis.

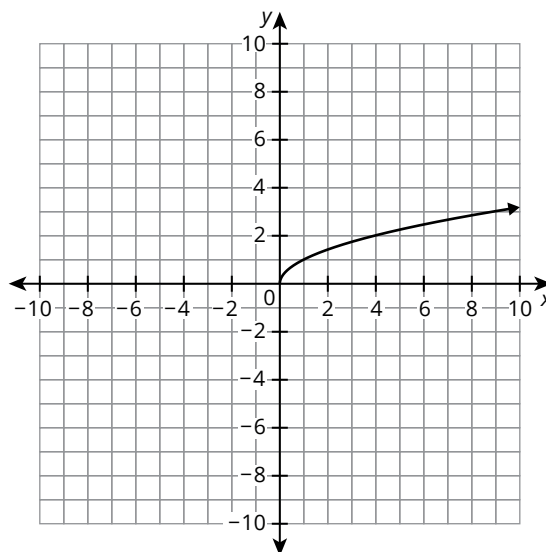


TOPIC 2 Radical Functions

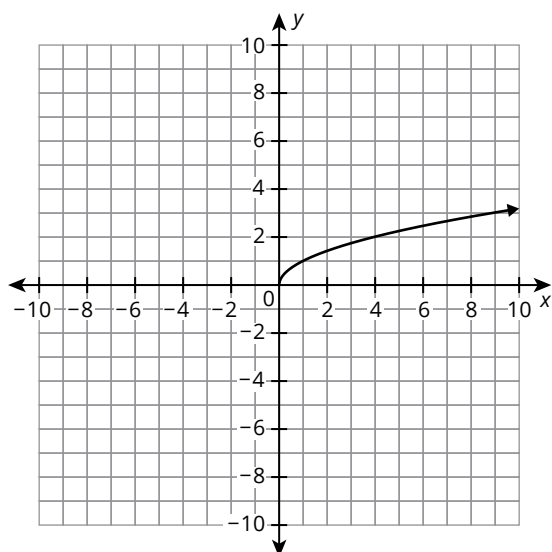
7. Horizontally compressed by a factor of 3 and reflected across the y -axis.



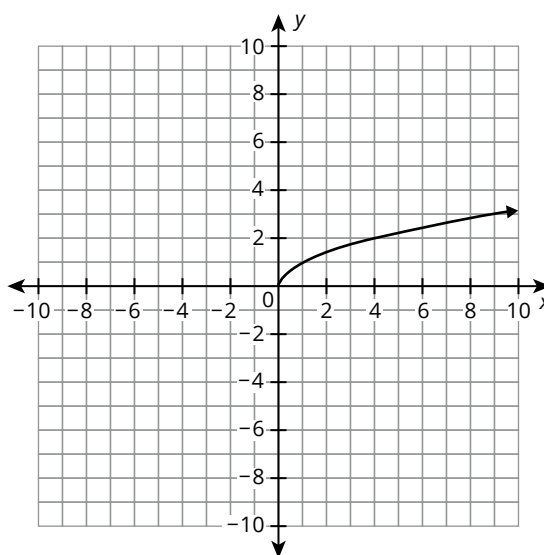
8. Horizontally stretched by a factor of 4.



9. Stretch the graph vertically by a factor of 3 and reflect the graph across the x -axis.

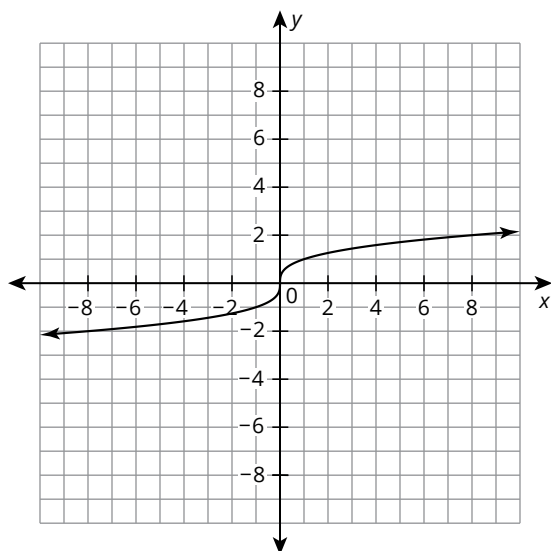


10. Translate the graph 4 units to the left, 2 units down, and reflect across the x -axis.

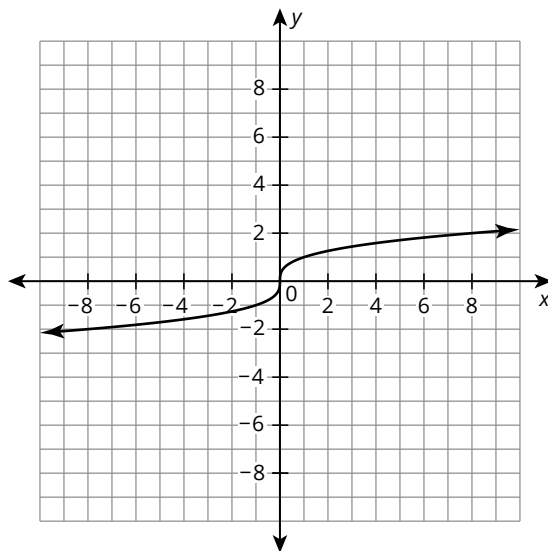


- B. Sketch the graph of the transformation of $f(x) = \sqrt[3]{x}$ as described in each exercise. Write the equation to describe each new function. The graph of $f(x) = \sqrt[3]{x}$ is shown on each grid.

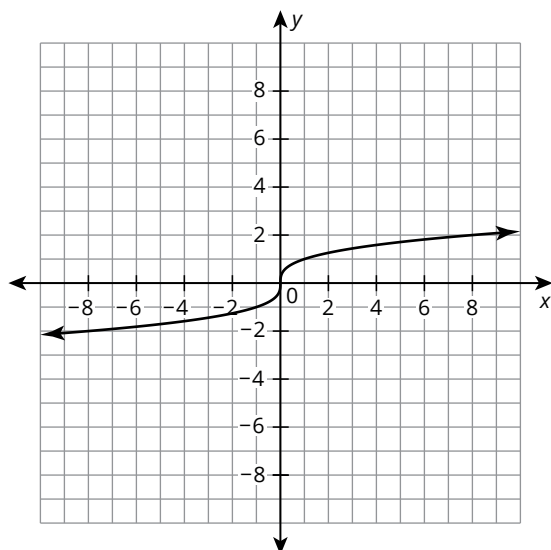
1. Translate the graph to the right 1 unit.



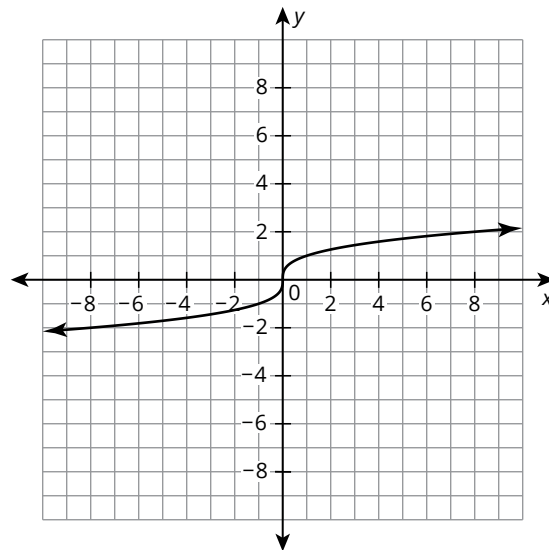
2. Translate the graph up 3 units.



3. Translate the graph to the left 8 units.

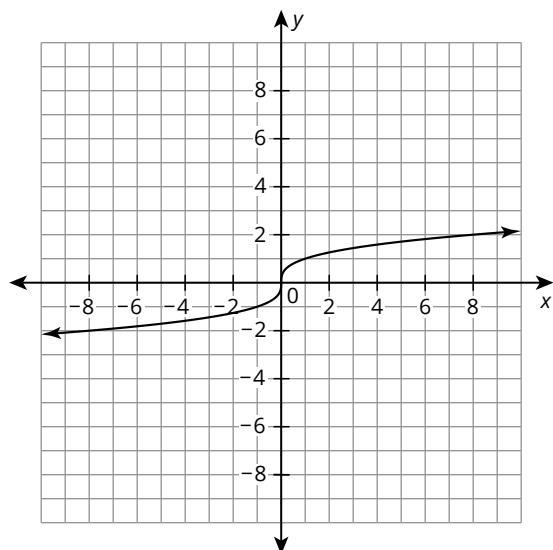


4. Translate the graph down 6 units.

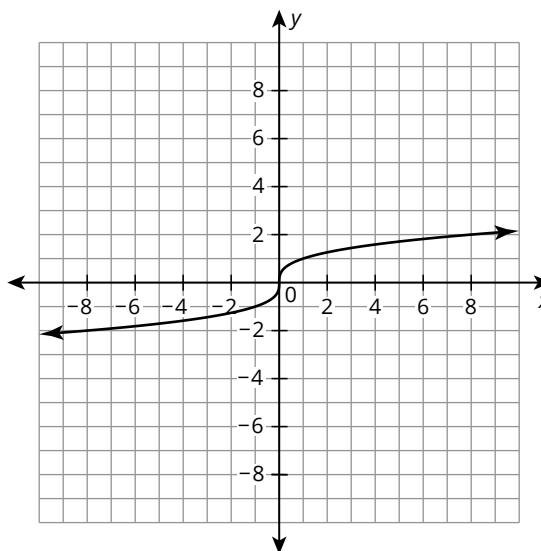


TOPIC 2 Radical Functions

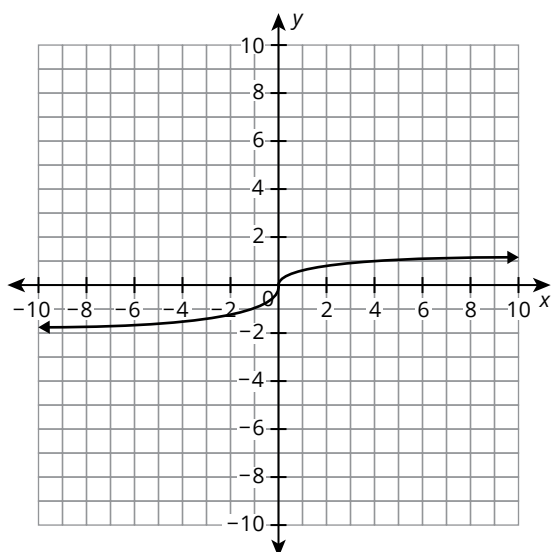
5. Compress the graph vertically by a factor of $\frac{1}{4}$.



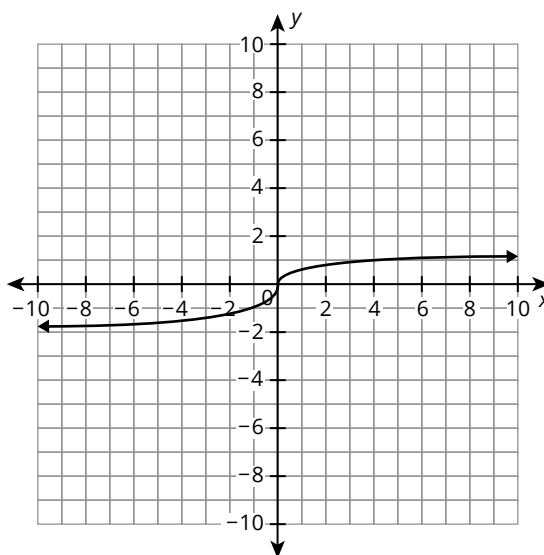
6. Reflect the graph over the y-axis.



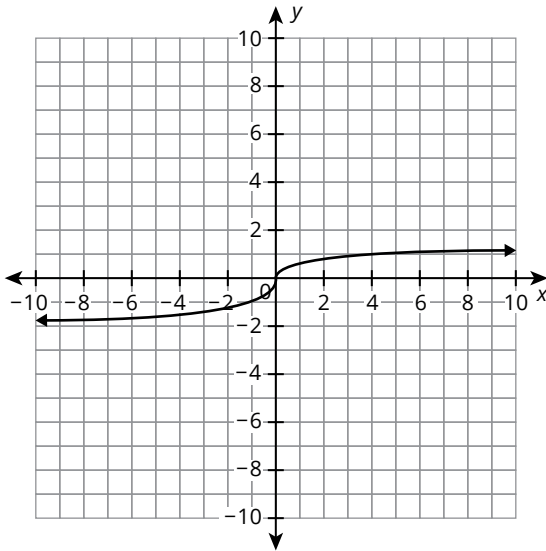
7. Stretch the graph horizontally by a factor of 3.



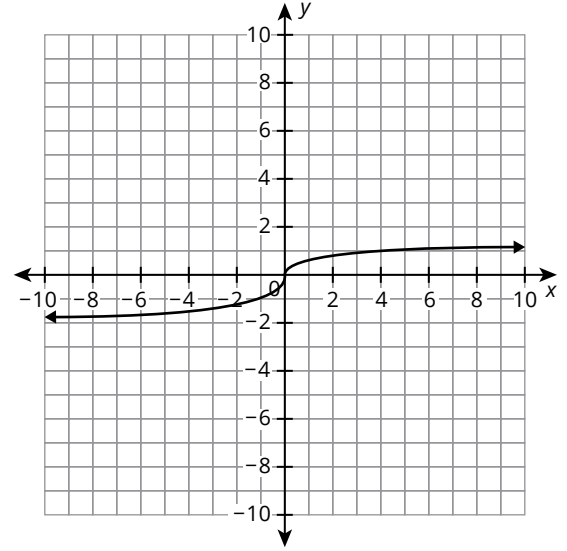
8. Compress the graph horizontally by a factor of 4.



9. Translate the graph up 2 units, right 4 units, reflect across the x -axis, and vertically stretch the graph by a factor of 3.



10. Translate the graph down 2 units, left 4 units, compress horizontally by a factor of 3, and reflect the graph across the x -axis.



- C. Use inequalities to describe how each transformation changes the domain of the function. In each exercise, $g(x)$ is a transformation of $f(x)$.

1. $f(x) = \sqrt{x}$
 $g(x) = \sqrt{x - 2}$

2. $f(x) = \sqrt{x - 4}$
 $g(x) = \sqrt{4 - x}$

3. $f(x) = \sqrt{x}$
 $g(x) = \sqrt{-x}$

4. $f(x) = \sqrt[3]{x}$
 $g(x) = \sqrt[3]{x - 3}$

TOPIC 2 Radical Functions

5. $f(x) = \sqrt[3]{x} + 5$
 $g(x) = \sqrt[3]{x} - 5$

7. $f(x) = \sqrt[3]{x - 3}$
 $g(x) = \sqrt[3]{x + 3}$

6. $f(x) = \sqrt[3]{x}$
 $g(x) = \sqrt[3]{-x}$

8. $f(x) = \sqrt[3]{x + 5}$
 $g(x) = \sqrt[3]{5 - x}$

D. Describe how each graph represented by $f(x)$ would be transformed to create the graph represented by $g(x)$.

1. $f(x) = \sqrt{x + 2}$
 $g(x) = \sqrt{x + 2} + 5$

3. $f(x) = \sqrt{x - 1}$
 $g(x) = 3\sqrt{x - 1}$

2. $f(x) = \sqrt{x}$
 $g(x) = \sqrt{-x}$

4. $f(x) = -\sqrt{x} - 4$
 $g(x) = -\sqrt{x} + 4$

5. $f(x) = \sqrt[3]{x-7} + 2$
 $g(x) = \sqrt[3]{x-4} - 3$

7. $f(x) = \sqrt[3]{x} + 5$
 $g(x) = \sqrt[3]{-x} + 5$

6. $f(x) = \sqrt[3]{x+6}$
 $g(x) = \frac{1}{2}\sqrt[3]{x+6}$

8. $f(x) = \sqrt[3]{2x}$
 $g(x) = \sqrt[3]{8x}$

E. Write an equation for each function by transforming the equation as described.

1. $f(x) = \sqrt{x}$
 translated to the right 8 units and
 up 2 units

3. $f(x) = -\sqrt{x+4}$
 translated to the left 3 units and
 down 2 units

5. $f(x) = \sqrt[3]{x}$
 translated to the left 6 units and
 down 3 units

2. $f(x) = \sqrt{2x}$
 reflected over the y-axis

4. $f(x) = \sqrt{x} - 9$
 translated to the right 5 units and
 stretched vertically by a factor of 2

6. $f(x) = \frac{2}{3}\sqrt[3]{x}$
 reflected over the x-axis

TOPIC 2 Radical Functions

- | | |
|---|---|
| <p>7. $f(x) = \sqrt[3]{x - 2} + 1$
translated to the right 7 units</p> <p>9. $f(x) = \sqrt[3]{x} + 9$
translated down 5 units and stretched
vertically by a factor of 2</p> | <p>8. $f(x) = -\sqrt[3]{x + 4} - 3$
translated up 7 units and compressed
vertically by $\frac{1}{2}$</p> <p>10. $f(x) = 3\sqrt[3]{x - 5} + 2$
translated up 4 units and shrink vertically
by a factor of $-\frac{1}{4}$</p> |
|---|---|

Extension

1. A skate park designer is creating a new ramp that follows the shape of a transformed square root function. The height of the ramp $h(x)$, in meters, at any horizontal position x , in meters, along the ramp is modeled by the function $h(x) = 2\sqrt{x - 1} + 0.5$. The designer wants to modify the ramp so that it starts at $x = 3$ instead of $x = 1$ and the so the new ramp reaches a height of 5 meters at $x = 11$. Write a function, $k(x)$ to model the height of the new ramp at any horizontal position x .

Spaced Practice

1. Consider the function $f(x) = 4x^2$.
 - a. Write the inverse function $f^{-1}(x)$.
 - b. Determine the domain and range of $f^{-1}(x)$.

2. Consider the parent rational function $f(x) = \frac{1}{x}$. Explain how the graph of $g(x) = \frac{5}{x+10} + 3$ compares to the graph of $f(x)$.

TOPIC 2 Radical Functions

3. Identify the number of real zeros of $x^3 - 7x^2 + 14x - 8 = 0$.
Explain your reasoning.

4. Determine the sum.
 $(3x^4 - 5x^3 + 2x^2 - x + 7) + (4x^3 + 6x^2 - 3x + 2)$

IV. Compositions of Functions

Topic Practice

A. Determine $(f \circ g)(x)$ and $(g \circ f)(x)$ for each pair of functions.

1. $f(x) = \frac{1}{2}x + 4$
 $g(x) = -6x - 2$

2. $f(x) = -x^2$
 $g(x) = 1 - x$

TOPIC 2 Radical Functions

3. $f(x) = \sqrt{x}$
 $g(x) = 4x^2$

5. $f(x) = x^2$
 $g(x) = \sqrt{x}$

4. $f(x) = 3x + 4$
 $g(x) = \frac{x-4}{3}$

6. $f(x) = \frac{2x+3}{5x+7}$
 $g(x) = \frac{3-7x}{5x-2}$

B. Use composition of functions to determine whether the given functions are inverses of each other.

1. $f(x) = 6 - 2x$
 $g(x) = 3 - \frac{1}{2}x$

2. $f(x) = \frac{1}{4}x - 2$
 $g(x) = 4x + 2$

TOPIC 2 Radical Functions

3. $f(x) = x^2 - 1, x \geq 0$
 $g(x) = \sqrt{x} + 1$

4. $f(x) = (x - 2)^2 + 3$
 $g(x) = 2 + \sqrt{x - 3}, x \geq 3$

5. $f(x) = \frac{1}{2x}$
 $g(x) = \frac{1}{2x}$

6. $f(x) = \frac{2x+3}{5x+7}$
 $g(x) = \frac{4x-1}{3x+2}$

TOPIC 2 Radical Functions

- C. Use the functions $f(x) = x - 6$, $g(x) = \sqrt{x}$, and $h(x) = \frac{1}{x+4}$ to evaluate each composition. Then, state the domain of the composite function using interval notation. Justify your reasoning.

1. $(f \circ g)(x)$

2. $(g \circ f)(x)$

3. $(h \circ f)(x)$

4. $(f \circ h)(x)$

5. $(g \circ h)(x)$

6. $(h \circ g)(x)$

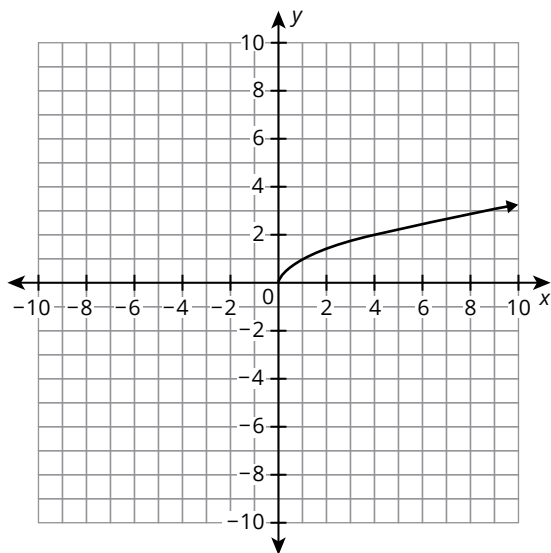
Extension

1. You have determined that the functions $f(x) = x^2$, when $x \geq 0$, and $g(x) = \sqrt{x}$ are inverses. Now, consider the transformations $h(x) = (x - 2)^2 + 3$, when for $x \geq 2$ and $j(x) = \sqrt{x - 3} + 2$. Determine whether $h(x)$ and $j(x)$ are still inverses. Justify your answer algebraically.

