ENGLISH

Bluebonnet Learning™
Secondary Mathematics
EDITION 1

Geometry

Skills Practice

STUDENT EDITION

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Secondary Mathematics
EDITION 1

Geometry

Skills Practice

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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Reasoning with Shapes

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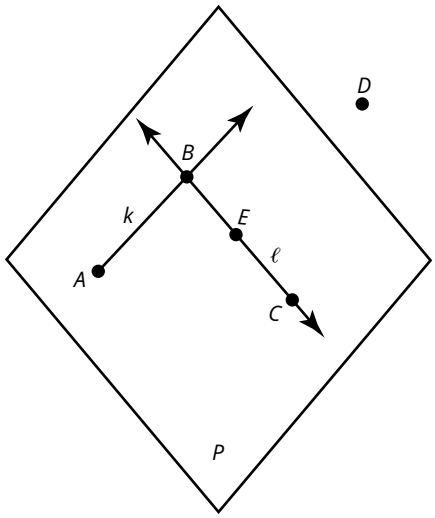
Name _____ Date _____

I. Points, Lines, Planes, Rays, and Line Segments

Topic Practice

A. Use the diagram to determine what each name represents and tell whether it represents an undefined or defined term.



1. P

2. $\overrightarrow{AB}$

3. $\overleftrightarrow{BC}$

4. ℓ

5. D

6. $\overleftrightarrow{BEC}$

7. $\overline{EC}$

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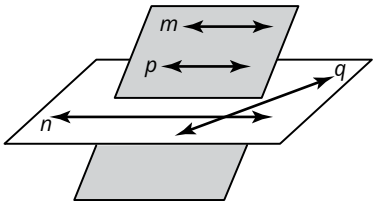
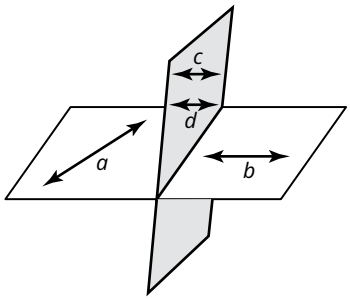
B. Identify all examples of either coplanar lines or skew lines.

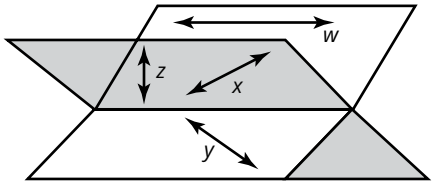
1. Coplanar lines



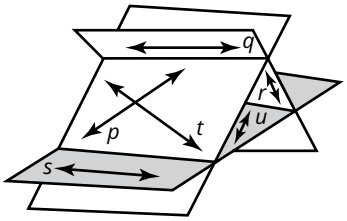
2. Coplanar lines



3. Coplanar lines

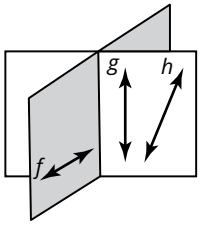


4. Coplanar lines

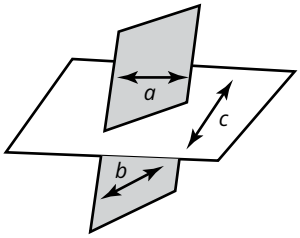


Identify all skew lines in each figure.

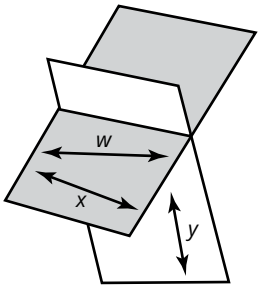
5. Skew lines



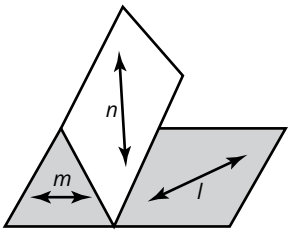
6. Skew lines



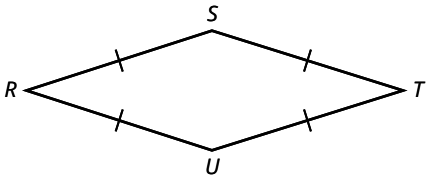
7. Skew lines

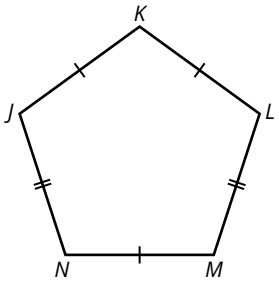


8. Skew lines



C. Use the diagrams to determine whether each statement is true or false.





1. $\overline{RS} \cong \overline{ST}$

2. $\overline{RU} \cong \overline{UT}$

3. $ST = RU$

4. $RS = RU$

5. $\overline{JK} \cong \overline{MN}$

6. $KL = JN$

7. $LM = JN$

8. $\overline{KJ} \cong \overline{NJ}$

Extension

1. Name a line and plane in multiple ways.

Spaced Practice

Determine the unknown side length of each right triangle.

1. 6, *b*, 10

2. 8, 15, *c*

3. *a*, 9, 12

II. Formal Reasoning in Euclidean Geometry

Topic Practice

A. Provide a counterexample for each statement to demonstrate it is not true.

1. All birds can fly.

2. All vegetables are green.

3. All mammals walk on four legs.

4. All numbers divisible by 3 are odd.

5. All numbers are either even or odd.

6. All fractions can be written as terminating decimals.

B. Write each statement as a conditional statement in the form "if *p*, then *q*."

1. The measure of an angle is 90°. So, the angle is a right angle.

2. Three points are all located on the same line. So, the points are collinear points.

3. Two lines are not on the same plane. So, the lines are skew.

4. Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

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5. Two angles share a common vertex and a common side. So, the angles are adjacent angles.

6. A ray divides an angle into two congruent angles. So, the ray is an angle bisector.

C. Identify the hypothesis and the conclusion of each conditional statement.

1. If two lines intersect at right angles, then the lines are perpendicular.

2. If the sum of two angles is 180°, then the angles are supplementary.

3. If the sum of two adjacent angles is 180°, then the angles form a linear pair.

4. If the measure of an angle is 180°, then the angle is a straight angle.

5. If two lines are located in the same plane, then the lines are coplanar lines.

6. If the sum of two angle measures is equal to 90°, then the angles are complementary angles.

D. Answer each question about the given conditional statement.

1. Conditional statement: If the measure of angle *ABC* is 45° and the measure of angle *XYZ* is 45°, then $\angle ABC \cong \angle XYZ$.

   What does it mean if the hypothesis is false and the conclusion is true? What is the validity of the conditional statement?

2. Conditional statement: If the measure of angle *XYZ* is less than 90°, then angle *XYZ* is acute.

   What does it mean if the hypothesis is true and the conclusion is false? What is the validity of the conditional statement?

3. Conditional statement: If $\angle 1$ and $\angle 2$ are two nonadjacent angles formed by two intersecting lines, then they are vertical angles.

   What does it mean if the hypothesis is true and the conclusion is true? What is the validity of the conditional statement?

4. Conditional statement: If the measure of $\angle LMN$ is 180°, then $\angle LMN$ is a straight angle.

   What does it mean if the hypothesis is false and the conclusion is false? What is the validity of the conditional statement?

E. Answer each question about spherical geometry.

1. What defines a line in spherical geometry, and how is this definition different from that of a line in Euclidean geometry?

2. Why do parallel lines not exist in spherical geometry, and what happens when two great circles intersect?

3. How many points of intersection can two different in spherical geometry have, and why is this significant?

4. What are the differences between the interior angle sums of triangles in Euclidean geometry and spherical geometry?

5. What is the range of possible sums for the interior angles of a spherical triangle, and how does this range reflect the geometry of the sphere?

6. What properties distinguish a right spherical triangle from a right triangle in Euclidean geometry, and how are the sides of these triangles different?

Extension

1. Research how how spherical geometry is related to fields such as aviation, astronomy, cartography, or telecommunications. Share your findings with your class.

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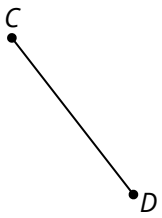
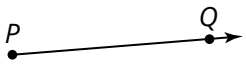
Spaced Practice

1. Use a symbol to represent the name of each geometric figure.

a.

b. 

c. 

III. Conjectures and Deductive Reasoning

Topic Practice

A. Determine whether inductive reasoning or deductive reasoning is used in each situation. Then determine whether the conclusion is correct and explain your reasoning.

1. Mateo sees a line of 10 school buses and notices that each is yellow. He concludes that all school buses must be yellow.

2. Victoria has been told that every taxi in New York City is yellow. When she sees a red car in New York City, she concludes that it cannot be a taxi.

3. Aaliyah has been told that lightning never strikes twice in the same place. During a lightning storm, she sees a tree struck by lightning and goes to stand next to it, convinced that it is the safest place to be.

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4. Liam is shown the first six numbers of a series of numbers: 7, 11, 15, 19, 23, 27. He concludes that the general rule for the series of numbers is $a_n = 4n + 3$.

5. Ava sees 5 red fire trucks. She concludes that all fire trucks are red.

6. Elijah is told that all garter snakes are not venomous. He sees a garter snake in his backyard and concludes that it is not venomous.

B. In each situation, identify whether each person is using inductive or deductive reasoning. Then compare and contrast the two types of reasoning.

1. When Harper babysat for the Johnsons for the first time, she was there 2 hours and was paid $30. The next time she was there for 5 hours and was paid $75. She decided that the Johnsons were paying her $15 per hour. The third time she went, she stayed for 4 hours. She tells her friend Emma that she makes $15 per hour babysitting. So, Emma predicted that Harper made $60 for her 4-hour babysitting job.

2. When Sofia was young, the only birds she ever saw were black crows. So, she told her little brother Diego that all birds are black. When Diego saw a bluebird for the first time, he was sure it had to be something other than a bird.

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3. Mia is flipping a coin and recording the results. She records the following results: heads, tails, heads, tails, heads, tails, heads. She tells her friend Juan that the coin alternates between heads and tails for each toss. Juan tells her that the next time the coin is flipped, it will definitely be tails.

4. Ethan likes to watch the long coal trains moving past his house. Over the weeks of watching he notices that every train going east is filled with coal, but the trains heading west are all empty. He tells his friend Samuel that all trains heading east have coal and all trains heading west are empty. When Samuel hears a train coming from the west, he concludes that it will certainly be filled with coal.

5. Noah earned $60 mowing 5 lawns last weekend for a homeowners association. Noah concluded that he earned $12 for each lawn. Noah told Kai that he planned to mow 7 lawns for the homeowners association next weekend. Kai concluded that Noah would earn $84 mowing the 7 lawns.

6. As a child, the only frogs Victoria ever saw were green. Victoria told Juan that all frogs are green. When Minh visited a zoo and saw a blue poison dart frog he concluded that it must be something other than a frog.

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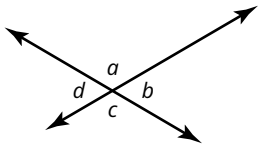
Extension

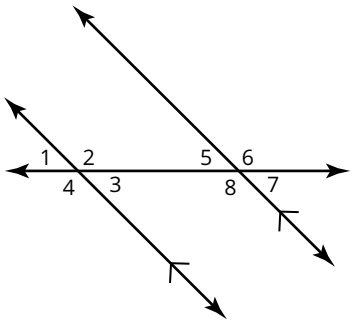
1. You know that the terms point, line, and plane are the building blocks of geometry. Research building block terms of other subjects such as biology or chemistry. Share your findings with your class.

Spaced Practice

1. Identify the vertical angles.



2. Identify the pairs of angles. Describe the measures of the angles in each pair.



a. Corresponding angles

b. Alternate interior angles

c. Alternate exterior angles

d. Same-side interior angles

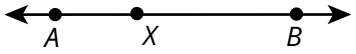
Name _____ Date _____

I. Constructing a Coordinate Plane

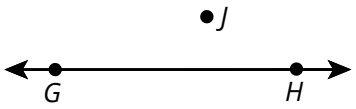
Topic Practice

A. Construct a line perpendicular to each given line and through the given point.

1. Construct a line that is perpendicular to $\overleftrightarrow{AB}$ and passes through point *X*.

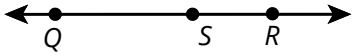


2. Construct a line that is perpendicular to $\overleftrightarrow{GH}$ and passes through point *J*.

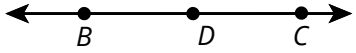


3. Construct a line that is perpendicular to $\overleftrightarrow{MN}$ and passes through point *O*.

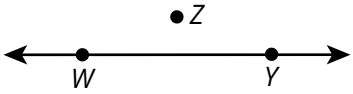
4. Construct a line that is perpendicular to $\overleftrightarrow{QR}$ and passes through point *S*.



5. Construct a line that is perpendicular to $\overleftrightarrow{BC}$ and passes through point *D*.

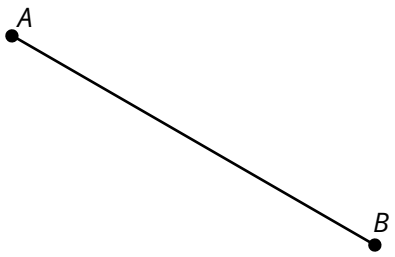


6. Construct a line that is perpendicular to $\overleftrightarrow{WY}$ and passes through point *Z*.

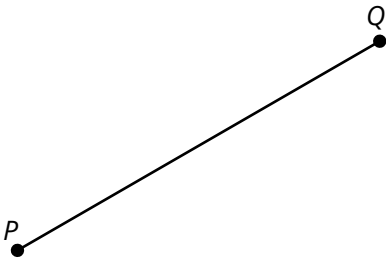


B. Construct a perpendicular bisector for each line segment using construction tools and label the midpoint *M*.

1.



2.



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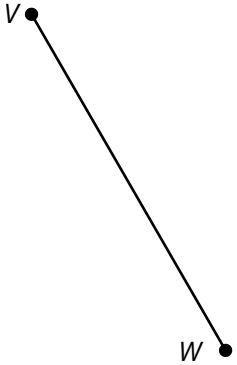

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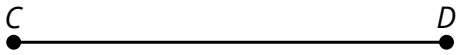
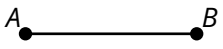


6.

C. Duplicate each line segment.

1.



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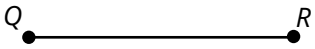

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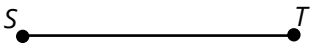

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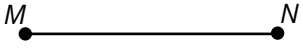

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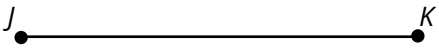
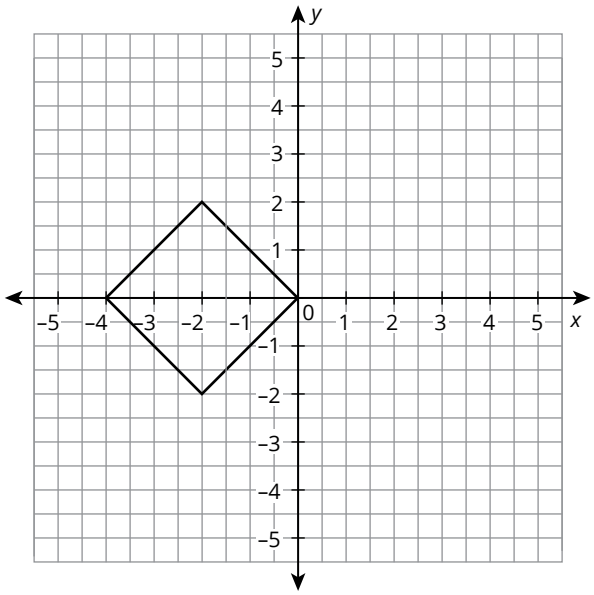


D. The figures shown were each constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape.

1.



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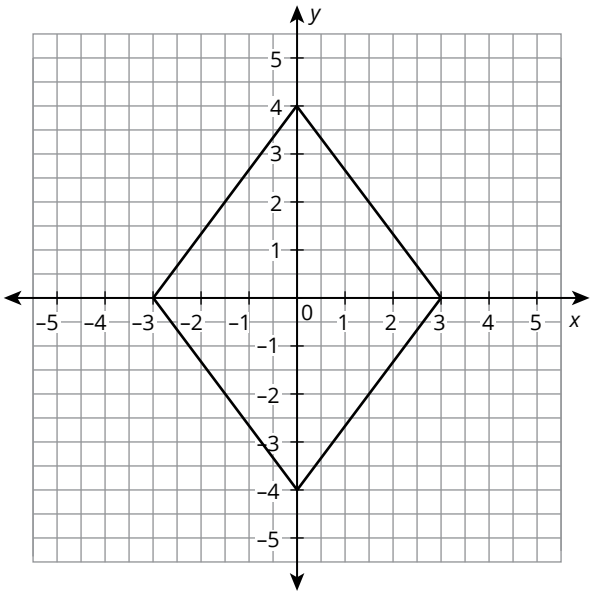

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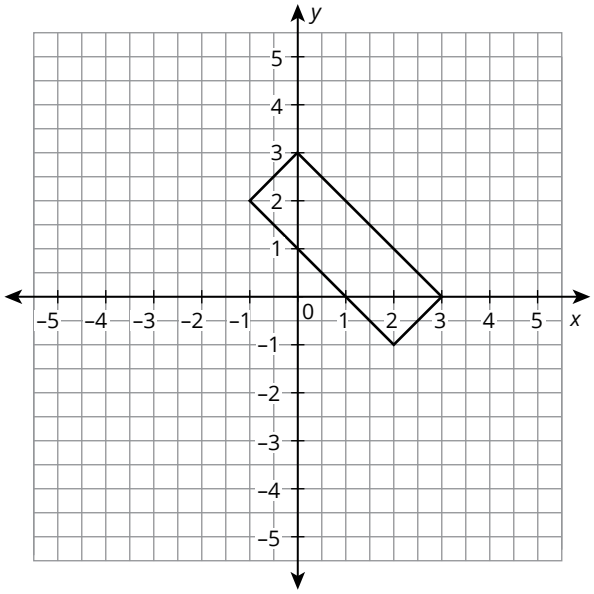

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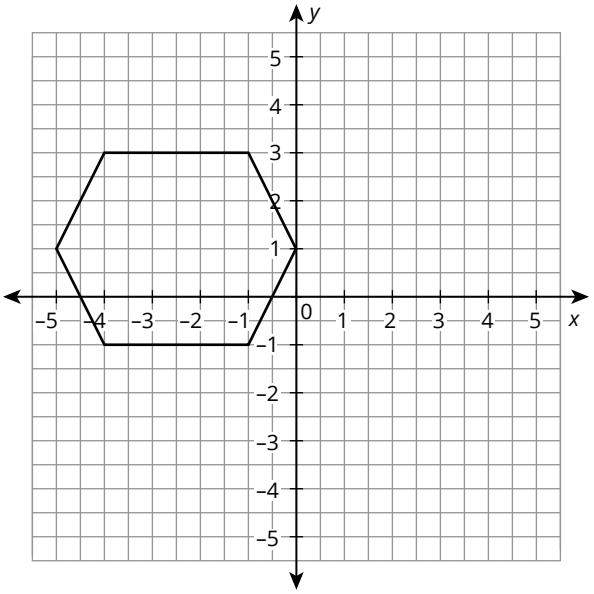

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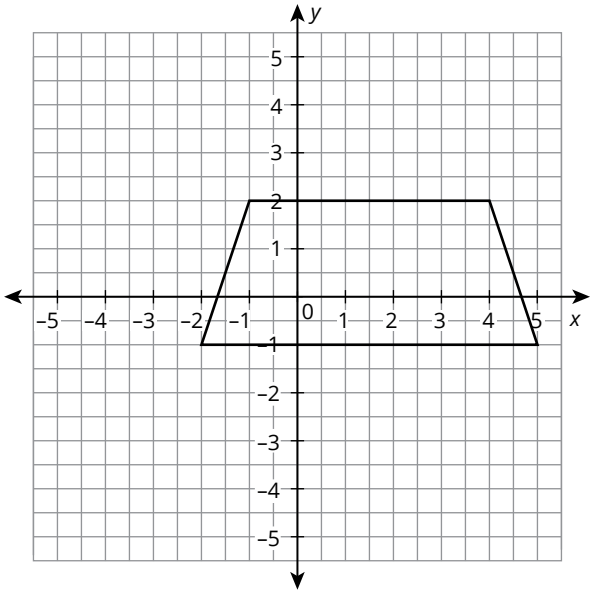
y
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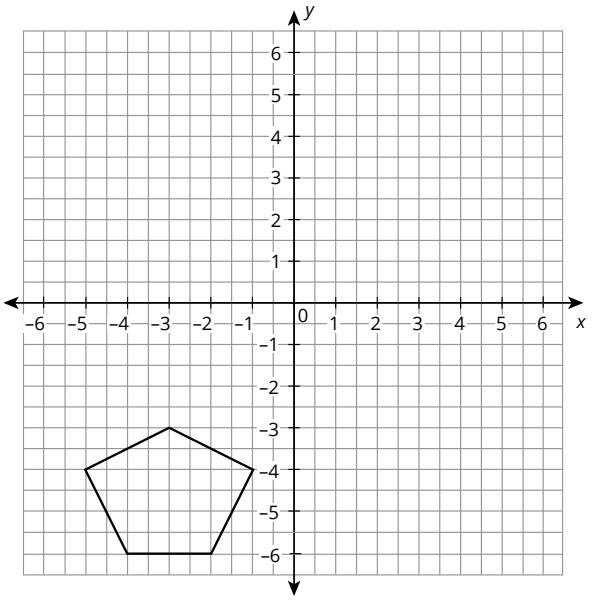
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−1
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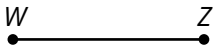
6

Extension

1. Research whether it is possible to trisect a segment using only construction tools. If possible, use construction tools to trisect $\overline{WZ}$ and explain your steps. If not possible, explain why.



Spaced Practice

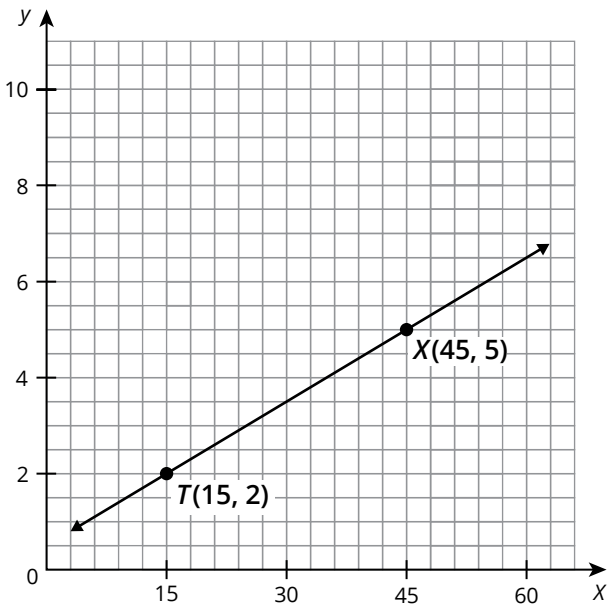
1. List three different properties of a square.

2. A right triangle has leg lengths of 6 in. and 8 in. Use the Pythagorean Theorem to determine the length of the hypotenuse. Show your work.

II. Parallel and Perpendicular Lines

Topic Practice

A. Derive the slope of $\overleftrightarrow{TX}$.



1. Sketch a vertical segment from point T to a point that has the same y-coordinate as point X. Label the endpoint of your segment as E.

2. Sketch a horizontal segment from point E to point X.

3. Determine the length of $\overline{TE}$ and explain how you can calculate TE using the y-coordinates of points T and E.

4. Determine the length of $\overline{EX}$ and explain how you can calculate EX using the x-coordinates of points E and X.

5. Is the slope of $\overleftrightarrow{TX}$ positive or negative? Explain your reasoning.

6. Write the ratio of the rise to the run for $\overleftrightarrow{TX}$.

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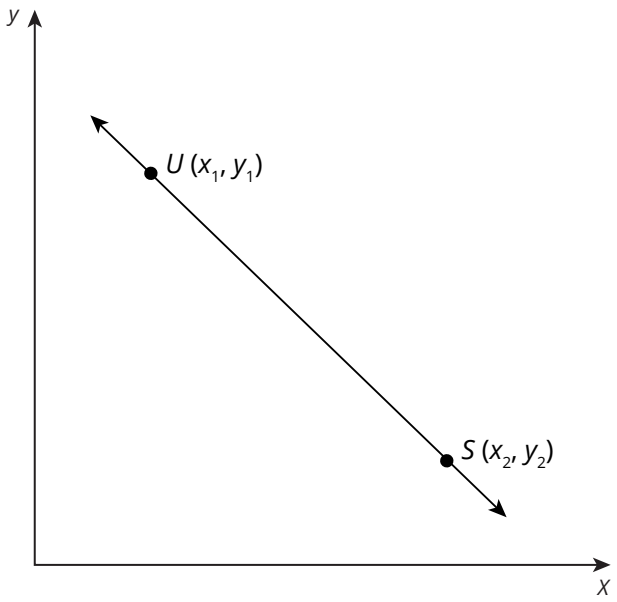
B. Derive the slope of $\overleftrightarrow{US}$.



1. Sketch a vertical segment from point *U* to a point that has the same *y*-coordinate as point *S*. Label the endpoint of your segment as *A*.

2. Sketch a horizontal segment from point *A* to point *S*.

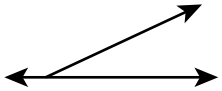
3. Use the coordinates of points *U* and *S* to write an expression that represents the length of $\overline{UA}$.

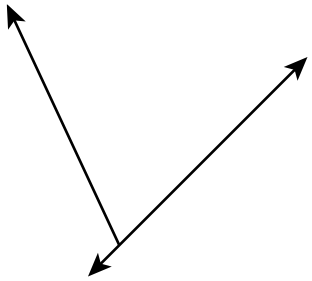
4. Use the coordinates of points *U* and *A* to write an expression that represents the length of $\overline{AS}$.

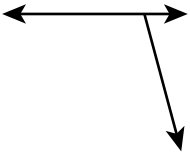
5. Is the slope of $\overleftrightarrow{US}$ positive or negative? Explain your reasoning.

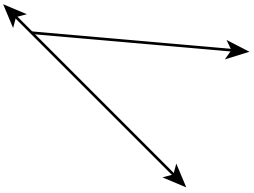
6. Write the ratio of the rise to the run for $\overleftrightarrow{US}$.

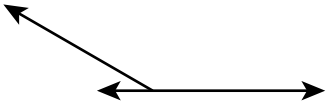
C. Duplicate each angle.

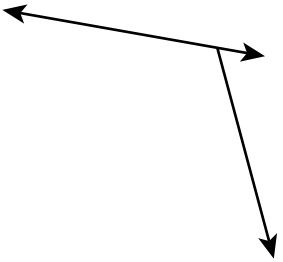
1. 

2. 

3. 

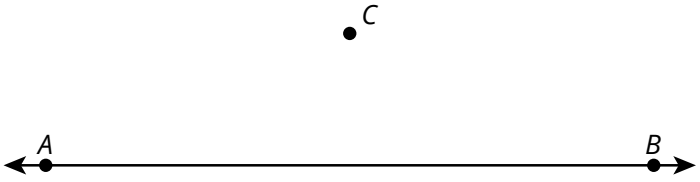
4. 

5. 

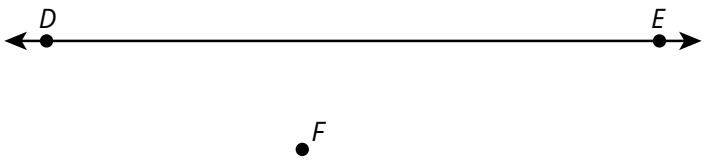
6. 

D. Construct a line parallel to each given line and through the given point.

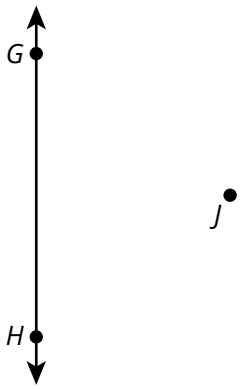
1. Construct a line that is parallel to $\overleftrightarrow{AB}$ and passes through point *C*.



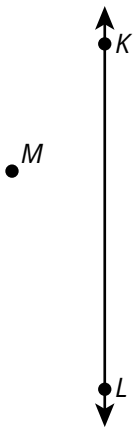
2. Construct a line that is parallel to $\overleftrightarrow{DE}$ and passes through point *F*.



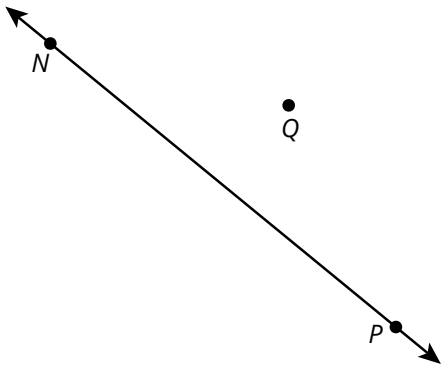
3. Construct a line that is parallel to $\overleftrightarrow{GH}$ and passes through point *J*.



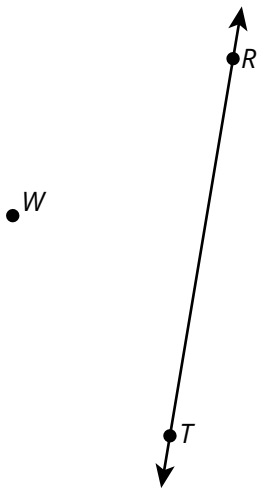
4. Construct a line that is parallel to $\overleftrightarrow{KL}$ and passes through point M.



5. Construct a line that is parallel to $\overleftrightarrow{NP}$ and passes through point Q.



6. Construct a line that is parallel to $\overleftrightarrow{RT}$ and passes through point W.



E. Determine an equation for each parallel line described.

1. What is the equation of a line parallel to $y = \frac{4}{5}x + 2$ that passes through (1, 2)?

2. What is the equation of a line parallel to $y = -5x + 3$ that passes through (3, 1)?

3. What is the equation of a line parallel to $y = 7x - 8$ that passes through (5, −2)?

4. What is the equation of a line parallel to $y = -\frac{1}{2}x + 6$ that passes through (−4, 1)?

5. What is the equation of a line parallel to $y = \frac{1}{3}x - 4$ that passes through (9, 8)?

6. What is the equation of a line parallel to $y = -4x - 7$ that passes through (2, −9)?

F. Determine an equation for each perpendicular line described.

1. What is the equation of a line perpendicular to $y = 2x - 6$ that passes through (5, 4)?

2. What is the equation of a line perpendicular to $y = -3x + 4$ that passes through $(-1, 6)$?

3. What is the equation of a line perpendicular to $y = -\frac{2}{5}x - 1$ that passes through $(2, -8)$?

4. What is the equation of a line perpendicular to $y = \frac{3}{4}x + 12$ that passes through (12, 3)?

5. What is the equation of a line perpendicular to $y = 6x - 5$ that passes through $(6, -3)$?

6. What is the equation of a line perpendicular to $y = \frac{5}{2}x - 1$ that passes through $(-1, -4)$?

G. Determine the equation of a vertical line that passes through each given point.

1. $(-2, 1)$

2. (3, 15)

3. $(9, -7)$

4. $(-11, -8)$

5. $(-5, -10)$

6. $(0, -4)$

H. Determine the equation of a horizontal line that passes through each given point.

1. (4, 7)
2. (−6, 5)
3. (−8, −3)
4. (2, −9)
5. (−7, 8)
6. (6, −2)

I. Determine an equation for each perpendicular or parallel line described.

1. Write an equation for a line that is parallel to the line given by $x = 5$ and passes through the point (−1, 2).
2. Write an equation for a line that is perpendicular to the line given by $y = -3$ and passes through the point (2, 4).
3. Write an equation for a line that is perpendicular to the x-axis and passes through the point (4, 3).
4. Write an equation for a line that is parallel to the y-axis and passes through the point (–1, 7).
5. Write an equation for a line that is parallel to the line given by $y = 4$ and passes through the point (0, 5).
6. Write an equation for a line that is perpendicular to the line given by $x = 9$ and passes through the point (−6, −3).

J. Determine the equation for the line with the given description.

1. Write the equation of a line parallel to $y = 4$ that passes through the point $(-3, 2)$.

2. Write the equation of a line perpendicular to $y = \frac{2}{3}x - 5$ that passes through the point $(-2, 6)$.

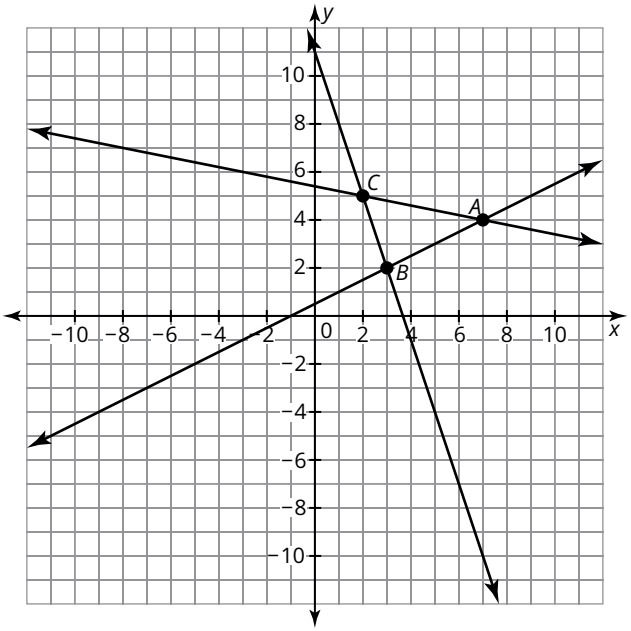
3. Write the equation of a line parallel to $y = -4x + 7$ that passes through the point $(0, 2)$.

4. Write the equation of a line perpendicular to $x = -3$ that passes through the point $(5, -2)$.

5. Write the equation of a line parallel to $y = 3x + 2$ that passes through the point $(1, -4)$.

Extension

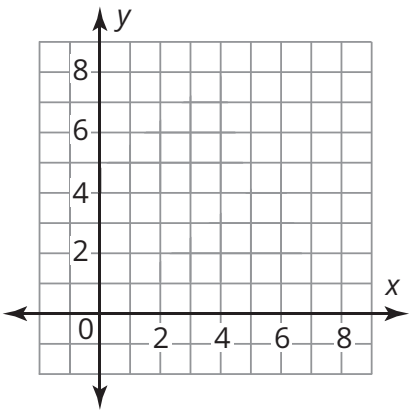
Triangle *ABC* is located on three lines such that the vertices occur at the points of intersection of pairs of the lines, as shown on the graph. If $\triangle ABC$ is rotated 90° counterclockwise around the origin to form $\triangle A'B'C'$, determine the equations of the lines that would contain $\triangle A'B'C'$. Explain your reasoning. Then draw the three lines that contain $\triangle A'BC'$ and label $\triangle A'B'C'$.



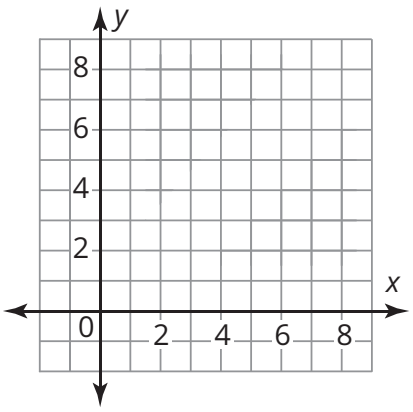
Spaced Practice

1. Identify each rigid motion as a translation, reflection, or rotation.

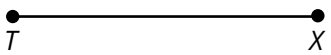
a.



b.



2. Duplicate the line segment.



3. Construct a line parallel to line m through point A.

III. Classifying Quadrilaterals on the Coordinate Plane

Topic Practice

A. Use the distance formula to calculate the distance between each pair of points. Round your answer to the nearest tenth, when necessary. Show all of your work.

1. (2, –5) and (6, 8)

2. (–14, –1) and (–3, 0)

3. (–6, 10) and (–2, –7)

4. (12, 8) and (4, 1)

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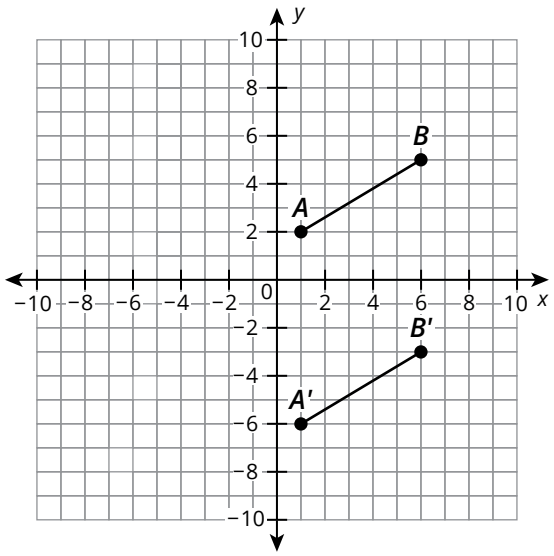
5. (–1, 18) and (3, –1)

6. (9, 0) and (0, –4)

B. Use the distance formula to verify that the given line segments are congruent.

1.



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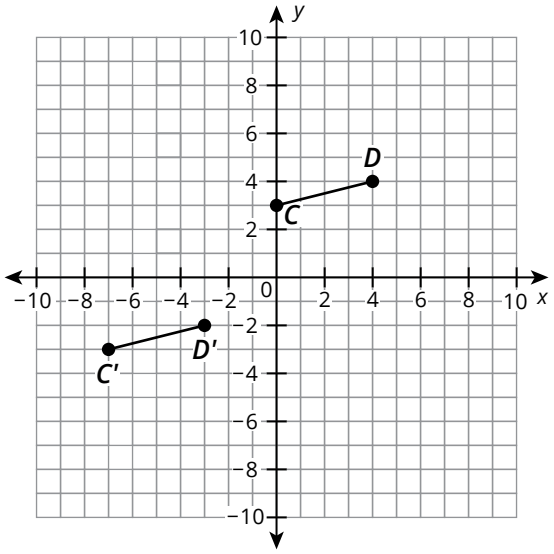

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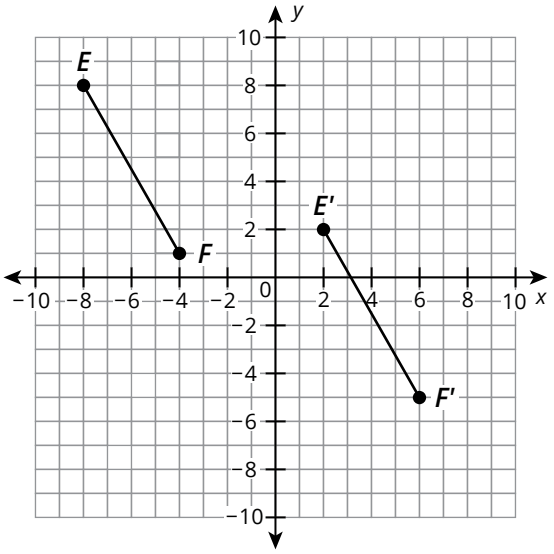
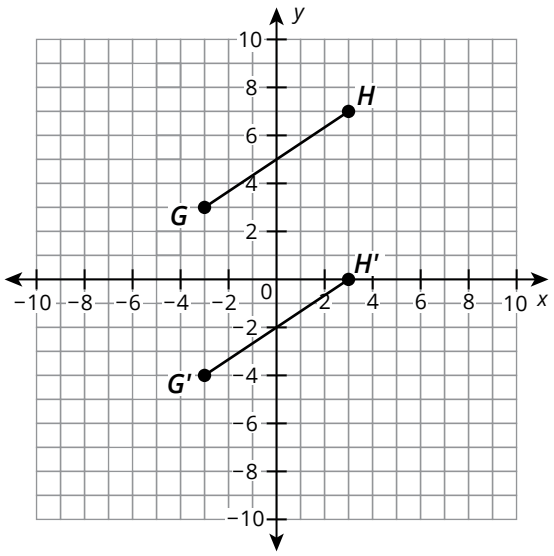
3.



4.



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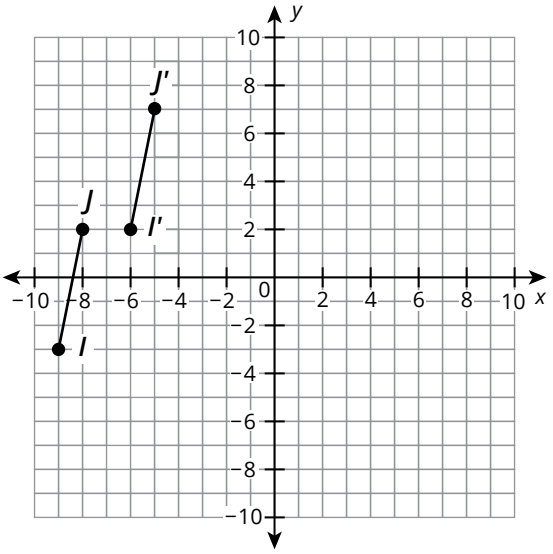

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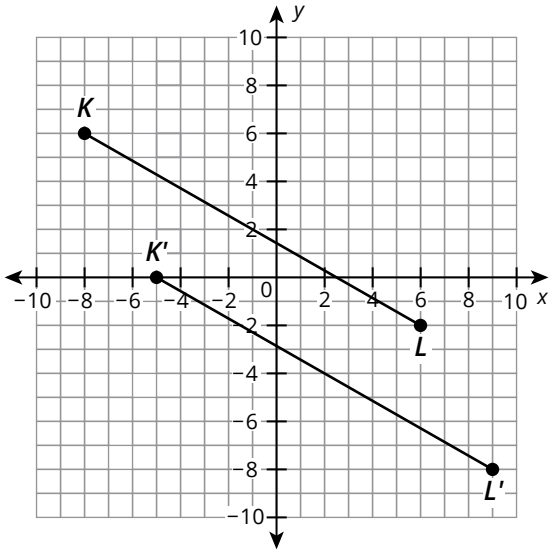
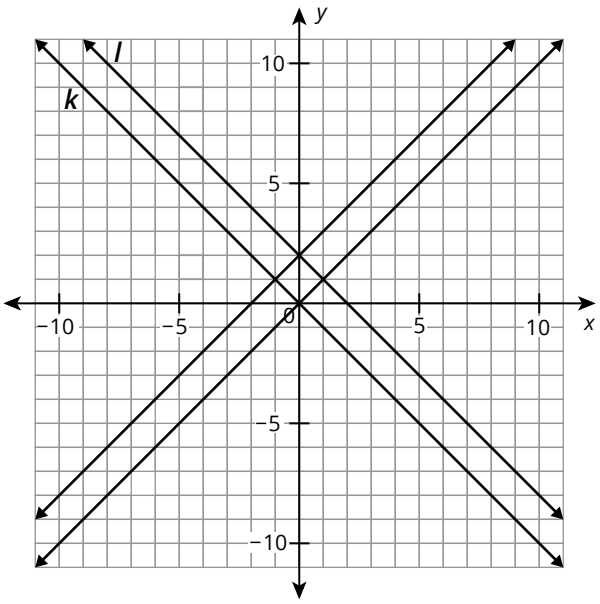
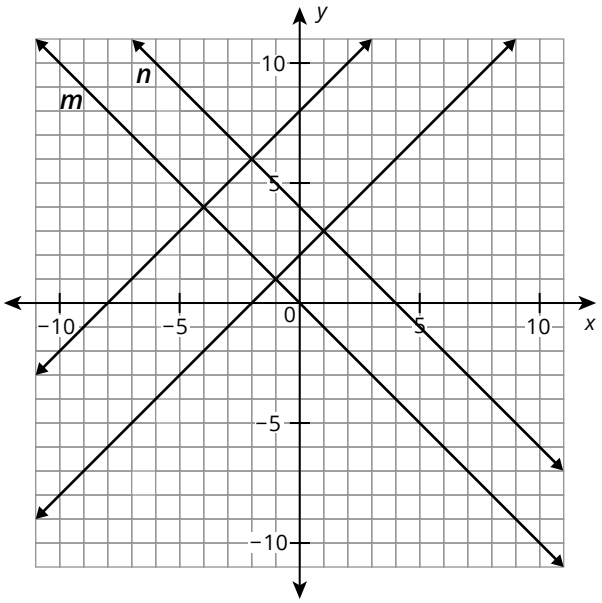


C. Use the distance formula to verify that the indicated lines are parallel.

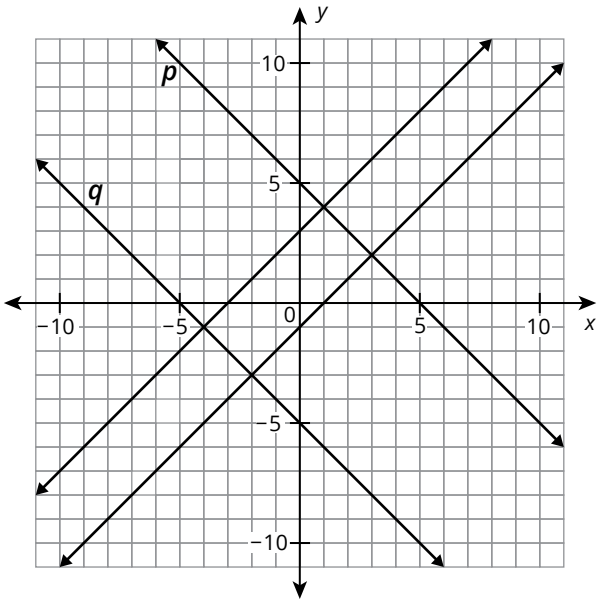
1. Line k and line ℓ



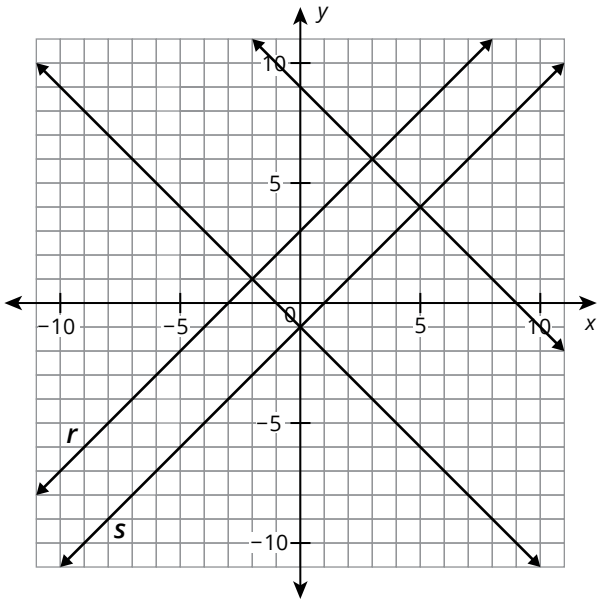
2. Line *m* and line *n*



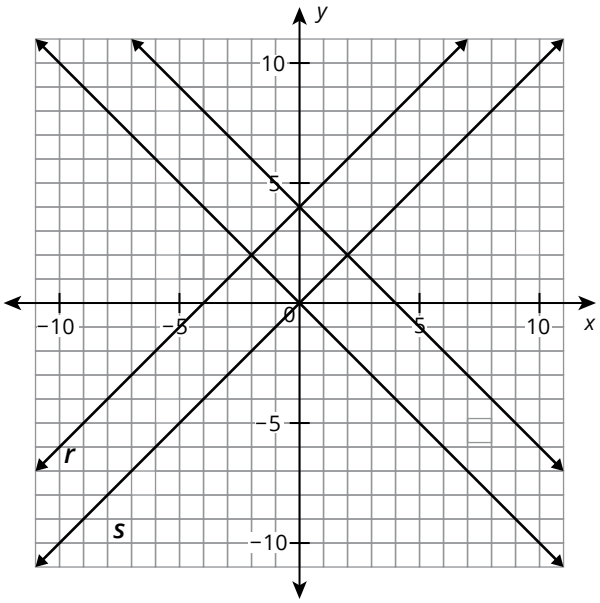
3. Line p and line q



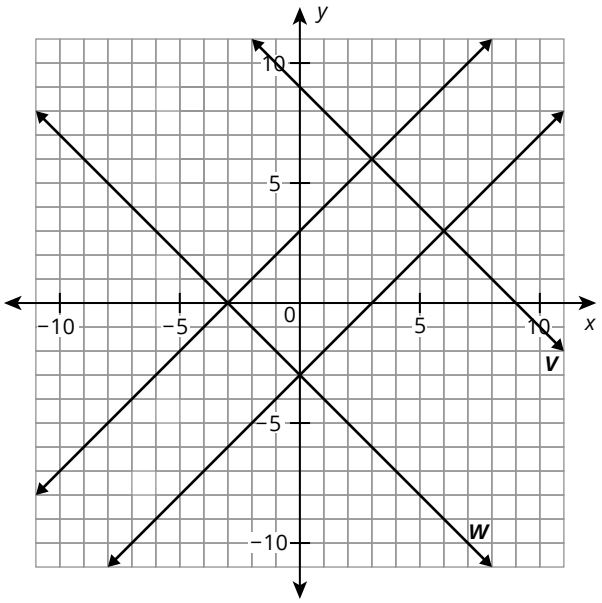
4. Line *r* and line *s*



5. Line *t* and line *u*

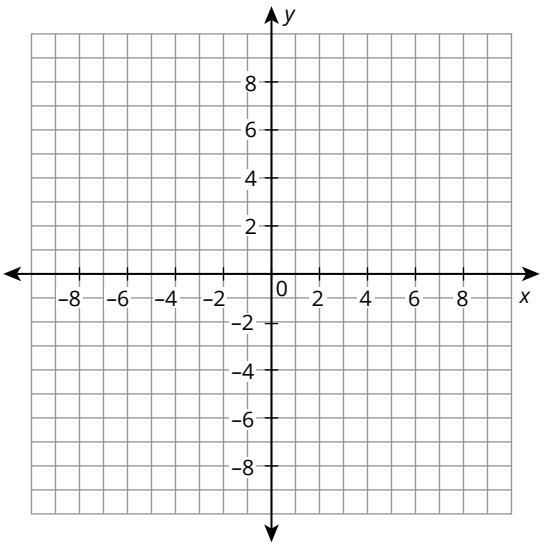


6. Line *v* and line *w*

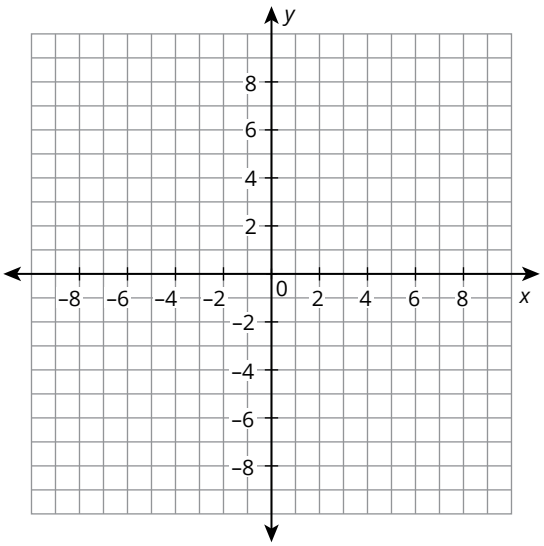


D. Graph quadrilateral *ABCD* using each set of given points. Determine if quadrilateral *ABCD* can be best described as a trapezoid, a square, a rectangle, or a rhombus.

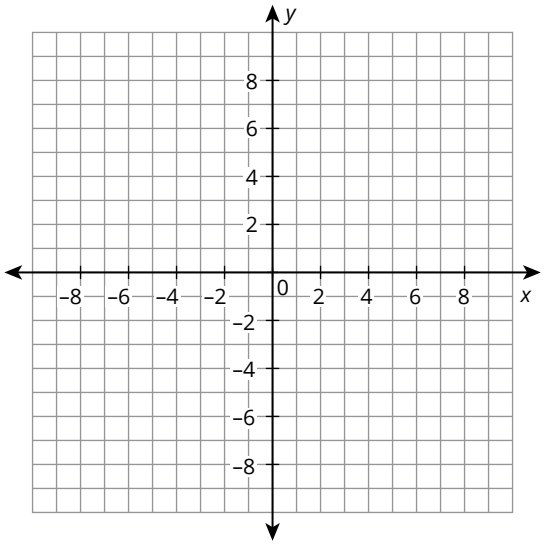
1. *A* (−4, −2), *B* (2, 4), *C* (8, −2), *D* (2, −8)



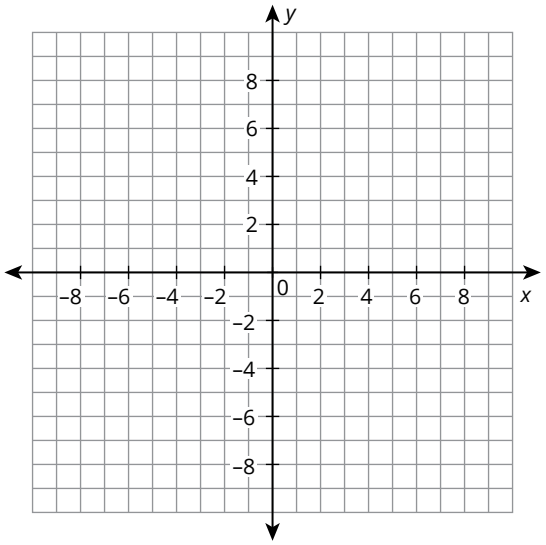
2. *A* (0, 6), *B* (3, 4), *C* (−1, −2), *D* (−4, 0)



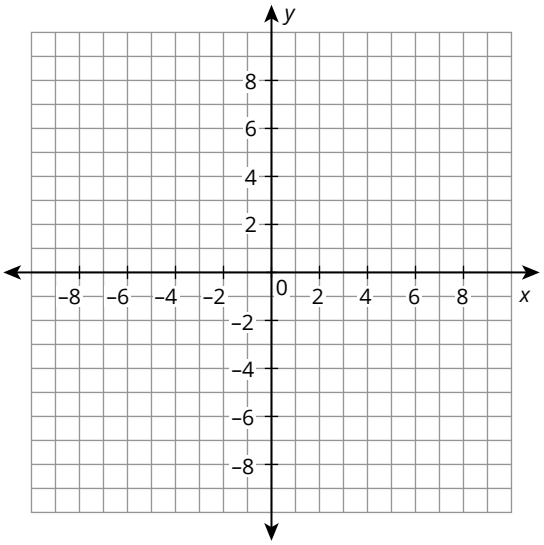
3. $A\ (1, 0)$, $B\ (2, 4)$, $C\ (10, 2)$, $D\ (5, -1)$



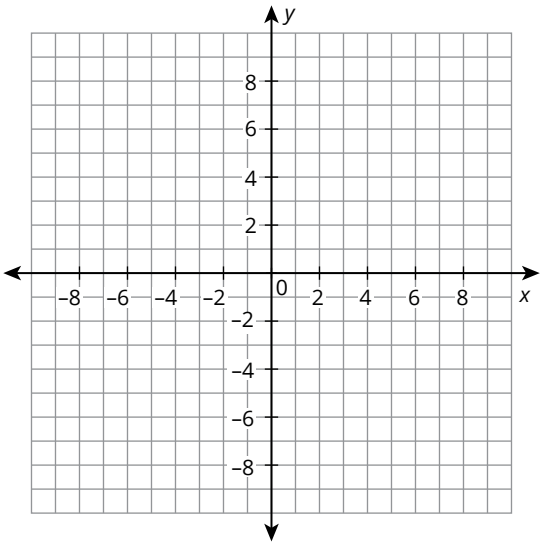
4. $A\ (-5, 3)$, $B\ (0, 6)$, $C\ (5, 3)$, $D\ (0, 0)$



5. $A(-3, 3)$, $B(-1, 5)$, $C(6, -2)$, $D(4, -4)$

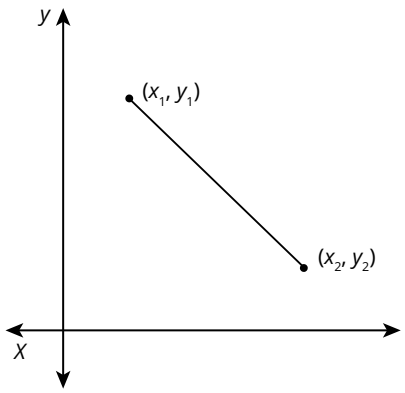


6. $A(-7, -7)$, $B(-4, -1)$, $C(5, -3)$, $D(4, -5)$



E. Derive and Use the midpoint formula to determine the midpoint of the given line segment.

1.



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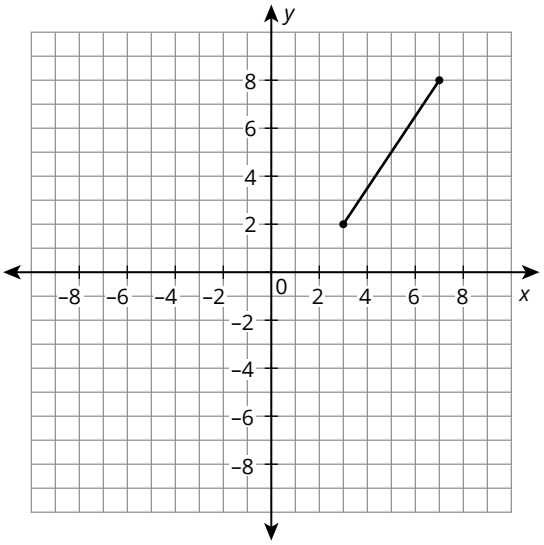

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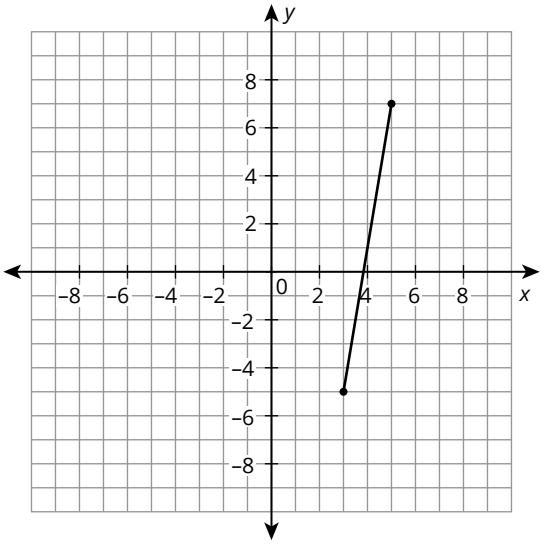

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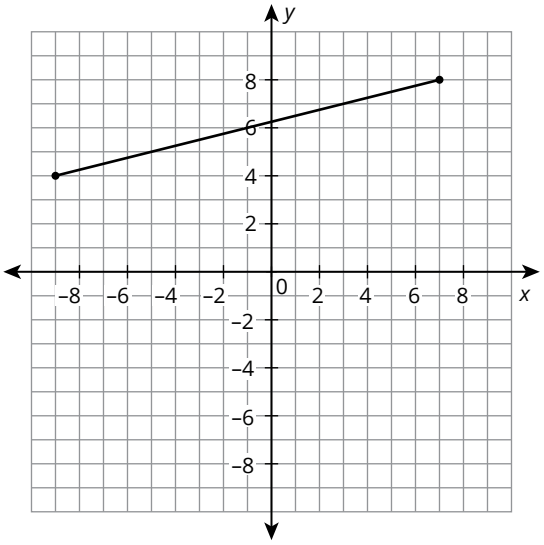
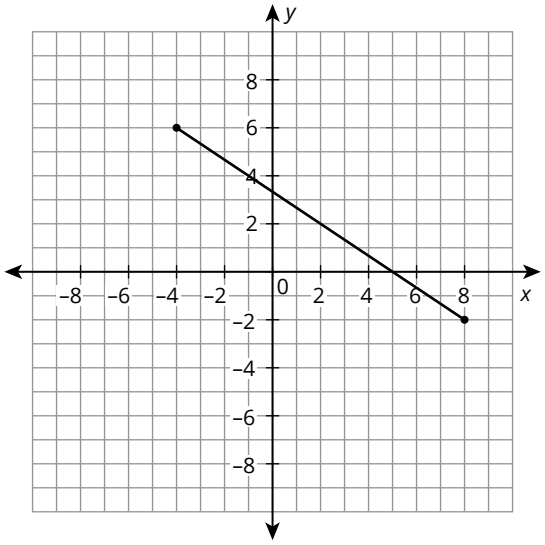


5.



6. Line segment *AB* with endpoints *A* (−10, −3) and *B* (2, −8)

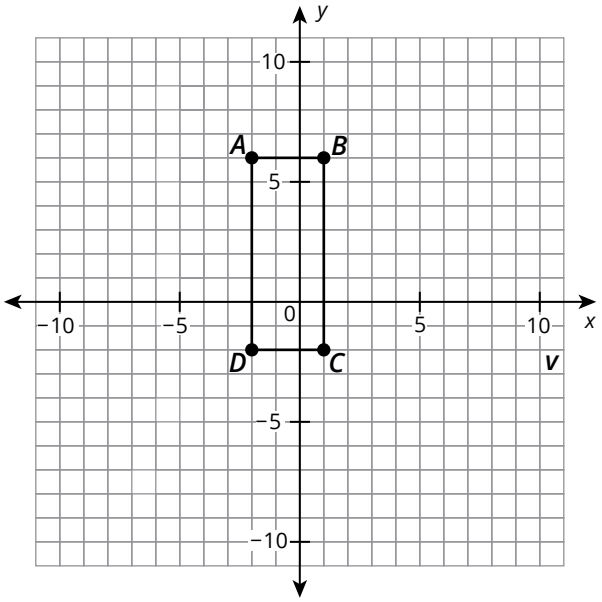
7. Line segment *CD* with endpoints *C* (−4, −8) and *D* (4, 8)

8. Line segment *EF* with endpoints *E* (−5.4, 7.2) and *F* (2.9, − 1.6)

9. Line segment *GH* with endpoints $G\left(-3\frac{1}{2}, -1\frac{3}{4}\right)$ and *H*(7, 5)

F. Calculate and label the midpoint of each side of the given quadrilateral. Determine the polygon formed by connecting the consecutive midpoints of each side. Then, use your determination to classify the given quadrilateral.

1.



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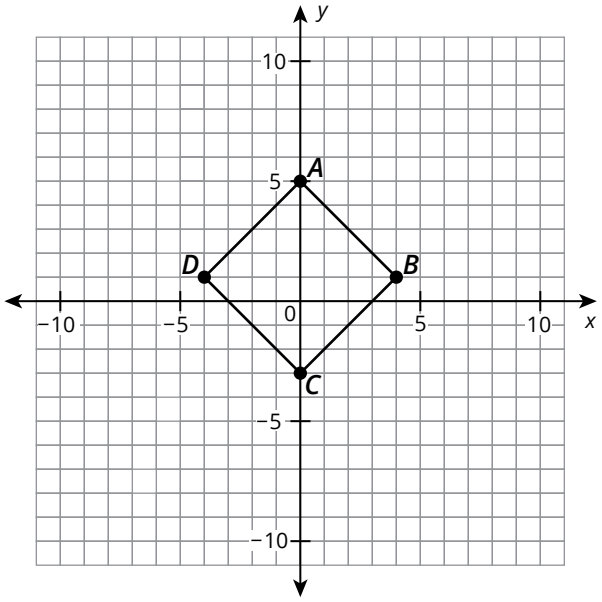

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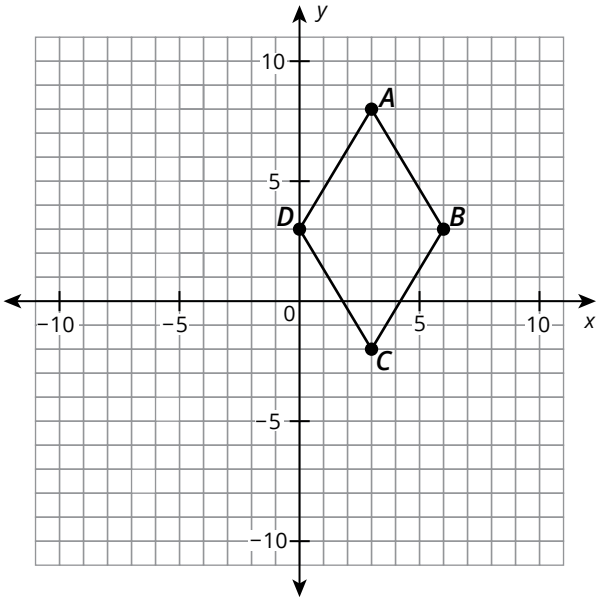

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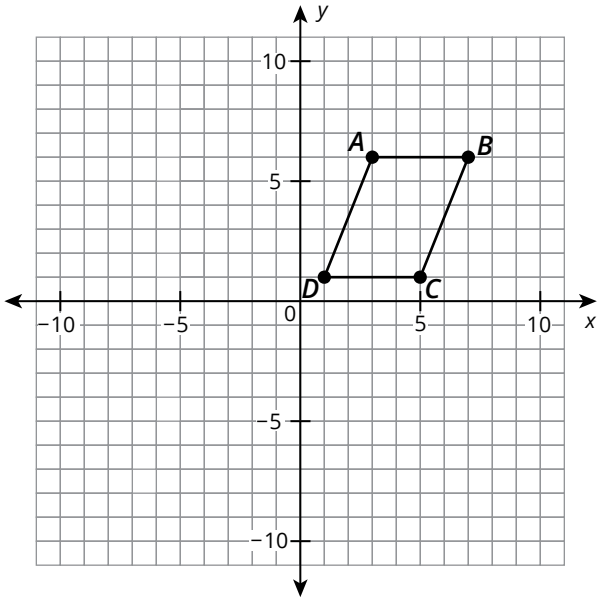

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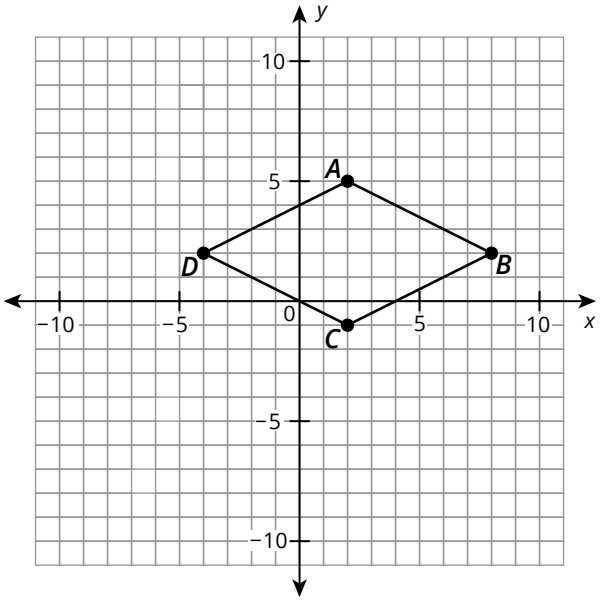
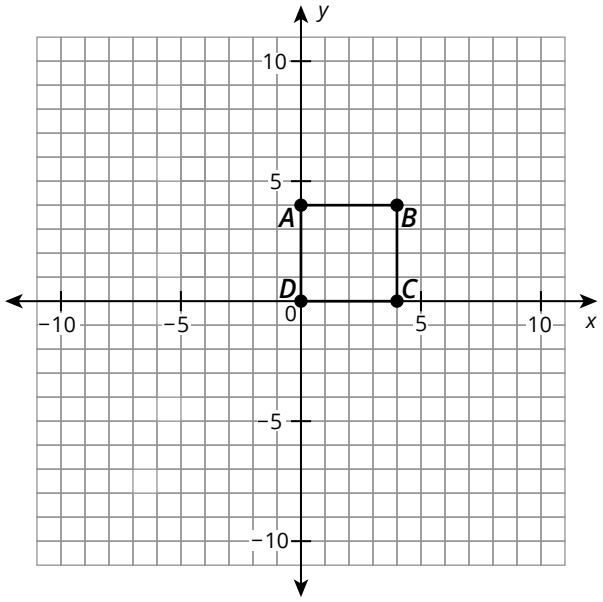


6.



Extension

1. The lines that connect points A, B, and C in a coordinate plane form a right triangle. Point A is located at $(-2, 5)$. Point B is located 6 units down from point A and an unknown distance to the left. Point C is located 4 units to the right of point A and an unknown distance down. The angle at point B is a right angle. The slope of the line between point B and point C is $-\frac{1}{3}$. The distance between point A and point B is $\sqrt{40}$. Determine the coordinates of point B and point C.

Spaced Practice

1. Write the equation of a line that passes through the point $(-8, 2)$ and is parallel to the line $3x - 2y = 12$.

2. Write the equation of a line that passes through the point $(5, -7)$ and is perpendicular to the line $-2x + 6y = -4$.

3. List the properties that are shared by each pair of polygons.

   a. squares and equilateral triangles

   b. rectangles and rhombi

IV. Classifying Triangles on the Coordinate Plane

Topic Practice

A. Write the converse of each conditional statement. Then, determine the validity of the statement. Explain your reasoning.

1. Conditional Statement: "If a number is divisible by 4, then it is even."

2. Conditional Statement: "If a person is on an airplane, then they are traveling to a different location."

3. Conditional Statement: "If a triangle is equilateral, then it has three congruent sides."

4. Conditional Statement: "If an animal is a dog, then it is a mammal. "

5. Conditional Statement: "If $x > 5$, then $x + 2 > 7$."

6. Conditional Statement: "If an angle measures 90°, then it is a right angle."

B. Consider the Pythagorean triple 3, 4, 5. Multiply by the given factor to generate a new Pythagorean triple and verify your answer using the Pythagorean Theorem.

1. Multiply by 6.

2. Multiply by 7.

3. Multiply by 10.

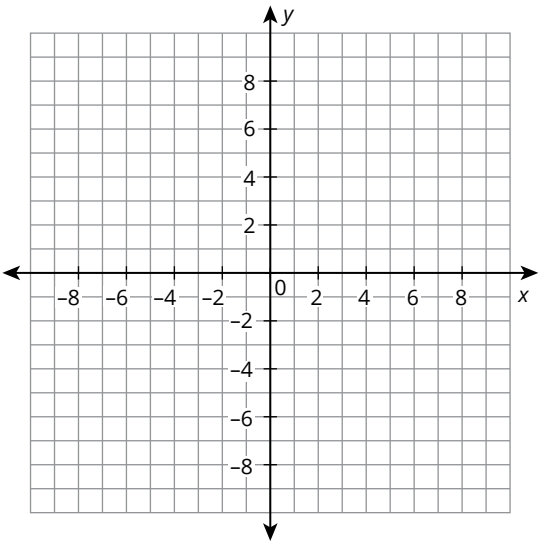
4. Multiply by 11.

5. Multiply by 4.

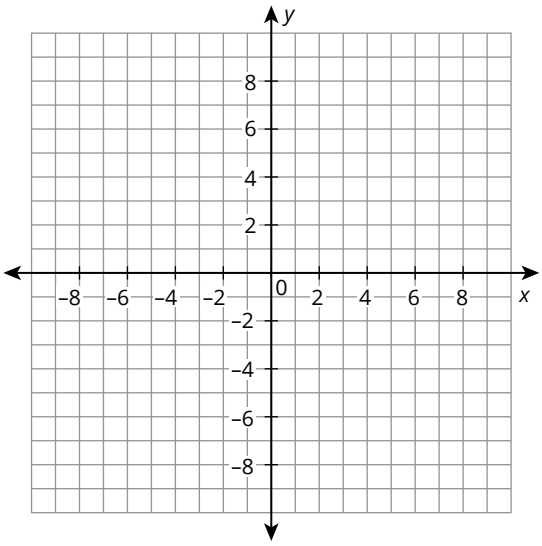
6. Multiply by 12.

C. Graph △ *ABC* using each set of given points. Determine if △ *ABC* is scalene, isosceles, or equilateral.

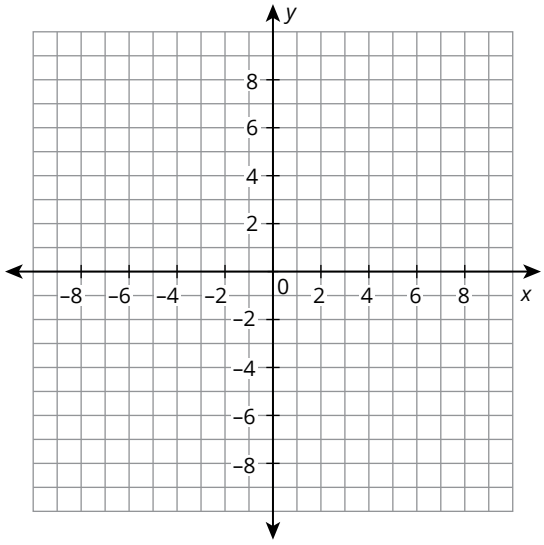
1. *A* (–3, 1), *B* (–3, –3), *C* (1, 0)



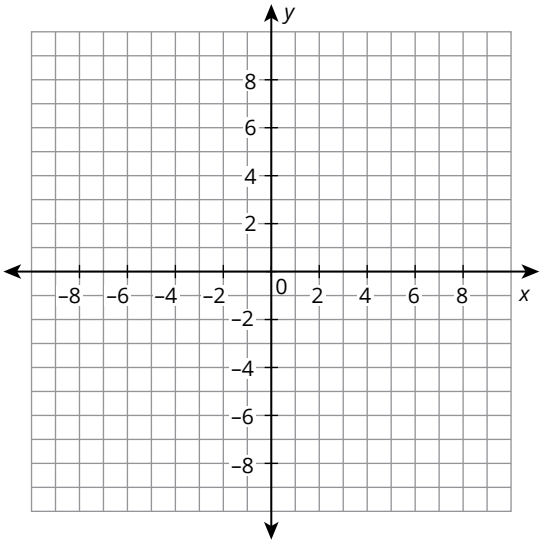
2. *A* (8, 5), *B* (8, 1), *C* (4, 3)



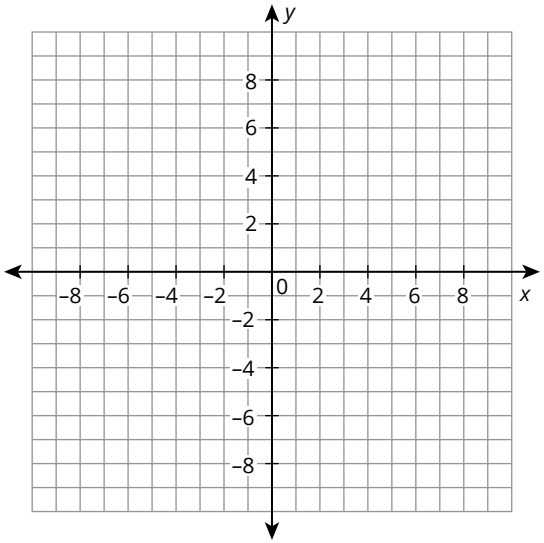
3. *A* (5, 8), *B* (5, 2), *C* (–3, 5)



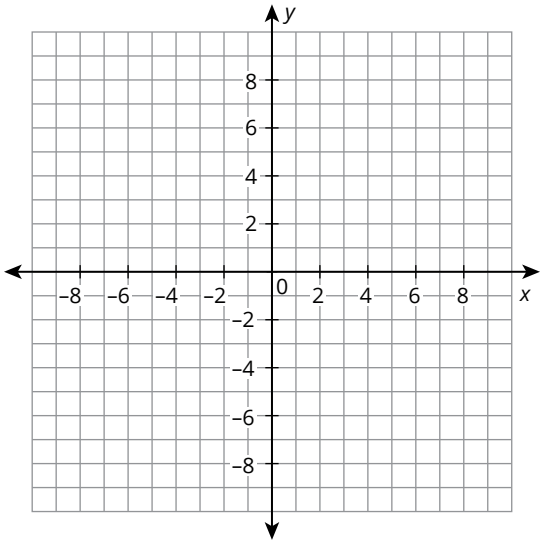
4. *A* (–2, –6), *B* (6, –6), *C* (2, –3)



5. A (0, 0), B (4, 0), C (3, 7)



6. A (−6, 4), B (0, 4), C (−2, −2)



D. Determine whether each set of side lengths creates an acute, right, or obtuse triangle. Explain your reasoning. If it is a right triangle, state whether it is a Pythagorean triple.

1. 3 m, 2.9 m, 5 m

2. 12 in., 16 in., 20 in.

3. 30 cm, 22 cm, 27 cm

4. 15 cm, 15 cm, $15\sqrt{2}$ cm

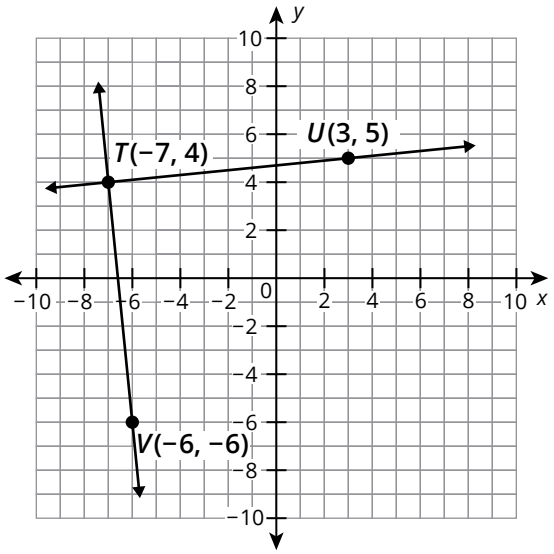
5. 6 ft, 3.1 ft, 9 ft

6. 7.4 in., 8.1 in., 9.8 in.

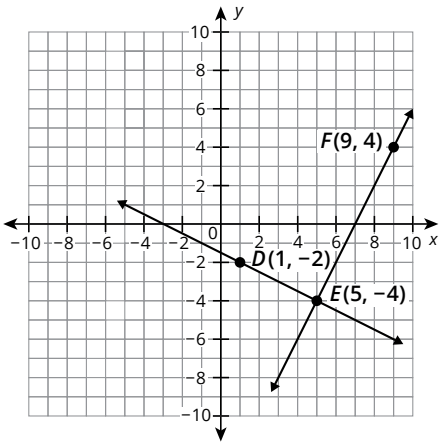
CC BY NC

E. Use the distance formula to verify that the pairs of lines are perpendicular.

1.



2.



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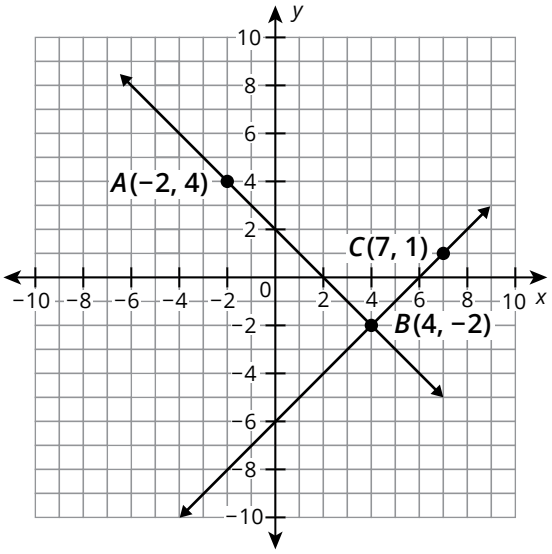

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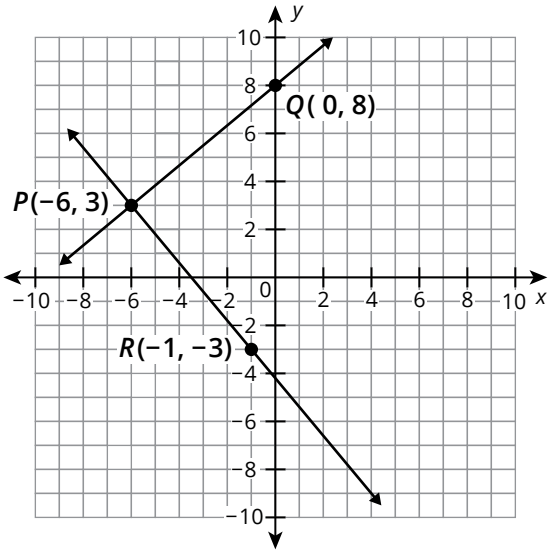

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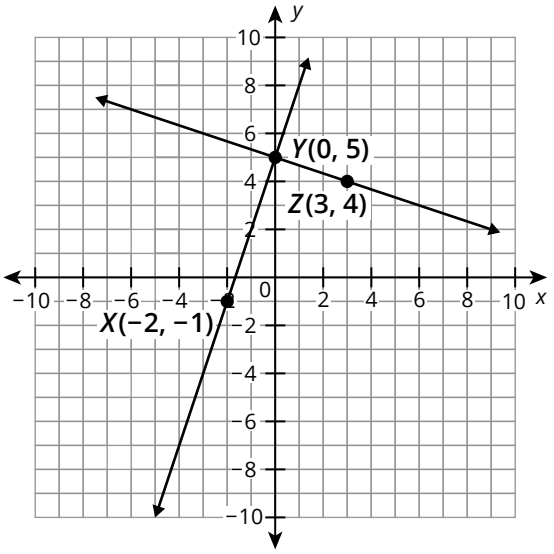
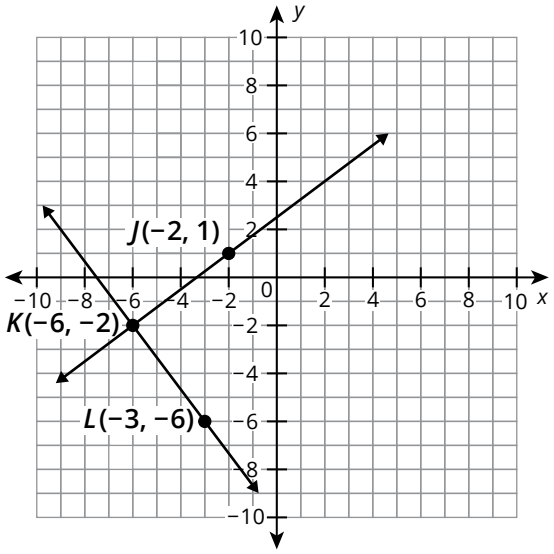
5.

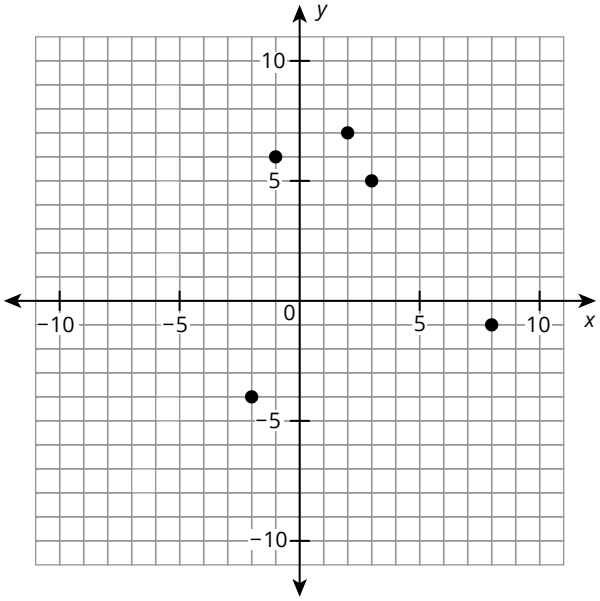


6.



Extension

1. Consider the five points graphed on the coordinate plane. Determine a sixth point that would lie in the exact middle of the given points. Graph and label the sixth point on the coordinate plane.



Spaced Practice

1. Write the equation of a line that passes through the point (3, –1) and is parallel to the line $14x - 7y = 4$.

2. Write the equation of a line that passes through the point (–6, 0) and is perpendicular to the line $3x + 5y = 2$.

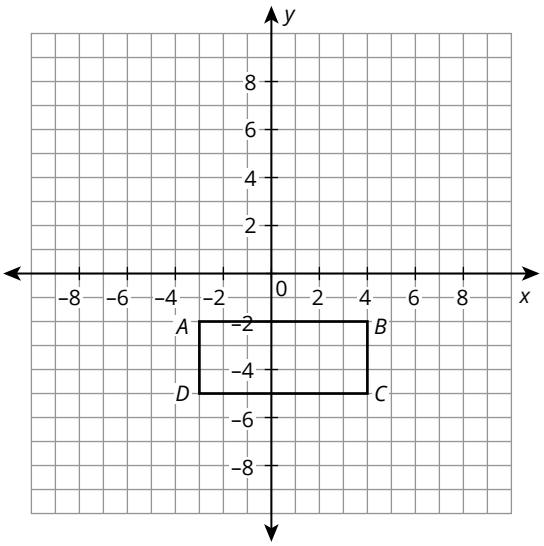
CC BY NC

V. Area and Perimeter on the Coordinate Plane

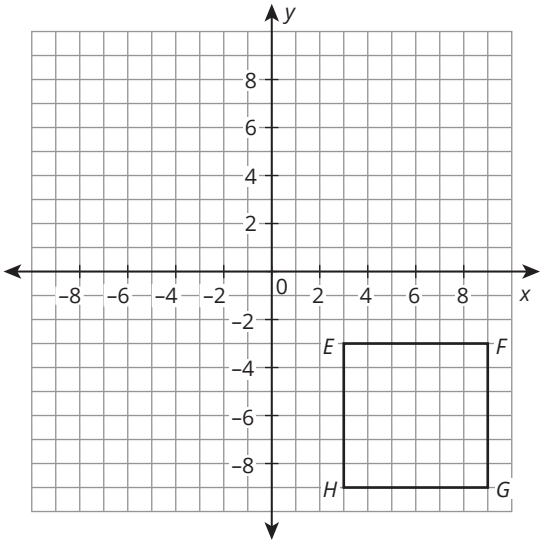
Topic Practice

A. Translate each given rectangle or square such that one vertex of the image is located at the origin and label the vertices of the image. Calculate the perimeter and area of the image. Round your answer to the nearest hundredth, if necessary.

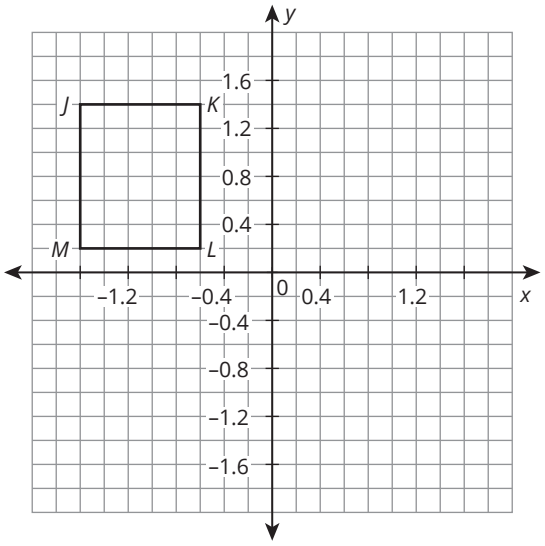
1. Rectangle *ABCD*



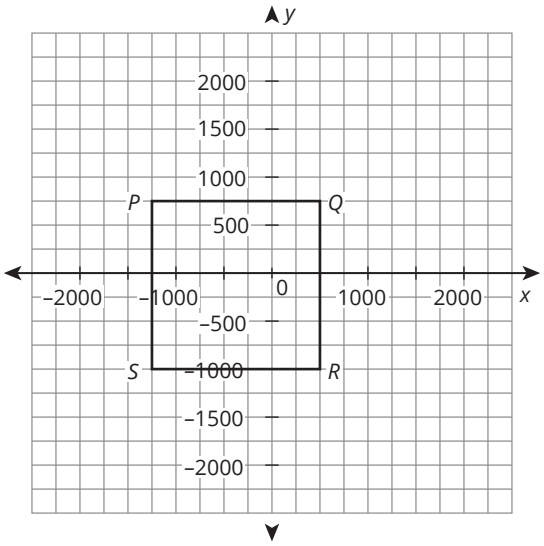
2. Square *EFGH*



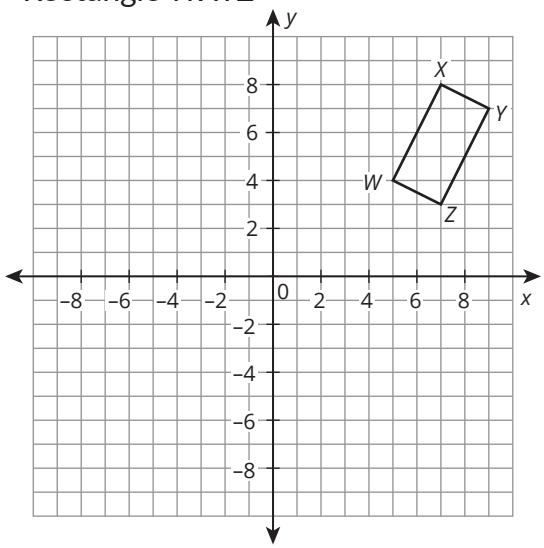
3. Rectangle *JKLM*



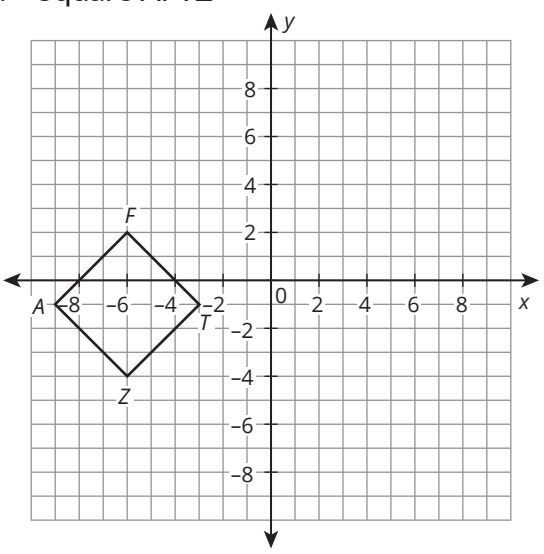
4. Square *PQRS*



5. Rectangle *WXYZ*



6. Square *AFTZ*



B. Describe how the given changes to the dimensions of the figure will affect its perimeter or area. Determine whether the change is proportional or non-proportional.

1. What happens to the area of a triangle with base 12 units and height 7 units when both the base and the height are increased by a factor of 3?

2. What happens to the perimeter of a triangle with side lengths of 3, 5, and 6 units when each side length is decreased by a factor of $\frac{1}{4}$?

3. What happens to the perimeter of a rectangle with length 29 units and width 18 units when its length is doubled?

4. What happens to the perimeter of a regular hexagon with side lengths of 5 units when each side length is increased by a factor of 4?

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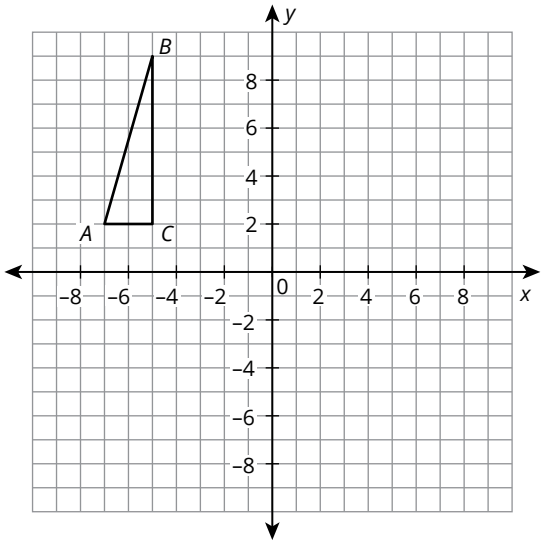
5. What happens to the area of a rectangle with length 12 units and width 15 units when its width is decreased by a factor of $\frac{1}{3}$?

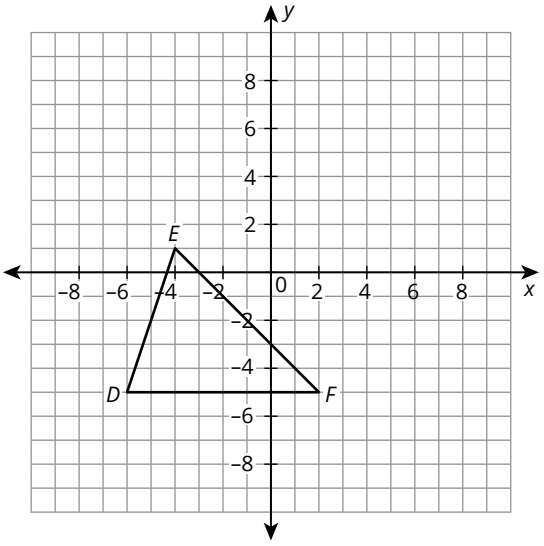
6. What happens to the area of a triangle with base 10 units and height 9 units when its base is doubled and height is tripled.

C. Determine the perimeter of each given figure on the coordinate plane. Round your answer to the nearest hundredth, if necessary.

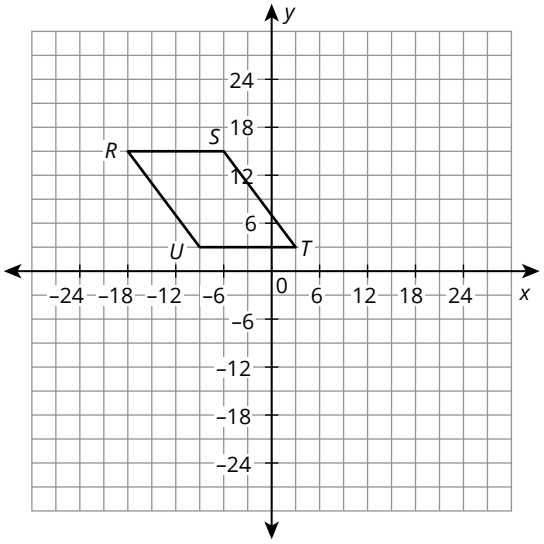
1. Triangle *ABC*



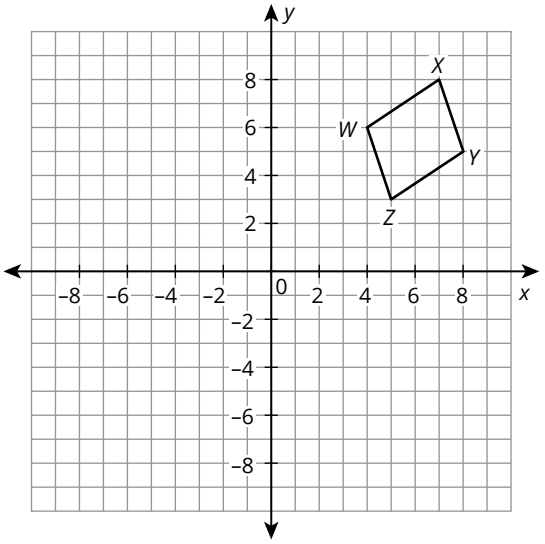
2. Triangle *DEF*



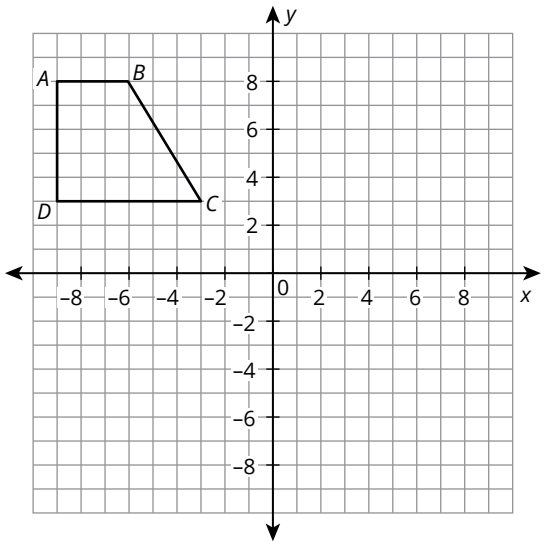
3. Parallelogram *RSTU*



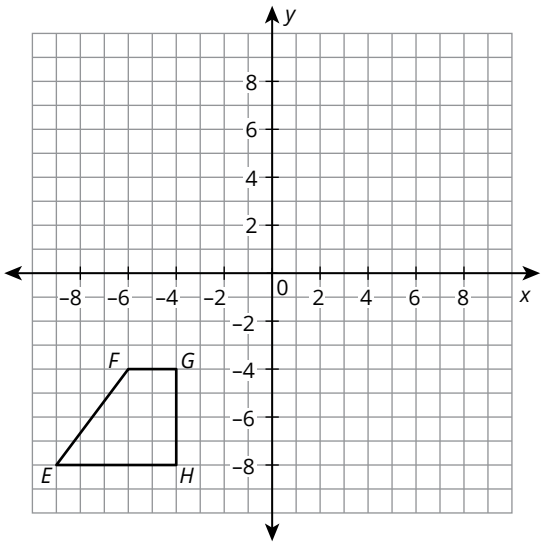
4. Parallelogram *WXYZ*



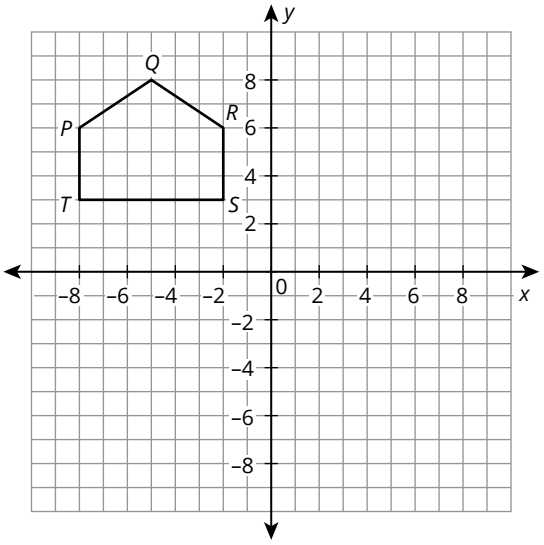
5. Trapezoid *ABCD*



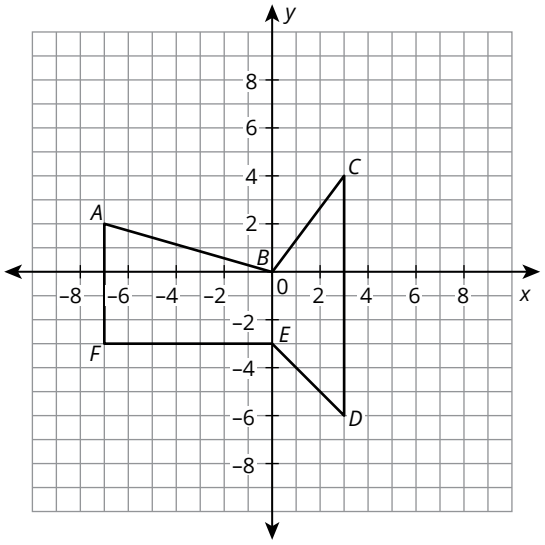
6. Trapezoid *EFGH*



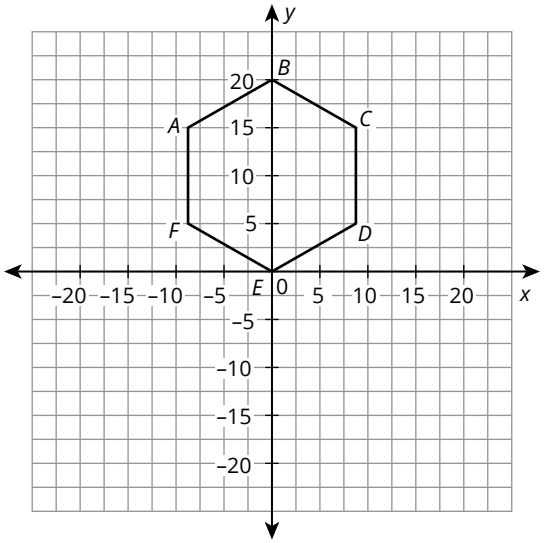
7. Figure *PQRST*



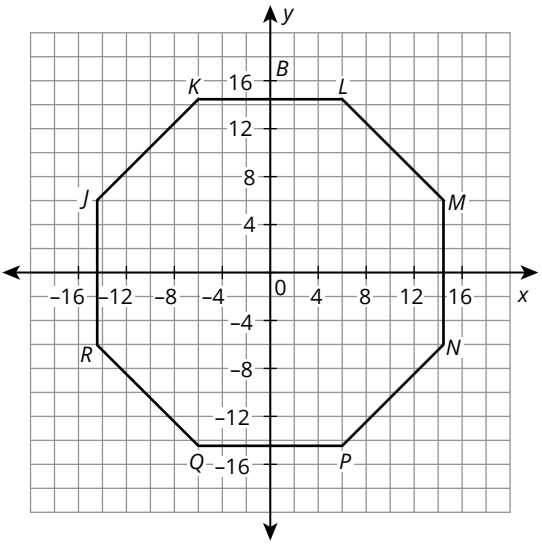
8. Figure *ABCDEF*



9. Regular hexagon *ABCDEF* with coordinates $A\ (-5\sqrt{3}, 15,)$ $B\ (0, 20)$, $C\ (5\sqrt{3}, 15,)$ $D\ (5\sqrt{3}, 5)$, $E\ (0, 0)$, and $F\ (-5\sqrt{3}, 5)$

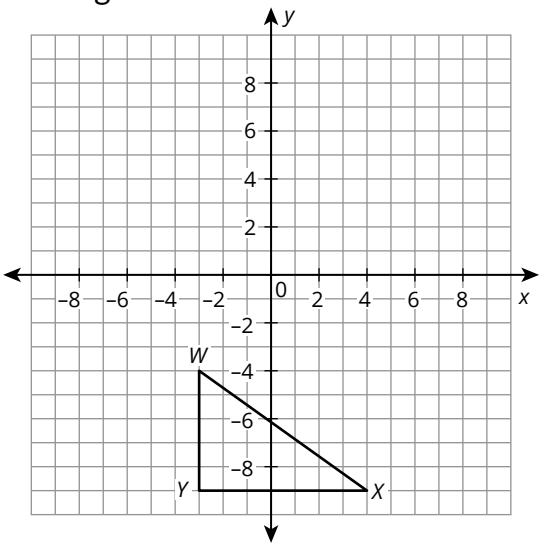


10. Regular octagon *JKLMNPQR* with coordinates $J\ (-6 - 6\sqrt{2}, 6)$, $K\ (-6, 6 + 6\sqrt{2})$, $L\ (6, 6 + 6\sqrt{2})$, $M\ (6 + 6\sqrt{2}, 6)$, $N\ (6 + 6\sqrt{2}, -6)$, $P\ (6, -6 - 6\sqrt{2})$, $Q\ (-6, -6 - 6\sqrt{2})$, and $R\ (-6 - 6\sqrt{2}, -6)$

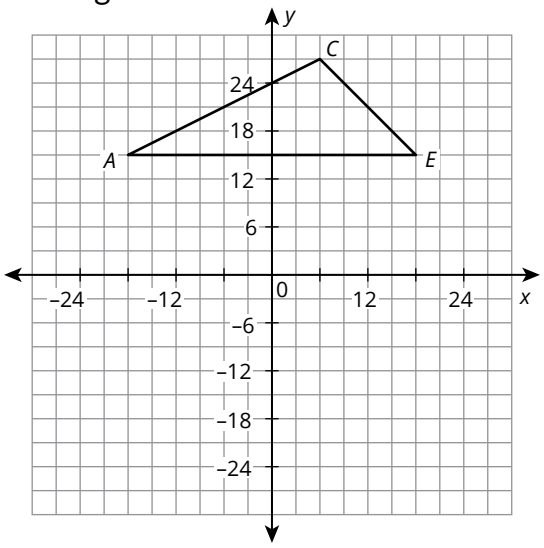


D. Determine the area of each given figure on the coordinate plane. Round your answer to the nearest hundredth, if necessary.

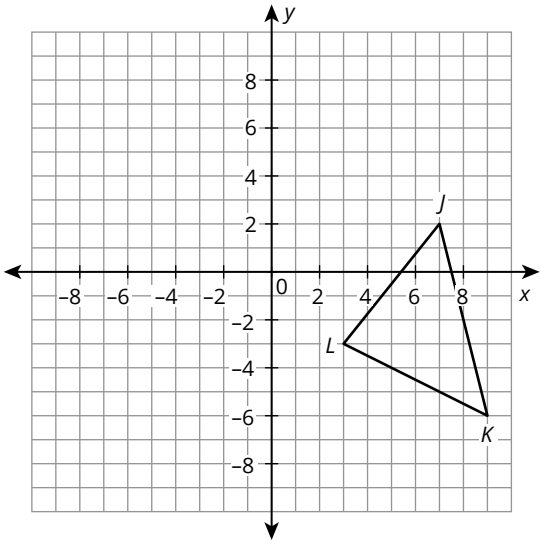
1. Triangle *WXY*



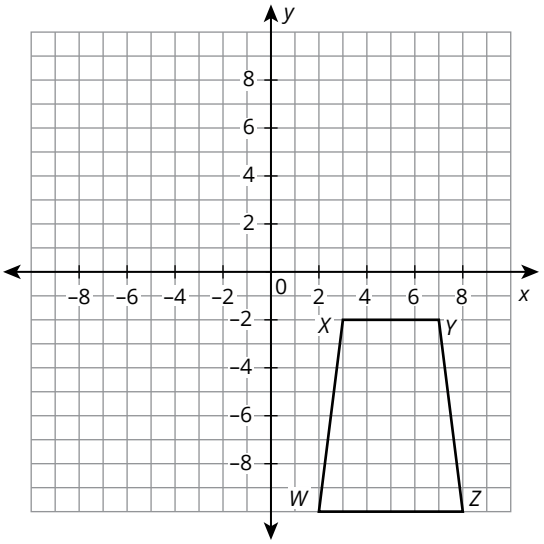
2. Triangle *ACE*



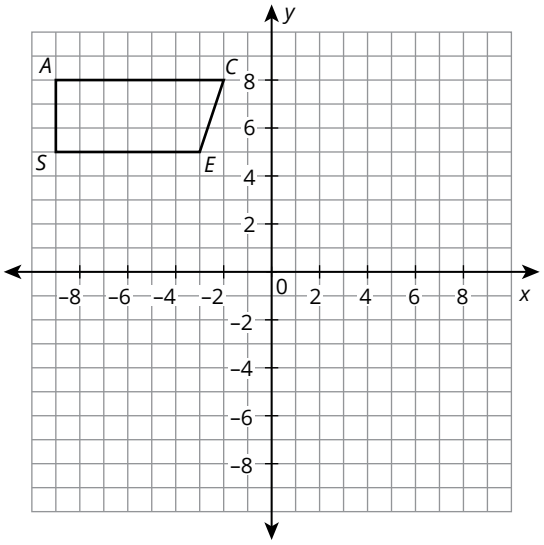
3. Triangle *JKL*



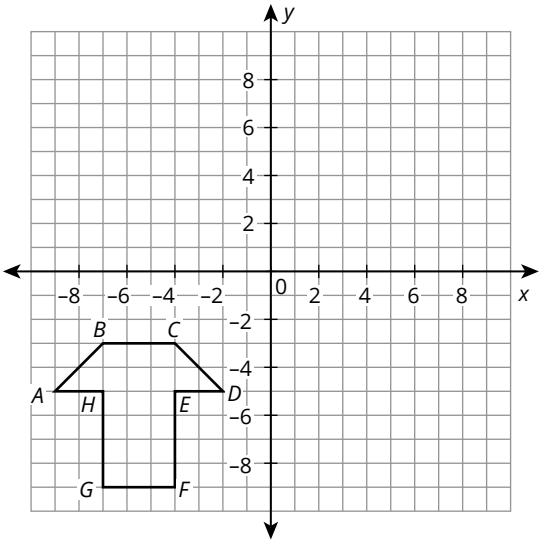
4. Trapezoid *WXYZ*



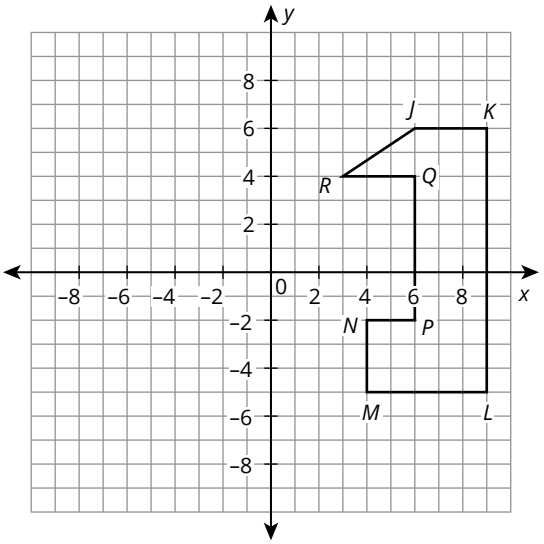
5. Trapezoid *ACES*



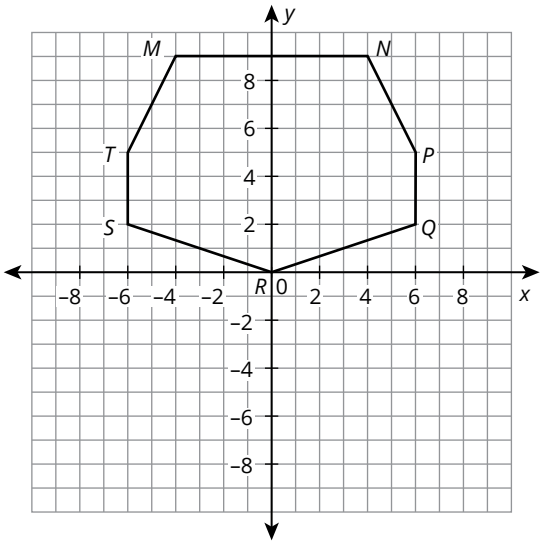
6. Figure *ABCDEFGH*



7. Figure *JKLMNPQR*

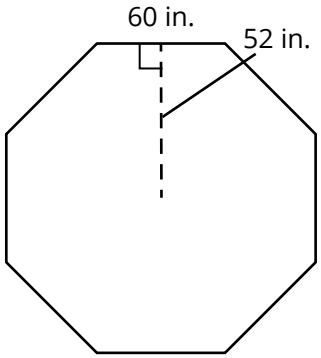


8. Figure *MNPQRST*

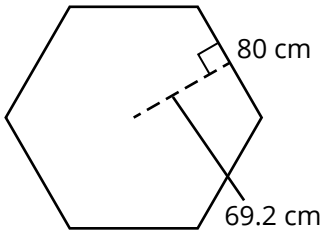


E. Calculate the area of each regular polygon.

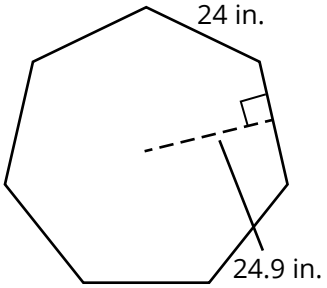
1.



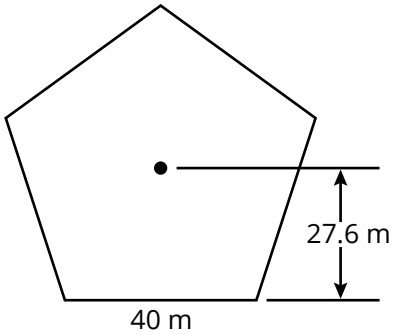
2.



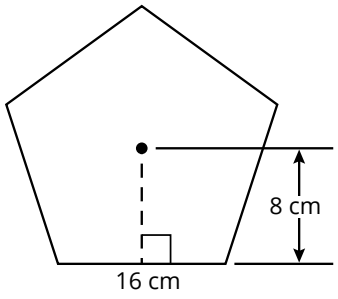
3.



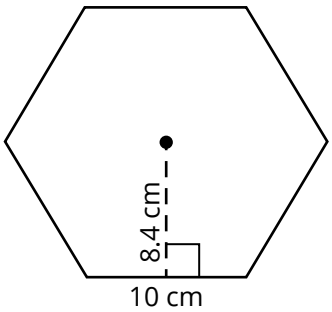
4.



5.

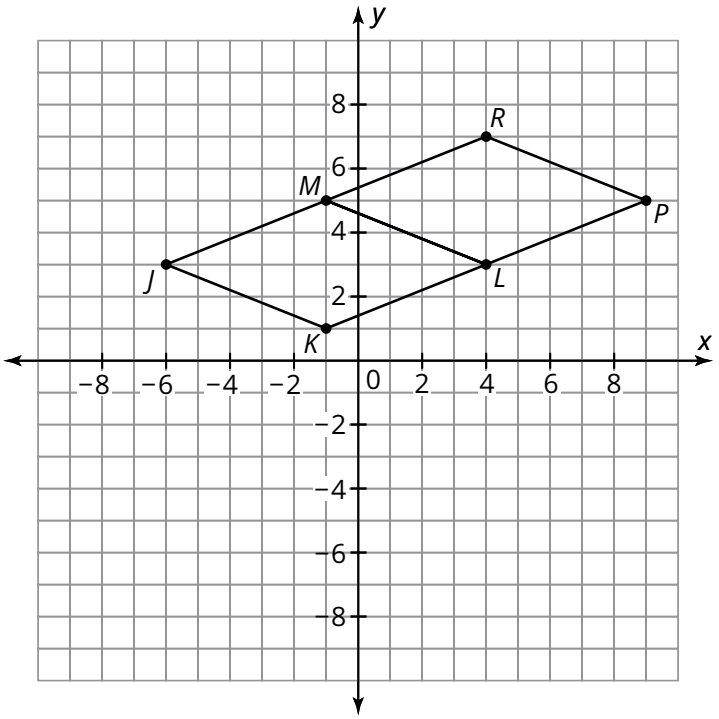


6.



Extension

1. Parallelograms *JKLM* and *JKPR* are given. Without calculating each area, determine whether or not the area of parallelogram *JKPR* is twice that of the area of parallelogram *JKLM*. Explain how you determined your answer.



Spaced Practice

1. The quadrilateral *ABCD* has the vertices *A* (−5, 4), *B* (0, 6), *C* (1, 3), and *D* (−4, 1). Determine whether it can be classified as a parallelogram. Justify your reasoning.

2. Triangle *DEF* has the vertices *D* (−2, 3), *E* (2, −1), and *F* (−5, −4.) Determine whether it is scalene, isosceles, or equilateral. Explain your reasoning.

CC BY NC

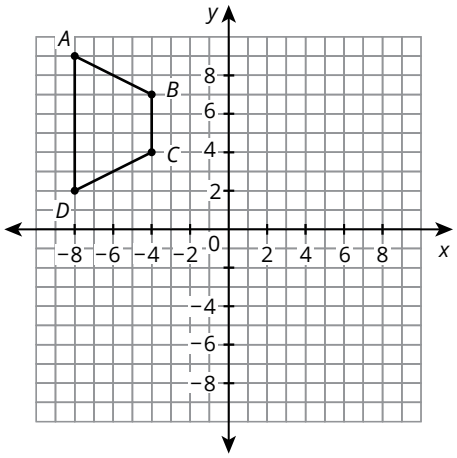
Name _____ Date _____

I. Translations On and Off the Coordinate Plane

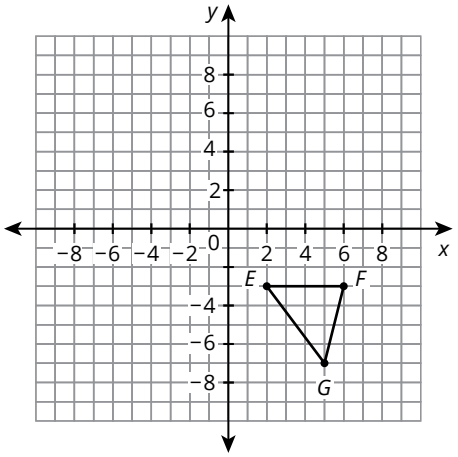
Topic Practice

A. Transform each given geometric figure on the coordinate plane as described. Then describe the transformation using coordinate notation.

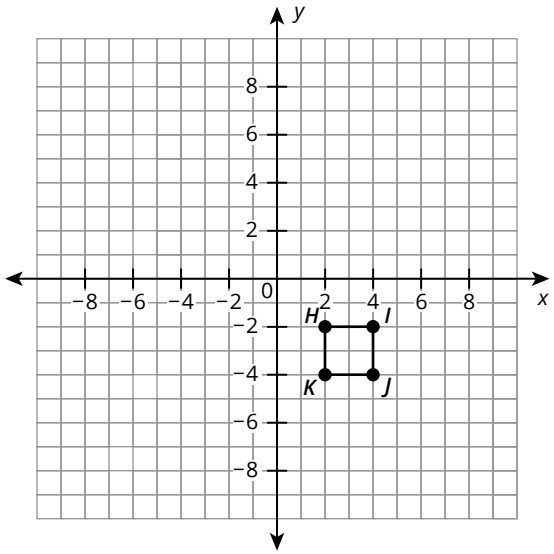
1. Translate trapezoid *ABCD* 11 units to the right.



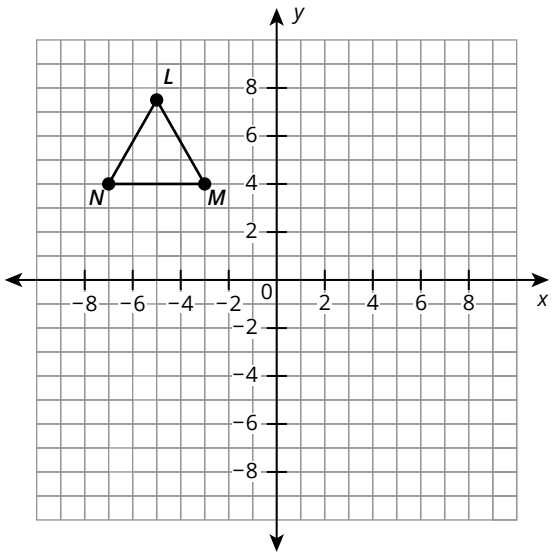
2. Translate triangle *EFG* 8 units up.



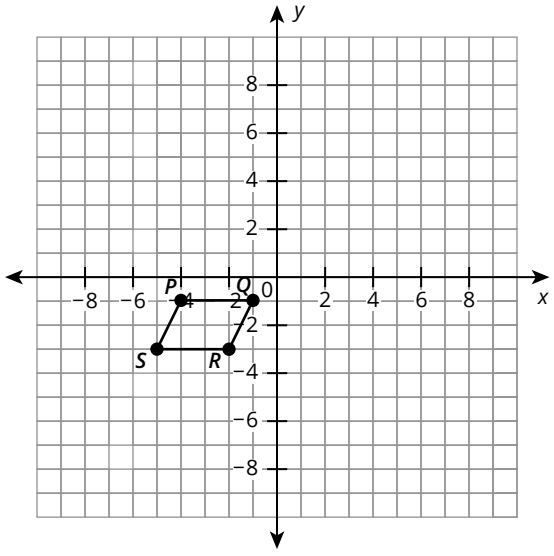
3. Translate square *HIJK* 5 units to the left.



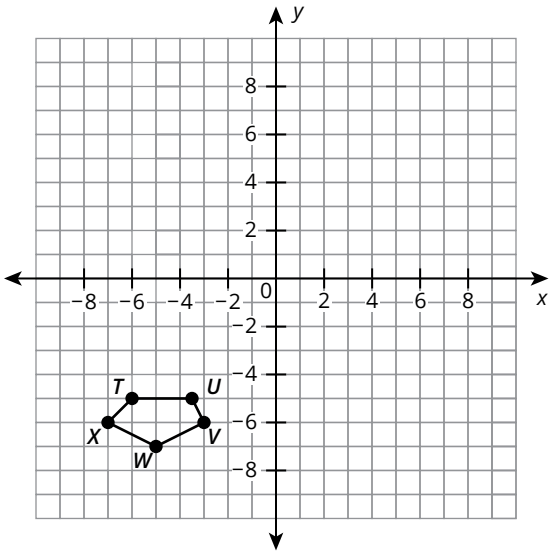
4. Translate isosceles triangle *LMN* 7 units down.



5. Translate parallelogram *PQRS* 4 units to the right.

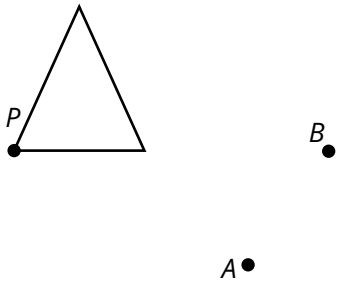


6. Translate pentagon *TUVWX* 10 units up.

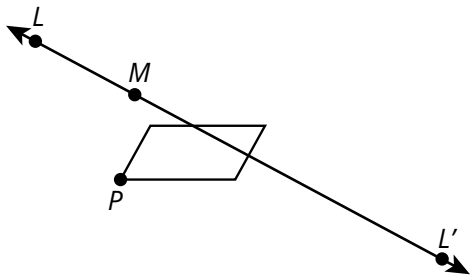


B. Complete each translation, given the function description.

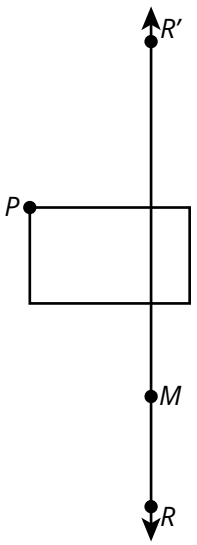
1. Every point on the triangle moves a distance equal to $\overline{BA}$ in the direction of $\overline{BA}$.



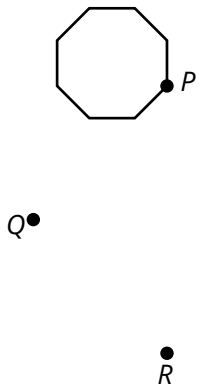
2. Every point on the parallelogram moves a distance equal to AB in the direction of $\overline{LM}$.



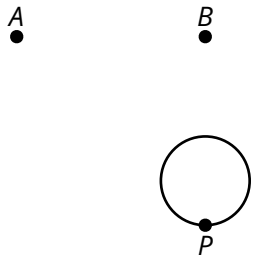
3. Every point on the rectangle moves a distance equal to $\overline{RM}$ in the direction of $\overline{RM}$.



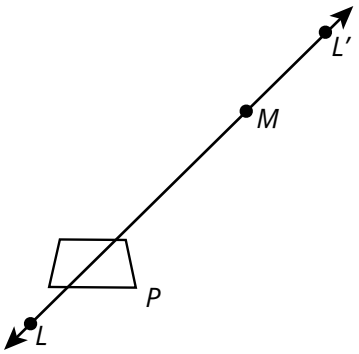
4. Every point on the octagon moves a distance equal to $\overline{QR}$ in the direction of $\overline{QR}$.



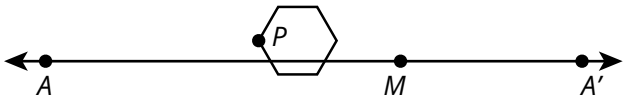
5. Every point on the circle moves a distance equal to $\overline{BA}$ in the direction of $\overline{BA}$.



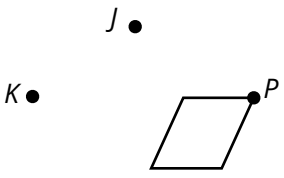
6. Every point on the trapezoid moves a distance equal to $\overline{ML'}$ in the direction of $\overline{ML'}$.



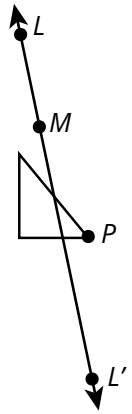
7. Every point on the hexagon moves a distance equal to $\overline{A'M}$ in the direction of $\overline{A'M}$.



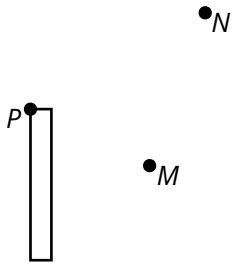
8. Every point on the rhombus moves a distance equal to $\overline{JK}$ in the direction of $\overline{JK}$.



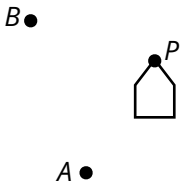
9. Every point on the triangle moves a distance equal to $\overline{LM}$ in the direction of $\overline{LM}$.



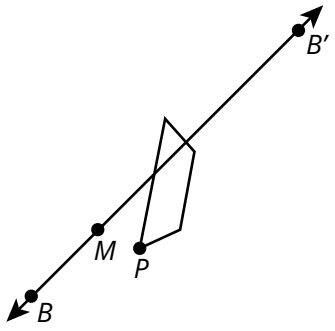
10. Every point on the rectangle moves a distance equal to $\overline{MN}$ in the direction of $\overline{MN}$.



11. Every point on the pentagon moves a distance equal to $\overline{BA}$ in the direction of $\overline{BA}$.

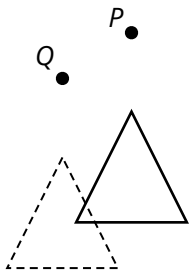


12. Every point on the trapezoid moves a distance equal to $\overline{BM}$ in the direction of $\overline{BM}$.

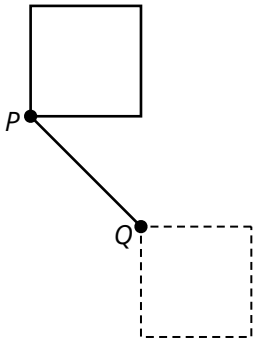


C. Describe the translation function that maps each image to its pre-image. Each solid figure is a pre-image and each dashed figure is an image.

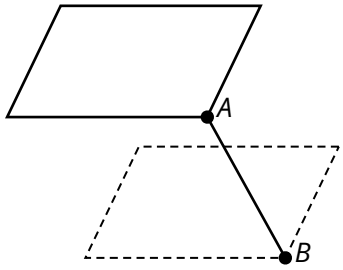
1.



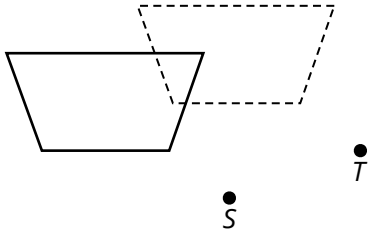
2.



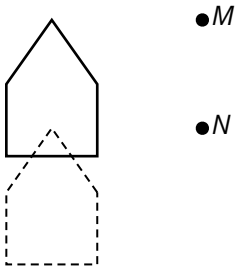
3.



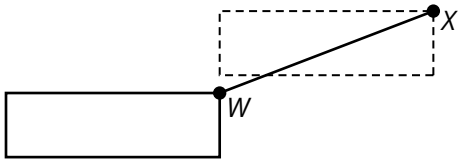
4.



5.

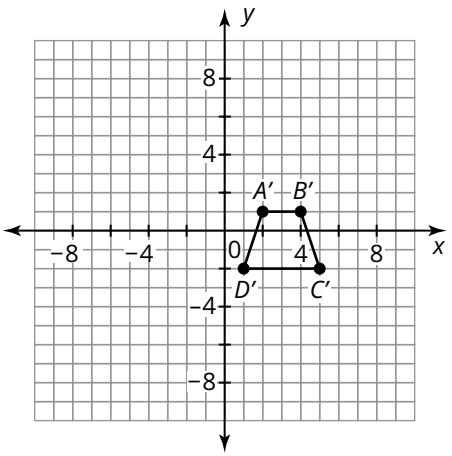


6.



Extension

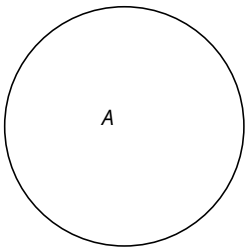
1. Trapezoid $A'B'C'D'$ is the image of trapezoid $ABCD$ after a transformation of $(x, y) \rightarrow (x + 9, y + 2)$. Identify the location of each vertice of the pre-image.



Spaced Practice

1. Construct a square inscribed in a circle using the given line segment.

2. Construct a regular hexagon inscribed inside circle A.

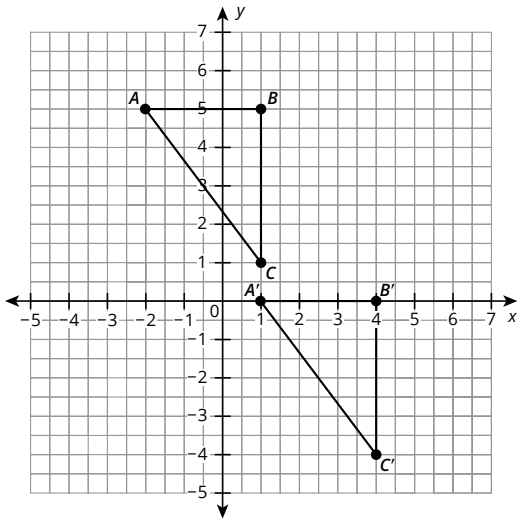


II. Sequences of Translations

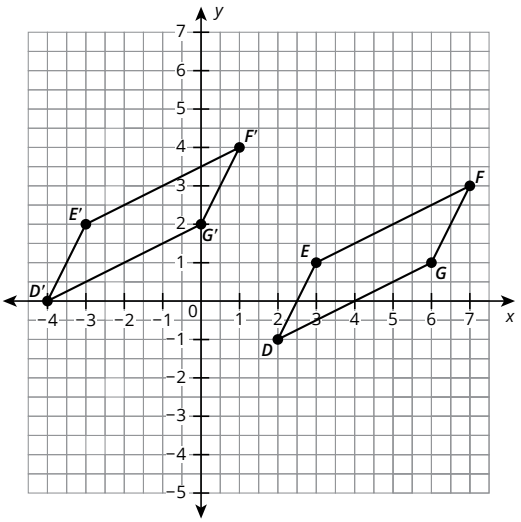
Topic Practice

A. Identify the sequence of translations that map the pre-image to the image.

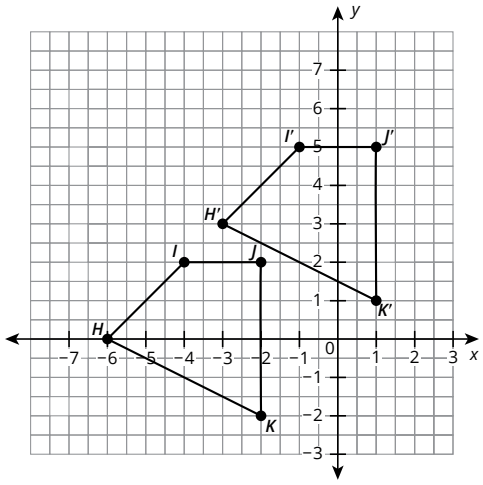
1.



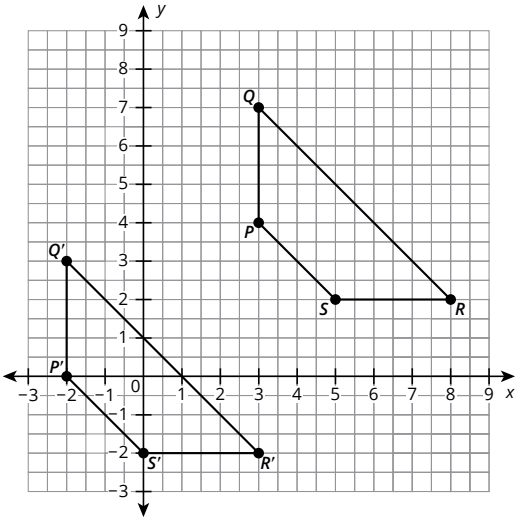
2.



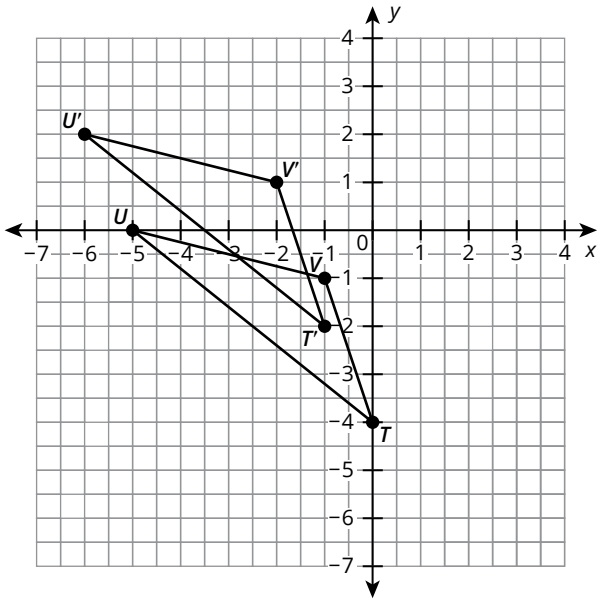
3.



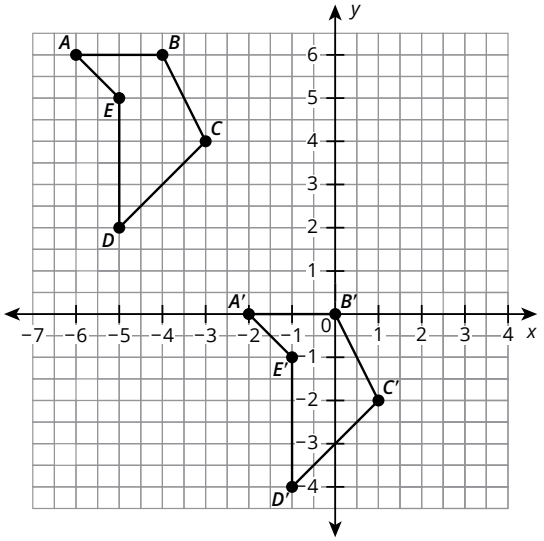
4.



5.