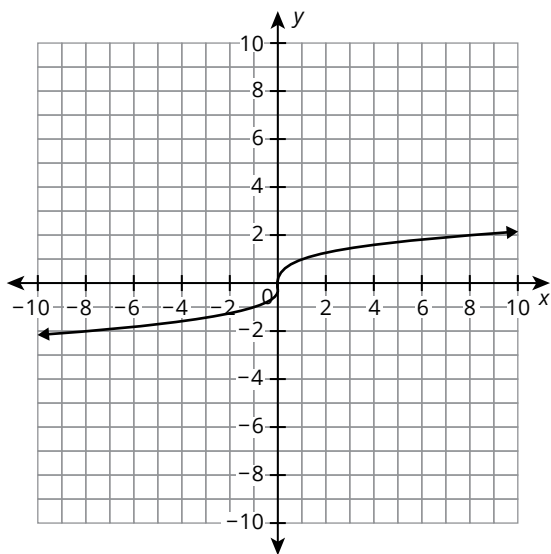
TOPIC 2 Radical Functions

Spaced Practice

1. Given $f(x) = \sqrt{x}$, graph $g(x) = 2f(x - 1) + 3$.



2. Given $f(x) = \sqrt[3]{x}$, graph $g(x) = -f(x + 3) - 4$.



3. Factor $x^3 - 27$.

Name _____ Date _____

I. Rewriting Radical Expressions

Topic Practice

A. Rewrite each expression using rational exponents.

1. $\sqrt{X^3Y}$

2. $\sqrt[3]{a^2b^4c^5}$

3. $\sqrt[4]{f^3g^6}$

4. $\sqrt[5]{(x+y)^2}$

5. $\sqrt[3]{\frac{r^2s}{t^4}}$

6. $\sqrt{a^5b}$

7. $\sqrt[4]{\frac{x^2}{y^3}}$

8. $\sqrt[5]{32f^4}$

TOPIC 3 Radical Equations

B. Rewrite each expression using radicals.

1. $u^{\frac{2}{3}}w^{\frac{5}{3}}$

2. $x^{\frac{1}{2}}y^{\frac{3}{2}}z^{\frac{7}{2}}$

3. $(a + b)^{\frac{3}{4}}$

4. $f^{\frac{4}{5}}g^{\frac{1}{5}}$

5. $r^{\frac{1}{2}}s^{\frac{3}{4}}$

6. $\frac{a^{\frac{3}{2}}b^{\frac{1}{4}}}{c^{\frac{5}{4}}}$

7. $x^{\frac{2}{5}}y^{\frac{6}{5}}$

8. $\frac{r^2s^{\frac{2}{3}}}{t^{\frac{1}{3}}u^{\frac{4}{3}}}$

C. Rewrite each expression.

1. $\sqrt{x^6y^8}$

2. $(a^3b^{12})^{\frac{1}{3}}$

3. $\sqrt[3]{(x - 2)^6}$

4. $\sqrt[3]{(5 + x)^{12}}$

5. $(25y^8)^{\frac{1}{2}}$

7. $\sqrt{16x^{10}y^8z^2}$

9. $(27x^{15}y^9z^3)^{\frac{1}{3}}$

6. $\sqrt{36z^4}$

8. $\sqrt{49x^{12}y^2z^6}$

10. $\sqrt[4]{16x^{12}y^4z^{16}}$

Extension

Perform each operation, extract all roots, and write your final answers in radical form, without radicals in the denominator.

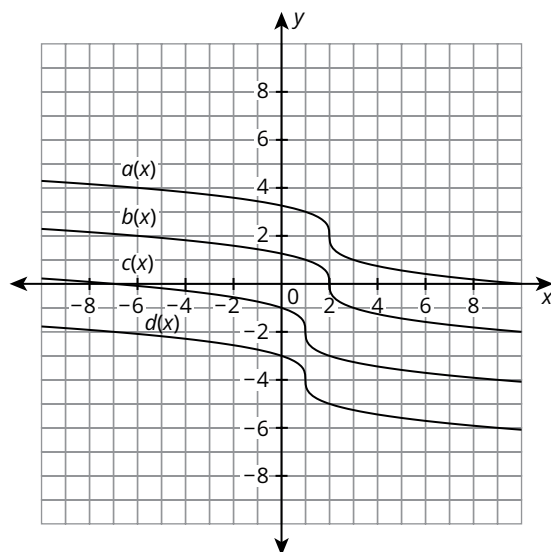
1. $\frac{8\sqrt{20x^7}}{3\sqrt[3]{8x^5}}$, given $x > 0$

2. $\frac{-2\sqrt[3]{24a^4}}{15\sqrt{27a^3}}$, given $a > 0$

3. $\frac{4\sqrt{28a^2b^2}}{5\sqrt{12ab^3}}$

Spaced Practice

- Each of the four curves shown on the graph are transformations of $f(x) = \sqrt[3]{x}$ with a restricted domain.



- Describe each transformation of $f(x) = \sqrt[3]{x}$ that was used to create the four functions in the design.

- b. Write each function used in the design. For each function, write the domain as an inequality.

2. Perform each operation. List any restrictions on the variables.

a. $\frac{5}{2x^2 - 5x - 3} - \frac{8}{x - 3}$

b. $\frac{x^2 - 16}{4(x + 1)} \cdot \frac{x^2 - 1}{x - 4}$

3. Transform the quadratic function from standard to vertex form.

$$f(x) = 2x^2 + 8x + 5$$

II. Solving Radical Equations

Topic Practice

A. Solve each equation. Check for extraneous solutions.

1. $\sqrt{3x} = 6$

3. $\sqrt{5x - 1} = 2$

5. $2\sqrt[3]{x} + 5 = 1$

7. $\sqrt{10x - 1} - 7 = -5$

2. $\sqrt{4x} = 8$

4. $\sqrt[3]{3x - 3} = 2$

6. $4\sqrt[3]{x} + 5 = -3$

8. $\sqrt{9x + 3} - 11 = -8$

B. Solve each equation. Check for extraneous solutions.

1. $3 = (2x + 27)^{\frac{1}{4}}$

3. $7 = (27 - x)^{\frac{1}{2}}$

2. $-48 = -2x^{\frac{3}{2}} + 6$

4. $3(x + 19)^{\frac{1}{2}} = 21$

5. $(13 - 4x)^{\frac{1}{3}} = 5$

7. $(x + 4)^{\frac{1}{2}} = x - 2$

6. $4(x - 26)^{\frac{1}{4}} = 12$

8. $(2x - 3)^{\frac{1}{2}} = x - 1$

C. Solve each equation. Check for extraneous solutions.

1. $3 + x = \sqrt{4x + 9}$

3. $2x - 2 = \sqrt{x + 2}$

5. $x = \sqrt[3]{2x^2 + 8x}$

7. $\sqrt{3x - 5} = 1 - \sqrt{2x}$

9. $x^{\frac{1}{3}} - 4 = 0$

11. $(4x - 1)^{\frac{1}{3}} - 2 = 3$

2. $x - 4 = \sqrt{2x - 9}$

4. $x + 2 = \sqrt{3x + 10}$

6. $-x = \sqrt[3]{x^2 - 12x}$

8. $\sqrt{x + 1} = \sqrt{2x + 1} + 2$

10. $(2x + 5)^{\frac{1}{2}} = 7$

12. $(x + 1)^{\frac{1}{2}} - 3 = -6$

TOPIC 3 Radical Equations

13. $(x + 2)^{\frac{1}{2}} = (10 - 3x)^{\frac{1}{2}}$

15. $(x + 1)^{\frac{1}{2}} + x^{\frac{1}{2}} = 5$

14. $(2x - 1)^{\frac{1}{2}} = -(x + 6)^{\frac{1}{2}}$

16. $x + (x + 13)^{\frac{1}{2}} = 7$

D. Solve each problem. Check for extraneous solutions.

1. The distance between any two points on a coordinate grid, d , can be calculated by using the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) represents the coordinates of one point and (x_2, y_2) represents the coordinates of the other point. Identify the point(s) on the x -axis $(x, 0)$, that is/are exactly 8 units from the point $(2, -3)$.
2. The distance between any two points on a coordinate grid, d , can be calculated by using the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) represents the coordinates of one point and (x_2, y_2) represents the coordinates of the other point. Identify the point(s) on the y -axis $(0, y)$, that is/are exactly 5 units from the point $(-3, -4)$.

3. The radius of a circle on a coordinate grid that is centered at the origin, r , can be calculated by using the equation $r = \sqrt{x^2 + y^2}$, where x represents the x -coordinate and y represents the y -coordinate of a point on the circle. Determine the x -coordinate(s) of a point(s) $(x, 6)$ on a circle with a radius of 8.

4. The minute you drive a newly purchased car off the lot, its resale value drops immediately. The equation $r = 1 - \sqrt[3]{\frac{v}{c}}$ models a car's immediate resale value, where v represents the immediate resale value of the car, c represents the original cost of the car, and r represents the depreciation rate. Determine the immediate resale value of the car if the original cost was \$29,500 and the depreciation rate is 7%. Round your answer to the nearest cent.

5. The speed, in meters per second, of a tsunami can be determined by using the formula, $s = \sqrt{9.8d}$, where d is the depth of the ocean in meters. Suppose a tsunami is traveling at a speed of 8.3 kilometers per second. How deep is the ocean at that point? (HINT: 1 kilometer = 1000 meters)

6. Maria deposited \$2580 in an account 3 years ago. The interest is compounded once a year, and the equation $r = \sqrt[3]{\frac{A}{2580}} - 1$, where A is the current balance, can be used to calculate the interest rate. If the interest rate is 3.5%, how much does Maria currently have in her account? Round your answer to the nearest cent. (HINT: Write the interest rate as its decimal equivalent before substituting it into the equation.)

TOPIC 3 Radical Equations

Extension

1. Solve $\sqrt{3x + 1} = 4 - \sqrt{5 - x}$ and check your solution.

Spaced Practice

1. Rewrite each expression by extracting all possible roots.

a. $\sqrt[3]{54x^5y^{12}}$

b. $\sqrt{44a^3b^7}$

2. Write a quadratic equation for the parabola that passes through the point $(-3, 27)$ with roots $(6, 0)$ and $(-2, 0)$.

3. Natalia is going to sell T-shirts at a concert. Company A tells her it will cost \$300 up front and \$4.50 per shirt. Company B tells her it will cost \$250 up front and \$4.00 per shirt.

- a. Write a function to represent the average cost per T-shirt for the two companies.

- b. Which company will have a lower average cost per T-shirt if she plans on getting 200 T-shirts made? Show all of your work and explain your reasoning.

III. Modeling with Radical Functions

Topic Practice

A. Determine whether each table represents a quadratic or square root function. Then, write a function to represent the data.

1.

x	y
-2	4
-1	1
0	0
1	1
2	4

2.

x	y
0	0
1	1
4	2
9	3
16	4

3.

x	y
-3	14
-2	9
-1	6
0	5
1	6

4.

x	y
0	0
1	2
4	4
9	6
16	8

TOPIC 3 Radical Equations

5.

x	y
-3	8
-2	3
-1	0
0	-1
1	-2

6.

x	y
0	1
1	2
4	3
9	4
16	5

B. Use the table and situation to answer each question.

The number of customers visiting a new coffee shop each week is shown in the table.

Number of Weeks	Number of Customers
0	100
2	120
4	180
6	280
8	420
10	600

1. What family of functions does the table represent? Explain your reasoning.
2. Use graphing technology to write an equation that represents the data in the table.
3. Write the domain and range of the problem situation using inequalities. Explain your reasoning.

TOPIC 3 Radical Equations

4. Use the equation from Question 2 to determine when the shop will have 500 customers.
5. Write an inverse function for the data. Explain any restrictions on the function based on the restrictions of the equation you wrote in Question 2.
6. Use the inverse equation to determine the number when the store will have 1000 customers.

Extension

The number of views on a viral video is increasing over time. The number of hours since the video was posted and the number of viewers is recorded in a table.

Number of Hours	Number of Views
0	200
2	208
4	264
6	416
8	712

1. Use graphing technology to write a cubic function to represent the data.
2. Write an inverse equation for the data. Explain what the equation means in terms of the problem situation.

TOPIC 3 Radical Equations

Spaced Practice

1. Solve the equation $\sqrt{x-3} = x-5$. Check for extraneous solutions.
2. Use compositions of functions to determine whether $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$ are inverses.
3. Determine the linear and quadratic factors of $x^3 + 3x^2 + 2x + 6$.

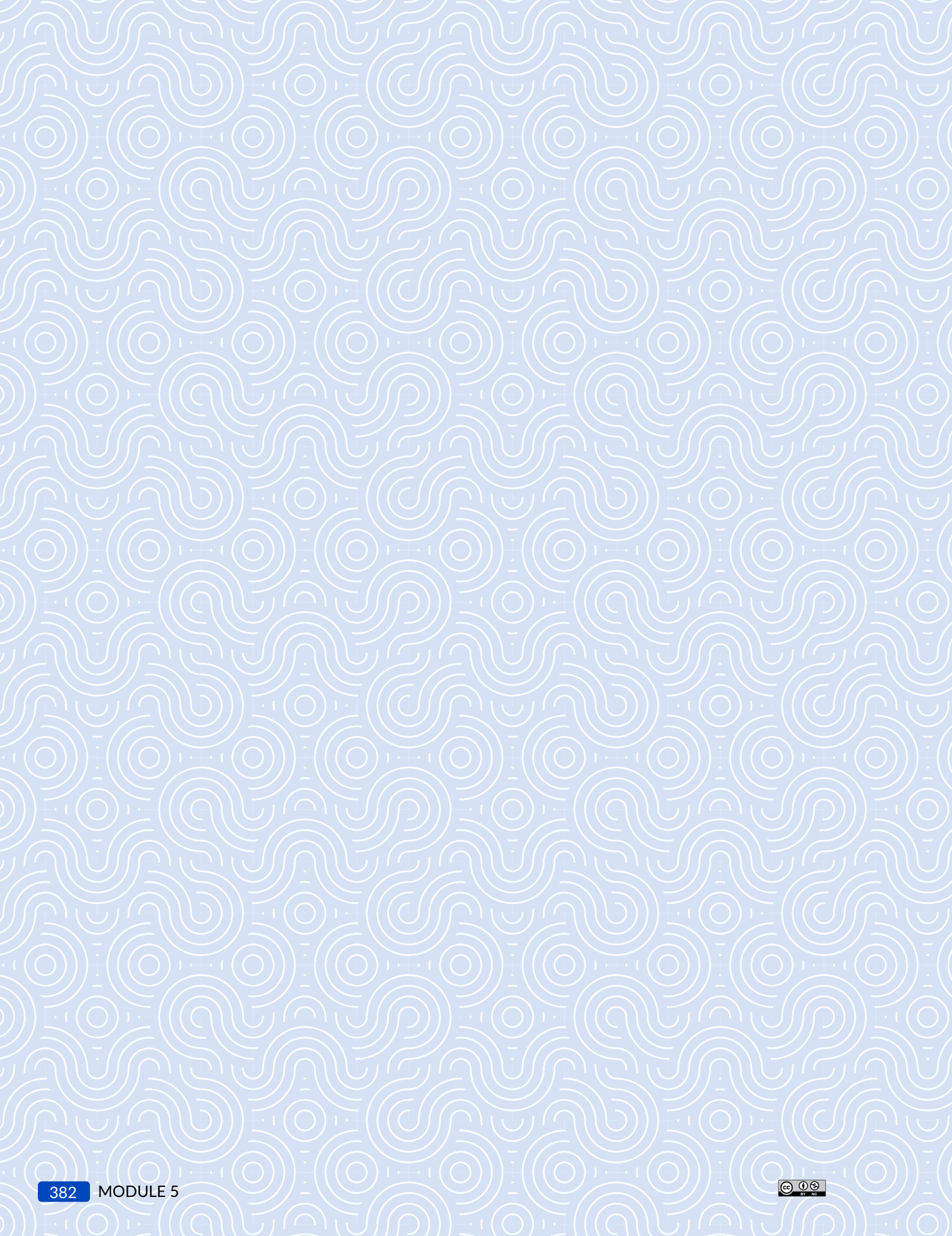
Exploring Exponentials and Logarithmics

TOPIC 1: Exponential and Logarithmic Functions

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TOPIC 2: Exponential and Logarithmic Equations

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Name _____ Date _____

I. Comparing Linear and Exponential Functions

- A. Write the explicit and recursive formula for each geometric sequence. Then, use the equation to determine the 10th term. Round answers to the nearest thousandth, if necessary.

1.

n	1	2	3	4	5	6	10
a_n	5	15	45	135	405	1,215	

2.

n	1	2	3	4	5	6	10
a_n	200	100	50	25	12.5	6.25	

3.

n	1	2	3	4	5	6	10
a_n	1	1.25	1.563	1.953	2.441	3.052	

4.

n	1	2	3	4	5	6	10
a_n	1	0.8	0.64	0.512	0.410	0.328	

TOPIC 1 Exponential and Logarithmic Functions

5.

n	1	2	3	4	5	6	10
a_n	0.4	0.8	1.6	3.2	6.4	12.8	

6.

n	1	2	3	4	5	6	10
a_n	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	

B. Write an exponential function to represent each geometric sequence.
 Evaluate the function for the given value of n . Round to the nearest thousandth, if necessary.

1. $a_n = 4 \cdot 2.5^{n-1}$
 $n = 10$

2. $a_n = 0.3 \cdot 8^{n-1}$
 $n = 3$
3. $a_n = 150 \cdot 0.8^{n-1}$
 $n = 2$

4. $a_n = 0.5 \cdot 1.25^{n-1}$
 $n = 24$
5. $a_n = 10 \cdot 4^{n-1}$
 $n = 7$

6. $a_n = 100 \cdot 0.5^{n-1}$
 $n = 5$

TOPIC 1 Exponential and Logarithmic Functions

- C. Write an exponential function $A(t)$, where t represents elapsed time, to represent each half-life situation. Then, use the function to complete each table. Round as necessary.

1.

Elapsed Time (hours)	0	2	4	6	8	20
Drug in Bloodstream (mg)	120	60	30	15	7.5	
Number of Half-Life Cycles	0	1	2	3	4	

2.

Elapsed Time (minutes)	0	5	10	15	20	100
Bacteria Subject to Growth Inhibitor	800	400	200	100	50	
Number of Half-Life Cycles	0	1	2	3	4	

3.

Elapsed Time (years)	0	14	21	28	42	56
Strontium in Rock Sample (grams)	16	8		4	2	1
Number of Half-Life Cycles	0	1		2	3	4

4.

Elapsed Time (years)	0	5,700	11,400	15,675	17,100	22,800
C-14 in Rock Sample (grams)	1	0.5	0.25		0.125	0.0625
Number of Half-Life Cycles	0	1	2		3	4

TOPIC 1 Exponential and Logarithmic Functions

5.

Elapsed Time (Days)	0	6	12	18	24	42
Rat Population Exposed to Virus	5000	2500	1250	625	313	
Number of Half-Life Cycles	0	1	2	3	4	

6.

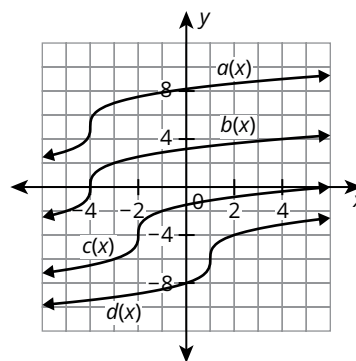
Elapsed Time (Hours)	0	2	4	6	8	16
Participants in Tennis Tournament	256	128	64	32	16	
Number of Half-Life Cycles	0	1	2	3	4	

Extension

A sheet of paper is approximately 0.1 mm thick. Suppose you could fold the paper in half as many times as you wished, doubling the thickness each time, without tearing the paper. How many times would you need to fold the paper in order for its thickness to reach the Moon?