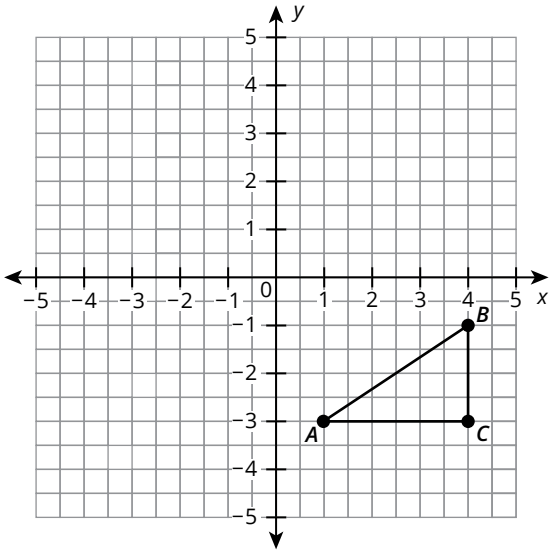


6.

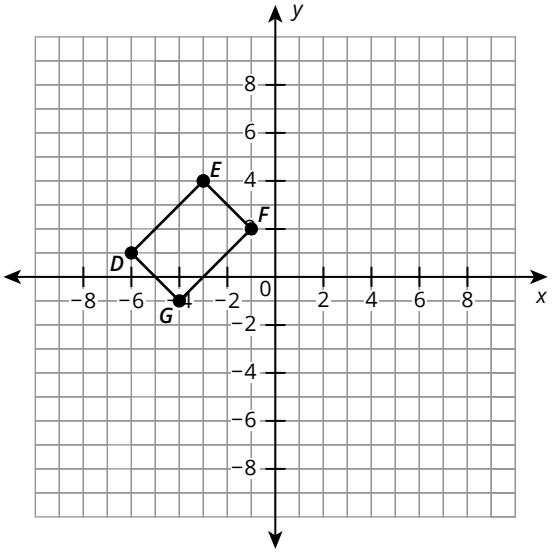


B. Transform each given geometric figure on the coordinate plane as described. Then describe the transformation using coordinate notation.

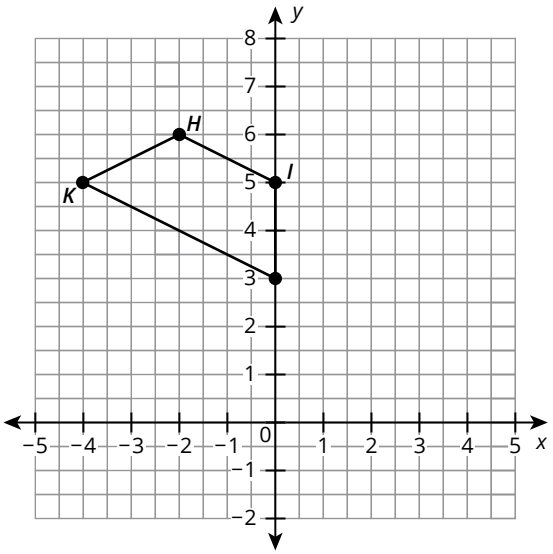
1. Translate triangle *ABC* 5 units left and 1 unit up.



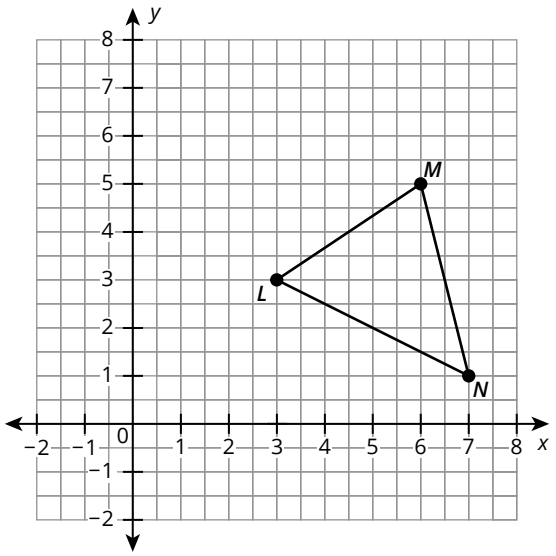
2. Translate rectangle *DEFG* 4 units down and 6 units right.



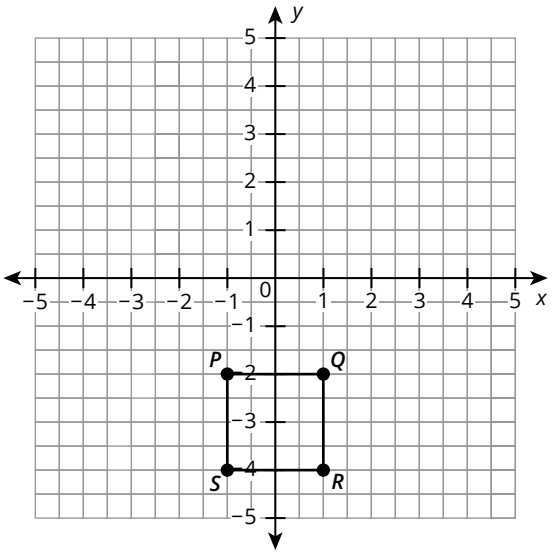
3. Translate trapezoid *HIJK* 2 units right and 3 units down.



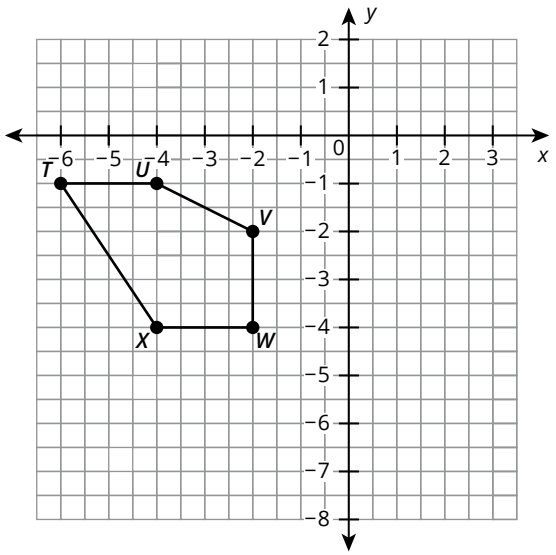
4. Translate triangle *LMN* 2 units up and 2 units left.



5. Translate square *PQRS* 1 unit right and 5 units up.

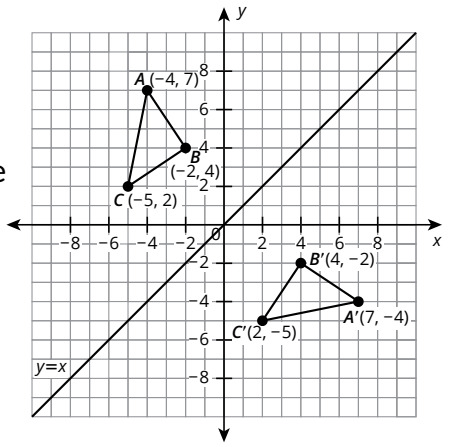


6. Translate pentagon *TUVWX* 2 units down and 5 units right.



Extension

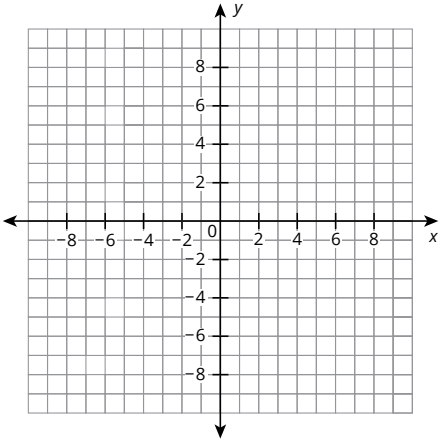
Consider triangle *ABC* with its vertices at *A*(−4, 7,) *B*(−2, 4) and *C*(−5, 2) and its image, when reflected along the line $y = x$.



a. Make a conjecture about the vertices from the pre-image to the reflected image.

b. Describe the transformation using coordinate notation.

c. Draw the image when triangle *ABC* is rotated 180° counterclockwise about the origin.



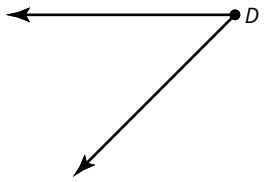
d. Make a comparison of the images when the pre-image is reflected along the line $y = x$ and rotated 180° counterclockwise about the origin.

Spaced Practice

1. Construct each figure described.

   a. Duplicate $\overline{AB}$.

   b. Duplicate $\angle D$.



2. Determine whether the two lines are parallel, perpendicular, or neither. Explain your reasoning.

   a. $2x - 3y = 15$ and $y = -\frac{3}{2}x - 10$

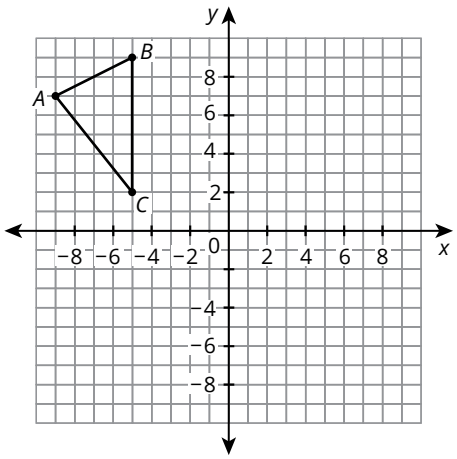
   b. $-\frac{6}{5}x + 4y = \frac{1}{5}$ and $-3(x - \frac{10}{3}y) = 17$

III. Reflections On and Off the Coordinate Plane

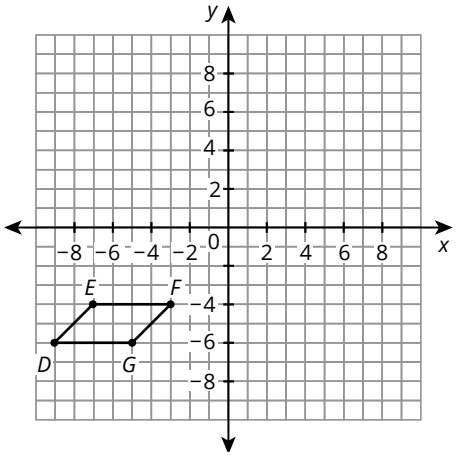
Topic Practice

A. Transform each given geometric figure on the coordinate plane as described. Then describe the transformation(s) using coordinate notation.

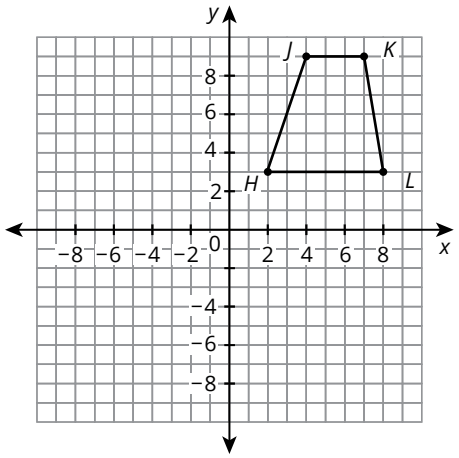
1. Reflect triangle *ABC* over the *y*-axis.



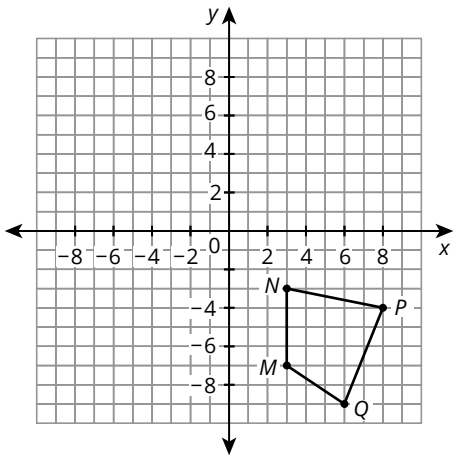
2. Reflect parallelogram *DEFG* over the *x*-axis.



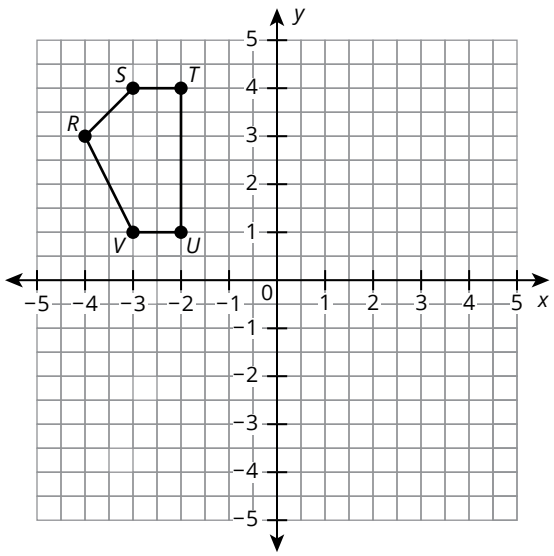
3. Reflect trapezoid *HJKL* over the *x*-axis.



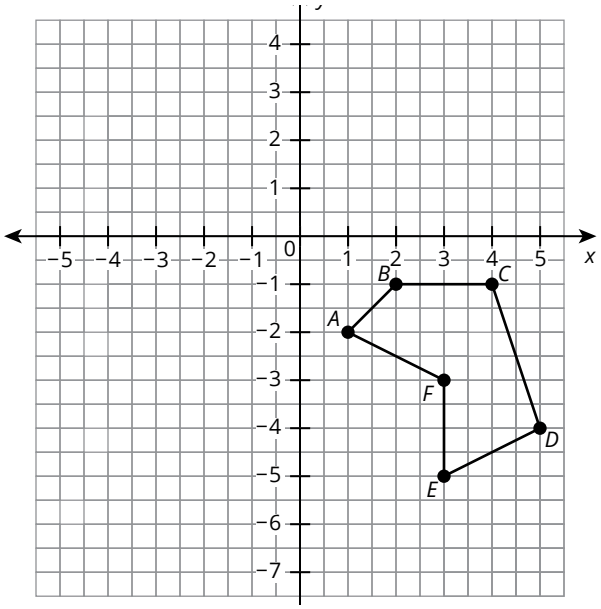
4. Reflect quadrilateral *MNPQ* over the *y*-axis and translate 1 unit up.



5. Reflect pentagon *RSTUV* over the *x*-axis and translate 4 units right.

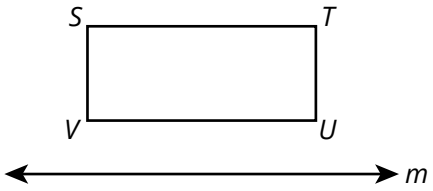


6. Reflect hexagon *ABCDEF* over the *y*-axis and translate 2 units down and 1 unit right.

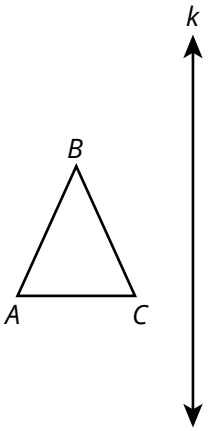


B. Complete each reflection using the description of the reflection function.

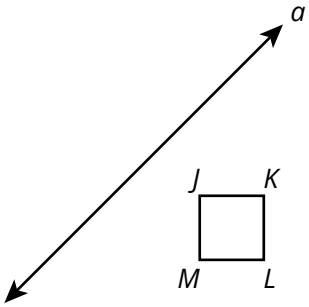
1. Reflect rectangle *STUV* across line *m*.



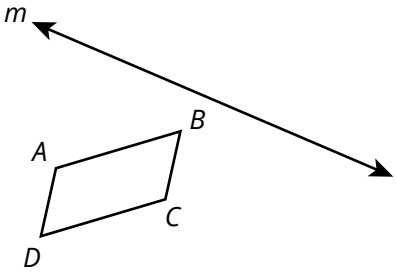
2. Reflect triangle *ABC* across line *k*.



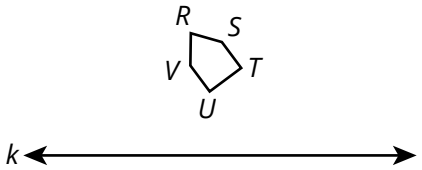
3. Reflect square *JKLM* across line *a*.



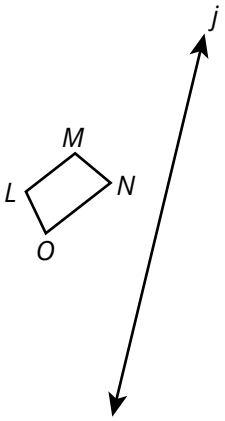
4. Reflect parallelogram *ABCD* across line *m*.



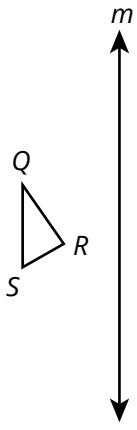
5. Reflect pentagon *RSTUV* across line *k*.



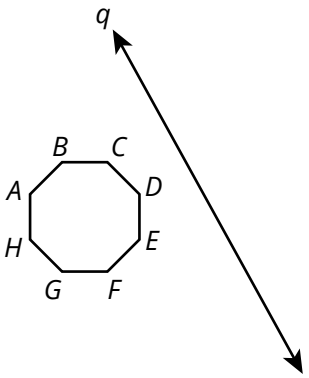
6. Reflect trapezoid *LMNO* across line *j*.



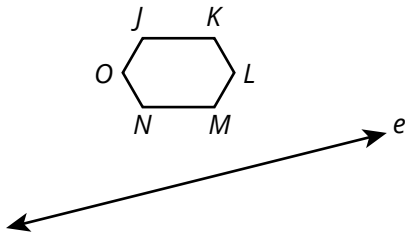
7. Reflect triangle *QRS* across line *m*.



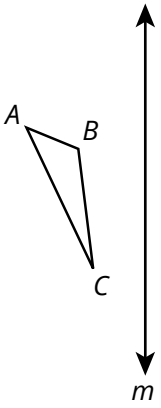
8. Reflect octagon *ABCDEFG* across line *q*.



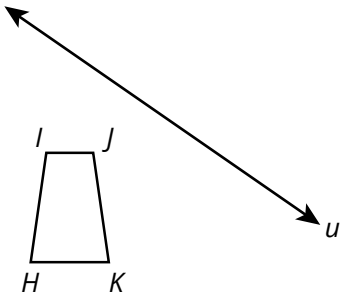
9. Reflect hexagon *JKLMNO* across line *e*.



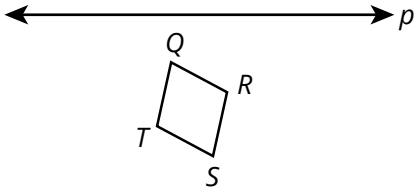
10. Reflect triangle *ABC* across line *m*.



11. Reflect trapezoid *HIJK* across line *u*.

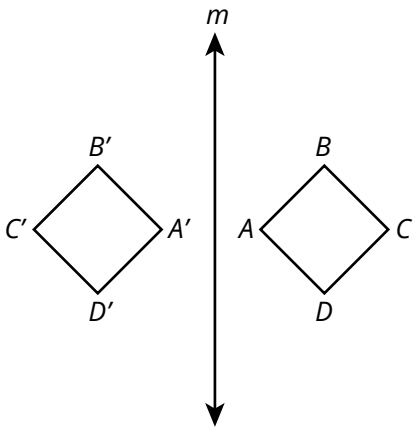


12. Reflect rhombus *QTRS* across line *p*.

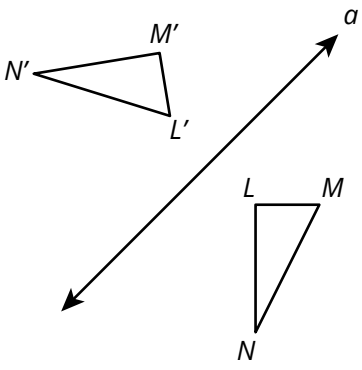


C. Describe the reflection function that maps each image to its pre-image.

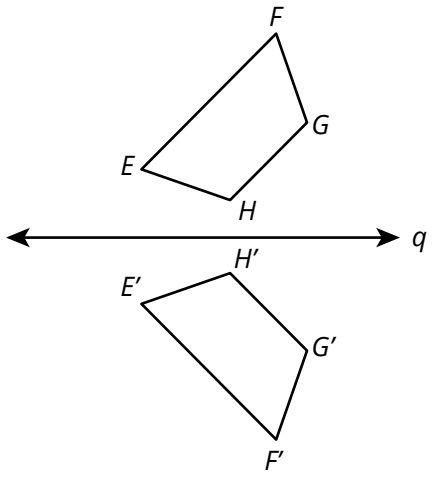
1.



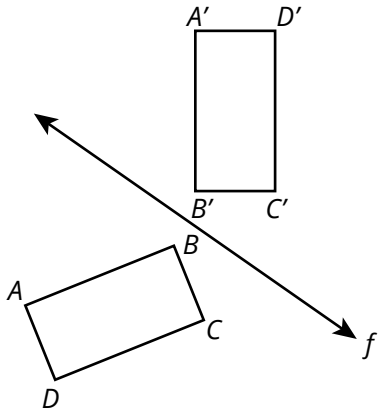
2.



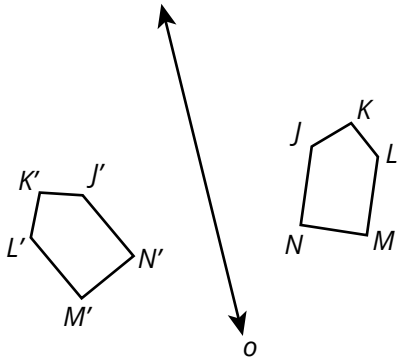
3.



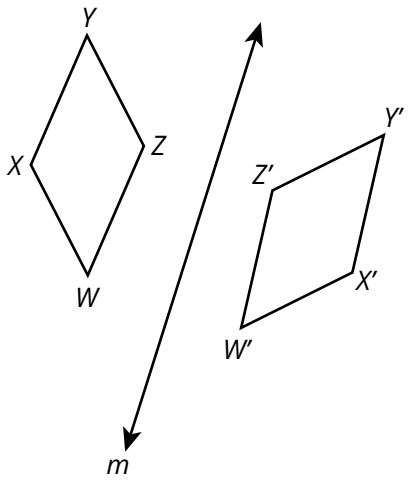
4.



5.

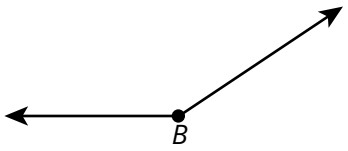


6.

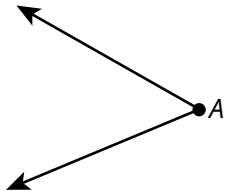


D. Construct the angle bisector of each angle.

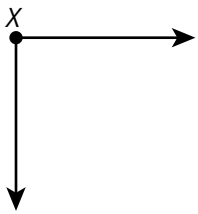
1.



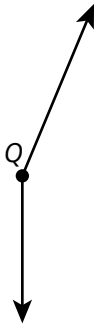
2.



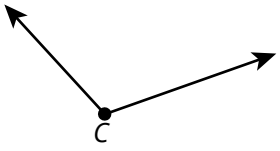
3.



4.



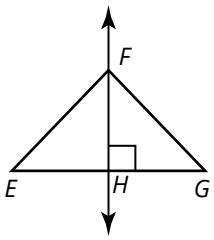
5.



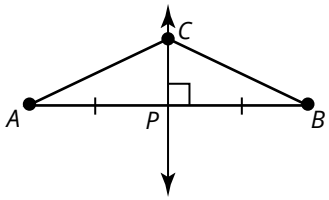
6.

E. Use the given perpendicular bisector to prove the perpendicular bisector theorem using a reflection across the perpendicular bisector.

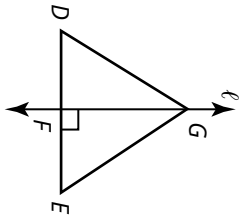
1. Given: line m is the perpendicular bisector of $\overline{EG}$.
   Prove: $\overline{FE} = \overline{FG}$.



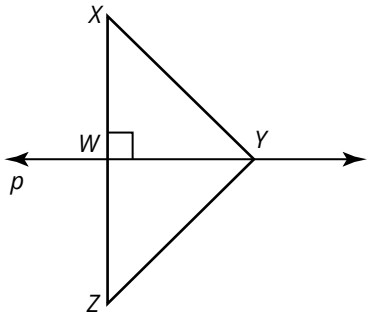
2. Given: line v is the perpendicular bisector of $\overline{AB}$.
   Prove: $\overline{CA} = \overline{CB}$



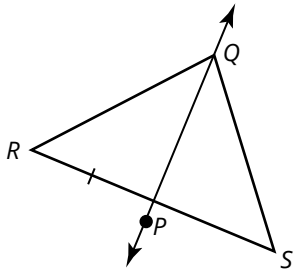
3. Given: line ℓ is the perpendicular bisector of $\overline{DE}$.
Prove: $\overline{GD} = \overline{GE}$



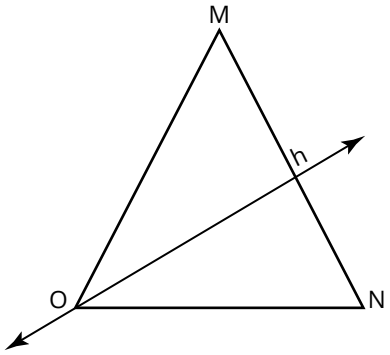
4. Given: line p is the perpendicular bisector of $\overline{XZ}$.
Prove: $\overline{YX} = \overline{YZ}$



5. Given: line k is the perpendicular bisector of $\overline{RS}$.
Prove: $\overline{QR} = \overline{QS}$

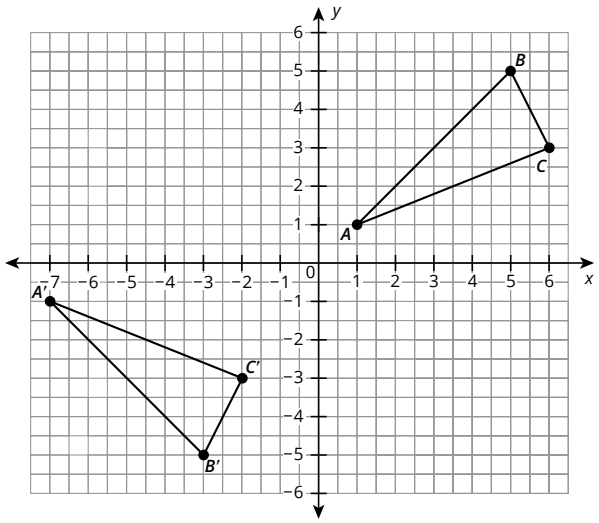


6. Given: line h is the perpendicular bisector of $\overline{MN}$.
Prove: $\overline{ON} = \overline{OM}$

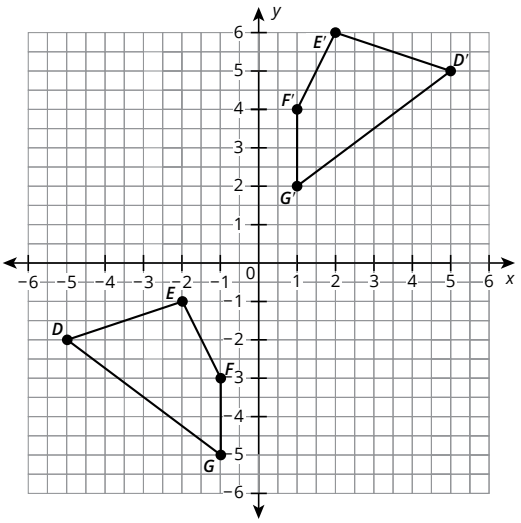


F. Identify the sequence of translations and reflections.

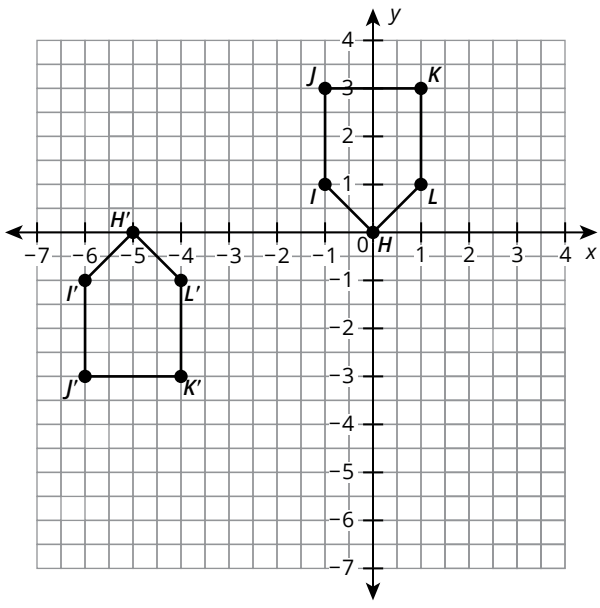
1.



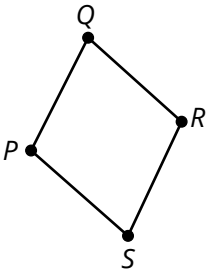
2.

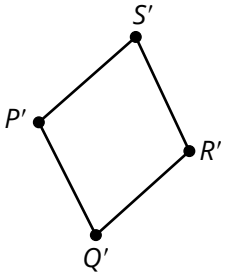


3.

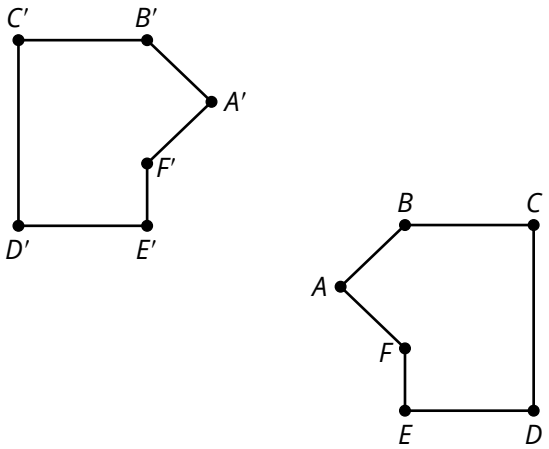


4.

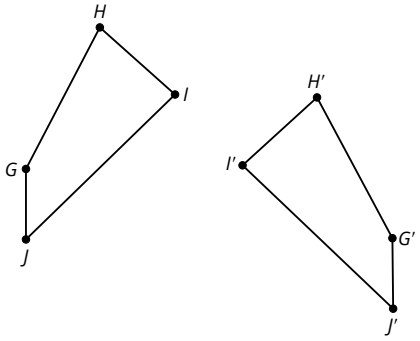




5.



6.

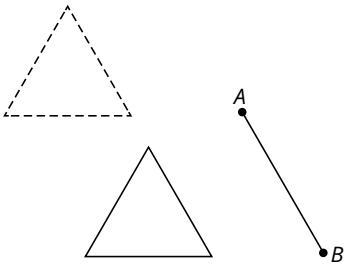


Extension

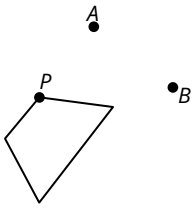
Describe the sequence of translations and reflections to each figure so that the word APPLE is shown in between the two lines with the figures in the same positions.

Spaced Practice

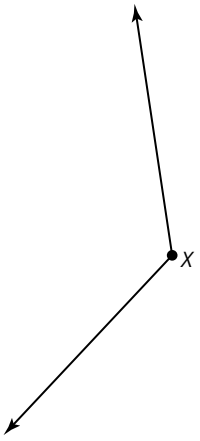
1. Write a function to describe the translation. The solid figure is the pre-image, and the dotted figure is the image.



2. Complete the translation given the function T_{BA}(Trapezoid) = Trapezoid'.



3. Construct the angle bisector of the given angle.



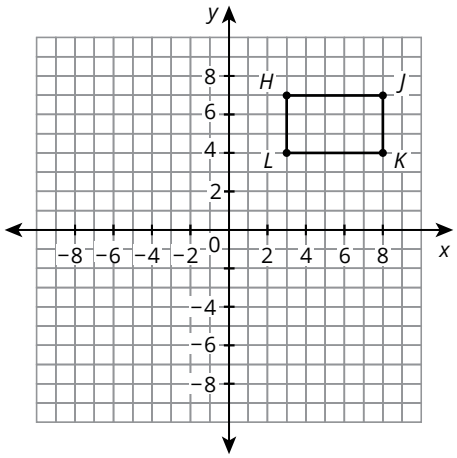
4. Construct a 22.5° angle using only a compass and straightedge. Summarize your steps.

IV. Rotations On and Off the Coordinate Plane

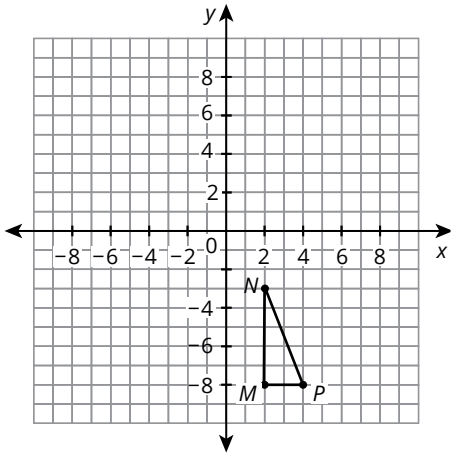
Topic Practice

A. Rotate each given geometric figure on the coordinate plane as described. Then describe the transformation using coordinate notation.

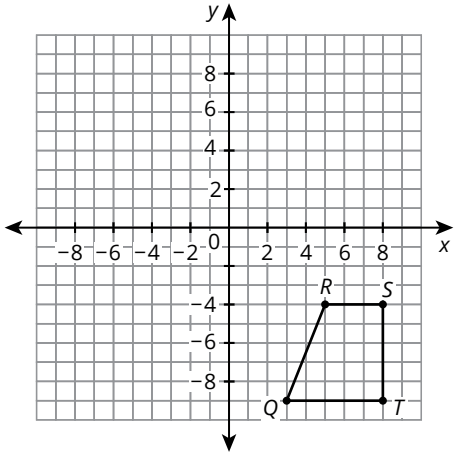
1. Rotate rectangle *HJKL* 90° counterclockwise about the origin.



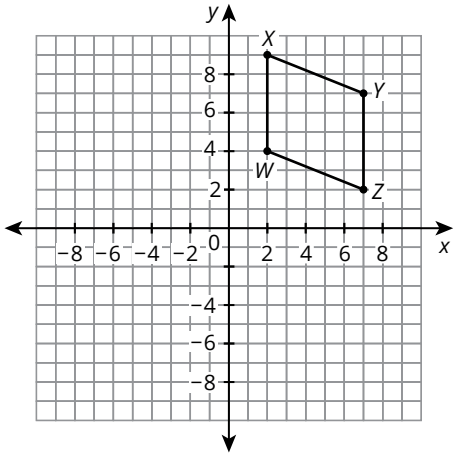
2. Rotate triangle *MNP* 180° counterclockwise about the origin.



3. Rotate trapezoid *QRST* 90° counterclockwise about the origin.

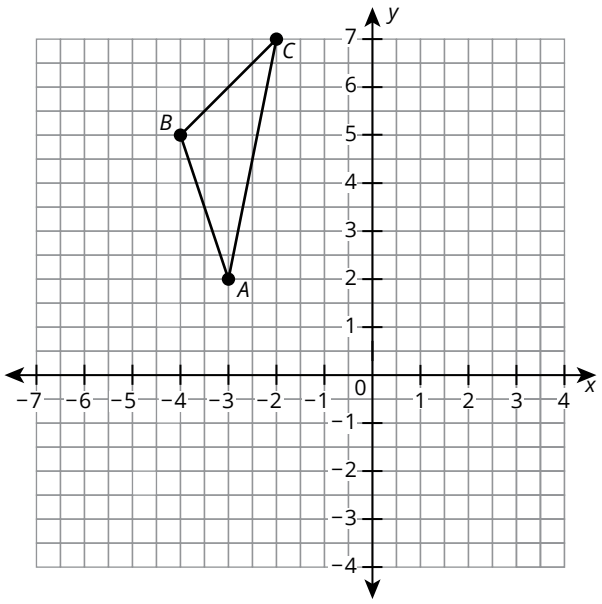


4. Rotate parallelogram *WXYZ* 180° counterclockwise about the origin.

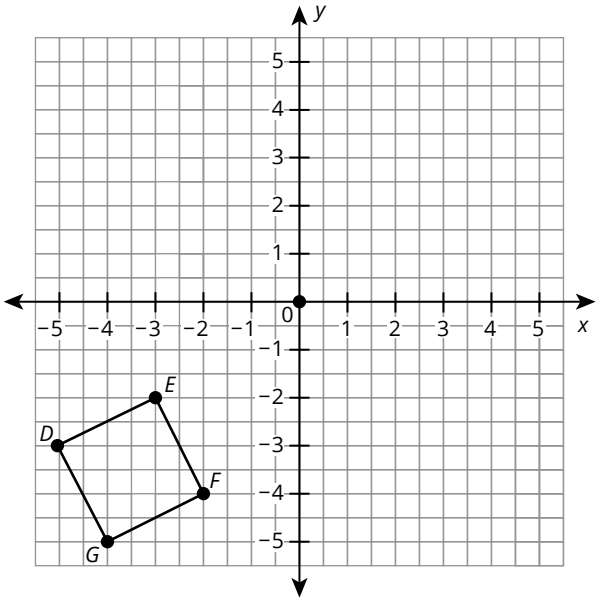


CC BY NC

5. Rotate triangle *ABC* 90° counterclockwise about the origin.

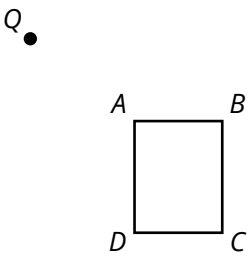


6. Rotate square *DEFG* 180° counterclockwise about the origin.

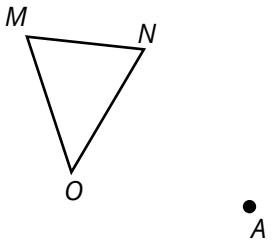


B. Complete each rotation, given the description of the rotation function.

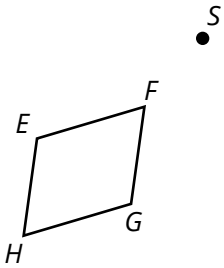
1. Rotate rectangle *ABCD* 65°, using point *Q* as the center of rotation.



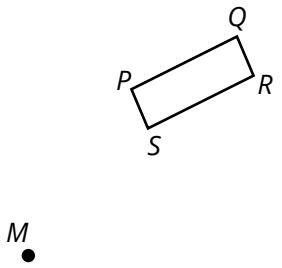
2. Rotate triangle *MNO* −45°, using point *A* as the center of rotation.



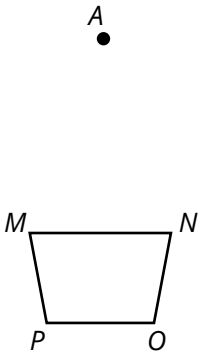
3. Rotate parallelogram *EFGH* 125°, using point *S* as the center of rotation.



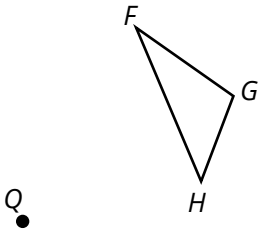
4. Rotate rectangle *PQRS* −95°, using point *M* as the center of rotation.



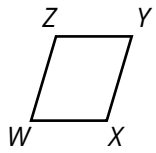
5. Rotate trapezoid *MNOP* −40°, using point *A* as the center of rotation.



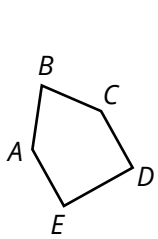
6. Rotate triangle *FGH* 150°, using point *Q* as the center of rotation.



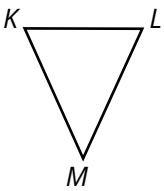
7. Rotate parallelogram *WXYZ* 60°, using point *J* as the center of rotation.



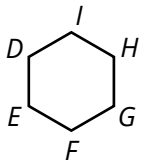
8. Rotate pentagon *ABCDE* −120°, using point *X* as the center of rotation.



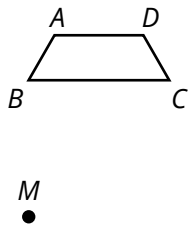
9. Rotate parallelogram *KLM* 20°, using point *T* as the center of rotation.



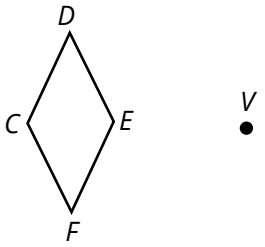
10. Rotate hexagon *DEFGHI* 100°, using point *Q* as the center of rotation.



11. Rotate trapezoid *ABCD* 45°, using point *M* as the center of rotation.

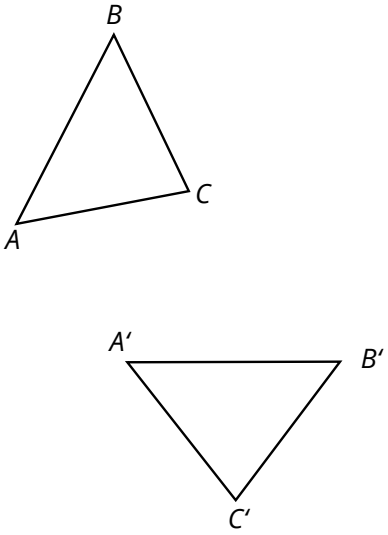


12. Rotate rhombus *CDEF* −90°, using point *V* as the center of rotation.



C. Determine the center of each rotation.

1.



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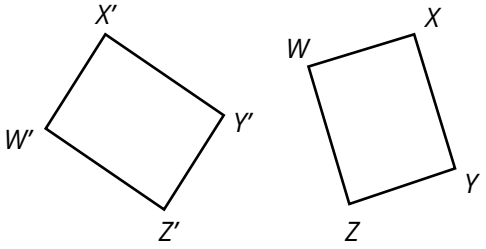

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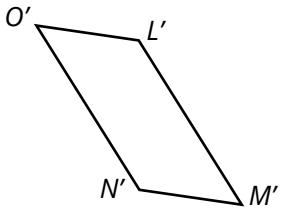

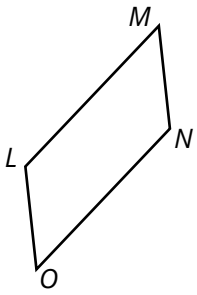

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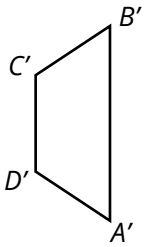

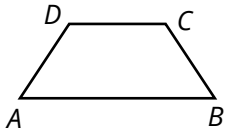

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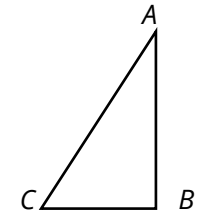

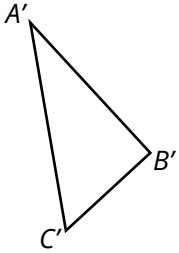

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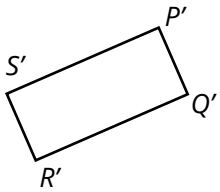
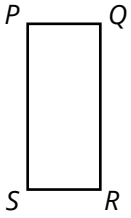
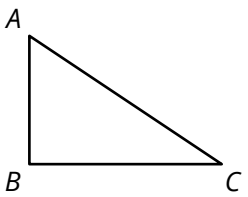
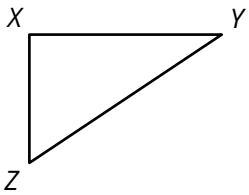




D. Identify the transformation used to create △*XYZ* and write congruency statements for the corresponding sides of the figures. For reflections, identify the line of reflection. For rotations, identify the center and angle of rotation.

1.





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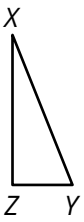

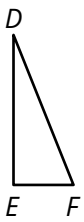

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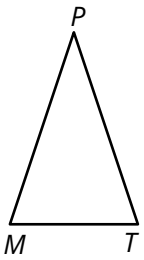

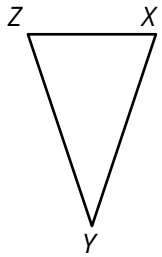

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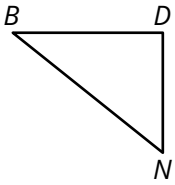

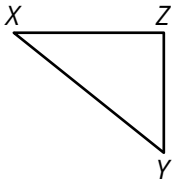

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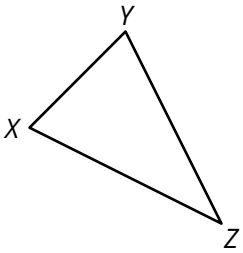

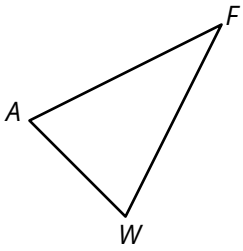

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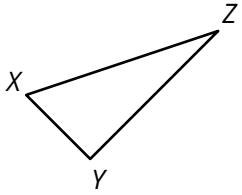

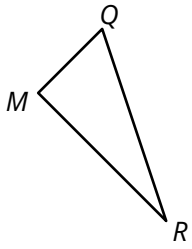

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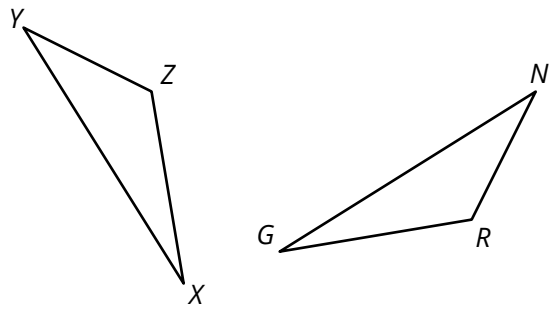

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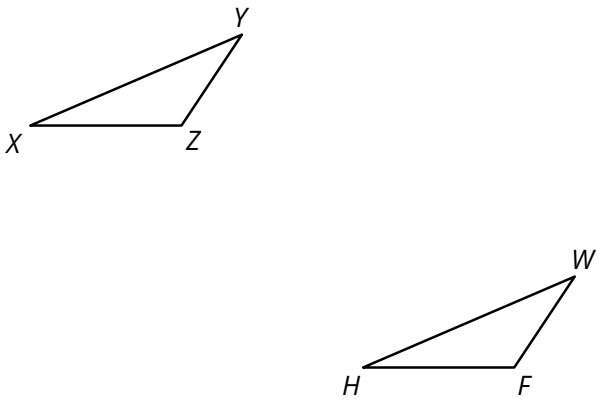

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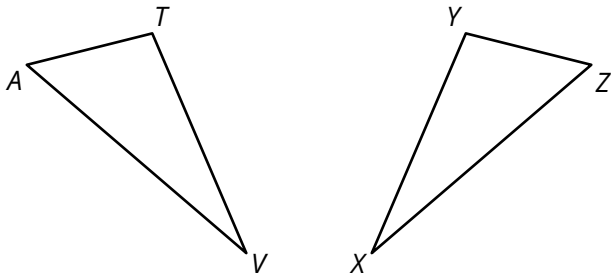

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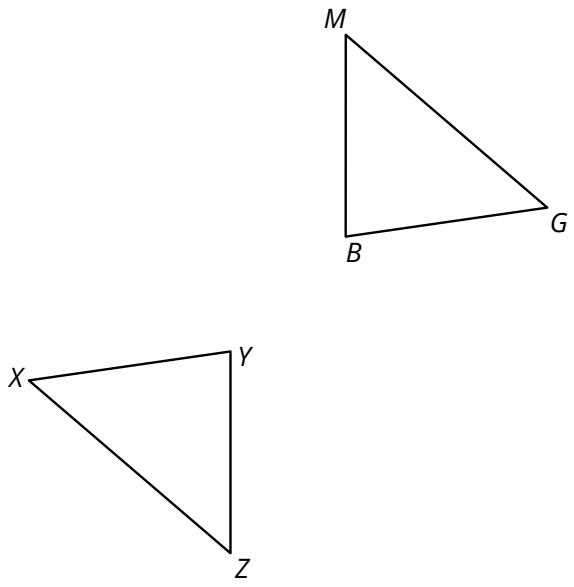
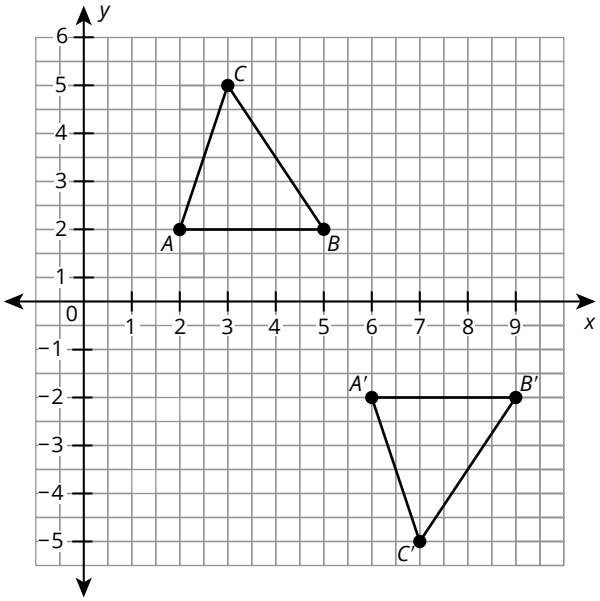


E. Draw and describe the sequence of rigid motion transformations which shows that the two figures in each pair are congruent.

1.



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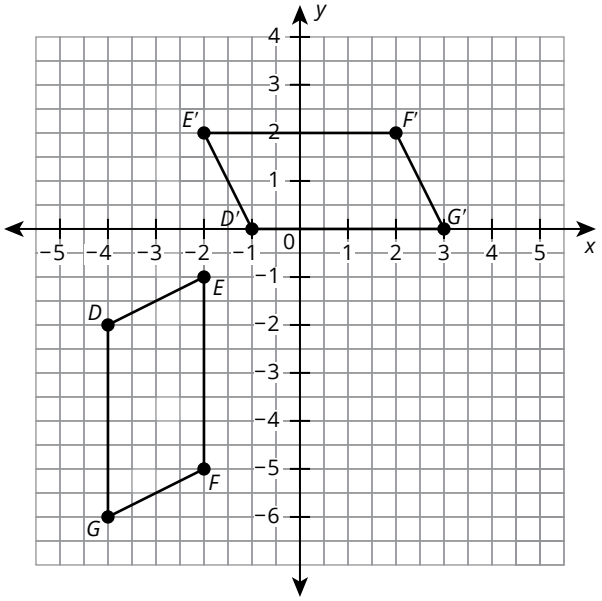

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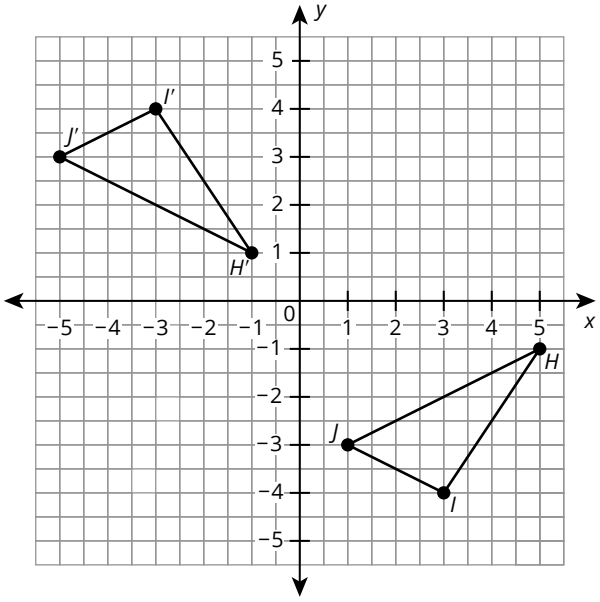

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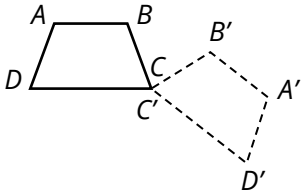

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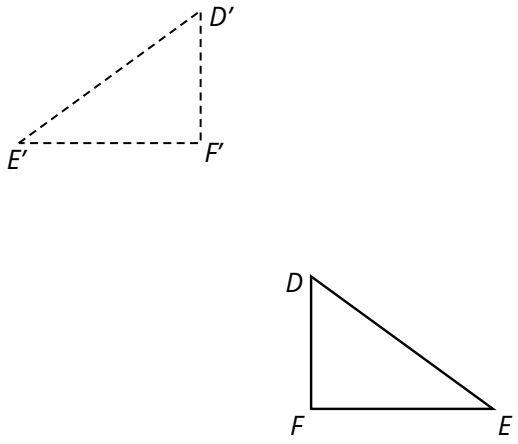

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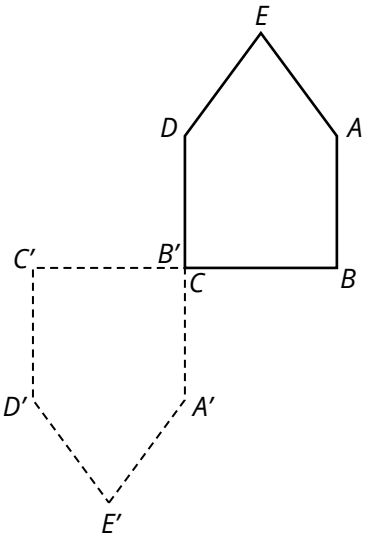

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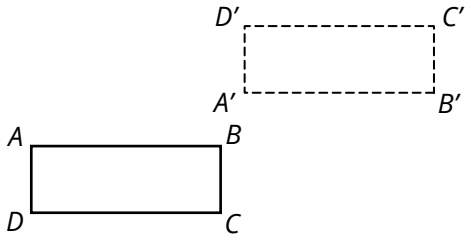

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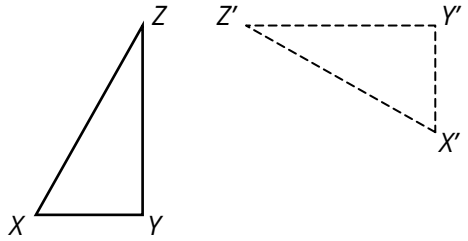

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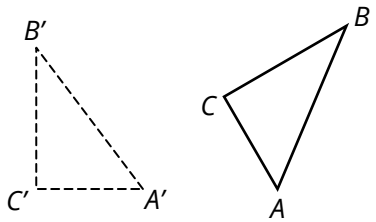
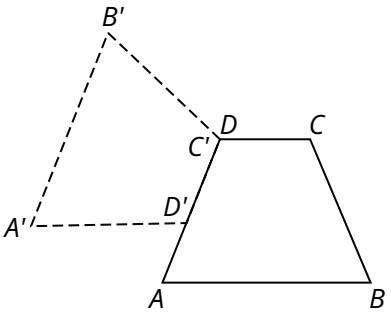
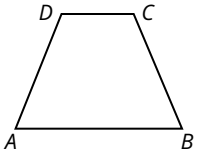
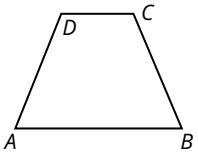


Extension

Determine and draw the sequence of transformations that could be used to transform Trapezoid *ABCD* into Trapezoid *A′B′C′D′*. Include the locations of lines and points that the figure is reflected around, translated on, or rotated about. Also include the rotation angle.

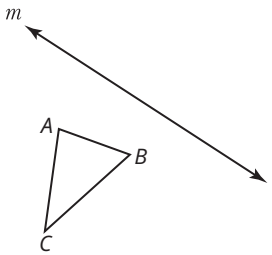




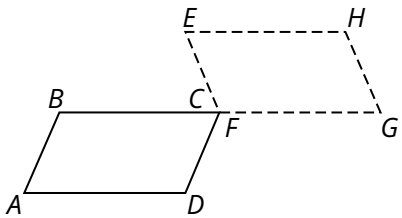


Spaced Practice

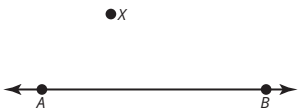
1. Reflect $\triangle ABC$ across line m.



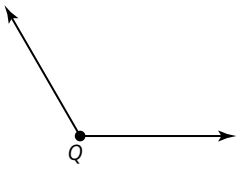
2. Describe the sequence of transformations that will map parallelogram $ABCD$ onto parallelogram $GHEF$.



3. Construct a line perpendicular to line AB that passes through point X.



4. Duplicate $\angle Q$.

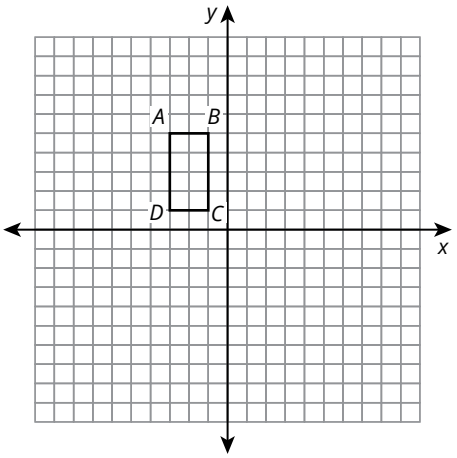


V. Sequences of Transformations

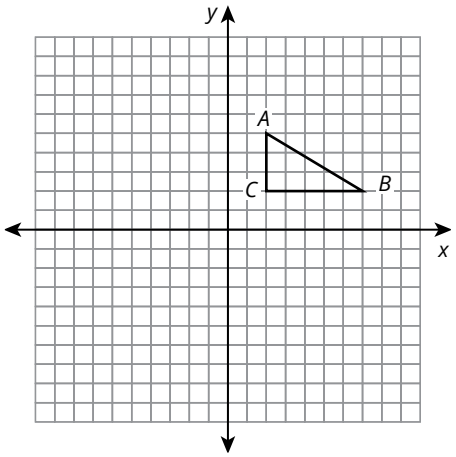
Topic Practice

A. Perform the sequence of transformations on the given geometric figure on the coordinate plane as described. Then, describe the transformations using coordinate notation.

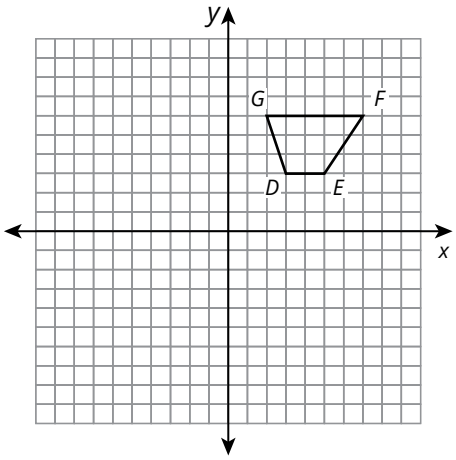
1. Translate rectangle *ABCD* 5 units to the right. Then rotate the image 270° counterclockwise about the origin. Label the final image $A''B''C''D''$.



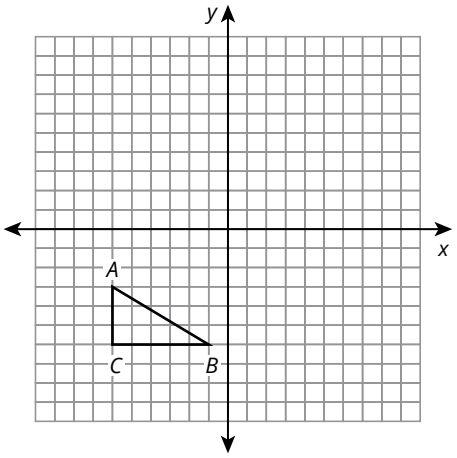
2. Reflect triangle *ABC* over the *x*-axis. Then translate the image 3 units up. Label the final image $A''B''C''$.



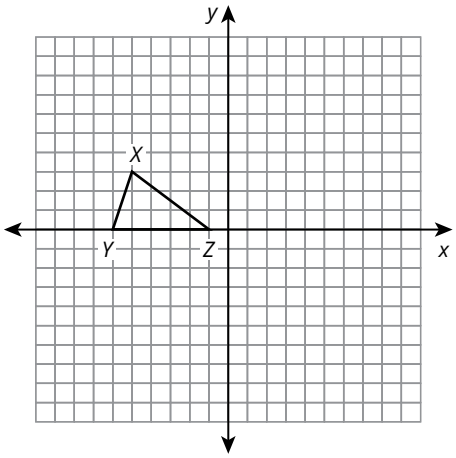
3. Rotate trapezoid *DEFG* 180° counterclockwise about the origin. Then translate the image 6 units up. Label the final image $D''E''F''G''$.



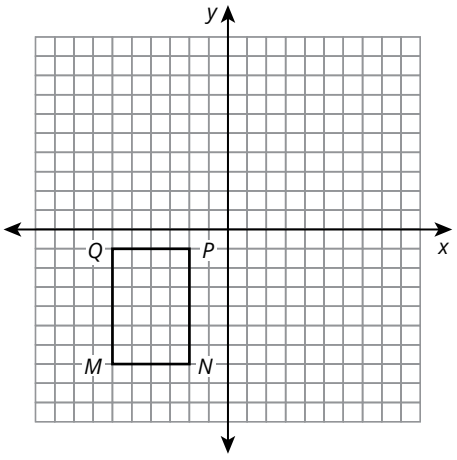
4. Translate triangle *ABC* 5 units up. Then reflect the image over the *y*-axis. Label the final image $A''B''C''$.



5. Translate triangle *XYZ* up 4 units, reflect the image over the *y*-axis, and then translate the second image left 2 units. Label the final image $X''Y''Z''$.



6. Rotate rectangle *MNPQ* 270° counterclockwise about the origin, reflect the image over the *y*-axis, and then reflect the second image over the *x*-axis. Label the final image $M''N''P''Q''$.



B. Determine the coordinates of each translated image without graphing.

1. The vertices of rectangle *DEFG* are *D*(−7, 1), *E*(−7, 8), *F*(1, 8), and *G*(1, 1). Translate the rectangle 10 units down to form rectangle *D′E′F′G′*.

2. The vertices of parallelogram *HJKL* are *H*(2, −6), *J*(3, −1), *K*(7, −1), and *L*(6, −6). Translate the parallelogram 7 units up to form parallelogram *H′J′K′L′*.

3. The vertices of trapezoid *MNPQ* are *M*(−6, −5), *N*(0, −5), *P*(−1, 2), and *Q*(−4, 2). Translate the trapezoid 4 units to the right to form trapezoid *M′N′P′Q′*.

4. The vertices of triangle *RST* are *R*(0, 3), *S*(2, 7), and *T*(3, −1). Translate the triangle 5 units to the left and 3 units up to form triangle *R′S′T′*.

5. The vertices of triangle *ABC* are *A*(5, 3), *B*(2, 8), and *C*(−4, 5). Rotate the triangle about the origin 90° counterclockwise to form triangle *A′B′C′*.

6. The vertices of rectangle *DEFG* are *D*(−7, 1), *E*(−7, 8), *F*(1, 8), and *G*(1, 1). Rotate the rectangle about the origin 180° counterclockwise to form rectangle *D′E′F′G′*.

7. The vertices of parallelogram *HJKL* are *H*(2, −6), *J*(3, −1), *K*(7, −1), and *L*(6, −6). Rotate the parallelogram about the origin 90° counterclockwise to form parallelogram *H′J′K′L′*.

8. The vertices of trapezoid *MNPQ* are *M*(−6, −5), *N*(0, −5), *P*(−1, 2), and *Q*(−4, 2). Rotate the trapezoid about the origin 180° counterclockwise to form trapezoid *M′N′P′Q′*.

9. The vertices of rectangle *DEFG* are *D*(−7, 1), *E*(−7, 8), *F*(1, 8), and *G*(1, 1). Reflect the rectangle over the *y*-axis to form rectangle $D'E'F'G'$.

10. The vertices of parallelogram *HJKL* are *H*(2, −6), *J*(3, −1), *K*(7, −1), and *L*(6, −6). Reflect the parallelogram over the *x*-axis to form parallelogram $H'J'K'L'$.

C. Determine the coordinates of each image without graphing.

1. The vertices of triangle *ABC* are *A*(−3, −2), *B*(−5, −7), and *C*(−7, −6). Reflect the triangle over the *y*-axis, and then translate it 6 units up to form triangle $A''B''C''$.

2. The vertices of rectangle *CDEF* are *C*(2, 1), *D*(5, 1), *E*(5, 6),and *F*(2, 6). Rotate the rectangle 90° counterclockwise about the origin, and then translate it 5 units down to form rectangle $C''D''E''F''$.

3. The vertices of parallelogram *MNPQ* are *M*(3, −2), *N*(7, −2), *P*(6, −4), and *Q*(2, −4). Translate the parallelogram 7 units up, and then reflect it over the *y*-axis to form parallelogram $M''N''P''Q''$.

4. The vertices of trapezoid *ABCD* are *A*(4, 5), *B*(7, 5), *C*(8, 2), and *D*(2, 2). Rotate the trapezoid 270° counterclockwise about the origin, and then reflect it over the *y*-axis to form trapezoid $A''B''C''D''$.

5. The vertices of rectangle *WXYZ* are *W*(2, −1), *X*(11, −1), *Y*(11, −5), and *Z*(2, −5). Translate the rectangle 8 units up, rotate it 90° counterclockwise about the origin, and then reflect it over the *x*-axis to form rectangle $W''X''Y''Z''$.

6. The vertices of triangle *EFG* are *E*(−2, 7), *F*(−1, −4), and *G*(−6, 4). Translate the triangle 4 units right, reflect it over the *x*-axis, and then reflect it over the *y* -axis to form triangle $E''F''G''$.

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D. Determine the coordinates of each pre-image without graphing.

1. Triangle *ABC* was translated five units right to form triangle $A'B'C'$. The vertices of triangle $A'B'C'$ are $A'(1, 1)$, $B'(2, 5)$, and $C'(5, -1)$.

2. Quadrilateral *WYXZ* was rotated 90° counterclockwise to form quadrilateral $W'X'Y'Z'$. The vertices of quadrilateral $W'X'Y'Z'$ are $W'(-2, -4)$, $X'(-6, -1)$, $Y'(-2, 2)$, and $Z(4, -3)$.

3. Rectangle *EFGH* was reflected across the x-axis to form rectangle $E'F'G'H'$. The vertices of rectangle $E'F'G'H$ are $E'(-2, -6)$, $F'(3, -6)$, $G'(3, -3)$, and $H'(-2, -3)$.

4. Triangle *LMN* was translated up 2 units and left 3 units to create triangle $L'M'N'$. The vertices of triangle $L'M'N'$ are $L'(-5, 2)$, $M'(1, 2)$, and $N'(-3, -2)$.

5. Square *JKLM* was reflected over the *x*-axis and translated left 2 units to create square $J'K'L'M'$. The vertices of square $J'K'L'M'$ are $J'(-5, 1)$, $K'(-2, 1)$, $L(-2, 4)$, and $M(-5, 4)$.

6. Trapezoid *RSTU* was reflected over the y-axis and rotated 90° counterclockwise to create trapezoid $R'S'T'U'$. The vertices of trapezoid $R'S'T'U'$ are $R'(-5, -1)$, $S'(-3, -4)$, $T'(1, -4)$, and $U'(3, -1)$.

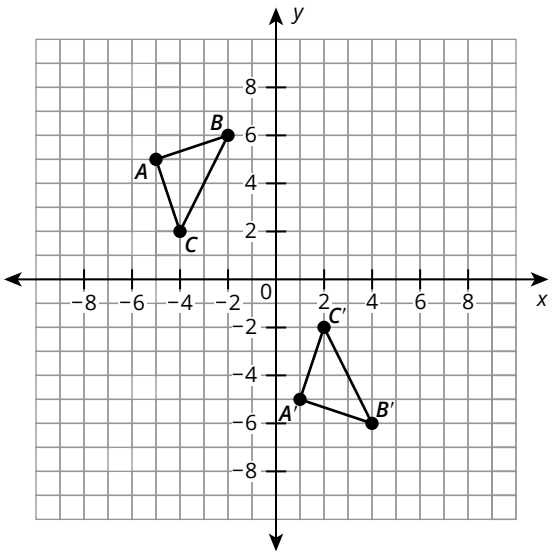
7. Rectangle *ABCD* was translated right 4 units and down 1 unit to create rectangle $A'B'C'D'$. The vertices of rectangle $A'B'C'D'$ are $A'(6, 4)$, $B'(6, 2)$, $C'(10, 2)$ and $D'(10, 4)$.

8. Triangle *FGH* was translated down 3 units and left 6 units to create triangle $F'G'H'$. The vertices of triangle $F'G'H'$ are $(-9, -1)$, $G'(-10, -4)$ and $H'(-6, -4)$.

9. Quadrilateral *JKLM* was translated right 6 units and up 5 units to create quadrilateral $J'K'L'M'$. The vertices of quadrilateral $J'K'L'M'$ are $J'(2, 6)$, $K'(4, 10)$, $L'(5, 8)$ and $M'(4, 5)$.

E. Identify the sequence of transformations.

1.



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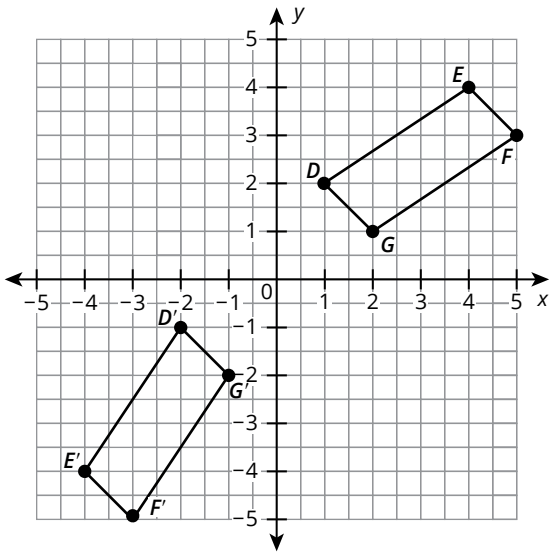
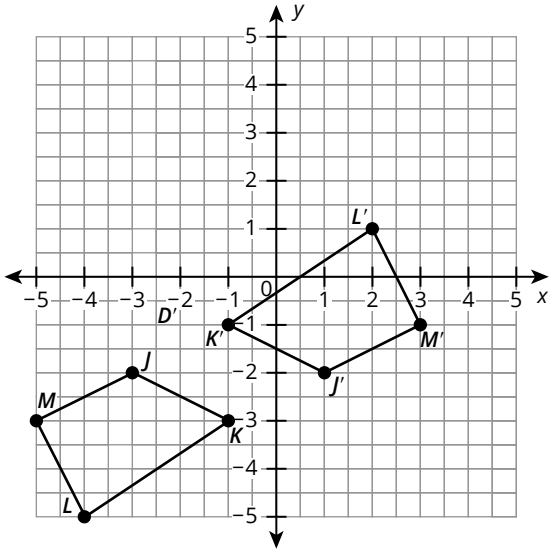
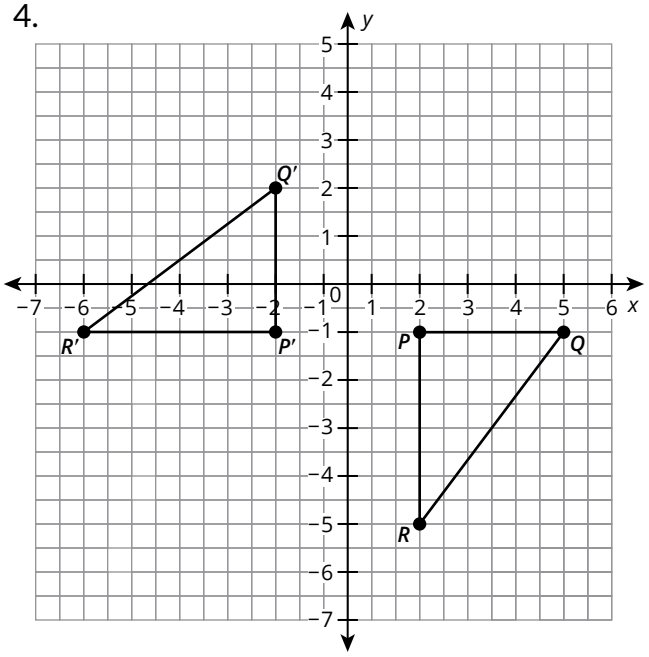


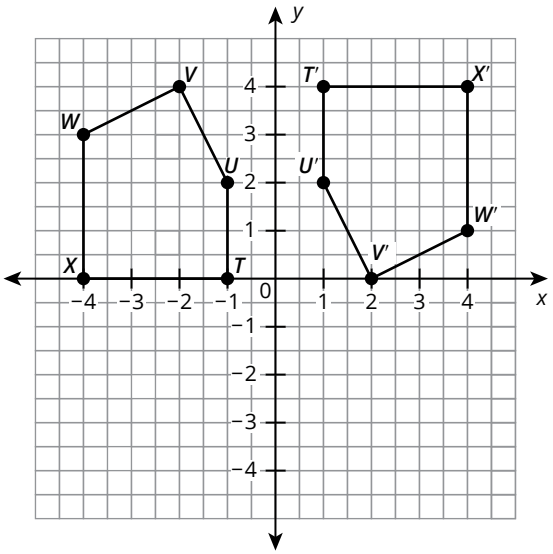
3.



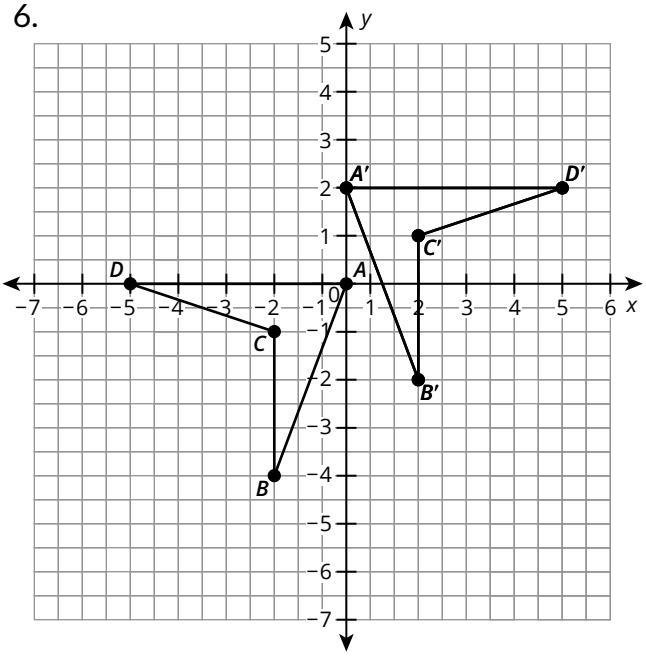
4.



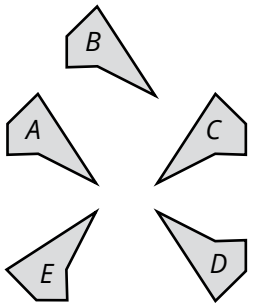
5.



6.



F. Describe a transformation or sequence of transformations that maps one figure onto the other. Be as specific as possible.



1. Figure *A* onto Figure *B*

2. Figure *A* onto Figure *C*

3. Figure *A* onto Figure *E*

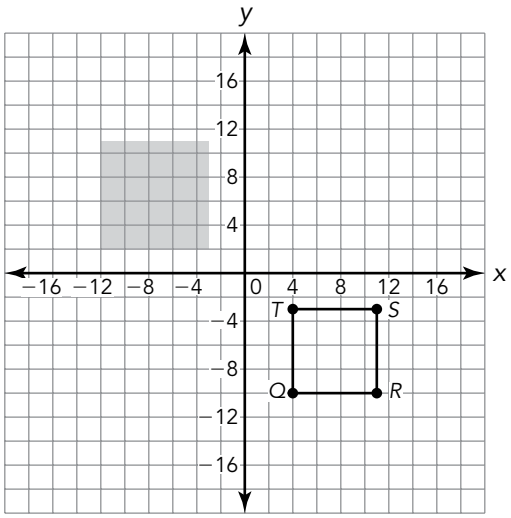
4. Figure *C* onto Figure *D*

5. Figure *D* onto Figure *A*

6. Figure *E* onto Figure *C*

Extension

1. Consider figure *QRST*.



Ramona states that there are three ways to transform *QRST* to place its image in the shaded area in Quadrant II.

- Translate *QRST* vertically 13 units up, then horizontally 15 units to the left.
- Reflect *QRST* over the *x*-axis then over the *y*-axis.
- Rotate *QRST* 180° counterclockwise (or clockwise) about the origin.

a. Without graphing, determine the coordinates of the vertices of the image for each of the transformations Ramona listed. Explain how you determined your answers.

b. Which transformation would you recommend that Ramona choose? Explain your reasoning.

c. Perform the transformation you chose in part (b).

Spaced Practice

1. What happens to the area of a triangle with base 12 units and height 7 units when its dimensions are increased by a factor of 3?

2. What happens to the perimeter of a triangle with side lengths of 3, 5, and 6 units when each side length is decreased by a factor of $\frac{1}{4}$?

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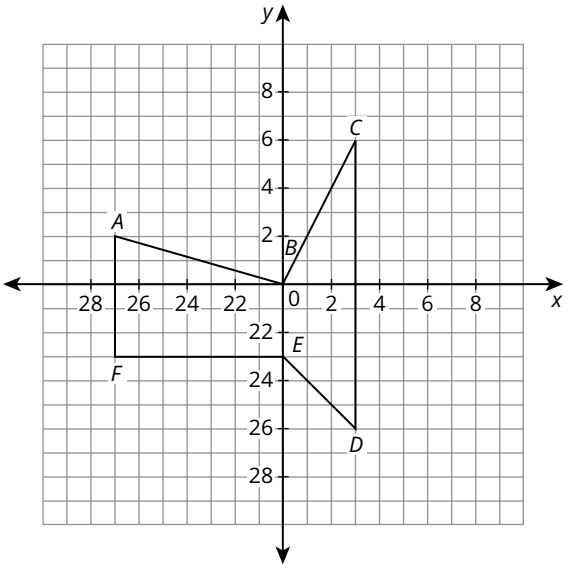
3. Calculate the perimeter of figure *ABCDEF*.



Name _____ Date _____

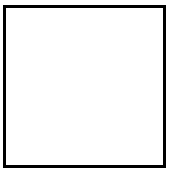
I. Reflectional and Rotational Symmetry

Topic Practice

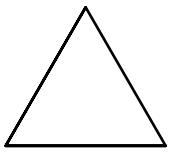
A. Draw the lines of symmetry for each plane figure.

1. Draw the line(s) of symmetry for the rectangle.

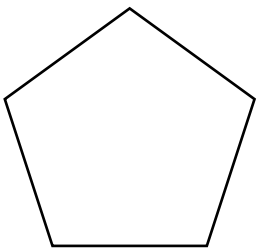
2. Draw the line(s) of symmetry for the square.



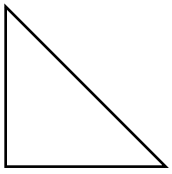
3. Draw the line(s) of symmetry for the equilateral triangle.



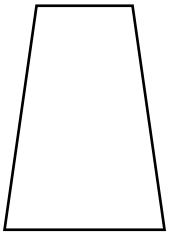
4. Draw the line(s) of symmetry for the regular pentagon.



5. Draw the line(s) of symmetry for the isosceles right triangle.

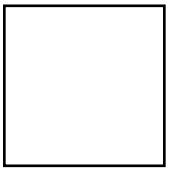


6. Draw the line(s) of symmetry for the isosceles trapezoid.

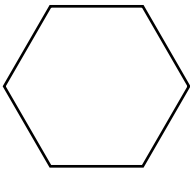


B. Determine the rotational symmetries for each plane figure. Determine the number of degrees you can rotate each shape to carry it onto itself, if possible.

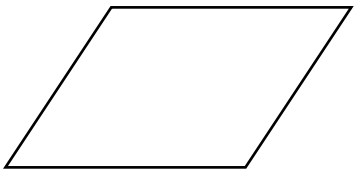
1. Square



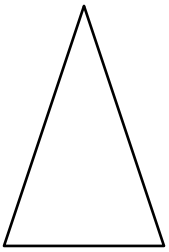
2. Regular hexagon



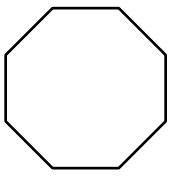
3. Parallelogram



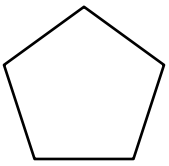
4. Isosceles triangle



5. Regular octagon

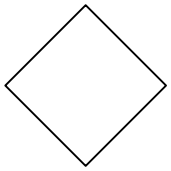


6. Regular pentagon

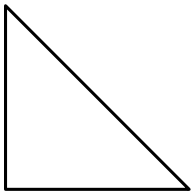


C. Describe the reflections and or rotations that will carry each figure onto itself.

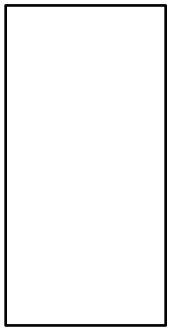
1. Square



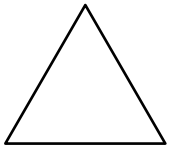
2. Isosceles right triangle



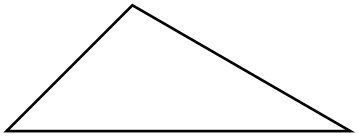
3. Rectangle



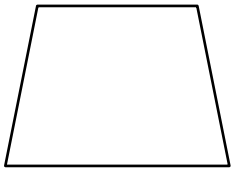
4. Equilateral triangle



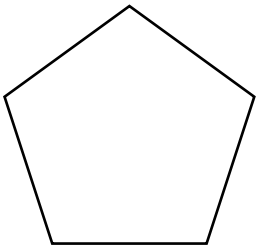
5. Scalene triangle



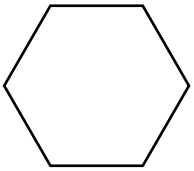
6. Isosceles trapezoid



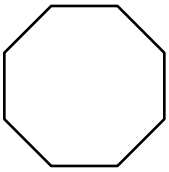
7. Regular pentagon



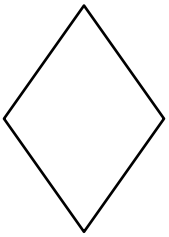
8. Regular hexagon



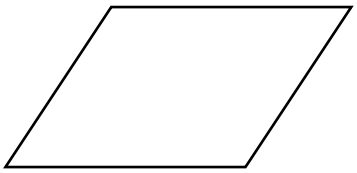
9. Regular octagon



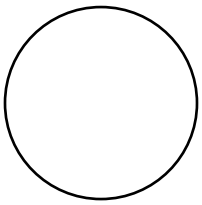
10. Rhombus



11. Parallelogram

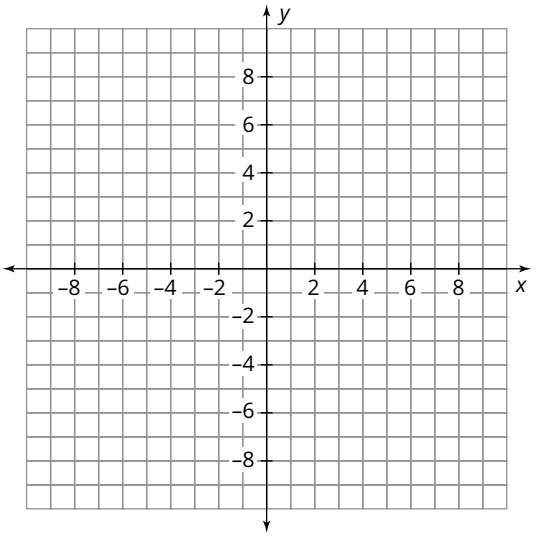


12. Circle



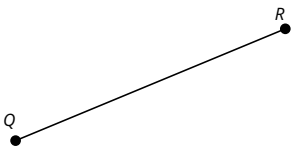
Extension

1. Graph the figure with vertices at $A(2, 6)$, $B(5, 6)$, $C(5, 3)$, $D(-2, -1)$. Determine the equation(s) for the line(s) of symmetry for the figure. Draw the line(s) of symmetry using a dashed line.

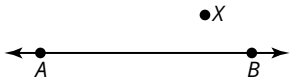


Spaced Practice

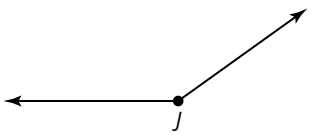
1. Locate the midpoint of the line segment using construction tools and label it point M.



2. Construct a line that is perpendicular to $\overleftrightarrow{AB}$ and passes through point X.



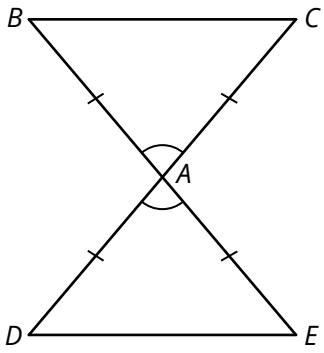
3. Construct the angle bisector of $\angle J$.

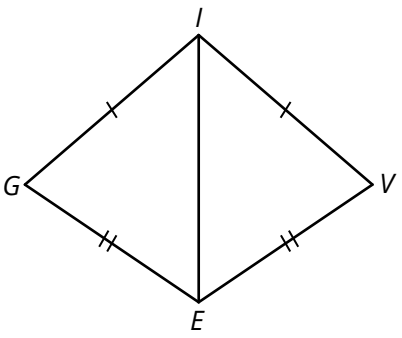


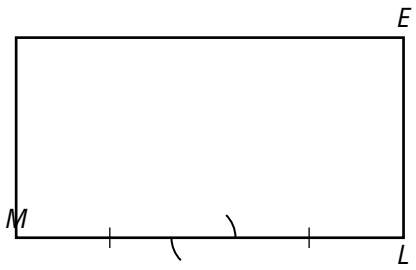
II. Proving Triangle Congruence Theorems

Topic Practice

A. Determine which congruence criteria proves the triangles are congruent. Explain your reasoning.

1. 

2. 

3. 

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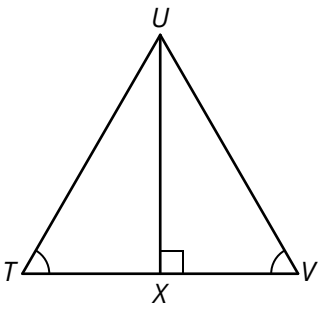
4. 

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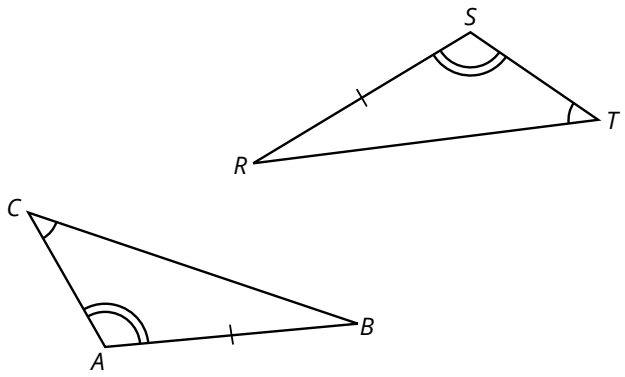

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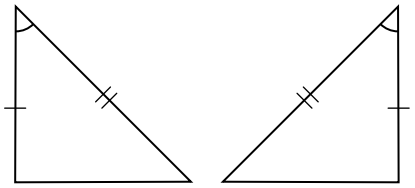
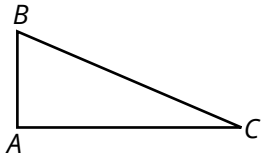
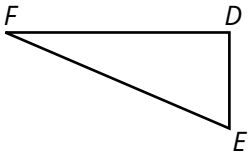


B. Use a ruler to determine whether each pair of given triangles is congruent by SSS.

1.





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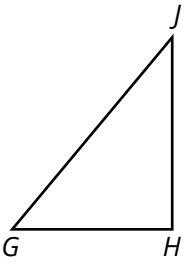

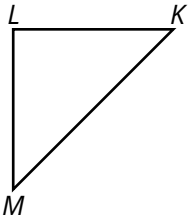

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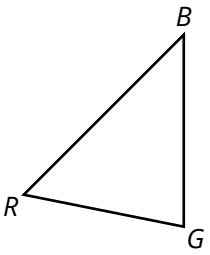

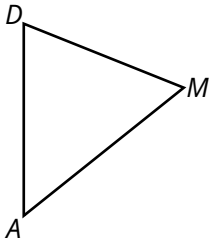

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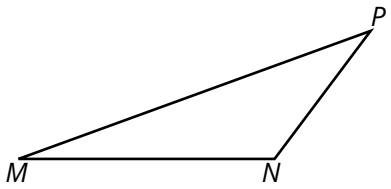

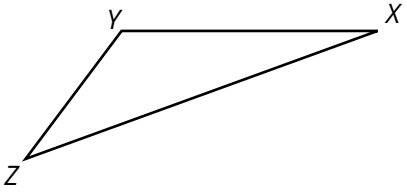

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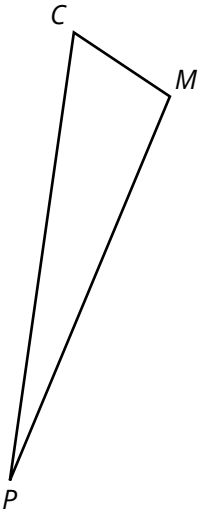

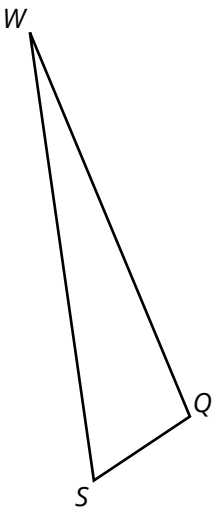

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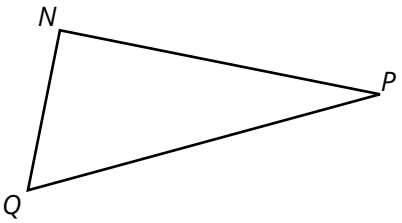
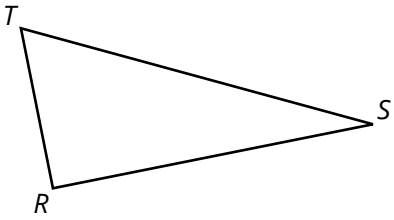
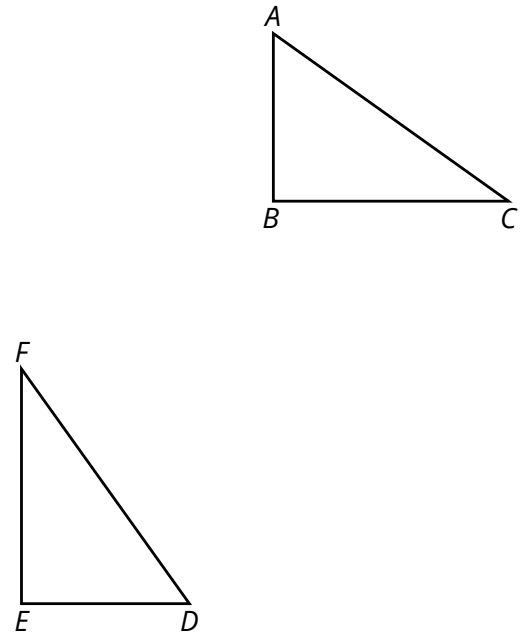




C. Use a ruler and a protractor to determine whether each pair of given triangles is congruent by SAS.

1.



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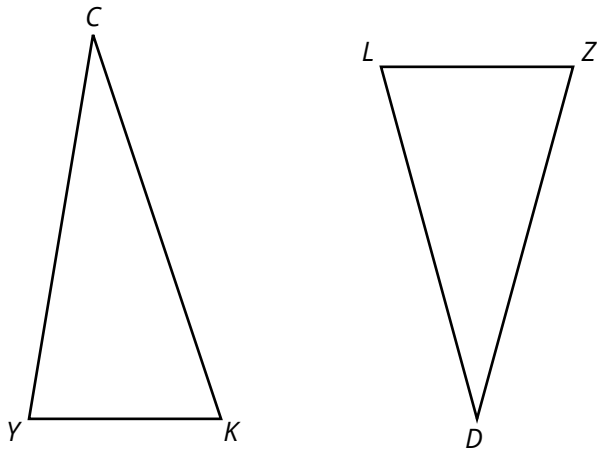

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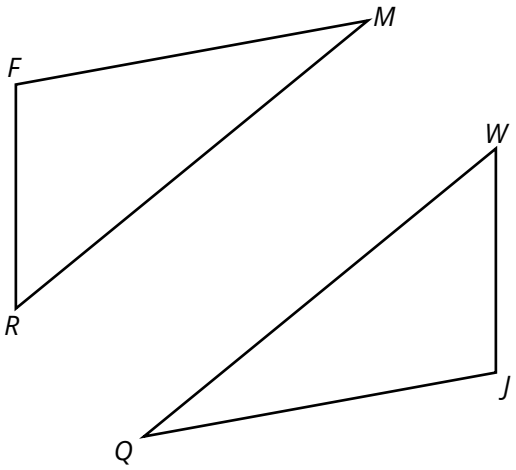

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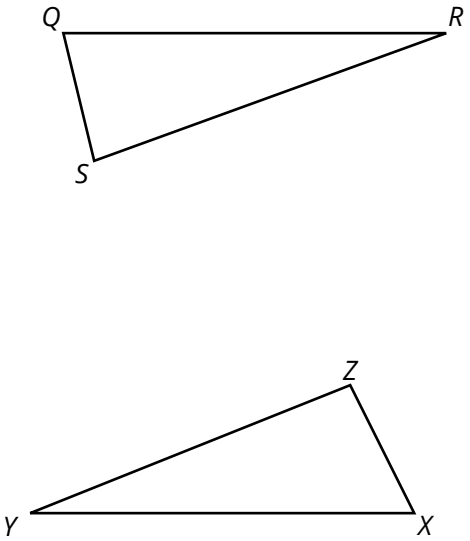

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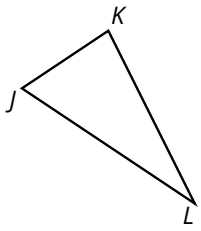

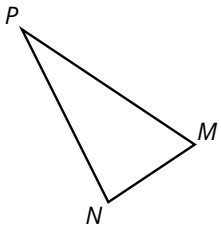

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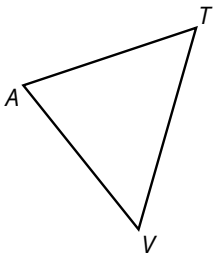
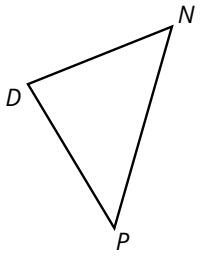
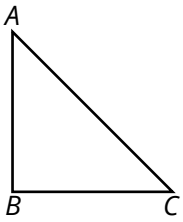
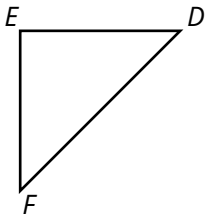




D. Use a ruler and a protractor to determine whether each pair of given triangles is congruent by ASA.

1.





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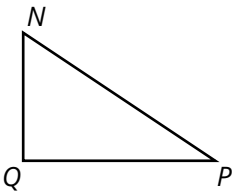

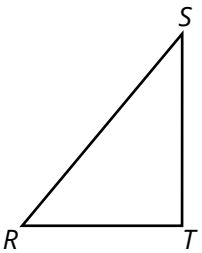

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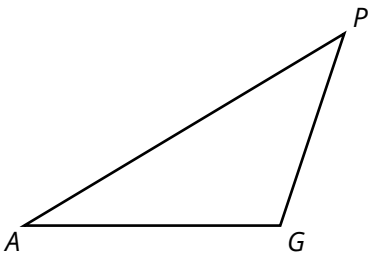

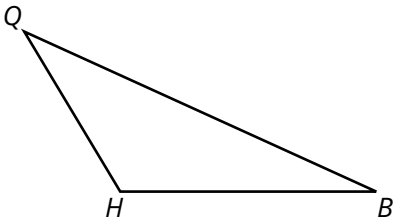

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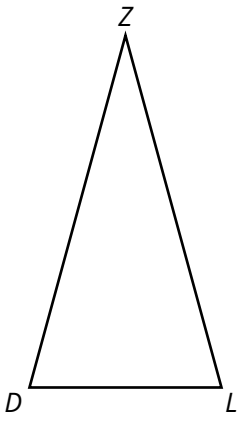

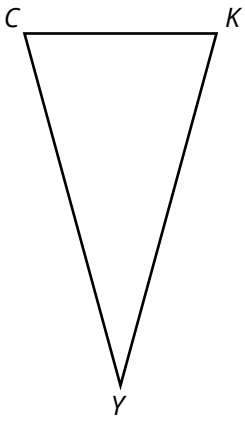

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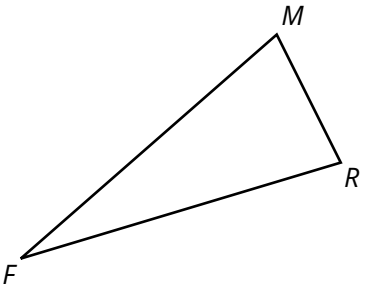

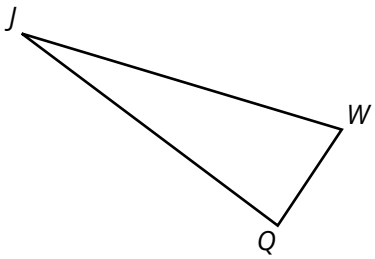

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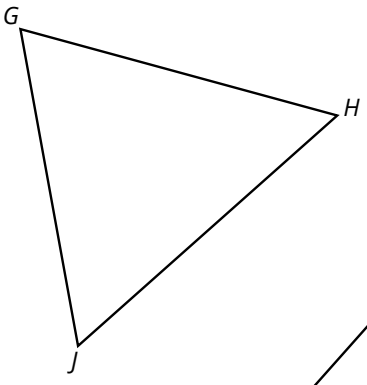
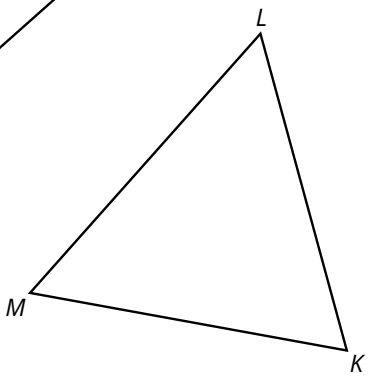
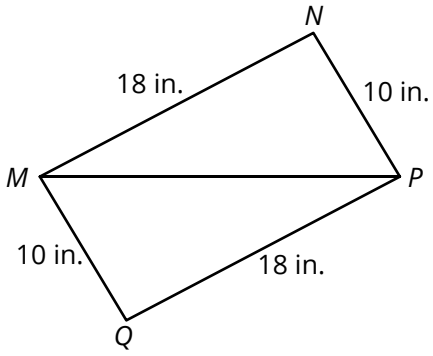


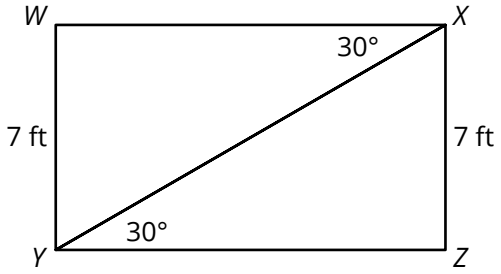


E. Determine whether there is enough information to prove that each pair of triangles is congruent by SSS or SAS. Write the congruence statements to justify your reasoning.

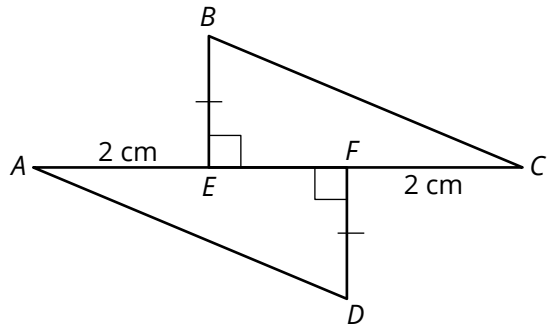
1. $\triangle MNP \stackrel{?}{=} \triangle PQM$



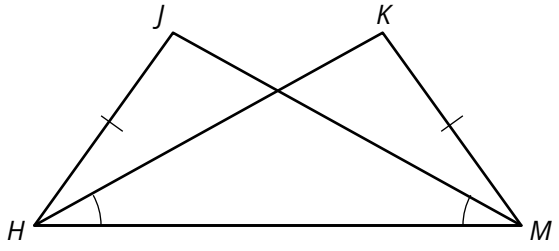
2. $\triangle WXY \stackrel{?}{=} \triangle ZYX$



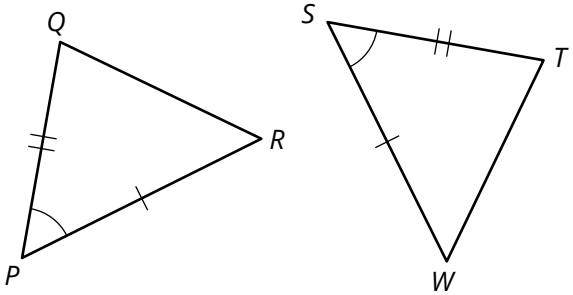
3. $\triangle BCE \stackrel{?}{=} \triangle DAF$



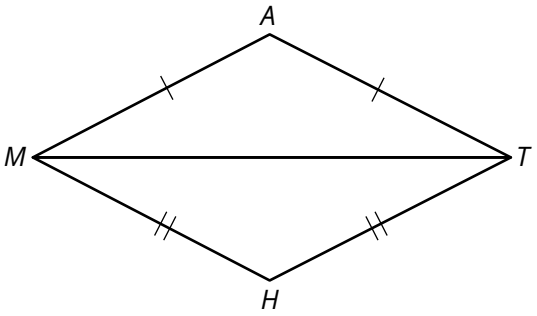
4. $\triangle HJM \stackrel{?}{=} \triangle MKH$



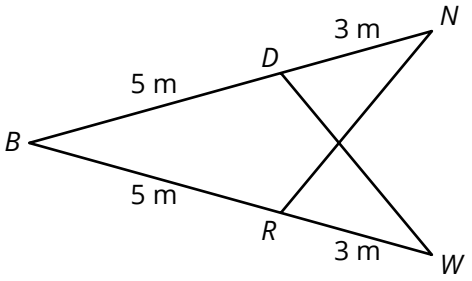
5. $\triangle PQR \stackrel{?}{=} \triangle STW$



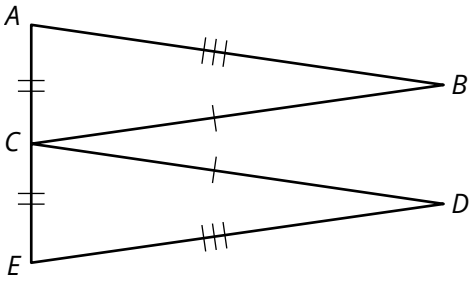
6. $\triangle MAT \stackrel{?}{=} \triangle MHT$



7. $\triangle BDW \stackrel{?}{=} \triangle BRN$

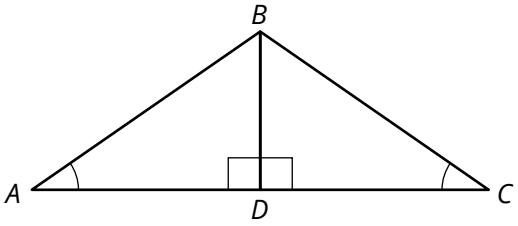


8. $\triangle ABC \stackrel{?}{=} \triangle EDC$

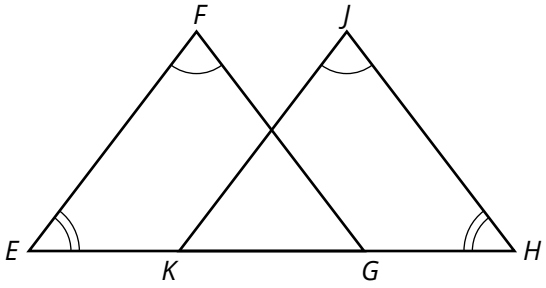


F. Determine whether there is enough information to prove that each pair of triangles is congruent by ASA or AAS. Write the congruence statements to justify your reasoning.

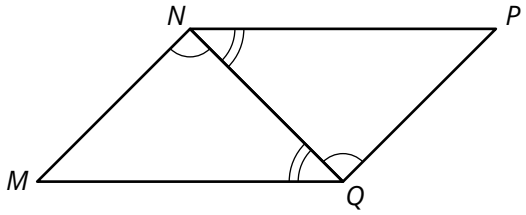
1. $\triangle ABD \stackrel{?}{=} \triangle CBD$



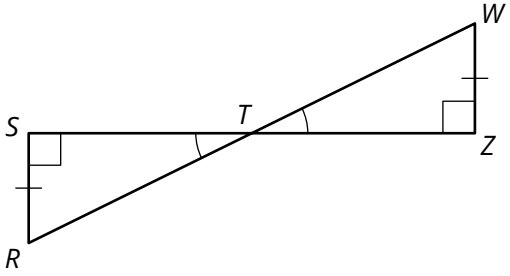
2. $\triangle EFG \stackrel{?}{=} \triangle HJK$



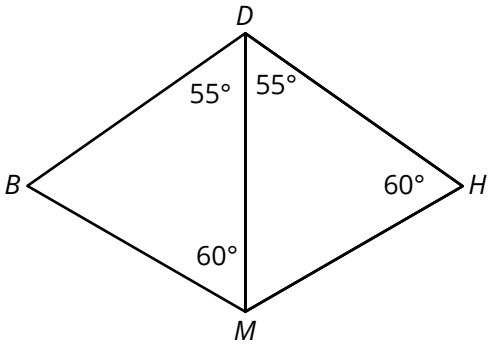
3. $\triangle MNQ \stackrel{?}{=} \triangle PQN$



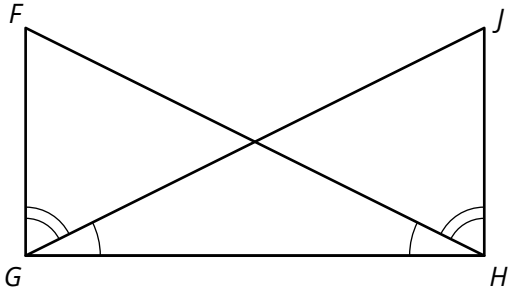
4. $\triangle RST \stackrel{?}{=} \triangle WZT$



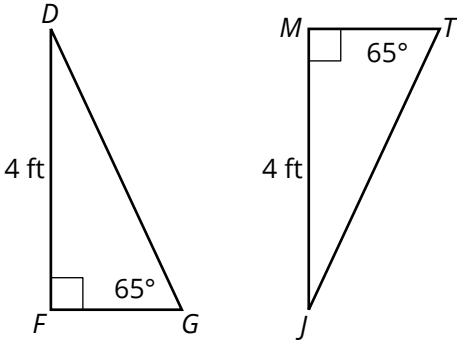
5. $\triangle BDM \stackrel{?}{=} \triangle HDM$



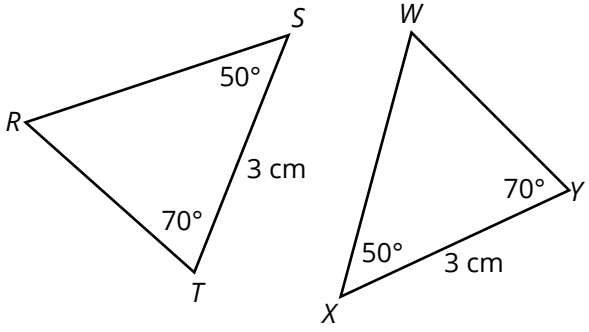
6. $\triangle FGH \stackrel{?}{=} \triangle JHG$



7. $\triangle DFG \stackrel{?}{=} \triangle JMT$

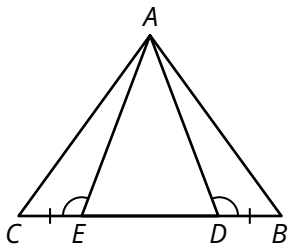


8. $\triangle RST \stackrel{?}{=} \triangle WXY$



Extension

1. Consider the given figure.



The table shows a series of statements that lead to a conclusion that shows that $\triangle CAD \cong \triangle BAE$. Fill in the table by writing sentences that give the reasoning for the truth of each statement.

Statement	Reasoning
$\overline{CE} \cong \overline{DB}$	
$\angle AEC \cong \angle ADB$	
$m\angle AEC + m\angle AED = 180°$	
$m\angle ADB + m\angle ADE = 180°$	
$\angle AED \cong \angle ADE$	
$\overline{AE} \cong \overline{AD}$	

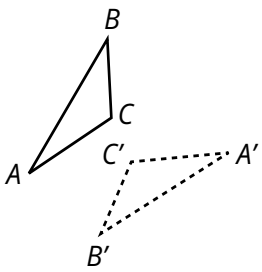
$\overline{ED} \cong \overline{ED}$	
$\overline{CD} \cong \overline{EB}$	
$\triangle CAD \cong \triangle BAE$	

Spaced Practice

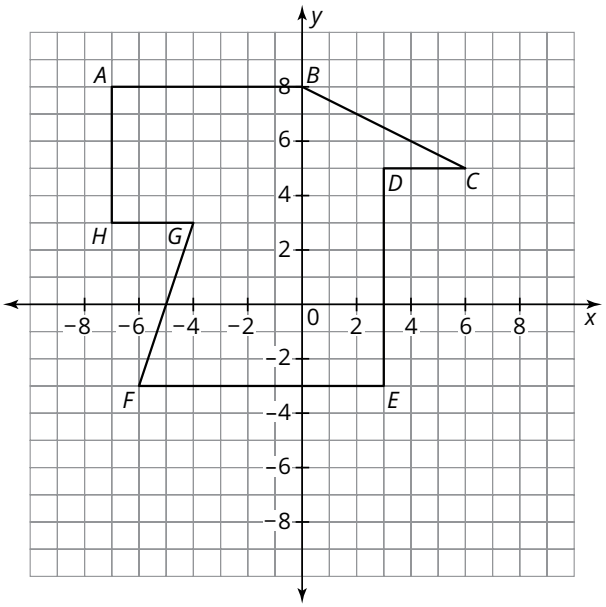
1. Write the postulate that confirms the statement.

   $m\angle BCD + m\angle DCE = m\angle BCE$

2. Identify a sequence of transformations that will carry each given pre-image onto the image shown with dashed lines.



3. A composite figure is graphed on the coordinate plane shown. Round your answers to the nearest hundredth, if necessary.



   a. Determine the perimeter of the figure.

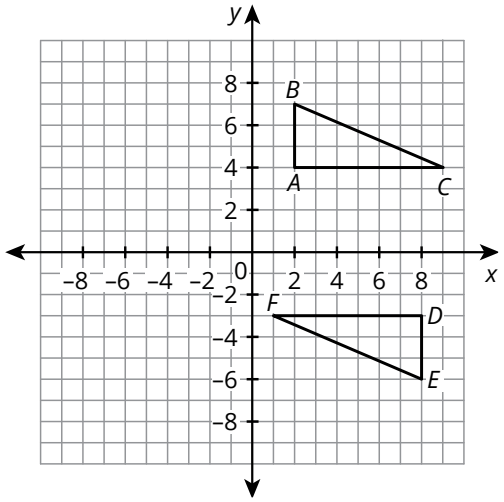
   b. Calculate the area of the figure.

III. Using Triangle Congruence to Solve Problems

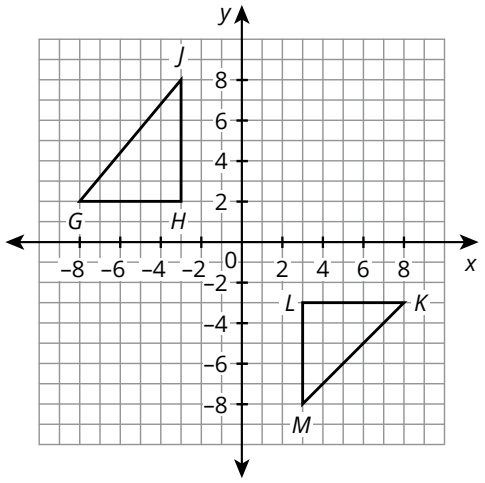
Topic Practice

A. Determine whether each pair of given triangles is congruent by SSS. Use the distance formula and/or protractor when necessary. When the triangles are congruent use CPCTC to identify the congruent corresponding sides and angles.

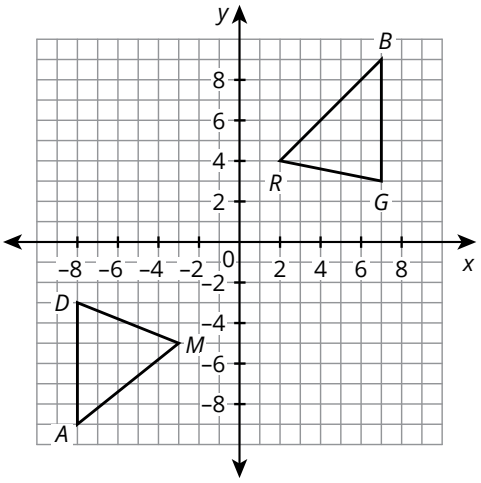
1.



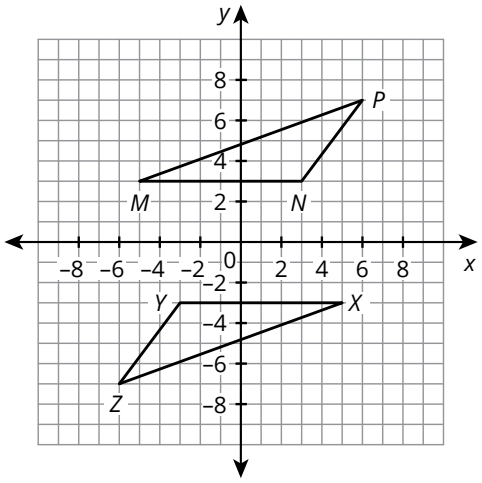
2.



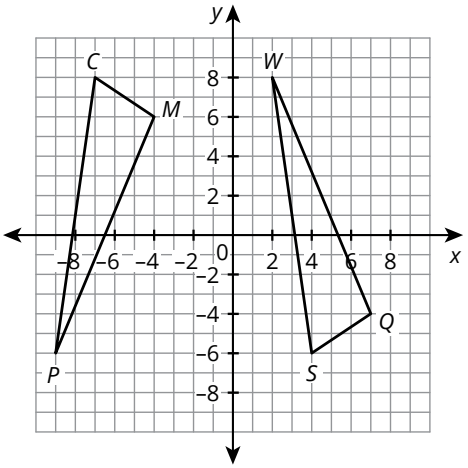
3.



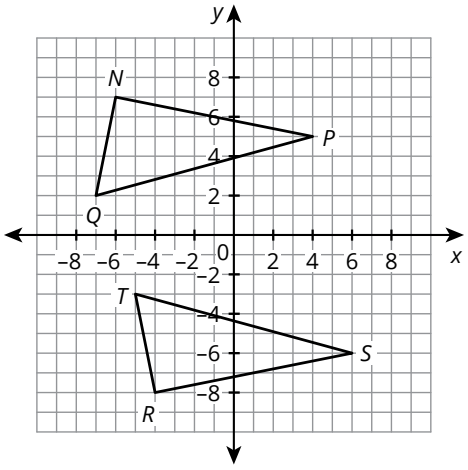
4.



5.

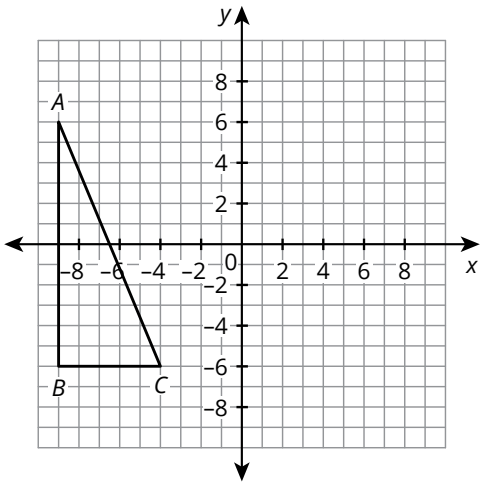


6.

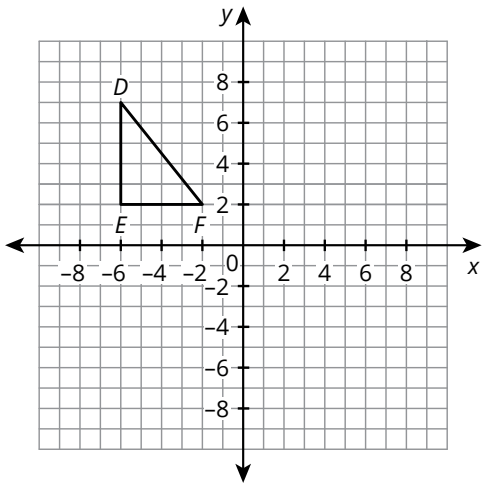


B. Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by SSS. Use the distance formula and/or protractor when necessary. Use CPCTC to identify the corresponding sides and angles.

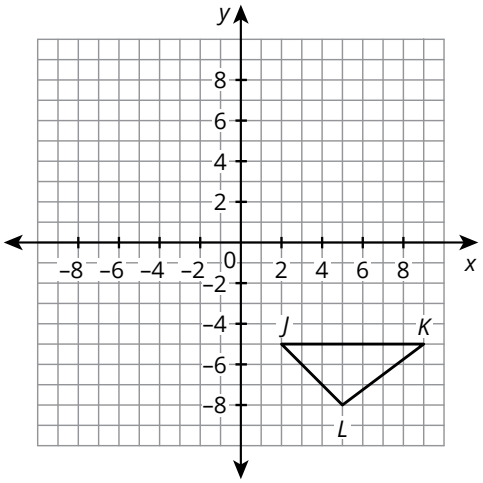
1. Reflect ΔABC over the y-axis to form ΔXYZ. Verify that $\Delta ABC \cong \Delta XYZ$ by SSS.



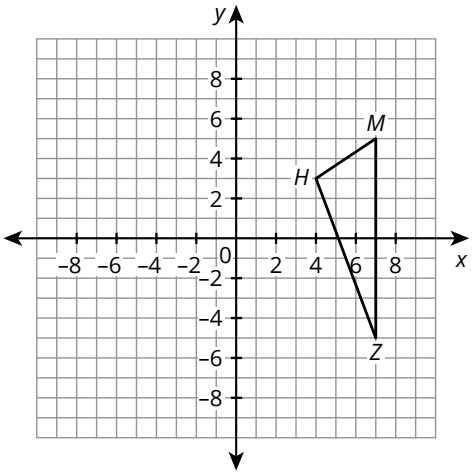
2. Rotate $\triangle DEF$ 180° clockwise about the origin to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by SSS.



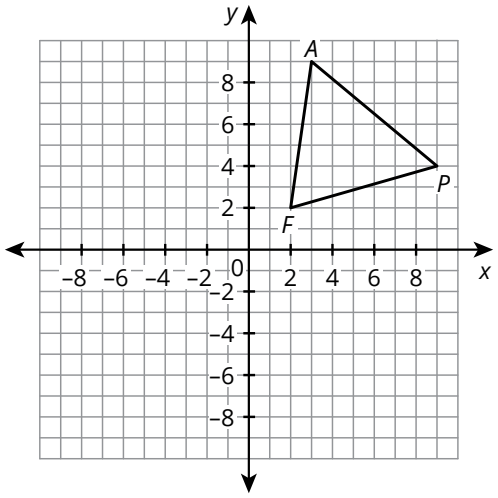
3. Reflect $\triangle JKL$ over the x-axis to form $\triangle MNP$. Verify that $\triangle JKL \cong \triangle MNP$ by SSS.



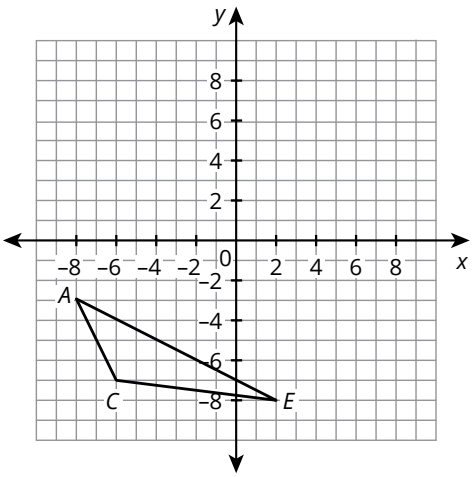
4. Translate △*HMZ* 10 units to the left and 1 unit down to form △*BNY*. Verify that △*HMZ* ≅ △*BNY* by SSS.



5. Rotate △*AFP* 90° counterclockwise about the origin to form △*DHW*. Verify that △*AFP* ≅ △*DHW* by SSS.

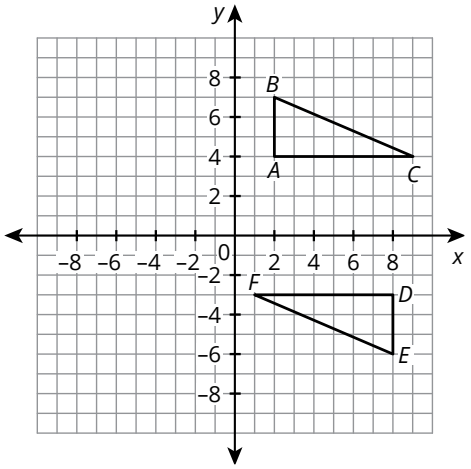


6. Translate $\triangle ACE$ 3 units to the right and 9 units up to form $\triangle JKQ$. Verify that $\triangle ACE \cong \triangle JKQ$ by SSS.

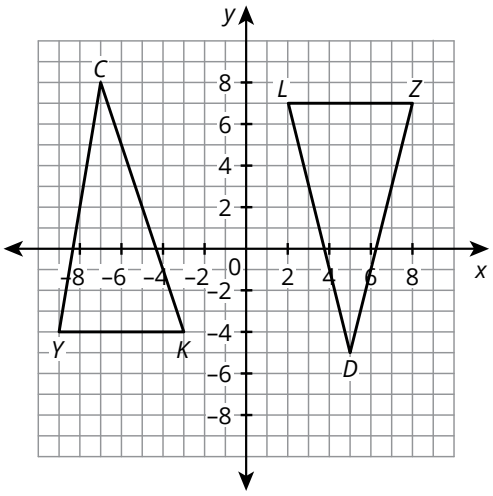


C. Determine whether each pair of given triangles is congruent by SAS. Use the distance formula and/or protractor when necessary. When the triangles are congruent use CPCTC to identify the corresponding sides and angles.

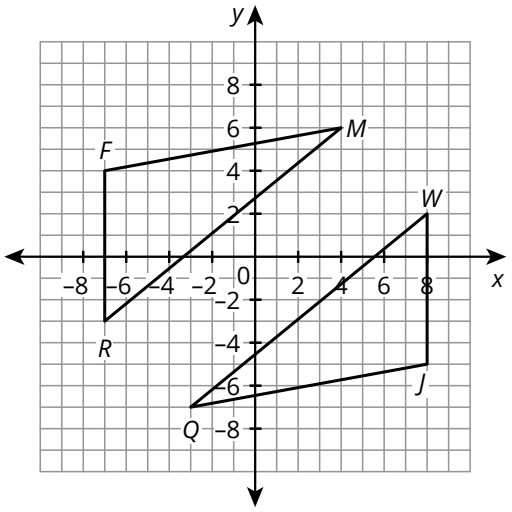
1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by SAS.



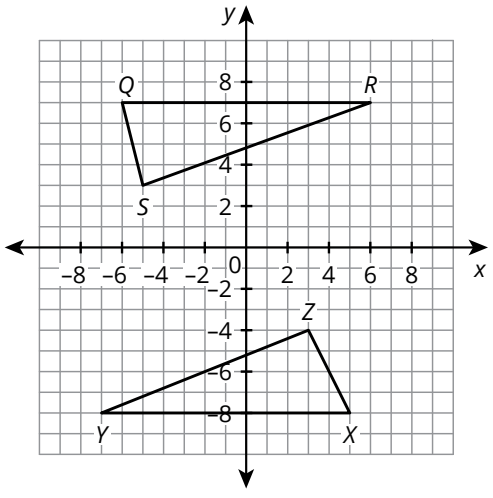
2. Determine whether $\triangle CKY$ is congruent to $\triangle DLZ$ by SAS.



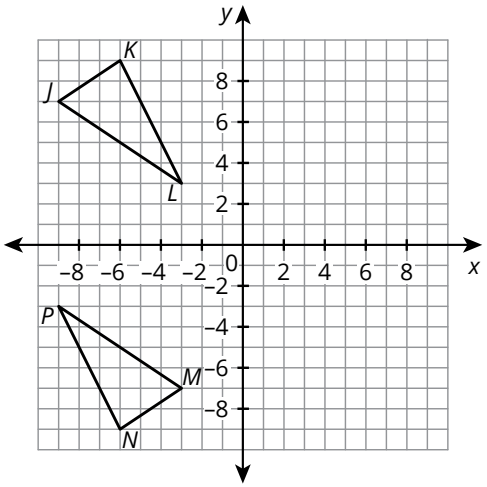
3. Determine whether $\triangle FMR$ is congruent to $\triangle JQW$ by SAS.



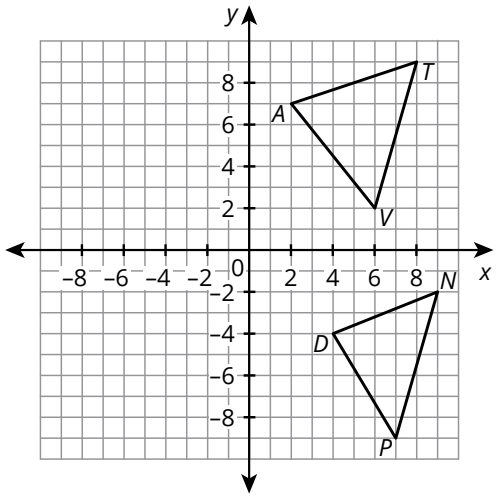
4. Determine whether ΔQRS is congruent to $\triangle XYZ$ by SAS.



5. Determine whether $\triangle JKL$ is congruent to $\triangle MNP$ by SAS.

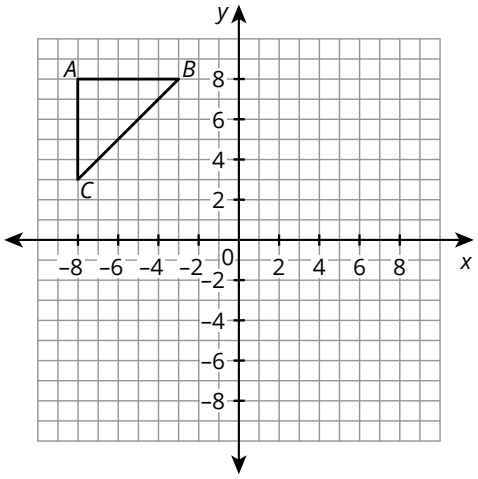


6. Determine whether $\triangle ATV$ is congruent to $\triangle DNP$ by SAS.

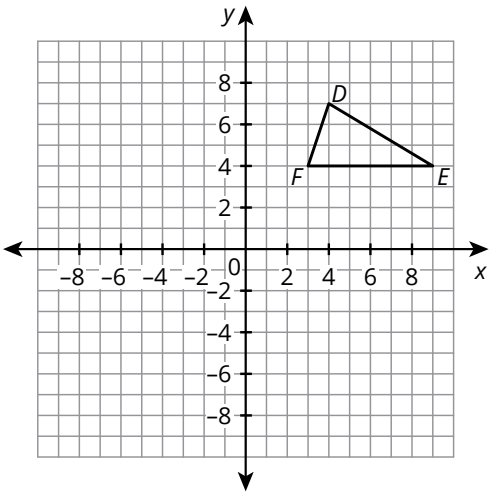


D. Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by SAS. Use the distance formula and/or protractor when necessary. Use CPCTC to identify the congruent sides and angles.

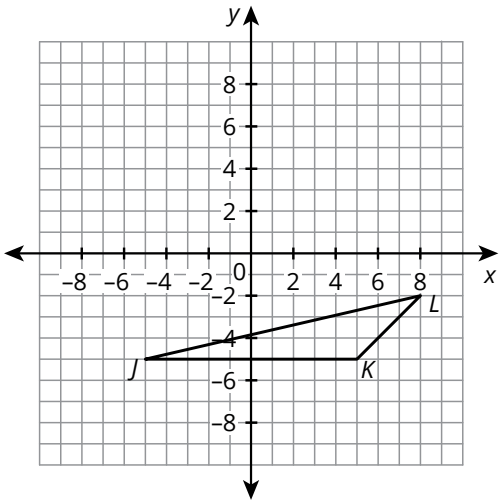
1. Reflect $\triangle ABC$ over the y-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by SAS.



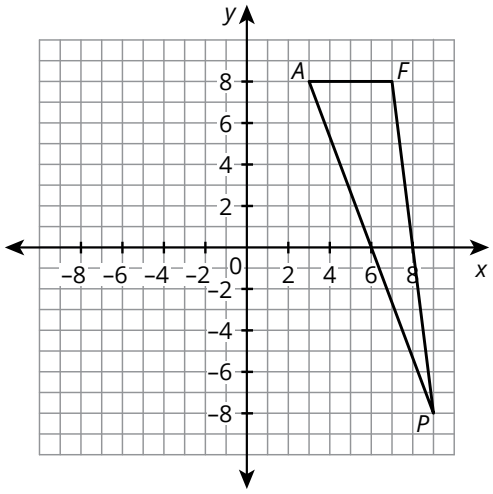
2. Translate ΔDEF 11 units to the right and 10 units up to form ΔQRS. Verify that $\Delta DEF \cong \Delta QRS$ by SAS.



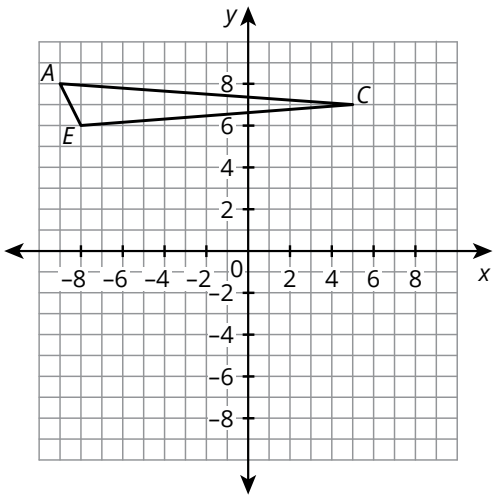
3. Reflect △*JKL* 180° counterclockwise about the origin to form △*MNP*. Verify that △*JKL* ≅ △*MNP* by SAS.



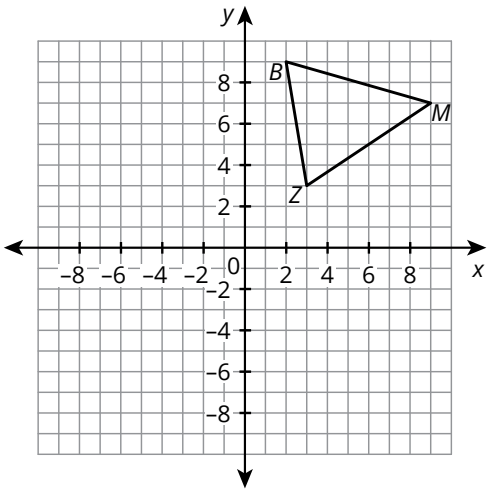
4. Reflect △*AFP* over the *y*-axis to form △*DHW*. Verify that △*AFP* ≅ △*DHW* by SAS.



5. Translate △*ACE* 4 units to the right and 4 units up to form △*JKQ*. Verify that △*ACE* ≅ △*JKQ* by SAS.

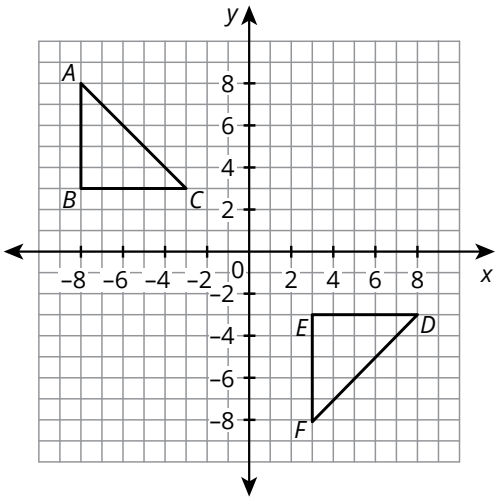


6. Rotate $\triangle BMZ$ 90° counterclockwise about the origin to form $\triangle DRT$. Verify that $\triangle BMZ \cong \triangle DRT$ by SAS.

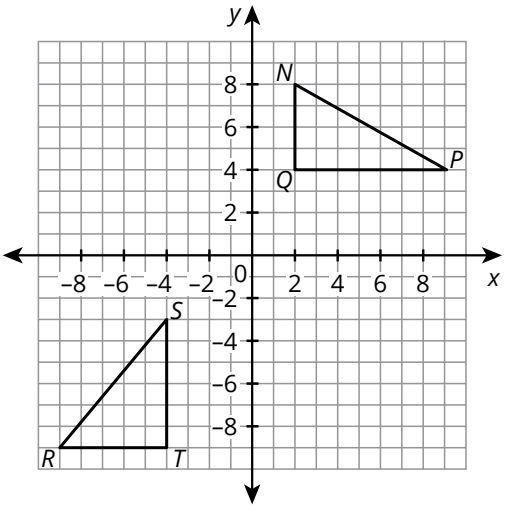


E. Determine whether each pair of given triangles is congruent by ASA. Use the distance formula and/or protractor when necessary. When the triangles are congruent use CPCTC to identify the congruent corresponding sides and angles.