Spaced Practice

1. Each of the four curves shown on the graph are transformations of $f(x) = \sqrt[3]{x}$ with a restricted domain.

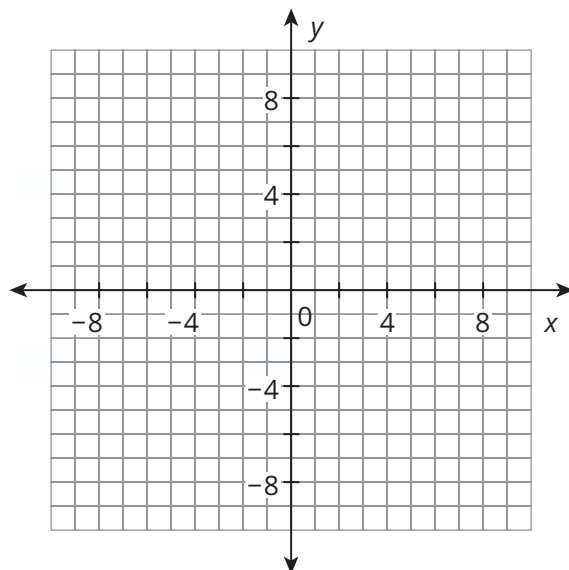
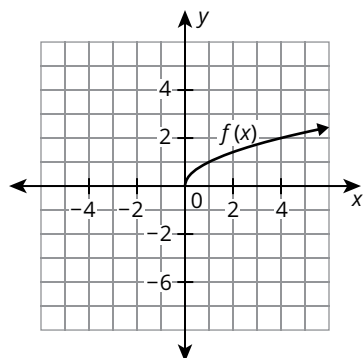


- a. Describe each of the four transformations of $f(x) = \sqrt[3]{x}$ that were used to create the four functions in the design.

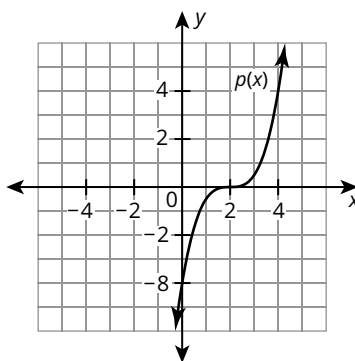
- b. Write each function used in the design. For each function, write the domain as an inequality.

TOPIC 1 Exponential and Logarithmic Functions

2. The graph of $f(x) = \sqrt{x}$ is shown with the restricted domain of $0 \leq x \leq 6$.



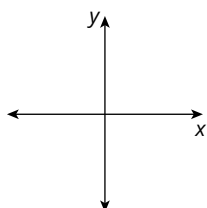
- a. Sketch a function $g(x)$ to represent the reflection of $f(x)$ across the y-axis. Write the function $g(x)$.
- b. Sketch the functions $f'(x)$ and $g'(x)$ to represent reflections of $f(x)$ and $g(x)$ across the x-axis. Write the functions $f'(x)$ and $g'(x)$.
3. Identify the extrema, zeros, and intercepts of the graph of $p(x)$.



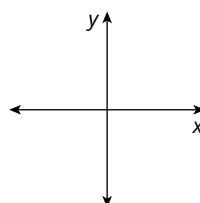
II. Properties of Exponential Graphs

A. Determine whether the given function represents exponential growth or decay. Explain your reasoning and include a sketch of the function.

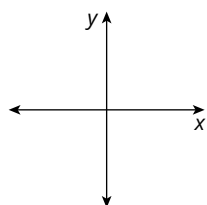
1. $f(x) = 8^x$



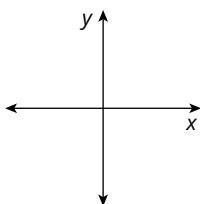
2. $f(x) = 0.2^x$



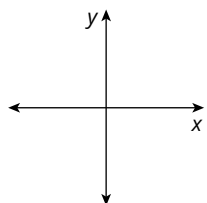
3. $f(x) = \left(\frac{5}{2}\right)^x$



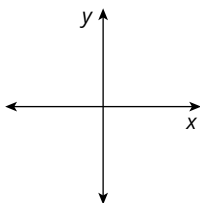
4. $f(x) = 25^x$



5. $f(x) = \left(\frac{1}{6}\right)^x$



6. $f(x) = 7.5^x$



TOPIC 1 Exponential and Logarithmic Functions

B. Write an exponential function with the given attributes from the graph.

- | | |
|---|---|
| 1. increasing over $(-\infty, \infty)$
reference point $(-1, \frac{1}{9})$ | 2. decreasing over $(-\infty, \infty)$
reference point $(1, \frac{2}{3})$ |
| 3. increasing over $(-\infty, \infty)$
reference point $(2, 2.25)$ | 4. decreasing over $(-\infty, \infty)$
reference point $(-2, 16)$ |
| 5. increasing over $(-\infty, \infty)$
reference point $(-3, \frac{1}{8})$ | 6. decreasing over $(-\infty, \infty)$
reference point $(-4, \frac{81}{16})$ |

C. Use the formula for compound interest to determine the amount of money in each account after interest is accrued.

1. An investor deposits \$1000 in an account that promises an annual interest rate of 5%, compounded at the end of each year. How much will be in the account after seven years?

2. At the start of the school year, Fairview High School deposits PTA dues in an account that offers an annual interest rate of 3.5%, compounded at the end of each year. If \$2500 is collected in PTA dues, how much money will the school have at the start of the next school year?

3. Kyle put \$300 of his birthday money in the bank. The bank offers an annual interest rate of 4%, compounded twice a year. How much money will Kyle have after three years?

4. An investing group has \$50,000 to invest. They put the money in an account that has an annual interest rate of 6%, compounded monthly. How much money will the group have at the end of 10 years?

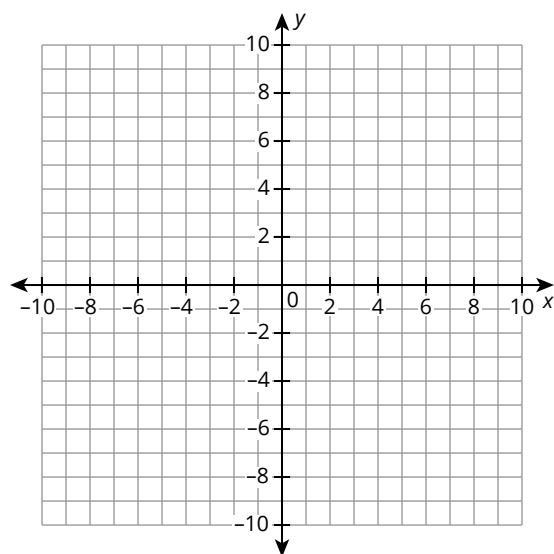
5. Interest is compounded quarterly at Money Bank at an annual rate of 5.5%. A new client opens an account with \$7200. How much money will be in the account at the end of six years?

TOPIC 1 Exponential and Logarithmic Functions

6. Sasha wants to earn the maximum interest on her money. She decides to deposit \$50 in two different banks for 90 days (3 months) to compare them before she deposits all of her money. She finds a bank that compounds interest daily at an annual rate of 2.2% and another bank that compounds interest monthly at an annual rate of 4.8%. With which bank will she earn more money?

- D. Graph each function and identify the key attributes (domain, range, y-intercept, increasing/decreasing).

1. $y = 2^x$



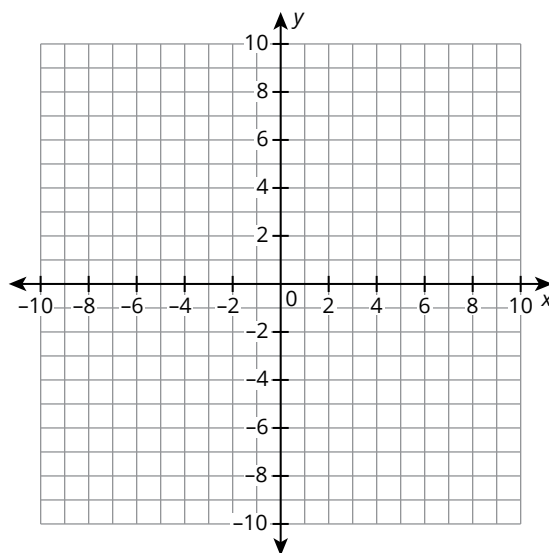
Domain:

Range:

y-intercept:

increasing/decreasing:

2. $y = e^x$



Domain:

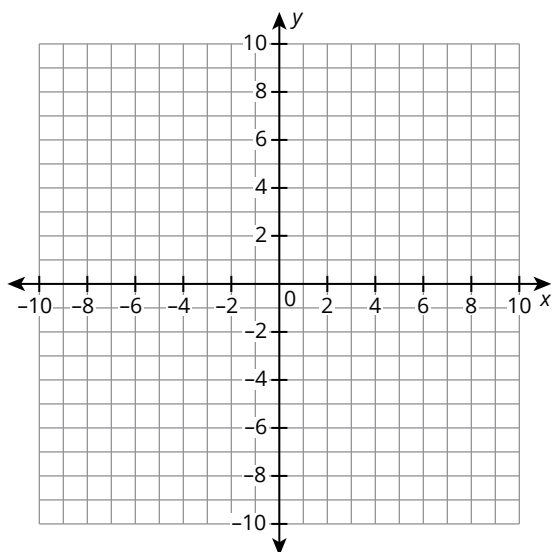
Range:

y-intercept:

increasing/decreasing:

TOPIC 1 Exponential and Logarithmic Functions

3. $y = 10^x$



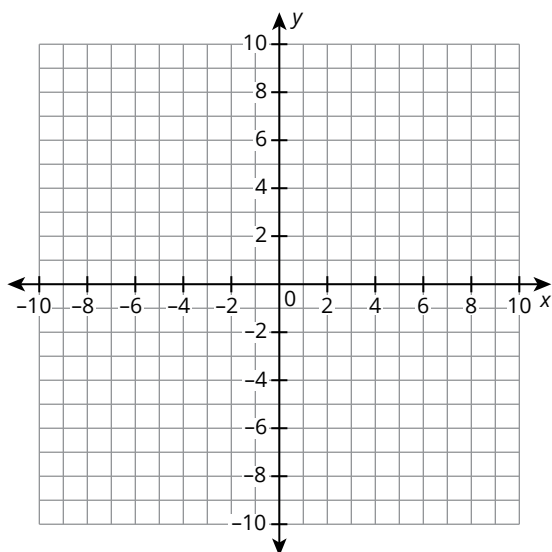
Domain:

Range:

y-intercept:

increasing/decreasing:

5. $Y = 10^{(x+1)}$



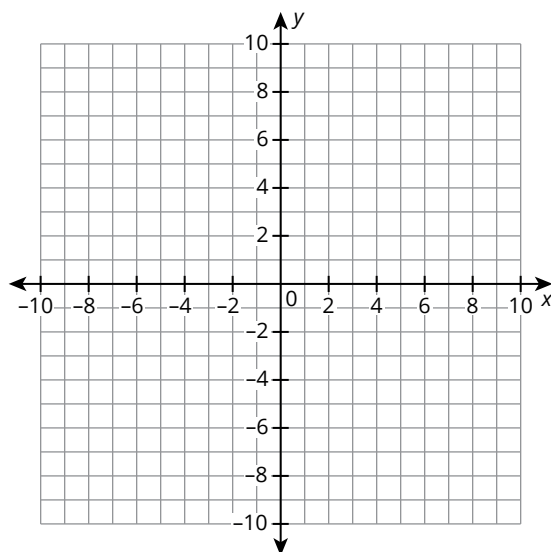
Domain:

Range:

y-intercept:

increasing/decreasing:

4. $y = e^{x-1}$



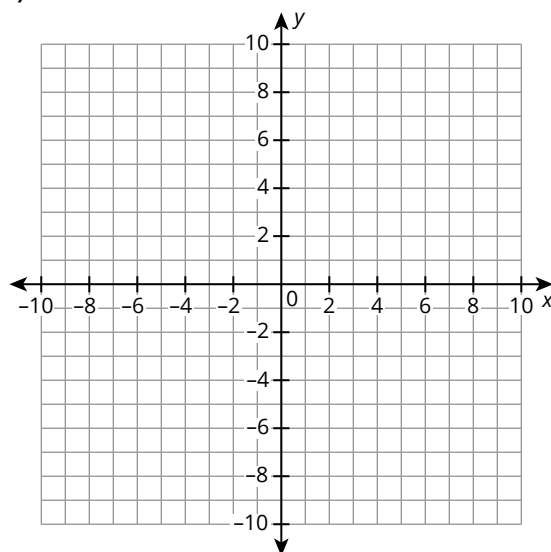
Domain:

Range:

y-intercept:

increasing/decreasing:

6. $y = 2 \cdot 2^x$



Domain:

Range:

y-intercept:

increasing/decreasing:

TOPIC 1 Exponential and Logarithmic Functions

E. Use the formula for population growth to predict the population of each city.

1. The population of Austin, Texas, is growing at a rate of 2.9% per year. If the population in 2016 was approximately 913,917, what is the predicted population for 2025?
2. The population of Boston, Massachusetts, is growing at a rate of 1.8% annually. The population in 2013 was approximately 636,500. What is the predicted population for 2025?
3. The population of Charlotte, North Carolina, in 2013 was approximately 775,000. If the annual rate of growth is about 3.2%, what is an approximation of Charlotte's population in 2000?
4. The population of Beijing, China, in 2012 was approximately 20,690,000 and is growing at an annual rate of about 5.5%. What is an approximation of Beijing's population in 1980?

5. The population of Detroit, Michigan, is decreasing at an annual rate of about 0.75%. Detroit's population in 2015 was approximately 678,250. What is the predicted population for 2030?

6. The population of Berlin, Germany, was about 3,290,000 in 2011. Its population is declining at an annual rate of about 0.2%. What is the predicted population for 2050?

Extension

Write the domain and range of the function in interval notation, inequalities, and set notation.

Spaced Practice

1. Nadine bought a house in 1995 for \$155,000. The table displays the value of the house over several years. The year 0 corresponds to the date of the year Nadine bought the house.

Year	Value of House (\$)
0	155,000
1	162,750
2	170,887.50
3	179,431.875

- a. Does the value of the house represent an example of exponential decay, exponential growth, or neither? Explain your reasoning.
- b. Write a function to represent the value of the house, V , as a function of the year, n .

2. The population of a small town in 2015 was 8,455. The table displays the population of the town for several years. The year 0 corresponds to the year 2015.

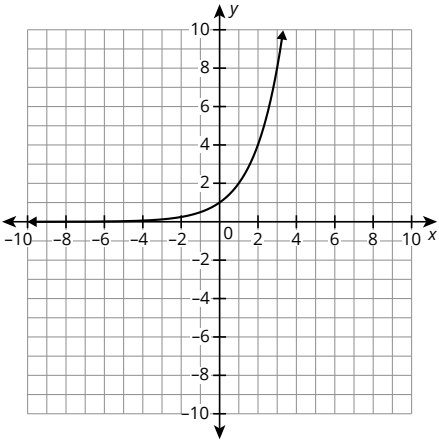
Year	Population
0	8455
1	6764
2	5411
3	
4	
5	

- Does the population represent an example of exponential decay, exponential growth, or neither? Explain your reasoning.
 - Write a function to represent the population of the town, P , as a function of the year, t .
 - Use the function you wrote in part (b) and the table to determine the population in year 5. Round all answers to the nearest whole number.
3. Solve each equation and check your solutions.
- $\sqrt[3]{4x + 3} = 2$
 - $\sqrt{x + 2} + = x$
 - $\frac{m - 4}{4} + \frac{m}{3} = 6$

III. Transformations of Exponential Functions

A. Complete the table to determine the corresponding points on $c(x)$, given reference points on $f(x)$. Then, graph $c(x)$ on the same coordinate plane as $f(x)$ and state the domain, range, and asymptotes of $c(x)$. Write the domain and range using interval notation.

1. $f(x) = 2^x$
 $c(x) = f(x - 1)$



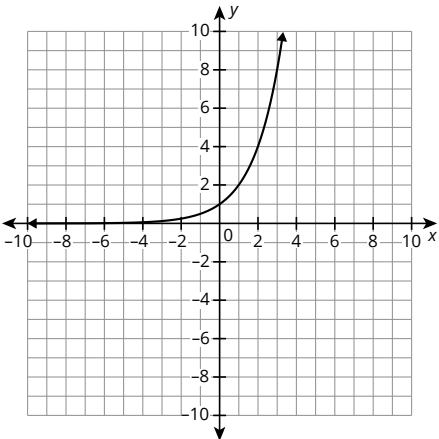
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

Domain:

Range:

Asymptote:

2. $f(x) = 2^x$
 $c(x) = 4f(x)$



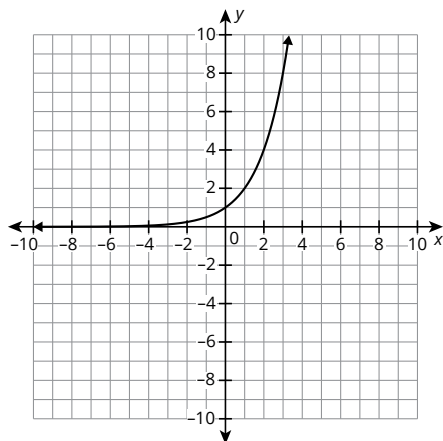
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

Domain:

Range:

Asymptote:

3. $f(x) = 2^x$
 $c(x) = f(x) - 3$



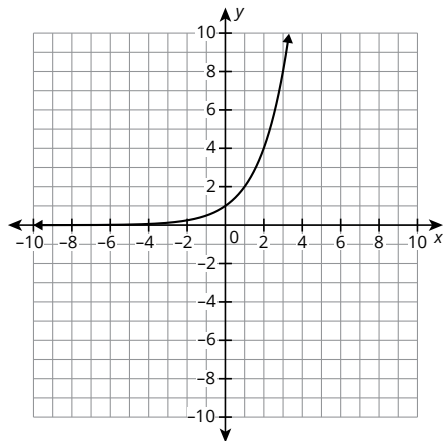
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

Domain:

Range:

Asymptote:

4. $f(x) = 2^x$
 $c(x) = f(x + 2)$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

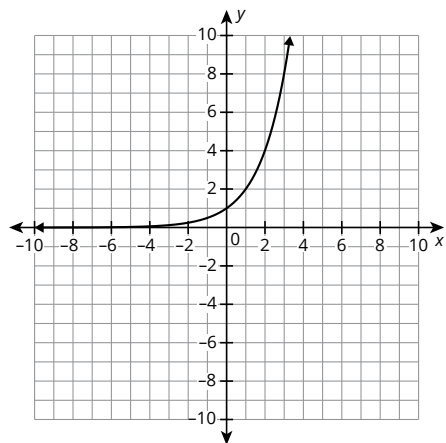
Domain:

Range:

Asymptote:

TOPIC 1 Exponential and Logarithmic Functions

5. $f(x) = 2^x$
 $c(x) = -f(x)$



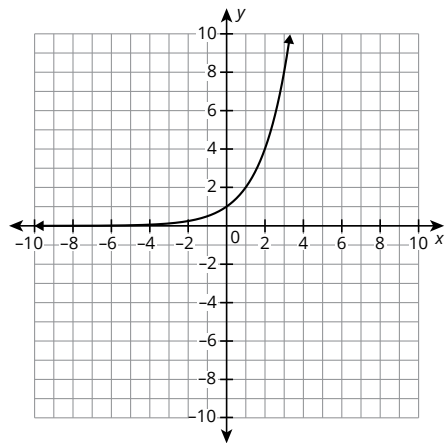
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

Domain:

Range:

Asymptote:

6. $f(x) = 2^x$
 $c(x) = f(x) + 4$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	
$(0, 1)$	
$(1, 2)$	

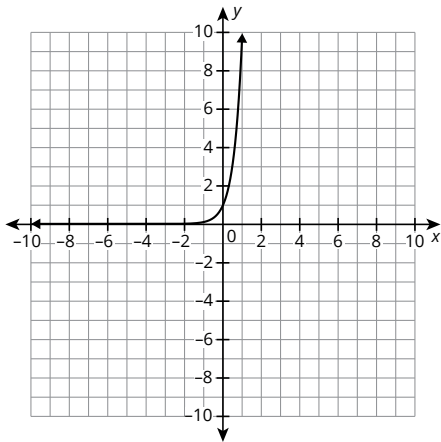
Domain:

Range:

Asymptote:

B. Complete the table to determine the corresponding points on $c(x)$, given reference points on $f(x)$. Then, graph $c(x)$ on the same coordinate plane as $f(x)$ and state the domain, range, and asymptotes of $c(x)$. Write the domain and range using interval notation.

1. $f(x) = 10^x$
 $c(x) = f(x) - 2$



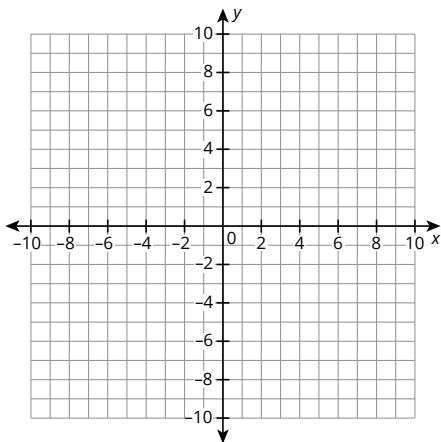
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$(1, 10)$	

Domain:

Range:

Asymptote:

2. $f(x) = 10^x$
 $c(x) = f(x + 3)$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$1, 10$	

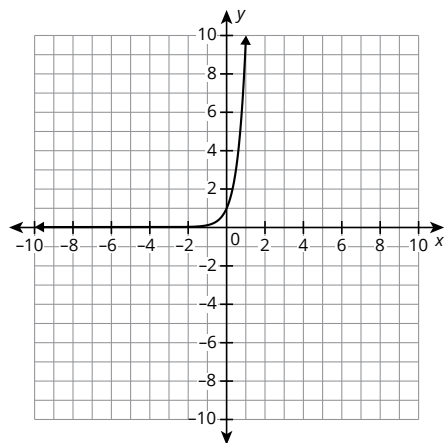
Domain:

Range:

Asymptote:

TOPIC 1 Exponential and Logarithmic Functions

3. $f(x) = 10^x$
 $c(x) = -4f(x)$



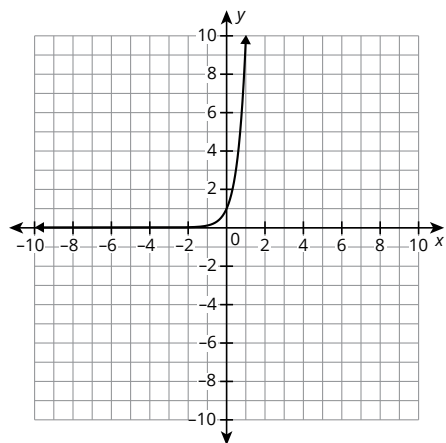
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$(1, 10)$	

Domain:

Range:

Asymptote:

4. $f(x) = 10^x$
 $c(x) = 3f(x)$



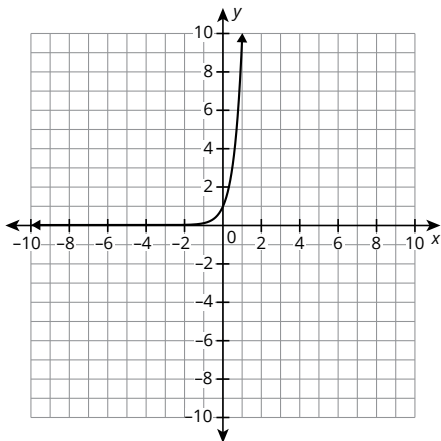
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$1, 10$	

Domain:

Range:

Asymptote:

5. $f(x) = 10^x$
 $c(x) = f(x - 3)$



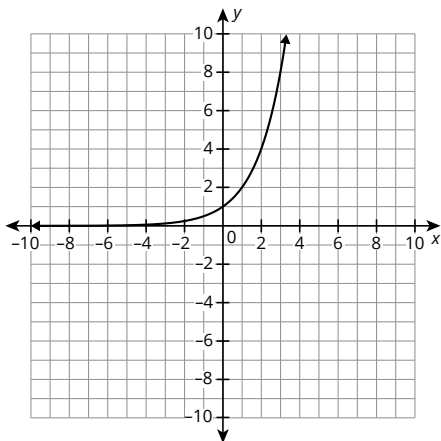
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$(1, 10)$	

Domain:

Range:

Asymptote:

6. $f(x) = 10^x$
 $c(x) = f(x) + 5$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, 0.1)$	
$(0, 1)$	
$(1, 10)$	

Domain:

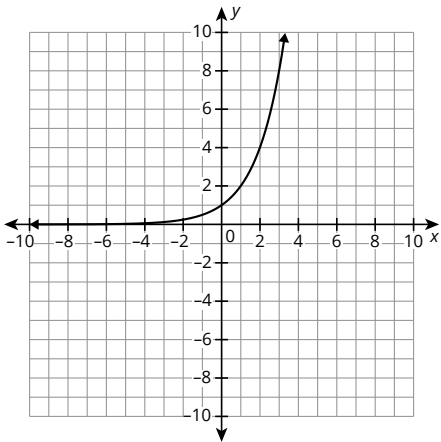
Range:

Asymptote:

TOPIC 1 Exponential and Logarithmic Functions

C. Complete the table to determine the corresponding points on $c(x)$, given reference points on $f(x)$. Then, graph $c(x)$ on the same coordinate plane as $f(x)$ and state the domain, range, and asymptotes of $c(x)$. Write the domain and range using interval notation.

1. $f(x) = e^x$
 $c(x) = f(x) - 2$



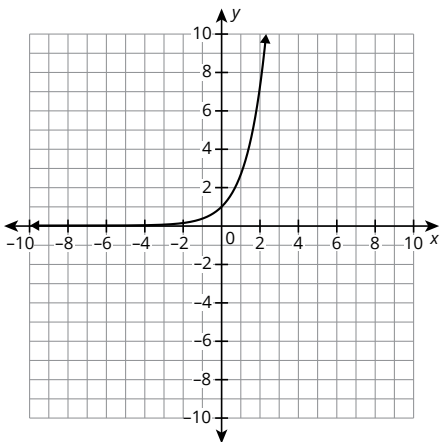
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

Domain:

Range:

Asymptote:

2. $f(x) = e^x$
 $c(x) = f(x + 3)$



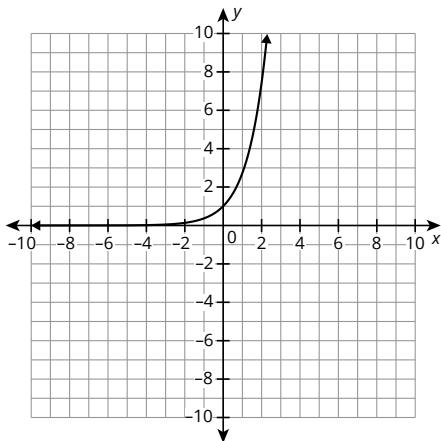
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

Domain:

Range:

Asymptote:

3. $f(x) = e^x$
 $c(x) = -4f(x)$



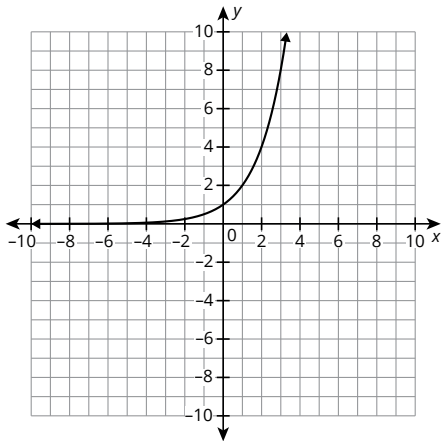
Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

Domain:

Range:

Asymptote:

4. $f(x) = e^x$
 $c(x) = 3f(x)$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

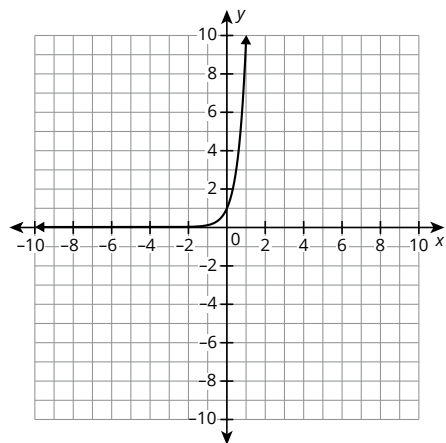
Domain:

Range:

Asymptote:

TOPIC 1 Exponential and Logarithmic Functions

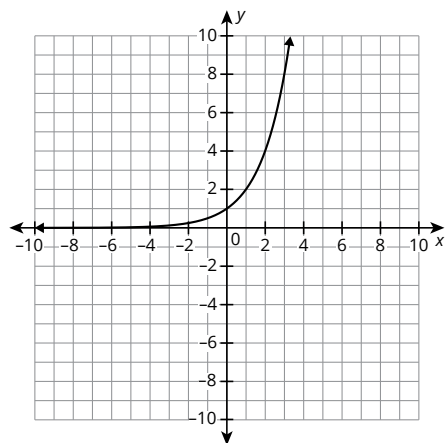
5. $f(x) = e^x$
 $c(x) = f(x - 3)$



Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

Domain:
Range:
Asymptote:

6. $f(x) = e^x$
 $c(x) = f(x) + 5$

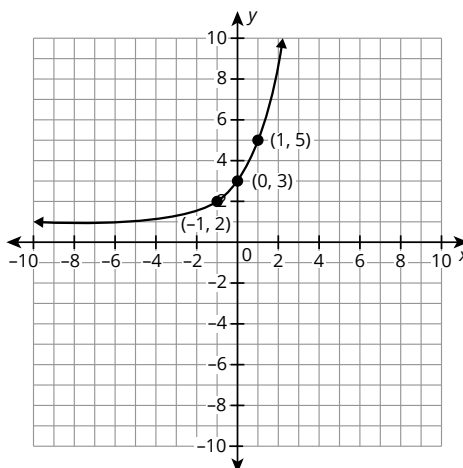
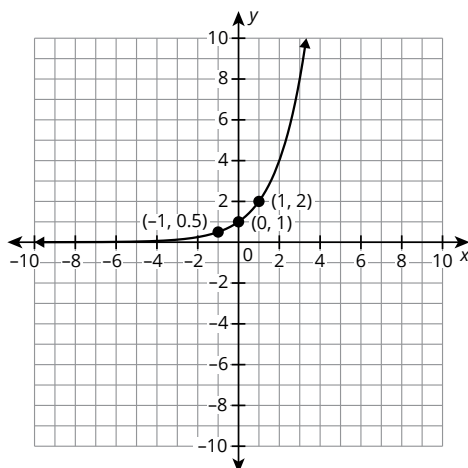


Reference Points on $f(x)$	Corresponding Points on $c(x)$
$(-1, e^{-1})$	
$(0, 1)$	
$(1, e)$	

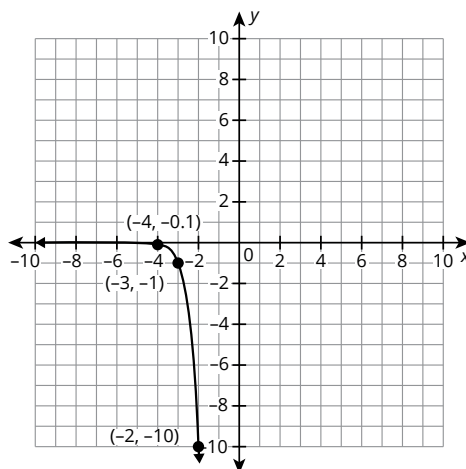
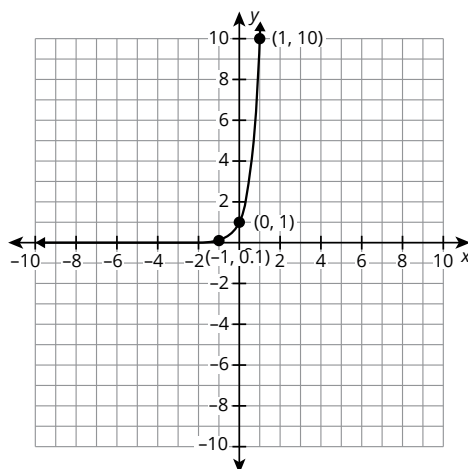
Domain:
Range:
Asymptote:

D. Describe the transformations performed on $f(x)$ to create $g(x)$. Then, write an equation for $g(x)$ in terms of $f(x)$.

1. $f(x) = 2^x$

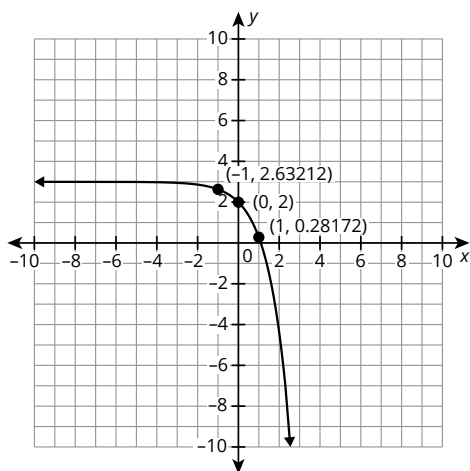
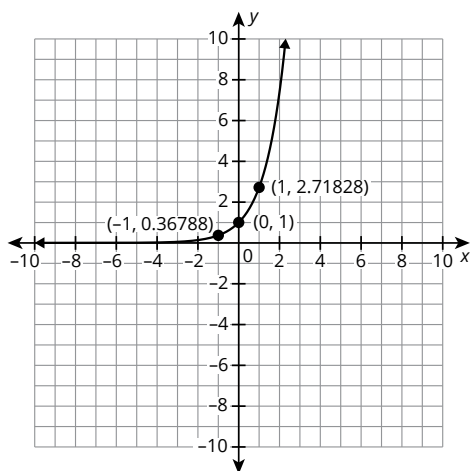


2. $f(x) = 10^x$

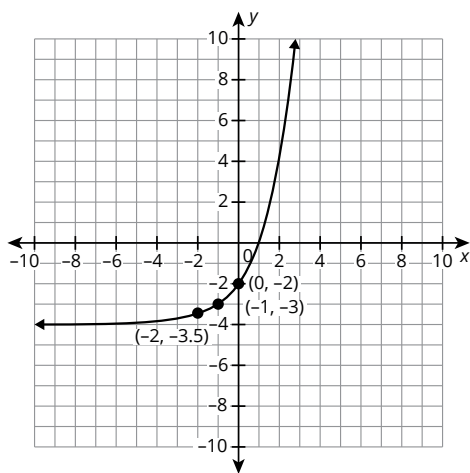
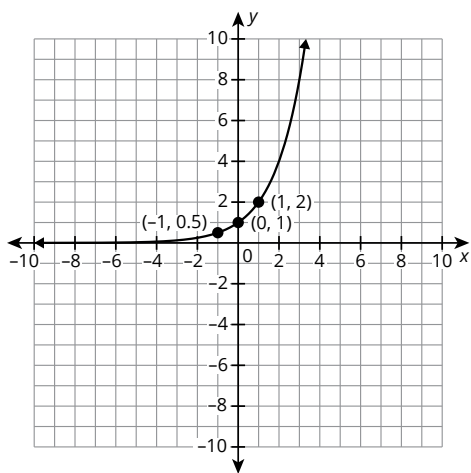


TOPIC 1 Exponential and Logarithmic Functions

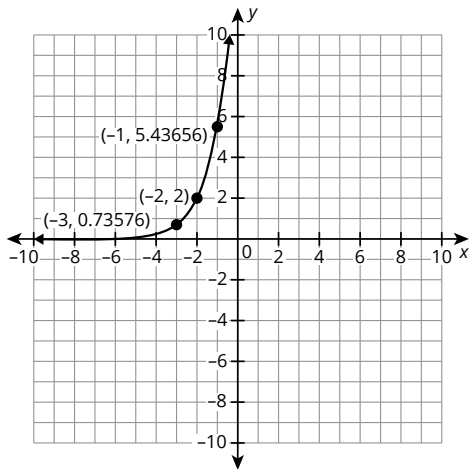
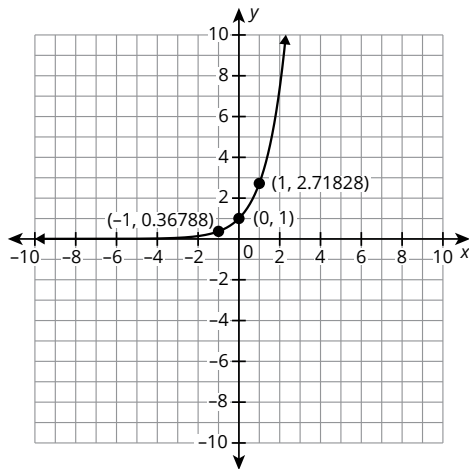
3. $f(x) = e^x$



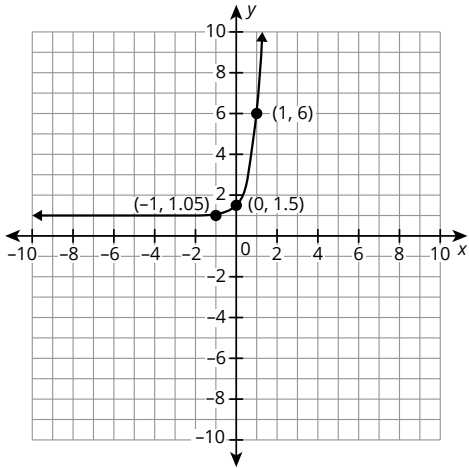
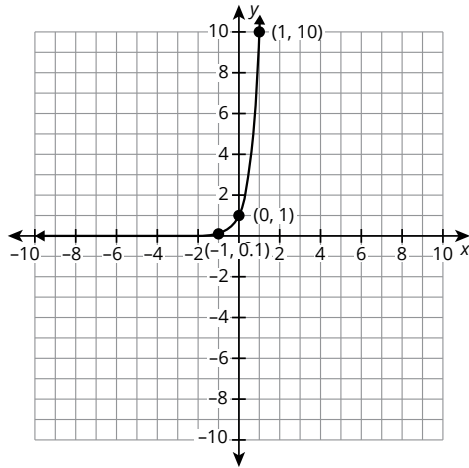
4. $f(x) = 2^x$



5. $f(x) = e^x$

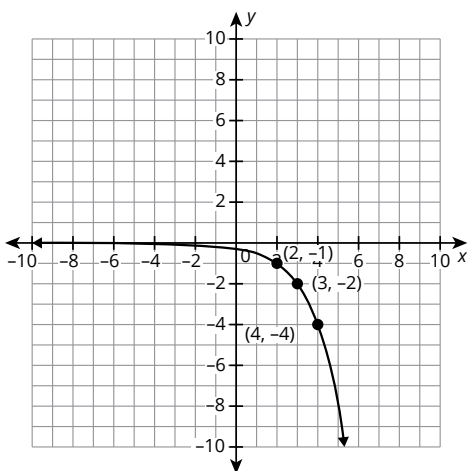
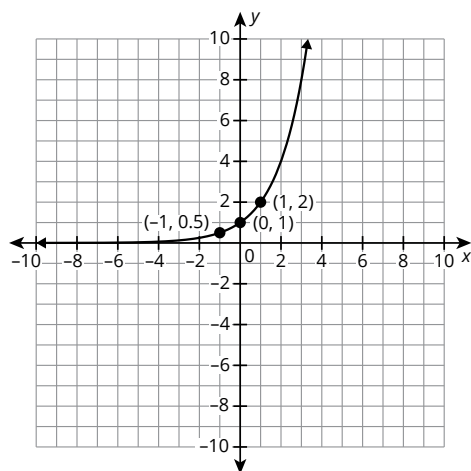


6. $f(x) = 10^x$

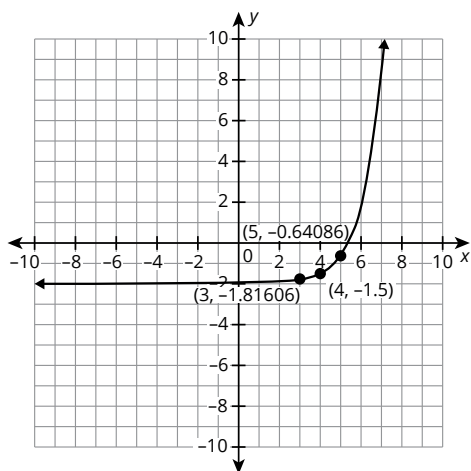
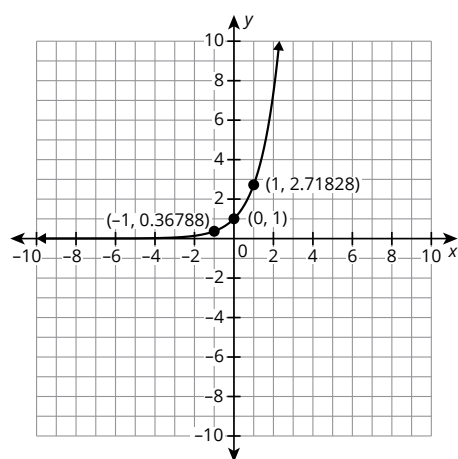


TOPIC 1 Exponential and Logarithmic Functions

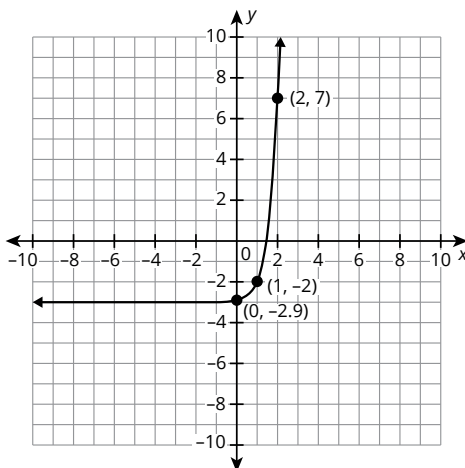
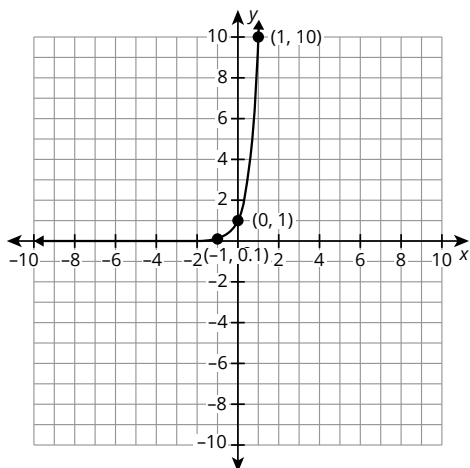
7. $f(x) = 2^x$



8. $f(x) = e^x$



9. $f(x) = 10^x$



E. Describe the transformations performed on $m(x)$ that produced $t(x)$. Then, write an exponential equation for $t(x)$.