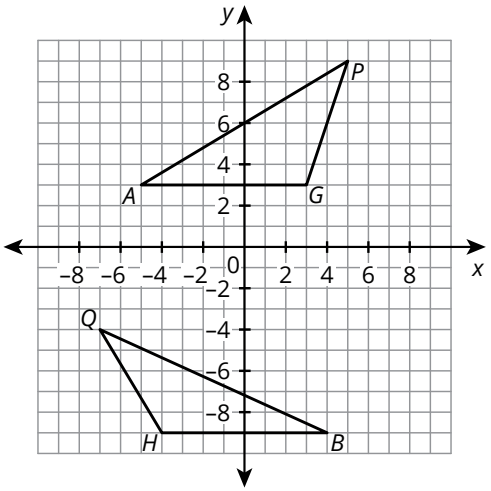
1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by ASA.



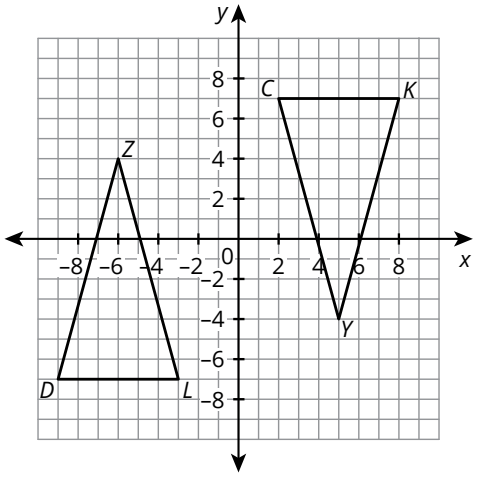
2. Determine whether $\triangle NPQ$ is congruent to $\triangle RST$ by ASA.



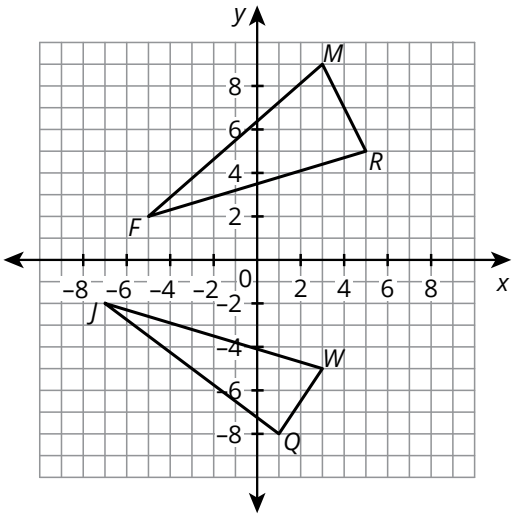
3. Determine whether $\triangle AGP$ is congruent to $\triangle BHQ$ by ASA.



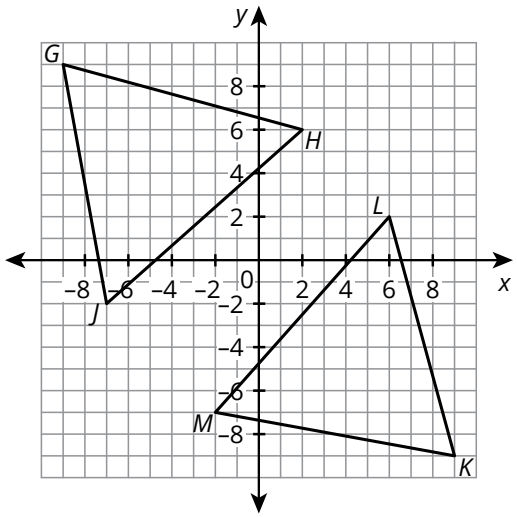
4. Determine whether $\triangle CKY$ is congruent to $\triangle DLZ$ by ASA.



5. Determine whether $\triangle FMR$ is congruent to $\triangle JQW$ by ASA.

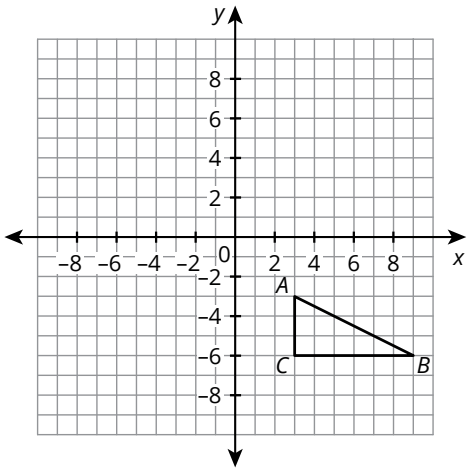


6. Determine whether $\triangle GHJ$ is congruent to $\triangle KLM$ by ASA.

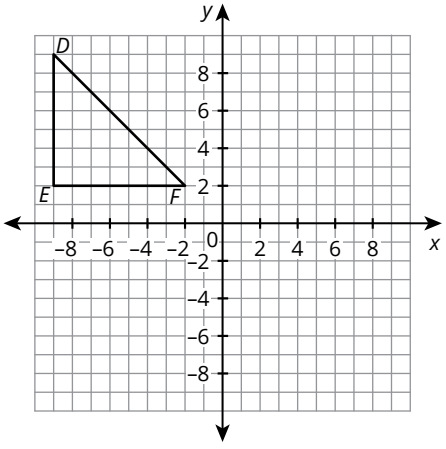


F. Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by ASA. Use the distance formula and/or protractor when necessary. Use CPCTC to identify the congruent corresponding sides and angles.

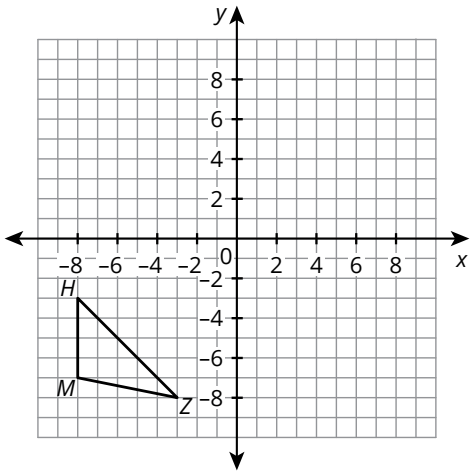
1. Reflect $\triangle ABC$ over the y-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by ASA.



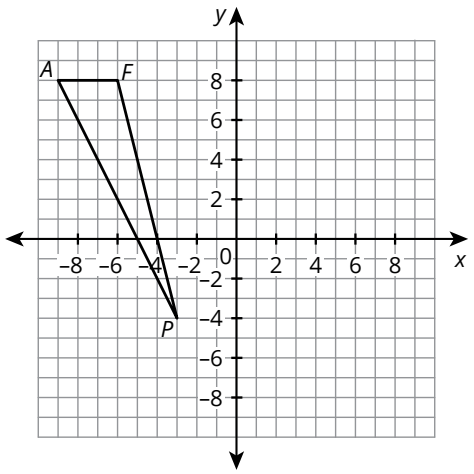
2. Rotate $\triangle DEF$ 90° clockwise about the origin to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by ASA.



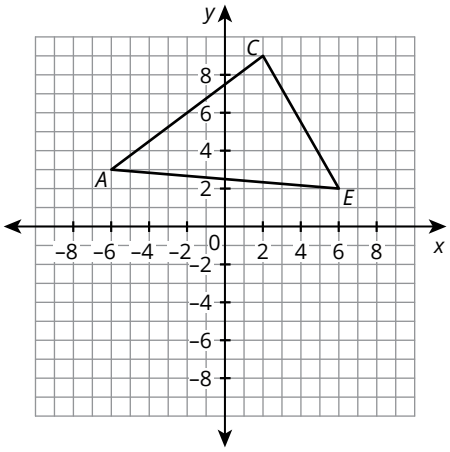
3. Translate △*HMZ* 6 units to the right and 10 units up to form △*BNY*. Verify that △*HMZ* ≅ △*BNY* by ASA.



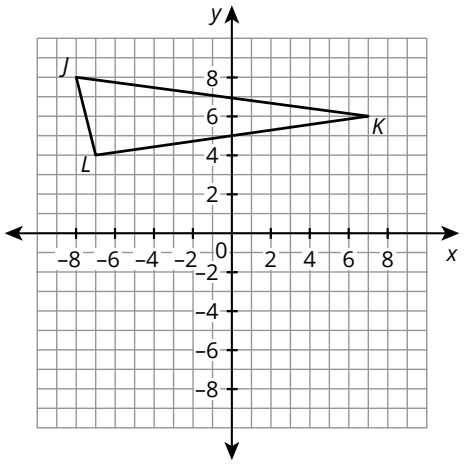
4. Reflect △*AFP* over the *y*-axis to form △*DHW*. Verify that △*AFP* ≅ △*DHW* by ASA.



5. Rotate △*ACE* 180° counterclockwise about the origin to form △*JKQ*. Verify that △*ACE* ≅ △*JKQ* by ASA.

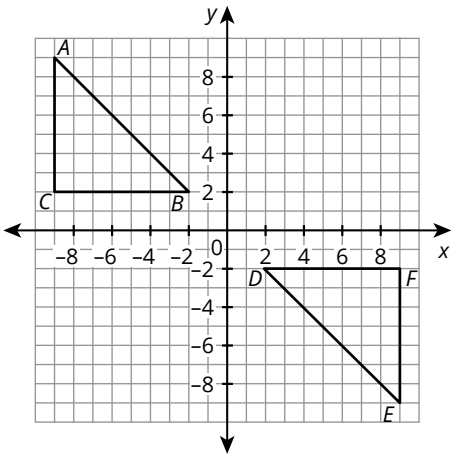


6. Reflect △*JKL* over the *x*-axis to form △*MNP*. Verify that △*JKL* ≅ △*MNP* by ASA.

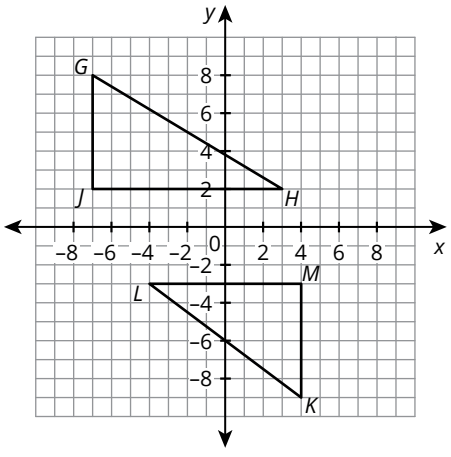


G. Determine whether each set of given triangles is congruent by AAS. Use the distance formula and/or protractor when necessary.

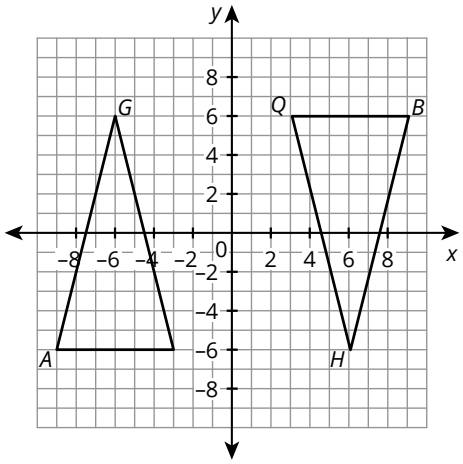
1. Determine whether $\triangle ABC$ is congruent to $\triangle DEF$ by AAS.



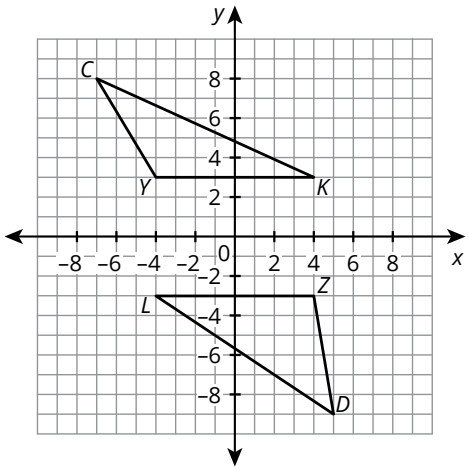
2. Determine whether $\triangle GHJ$ is congruent to $\triangle KLM$ by AAS.



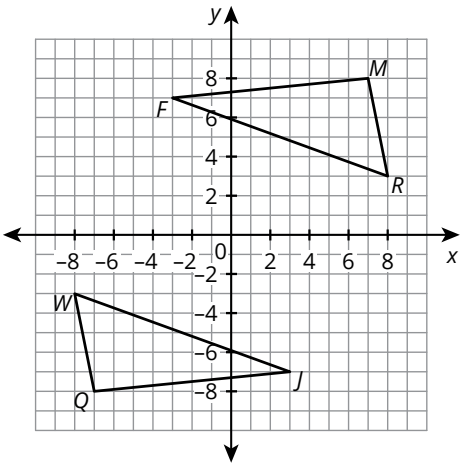
3. Determine whether $\triangle AGP$ is congruent to $\triangle BHQ$ by AAS.



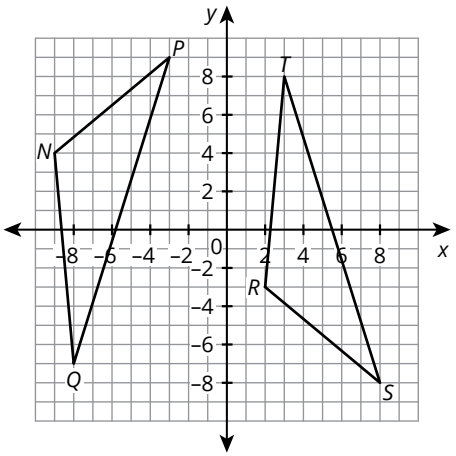
4. Determine whether ΔCKY is congruent to ΔDLZ by AAS.



5. Determine whether $\triangle FMR$ is congruent to $\triangle JQW$ by AAS.

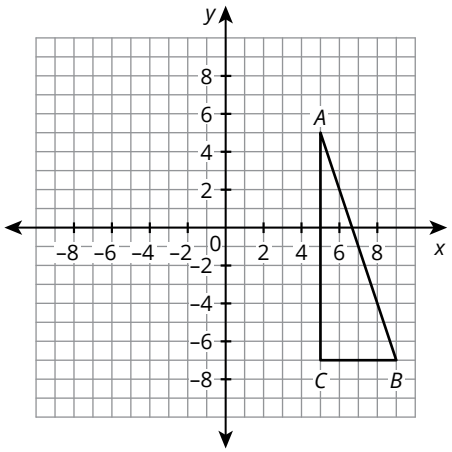


6. Determine whether $\triangle NPQ$ is congruent to $\triangle RST$ by AAS.

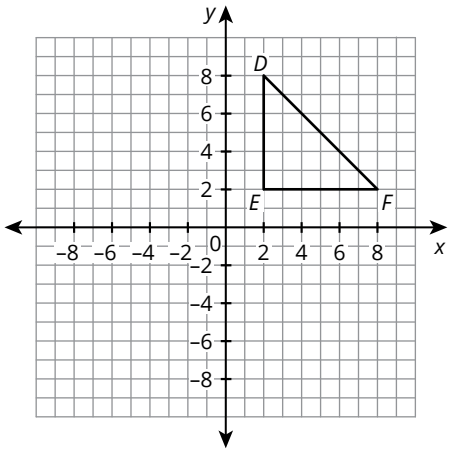


H. Perform the transformation described on each given triangle. Then, verify that the triangles are congruent by AAS. Use the distance formula and/or protractor when necessary. Use CPCTC to identify the congruent sides and angles.

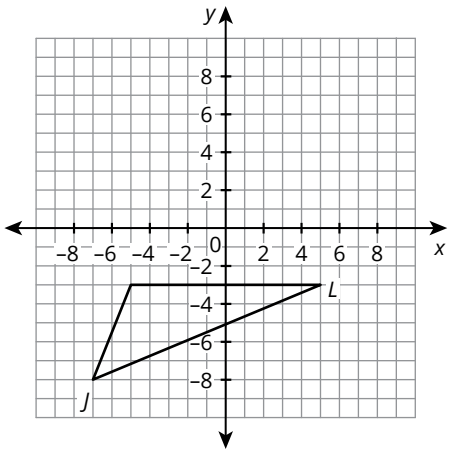
1. Reflect $\triangle ABC$ over the y-axis to form $\triangle XYZ$. Verify that $\triangle ABC \cong \triangle XYZ$ by AAS.



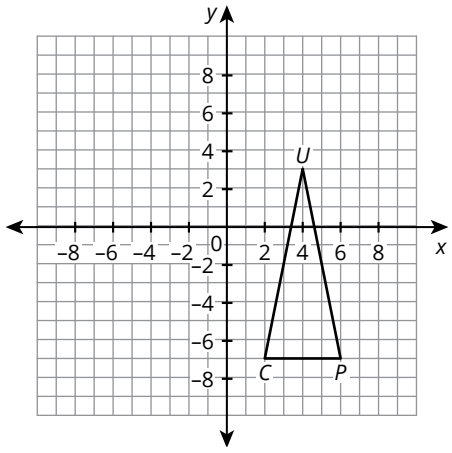
2. Translate $\triangle DEF$ 11 units to the left and 11 units down to form $\triangle QRS$. Verify that $\triangle DEF \cong \triangle QRS$ by AAS.



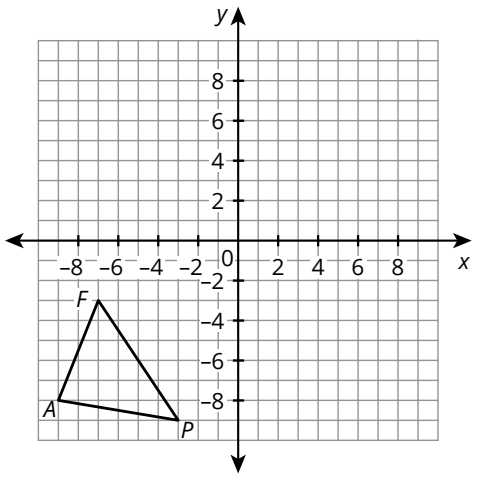
3. Rotate △*JKL* 180° counterclockwise about the origin to form Δ*MNP*. Verify that △*JKL* ≅ △*MNP* by AAS.



4. Translate △*CUP* 9 units to the left and 4 units down to form △*JAR*. Verify that △*CUP* ≅ △*JAR* by AAS.



5. Reflect $\triangle AFP$ over the x-axis to form $\triangle DHW$. Verify that $\triangle AFP \cong \triangle DHW$ by AAS.



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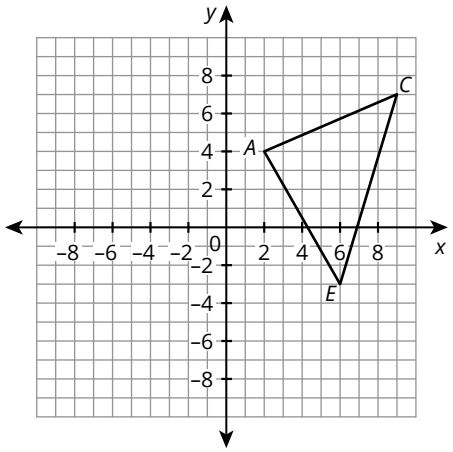
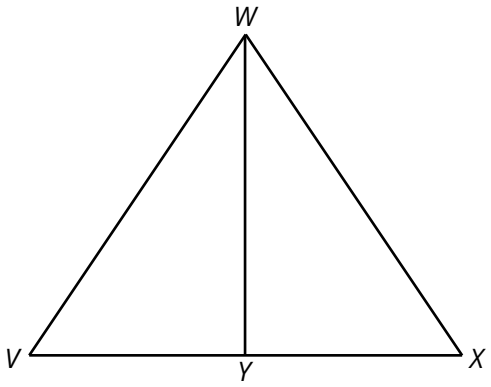
6. Rotate △*ACE* 270° clockwise about the origin to form △*JKQ*. Verify that △*ACE* ≅ △*JKQ* by AAS.

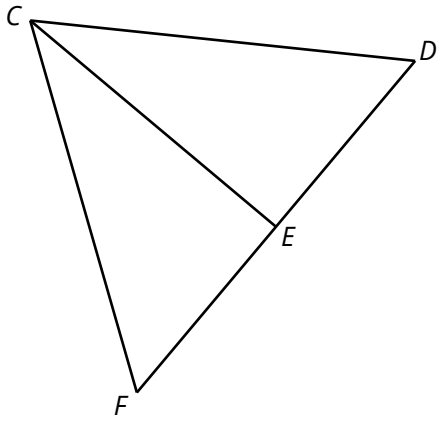


I. Mark each diagram with the given information. State the congruence theorem that proves the triangles in each diagram are congruent. If not enough information is given, name an example of information that could be given for you to use to prove congruency. Explain your reasoning.

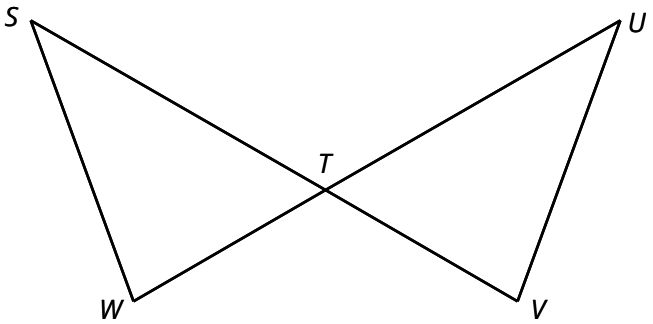
1. Given: $\overline{VW} \cong \overline{XW}$
   Prove: $\triangle VYW \cong \triangle XYW$



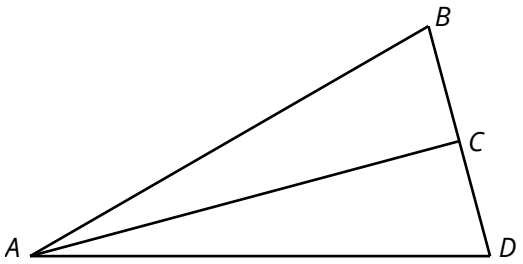
2. Given: $\overline{CE}$ is a perpendicular bisector of $\overline{FD}$
   Prove: $\triangle FEC \cong \triangle DEC$



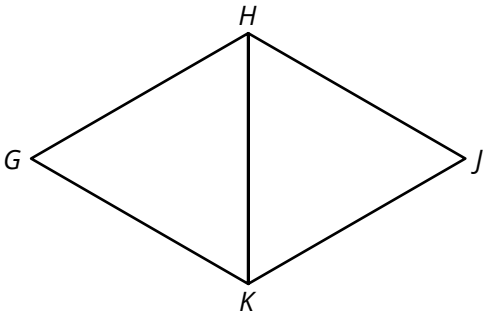
3. Given: $\angle WST \cong \angle VUT < \overline{ST} \cong \overline{UT}$
   Prove: $\Delta WST \cong \Delta VUT$



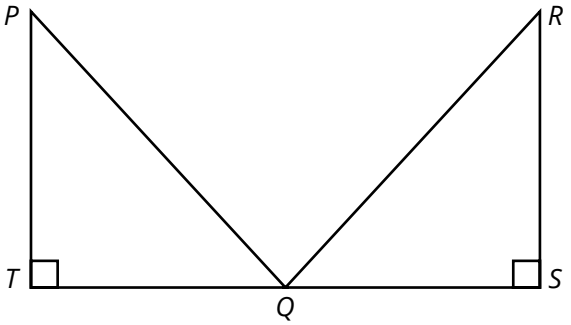
4. Given: $\triangle ABD$ is isosceles with $\overline{AB} \cong \overline{AD}$
Prove: $\triangle ABC \cong \triangle ADC$



5. Given: $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$
Prove: $\triangle GHK \cong \triangle JKH$

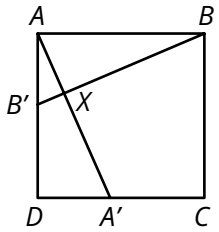


6. Given: $\overline{PT} \cong \overline{RS}$, $\angle PQT \cong \angle RQS$
Prove: $\triangle TPQ \cong \triangle SRQ$



Extension

1. Figure *ABCD* is a square. The line segments AA' and BB' are perpendicular to each other. Determine how it can be proven that $ABB' \cong DAA'$ using either SAS, SSS, or ASA.



Spaced Practice

1. Draw two triangles that correspond with the congruence statement $\triangle MTA \cong \triangle BGC$. Then list the six pairs of congruent corresponding parts.

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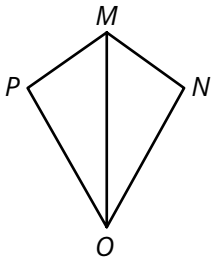
2. For the figure shown, determine whether there is enough information to conclude that the triangles are congruent. If so, state the theorem you used.

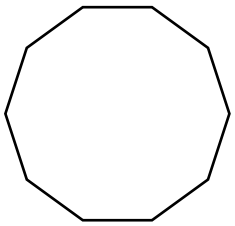


Given: $\angle MOP \cong \angle MON$

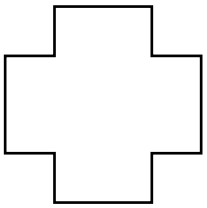
Is $\triangle POM \cong \triangle NOM$?

3. Describe the sequence of rotations and/or reflections that can carry each figure onto itself. Use the figure's reflectional and rotational symmetry, if any, to justify your answer.

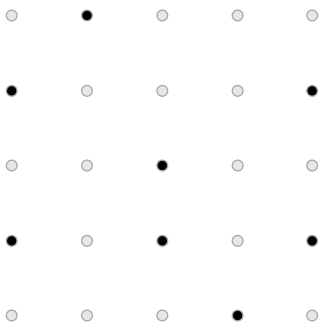
a.



b.



4. Use the dark points to determine the location of the vertices of a square.



Justifying Mathematical Ideas and Arguments

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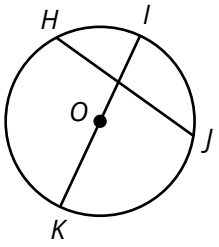
Name _____ Date _____

I. Using Circles to Make Conjectures

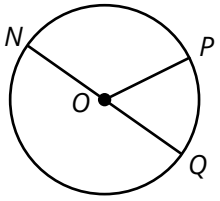
Topic Practice

A. Classify each arc as a major arc, a minor arc, or a semicircle.

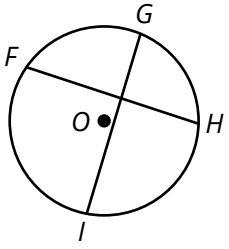
1. $\overset{\frown}{HI}$



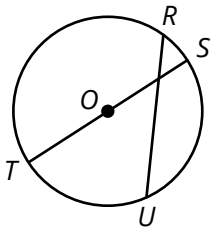
2. $\overset{\frown}{NPQ}$



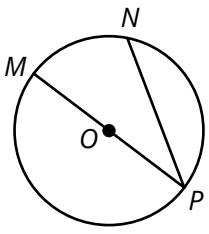
3. $\overset{\frown}{FHI}$



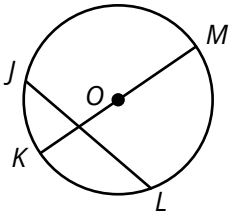
4. $\overset{\frown}{TRS}$



5. $\overset{\frown}{NP}$

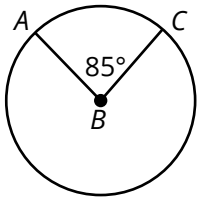


6. $\overset{\frown}{JML}$

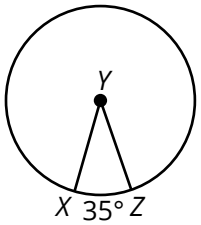


B. Determine the measure of the indicated central angle or arc.

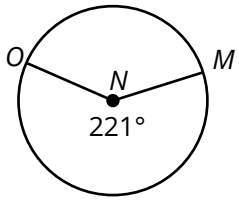
1. $m\overset{\frown}{AC}$ = _____



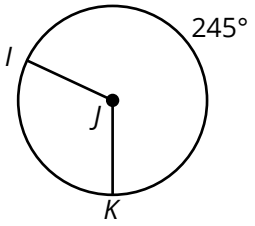
2. $m\angle XYZ$ = _____



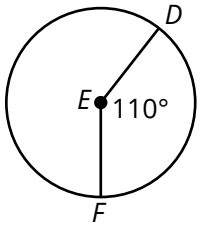
3. $m\overset{\frown}{MO}$ = _____



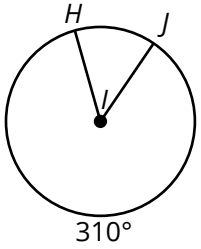
4. $m\angle IJK$= _____



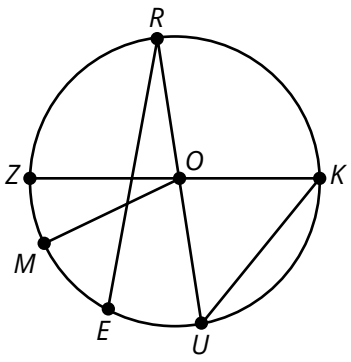
5. $m\overset{\frown}{DF}$ = _____



6. $m\angle HIJ$ = _____



C. Identify each angle as an inscribed angle or a central angle.



1. $\angle URE$

2. $\angle ZOM$

3. $\angle KOM$

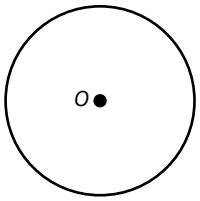
4. $\angle ZKU$

5. $\angle MOU$

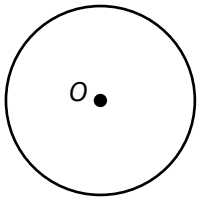
6. $\angle ROK$

D. Draw the part of a circle that is described.

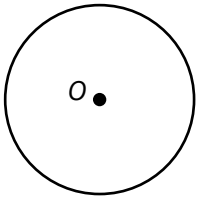
1. Draw chord $\overline{AB}$.



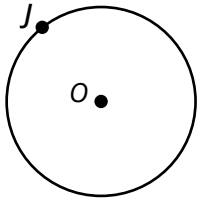
2. Draw radius $\overline{OE}$.



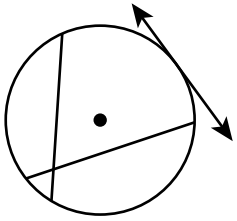
3. Draw secant $\overleftrightarrow{GH}$.



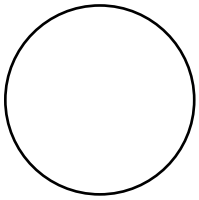
4. Draw a tangent at point *J*.



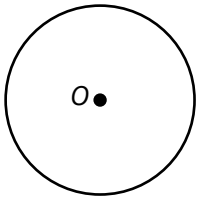
5. Label the point of tangency *A*.



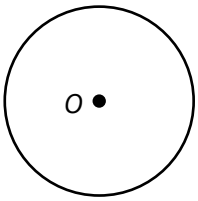
6. Label center *C*.



7. Draw inscribed angle $\angle FDG$.

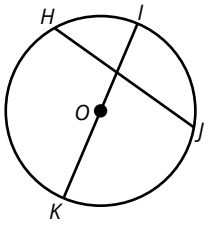


8. Draw central angle $\angle HOI$.

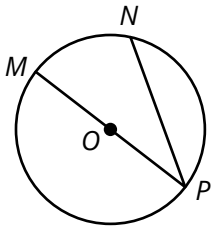


E. Identify the indicated part of each circle. Explain your answer.

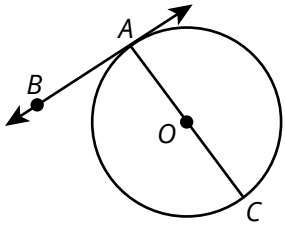
1. O



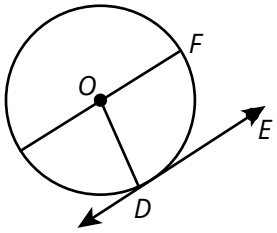
2. $\overline{NP}$



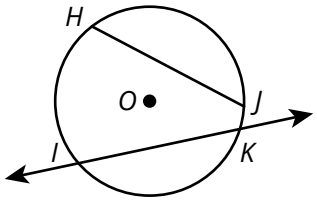
3. $\overleftrightarrow{AB}$



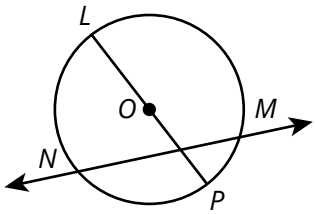
4. D



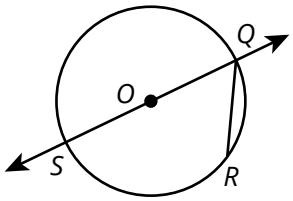
5. $\overline{JH}$



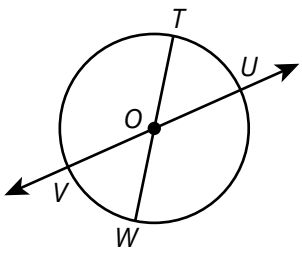
6. $\overleftrightarrow{MN}$



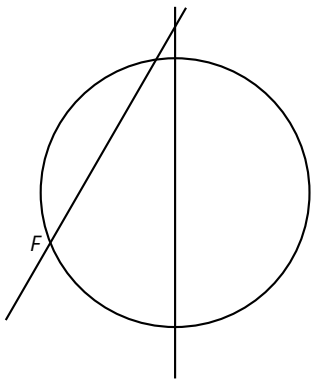
7. $\angle SQR$



8. $\angle TOU$



F. Given a circle with a vertical diameter. Perform each task.



1. Construct the perpendicular bisector of the diameter.

2. Draw a line through two the points on a circle to form a secant and chord that is not a diameter.

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3. Make a conjecture about the type of triangle formed.

   Conjecture: What type of triangle is formed by the points A, C, and D?

4. Make a conjecture about vertical angles using the diagram they created.

   Conjecture: What can you say about the vertical angles formed at the intersection of the secant $\overline{CD}$ and the diameter $\overline{AB}$?

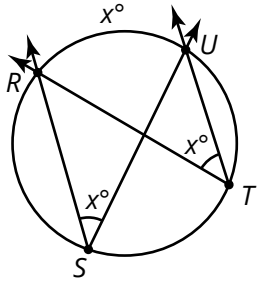
5. Construct a parallel line through the center of the circle parallel to the secant.

6. Make a conjecture about an angle pair.

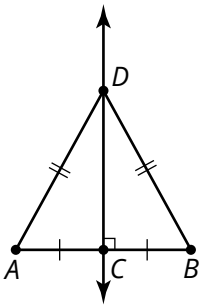
   Conjecture: What can you say about the angle between the parallel line $\overline{EF}$ and the secant $\overline{CD}$ compared to the angle formed between $\overline{AB}$ and $\overline{CD}$?

G. Write a conjecture about each geometric example.

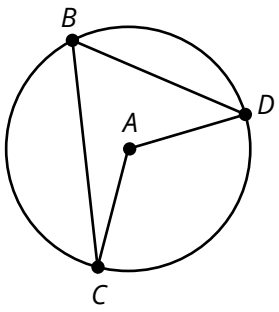
1.



2.

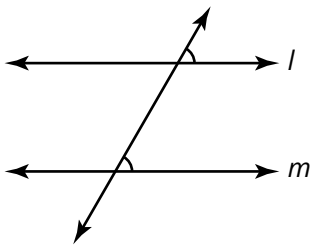


3.

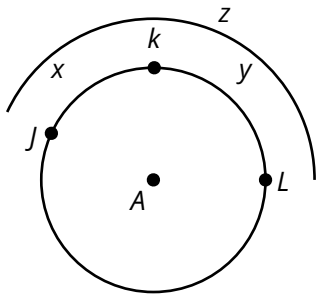


$m\angle CBD = \frac{1}{2} m\angle CAD$

4.

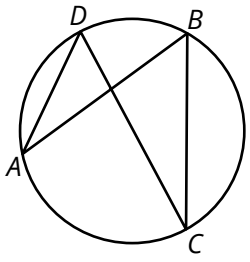


5.



$m\overset{\frown}{JK} + m\overset{\frown}{KL} = m\overset{\frown}{JKL}$

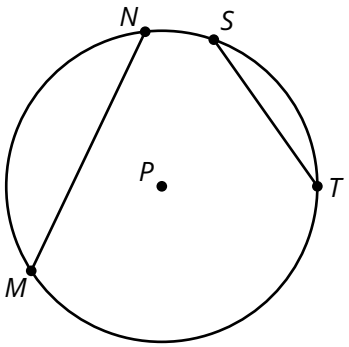
6.



$m\angle ADC = m\angle ABC$

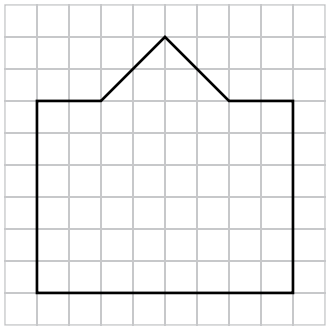
Extension

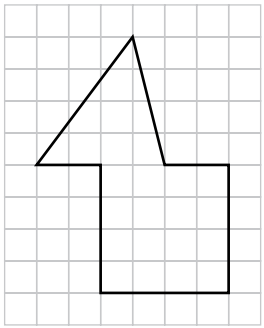
Consider the circle with a center at point *P*. Construct the perpendicular bisectors of chord $\overline{MN}$ and chord $\overline{ST}$. Then make a conjecture about the perpendicular bisectors of chords in a circle.

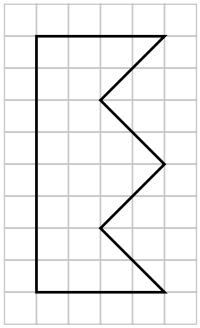


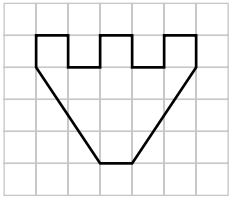
Spaced Practice

Determine the area of each composite figure. Each grid square measures 1 unit by 1 unit.

1. 

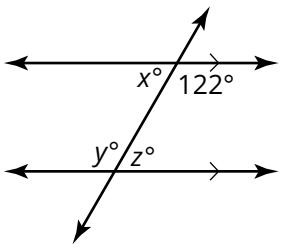
2. 

3. 

4. 

5. Determine each unknown measure in the figure.

   Explain your reasoning.



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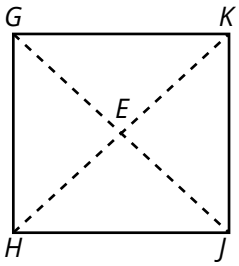
II. Conjectures About Quadrilaterals

Topic Practice

A. Complete each statement using the given diagram.

1. Complete each statement for square *GKJH*.



   a. $\overline{GK} \cong$ _____ $\cong$ _____ $\cong$ _____

   b. $\angle KGH \cong \angle$_____ $\cong \angle$_____ $\cong \angle$_____
   $\cong \angle$_____ $\cong \angle$_____ $\cong \angle$_____ $\cong \angle$_____

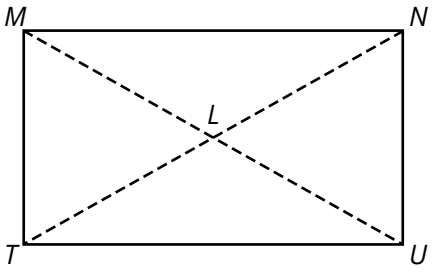
   c. $\angle KEG$, $\angle$_____, $\angle$_____, $\angle$_____, $\angle$_____,
   $\angle$_____, $\angle$_____, and $\angle$_____ are right angles.

   d. $\overline{GK} \parallel$ _____ and $\overline{GH} \parallel$ _____

   e. $\overline{GE} \cong$ _____ $\cong$ _____ $\cong$ _____

   f. $\angle KGE \cong \angle$_____ $\cong \angle$_____ $\cong \angle$_____ $\cong \angle$_____ $\cong$
   $\angle$_____ $\cong \angle$_____ $\cong \angle$_____

2. Complete each statement for rectangle *TMNU*.



   a. $\overline{MN} \cong$ _____ and $\overline{MT} \cong$ _____

   b. $\angle NMT \cong \angle$_____ $\cong \angle$_____
   $\cong \angle$_____

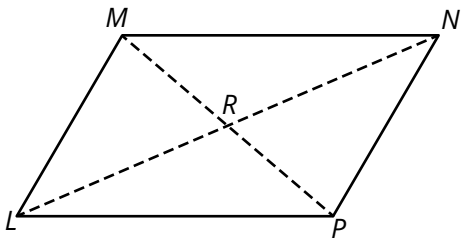
   c. $\angle MTU$, $\angle$_____, $\angle$_____, and $\angle$_____ are right angles.

   d. $\overline{MN} \parallel$ _____ and $\overline{MT} \parallel$ _____

   e. $\overline{MU} \cong$ _____

   f. $\overline{ML} \cong$ _____ $\cong$ _____ $\cong$ _____

3. Complete each statement for parallelogram *MNPL*.



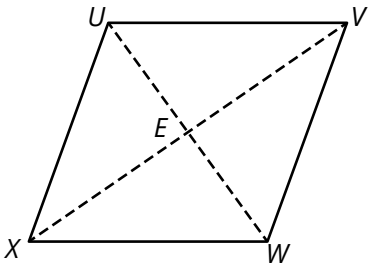
a. $\overline{MN} \cong$ _____ and $\overline{ML} \cong$ _____

b. $\angle NML \cong \angle$_____ and $\angle MLP \cong \angle$_____

c. $\overline{MN} \parallel$ _____ and $\overline{ML} \parallel$ _____

d. $\overline{MR} \cong$ _____ and $\overline{LR} \cong$ _____

4. Complete each statement for rhombus *UVWX*.



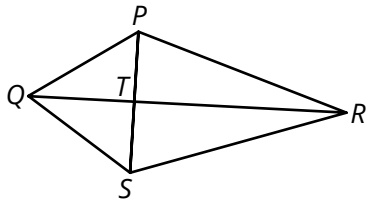
a. $\overline{UV} \cong$ _____ $\cong$ _____ $\cong$ _____

b. $\angle UVW \cong \angle$_____ and $\angle XUV \cong \angle$_____

c. $\overline{UV} \parallel$ _____ and $\overline{UX} \parallel$ _____

d. $\overline{UE} \cong$ _____ and $\overline{XE} \cong$ _____

5. Complete each statement for kite *PRSQ*.



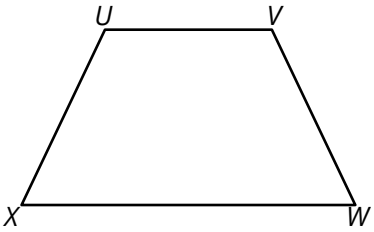
a. $\overline{PQ} \cong$ _____ and $\overline{PR} \cong$ _____

b. $\angle QPR \cong \angle$_____

c. $\overline{PT} \cong$ _____

d. $\angle PQT \cong \angle$_____ and $\angle PRT \cong \angle$_____

6. Complete each statement for isosceles trapezoid *UVWX*.



a. The bases are _____ and _____.

b. The pairs of base angles are $\angle$_____ and $\angle$_____, and $\angle$_____ and $\angle$_____.

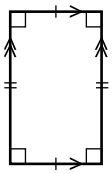
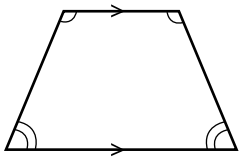
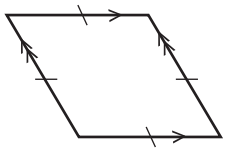
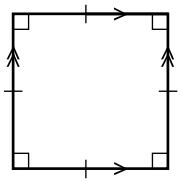
c. The legs are _____ and _____.

d. The vertices are _____, _____, _____ and _____.

B. List all of the quadrilaterals that have the given characteristic.

1. all sides congruent
2. diagonals congruent
3. no parallel sides
4. diagonals bisect each other
5. two pairs of parallel sides
6. all angles congruent

C. Name the type of quadrilateral that best describes each figure. Explain your answer.

1. 
2. 
3. 
4. 

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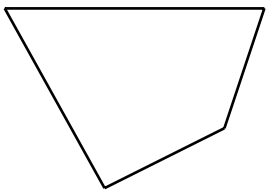
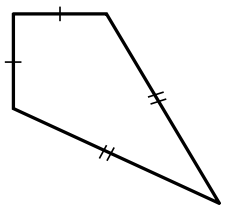
5. 

6. 

D. Use the given information to answer each question. Explain your reasoning.

1. Michael drew a quadrilateral. He used a protractor to measure all four angles of the quadrilateral. How many pairs of angles must be congruent for the quadrilateral to be a parallelogram?

2. Olivia makes the following statement: "Every rhombus is a parallelogram." Do you agree?

3. Javier connected a square and two congruent right triangles together to form an isosceles trapezoid. Draw a diagram to represent the isosceles trapezoid.

4. Daniela told Fernando that an isosceles trapezoid must also be a parallelogram because there is only one a pair of parallel sides in an isosceles trapezoid. Is Daniela correct? Explain.

5. Alexander cut a quadrilateral out of a piece of cardstock but is not sure if the figure is a parallelogram or a rhombus. He measures the lengths of the opposite sides and determines them to be congruent. He measures the opposite angles of the quadrilateral and determines them also to be congruent. He measures one angle and is able to determine that the quadrilateral is a rhombus. What angle did he measure?

6. Koda created a kite out of two sticks and some fabric. The sticks were 10 inches and 15 inches long. She tied the sticks together so they were perpendicular and attached the fabric. When she measured the kite, she noticed that the distance from where the sticks meet to the top of the kite was 5 inches. What is the area of the kite Koda created?

7. Destiny plans to make the base of her sculpture in the shape of a rhombus. She cuts out four pieces of wood to create a mold for concrete. The pieces of wood are the following lengths: 5 inches, 5 inches, 3 inches, and 3 inches. Will the base of Destiny's sculpture be a rhombus?

8. Gabriela drew what she thought was an isosceles trapezoid. She measured the base angles and determined that they measured 81°, 79°, 101°, and 99°. Could her drawing be an isosceles trapezoid? Explain.

9. Hannah constructed a kite with a perimeter of 38 centimeters so that the sum of the two shorter sides is 10 centimeters. What are the lengths of each of the two longer sides?

10. Jacob has a picture in the shape of a quadrilateral that he cut out of a magazine. How could Jacob use a ruler to prove that the picture is a parallelogram?

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11. Three angles of a parallelogram have the following measures: 58°, 122°, and 58°. What is the measure of the fourth angle?

12. Madison constructed a kite that has side lengths of 8 inches and 5 inches. What are the lengths of the other two sides? Explain.

E. Write a conjecture about the diagonals and interior angles of each quadrilateral. Draw examples to test your conjecture.

1. Parallelogram that is not a rhombus or a rectangle

2. Rectangle

3. Kite

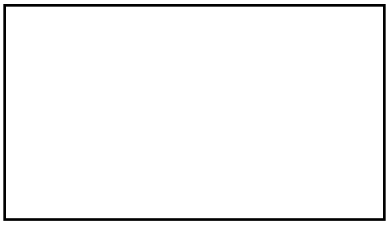
4. Square

5. Isosceles trapezoid

6. Rhombus

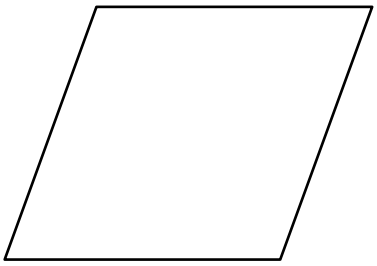
F. Draw the midsegments of each quadrilateral.

1. Rectangle

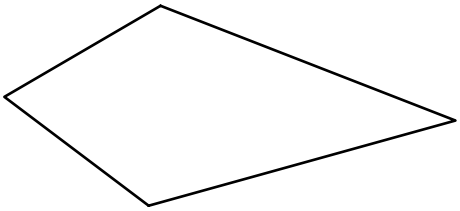


2. Parallelogram

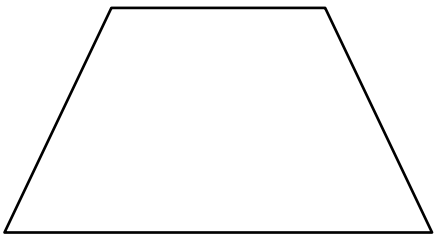
3. Rhombus



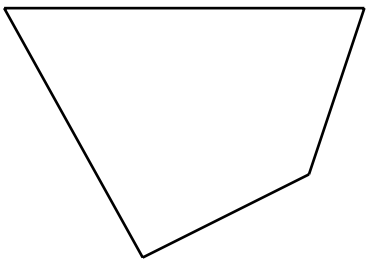
4. Kite



5. Trapezoid

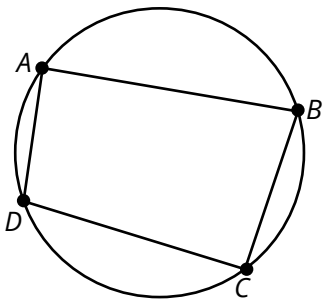


6. Quadrilateral



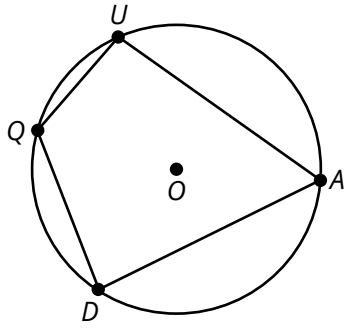
G. Determine the measure of the indicated angle.

1. In quadrilateral $ABCD$, $m\angle B = 75°$. Determine $m\angle D$.



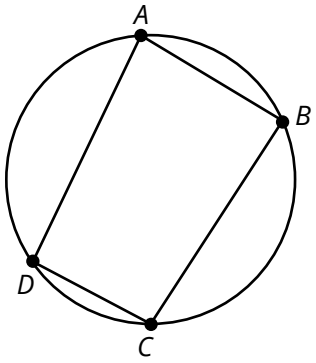
$m\angle D =$ _____

2. In quadrilateral $QUAD$, the $m\angle A = 62°$. Determine the measure of $m\angle Q$.



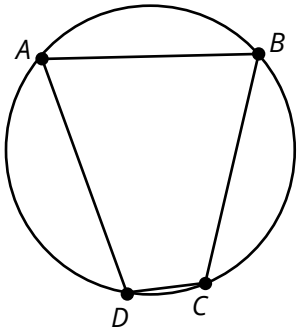
$m\angle Q =$ _____

3. In quadrilateral $ABCD$, $m\angle A = 81°$. Determine $m\angle C$.



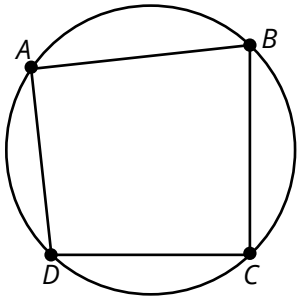
$m\angle C =$ _____

4. In quadrilateral $ABCD$, $m\angle D = 112°$. Determine $m\angle B$.



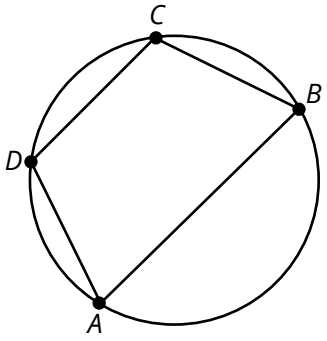
$m\angle B =$ _____

5. In quadrilateral $ABCD$, $m\angle D = 93°$. Determine $m\angle B$.



$m\angle B =$ _____

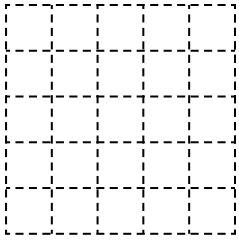
6. In quadrilateral $ABCD$, $m\angle A = 72°$. Determine $m\angle C$.



$m\angle C =$ _____

Extension

Create a Zukei puzzle for an isosceles trapezoid in which the bases do not lie on the grid lines. Use a minimum of 10 dots. Make sure that your puzzle has only one correct answer.

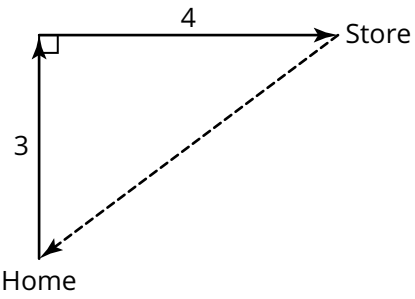


Spaced Practice

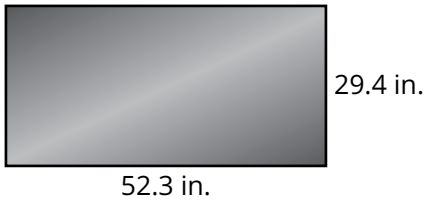
1. Write a conjecture about alternate interior angles. Draw an example to test your conjecture.

2. Draw examples of inscribed angles that intercept the same arc of a circle. What conjecture can you make about the measures of the inscribed angles?

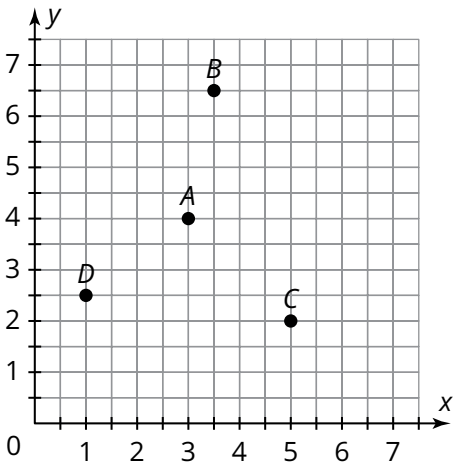
3. Jay walks 3 blocks north and then 4 blocks east to get to the store. If he walks straight back home, how far does Jay walk in all?



4. TV screen sizes are given by their diagonal measure from a top corner to the opposite bottom corner. What is the approximate size of this TV to the nearest inch?



5. Use the coordinate plane to approximate each distance. Write each answer as a decimal to the nearest hundredth.



   a. The distance between point *A* and point *C*

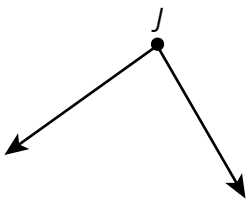
   b. The distance between point *D* and point *B*

III. Constructing an Inscribed Regular Polygon

Topic Practice

A. Using construction tools, duplicate each angle.

1.



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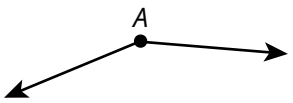

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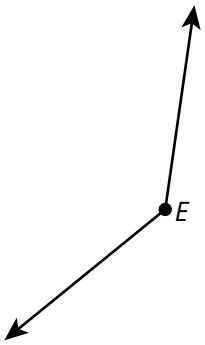

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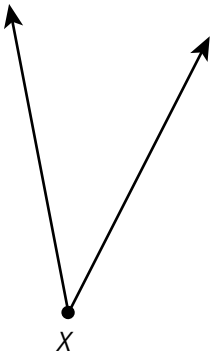

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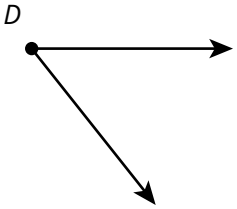

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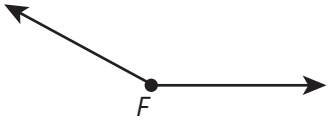
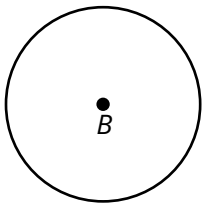


CC BY NC

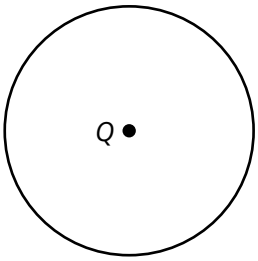
B. Construct each figure.

1. an equilateral triangle (not inscribed in a circle)

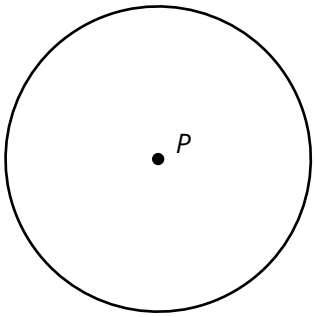
2. a right triangle inscribed in a circle



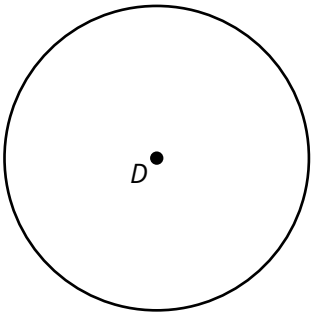
3. an equilateral triangle inscribed in a circle



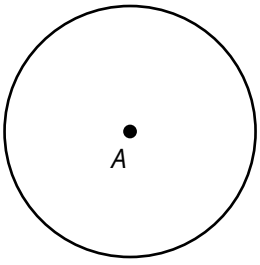
4. a regular hexagon inscribed in a circle



5. a square inscribed in a circle



6. a regular octagon inscribed in a circle



Extension

Determine two different constructions you could use to construct an angle with a measure of 45° using only a compass and straightedge. Summarize the processes and show both constructions.

Spaced Practice

1. Identify the quadrilaterals that match each description.

   a. opposite sides are parallel

   b. at least one pair of opposite angles is congruent

2. Write an equation for a line that is perpendicular to the given line.

   a. $y = 4x - 1$

   b. $y = x + 2$

3. Write an equation for a line that is parallel to the given line.

   a. $x = 3$

   b. $y = -\frac{1}{2}x - \frac{5}{2}$

IV. Conjectures About Triangles

Topic Practice

A. Consider each conditional statement. Write the converse and determine if that statement is a biconditional statement. If false, provide a counterexample.

1. If two segments are congruent, then they have the same length.

2. If Ana lives in Miami, then she lives in Florida.

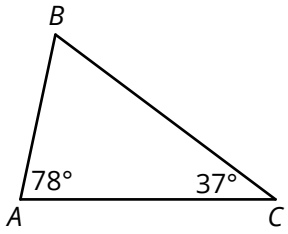
3. If an angle is acute, then it has a measure of less than 90°.

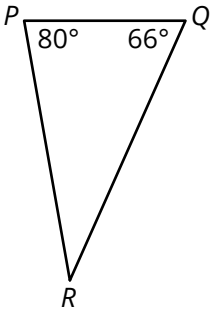
4. If it is raining, then there are clouds in the sky.

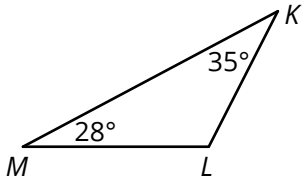
5. If $x = 6$, then $x^2 = 36$.

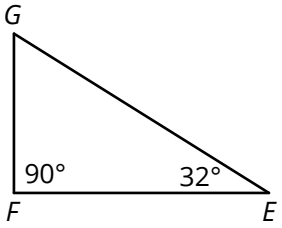
6. If a quadrilateral is a square, all the sides are congruent.

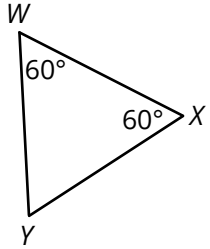
B. Use a conjecture about the measures of the interior angles of a triangle to determine the unknown angle measure in each figure.

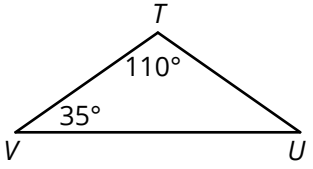
1. 

2. 

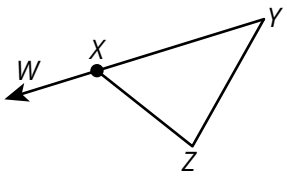
3. 

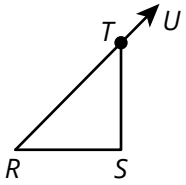
4. 

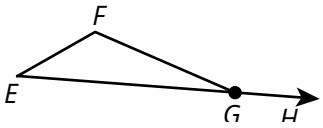
5. 

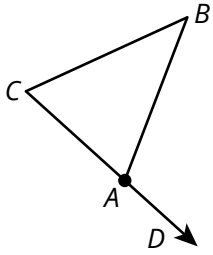
6. 

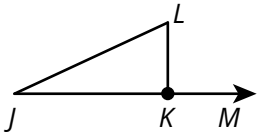
C. Identify the interior angle(s), the exterior angle(s), and the remote interior angle(s) of each triangle.

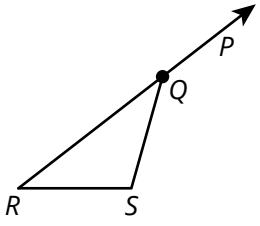
1. 

2. 

3. 

4. 

5. 

6. 

D. Write a conjecture about each geometric object. Use constructions or draw examples to test your conjecture.

1. base angles of an isosceles triangle

2. the sum of the measures of the interior angles of a triangle

3. an exterior angle of a triangle

4. alternate interior angles

5. the side lengths of a triangle

6. a midsegment of a triangle

E. Apply the Triangle Inequality theorem in each question.

1. Given the lengths of three sides of a triangle as 5, 7, and 10 units, determine if these sides lengths can form a triangle.

2. A triangle has sides measuring 6, 8, and 13 units. Is this a valid triangle? Explain why or why not.

3. In a triangle, the lengths of two sides are given as 8 and 12 units. If the length of the third side is x, what range of values can x take for this to be a valid triangle?

4. A triangle has side lengths of 3, 4, and 5 units. Based on the Triangle Inequality Theorem, explain what type of triangle it is (scalene, isosceles, or right) and why.

5. Determine if the following side lengths can form a valid triangle: 5, 9, and 14.

6. Find the range of possible lengths for the third side x in a triangle, given that the two sides are 8 and 15 units, and the perimeter of the triangle is 30 units.

   Solve for x:

   $$8 + 15 + x = 30$$

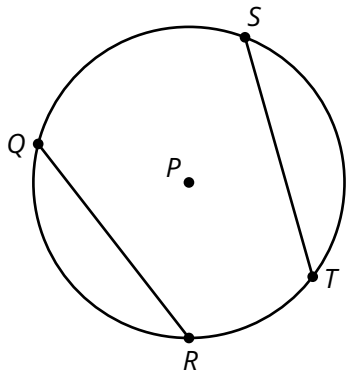
   $$23 + x = 30$$

   $$x = 7$$

   Check for validity of the triangle:

Extension

1. Consider the circle with center P. Chords $\overline{QR}$ and $\overline{ST}$ are congruent. Draw $\triangle QPR$ and $\triangle SPT$.



   a. Is $\triangle QPR$ congruent to $\triangle SPT$? Explain your reasoning.

   b. Draw the altitude of each triangle. How do the two altitudes compare? Explain your reasoning.

   c. Write a conjecture about the distance of congruent chords from the center of the circle. Then write the converse of the conjecture.

Spaced Practice

1. State as many properties as you can about a kite.

2. State as many properties as you can about a rectangle.

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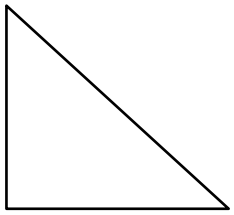
V. Points of Concurrency

Topic Practice

A. Construct the circumcenter of each triangle.

1.



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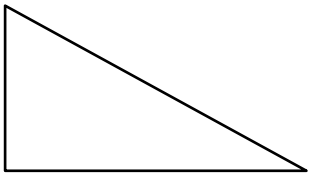

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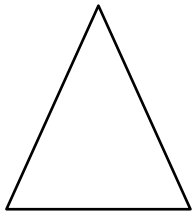

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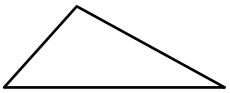

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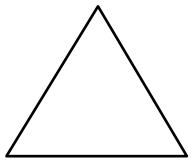

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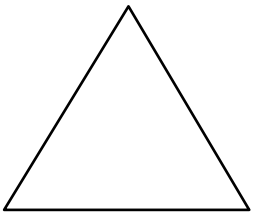
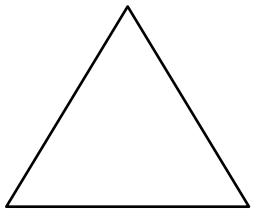


B. Construct the incenter of each triangle.

1.



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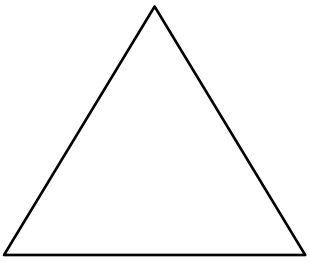
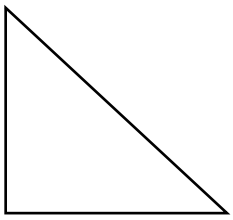
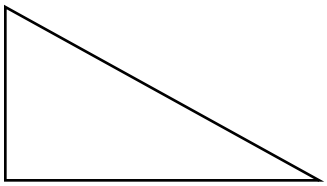
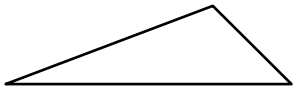
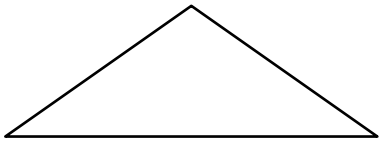


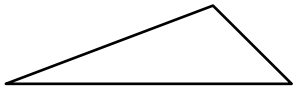
3. 

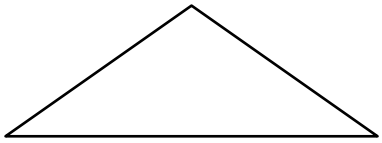
4. 

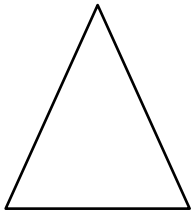
5. 

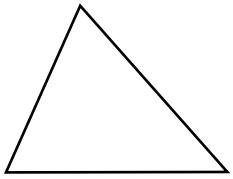
6. 

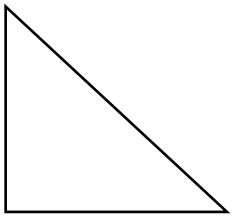
C. Construct the centroid of each triangle.

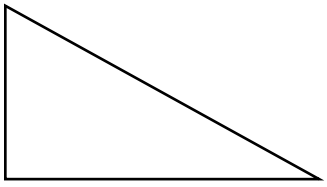
1. 

2. 

3. 

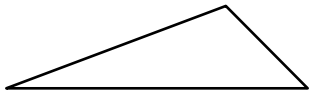
4. 

5. 

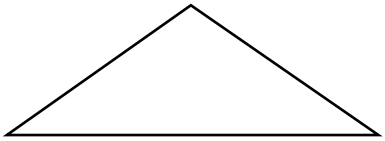
6. 

D. Construct the orthocenter of each triangle.

1.



2.