1. $m(x) = 2^x$

$t(x) = -m(x + 1) + 3$

2. $m(x) = 10^x$

$t(x) = 3m(x - 1) - 2$

3. $m(x) = e^x$

$t(x) = \frac{1}{2}m(x - 2) + 4$

4. $m(x) = 2^x$

$t(x) = 3m(x - 4) - 1$

5. $m(x) = 10^x$
 $t(x) = -m(x + 5) - 2$

6. $m(x) = e^x$
 $t(x) = -2m(x + 8) - 3$

Extension

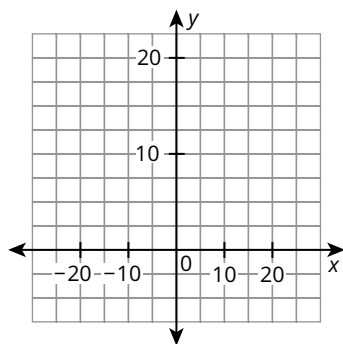
- Consider the function $f(x) = 2^x$. The table shows the corresponding point on $g(x)$ for each reference point on $f(x)$ from the transformation of $f(x)$. Determine the function $g(x)$.

Reference Point on $f(x)$	Corresponding Point on $g(x)$
$(-2, 0.25)$	$(1, 1.5)$
$(-1, 0.5)$	$(0, 1)$
$(0, 1)$	$(-1, 0)$
$(1, 2)$	$(-2, -2)$
$(2, 4)$	$(-3, -6)$

Spaced Practice

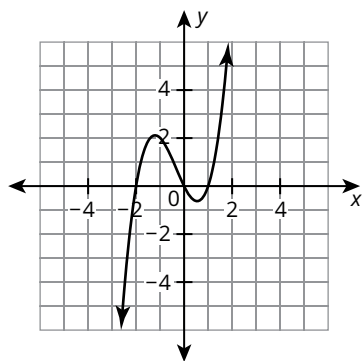
1. Consider the power function, $g(x) = x^2$.

a. Sketch the graph of $g(x)$.



b. Determine whether $g(x)$ is invertible. Explain your reasoning.

2. Determine whether the inverse of the graphed function is a function. Explain your reasoning.



TOPIC 1 Exponential and Logarithmic Functions

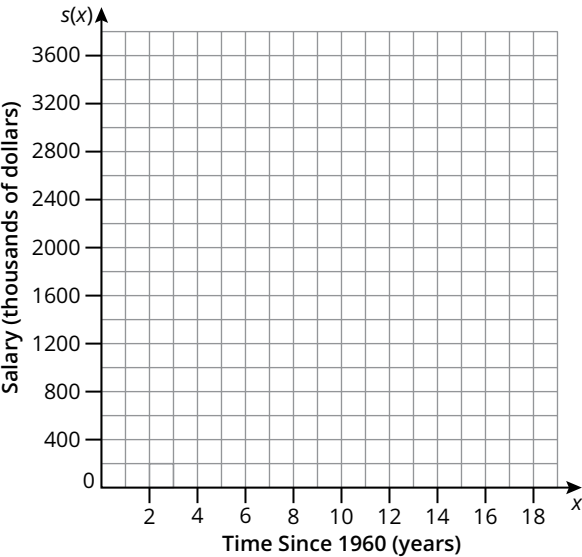
3. Consider the function $f(x) = 16x^3$.
 - a. Determine the domain and range of $f(x)$.
 - b. Write the inverse function $f^{-1}(x)$.
 - c. Determine the domain and range of $f^{-1}(x)$.
4. Algebraically determine whether $f(x) = \frac{3}{4}x + 2$ and $g(x) = \frac{4}{3}x - \frac{1}{2}$ are inverses. Show your work.

IV. Modeling With Exponential Functions

A. Graph the points and analyze the data. Then, use graphing technology to determine the exponential regression function.

1. The average salary in thousands of dollars, $s(x)$, of a company's presidents for various years since 1960 is displayed in the table.

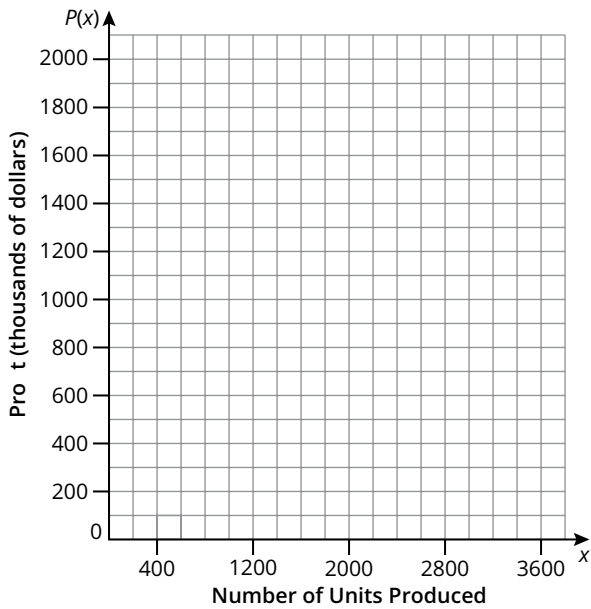
Time Since 1960 (years)	0	5	10	15	16	17	18
Average Salary (thousands of dollars)	170	325	750	1900	2000	2200	2600



TOPIC 1
 Exponential and Logarithmic Functions

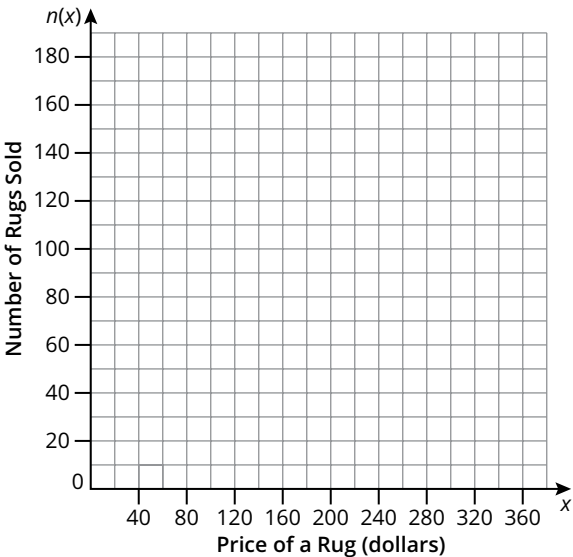
2. The number of units produced, x , of a product and the profit associated with making that many units, $P(x)$, in thousands of dollars, are shown in the table.

Number of Units Produced	1080	1350	1500	1725	1960	2400	2650	2800
Profit (thousands of dollars)	1180	1170	1280	1560	1720	1800	1940	1800



3. The table shows the number of oriental rugs a company sells, $n(x)$, and the price of each rug, x .

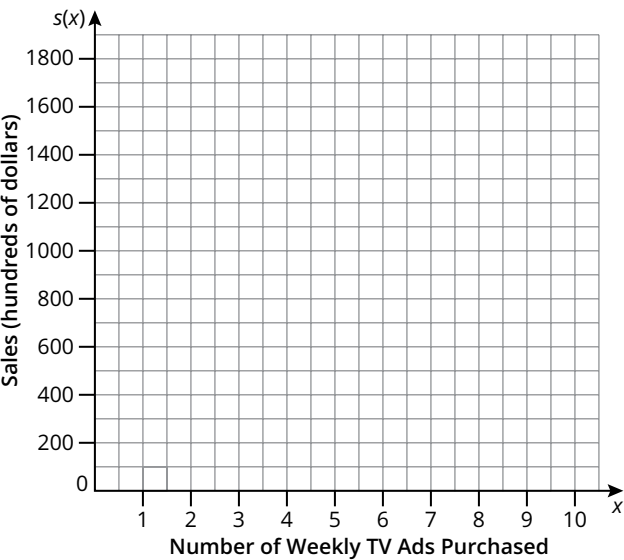
Price (dollars)	120	180	200	220	240	260	360
Number of Rugs	125	100	85	75	60	40	20



TOPIC 1
 Exponential and Logarithmic Functions

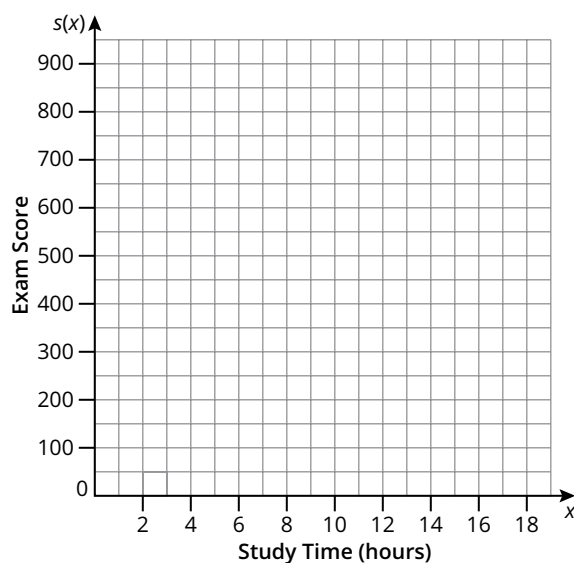
4. A company conducts a study to see if the number of weekly TV ads that they buy, x , is related to the sales of the product, $s(x)$, in hundreds of dollars. The results are recorded in the table.

Number of Weekly TV Ads Purchased	5	6	7	8	9	10
Sales (hundreds of dollars)	1000	1100	1350	1520	1700	1600



5. A college recorded the number of hours students studied for a math entrance exam, x , and their scores on the exam, $s(x)$. The results are shown in the table.

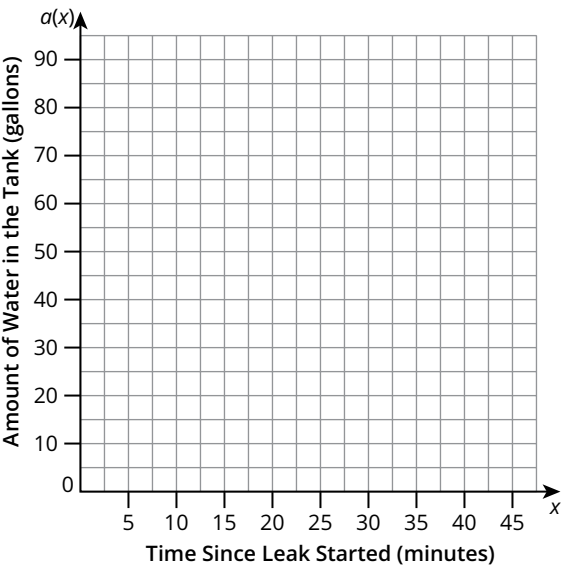
Study Time (hours)	1	3	4	9	10	12	13	14	16
Exam Score	350	400	490	580	650	600	700	730	870



TOPIC 1
 Exponential and Logarithmic Functions

6. A 75-gallon tank begins to leak and lose water. The table shows the amount of water left in the tank $a(x)$ after x minutes since the leak started.

Time Since Leak Started (minutes)	0	5	10	15	20	25	30
Amount of Water in the Tank (gallons)	75	45	27	18	13	8	7



B. Use the given tables to answer the questions.

1. A car dealership studies the average fuel economy of one of its vehicles, $f(x)$, in miles per gallon, for various speeds, x , in miles per hour. The table shows the results.

Speed (miles per hour)	20	25	30	35	40	45	50	55	60	65
Fuel Economy (miles per gallon)	31.6	30.6	29.1	29.2	30.1	29.8	29.1	29.3	28.5	27.5

A new customer wants to average 28.5 miles per gallon. Write an exponential regression function to model the data and use the function to predict the fuel economy for a speed of 40 miles per hour.

TOPIC 1
 Exponential and Logarithmic Functions

2. A small business conducts a study to see if the amount of money spent on monthly advertising, x , in hundreds of dollars, is related to the amount of monthly sales $s(x)$, in hundreds of dollars. The table shows the results.

Monthly Advertising Expenditures (hundreds of dollars)	5.6	5.8	5.9	6.1	6.2	6.3	6.4	6.5	6.6
Monthly Sales (hundreds of dollars)	105	110	114	113	117	116	121	120	125

The company wants to have sales of \$15,000. Write an exponential regression function to model the data and use the function to predict the amount of money the company will have to spend for advertising each month in order to generate \$15,000 in sales.



3. The amount of medicine in the body, $m(x)$, in milligrams/milliliter, x minutes after taking it, is shown in the table.

Time Since Taking Medicine (minutes)	0	1	4	7	10	13	16	19
Amount of Medicine (milligrams/milliliter)	10.5	9.8	8.4	7.3	6.3	5.5	4.8	4.6

Write an exponential regression function to model the data, and use the function to predict the number of minutes that the medicine has been in the system if there is 7.5 milligrams/milliliter left.

4. A hospital notices that flu cases are on the rise and begins to track the number of flu patients. The table shows the number of flu cases, $f(x)$, and the time in days since the hospital started tracking the data, x .

Time (days)	0	2	4	6	8	10
Number of Flu Cases	1500	1610	1650	1590	1710	1720

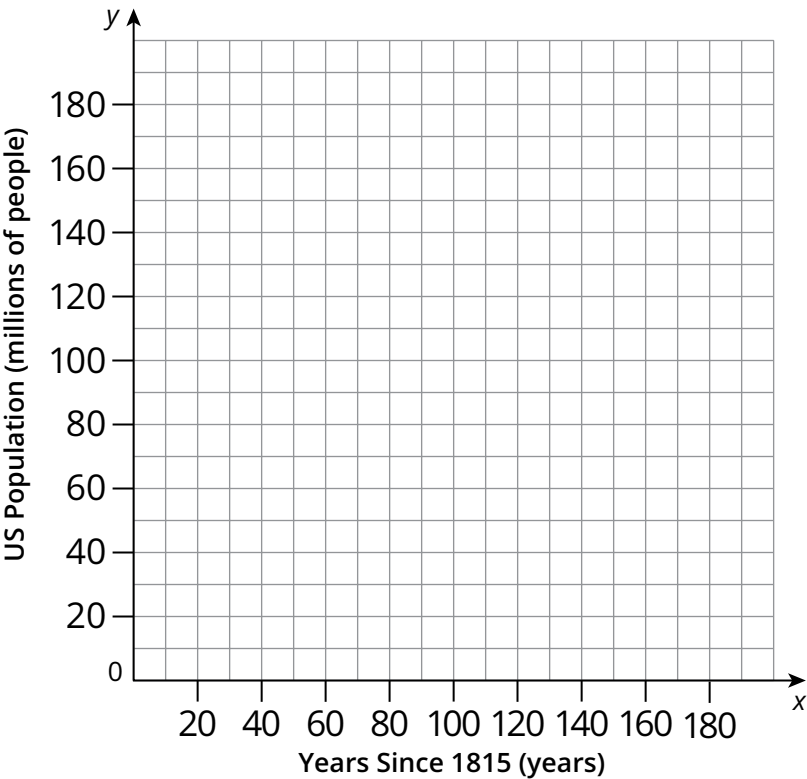
Write an exponential regression function to model the data, and use the function to predict the number of flu cases the hospital can expect 12 days after it started tracking.

TOPIC 1
 Exponential and Logarithmic Functions

C. The U.S. population from 1815 through 1975 is listed in the table of values in 10 year intervals.

Year	US Population (millions of people)	Year	US Population (millions of people)
1815	8.3	1905	83.2
1825	11	1915	98.8
1835	14.7	1925	114.2
1845	19.7	1935	127.1
1855	26.7	1945	140.1
1865	35.2	1955	164
1875	44.4	1965	190.9
1885	55.9	1975	214.3
1895	68.9		

1. Create a scatterplot of the data on the coordinate plane.



2. Use technology to determine a linear regression function and the coefficient of determination, r^2 .
3. Use technology to determine an exponential regression function and the coefficient of determination, r^2 .
4. Compare the fit of each regression function. Which function is the best fit?

TOPIC 1 Exponential and Logarithmic Functions

5. Use the best-fit function to predict the US population in the year 2000.
6. Use the other regression function to predict the US population in the year 2000. How do these predictions compare to the prediction from the best-fit function?
7. The actual US population in 2000 was approximately 282.2 million people. Which regression function was the most accurate? What does this suggest about the function that best fits the data?

Extension

1. Analyze the scatterplot shown.
 - a. Determine the function that best models the graph.
 - b. Plot the point $(8, 6)$ on the graph. Does your answer to part (a) change? Why or why not?
 - c. If you were doing research and a situation arose in which a data point that gets added to the graph changes the model, what is one thing you might do to investigate further?

TOPIC 1 Exponential and Logarithmic Functions

Spaced Practice

1. A home recently experienced an infestation of insects. The insect population over time is shown in the table. Write the function that represents the insect population over time.

Insect Population	
Day	Number of Insects
1	240
2	360
3	540
4	810

2. The time t (in hours) it takes to travel a fixed distance varies inversely with speed s (in miles per hour). Suppose a driver takes 6 hours to travel 240 miles at a speed of 40 mph.
 - a. Determine the value of k .
 - b. Formulate an equation to represent the real-world situation.
 - c. Suppose the driver increases the speed to 60 mph. How long will the trip take?

V. Logarithmic Functions

A. Write each exponential equation as a corresponding logarithmic equation.

1. $3^2 = 9$

3. $4^{-3} = \frac{1}{64}$

5. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

2. $5^4 = 625$

4. $10^{-5} = \frac{1}{100,000}$

6. $\left(\frac{1}{11}\right)^{-2} = 121$

B. Write each logarithmic equation as a corresponding exponential equation.

1. $\log_7\left(\frac{1}{49}\right) = -2$

3. $\log_2(128) = 7$

5. $\log_{\frac{1}{2}}\left(\frac{1}{125}\right) = 3$

2. $\log_{\frac{1}{3}}\left(\frac{1}{729}\right) = 6$

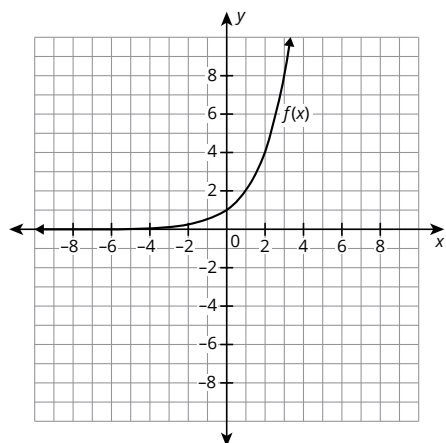
4. $\log_6\left(\frac{1}{1296}\right) = -4$

6. $\log_9(729) = 3$

TOPIC 1 Exponential and Logarithmic Functions

- C. Graph the inverse of each exponential function $f(x)$. Then, describe the domain, range, and asymptotes of the inverse.

1. $f(x) = 2^x$

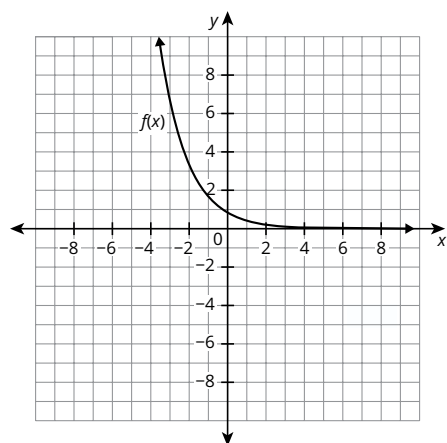


Domain:

Range:

Asymptote:

3. $f(x) = \frac{1}{2}^x$

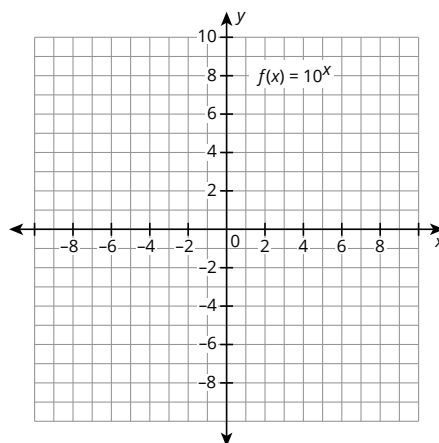


Domain:

Range:

Asymptote:

2. $f(x) = 10^x$

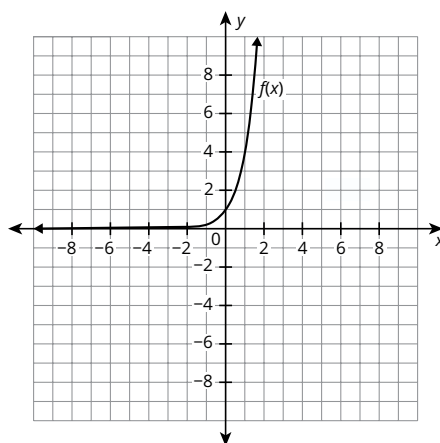


Domain:

Range:

Asymptote:

4. $f(x) = 4^x$

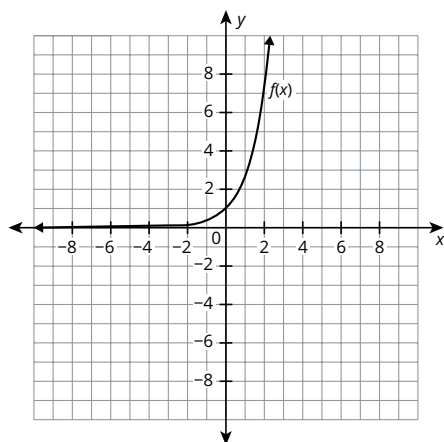


Domain:

Range:

Asymptote:

5. $f(x) = e^x$

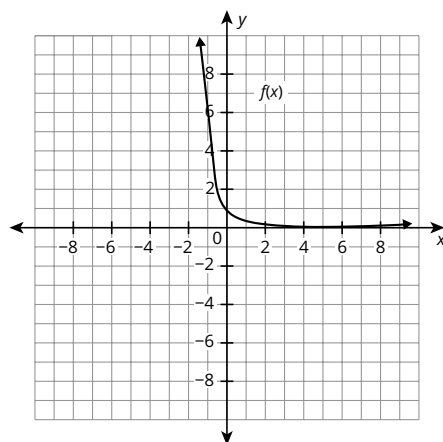


Domain:

Range:

Asymptote:

6. $f(x) = \frac{1}{4}^x$



Domain:

Range:

Asymptote:

7. Analyze the domain and range of each function and its inverse. Describe the relationship between the restricted domain and range of the function and the domain and range of the inverse function.

Extension

Evaluate each logarithm.

1. $\log_{49} 7 + 2 \log_8 64$
2. $\log_{100} 10 + \log_{81} 3$

TOPIC 1 Exponential and Logarithmic Functions

Spaced Practice

1. In the year 2012, Graceville had a population of 1.4 million people and an annual growth rate of 1.35%.
 - a. Write a function in the form $N(t) = N_0 e^{rt}$ to model Graceville's population with respect to t , the number of years since 2012.
 - b. Use your model to predict Graceville's population in the year 2025.
 - c. Use your model to estimate Graceville's population in the year 1989.
2. Multiply $8\sqrt[3]{a^2}(-2\sqrt{a} + 5\sqrt[3]{a^4})$, given $a \geq 0$. Extract all roots and write your answer in radical form.

VI. Transformations of Logarithmic Functions

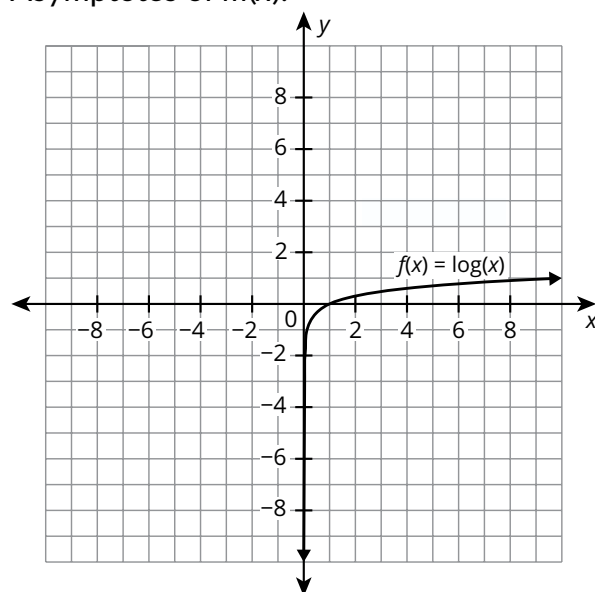
- A. The graph of $f(x) = \log(x)$ is shown. Use the graph of $f(x)$ to sketch the transformed function $m(x)$ on the coordinate plane. Then, state the domain, range, and asymptotes of $m(x)$. Use set notation to write the domain and range.

1. $m(x) = f(x) + 2$

Domain of $m(x)$:

Range of $m(x)$:

Asymptotes of $m(x)$:

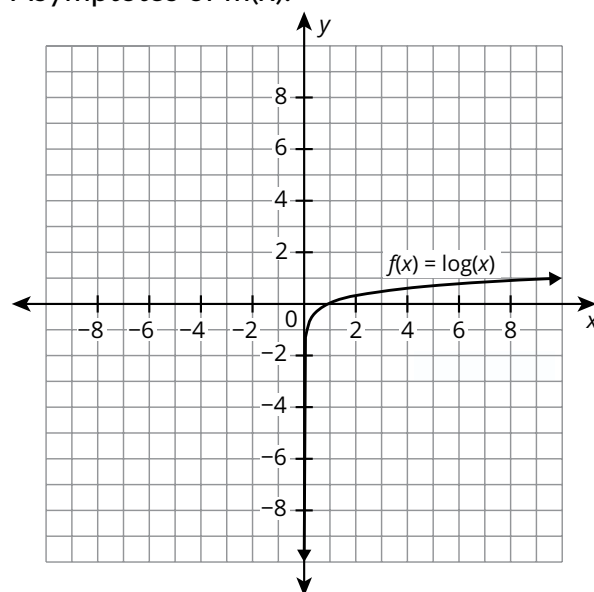


2. $m(x) = f(x - 4)$

Domain of $m(x)$:

Range of $m(x)$:

Asymptotes of $m(x)$:



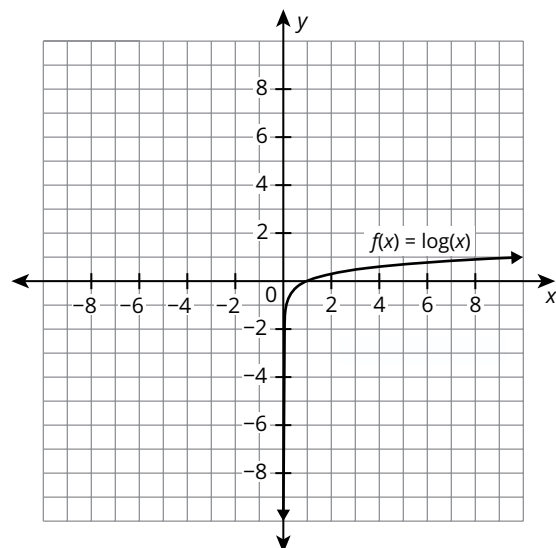
TOPIC 1 Exponential and Logarithmic Functions

3. $m(x) = f(x + 1) - 3$

Domain of $m(x)$:

Range of $m(x)$:

Asymptotes of $m(x)$:

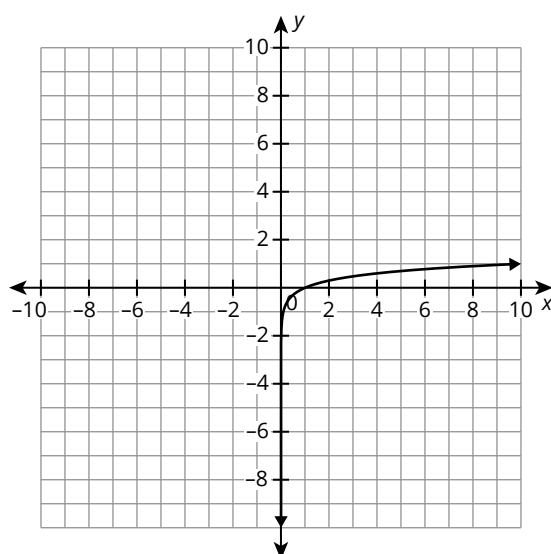


4. $m(x) = -f(x - 3)$

Domain of $m(x)$:

Range of $m(x)$:

Asymptotes of $m(x)$:

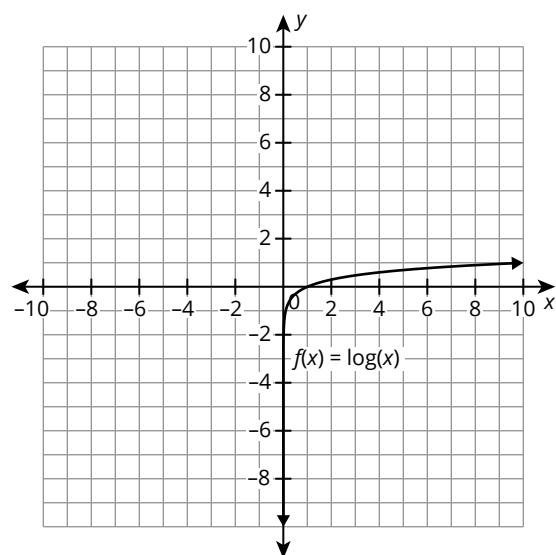


5. $m(x) = .5f(x) + 1$

Domain of $m(x)$:

Range of $m(x)$:

Asymptotes of $m(x)$:

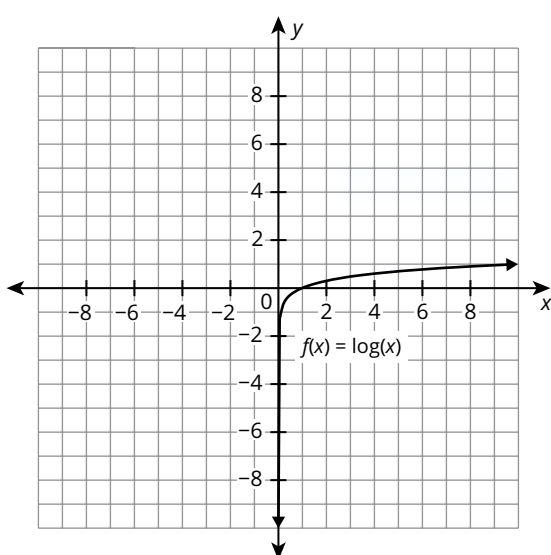


6. $m(x) = -f(x - 2) + 2$

Domain of $m(x)$:

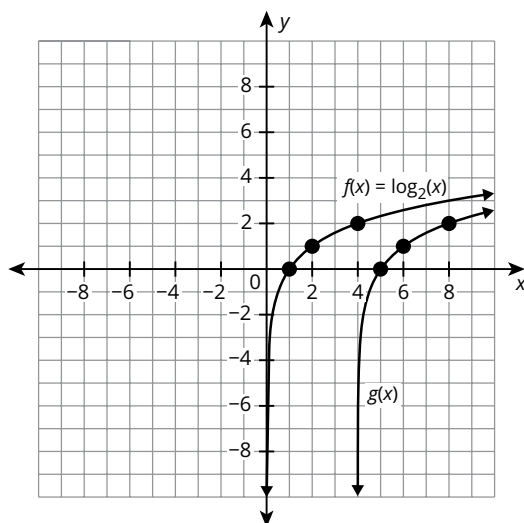
Range of $m(x)$:

Asymptotes of $m(x)$:

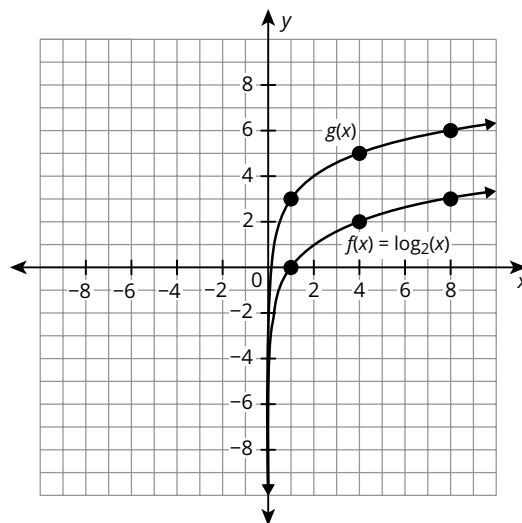


B. Analyze the graphs of $f(x)$ and $g(x)$. Describe the transformations performed on the graph of $f(x)$ to produce the graph of the transformed function $g(x)$. Then, write an equation for $g(x)$.

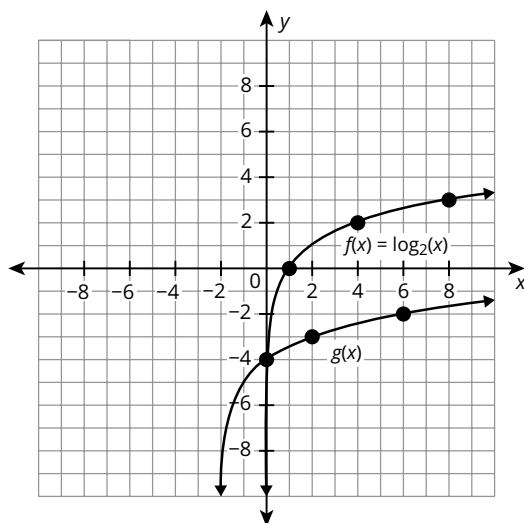
1.



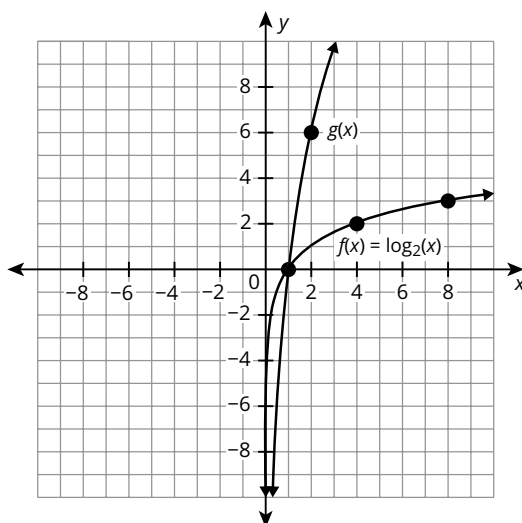
2.



3.

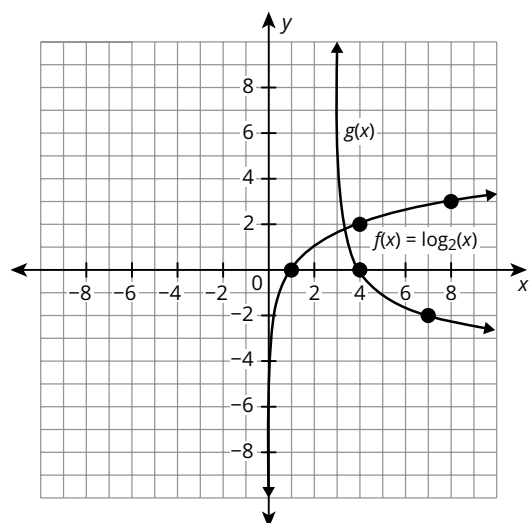


4.

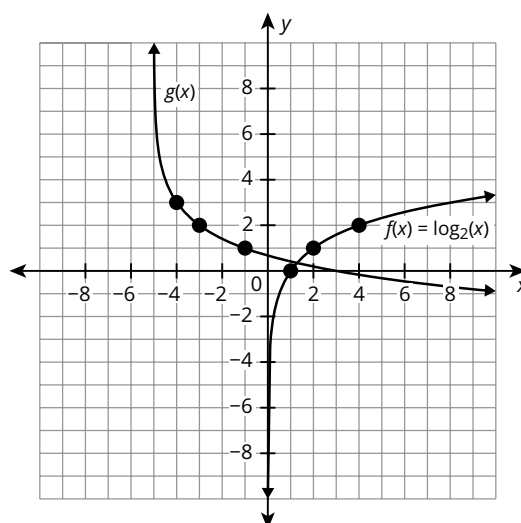


TOPIC 1 Exponential and Logarithmic Functions

5.

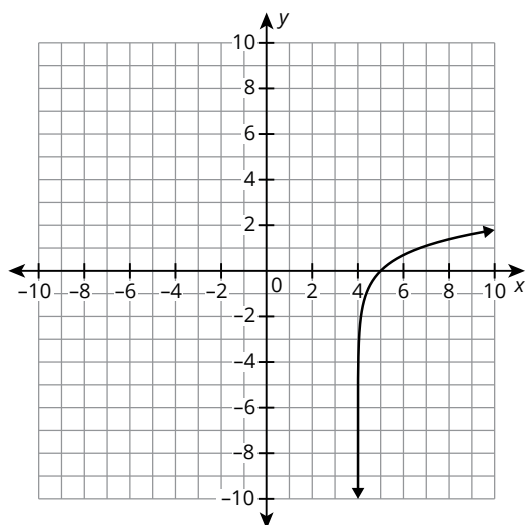


6.

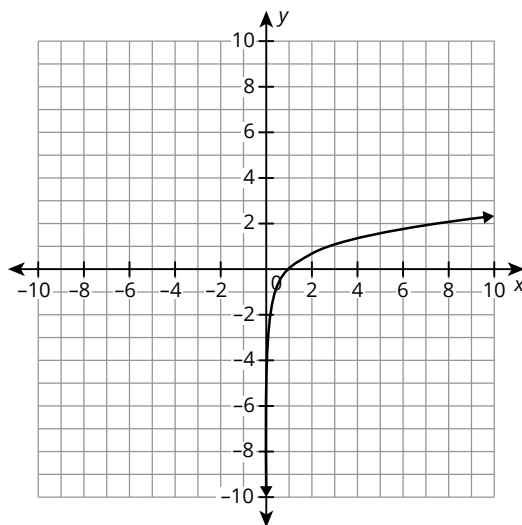


C. Analyze the graphs of $f(x)$ and $g(x)$. Describe the transformations performed on the graph of $f(x)$ to produce the graph of the transformed function $g(x)$. Then, write an equation for $g(x)$.

1.

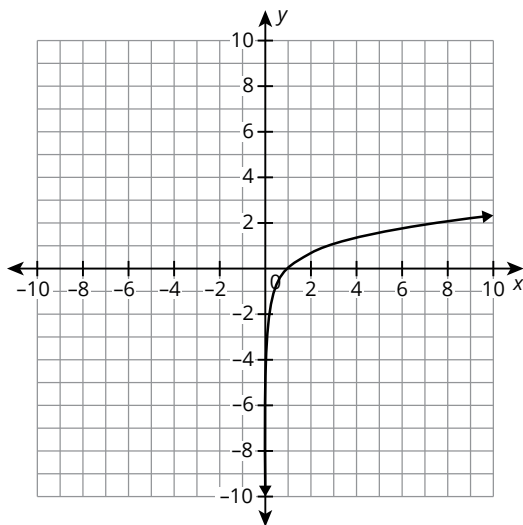


2.

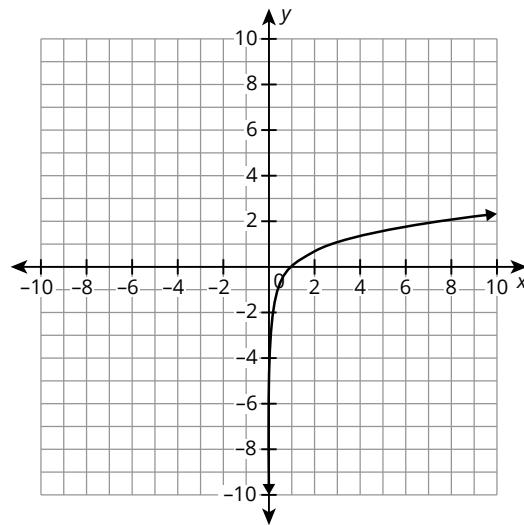


TOPIC 1 Exponential and Logarithmic Functions

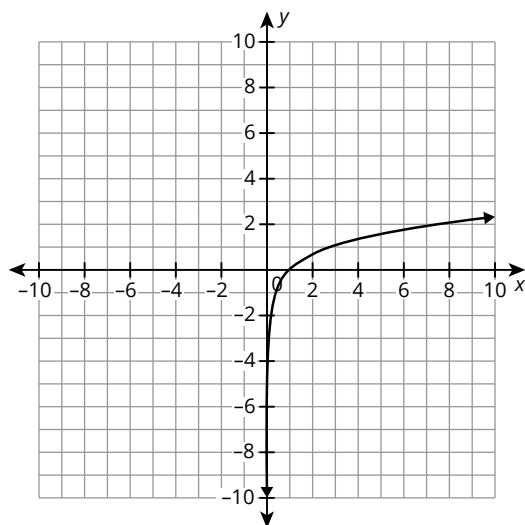
3.



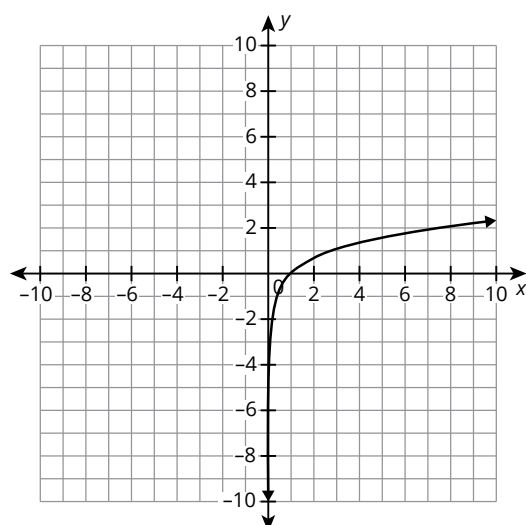
4.



5.



6.



TOPIC 1 Exponential and Logarithmic Functions

D. Write a transformed logarithmic function, $c(x)$, in terms of $f(x) = \log_2(x)$ with the characteristics given.

1. vertical asymptote at $x = 6$

2. x -intercept at $(-3, 0)$

3.