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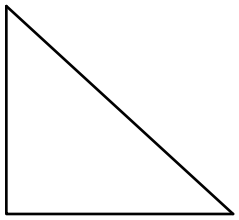

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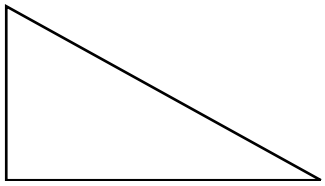

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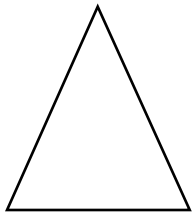

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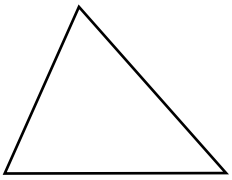
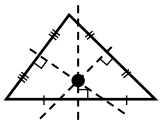
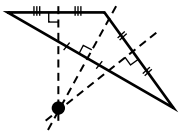
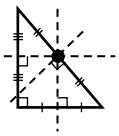
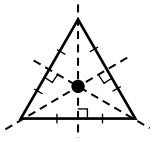
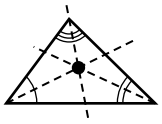
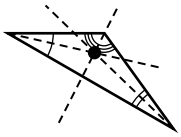
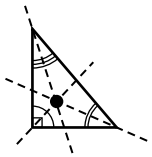
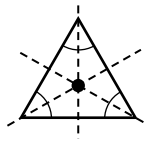
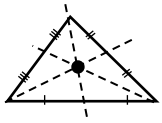
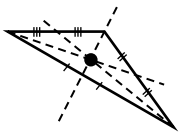
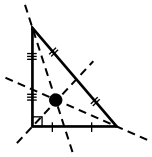
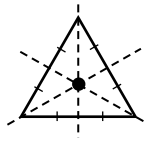
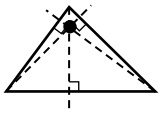
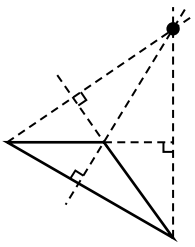
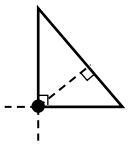
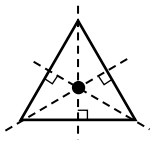


E. Use the constructions in the table to complete each conjecture about the location of each point of concurrency by filling in the blank of each sentence.

	Acute (not equilateral)	Obtuse	Right	Equilateral
Circumcenter				
Incenter				
Centroid				
Orthocenter				

1. The circumcenter of an obtuse triangle lies on the _____ of the triangle.
2. The centroid of a right triangle lies on the _____ of the triangle.
3. The orthocenter of an acute triangle lies on the _____ of the triangle.
4. The circumcenter of a right triangle lies on the _____ of the triangle.
5. The orthocenter of a right triangle lies on the _____ of the triangle.
6. The incenter of an acute triangle lies on the _____ of the triangle.
7. The centroid of a right triangle lies on the _____ of the triangle.
8. The incenter of an equilateral triangle lies on the _____ of the triangle.

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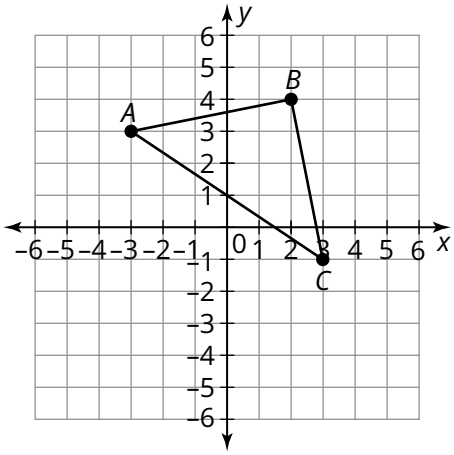
Extension

1. The Euler Line is a line that represents the relationship between the centroid, circumcenter, incenter, and orthocenter of any triangle.

   a. Construct an isosceles triangle and determine all four points of concurrency. How do the points relate to the Euler Line?

   b. What happens if the triangle is equilateral?

2. Determine the coordinates of the centroid of the triangle on the coordinate grid.



Spaced Practice

1. Write a conjecture about points on the perpendicular bisector of a line segment. Then, write the converse of the conjecture.

2. Write a conjecture about the sides of an equilateral triangle. Then, write the converse of the conjecture.

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3. Determine whether each statement is biconditional.

   a. If a figure is a square, then it is a rectangle.

   b. If two angles are congruent, then they have the same measure.

Name _____ Date _____

I. Forms of Proof

Topic Practice

A. Identify the property demonstrated in each example.

1. $m\angle ABC = m\angle XYZ$

   $m\angle ABC - m\angle RST = m\angle XYZ - m\angle RST$

2. $m\overline{QT} = m\overline{TU}$

   $m\overline{QT} + m\overline{WX} = m\overline{TU} + m\overline{WX}$

3. $\angle JKL \cong \angle JKL$

4. $GH = MN$ and $MN = OP$, so $GH = OP$

5. $m\overline{XY} = 4$ cm and $m\overline{BC} = 4$ cm, so $m\overline{XY} = m\overline{BC}$

6. $\overline{PR} \cong \overline{PR}$

7. $GH = JK$

   $GH - RS = JK - RS$

8. $m\angle 1 = 134°$ and $m\angle 2 = 134°$, so

   $m\angle 1 = m\angle 2$

9. $m\angle ABC = m\angle DEF$

   $m\angle ABC + m\angle QRS = m\angle DEF + m\angle QRS$

10. $GH = GH$

11. $ED = 3$ in. and $PQ = 3$ in., so $ED = PQ$

12. $\angle EFG \cong \angle LMN$ and $\angle LMN \cong \angle SPT$, so $\angle EFG \cong \angle SPT$

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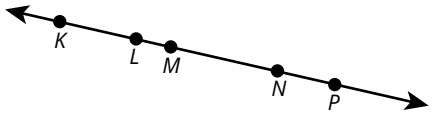
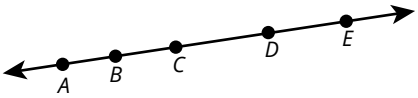
B. Complete each statement. The write the postulate you used.

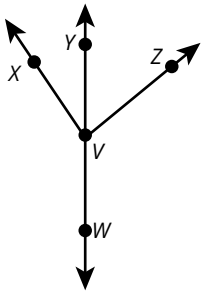
1. m_____ + m_____ = m_____



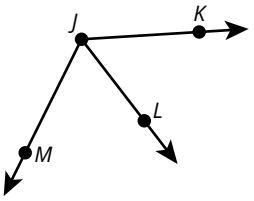
2. m$\overline{AB}$ + m_____ = m$\overline{AC}$



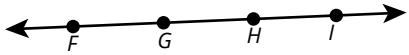
3. m∠*YVZ* + m∠_____ = 180°



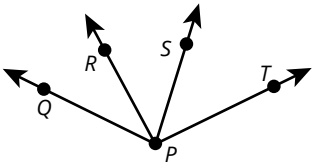
4. m∠_____ + m∠_____ = m∠*MJK*



5. m_____ + m$\overline{GI}$ = m_____



6. m∠_____ + m∠*RPS* = m∠*RPT*



C. Write a statement that fits each given description.

1. Write a segment statement using the reflexive property of congruence.

2. Write angle statements using the addition property of equality.

3. Write angle statements using the substitution property of equality.

4. Write segment statements using the transitive property of congruence.

5. Write segment statements using the subtraction property of equality.

6. Write an angle statement using the reflexive property of equality.

7. Write a statement using the segment addition postulate.

8. Write a statement using the angle addition postulate.

9. Write a statement using the linear pair postulate.

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D. Identify the hypothesis (given) and the conclusion (prove) of each conditional statement.

1. If two lines intersect at right angles, then the lines are perpendicular.

2. If the sum of two angles is 180°, then the angles are supplementary.

3. If the sum of two adjacent angles is 180°, then the angles form a linear pair.

4. If the measure of an angle is 180°, then the angle is a straight angle.

5. If two lines are located in the same plane, then the lines are coplanar lines.

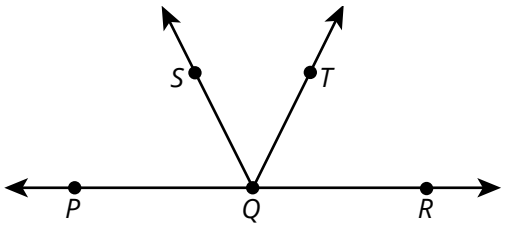
6. If the sum of two angle measures is equal to 90°, then the angles are complementary angles.

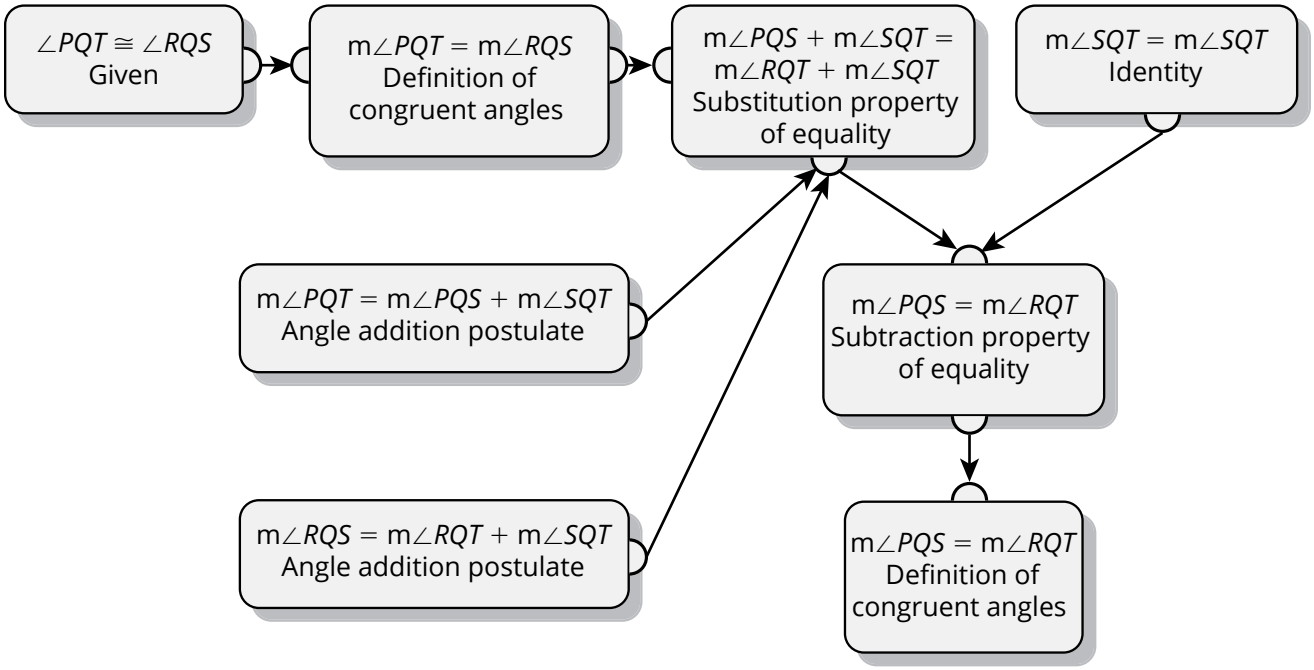
E. Write each given proof as the indicated proof.

1. Write the flow chart proof as a two-column proof.

   Given: $\angle PQT \cong \angle RQS$

   Prove: $\angle PQS \cong \angle RQT$



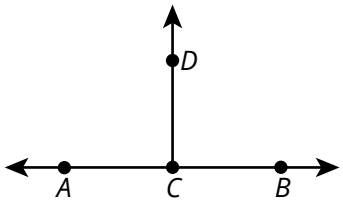


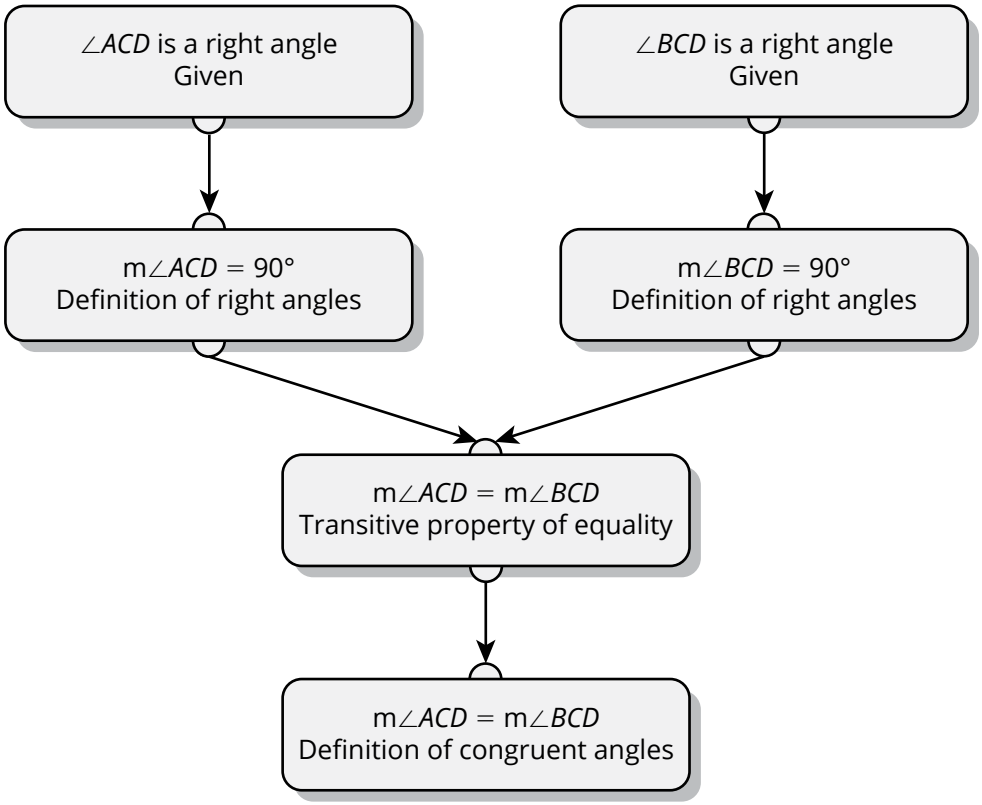
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

2. Write the flow chart proof of the Right Angle Congruence Theorem as a two-column proof.

   Given: Angles *ACD* and *BCD* are right angles.

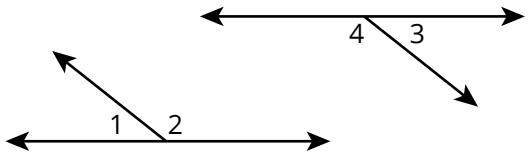
   Prove: $\angle ACD \cong \angle BCD$





Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

3. Write the two-column proof of the congruent supplement theorem as a paragraph proof.



Given: $\angle 1$ is supplementary to $\angle 2$, $\angle 3$ is supplementary to $\angle 4$, and $\angle 2 \cong \angle 4$

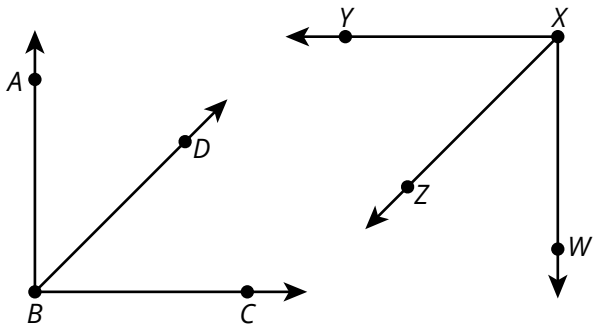
Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $\angle 1$ is supplementary to $\angle 2$	1. Given
2. $\angle 3$ is supplementary to $\angle 4$	2. Given
3. $\angle 2 \cong \angle 4$	3. Given
4. $m\angle 2 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = 180°$	5. Definition of supplementary angles
6. $m\angle 3 + m\angle 4 = 180°$	6. Definition of supplementary angles
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	7. Substitution property of equality
8. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	8. Substitution property of equality
9. $m\angle 1 = m\angle 3$	9. Subtraction property of equality
10. $\angle 1 \cong \angle 3$	10. Definition of congruent angles

4. Write the two-column proof of the congruent complement theorem as a paragraph proof.

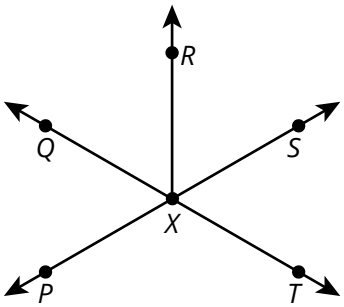
   Given: Angles *ABD* and *DBC* are complementary, angles *WXZ* and *ZXY* are complementary, and $\angle DBC \cong \angle ZXY$

   Prove: $\angle ABD \cong \angle WXZ$



Statements	Reasons
1. $\angle ABD$ is complementary to $\angle DBC$	1. Given
2. $\angle WXZ$ is complementary to $\angle ZXY$	2. Given
3. $\angle DBC \cong \angle ZXY$	3. Given
4. $m\angle ABD + \angle DBC = 90°$	4. Definition of complementary angles
5. $m\angle WXZ + \angle ZXY = 90°$	5. Definition of complementary angles
6. $m\angle DBC = m\angle ZXY$	6. Definition of congruent angles
7. $m\angle ABD + \angle DBC = m\angle WXZ + \angle ZXY$	7. Substitution property of equality
8. $m\angle ABD + \angle DBC = m\angle WXZ + \angle DBC$	8. Substitution property of equality
9. $m\angle ABD = m\angle WXZ$	9. Subtraction property of equality
10. $\angle ABD \cong \angle WXZ$	10. Definition of congruent angles

5. Write the paragraph proof as a flow chart proof.

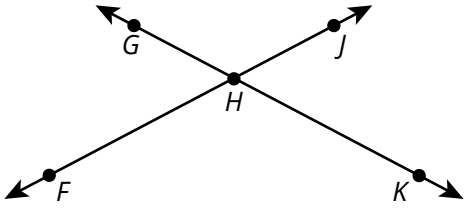


Given: $m\angle QXR = m\angle SXR$

Prove: $m\angle PXR = m\angle TXR$

By the angle addition postulate, $m\angle TXR = m\angle TXS + m\angle SXR$. It is given that $m\angle QXR = m\angle SXR$, so by Substitution property of equality, $m\angle TXR = m\angle TXS + m\angle QXR$. Angles PXQ and TXS are vertical angles by the definition of vertical angles. Vertical angles are congruent by the vertical angle theorem, so $\angle PXQ \cong TXS$, and by the definition of congruent angles, $m\angle PXQ = m\angle TXS$. Using Substitution property of equality, you can write $m\angle TXR = m\angle PXQ + m\angle QXR$. By the angle addition postulate, $m\angle PXR = m\angle PXQ + m\angle QXR$. So, you can use the substitution property of equality to write $m\angle PXR = m\angle TXR$.

6. Write the paragraph proof as a flow chart proof.

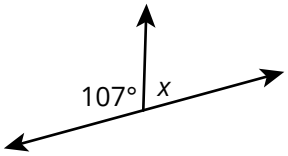


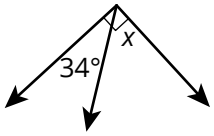
Given: $\overline{GH} \cong \overline{HJ}$ and $\overline{FH} \cong \overline{HK}$

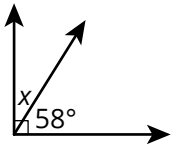
Prove: $\overline{GK} \cong \overline{FJ}$

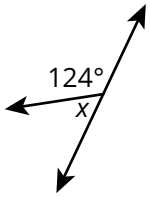
By the segment addition postulate, $GK = GH + HK$. You are given that $\overline{GH} = \overline{HJ}$, so $GH = HJ$ by the definition of congruent segments, and you can use the Substitution property of equality to write $GK = HJ + HK$. You are also given that $\overline{FH} \cong \overline{HK}$, so $FH = HK$ by the definition of congruent segments, and you can use the Substitution property of equality to write $GK = HJ + FH$. By the segment addition postulate, $FJ = FH + HJ$. So, you can use the Substitution property of equality to write $GK = FJ$. By the definition of congruent segments, $\overline{GK} \cong \overline{FJ}$.

F. Solve for *x*.

1. 

2. 

3. 

4. 

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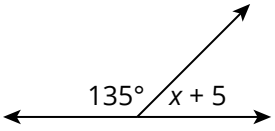
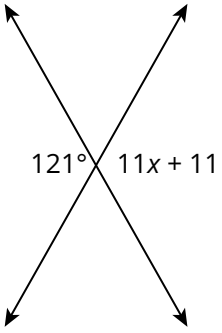
7. 

8. 

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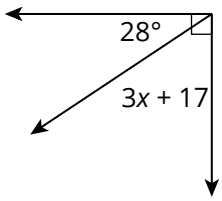

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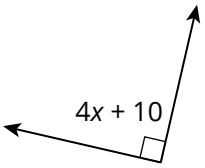

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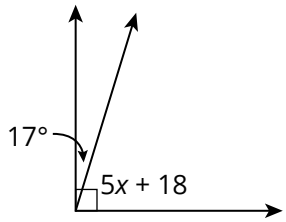

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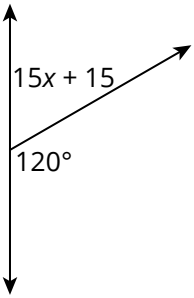


G. Use the given information to determine the measures of the angles in each pair.

1. The measure of the complement of an angle is three times the measure of the angle. What is the measure of each angle?

2. The measure of the supplement of an angle is one fourth the measure of the angle. What is the measure of each angle?

3. The measure of the supplement of an angle is twice the measure of the angle. What is the measure of each angle?

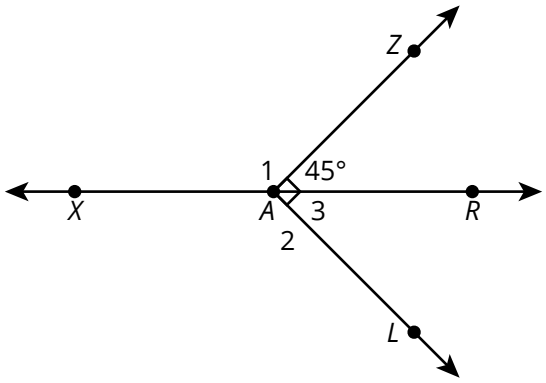
4. The measure of the complement of an angle is one fifth the measure of the angle. What is the measure of each angle?

5. The measure of one angle in a linear pair is four times the measure of the other angle. What is the measure of each angle?

6. The measure of the second angle in a linear pair is twenty more than three times the first angle. What is the measure of each angle?

H. Solve for each missing angle. Images are not drawn to scale.

1.



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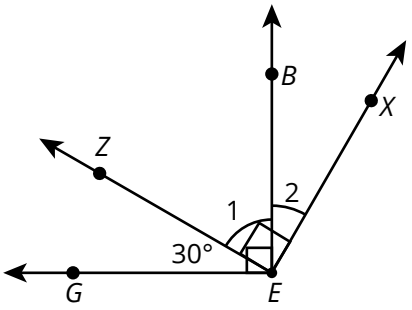

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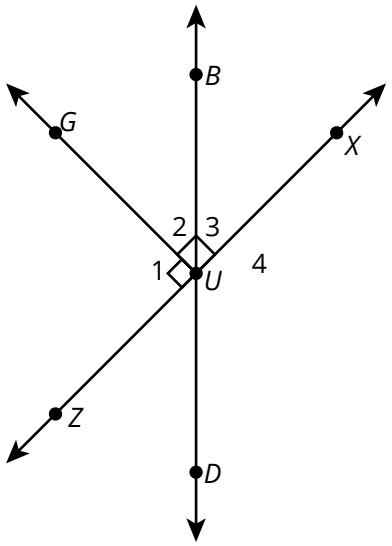

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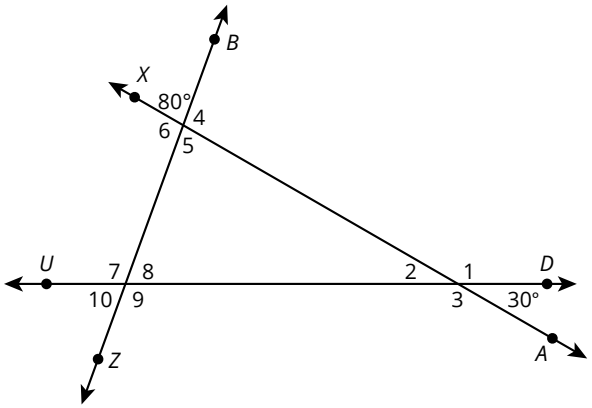
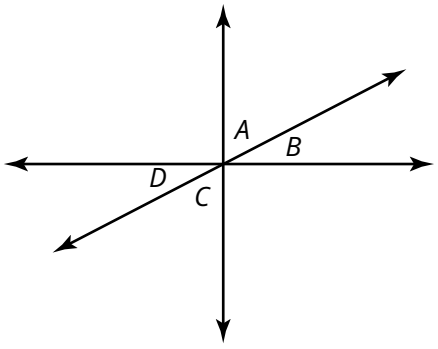


Extension

Create a two-column proof.

Given: $\angle A$ and $\angle B$ are complementary.

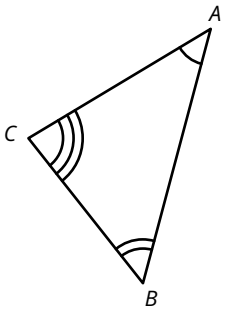
Prove: $\angle C$ and $\angle D$ are complementary.



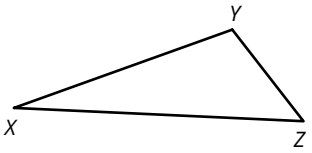
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Spaced Practice

1. Use a compass and straightedge to construct the incenter of $\triangle ABC$.



2. Use a compass and straightedge to construct the circumcenter of $\triangle XYZ$.

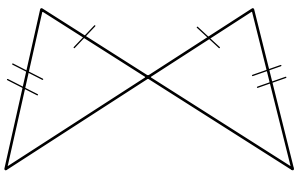


3. Write a conjecture about corresponding angles. Draw an example to test your conjecture.

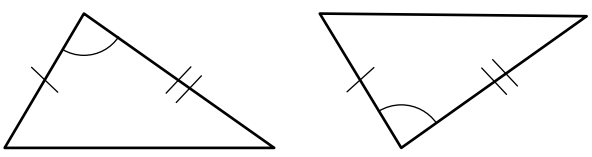
4. Draw an example of an inscribed angle and a central angle that intercepts the same arc of a circle. What conjecture can you make about the measure of the inscribed angle?

5. Determine whether each pair of triangles is congruent. If so, state whether they are congruent by SSS, SAS, or ASA. If not, explain why.

a.



b.

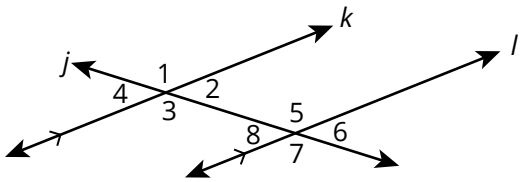


II. Proving Parallel Line Theorems

Topic Practice

A. If possible, write congruence statements for the pairs of corresponding angles in each figure.

1.



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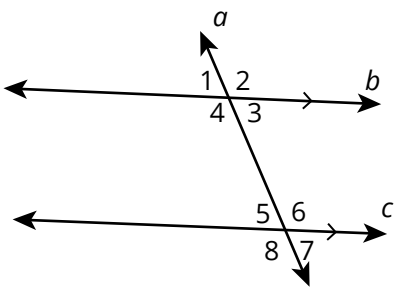

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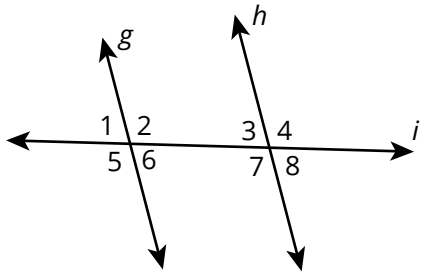

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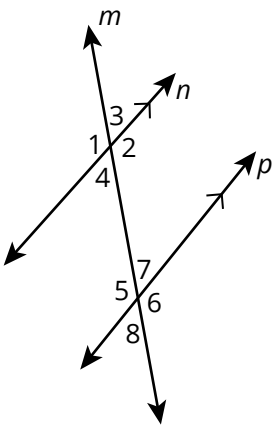


B. Draw and label a diagram to illustrate each theorem.

1. same-side interior angles theorem

2. alternate exterior angles theorem

3. alternate interior angles theorem

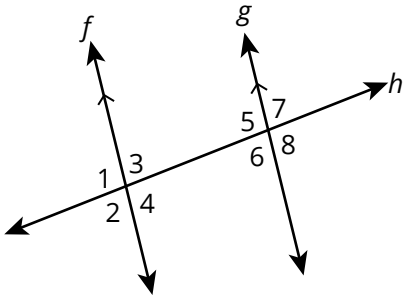
4. same-side exterior angles theorem

5. corresponding angles converse theorem

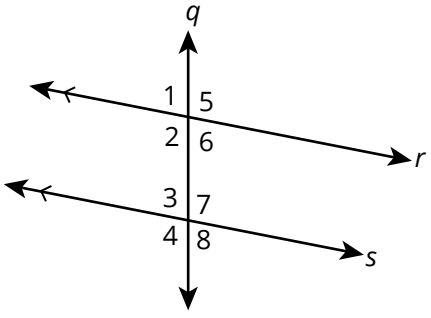
6. same-side interior angles converse theorem

C. Identify the theorem applied in each statement.

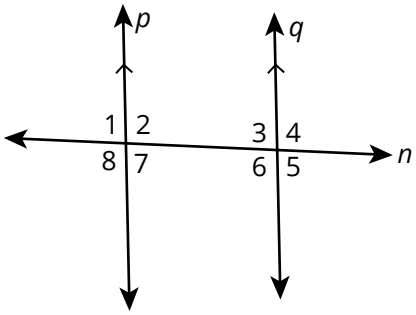
1. $\angle 3 \cong \angle 6$



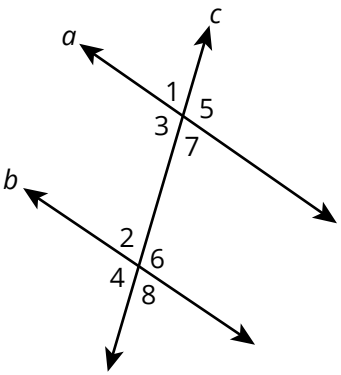
2. $m\angle 1 + m\angle 4 = 180°$



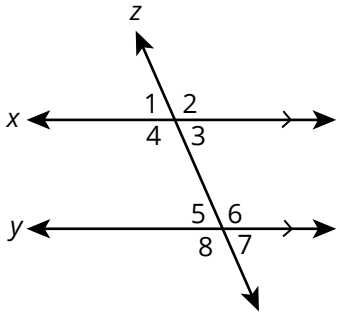
3. $\angle 1 \cong \angle 5$



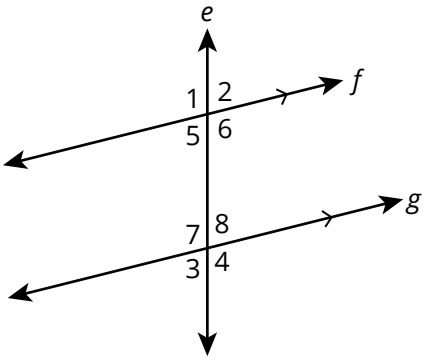
4. $\angle 4 \cong \angle 6$



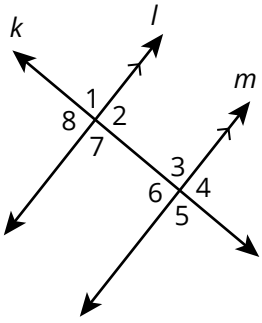
5. $m\angle 4 + m\angle 5 = 180°$



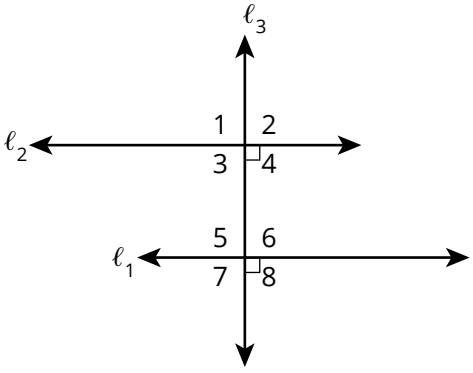
6. $\angle 5 \cong \angle 8$



7. $\angle 6 \cong \angle 8$



8. $\ell_1 \parallel \ell_2$



D. Determine the unknown angle measure described in each statement. Explain your reasoning.

1. A traversal intersects two parallel lines a and b, creating alternate interior angles 1 and 2. Given $m\angle 1 = 75°$, determine $m\angle 2$.

2. Two lines intersect at point C, creating vertical angles A and B. If $m\angle A = 50°$, determine $m\angle B$.

3. A transversal intersects two parallel lines m and n, creating same-side interior angles 3 and 4. Given $m\angle 3 = 120°$, determine $m\angle 4$.

4. A transversal intersects two parallel lines n and m, creating corresponding angles E and F. Given $m\angle E = 110°$ determine $m\angle F$.

5. Two lines intersect at point P, forming a linear pair with adjacent angles 1 and 2. Given $m\angle 1 = 35°$, determine $m\angle 2$.

6. A transversal intersects two parallel lines, r and s, creating alternate interior angles 1 and 2. Given $m\angle 1 = 135°$, determine $m\angle 2$.

E. Answer each question about the theorems.

The corresponding angles theorem which states: If two parallel lines are intersected by a transversal, then corresponding angles are congruent.

1. Write the converse of this theorem.

2. Write a biconditional statement about this theorem and its converse.

The same-side interior angles theorem which states: If two parallel lines are intersected by a transversal, then the interior angles on the same side of the transversal are supplementary.

3. Write the converse of this theorem.

4. Write a biconditional statement about this theorem and its converse.

The alternate interior angles theorem which states: If two parallel lines are intersected by a transversal, then the alternate interior angles are congruent.

5. Write the converse of this theorem.

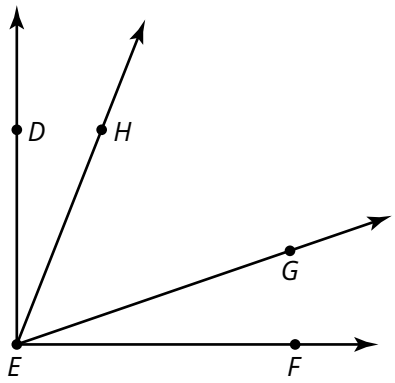
6. Write a biconditional statement about this theorem and its converse.

Extension

Prove the conditional statement using any method you choose.

Given: $\angle DEG > \angle HEF$

Prove: $\angle DEH > \angle GEF$

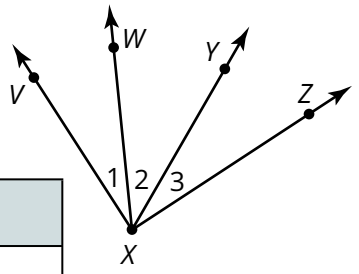


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Spaced Practice

1. Identify the property that justifies the statement: If $m\angle A = m\angle B$, then $m\angle A - m\angle C = m\angle B - m\angle C$.

2. Enter the reasons to complete the two-column proof.



Given: m∠1 = m∠3

Prove: m∠*WXZ* = m∠*YXV*

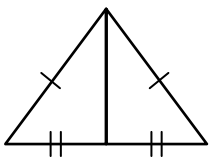
Statements	Reasons
1. m∠1 = m∠3	1. Given
2. m∠*WXZ* = m∠3 + m∠2	2.
3. m∠*WXZ* = m∠1 + m∠2	3.
4. m∠1 + m∠2 = m∠*YXV*	4.
5. m∠*WXZ* = m∠*YXV*	5. Substitution property

3. State as many properties as you can about a rhombus.

4. Describe how to construct rhombus *JKLM* given diagonal $\overline{JL}$.

5. Determine whether each pair of triangles is congruent. If so, state whether they are congruent by SSS, SAS, or ASA. If not, explain why.

a.



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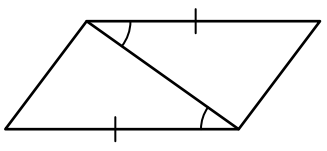
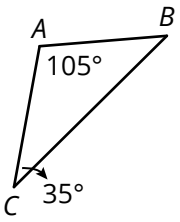


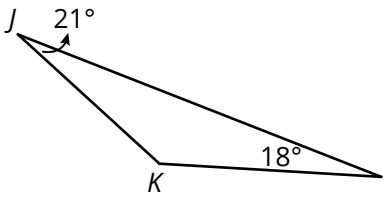
III. Interior and Exterior Angles of Polygons

Topic Practice

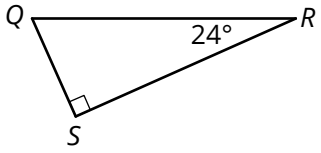
A. Determine the unknown interior angle measure(s) in each triangle.

1. 

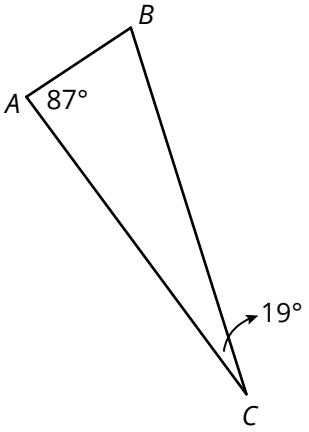
m∠*ABC* =

2. 

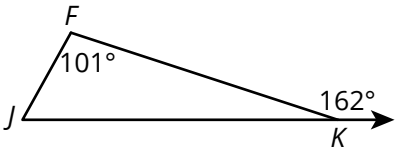
m∠*JKL* =

3. 

m∠*RQS* =

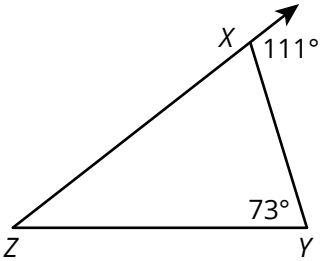
4. 

m∠*ABC* =

5. 

m∠*FJK* =

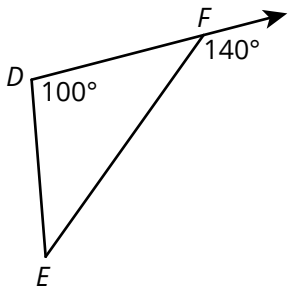
m∠*FKJ* =

6. 

m∠*XZY* =

m∠*ZXY* =

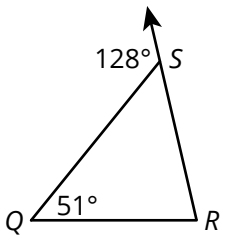
7.



m∠*DEF* =

m∠*DFE* =

8.

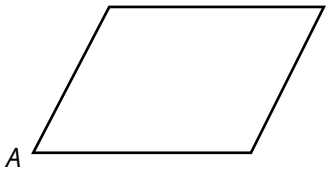


m∠*QRS* =

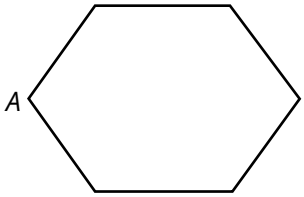
m∠*QSR* =

B. Draw all possible diagonals from vertex *A* for each polygon. Then write the number of triangles formed by the diagonals.

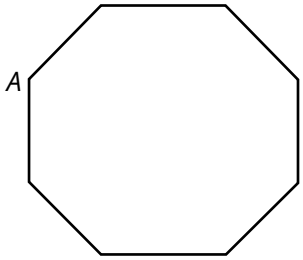
1.



2.



3.



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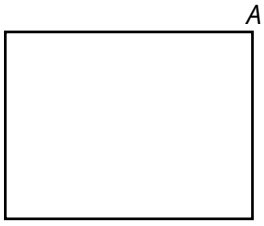

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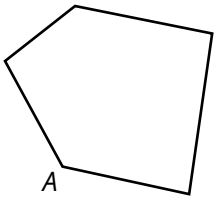

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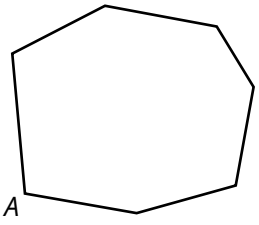
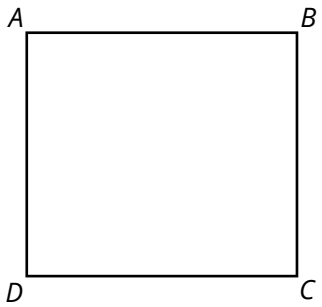
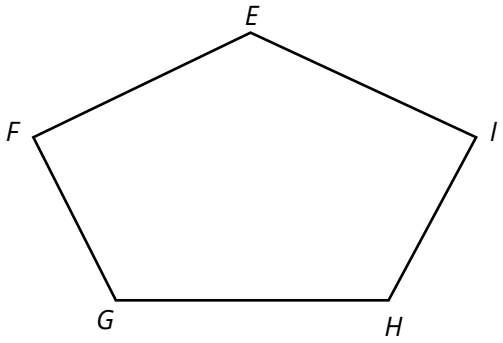


C. Use the triangles formed by diagonals to calculate the sum of the interior angle measures of each polygon.

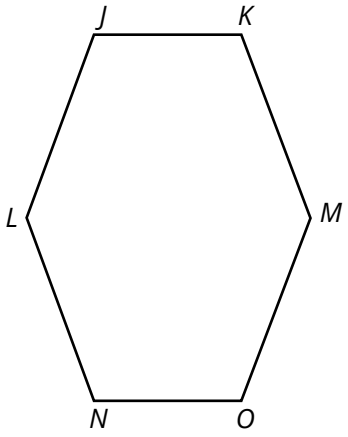
1. Draw all of the diagonals that connect to vertex *A*. What is the sum of the interior angles of square *ABCD*?



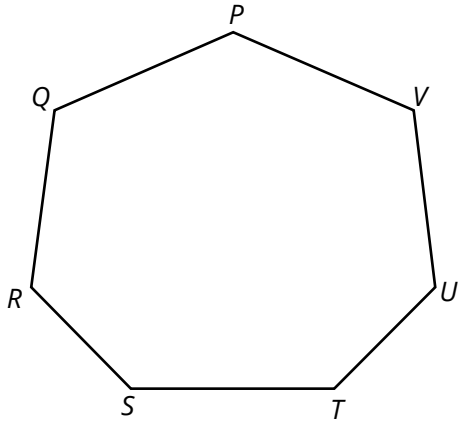
2. Draw all of the diagonals that connect to vertex *E*. What is the sum of the interior angles of figure *EFGHI*?



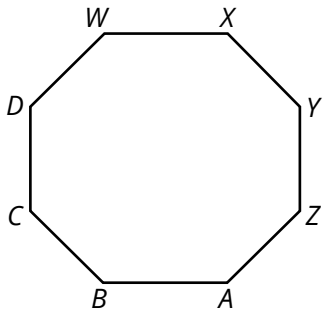
3. Draw all of the diagonals that connect to vertex *J*. What is the sum of the interior angles of figure *JKMONL*?



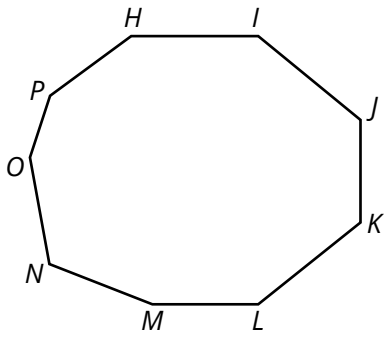
4. Draw all of the diagonals that connect to vertex *P*. What is the sum of the interior angles of the figure *PQRSTUV*?



5. Draw all of the diagonals that connect to vertex *W*. What is the sum of the interior angles of the figure *WXYZABCD*?



6. Draw all of the diagonals that connect to vertex *H*. What is the sum of the interior angles of the figure *HIJKLMNOP*?



D. Calculate the sum of the interior angle measures of each polygon.

1. A polygon has 8 sides.

2. A polygon has 9 sides.

3. A polygon has 13 sides.

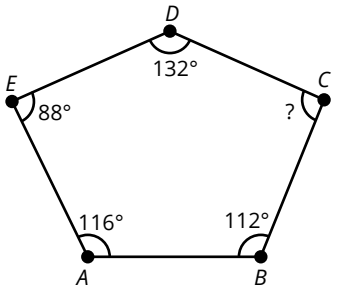
4. A polygon has 16 sides.

5. A polygon has 20 sides.

6. A polygon has 25 sides.

E. Determine the measures of the unknown angles.

1.



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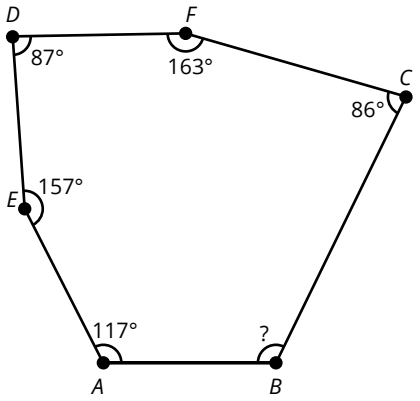

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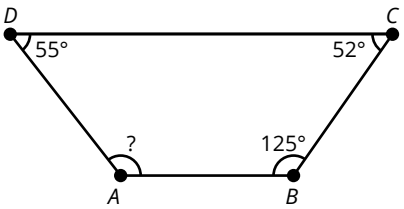

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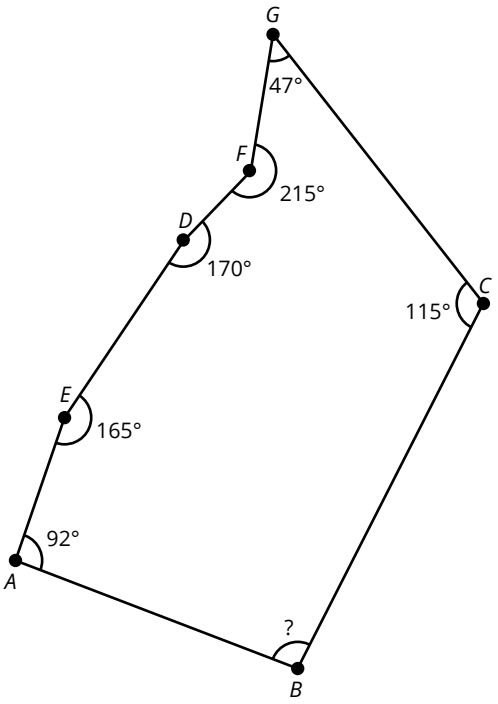

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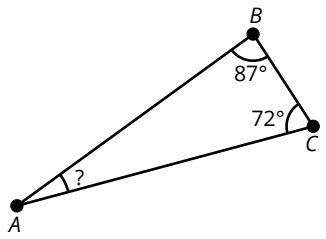

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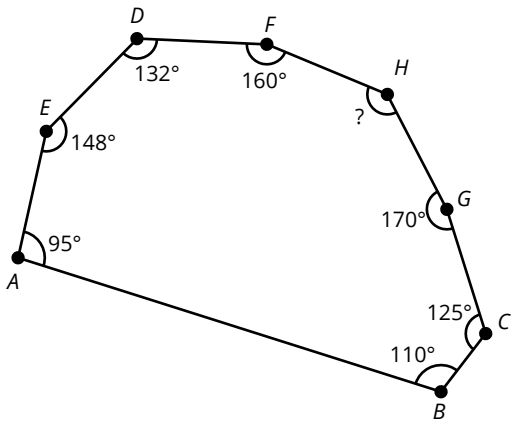


F. The sum of the measures of the interior angles of a polygon is given. Determine the number of sides for each polygon.

1. 1080°

2. 1800°

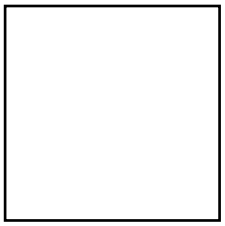
3. 540°

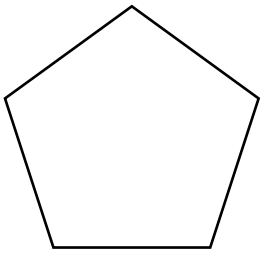
4. 1260°

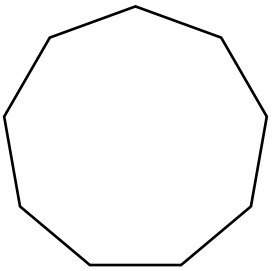
5. 3780°

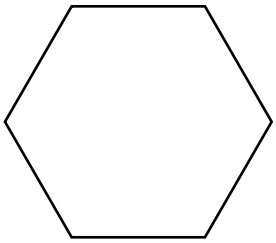
6. 6840°

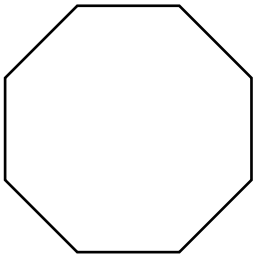
G. Calculate the measure of the exterior angles for each regular polygon.

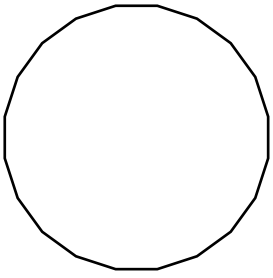
1. 

2. 

3. 

4. 

5. 

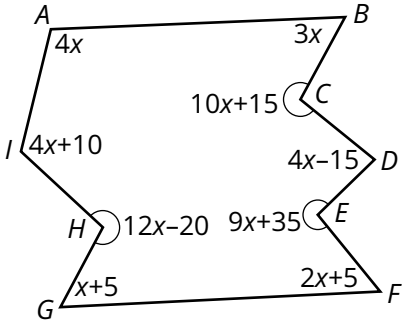
6. 

Extension

1. Consider the nonagon shown.

   a. Determine the value of x.

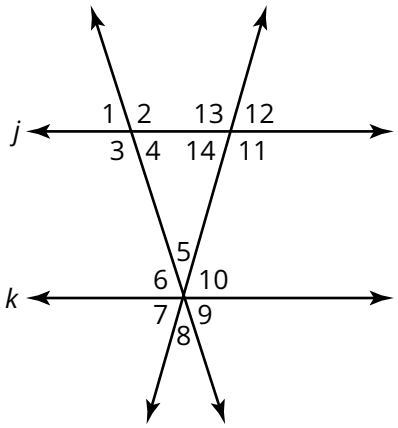
   b. Determine the value of all the interior angles of the nonagon.



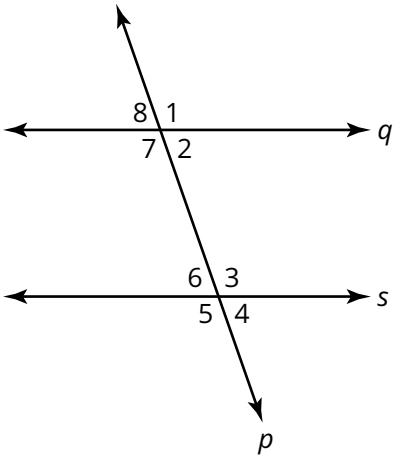
Spaced Practice

1. Given: $\angle 12 \cong \angle 7$, $m\angle 12 = 65°$, $m\angle 8 = 50°$

   Using the diagram in conjunction with postulates and theorems, determine the measures of the unknown angles.

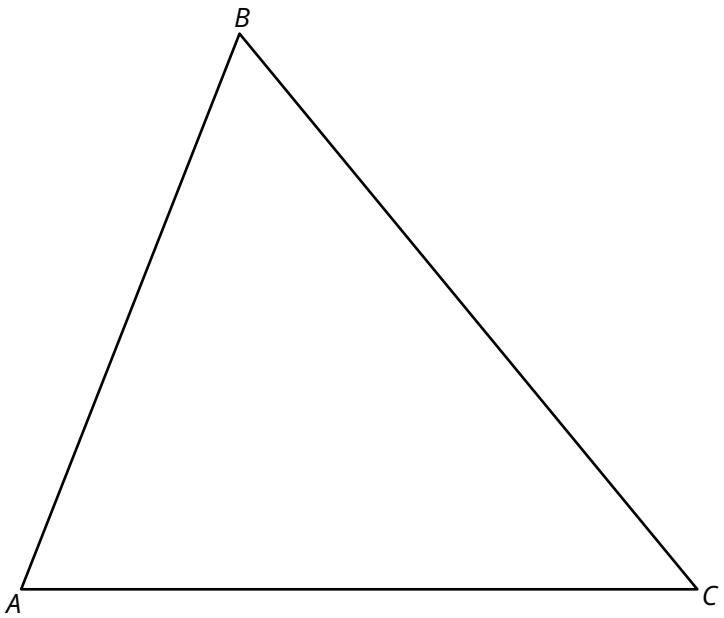


2. If $\angle 1$ and $\angle 4$ are supplementary, which theorem leads to the conclusion that $q \parallel s$?



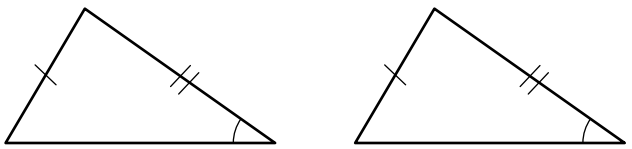
3. Write a conjecture about the angle measures of an equilateral triangle. Then, write the converse of the conjecture.

4. Use a compass and straightedge to construct the orthocenter of $\triangle ABC$.

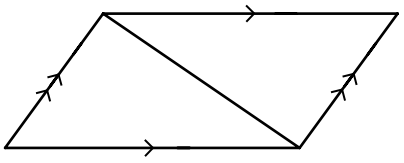


5. Determine whether each pair of triangles is congruent. If so, state whether they are congruent by SSS, SAS, or ASA. If not, explain why.

a.



b.



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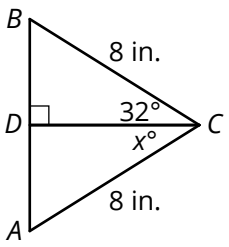
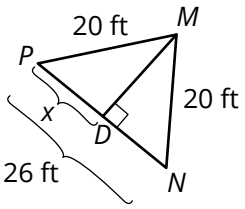
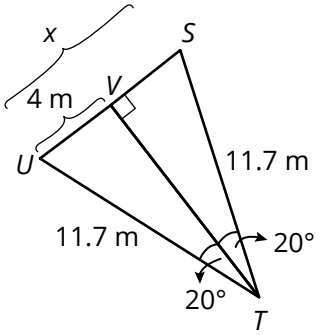
IV. Perpendicular Bisector and Isosceles Triangle Theorems

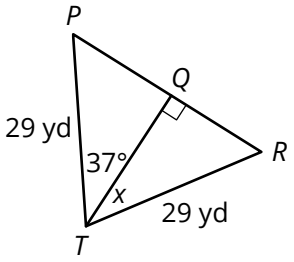
Topic Practice

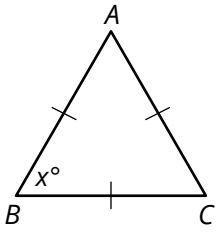
A. Determine the value of *x* in each isosceles triangle.

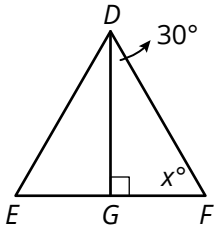
1. 

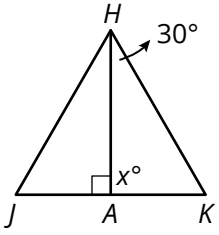
2. 

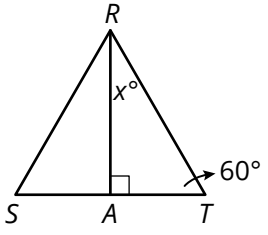
3. 

4. 

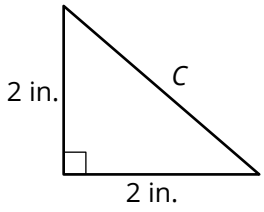
5. 

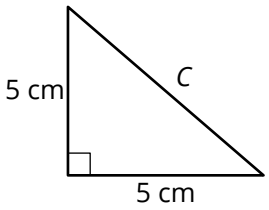
6. 

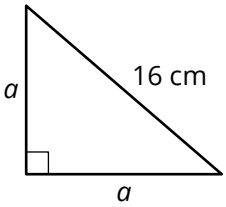
7. 

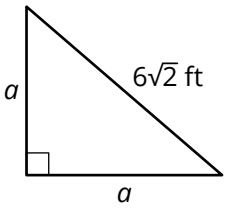
8. 

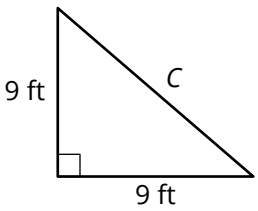
B. Determine the missing length(s) in each triangle. Write your answer as a radical in simplest form.

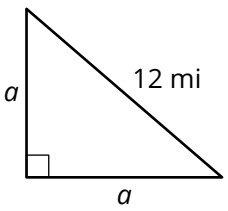
1. 

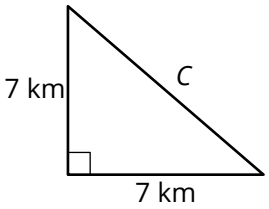
2. 

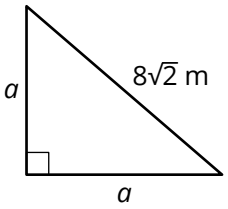
3. 

4. 

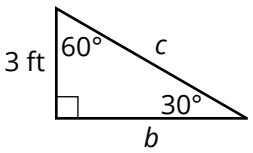
5. 

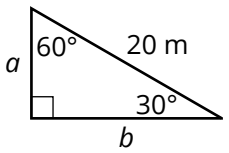
6. 

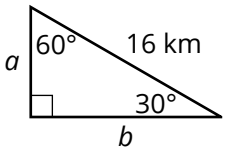
7. 

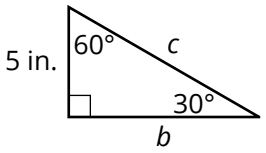
8. 

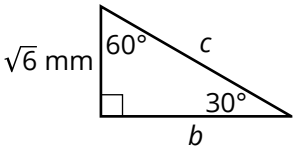
C. Determine the length of the missing side(s) in each triangle. Write your answers as a radical in simplest form.

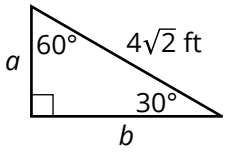
1. 

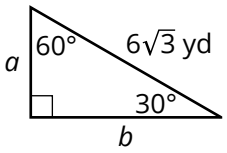
2. 

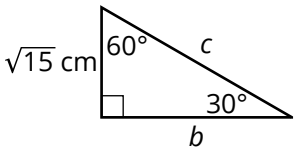
3. 

4. 

5. 

6. 

7. 

8. 

D. Use the given information to answer each question. Round your answer to the nearest tenth, if necessary.

1. Michael is flying a kite on the beach. The string forms a 45° angle with the ground. If he has let out 16 meters of line, how high above the ground is the kite?

2. Olivia is picking oranges from the tree in her yard. She rests a 12-foot ladder against the tree at a 45° angle. How far is the top of the ladder from the ground?

3. Daniela is building a square bookshelf. She wants to add a diagonal support beam to the back to strengthen it. The diagonal divides the bookshelf into two 45° −45° −90° triangles. If each side of the bookshelf is 4 feet long, what must the length of the support beam be?

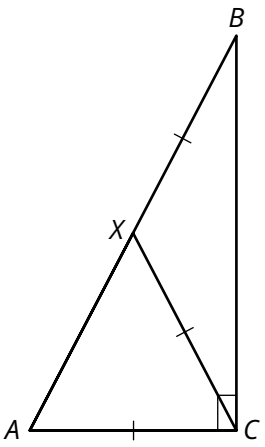
4. A park is a square with side lengths of 512 meters. One of the paths through the park runs diagonally from the northeast corner to the southwest corner, and it divides the park into two 45° −45° −90° triangles. How long is that path?

5. A sporting goods store sells pennants in the shape of 30°−60°−90° triangles. The length of the longest side of each pennant is 16 inches. Determine the area of the pennant.

6. A factory produces solid drafting triangles in the shape of 30°−60°−90° triangles. The length of the side opposite the right angle is 15 centimeters. What is the area of the drafting triangle?

Extension

Explain why m∠XAC = 60°.



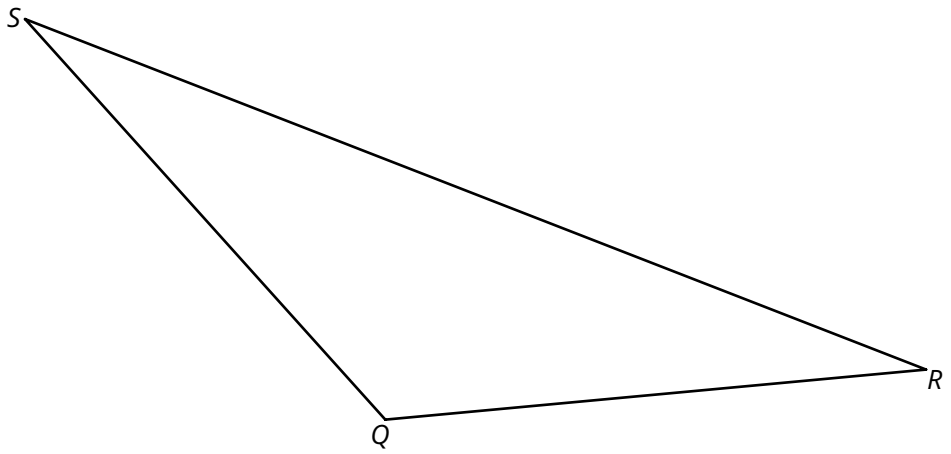
Spaced Practice

1. Determine the measure of an interior angle of a regular 18-gon.

2. If a regular polygon has 15 sides, what is the measure of each exterior angle? Explain your reasoning.

3. Write a conjecture about the exterior angles of triangles at a vertex. Then, write the converse of the conjecture.

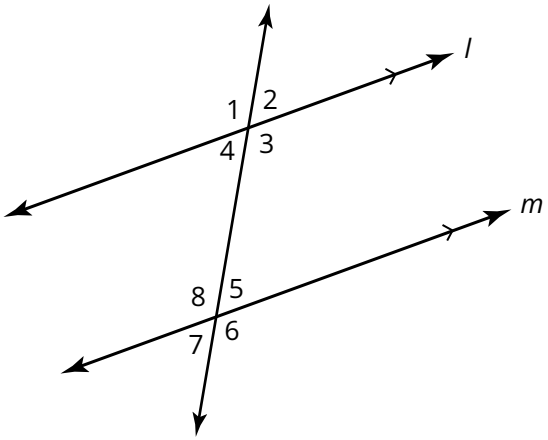
4. Use a compass and straightedge to construct the centroid of $\triangle QRS$.



5. Given that $l \parallel m$ and each angle measure, determine the measures of the remaining unknown angles.

a. $m\angle 1 = 105°$

b. $m\angle 7 = 22°$



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V. Inverse, Contrapositive, Direct Proof, and Indirect Proof

Topic Practice

A. Write the converse of each conditional statement. Then, determine whether the converse is true.

1. If two lines do not intersect and are not parallel, then they are skew lines.

2. If two lines are coplanar and do not intersect, then they are parallel lines.

3. If a triangle has one angle whose measure is greater than 90°, then the triangle is obtuse.

4. If a triangle has two sides with equal lengths, then it is an isosceles triangle.

5. If the lengths of the sides of a triangle measure 5 mm, 12 mm, and 13 mm, then it is a right triangle.

6. If the lengths of the sides of a triangle are 3 cm, 4 cm, and 5 cm, then the triangle is a right triangle.

7. If the corresponding sides of two triangles are congruent, then the triangles are congruent.

8. If the corresponding angles of two triangles are congruent, then the triangles are similar.

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B. Write the inverse of each conditional statement. Then, determine whether the inverse is true.

1. If a triangle is an equilateral triangle, then it is an isosceles triangle.

2. If a triangle is a right triangle, then the sum of the measures of its acute angles is 90°.

3. If the sum of the internal angles of a polygon is 180°, then the polygon is a triangle.

4. If a polygon is a triangle, then the sum of its exterior angles is 360°.

5. If two angles are the acute angles of a right triangle, then they are complementary.

6. If two angles are complementary, then the sum of their measures is 90°.

7. If a polygon is a square, then it is a rhombus.

8. If a polygon is a trapezoid, then it is a quadrilateral.

C. Write the contrapositive of each conditional statement. Then, determine whether the contrapositive is true.

1. If one of the acute angles of a right triangle measures 45°, then it is an isosceles right triangle.

2. If one of the acute angles of a right triangle measures 30°, then it is a 30°–60°–90° triangle.

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3. If a quadrilateral is a rectangle, then it is a parallelogram.

4. If a quadrilateral is an isosceles trapezoid, then it has two pairs of congruent base angles.

5. If the sum of the measures of two angles is 180°, then the angles are supplementary.

6. If two angles are supplementary, then the sum of their measures is 180°.

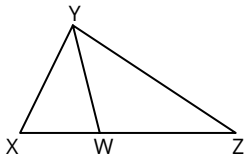
7. If the radius of a circle is 8 meters, then the diameter of the circle is 16 meters.

8. If the diameter of a circle is 12 inches, then the radius of the circle is 6 inches.

D. Create an indirect proof to prove each statement.

1. Given: $\overline{WY}$ bisects $\angle XYZ$ and $\overline{XW} \ncong \overline{ZW}$

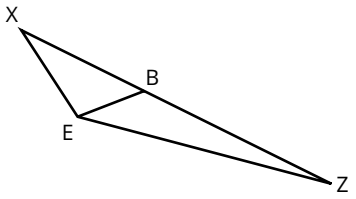
   Prove: $\overline{XY} \ncong \overline{ZY}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

2. Given: $m\angle EBX \neq m\angle EBZ$

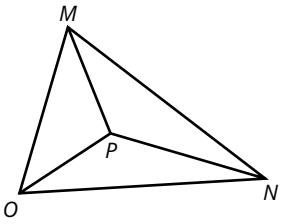
   Prove: $\overline{EB}$ is not an altitude of $\triangle EZX$.



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

3. Given: $\angle OMP \cong \angle MOP$ and $\overline{NP}$ does not bisect $\angle ONM$.

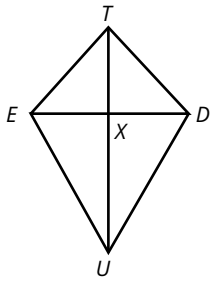
   Prove: $\overline{NM} \ncong \overline{NO}$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

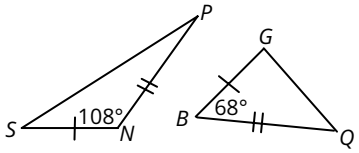
4. Given: $\overline{ET} \cong \overline{DT}$ and $\overline{EU} \ncong \overline{DU}$

   Prove: $\overline{EX} \ncong \overline{DX}$



E. For each pair of triangles, use the hinge theorem or its converse to write a conclusion using an inequality,

1.