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $c(x)$
$(\frac{1}{2}, -1)$	$\rightarrow$	$(\frac{1}{2}, -3)$
$(1, 0)$	$\rightarrow$	$(1, 0)$
$(2, 1)$	$\rightarrow$	$(2, 3)$

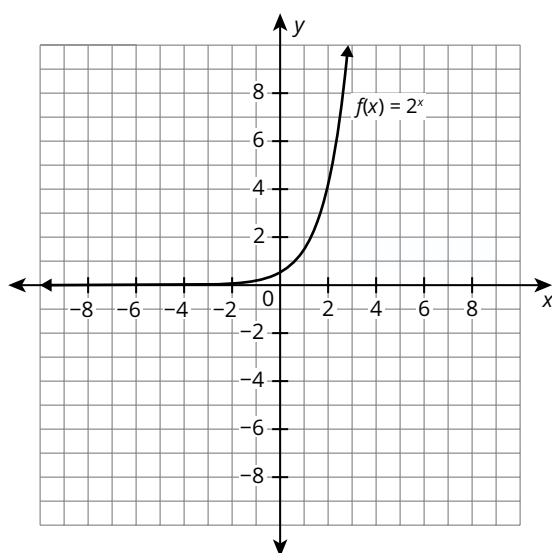
4. vertical asymptote at $x = -2$

5. domain: $(4, \infty)$
 point: $(5, -4)$

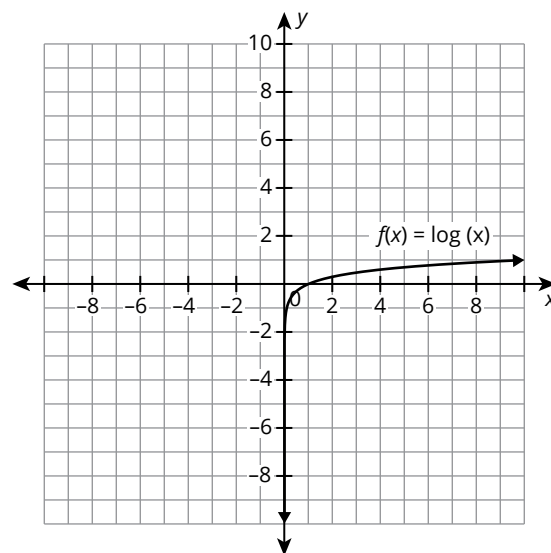
6.

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $c(x)$
$(\frac{1}{2}, -1)$	$\rightarrow$	$(\frac{1}{2}, 3)$
$(1, 0)$	$\rightarrow$	$(1, 2)$
$(2, 1)$	$\rightarrow$	$(2, 1)$

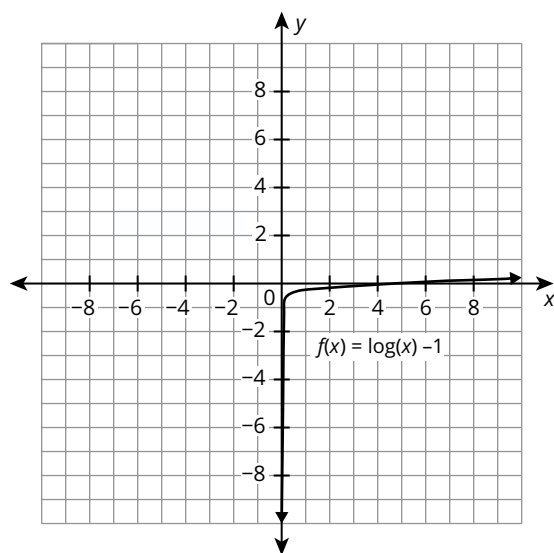
1.



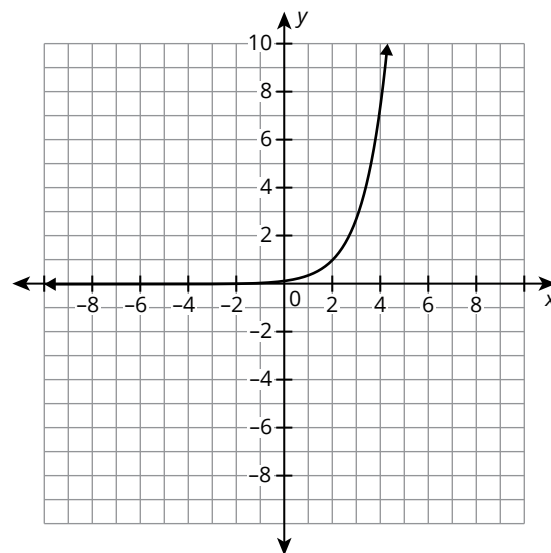
2.



3.

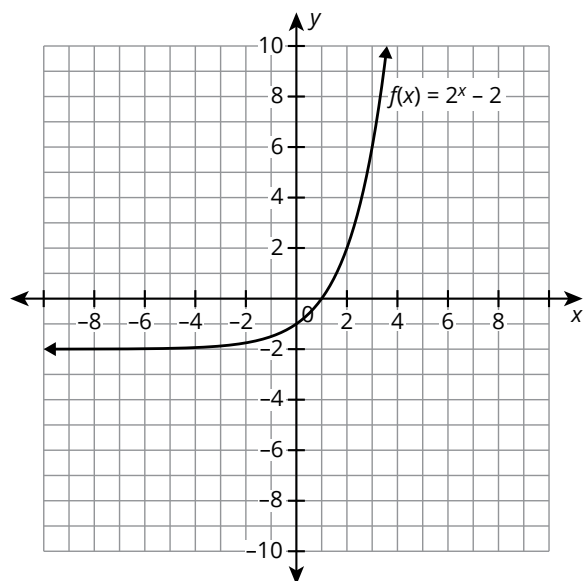


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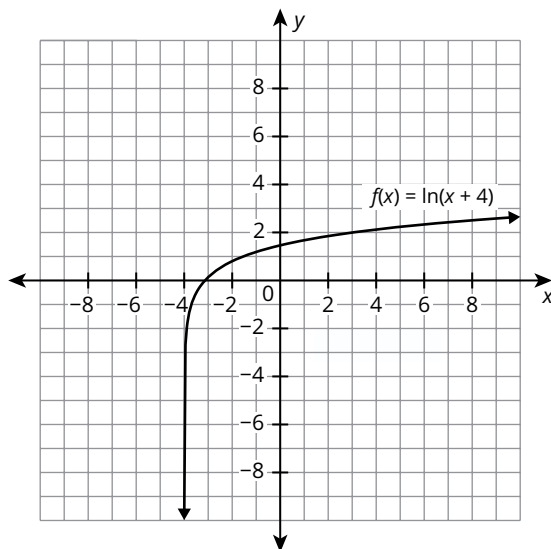


TOPIC 1 Exponential and Logarithmic Functions

5.



6.



F. Consider the function $y = f(x)$ and the transformed function $g(x)$.
Write an equation for $g^{-1}(x)$ in terms of $f^{-1}(x)$.

1. $g(x) = f(x) + 3$

2. $g(x) = f(x - 2)$

3. $g(x) = f(x + 6)$

4. $g(x) = f(x) + 5$

5. $g(x) = f(x + 1) - 3$

6. $g(x) = f(x - 4) + 2$

Extension

1. Consider the function $f(x) = 2^x$ and its inverse function $f^{-1}(x) = \log_2(x)$. Complete a table of coordinate points for each transformation using the reference points $(-1, \frac{1}{2})$, $(0, 1)$, and $(1, 2)$ on $f(x)$. Write $h^{-1}(x)$ in terms of $f^{-1}(x)$. Then, identify the transformation on $f(x)$ and the effect on the inverse.

a.

$h(x) = 2f(x)$	$h^{-1}(x) =$

Transformation on $f(x)$:

Effect on the inverse:

b.

$h(x) = \frac{1}{4}f(x)$	$h^{-1}(x) =$

Transformation on $f(x)$:

Effect on the inverse:

TOPIC 1 Exponential and Logarithmic Functions

Spaced Practice

1. Formulate a square root equation given a table of values.

x	-2	-1	2	7	14
y	0	1	2	3	4

2. Formulate a quadratic equation given a table of values.

x	0	1	2	3	4
y	-7	-6	-3	2	9

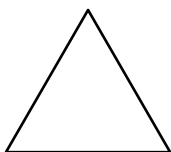
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I. Logarithmic Expressions

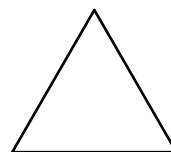
Topic Practice

A. Arrange the given terms using a Triangle of Power to create a true exponential equation and a true logarithmic equation.

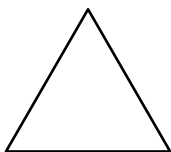
1. 25, 5, 2



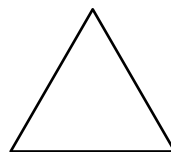
2. 3, 11, 1331



3. 2, 256, 8

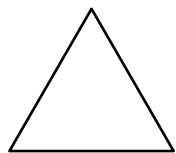


4. 1296, 4, 6

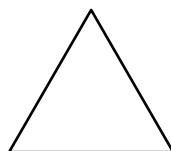


TOPIC 2 Exponential and Logarithmic Equations

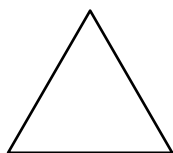
5. $-3, \frac{1}{512}, 8$



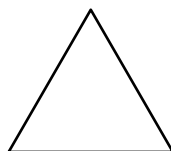
6. $\frac{1}{2}, 4, 16$



7. $17, \frac{1}{289}, -2$



8. $\frac{1}{3}, 9, 729$



B. Arrange the given terms to create a true exponential equation and a true logarithmic equation.

1. $64, 8, 2$

2. $3, 12, 1728$

3. $2, 128, 7$

4. $2401, 4, 7$

5. $-3, \frac{1}{216}, 6$

6. $\frac{1}{2}, 5, 25$

7. $16, \frac{1}{256}, -2$

8. $\frac{1}{3}, 11, 1331$

C. Rewrite each logarithmic equation as an exponential equation and each exponential equation as a logarithmic equation.

1. $8^3 = 512$

2. $\log_3 243 = 5$

3. $2^6 = 64$

4. $\log_{16} 4 = \frac{1}{2}$

5. $5^{-3} = \frac{1}{125}$

6. $\log_{\frac{1}{7}} \frac{1}{49} = 2$

7. $10^{-1} = 0.1$

8. $\log_{0.6} 0.216 = 3$

TOPIC 2 Exponential and Logarithmic Equations

D. Estimate the logarithm to the tenths place. Explain how you determined your answer.

1. $\log_2 48$

2. $\log_3 2$

3. $\log_8 \left(\frac{1}{495} \right)$

4. $\log_6 53$

5. $\log_9 6000$

6. $\log_7 \left(\frac{1}{10} \right)$

7. $\log_{\frac{1}{2}} \left(\frac{1}{24} \right)$

8. $\log_{\frac{4}{3}} 0.85$

E. Determine the appropriate base of each logarithm. Explain your reasoning.

1. $\log_b 29 = 2.1$

2. $\log_b 35 = 1.9$

TOPIC 2 Exponential and Logarithmic Equations

3. $\log_b\left(\frac{1}{15}\right) = -1.9$

4. $\log_b 80 = 3.9$

5. $\log_b 6 = 0.9$

6. $\log_b 66 = 3.1$

Extension

Solve the equation $4 = \log_2(x^2 - 6x)$.

Spaced Practice

1. Given: $f(x) = 2^x$ and $g(x) = -\frac{1}{2}f(x - 4)$.
 - a. Describe the transformation of $f(x)$ that produces $g(x)$.
 - b. Write $g(x)$ as an exponential function.

2. Consider $s(x) = 10^{x-1}$, which is a transformation of the function $f(x) = 10^x$.
 - a. Describe the transformation(s) of $f(x)$ to produce $s(x)$.
 - b. Write the equations of the inverse functions $f^{-1}(x)$ and $s^{-1}(x)$.
 - c. Describe the transformation(s) on the graph of $f^{-1}(x)$ to produce $s^{-1}(x)$.

TOPIC 2 Exponential and Logarithmic Equations

3. Consider the function $h(x)$, which is formed by translating the function $g(x) = \log_2 x$ right 2 units and down 1 unit.
- Write $h(x)$ in terms of $g(x)$.
 - Complete the table by determining the corresponding point on $h(x)$ for each reference point on $g(x)$.

Reference Point on $g(x)$	Corresponding Point on $h(x)$
$(\frac{1}{2}, -1)$	
$(1, 0)$	
$(2, 1)$	
$(4, 2)$	

- Write $h(x)$ as a logarithmic function.
 - List the domain, range, and any asymptotes of the logarithmic function $h(x)$.
4. Determine the quadratic equation that goes through the points $(-1, 10)$, $(2, 4)$, and $(3, -6)$.

II. Solving Exponential Equations

Topic Practice

A. Solve each exponential equation.

1. $6^x = 6^4$

3. $4096 = 4^x$

5. $5 \cdot 6^x = 1080$

7. $-36 \cdot 9^x = -\frac{4}{9}$

2. $x^6 = \left(\frac{2}{3}\right)^6$

4. $8^x = \frac{1}{4096}$

6. $40 = 5 \cdot \left(\frac{1}{2}\right)^x$

8. $19 \cdot 27^x = 57$

B. Solve each exponential equation.

1. $2^{2x-1} = 8^{3x}$

2. $2^{x-7} - 15 = 1$

TOPIC 2 Exponential and Logarithmic Equations

3. $3 = 9^{-x+5}$

4. $16^{-3x-2} = 32^{x+1}$

5. $64 = 8^{3x-4}$

6. $\frac{1}{9} = 3^{5x+3}$

7. $\frac{1}{32}^{4x+3} = \frac{1}{8}^{-8}$

8. $36^{3x-4} = 216^{2x-3}$

C. Use a calculator to solve each exponential equation.

1. $6^{x-5} = 24$

2. $12^{x+4} = 65$

3. $7^{3x} = 15$

5. $4^{x+3} - 7 = 32$

7. $4\left(\frac{2}{3}\right)^{3x} = 248$

9. $2(5)^{2x+1} + 4 = 18$

4. $8^{5x} = 71$

6. $11^{x-4} + 8 = 59$

8. $9\left(\frac{3}{5}\right)^{2x} = 999$

10. $-8(2)^{x-9} - 5 = -77$

TOPIC 2 Exponential and Logarithmic Equations

D. Solve each exponential equation. Explain why you chose the method that you used.

1. $11^{x-4} = 343$

2. $5^{2x+1} = 3125$

3. $16\left(\frac{8}{9}\right)^{5x} = 752$

4. $23^{5x} = 735$

5. $\left(\frac{7}{12}\right)^{4x} = 49$

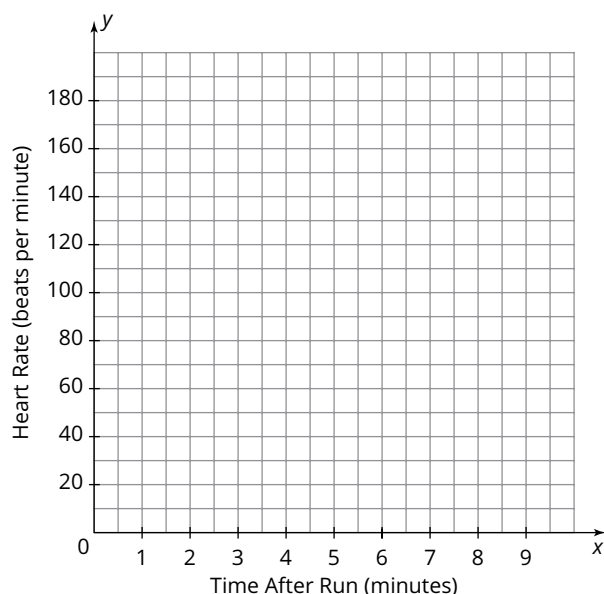
6. $14^{5x-12} - 9 = 2735$

TOPIC 2 Exponential and Logarithmic Equations

Extension

Alyssa has been training for a marathon. Immediately after her morning run, her heart rate is 180 beats per minute. One minute later, her heart rate has fallen to 147 beats per minute, and 2 minutes after her run, it is down to 124 beats per minute. Alyssa's resting heart rate is 70 beats per minute.

1. Determine the type of function that would best model the given data. Explain your reasoning.
2. Write the function $R(t)$ to model Alyssa's heart rate t minutes after her morning run.
3. Graph the function $R(t)$.



4. Use the function $R(t)$ to predict Alyssa's heart rate 5 minutes after her morning run.

Spaced Practice

1. Rewrite the exponential equations as logarithmic equations and the logarithmic equations as exponential equations.

a. $\log 10,000 = 4$

b. $\log_9 \left(\frac{1}{81} \right) = -2$

c. $\left(\frac{1}{2} \right)^{-6} = 64$

d. $9^{\frac{3}{2}} = 27$

2. Estimate $\log_4 5$ to the nearest tenth. Explain your reasoning

TOPIC 2 Exponential and Logarithmic Equations

3. The Richter scale is used to rate the magnitude of an earthquake, or the amount of energy released. An earthquake's magnitude, M , is determined using the equation, $M = \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the earthquake being measured (measured by the amplitude of a seismograph reading taken 100 km from the epicenter of the earthquake), and I_0 is the intensity of a standard earthquake, or "threshold quake," whose seismograph amplitude is 10^{-4} cm.
- a. An earthquake in San Francisco measured 7.9 on the Richter scale, while an earthquake in Chile measured 8.8. How many times more intense was the Chilean earthquake?
- b. An earthquake in Mexico City measured 8.0 on the Richter scale. An earthquake was recorded in Haiti that was three times stronger. What was the magnitude of the Haitian earthquake?
- c. Do your answers to part a and b seem reasonable?

III. Solving Logarithmic Equations

Topic Practice

A. Solve for the unknown.

1. $\log_7 343 = n$

2. $\log_{\frac{1}{4}} 64 = n$

3. $\log_n 1024 = 5$

4. $\log_n \left(\frac{1}{625} \right) = -4$

5. $\log_n (\sqrt[4]{8}) = \frac{3}{4}$

6. $\log n = 6$

7. $\log_4 16 = n$

8. $\log_{81} \left(\frac{1}{9} \right) = n$

B. Solve each logarithmic equation.

1. $\log_9 (x) = 4$

2. $\log_5 (x) = 4.2$

TOPIC 2 Exponential and Logarithmic Equations

3. $\log_5 125 = x$

5. $\log_x (\sqrt[5]{16}) = \frac{4}{5}$

7. $\log 1000 = x$

4. $\log_{\frac{1}{2}} 64 = x$

6. $\log_x \left(\frac{1}{3125} \right) = 5$

8. $\log 0.01 = x$

C. Solve each logarithmic equation.

1. $\log_2 (x - 5) = 4$

3. $\log_2 (8) = 2x - 1$

2. $\log_3 (-3x + 15) = 4$

4. $\log_7 (6x + 1) = 3$

5. $\log_9(27) = 5x + 4$

6. $\log_5(7x - 3) = 2$

7. $\log(-5x) = 1$

8. $\log_8(2x + 7) = 0$

D. Solve each logarithmic equation. Check your answer(s).

1. $\log_2(x^2 - x) = 1$

2. $\log_{15}(x^2 - 2x) = 1$

3. $\log_6(x^2 + 5x) = 2$

4. $\log_2(x^2 + 6x) = 4$

TOPIC 2 Exponential and Logarithmic Equations

5. $\log_4(x^2 - 12x) = 3$

6. $\log_{10}(x^2 + 15x) = 2$

7. $\log_3(3x^2 + 18x) = 4$

8. $\log_4(2x^2 - 28x) = 3$

Extension

1. Solve $\log(x - 2) = \log(3x - 4)$.

Spaced Practice

1. Solve each rational equation.

a. $4 = \frac{x+2}{x-10}$

b. $\frac{6}{x-1} = \frac{4}{x+5}$

c. $\frac{x+2}{x+3} = \frac{x+7}{x-2}$

d. $\frac{8x+4}{x} + \frac{x}{2} = \frac{x-2}{2}$

IV. Applications of Exponential and Logarithmic Equations

Topic Practice

- A. Write an exponential equation to answer each question. Show your work.
1. A bacteria culture starts with 100 bacteria and grows continuously at a rate of 15% per day. How many bacteria are in the culture after 8 days?
 2. The population of a small town was 20,315 people in 2020 and has been continuously growing at a rate of 2.8% each year. What will the population be in 2030?
 3. Andrew invested \$600 in a savings account earning 5% interest compounded continuously. What would the value of Andrew's account be after five years?

5. A scientist is studying a certain type of bacteria. The study begins with 200 bacteria in a petri dish. If the bacteria grow continuously at a rate of 18% per day, when will there be 1000 bacteria in the petri dish?

TOPIC 2 Exponential and Logarithmic Equations

6. Matthew invested \$10,000 in a savings account earning 3% interest compounded continuously. In how many years will the value of the account be \$12,000?
- B. Use the formula $M = \log\left(\frac{I}{I_0}\right)$, where M is the magnitude of an earthquake on the Richter scale, I_0 represents the intensity of a zero-level earthquake the same distance from the epicenter, and I is the number of times more intense an earthquake is than a zero-level earthquake, to formulate an equation and solve each problem. A zero-level earthquake has a seismographic reading of 0.001 millimeter at a distance of 100 kilometers from the center. Check your solution to determine the reasonableness of your answer.**
1. An earthquake southwest of Chattanooga, Tennessee in 2003 had a seismographic reading of 79.43 millimeters registered 100 kilometers from the center. What was the magnitude of the Tennessee earthquake of 2003 on the Richter scale?

2. An earthquake in Illinois in 2008 had a seismographic reading of 158.5 millimeters registered 100 kilometers from the center. What was the magnitude of the Illinois earthquake of 2008 on the Richter scale?
3. An earthquake off the northern coast of California in 2005 had a seismographic reading of 15,849 millimeters registered 100 kilometers from the center. What was the magnitude of the California earthquake in 2005 on the Richter scale?

TOPIC 2 Exponential and Logarithmic Equations

4. The devastating earthquake in Haiti in 2010 had a magnitude of 7.0 on the Richter scale. What was its seismographic reading in millimeters 100 kilometers from the center?
5. Calculate the value of the seismographic reading for an earthquake of magnitude 6.4 on the Richter scale.

6. Calculate the value of the seismographic reading for an earthquake of magnitude 8.1 on the Richter scale.

C. Use the given formula to solve each problem. Then, determine the reasonableness of your answer by checking your solution.

1. The formula for the population of a species is $n = k \log(A)$, where n represents the population of a species, A is the area of the region in which the species lives, and k is a constant that is determined by field studies. Based on population samples, an area that is 1000 square miles contains 360 wolves. Calculate the value of k . Then, use the formula to find the number of wolves remaining in 15 years when only 300 square miles of this area is still inhabitable.

TOPIC 2 Exponential and Logarithmic Equations

2. The formula for the population of a species is $n = k \log(A)$, where n represents the population of a species, A is the area of the region in which the species lives, and k is a constant that is determined by field studies. Based on population samples, a rainforest that is 100 square miles contains 342 monkeys. Calculate the value of k . Then, use the formula to find the number of monkeys remaining in 5 years if only 40 square miles of the rainforest survives due to the current level of deforestation.

4. The formula $y = a + b \cdot \ln(t)$, where t represents the time in hours, y represents the amount of fresh water produced in t hours, a represents the amount of fresh water produced in one hour, and b is the rate of production, models the amount of fresh water produced from salt water during a desalinization process. At a desalination plant, 18.65 cubic yards of fresh water can be produced in one hour with a rate of production of 34.5. How long will it take for the plant to produce 250 cubic yards of fresh water?

TOPIC 2 Exponential and Logarithmic Equations

5. The relationship between the age of an item in years and its value is given by the equation $t = \frac{\log\left(\frac{V}{C}\right)}{\log(1-r)}$, where t represents the age of the item in years, V represents the value of the item after t years, C represents the original value of the item, and r represents the yearly rate of depreciation expressed as a decimal. A luxury car was originally purchased for \$110,250 and is currently valued at \$65,200. The average rate of depreciation for this car is 10.3% per year. How old is the car to the nearest tenth of a year?
6. The relationship between the age of an item in years and its value is given by the equation $t = \frac{\log\left(\frac{V}{C}\right)}{\log(1-r)}$, where t represents the age of the item in years, V represents the value of the item after t years, C represents the original value of the item, and r represents the yearly rate of depreciation expressed as a decimal. A 4-year-old car was originally purchased for \$35,210. Its current value is \$16,394. What is this car's annual rate of depreciation to the nearest tenth?

Extension

Solve the exponential equation.

1. $(\sqrt[3]{9})^{2x-1} + 95 = 500$

Spaced Practice

1. In music, the cent, c , is a unit of measure that is used to measure the differences in frequencies between musical notes. The formula $c = 1200 \log_2 \left(\frac{f_1}{f_2} \right)$ can be used to determine the number of cents, c , between a musical note of higher frequency, f_1 , and a musical note of a lower frequency, f_2 .
 - a. Determine the number of cents between the notes A5 and D3 if A5 has a frequency of 880 Hz (hertz) and D3 has a frequency of 146.83 Hz.
 - b. Determine the frequency (in hertz) of the note F2 if the notes F2 and B2 are separated by 600 cents and B2 has a frequency of 123.47 Hz. The note B2 has a higher frequency than the note F2.

TOPIC 2 Exponential and Logarithmic Equations

2. A scientist begins with a population of 20 bacteria in a culture. She records the population every hour. The hour 0 corresponds to the hour the study began.
 - a. Does the population represent an example of exponential decay, exponential growth, or neither? Explain your reasoning.
 - b. Write a function to represent the population of bacteria, B , as a function of the hour, h .
 - c. Use your function to complete the table.

Hour	Bacteria Population
0	20
1	50
2	125
3	
4	
5	

3. Solve $\frac{a-4}{5a} = \frac{1}{5a+1}$.

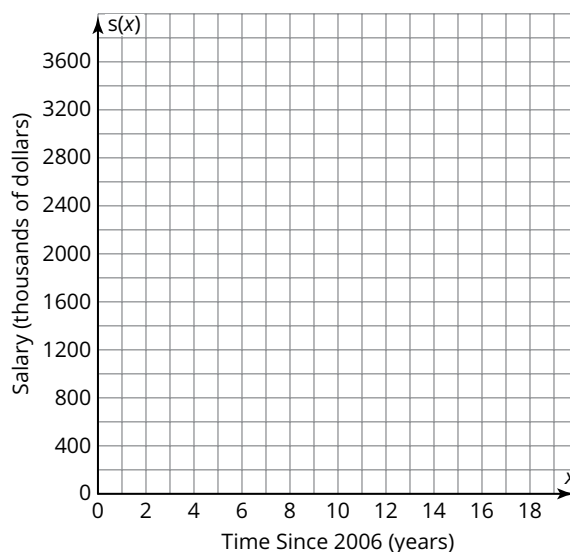
V. Modeling with Linear, Quadratic, and Exponential Functions

Topic Practice

A. Graph the points and analyze the data to determine the best regression model for the data. Explain your reasoning. Then, use graphing technology to determine the regression function.

1. The average salary in thousands of dollars, $s(x)$, of a company's presidents for various years since 2006 is displayed in the table.

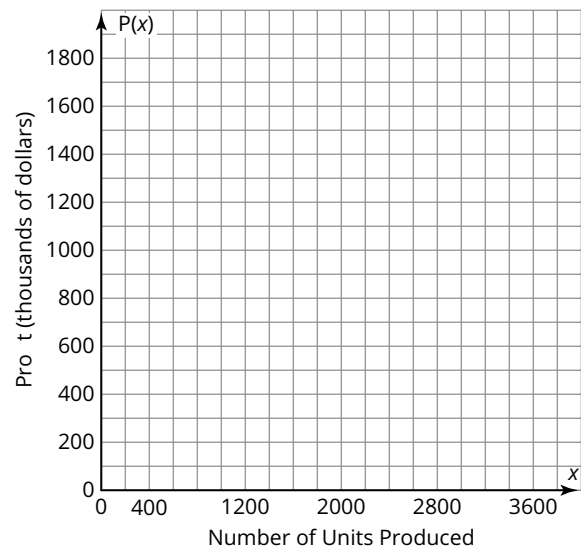
Time Since 2006 (years)	0	5	10	15	16	17	18
Average Salary (thousands of dollars)	170	325	750	1900	2000	2200	2600



TOPIC 2 Exponential and Logarithmic Equations

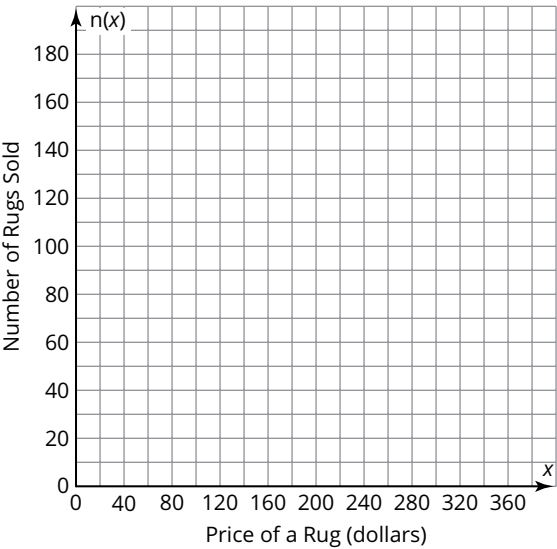
2. The number of units produced, x , of a product and the profit associated with making that many units, $P(x)$, in thousands of dollars, are shown in the table.

Number of Units Produced	1280	1350	1500	1725	1960	2400	2650	2800
Profit (thousands of dollars)	1180	1170	1280	1560	1720	1960	1940	1800



3. The table shows the number of oriental rugs a company sells, $n(x)$, and the price of each rug, x .

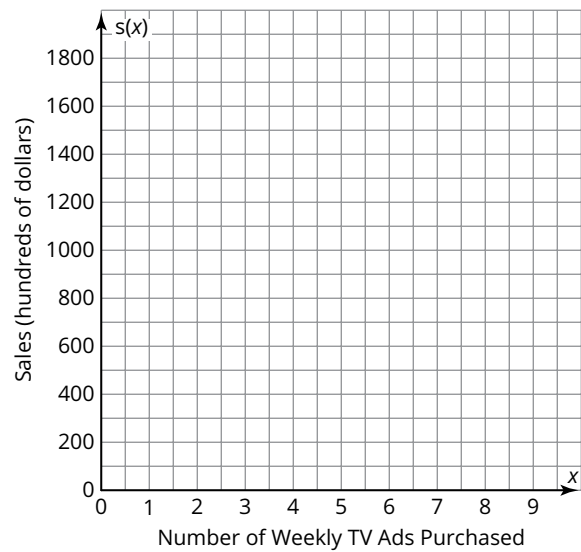
Price (dollars)	160	180	200	220	240	260	280
Number of Rugs	125	100	85	75	60	40	20



TOPIC 2 Exponential and Logarithmic Equations

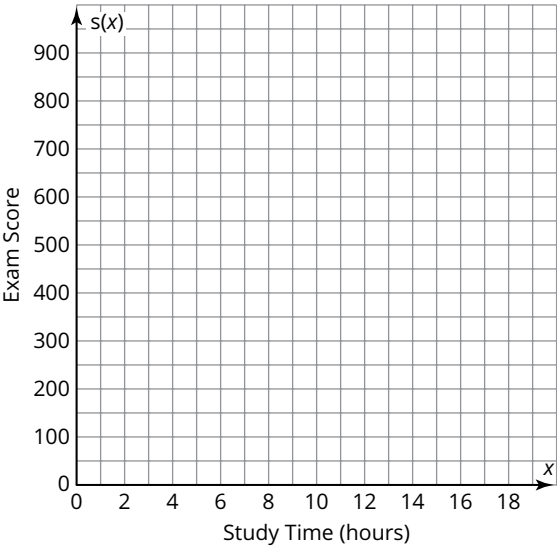
4. A company conducts a study to see if the number of weekly TV ads that they buy, x , is related to the sales of the product, $s(x)$, in hundreds of dollars. The results are recorded in the table.

Number of Weekly TV Ads Purchased	5	6	7	8	9	10
Sales (hundreds of dollars)	1000	1200	1400	1520	1000	950



5. A college recorded the number of hours students studied for a math entrance exam, x , and their scores on the exam, $s(x)$. The results are shown in the table.

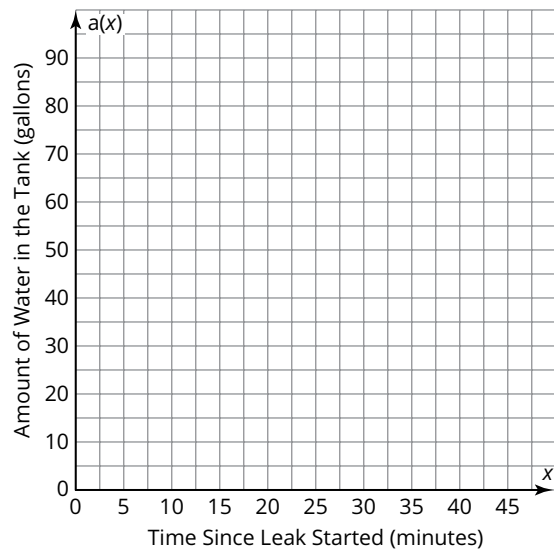
Study Time (hours)	1	3	4	9	10	12	13	14	16
Exam Score	350	400	490	580	650	600	700	730	770



TOPIC 2 Exponential and Logarithmic Equations

6. A 75-gallon tank begins to leak and lose water. The table shows the amount of water left in the tank $a(x)$ after x minutes since the leak started.

Time Since Leak Started (minutes)	0	5	10	15	20	25	30
Amount of Water in the Tank (gallons)	75	45	27	18	13	8	7



- B. Use the given tables to determine whether a linear, quadratic, or exponential function best represents the data. Then, write a regression function for the data and use the function to answer the question.**

1. A car dealership studies the average fuel economy of one of its vehicles, $f(x)$, in miles per gallon, for various speeds, x , in miles per hour. The table shows the results.

Speed (miles per hour)	20	25	30	35	40	45	50	55	60	65
Fuel Economy (miles per gallon)	25.6	27.8	29.1	29.2	30.1	29.8	30.1	29.9	28.5	26.1

A new customer wants to average 28.5 miles per gallon. Write a regression function that best models the data, and use the function to predict the fuel economy for a speed of 40 miles per hour.

TOPIC 2 Exponential and Logarithmic Equations

2. A small business conducts a study to see if the amount of money spent on monthly advertising, x , in hundreds of dollars, is related to the amount of monthly sales $s(x)$, in hundreds of dollars. The table shows the results.

Monthly Advertising Expenditures (hundreds of dollars)	5.6	5.8	5.9	6.1	6.2	6.3	6.4	6.5	6.6
Monthly Sales (hundreds of dollars)	105	110	114	113	117	116	121	120	125

The company wants to have sales of \$12,500. Write a regression function that best models the data, and use the function to predict the amount of money the company will have to spend for advertising each month in order to generate \$12,500 in sales.



3. The amount of medicine in the body, $m(x)$, in milligrams/milliliter, x minutes after taking it, is shown in the table.

Time Since Taking Medicine (minutes)	0	1	4	7	10	13	16	19
Amount of Medicine (milligrams/milliliter)	10.5	9.8	8.4	7.3	6.3	5.5	4.8	4.6

Write a regression function that best models the data, and use the function to predict the number of minutes that the medicine has been in the system when there is 7.5 milligrams/milliliter left.

TOPIC 2 Exponential and Logarithmic Equations

4. A hospital notices that flu cases are on the rise and begins to track the number of flu patients. The table shows the number of flu cases, $f(x)$, and the time in days since the hospital started tracking the data, x .

Time (days)	0	2	4	6	8	10
Number of Flu Cases	1500	1610	1650	1590	1550	1490

Write a regression function that best models the data, and use the function to predict the number of flu cases the hospital can expect 12 days after it started tracking.

5. The relationship between the length of time in months that a new television has been for sale and the price of the television in dollars is shown in the table.

Time (months)	0	6	8	12
Price (dollars)	750	530	450	275

Write a regression function that best models the data, and use the function to predict the cost of the TV after 15 months.

6. The relationship between the number of years a family has owned an antique table and the value of the table in dollars is shown in the table.

Time (years)	0	22	50	100
Price (dollars)	25	9755	15,750	8550

Write a regression function that best models the data, and use the function to predict the value of the table in 60 years.

TOPIC 2 Exponential and Logarithmic Equations

C. Consider each table and the three functions shown. Choose the function that best represents each table of data. Explain your reasoning.

1.

x	y
1	3.4
2	4.0
3	4.6
4	5.2

- $f(x) = -0.6x + 6$
- $g(x) = 0.6x + 2.6$
- $h(x) = 0.6x + 5$

2.

x	y
1	3.0
2	4.5
3	6.75
4	10.125

- $f(x) = 2(1.4)^x$
- $g(x) = -3(1.4)^x$
- $h(x) = 1.4^x$

3.

x	y
1	18
2	30
3	24
4	20

- $f(x) = 2x^2 - 12x + 28$
- $g(x) = -2x^2 + 16x + 15$
- $h(x) = -x^2 + 8x + 10$

4.

x	y
1	13.8
2	11.0
3	8.5
4	6.0

- $f(x) = -2.5x + 16$
- $g(x) = 2.5x + 16$
- $h(x) = -3.5x + 16$

5.

x	y
1	136
2	121.5
3	109.35
4	98.415

- $f(x) = -150 \cdot 0.9^x$
- $g(x) = 150 \cdot 0.9^x$
- $h(x) = 150 \cdot 0.1^x$
-

TOPIC 2 Exponential and Logarithmic Equations

6.

x	y
1	10
2	7
3	6
4	7

- $f(x) = x^2 - 2x + 5$
- $g(x) = x^2 - 6x + 14$
- $h(x) = x^2 + 6x - 5$
-

Extension

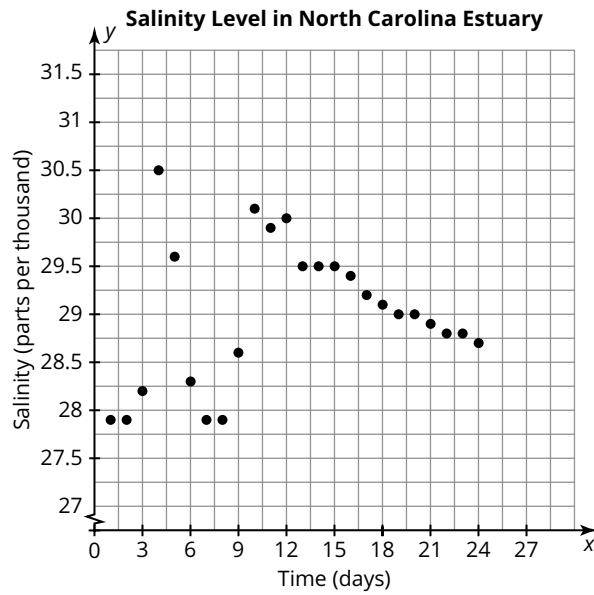
The table shows the salinity levels in an estuary in North Carolina over a period of 24 days. A scatterplot of this data is also shown.

Time (days)	1	2	3	4	5	6	7	8
Salinity (parts per thousand)	27.9	27.9	28.2	30.5	29.6	28.3	27.9	27.9

Time (days)	9	10	11	12	13	14	15	16
Salinity (parts per thousand)	28.6	30.1	29.9	30	29.5	29.5	29.5	29.4

Time (days)	17	18	19	20	21	22	23	24
Salinity (parts per thousand)	29.2	29.1	29.0	29.0	28.9	28.8	28.8	28.7

TOPIC 2 Exponential and Logarithmic Equations



1. Consider the interval $0 < x \leq 10$. Predict the type of function that would best model the data. Then use technology to determine the regression function for the data.
2. Consider the interval $10 < x \leq 24$. Predict the type of function that would best model the data. Then use technology to determine the regression function for the data.

Spaced Review

- Determine an exponential regression function that shows the relationship between the time in months and the population of a town.

Time (months)	1	2	3	4	5
Population	1000	1050	1175	1340	1450

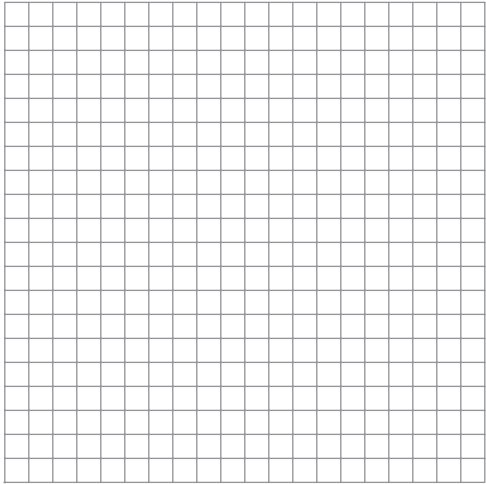
- Determine a quadratic regression function that shows the relationship between a vehicle's mileage in thousands of miles and the cost for repairs in dollars during the vehicle's last inspection.

Mileage (thousands of miles)	50	100	150	200
Cost for Repairs (dollars)	200	350	675	1425

3. Aubrey purchased some stock at the beginning of the year. Since that time, the price per share has continued to decline. The data shows the decline in the price per share each month since Aubrey purchased it.

Time (months)	0	1	2	3	4	5	6
Price Per Share (dollars)	218	215	210	199	184	150	87

4. Drew wants to invest \$5000 in a savings account. Sun Bank is offering 4.5% interest compounded monthly. Brightside Bank is offering 4.75% interest compounded quarterly.
- a. For each bank, write an exponential function to represent the amount of money Drew would have in the account after t years.



- b. Determine which bank Drew should choose if she plans to invest her money for 5 years. If Drew decides to leave the money in the bank for a longer period of time, will the other bank be a better deal in the long run? Explain your reasoning.

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