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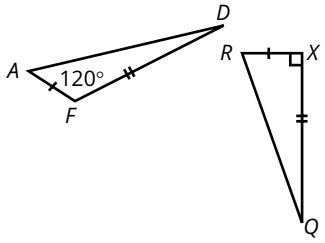

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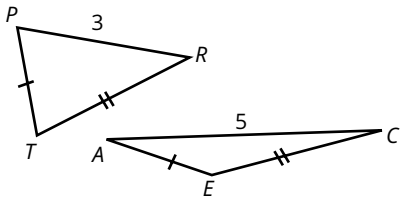

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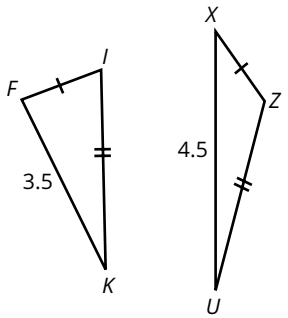

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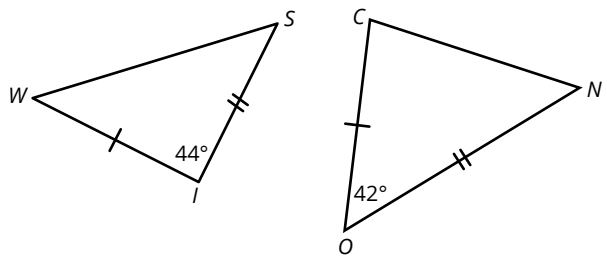

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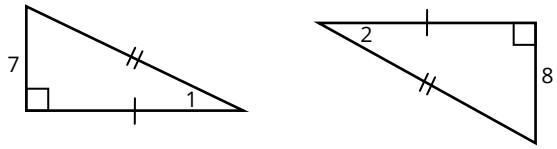

Extension

The law of syllogism is a logical rule that states if "If *A* implies *B*" is true and "If *B* implies *C*" is true, then you can conclude "If *A* implies *C*" is also true. Use the law of syllogism to write a third true conditional statement.

1. Given:

   If it is a weekend, then I will sleep in

   If I sleep in, then I will miss breakfast.

2. Given:

   If a number is divisible by 4, then it is divisible by 2.

   If a number is divisible by 2, then it is an even number.

Spaced Practice

1. Determine the unknown side length(s) in each triangle. Write your answer as a radical in simplest form.

   a. A rectangular garden has a diagonal that measures 10 meters. The garden is shaped to equal length and width, forming a 45−45−90 triangle. Determine the length of one side of the garden.

   b. A ladder is leaning against a wall. The distance from the base of the ladder to the wall is 6 meters, and the angle between the ground and the ladder is 30°. Determine the length of the ladder.

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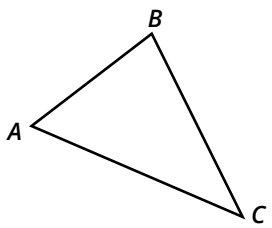
2. A regular hexagon has a side length of 10 cm. Determine the area of the hexagon. Round your answer to the nearest tenth.

3. Determine the sum of the interior angles of a regular decagon (10-sided polygon).

4. Determine the measure of the exterior angle of a regular dodecagon.

5. Using ΔABC and constructions of congruent segments, verify the Triangle Inequality theorem.

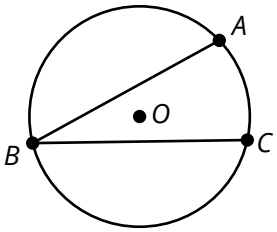


VI. Angle Relationships Inside and Outside Circles

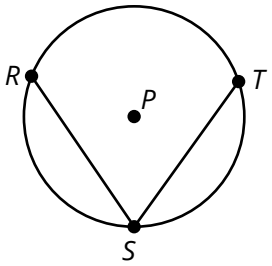
Topic Practice

A. Determine the measure of each inscribed angle or arc.

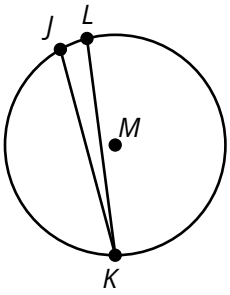
1. $m\overset{\frown}{AC} = 56°$



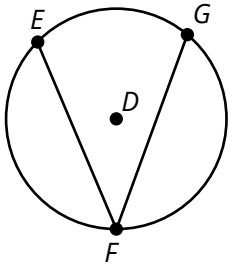
2. $m\angle RST = 70°$



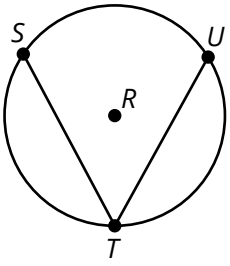
3. $m\overset{\frown}{JL} = 15°$



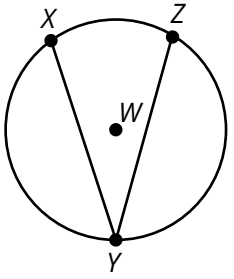
4. $m\angle EFG = 43°$



5. $m\angle STU = 57°$

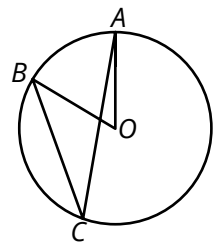


6. $m\overset{\frown}{XZ} = 67°$

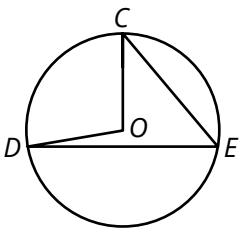


B. Determine the measure of each angle.

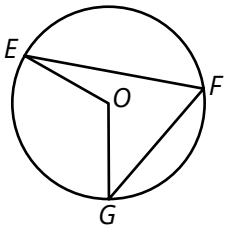
1. The measure of $\angle AOB$ is 62°. What is the measure of $\angle ACB$?



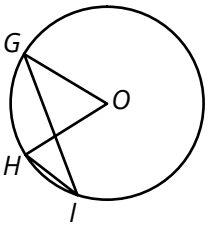
2. The measure of $\angle COD$ is 98°. What is the measure of $\angle CED$?



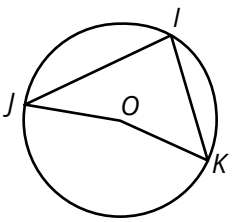
3. The measure of $\angle EOG$ is 128°. What is the measure of $\angle EFG$?



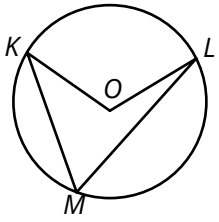
4. The measure of $\angle GOH$ is 74°. What is the measure of $\angle GIH$?



5. The measure of $\angle JOK$ is 168°. What is the measure of $\angle JIK$?

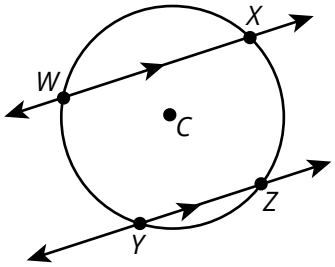


6. The measure of $\angle KOL$ is 148°. What is the measure of $\angle KML$?

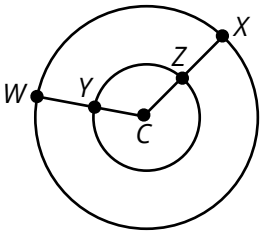


C. Use the given information to answer each question.

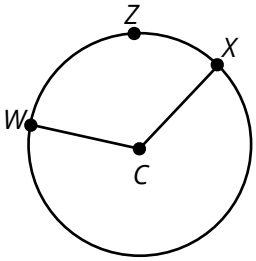
1. In circle C, $m\overset{\frown}{XZ} = 86°$. What is $m\overset{\frown}{WY}$?



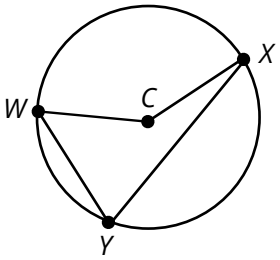
2. In circle C, $m\angle WCX = 102°$. What is $m\overset{\frown}{YZ}$?



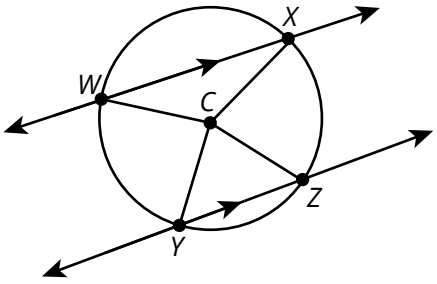
3. In circle C, $m\overset{\frown}{WZ} = 65°$ and $m\overset{\frown}{XZ} = 38°$. What is $m\angle WCX$?



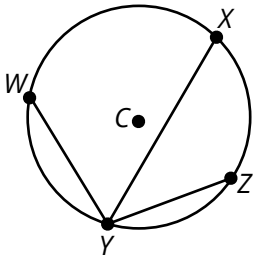
4. In circle C, $m\angle WCX = 105°$. What is $m\angle WYX$?



5. In circle C, $m\angle WCY = 83°$. What is $m\angle XCZ$?

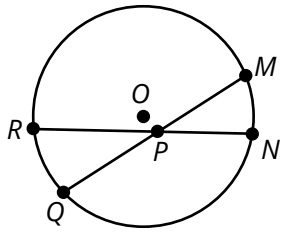


6. In circle C, $m\angle WYX = 50°$ and $m\angle XYZ = 30°$. What is $m\overset{\frown}{WXZ}$?

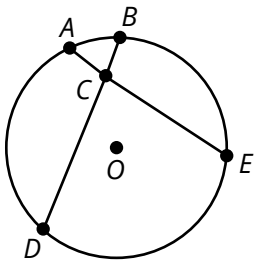


D. Write an expression for the measure of the given angle.

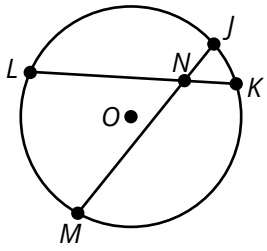
1. m∠*RPM*



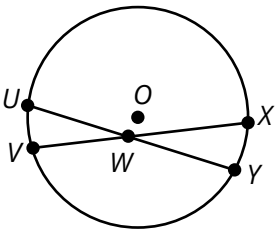
2. m∠*ACD*



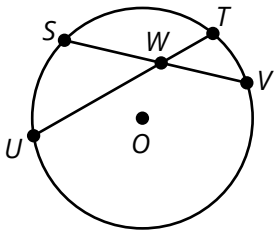
3. m∠*JNK*



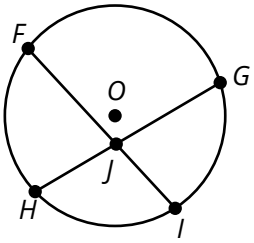
4. m∠*UWV*



5. m∠*SWT*

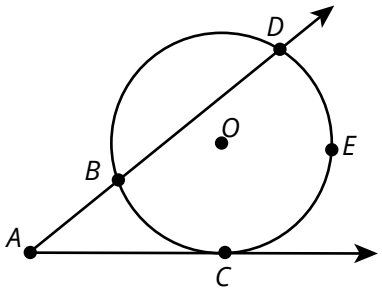


6. m∠*HJI*

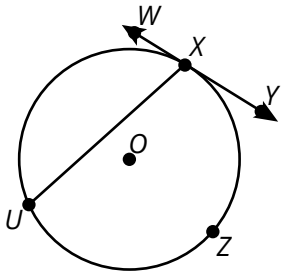


E. Write an expression for the measure of the given angle.

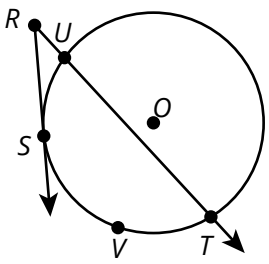
1. m∠*DAC*



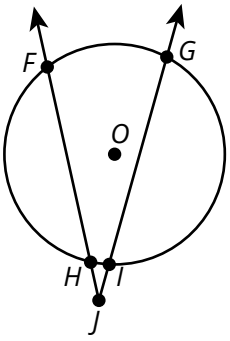
2. m∠*UXY*



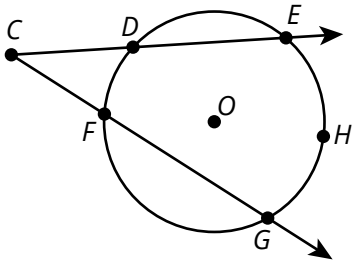
3. m∠*SRT*



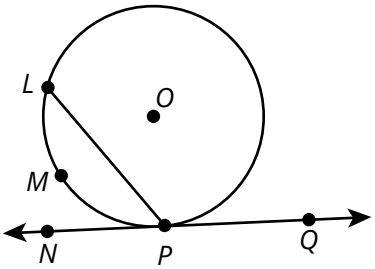
4. m∠*FJG*



5. m∠*ECG*



6. m∠*LPN*

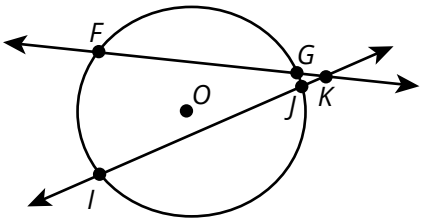


F. Use the diagram shown to determine the measure of each angle or arc.

1. Determine $m\overset{\frown}{FI}$.

   $m\angle K = 20°$

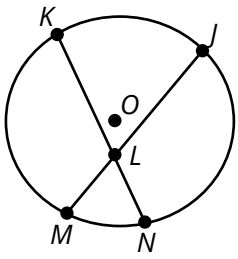
   $m\overset{\frown}{GJ} = 80°$



2. Determine $m\angle KLJ$.

   $m\overset{\frown}{KM} = 120°$

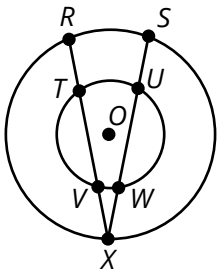
   $m\overset{\frown}{JN} = 100°$



3. Determine m∠X.

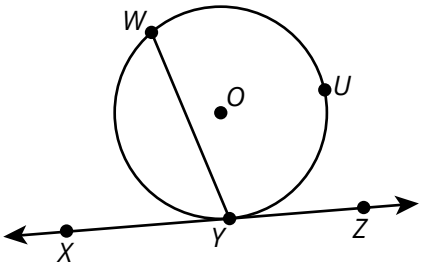
   m$\overset{\frown}{VW}$ = 50°

   m$\overset{\frown}{TU}$ = 85°



4. Determine m∠WYX.

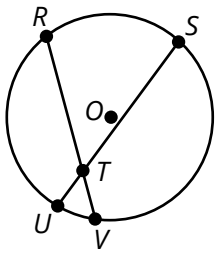
   m$\overset{\frown}{WUY}$ = 300°



5. Determine $m\overset{\frown}{RS}$.

   $m\overset{\frown}{UV} = 30°$

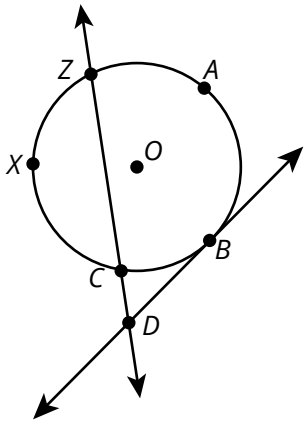
   $m\angle RTS = 80°$



6. Determine $m\angle D$.

   $m\overset{\frown}{ZXC} = 150°$

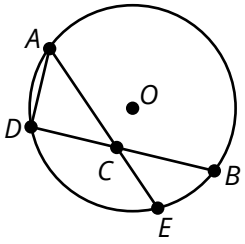
   $m\overset{\frown}{CB} = 30°$



G. Complete the proof to prove each statement.

1. Given: Chords $\overline{AE}$ and $\overline{BD}$ intersect at point C.

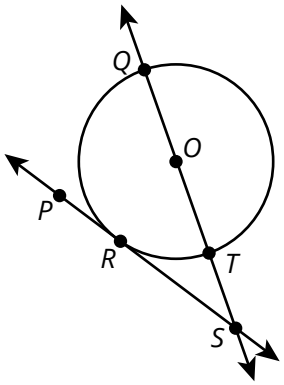
   Prove: $m\angle ACB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DE})$



Statements	Reasons
1. Chords $\overline{AE}$ and $\overline{BD}$ intersect at point C.	1.
2. Draw Chord $\overline{AD}$	2.
3. $m\angle ACB = m\angle D + m\angle A$	3. Exterior angle theorem
4. $m\angle A = \frac{1}{2}m\overset{\frown}{DE}$	4. Inscribed angle theorem
5. $m\angle D = \frac{1}{2}m\overset{\frown}{AB}$	5.
6.	6. Substitution property of equality
7. $m\angle ACB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DE})$	7. Distributive property

2. Given: Secant $\overleftrightarrow{QT}$ and tangent $\overleftrightarrow{SR}$ intersect at point S.

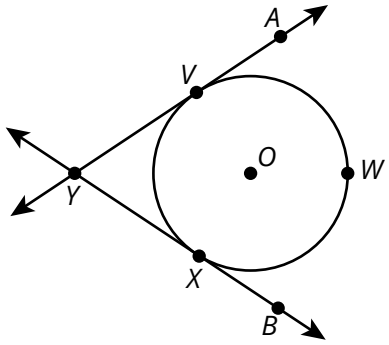
   Prove: $m\angle QSR = \frac{1}{2}(m\overset{\frown}{QR} - m\overset{\frown}{RT})$



Statements	Reasons
1.	1. Given
2.	2. Construction
3. $m\angle QRP = m\angle RQS + m\angle QSR$	3.
4. $m\angle QSR = m\angle QRP - m\angle RQS$	4. Subtraction property of equality
5. $m\angle QRP = \frac{1}{2}m\overset{\frown}{RQ}$	5.
6. $m\angle RQS = \frac{1}{2}m\overset{\frown}{RT}$	6. Inscribed angle theorem
7. $m\angle QSR = \frac{1}{2}m\overset{\frown}{RQ} - \frac{1}{2}mRT$	7.
8. $m\angle QSR = \frac{1}{2}(m\overset{\frown}{RQ} - m\overset{\frown}{RT})$	8.

3. Given: Tangents $\overleftrightarrow{VY}$ and $\overleftrightarrow{XY}$ intersect at point Y.

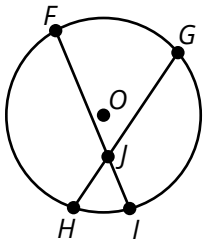
   Prove: $m\angle Y = \frac{1}{2}(m\overset{\frown}{VWX} - m\overset{\frown}{VX})$



Statements	Reasons
1. Tangents $\overleftrightarrow{VY}$ and tangent $\overleftrightarrow{XY}$ intersect at point Y.	1. Given
2.	2. Construction
3.	3. Exterior angle theorem
4.	4. Subtraction property of equality
5.	5. Exterior angle of a circle theorem
6.	6. Inscribed angle theorem
7.	7. Substitution property of equality
8. $m\angle Y = \frac{1}{2}(m\overset{\frown}{VWX} - m\overset{\frown}{VX})$	8. Distributive property

4. Given: Chords $\overline{FI}$ and $\overline{GH}$ intersect at point J.

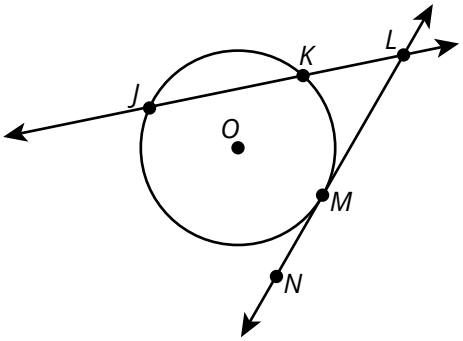
   Prove: $m\angle FJH = \frac{1}{2}(m\overset{\frown}{FH} + m\overset{\frown}{GI})$



Statements	Reasons
1. Chords $\overline{FI}$ and $\overline{GH}$ intersect at point J.	1. Given
2. Draw chord $\overline{FG}$	2.
3. $m\angle FJH = m\angle G + m\angle F$	3.
4. $m\angle F = \frac{1}{2}m\overset{\frown}{GI}$	4.
5. $m\angle G = \frac{1}{2}m\overset{\frown}{FH}$	5.
6. $m\angle FJH = \frac{1}{2}m\overset{\frown}{FH} + \frac{1}{2}m\overset{\frown}{GI}$	6.
7. $m\angle FJH = \frac{1}{2}(m\overset{\frown}{FH} + m\overset{\frown}{GI})$	7. Distributive property

5. Given: Secant $\overleftrightarrow{JL}$ and tangent $\overleftrightarrow{NL}$ intersect at point L.

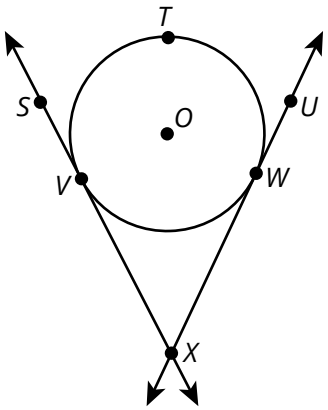
   Prove: $m\angle L = \frac{1}{2}(m\overset{\frown}{JM} - m\overset{\frown}{KM})$



Statements	Reasons
1. Secant $\overleftrightarrow{JL}$ and tangent $\overleftrightarrow{NL}$ intersect at point L.	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8. $m\angle L = \frac{1}{2}(m\overset{\frown}{JM} - m\overset{\frown}{KM})$	8. Distributive property

6. Given: Tangents $\overleftrightarrow{SX}$ and $\overleftrightarrow{UX}$ intersect at point X.

   Prove: $m\angle X = \frac{1}{2}(m\overset{\frown}{VTW} - m\overset{\frown}{VW})$



Statements	Reasons
1. Tangents $\overleftrightarrow{SX}$ and tangent $\overleftrightarrow{UX}$ intersect at point X.	1.
2. Draw chord $\overline{VW}$	2.
3. $m\angle SVW = m\angle X + m\angle VWX$	3.
4. $m\angle X = m\angle SVX - m\angle VWX$	4.
5. $m\angle SVW = \frac{1}{2}m\overset{\frown}{VTW}$	5.
6. $m\angle VWX = \frac{1}{2}m\overset{\frown}{VW}$	6.
7. $m\angle X = \frac{1}{2}m\overset{\frown}{VTW} - \frac{1}{2}m\overset{\frown}{VW}$	7.
8. $m\angle X = \frac{1}{2}(m\overset{\frown}{VTW} - m\overset{\frown}{VW})$	8.

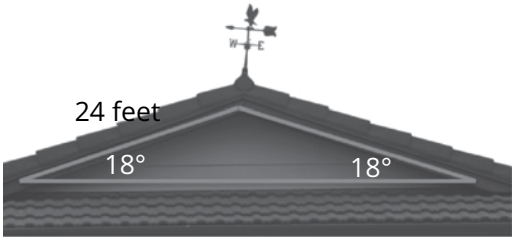
Extension

Ruben says that, in a right triangle, if you draw a line segment from the right angle's vertex to the center of the hypotenuse, the segment that you draw is always half the length of the hypotenuse. How can you show that Ruben's conjecture is correct?

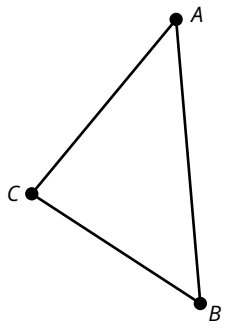
Spaced Practice

1. A hazard sign is placed near a steep stairwell. The sign design is an equilateral triangle. Suppose the height of the sign is 15 inches. Determine the area of the sign.

2. The roof of a house is in the shape of an triangle. If the builder uses 4 foot shingles in rows on the roof, how many shingles will the builder need for the first row on both sides of the roof? Explain your reasoning.

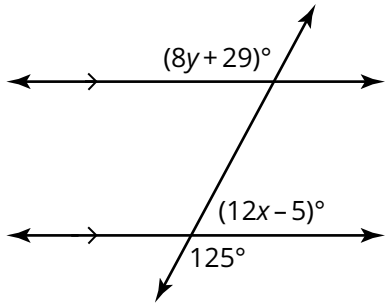


3. Use a compass and straightedge to construct the incenter of $\triangle ABC$.



4. Which type(s) of quadrilateral have two congruent diagonals?

5. Solve for x and y.



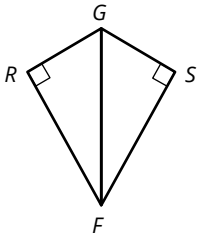
Name _____ Date _____

I. Using Triangle Congruence to Determine Relationships Between Segments

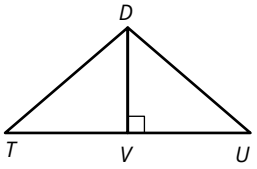
Topic Practice

A. For each figure, determine if there is enough information to prove that the two triangles are congruent. If so, name the right triangle congruence theorem used.

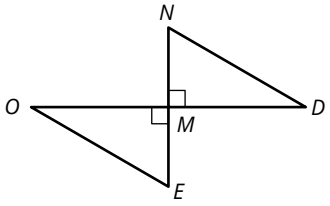
1. Given: $\overline{GF}$ bisects $\angle RGS$, and $\angle R$ and $\angle S$ are right angles.
Is $\triangle FRG \cong \triangle FSG$?



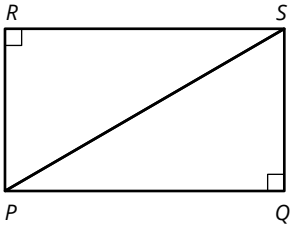
2. Given: $\overline{DV} \perp \overline{TU}$
Is $\triangle DVT \cong \triangle DVU$?



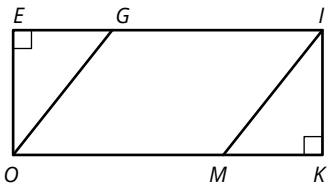
3. Given: $\overline{NM} \cong \overline{EM}$, $\overline{DM} \cong \overline{OM}$, and $\angle NMD$ and $\angle EMO$ are right angles.
Is $\triangle NMD \cong \triangle EMO$?



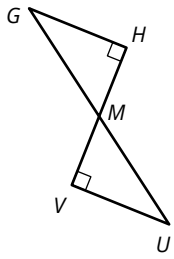
4. Given: $\overline{RP} \cong \overline{QS}$ and $\angle R$ and $\angle Q$ are right angles.
Is $\triangle SRP \cong \triangle PQS$?



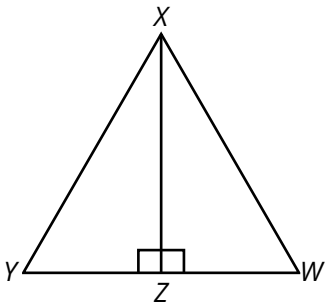
5. Given: $\overline{GO} \cong \overline{MI}$, and $\angle E$ and $\angle K$ are right angles.
Is $\triangle GEO \cong \triangle MKI$?



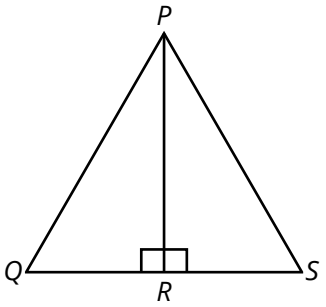
6. Given: $\overline{HM} \cong \overline{VM}$, and $\angle H$ and $\angle V$ are right angles.
Is $\triangle GHM \cong \triangle UVM$?



7. Given: $\overline{XY} \cong \overline{WY}$
Is $\triangle XYZ \cong \triangle XWZ$?

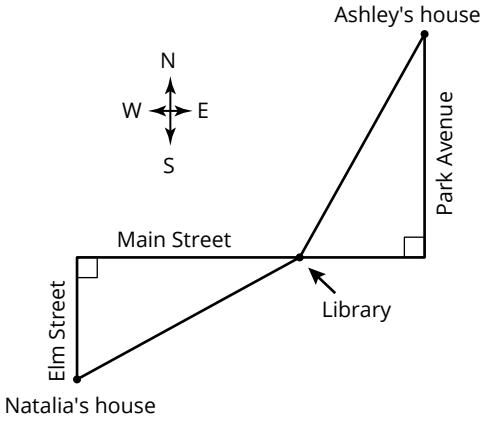


8. Given: $\triangle PQR$ and $\triangle PSR$ are right triangles. The leg $\overline{PR}$ is shared with both triangles.
Is $\triangle PQR \cong \triangle PSR$?

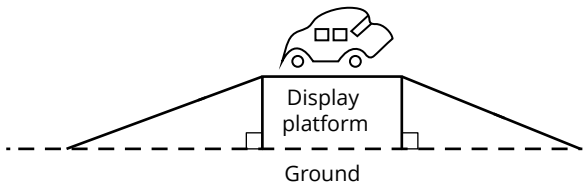


B. Use the given information to answer each question.

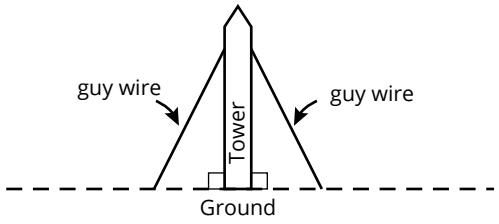
1. Two friends are meeting at the library. Natalia leaves her house and walks north on Elm Street and then east on Main Street to reach the library. Ashley leaves her house and walks south on Park Avenue and then west on Main Street to reach the library. Natalia walks the same distance on Elm Street as Ashley walks on Main Street, and she walks the same distance on Main Street as Ashley walks on Park Avenue. Is there enough information to determine whether Natalia's walking distance is the same as Ashley's walking distance? Explain.



2. An auto dealership displays one of their cars by driving it up a ramp onto a display platform. Later they will drive the car off the platform using a ramp on the opposite side. Both ramps form a right triangle with the ground and the platform. Is there enough information to determine whether the two ramps have the same length? Explain your reasoning



3. A radio station built a new transmission antenna to provide its listeners with better reception. The antenna was built perpendicular to the ground, and to keep the antenna from swaying in the wind, two guy wires were attached from it to the ground on opposite sides of the antenna. Is there enough information to determine if the guy wires have the same length? Explain your reasoning.



4. Two ladders resting on level ground are leaning against the side of a house. The bottom of each ladder is exactly 2.5 feet directly out from the base of the house. The point at which each ladder rests against the house is 10 feet directly above the base of the house. Is there enough information to determine whether the two ladders have the same length? Explain your reasoning.

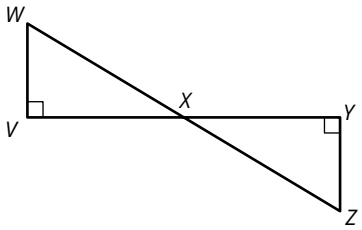
5. Tiara and Chris are both going for a run. Tiara leaves her home and runs 1 mile north, then 2 miles west, and then back to her house via the shortest route. Chris leaves her home and runs 2 miles east, then 1 mile south, and then back to her house via the shortest route. Is there enough information to determine whether Tiara's running distance is the same as Chris' running distance? Explain your reasoning.

6. When the Texas Flag is displayed out-of-doors, it must be on either a flagpole or a staff. Some places display the Texas Flag on a flagpole and will illuminate it with lights. Let's assume that a Texas Flag is being displayed and illuminated by two lights on either side of the flagpole and each light is aimed at an angle of 30 degrees from the ground. Is there enough information to determine that the two triangles formed by the flagpole, the ground, and the lights are congruent? Explain your reasoning.

CC BY NC

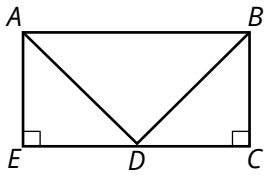
C. Complete, correct, or write the 2-column proof. Fill in the unknown Statement(s) and Reason(s).

1. Fill in the unknown statement and reasons.
Given: $\overline{WZ}$ bisects $\overline{VY}$, $\overline{VW} \perp \overline{VY}$, and $\overline{YZ} \perp \overline{VY}$
Prove: $\triangle WVX \cong \triangle ZYX$



Statements	Reasons
1. $\overline{WV} \perp \overline{VY}$ and $\overline{YZ} \perp \overline{VY}$	1. Given
2. $\angle WVX$ and $\angle ZYX$ are right angles.	2.
3. $\triangle WVX$ and $\triangle ZYX$ are right triangles.	3.
4.	4. Given
5. $\overline{VX} \cong \overline{YX}$	5. Definition of segment bisector
6. $\angle WXV \cong \angle ZXY$	6.
7. $\triangle WVX \cong \triangle ZYX$	7.

2. Analyze the given proof. Determine which statement(s) or reason(s) are incorrect. Write the correct statement(s) and/or reason(s).



Given: Point D is the midpoint of $\overline{EC}$, $\triangle ADB$ is an isosceles triangle with base $\overline{AB}$, and $\angle E$ and $\angle C$ are right angles.

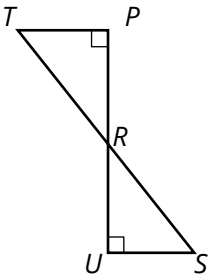
Prove: $\triangle AED \cong \triangle BCD$

Statements	Reasons
1. $\angle E$ and $\angle C$ are right angles.	1. Given
2. $\triangle AED$ and _____ are right triangles.	2. Definition of right triangles
3. Point D is the midpoint of $\overline{EC}$.	3. Given
4. _____ $\cong$ _____	4. Definition of _____
5. $\triangle ADB$ is an isosceles triangle with base $\overline{AB}$.	5. Given
6. $\overline{AD} \cong \overline{BD}$	6. Definition of _____
7. $\triangle AED \cong \triangle BCD$	7. HL congruence theorem

3. Using the statements given in the box below, determine where each statement belongs in the 2-column proof.

   Given: $\overline{SU} \perp \overline{UP}$, $\overline{TP} \perp \overline{UP}$, and $\overline{UR} \cong \overline{PR}$

   Prove: $\triangle SUR \cong \triangle TPR$



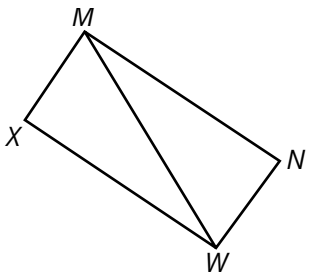
Statements	Reasons
• LA congruence theorem	• $\overline{SU} \perp \overline{UP}$ and $\overline{TP} \perp \overline{UP}$
• Definition of perpendicular lines	• $\overline{UR} \cong \overline{PR}$
• Given	• $\angle PRT \cong \angle URS$
• Definition of vertical angles	• $\angle PRT$ and $\angle URS$ are vertical angles.
• Vertical angle theorem	• $\angle U$ and $\angle P$ are right angles.
• Definition of right triangles	• $\triangle SUR \cong \triangle TPR$
• Given	• $\triangle SUR$ and $\triangle TPR$ are right triangles.

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

4. Create a 2-column proof.

   Given: Rectangle *MNWX* and $\angle NMW \cong \angle XWM$

   Prove: $\triangle MNW \cong \triangle WXM$

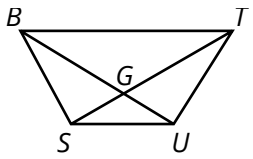


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

5. Given: $\overline{RS}$ is the $\perp$ bisector of $\overline{PQ}$.
   Prove: $\angle SPT \cong \angle SQT$

Statements	Reasons
1. $\overline{RS}$ is the $\perp$ bisector of $\overline{PQ}$.	1. Given
2. $\overline{RS} \perp \overline{PQ}$	2. Definition of _____
3. $\angle PTS$ and $\angle QTS$ are right triangles.	3. Definition of _____
4. $\triangle PTS$ and $\triangle QTS$ are right triangles.	4. Definition of right triangles
5. $\overline{RS}$ bisects $\overline{PQ}$.	5. Definition of _____
6. $\overline{PT} \cong \overline{QT}$	6. Definition of bisect
7. $\overline{TS} \cong \overline{TS}$	7. Reflexive property of equality
8. $\triangle PTS \cong \triangle QTS$	8. _____ congruence theorem
9. $\angle SPT \cong \angle SQT$	9. CPCTC

6. Given: $\triangle UGT \cong \triangle SGB$
Prove: $\angle TUS \cong \angle BSU$



Statements	Reasons
1. $\triangle UGT \cong \triangle SGB$	1. Given
2. $\overline{TU} \cong \overline{BS}$	2. _____
3. $\overline{SG} \cong \overline{UG}$	3. _____
4. $\overline{GT} \cong \overline{GB}$	4. _____
5. $SG \cong UG$	5. Definition of congruent segments
6. $GT \cong GB$	6. Definition of congruent segments
7. $SG + GT = UG + GB$	7. _____
8. $SG + GT = ST$	8. Segment addition postulate
9. $UG + GB = UB$	9. Segment addition postulate
10. _____	10. Substitution property
11. $\overline{ST} \cong \overline{UB}$	11. Definition of congruent segments
12. $\angle STU \cong \angle UBS$	12. CPCTC
13. $\triangle STU \cong \triangle UBS$	13. _____
14. _____	14. CPCTC

Extension

Complete the following two-column proof for the given geometric statement. Fill in the missing Statement(s) and Reason(s).

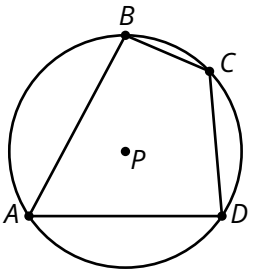
Given: $\triangle ABC$ is an isosceles triangle with $AB = AC$, and $\angle B \cong \angle C$.

Prove: $\triangle ABC \cong \triangle ACB$.

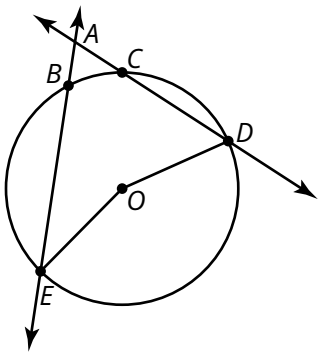
Statement	Reasons
1. $\triangle ABC$ an isosceles triangle.	1. Given
2. $AB = AC$	2.
3. $\angle B \cong \angle C$	3.
4.	4. Reflexive property of congruence
5. $\triangle ABC \cong \triangle ACB$	5.

Spaced Practice

1. In the figure shown, quadrilateral $ABCD$ is inscribed in circle P, $m\overset{\frown}{BC} = 55°$, $m\overset{\frown}{CD} = 109°$, and $m\overset{\frown}{DA} = 91°$. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?

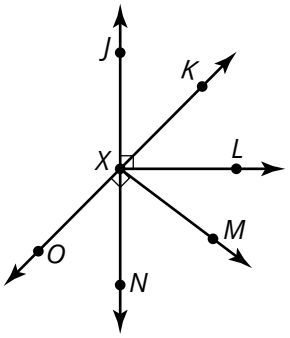


2. Consider circle O. The measure of $\angle EOD$ is 155°. The measure of $\angle A$ is 40°. Determine $m\overset{\frown}{BC}$.

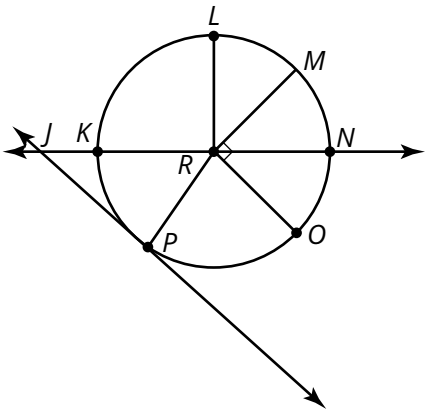


3. Identify the property that justifies the statement: If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

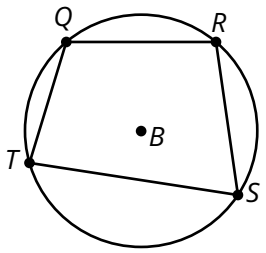
4. Consider the figure. What theorem tells you that $\angle KXL \cong \angle MXN$?



5. Consider circle R with diameter $\overline{KN}$. Line JP is tangent to circle R at point P. The measure of $\angle J$ is 42°, the measure of $\angle LRM$ is 45°, and the measure of $\angle PRO$ is 80°. Determine the measure of $\overset{\frown}{KL}$.



6. In the figure shown, quadrilateral $QRST$ is inscribed in circle B, $m\angle R = 98°$, and $m\angle Q$ is 14° more than $m\angle R$. What are $m\angle Q$, $m\angle S$, and $m\angle T$?

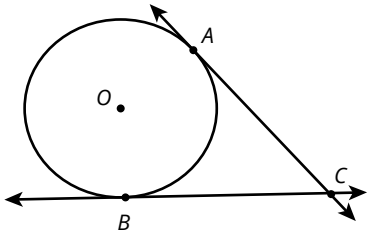


II. Using Congruent Triangles to Determine Relationships Between Chords and Tangents

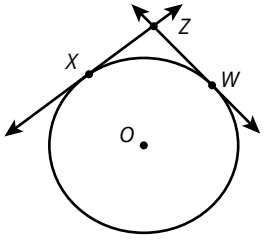
Topic Practice

A. Write a statement to show the congruent segments.

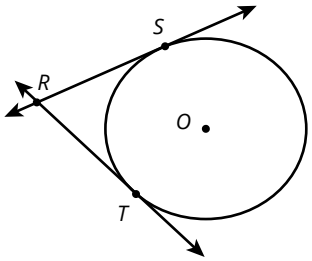
1. $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ are tangent to circle O.



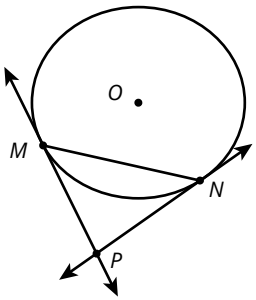
2. $\overleftrightarrow{XZ}$ and $\overleftrightarrow{WZ}$ are tangent to circle O.



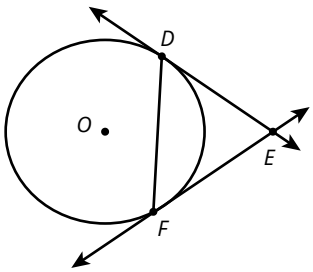
3. $\overleftrightarrow{RS}$ and $\overleftrightarrow{RT}$ are tangent to circle O.



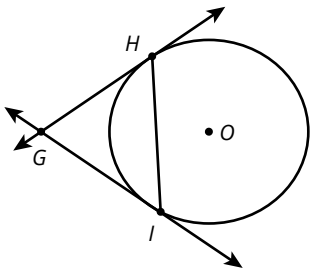
4. $\overleftrightarrow{MP}$ and $\overleftrightarrow{NP}$ are tangent to circle O.



5. $\overleftrightarrow{DE}$ and $\overleftrightarrow{FE}$ are tangent to circle O.

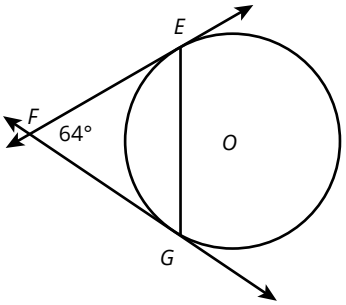


6. $\overleftrightarrow{GH}$ and $\overleftrightarrow{GI}$ are tangent to circle O.

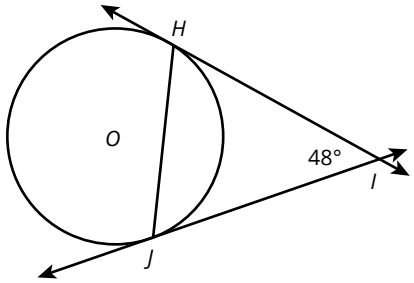


B. Calculate the measure of each angle. Explain your reasoning.

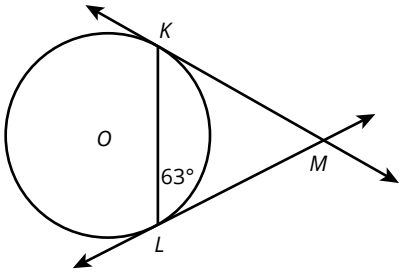
1. If $\overline{EF}$ and $\overline{GF}$ are tangent segments, what is the measure of $\angle EGF$?



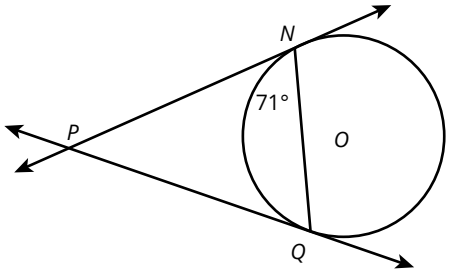
2. If $\overline{HI}$ and $\overline{JI}$ are tangent segments, what is the measure of $\angle HJI$?



3. If $\overline{KM}$ and $\overline{LM}$ are tangent segments, what is the measure of $\angle KML$?



4. If $\overline{NP}$ and $\overline{QP}$ are tangent segments, what is the measure of $\angle NPQ$?



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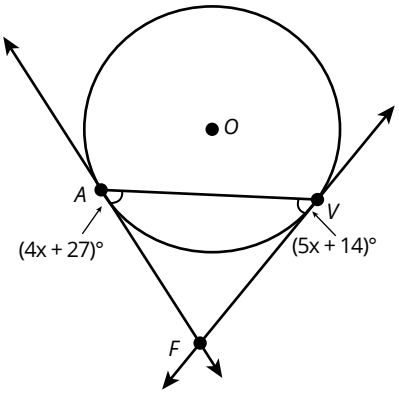
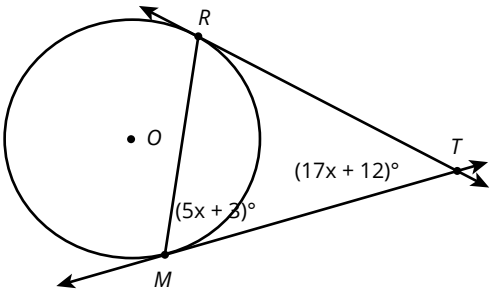
5. If $\overline{AF}$ and $\overline{VF}$ are tangent segments, what is the measure of $\angle AVF$?

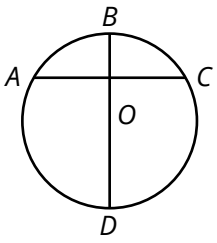


6. If $\overline{RT}$ and $\overline{MT}$ are tangent segments, what is the measure of $\angle RTM$?

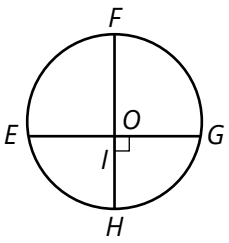


C. Use the given information to answer each question. Explain your reasoning.

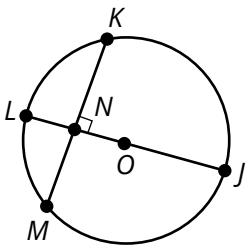
1. If diameter $\overline{BD}$ bisects $\overline{AC}$, what is the angle of intersection?



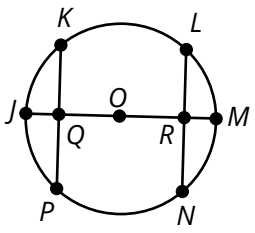
2. If diameter $\overline{FH}$ intersects $\overline{EG}$ at a right angle, how does the length of $\overline{EI}$ compare to the length of $\overline{IG}$?



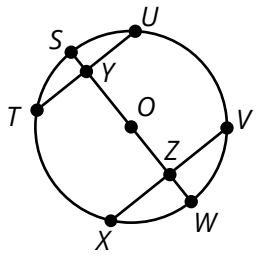
3. How do the measures of $\overset{\frown}{KL}$ and $\overset{\frown}{LM}$ compare?



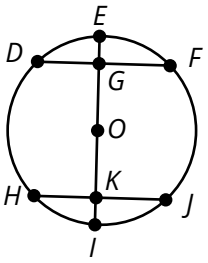
4. 4. If $\overline{KP} \cong \overline{LN}$, how does the length of $\overline{QO}$ compare to the length of $\overline{RO}$?



5. If $\overline{YO} \cong \overline{ZO}$, what is the relationship between $\overline{TU}$ and $\overline{XV}$?

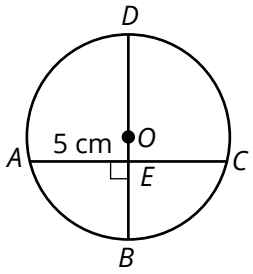


6. If $\overline{GO} \cong \overline{KO}$ and diameter $\overline{EI}$ is perpendicular to both chords $\overline{DF}$ and $\overline{HJ}$, what is the relationship between $\overline{GF}$ and $\overline{KJ}$?

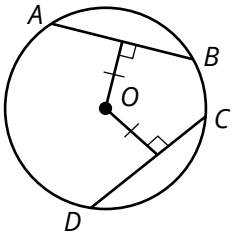


D. Determine each measurement.

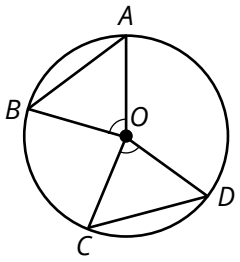
1. If $\overline{BD}$ is a diameter, what is the length of $\overline{EC}$?



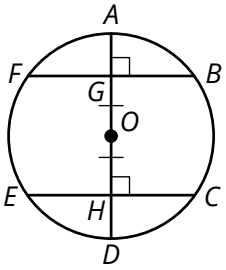
2. If the length of $\overline{AB}$ is 13 millimeters, what is the length of $\overline{CD}$?



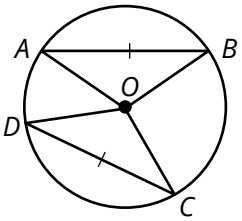
3. If the length of $\overline{AB}$ is 24 centimeters, what is the length of $\overline{CD}$?



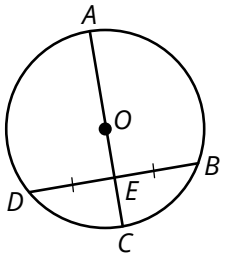
4. If the length of $\overline{BF}$ is 32 inches, what is the length of $\overline{CH}$?



5. If the measure of $\angle AOB = 155°$, what is the measure of $\angle DOC$?

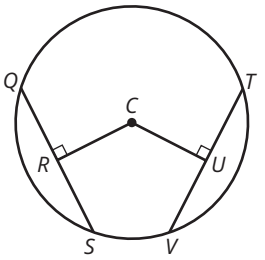


6. If segment $\overline{AC}$ is a diameter, what is the measure of $\angle AED$?



Extension

1. The circle shown has a diameter of 40 centimeters. The length of $\overline{RC}$ is 12 centimeters, and the length of $\overline{UV}$ is 16 centimeters.



   a. Determine the length of $\overline{CU}$. Explain your reasoning.

   b. Determine the length of $\overline{QS}$. Explain your reasoning.

Spaced Practice

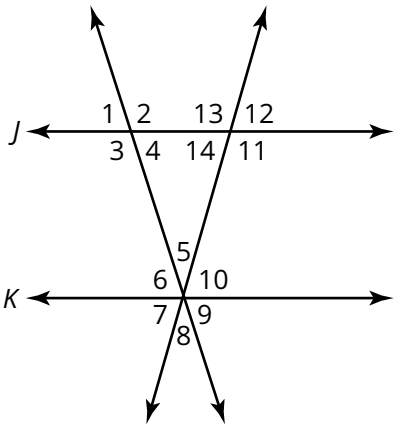
1. Koda tells you she is thinking of a quadrilateral that is either a rectangle or a square, but not both. She wants you to guess which quadrilateral she is thinking of and allows you to ask one question about the quadrilateral. What question should you ask?

2. Consider the kite shown. The kite without the tail is a quadrilateral. Classify the quadrilateral based only on each piece of given information.

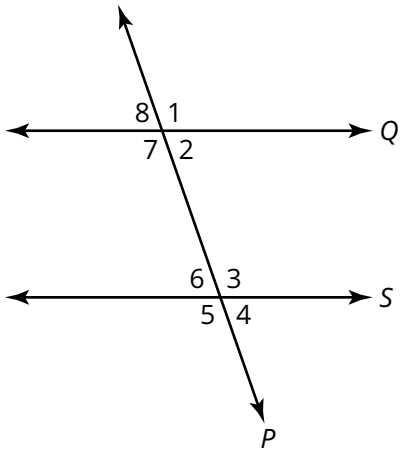
   a. The diagonals of the quadrilateral are perpendicular to each other and bisect each other.

   b. The four angles and the four sides of the quadrilateral are congruent.

3. Given: $\angle 3$ is supplementary to $\angle 6$, $\angle 1 \cong \angle 12$, and $m\angle 12 = 52°$. Using the diagram in conjunction with postulates and theorems, determine the measures of the unknown angles.



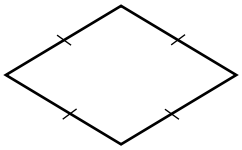
4. If $\angle 1 \cong \angle 5$, which theorem leads to the conclusion that $Q \parallel S$?

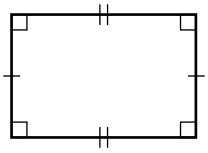


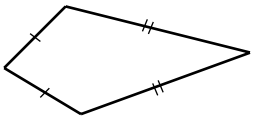
III. Properties of Quadrilaterals

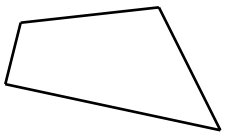
Topic Practice

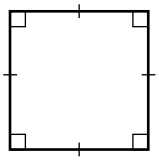
A. Identify all of the terms from the following list that apply to each figure: quadrilateral, parallelogram, rectangle, square, trapezoid, rhombus, or kite.

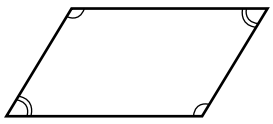
1. 

2. 

3. 

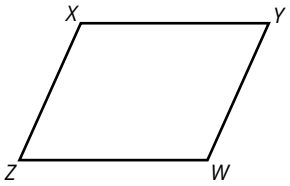
4. 

5. 

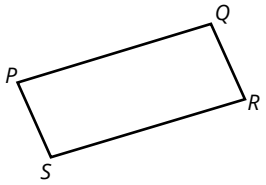
6. 

B. Determine the missing statement needed to prove each quadrilateral is a parallelogram by the Parallelogram/ Congruent-Parallel Side Theorem.

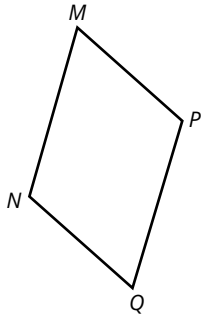
1. $\overline{XY} \parallel \overline{ZW}$



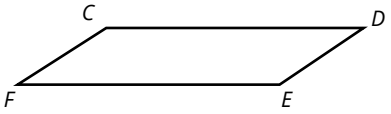
2. $\overline{PS} \cong \overline{QR}$



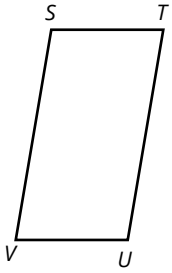
3. $\overline{MN} \cong \overline{PQ}$



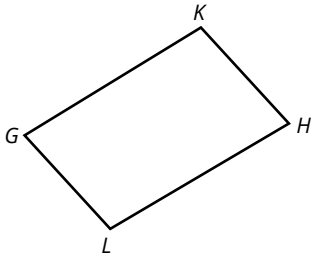
4. $\overline{CF} \parallel \overline{DE}$



5. $\overline{ST} \parallel \overline{VU}$

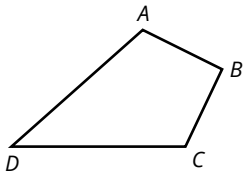


6. $\overline{KG} \parallel \overline{HL}$

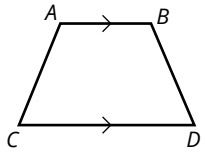


C. Use the given figure to answer each question.

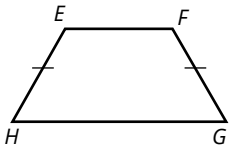
1. The figure shown is a kite with $\angle DAB \cong \angle DCB$. Which sides of the kite are congruent?



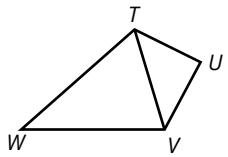
2. The figure shown is an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$. Which sides are congruent?



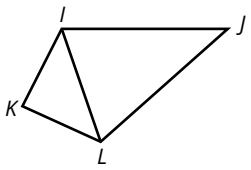
3. The figure shown is an isosceles trapezoid with $\overline{EH} \cong \overline{FG}$. Which sides are parallel?



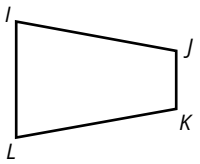
4. Given that *TUVW* is a kite, which angles are congruent?



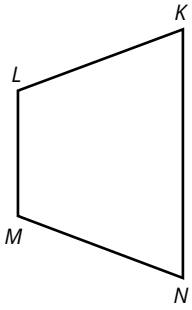
5. Given that *IJLK* is a kite, what kind of triangles are formed by diagonal $\overline{IL}$?



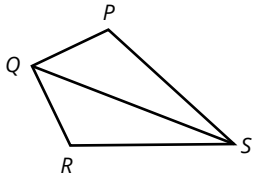
6. The figure shown is an isosceles trapezoid with $\overline{IJ} \cong \overline{LK}$. What are the bases?



7. The figure shown is an isosceles trapezoid with $\overline{LM} \parallel \overline{KN}$. What are the pairs of base angles?



8. Given that *PQRS* is a kite, which angles are congruent?

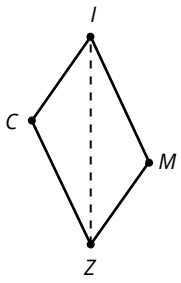


D. Consider the parallelogram proofs in each question. Follow the instructions to complete each proof.

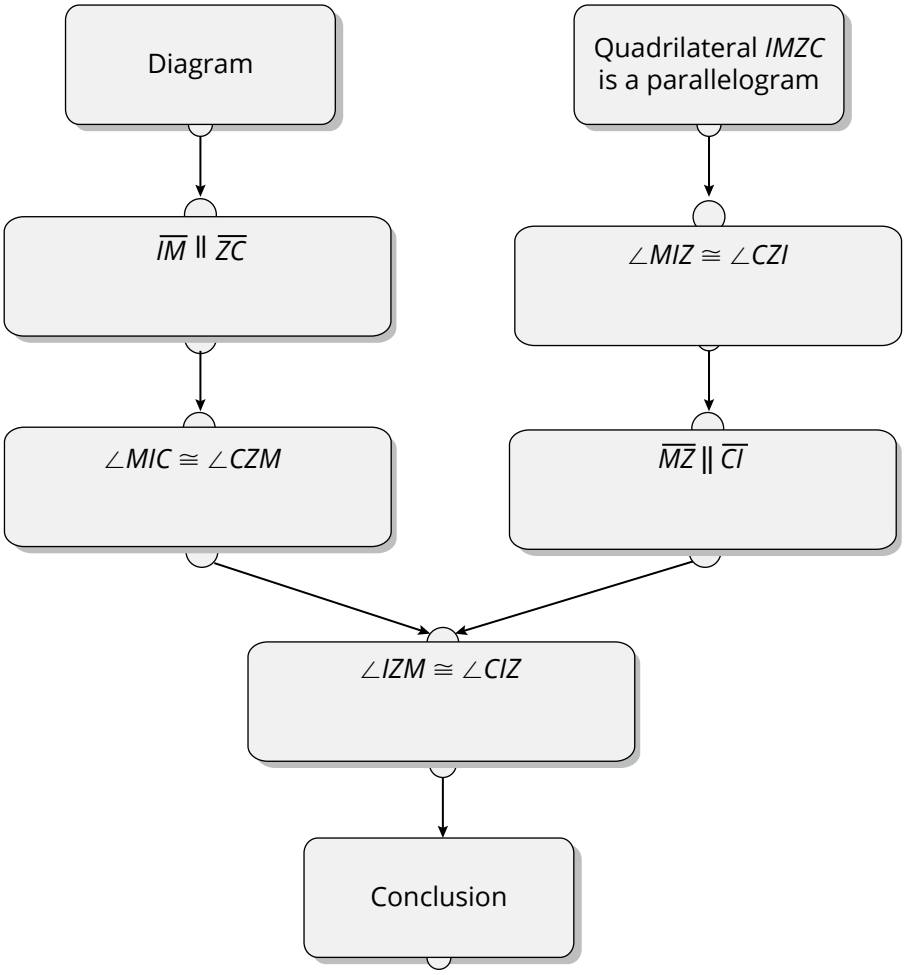
1. Given the diagram below and the following statement:
   - Quadrilateral *IMZC* is a parallelogram.

   Prove that $\angle MIC \cong \angle CZM$.

   Use the following Statements and Reasons to create a flowchart proof. Reasons may be used more than once.



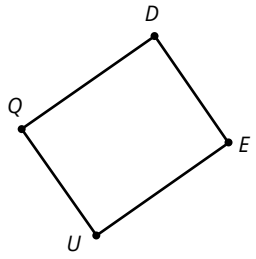
Reasons
• Definition of a parallelogram • Congruent angle addition theorem • Alternate interior angle theorem



2. Given the diagram below and the following statements:
   - Quadrilateral *QDEU* is a parallelogram.
   - $\angle Q$ is a right angle.

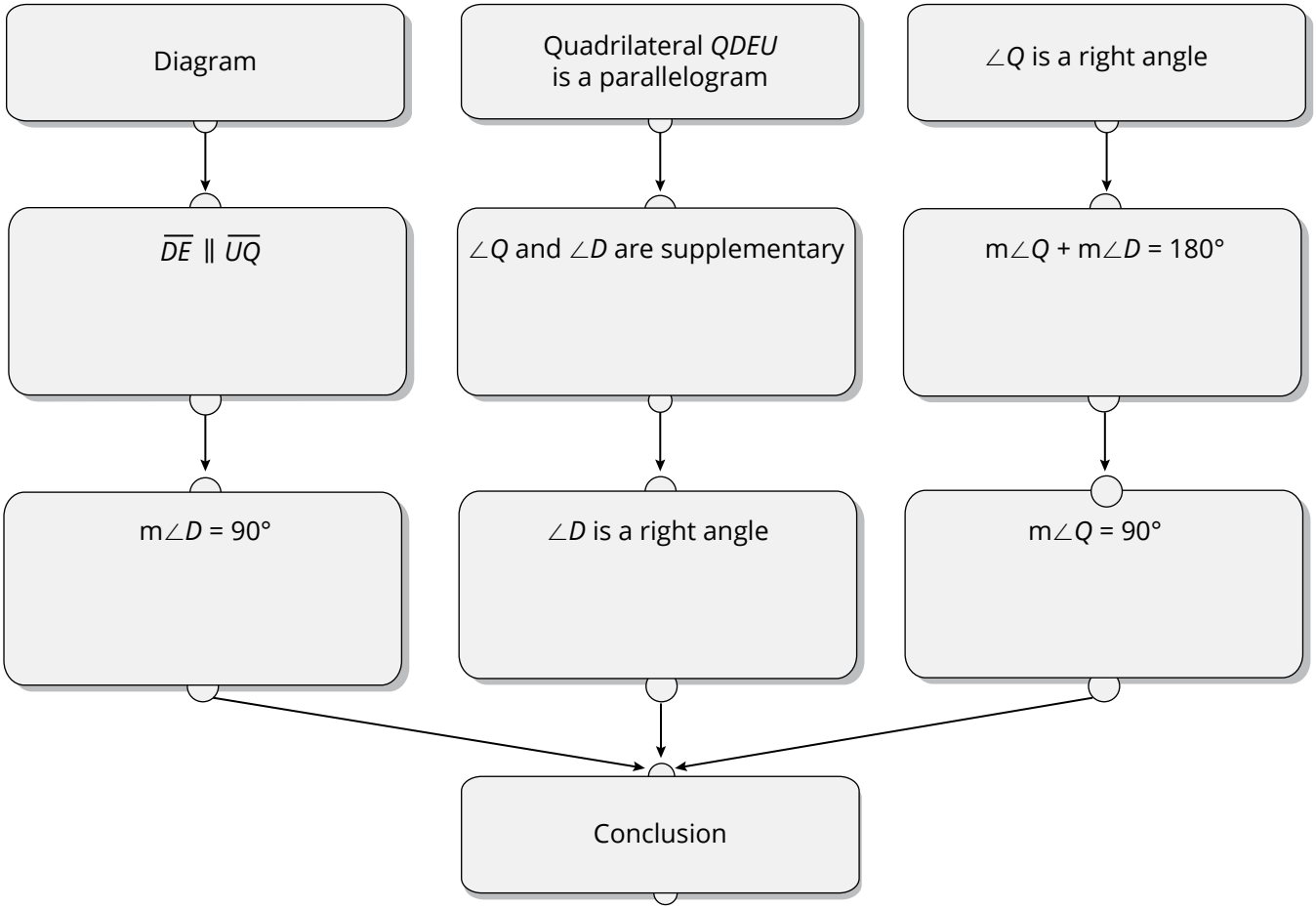
   Prove that $\angle D$ is a right angle.

   Use the Reason's to fill out the 2-column table proof. Reasons may be used more than once.

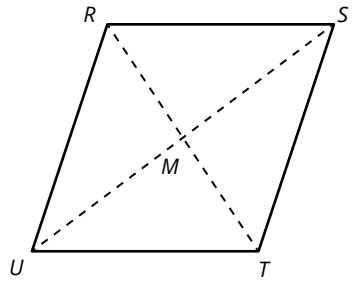


Reasons

- Definition of a right angle
- Same side interior angle theorem
- Definition of supplementary angles
- Subtraction property of equality
- Definition of a parallelogram



3. Create a two-column proof to prove $\overline{RT} \perp \overline{SU}$ in rhombus *RSTU*.

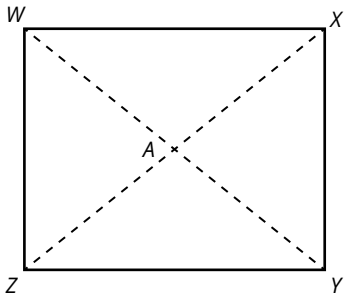


Reasons

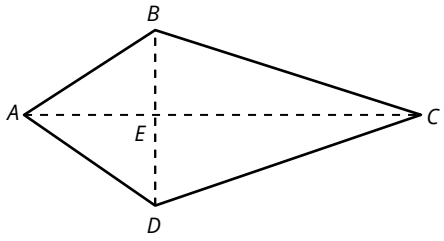
- Reflexive property of congruence
- SSS congruence theorem
- Given
- Diagonals of a rhombus bisect each other
- If two angles are supplementary and congruent, then they are right angles.
- CPCTC
- Definition of linear pair
- Definition of perpendicular
- Definition of a rhombus

Statements	Reasons
1. Rhombus *RSTU* with diagonals $\overline{RT}$ and $\overline{SU}$ intersecting at point *M*	1.
2. $\overline{RS} \cong \overline{RU}$	2.
3. $\overline{UM} \cong \overline{SM}$	3.
4. $\overline{RM} \cong \overline{RM}$	4.
5. $\triangle RMU \cong \triangle RMS$	5.
6. $\angle RMU$ and $\angle RMS$ are supplementary.	6.
7. $\angle RMU \cong \angle RMS$	7.
8. $\angle RMU$ and $\angle RMS$ are right angles.	8.
9. $\overline{RT} \perp \overline{SU}$	9.

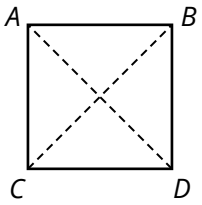
4. Write a paragraph proof to prove opposite sides are parallel in rectangle *WXYZ*.



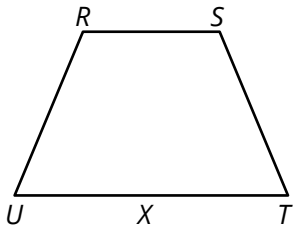
5. Write a paragraph proof to prove $\angle ABC \cong \angle ADC$ in kite *ABCD*.



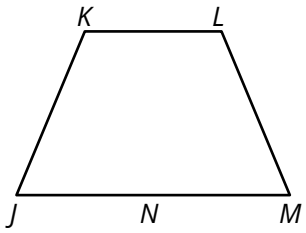
6. Using the diagram of quadrilateral *ABCD*, prove that *ABCD* is a parallelogram. Given that $AB \parallel CD$ and $AB \cong CD$, show that *ABCD* satisfies the properties of a parallelogram.



7. Write a paragraph proof to prove $\angle U \cong \angle T$ in isosceles trapezoid *RSTU*.



8. Write a two-column proof to prove $\angle K \cong \angle L$ in isosceles trapezoid *JKLM*.



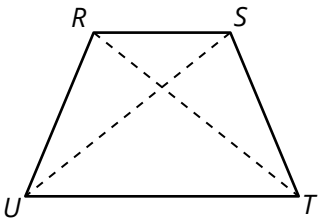
Reasons

- Transitive property of congruence
- Same-side alternate interior angles between parallel lines are supplementary
- Alternate interior angles theorem
- Transitive property of congruence
- Given
- Base angles in an isosceles triangle are congruent
- Opposite sides of a parallelogram are congruent
- Opposite angles in a parallelogram are congruent
- Definition of an isosceles triangle
- Definition of a trapezoid
- Definition of a parallelogram
- Construction
- Transitive property of congruence
- Supplements of congruent angles are congruent

Statements	Reasons
1. Isosceles trapezoid *JKLM* with $\overline{KJ} \cong \overline{LM}$	1.
2. $\overline{KJ} \parallel \overline{JM}$	2.
3. $\angle L$ is supplementary to $\angle M$ $\angle K$ is supplementary to $\angle J$	3.
4. Construct $\overline{KN} \parallel \overline{LM}$	4.
5. Quadrilateral *KLMN* is a parallelogram.	5.
6. $\overline{KN} \cong \overline{LM}$	6.
7. $\overline{KJ} \cong \overline{KN}$	7.
8. $\triangle JKN$ is isosceles	8.
9. $\angle J \cong \angle KNJ$	9.
10. $\angle LKN \cong \angle KNJ$	10.

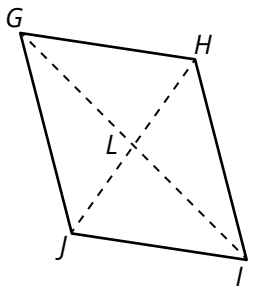
Statements	Reasons
11. $\angle J \cong \angle LKN$	11.
12. $\angle LKN \cong \angle M$	12.
13. $\angle J \cong \angle M$	13.
14. $\angle K \cong \angle L$	14.

9. Write a two-column proof to prove that diagonals $\overline{RT}$ and $\overline{SU}$ in isosceles trapezoid $RSTU$ are congruent if $\overline{RU} \cong \overline{ST}$.



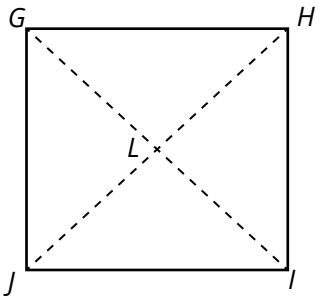
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

10. Create a two-column proof to prove $\overline{GL} \cong \overline{IL}$ and $\overline{JL} \cong \overline{HL}$ in parallelogram *GHIJ*.

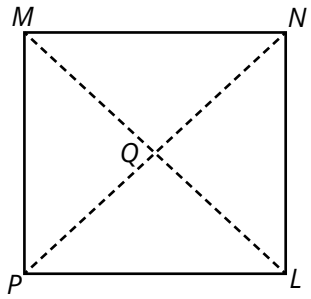


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

11. Write a paragraph proof to prove $\overline{GL} \cong \overline{IL}$ and $\overline{HL} \cong \overline{JL}$ in square *GHIJ*.

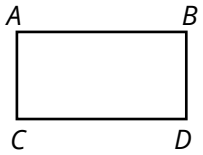


12. Create a two-column proof to prove $\overline{ML} \perp \overline{NP}$ in square $MNLP$. Complete the 2-column proof by writing in the correct reason(s) for each Statement.

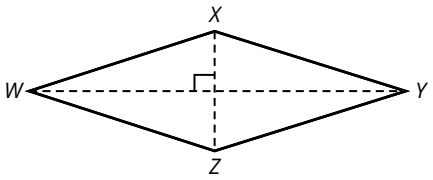


Statements	Reasons
1. Square $MNLP$ with diagonals $\overline{ML}$ and $\overline{NP}$ intersecting at point Q	1.
2. $\overline{MN} \cong \overline{PM}$	2.
3. $\overline{PQ} \cong \overline{QN}$	3.
4. $\overline{MQ} \cong \overline{MQ}$	4.
5. $\triangle PMQ \cong \triangle NMQ$	5.
6. $\angle PQM$ and $\angle NQM$ are supplementary.	6.
7. $\angle PMQ \cong \angle NMQ$	7.
8. $\angle PQM$ and $\angle MQN$ are right angles.	8.
9. $\overline{ML} \parallel \overline{NP}$	9.

13. Prove that quadrilateral *ABCD* is a rectangle using supplementary exterior angles.



14. Prove that a quadrilateral *WXYZ* is a rhombus using diagonals that bisect each other at a right angle.



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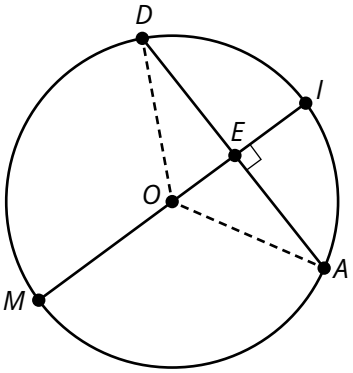
Extension

Consider the circle shown. Diameter $\overline{MI}$ is perpendicular to chord $\overline{DA}$.

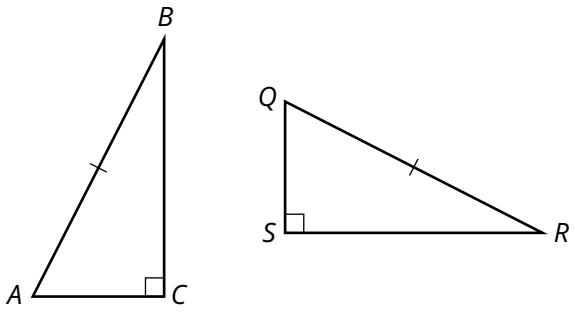


1. Write a conjecture about the relationship between a diameter and a chord when they are perpendicular. Prove that your conjecture is true in all cases.

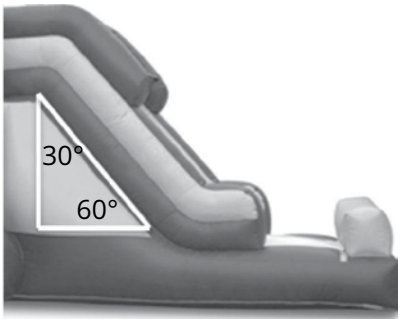
2. Chord DA is also a perpendicular bisector of radius OI. What type of quadrilateral is formed when you create segments DI and IA? Explain your reasoning.

Spaced Practice

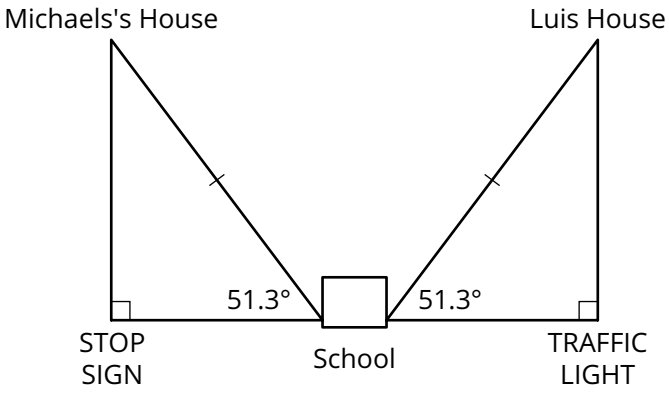
1. Determine the information that is needed to show that $\triangle ABC \cong \triangle QRS$ by the *HL* congruence theorem.



2. An inflatable slide is modeled from a triangle as shown. The length of the shorter leg of the triangle is 12 feet. Use the 30°-60°-90° triangle theorem to determine the length of the hypotenuse and the longer leg of the triangle.



3. Michael and Luis both live the same distance from the school. Michael's house is 51.3° northwest of the school, and Luis's house is 51.3° northeast of the school. If the distance from the school to the stop sign is 0.5 mile, how far is it from the school to the traffic light? Explain your reasoning.



Investigating Proportionality

TOPIC 1: Similarity

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TOPIC 2: Trigonometry

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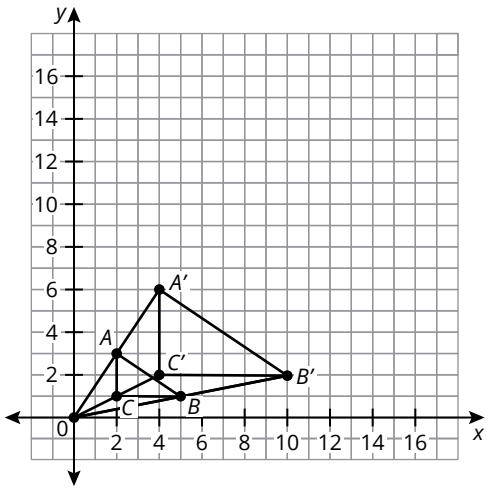
Name _____ Date _____

I. Dilating Figures to Create Similar Figures

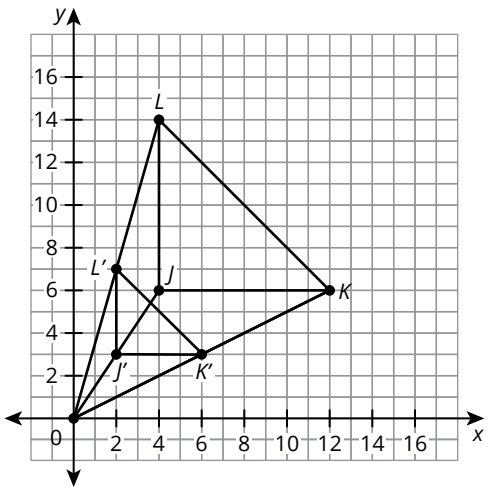
Topic Practice

A. Given each pre-image and image, determine the scale factor.

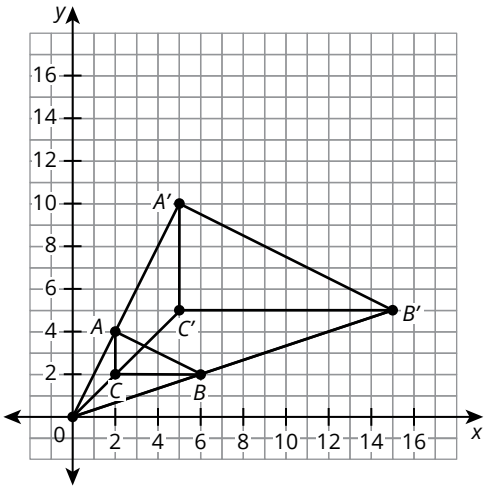
1.



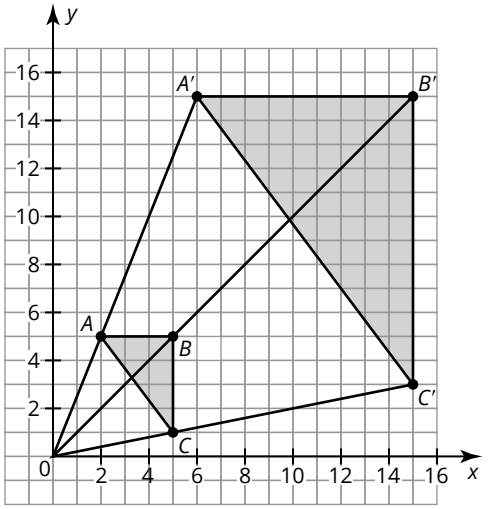
2.



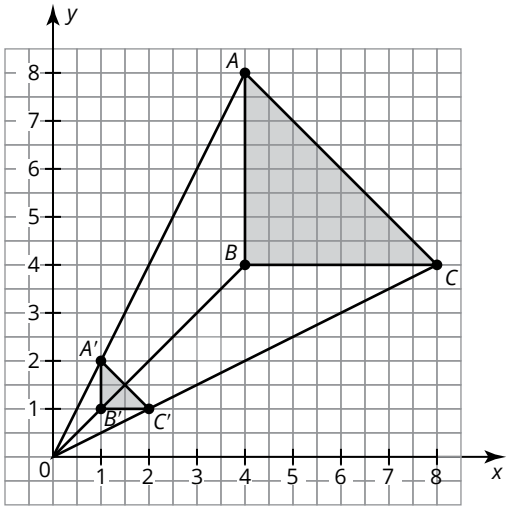
3.



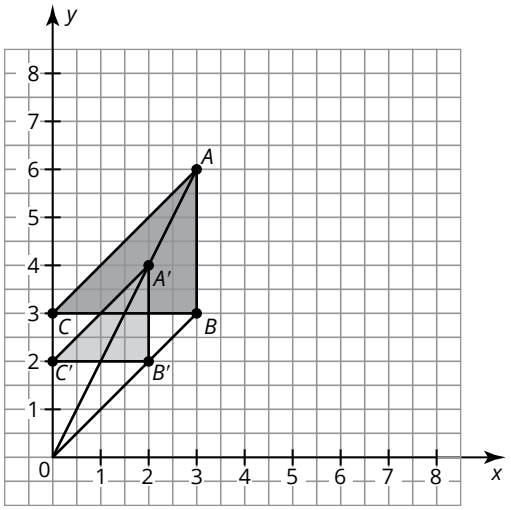
4.



5.



6.



B. Determine the coordinates of the image. Write the transformation in coordinate notation.

1. Triangle *ABC* has vertices *A*(1, 2), *B*(3, 6), and *C*(9, 7). What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

2. Triangle *DEF* has vertices *D*(8, 4), *E*(2, 6), and *F*(3, 1). What are the vertices of the image after a dilation with a scale factor of 5 using the origin as the center of dilation?

3. Triangle *GHI* has vertices *G*(0, 5), *H*(4, 2), and *I*(3, 3). What are the vertices of the image after a dilation with a scale factor of 9 using the origin as the center of dilation?

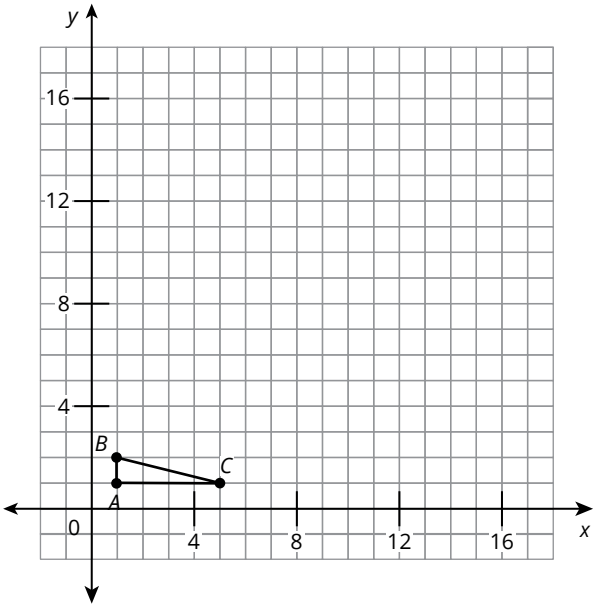
4. Triangle *JKL* has vertices *J*(6, 2), *K*(1, 3), and *L*(7, 0). What are the vertices of the image after a dilation with a scale factor of 12 using the origin as the center of dilation?

5. Quadrilateral *ABCD* has vertices *A*(8, 4), *B*(14, 16), *C*(6, 10), and *D*(4, 6). What are the vertices of the image after a dilation with a scale factor of $\frac{1}{2}$ using the origin as the center of dilation?

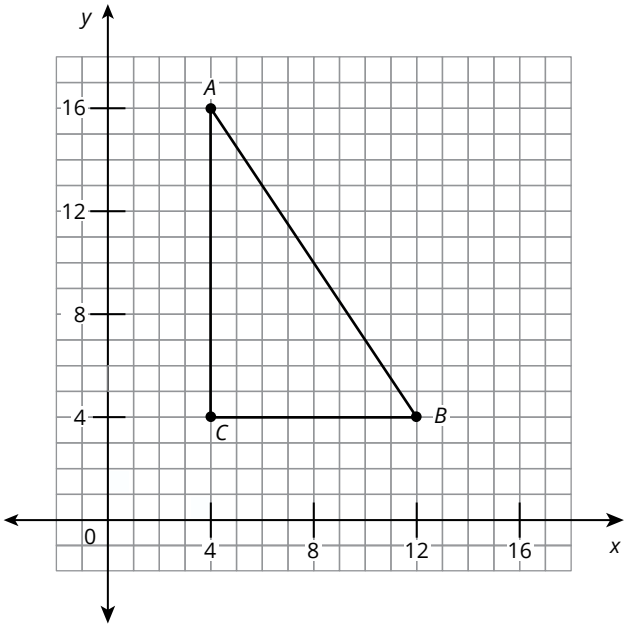
6. Polygon *DEFGH* has vertices *D*(25, 25), *E*(20, 10), *F*(15, 10), *G*(10, 20), and *H*(20, 30). What are the vertices of the image after a dilation with a scale factor of $\frac{1}{5}$ using the origin as the center of dilation?

C. Given the pre-image, scale factor, and center of dilation, use a straightedge to graph the image.

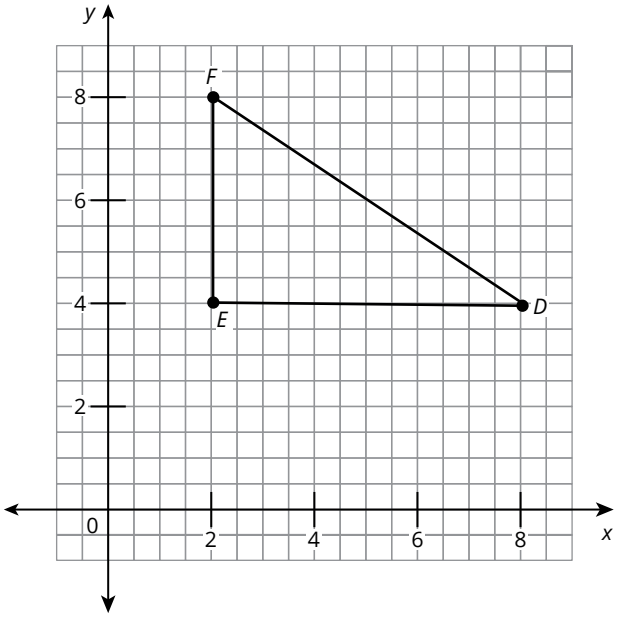
1. The scale factor is 3 and the center of dilation is the origin.



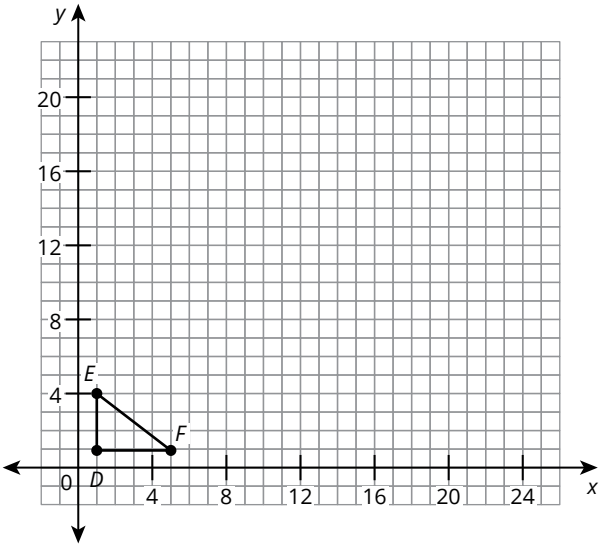
2. The scale factor is $\frac{1}{4}$ and the center of dilation is the origin.



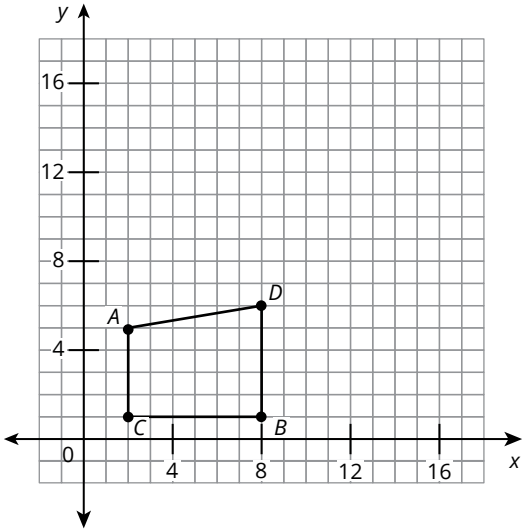
3. The scale factor is $\frac{1}{2}$ and the center of dilation is the origin.



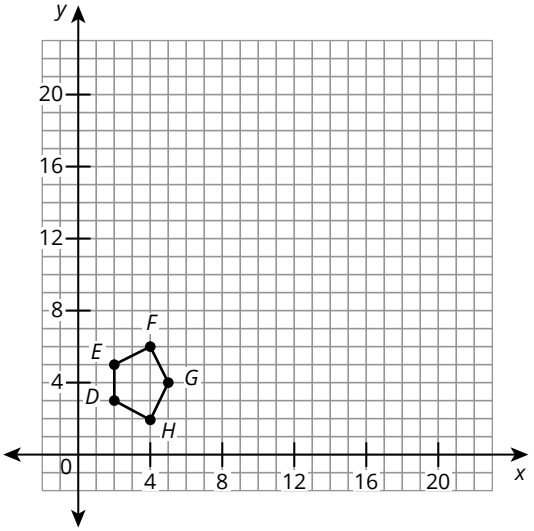
4. The scale factor is 5 and the center of dilation is the origin.



5. The scale factor is 1.5 and the center of dilation is the origin.



6. The scale factor is 2.5 and the center of dilation is the origin.



D. Dilate $\triangle ABC$ by the given scale factor(s) using the given center of dilation.

1. The scale factor is $k = \frac{1}{4}$ and the center of dilation is the origin. Determine the coordinates of A', B', and C'.

   $A(4, -3)$, $B(6, -4)$, $C(7, 0)$

2. The scale factor is $k = \frac{1}{3}$ and the center of dilation is the origin. Determine the coordinates of A', B', and C'.

   $A(-3, 2)$, $B(1, -4)$, $C(3, 1)$

3. The scale factor is $k = \frac{3}{2}$ and the center of dilation is the origin. Determine the coordinates of A', B', and C'.

   $A(8, 0)$, $B(2, 4)$, $C(6, 2)$

4. The scale factor is $k = \frac{3}{4}$ and the center of dilation is the origin. Determine the coordinates of A', B', and C'.

   $A(6, 3)$, $B(4, 5)$, $C(3, 7)$

5. The scale factors are $k_1 = \frac{1}{2}$ and $k_2 = 2$ and the center of dilation is the origin. Determine the coordinates of A', B', and C' using k_1, then determine the coordinates of A'', B'', and C'' by applying k_2 to the coordinates of A', B', and C'.

   $A(9, 4)$, $B(3, 1)$, $C(6, 2)$

6. The scale factors are $k_1 = \frac{1}{4}$ and $k_2 = 5$ and the center of dilation is the origin. Determine the coordinates of A', B', and C' using k_1, then determine the coordinates of A'', B'', and C'' by applying k_2 to the coordinates of A', B', and C'.

   $A(4, 3)$, $B(7, -2)$, $C(1, 6)$

E. Determine the coordinates of the image after the composition of dilations. Label the image coordinates A'', B'', and C''.

1. Triangle ABC has vertices $A(4, 6)$, $B(-4, 2)$, and $C(2, -6)$. Dilate $\triangle ABC$ by a scale factor of 2, then dilate the result by a scale factor of $\frac{1}{4}$ using the origin as the center of dilation.

2. Triangle ABC has vertices $A(2, -1)$, $B(4, 2)$, and $C(6, 3)$. Dilate $\triangle ABC$ by a scale factor of $\frac{1}{2}$, then dilate the result by a of scale factor of 3 using $O(1, 1)$ as the center of dilation.

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3. Triangle ABC has vertices $A(-3, 5)$, $B(0, 0)$, and $C(4, -2)$. Dilate $\triangle ABC$ by a scale factor of 4, then dilate the result by a scale factor of $\frac{1}{2}$ using $O(-1, -1)$ as the center of dilation.

4. Triangle ABC has vertices $A(0, 3)$, $B(2, -1)$, and $C(5, 2)$. Dilate $\triangle ABC$ by a scale factor of $\frac{2}{3}$, then dilate the result by a scale factor of $\frac{3}{2}$ using $O(2, 2)$ as the center of dilation.

5. Triangle ABC has vertices $A(4, -3)$, $B(6, -4)$, and $C(7, 0)$. Dilate $\triangle ABC$ by a scale factor of 4, then dilate the result by a scale factor of $\frac{1}{2}$ using the origin as the center of dilation.

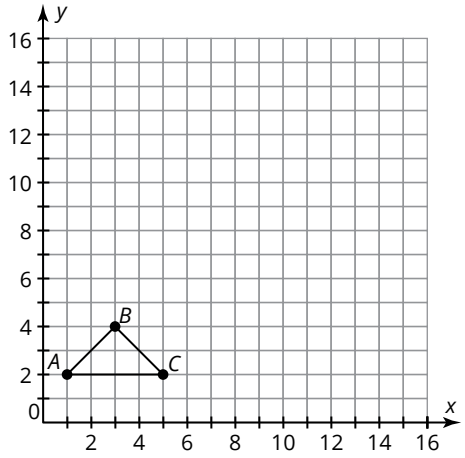
6. Triangle ABC has vertices $A(7, 8)$, $B(10, 12)$, and $C(13, 9)$. Dilate $\triangle ABC$ by a scale factor of 2, then dilate the result by a scale factor of $\frac{1}{2}$ using $O(5, 5)$ as the center of dilation.

F. Identify the image after a composition of rigid motions and non-rigid motions.

1. $A(1, 2)$, $B(3, 4)$, $C(5, 2)$

   Dilation: Scale Factor of 2

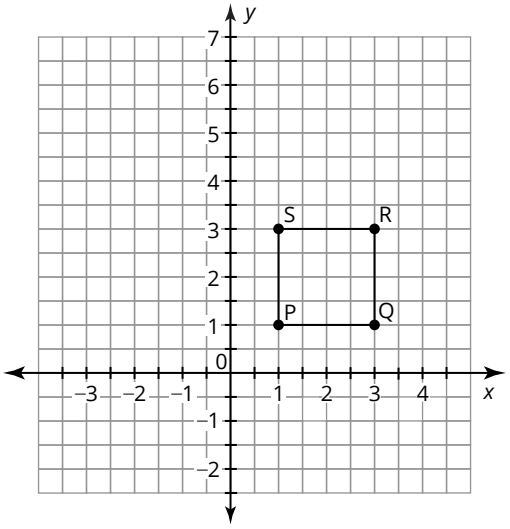
   Translation: 4 units right, 3 units up



2. $P(1, 1)$, $Q(3, 1)$, $R(3, 3)$, $S(1, 3)$

   Reflection: over the y-axis

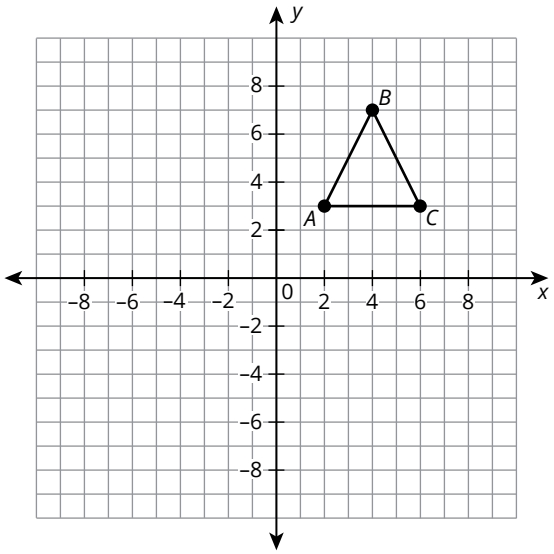
   Translation: 5 units right, 2 units down



3. $A(2, 3)$, $B(4, 7)$, $C(6, 3)$

Rotation: 90° CC around the origin.

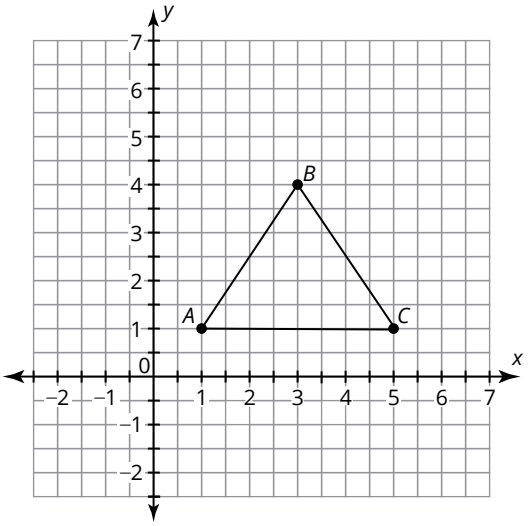
Reflection: over x-axis



4. $A(1, 1)$, $B(3, 4)$, $C(5, 1)$

Dilation: Scale Factor of 2

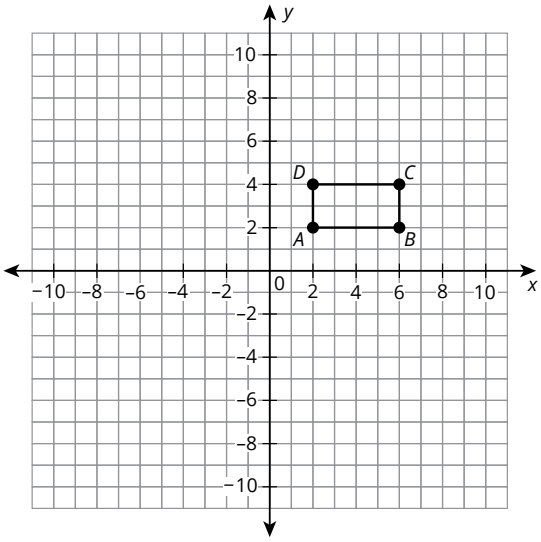
Translation: 3 units left, 2 units down



5. $A(2, 2)$, $B(6, 2)$, $C(6, 4)$, $D(2, 4)$

Rotation: 180° around the origin

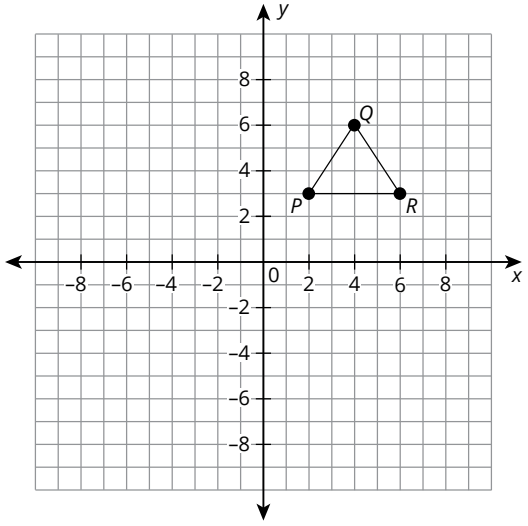
Translation: 4 units left, 3 units up



6. $P(2, 3)$, $Q(4, 6)$, $R(6, 3)$

Reflection: $y = x$

Rotation: 90° clockwise around the origin.



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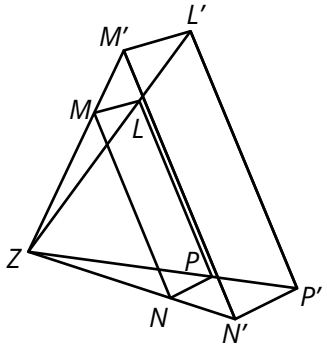
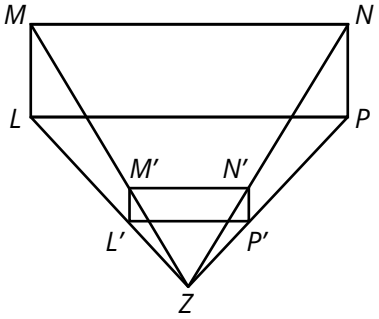
G. Given each pre-image and image, determine the scale factor, identify the congruent corresponding angles of the image and pre-image, and determine the ratio of the lengths of the corresponding sides.

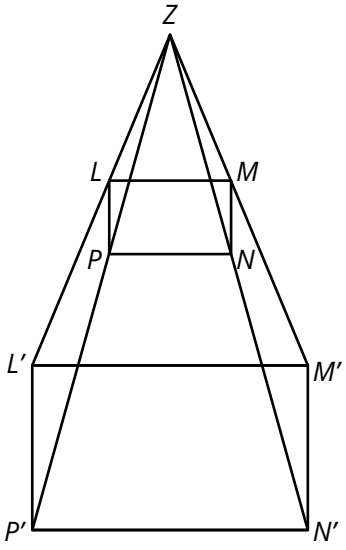
1. Given $ZN = 2$ in and $ZN' = 3$



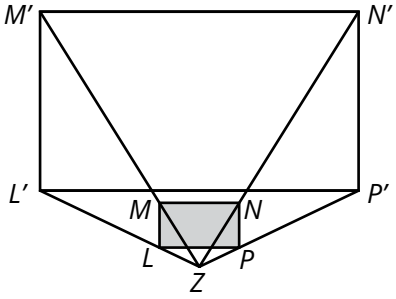
2. Given $ZN = 3$ in and $ZN' = 1$



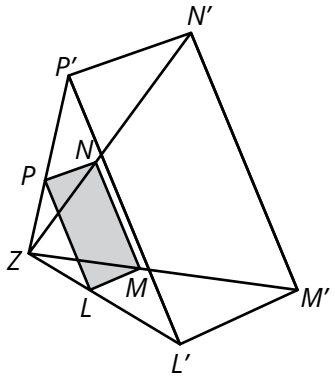
3. Given $ZN = 1.5$ in and $ZN' = 3.3$



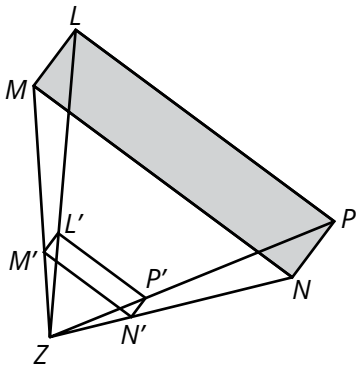
4. Given $ZN = 2.2$ in and $ZN' = 8.8$



5. Given $ZN = 2$ in and $ZN' = 5$

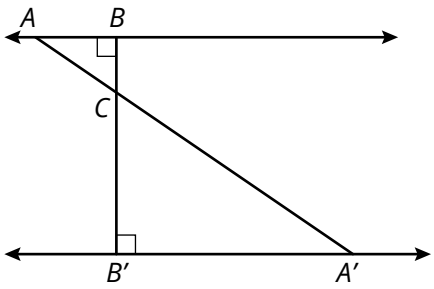


6. Given $ZN = 10$ in and $ZN' = 3$

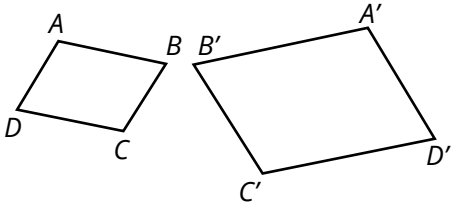


H. Determine the sequence of transformations that carries the pre-image onto the image.

1.



2.



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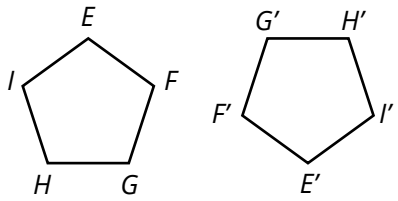
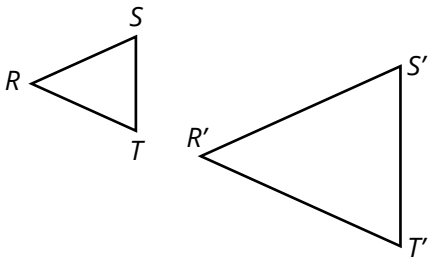
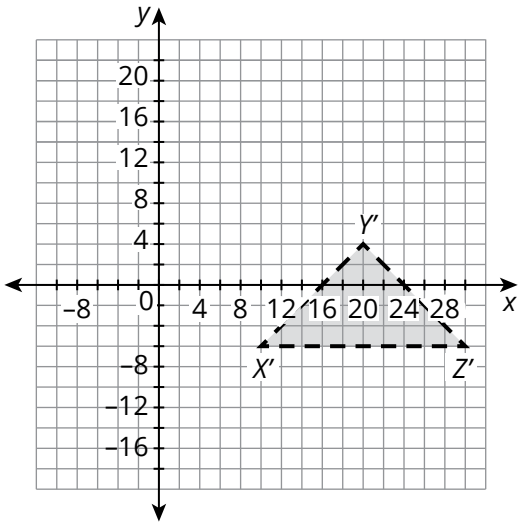
3.


E
I
F
H
G
G′
H′
F′
I′
E′

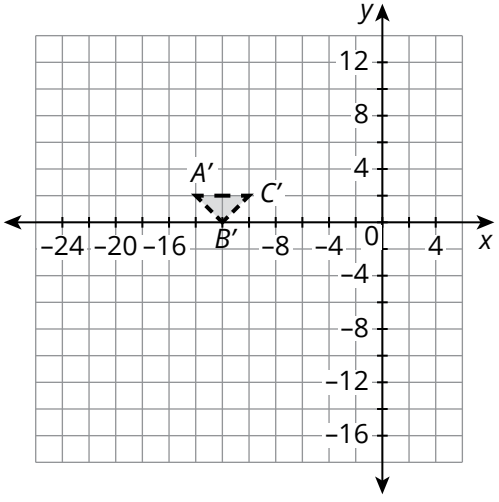
4.


S
R
T
S′
R′
T′

5.

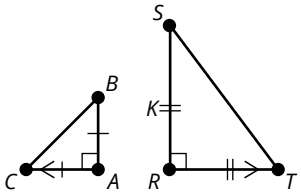

y
x
20
16
12
8
4
−8
0
4
8
12
16
20
24
28
−8
−12
−16
Y′
X′
Z′

6.

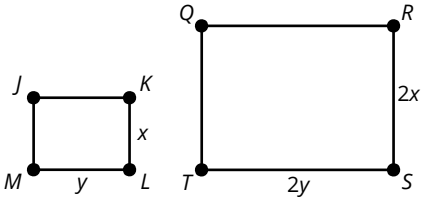

y
x
12
8
4
A′
C′
−24
−20
−16
B′
−8
−4
0
4
−4
−8
−12
−16

I. Use the definition of similarity in terms of transformations to determine whether each pair of figures are similar. Justify your reasoning.

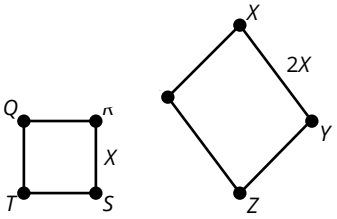
1. Is $\triangle ABC$ similar to $\triangle RST$?



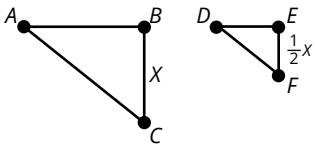
2. Is rectangle *JKLM* similar to rectangle *QRST*?



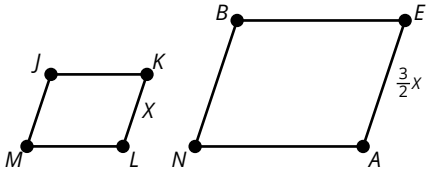
3. Is square *QRST* similar to square *WXYZ*?



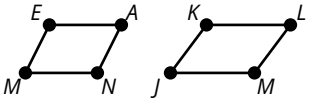
4. Is $\triangle ABC$ similar to $\triangle DEF$?



5. Is quadrilateral *JKLM* similar to quadrilateral *BEAN*?



6. Is quadrilateral *MEAN* similar to quadrilateral *JKLM*?

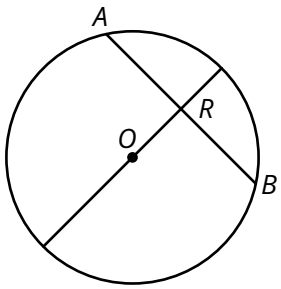


Extension

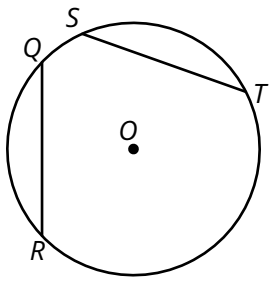
What is the area ratio between any two similar triangles? Justify your answer by providing an example of an enlargement, a reduction, and a congruence.

Spaced Practice

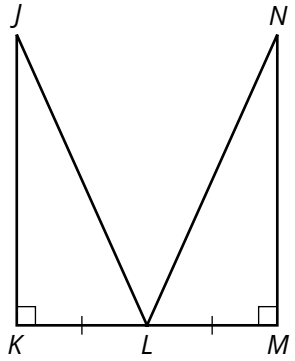
1. Consider the circle with center at point O. Determine the information that is needed to show that $\overline{AR} \cong \overline{BR}$.



2. Consider the circle with center at point O. Determine the information that is needed to show that $\overset{\frown}{RQ} \cong \overset{\frown}{ST}$.



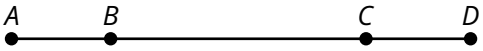
3. Determine the information that is needed to show that $\triangle JKL \cong \triangle NML$ by the LA congruence theorem.



4. Provide the reasons to complete the proof.

Given: $AC = BD$

Prove: $AB = CD$



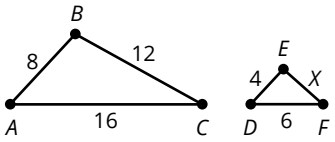
Statements	Reasons
1. $AC = BD$	
2. $AC = AB + BC$ $BD = BC + CD$	
3. $AB + BC = BC + CD$	
4. $AB = CD$	

II. Establishing Triangle Similarity Criteria

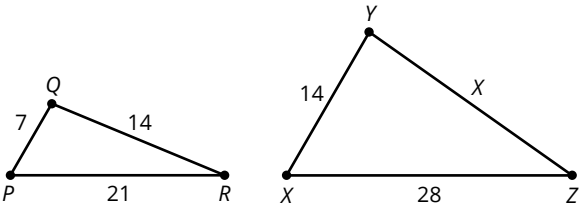
Topic Practice

A. Apply the proportionality of corresponding sides of similar triangles to solve problems.

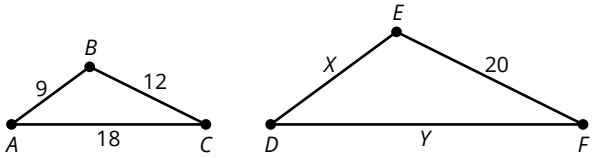
1. Determine *EF*.



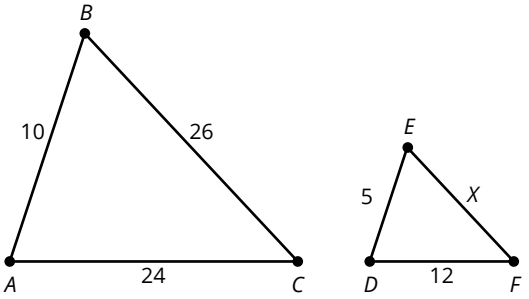
2. Determine the value of *x*.



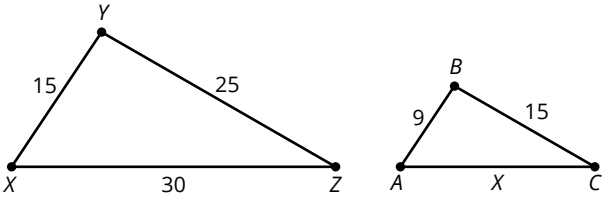
3. Determine the value of *x* and *y*.



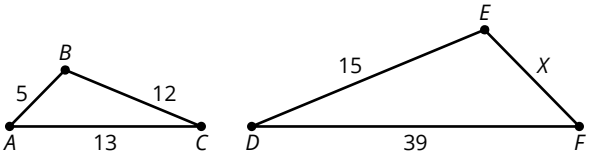
4. Determine *EF*.



5. Determine *AC*.

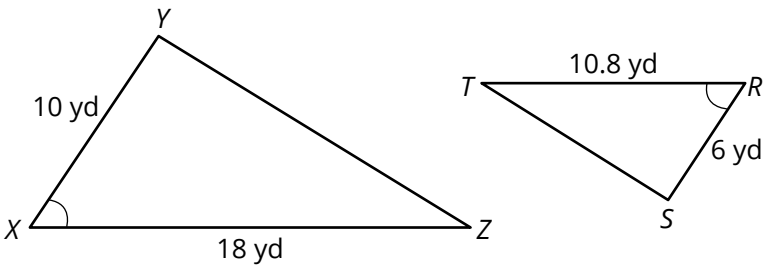


6. Determine the value of *x*.



B. Determine whether each pair of triangles is similar. Explain your reasoning.

1.



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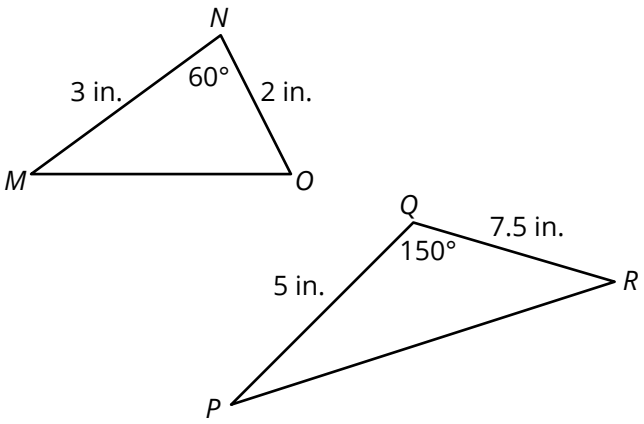

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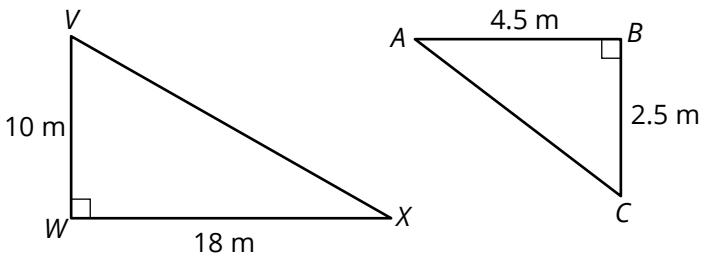

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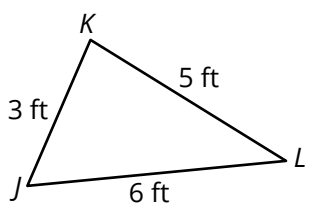
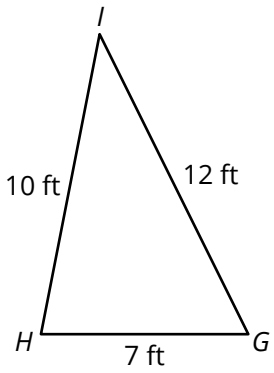
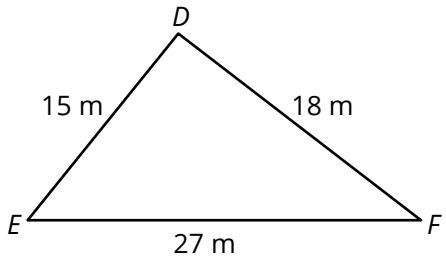
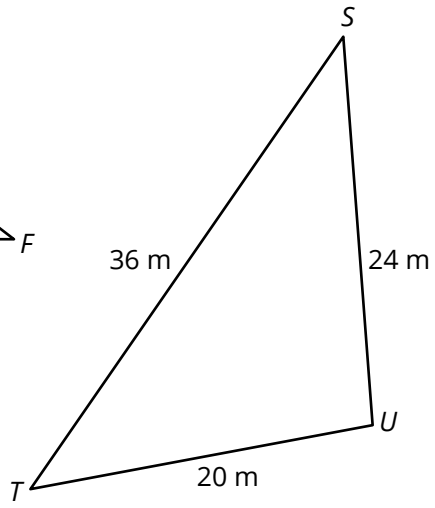

K
5 ft
3 ft
L
J
6 ft

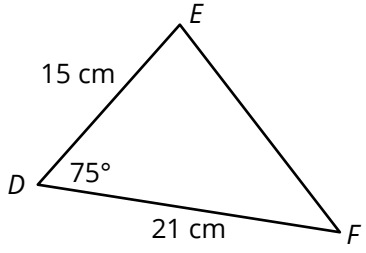

I
10 ft
12 ft
H
7 ft
G

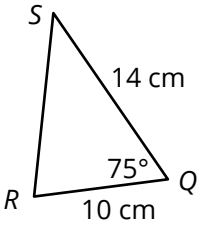
5.


D
15 m
18 m
E
27 m
F

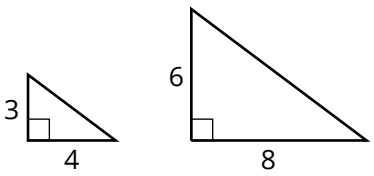

S
36 m
24 m
U
T
20 m

6.

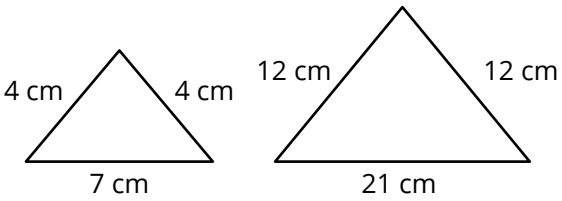

E
15 cm
D
75°
21 cm
F


S
14 cm
75°
R
Q
10 cm

7.



8.



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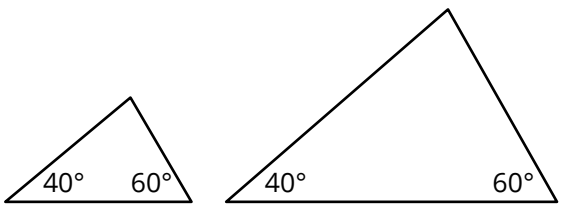

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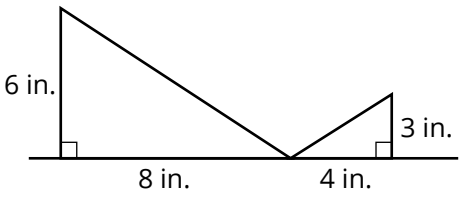
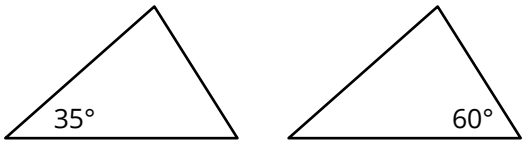
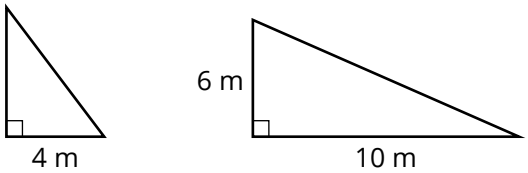


C. Determine what additional information you would need to prove that the triangles are similar using the given theorem.

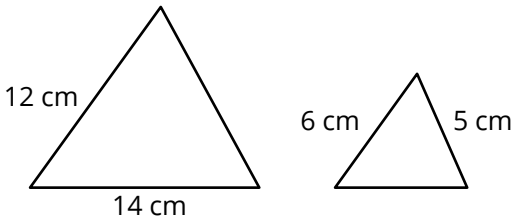
1. What information would you need to use the Angle-Angle similarity theorem to prove that the triangles are similar?



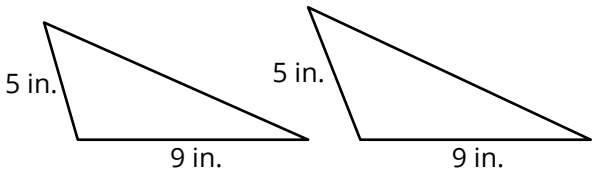
2. What information would you need to use the Side-Angle-Side similarity theorem to prove that the triangles are similar?



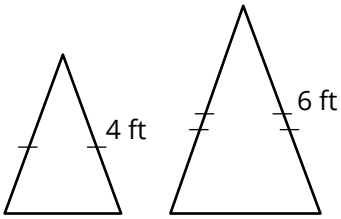
3. What information would you need to use the Side-Side-Side similarity theorem to prove that these triangles are similar?



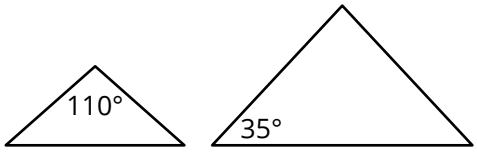
4. What information would you need to use the Side-Angle-Side similarity theorem to prove that the triangles are similar?



5. What information would you need to use the Side-Side-Side similarity theorem to prove that these triangles are similar?

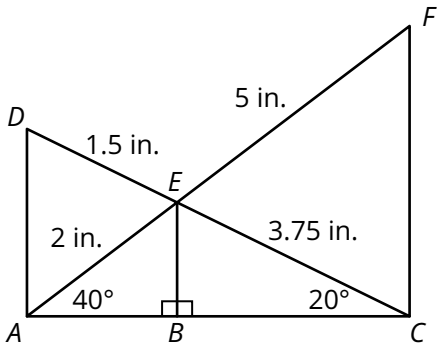


6. What information would you need to use the Angle-Angle similarity theorem to prove that the triangles are similar?



Extension

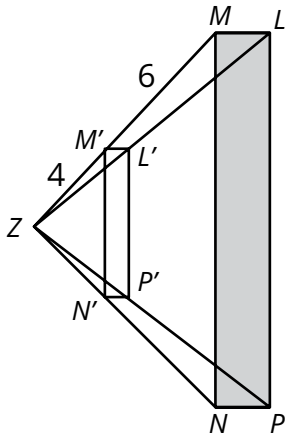
Determine whether there are any similar triangles in the figure. Explain your reasoning.



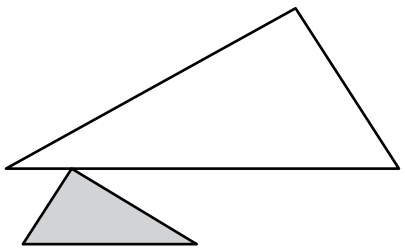
CC BY NC

Spaced Practice

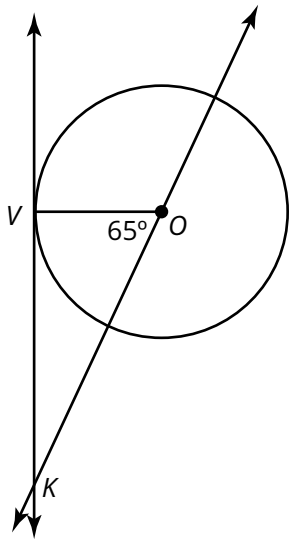
1. Given the pre-image and image, determine the scale factor.



2. Demonstrate that the two triangles are similar using a sequence of dilations and rigid motions.

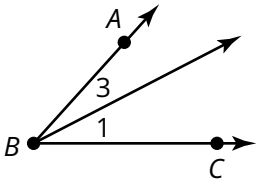


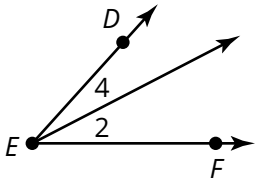
3. If $\overline{KV}$ is a tangent segment and $\overline{OV}$ is a radius, what is the measure of $\angle VKO$? Explain your reasoning.



4. Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$

   Prove: $m\angle ABC = m\angle DEF$





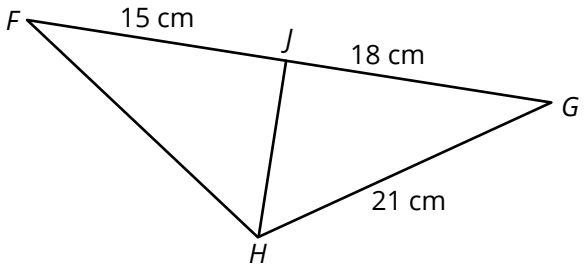
Statements	Reasons
1. $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$	
2. $m\angle 1 + m\angle 3 = m\angle ABC$ $m\angle 2 + m\angle 4 = m\angle DEF$	
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	
4. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	
5. $m\angle ABC = m\angle DEF$	

III. Theorems About Proportionality

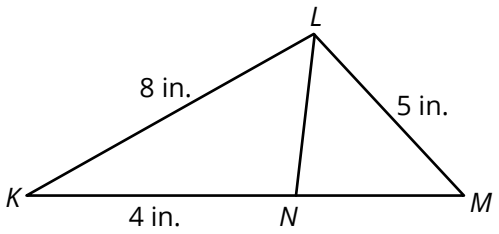
Topic Practice

A. Using the angle bisector/proportional side theorem, calculate the length of the indicated segment in each figure.

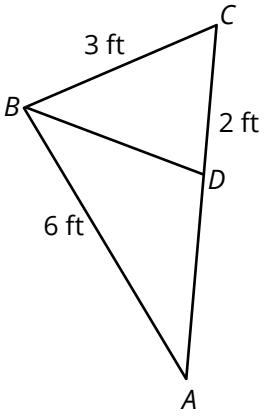
1. $\overline{HJ}$ bisects $\angle H$. Calculate HF.



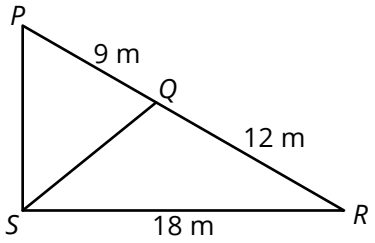
2. $\overline{LN}$ bisects $\angle L$. Calculate NM.



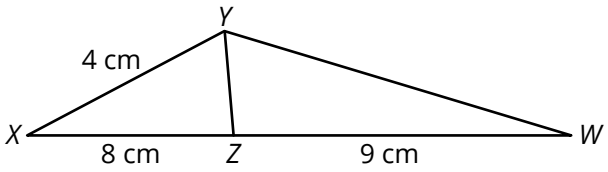
3. $\overline{BD}$ bisects $\angle B$. Calculate AD.



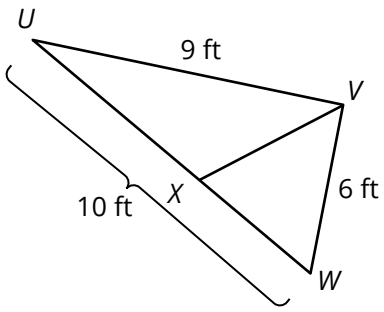
4. $\overline{SQ}$ bisects $\angle S$. Calculate SP.



5. $\overline{YZ}$ bisects $\angle Y$. Calculate YW.

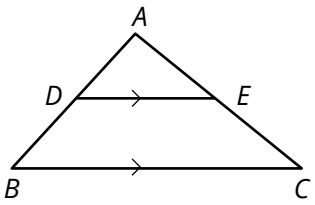


6. $\overline{VX}$ bisects $\angle V$. Calculate XW.



B. Use the diagram and given information to write a statement that can be justified using the proportional segments theorem, its converse, the Triangle Proportionality theorem, or its converse. State the theorem used.

1.



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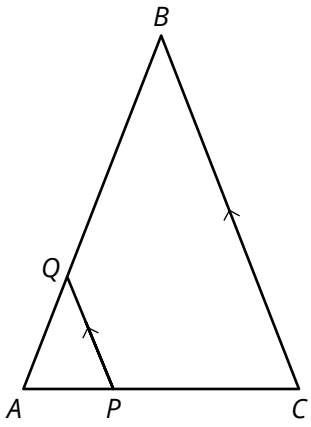
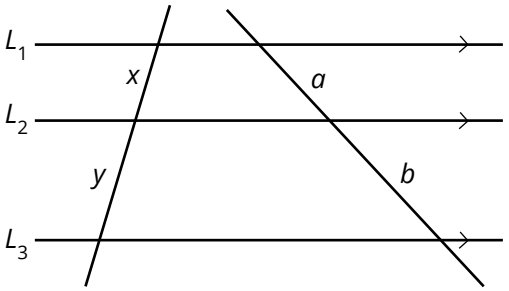
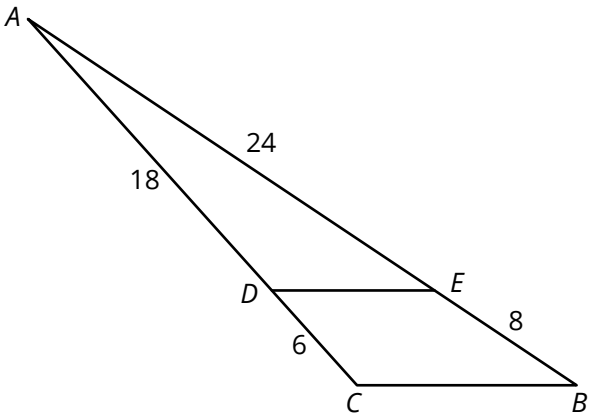


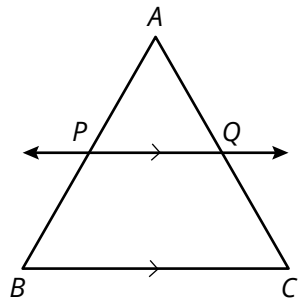
3.


L_1
L_2
L_3
x
y
a
b

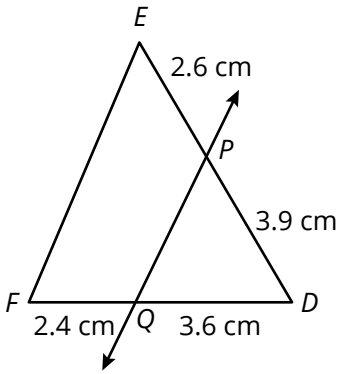
4.


A
24
18
D
E
8
6
C
B

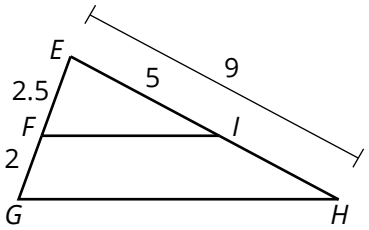
5.


A
P
Q
B
C

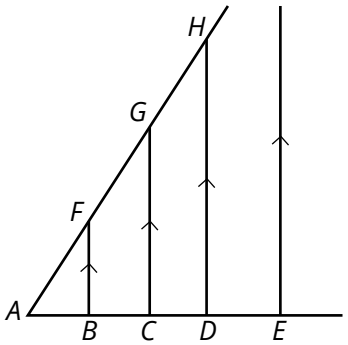
6.


E
2.6 cm
P
3.9 cm
F
D
2.4 cm
Q
3.6 cm

7.

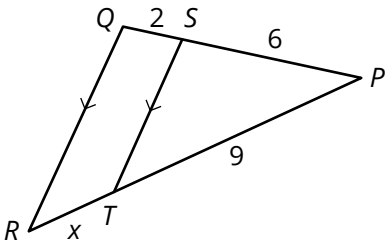

E
2.5
5
9
F
2
I
G
H

8.


H
G
F
A
B
C
D
E

C. Use the Triangle Proportionality theorem and the proportional segments theorem to determine the value of *x* in each diagram.

1.



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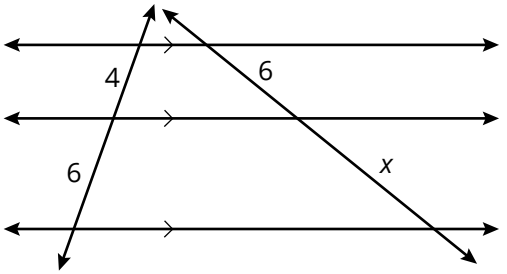

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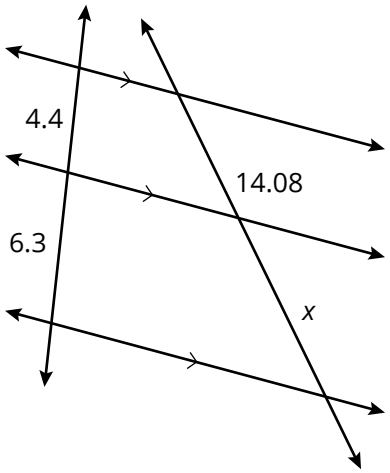

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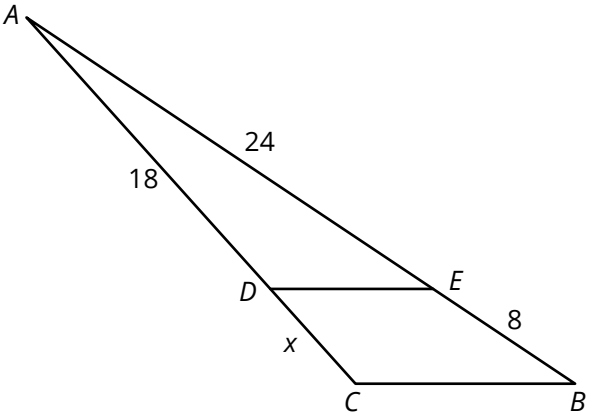
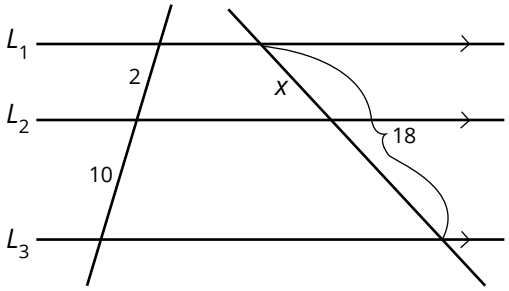
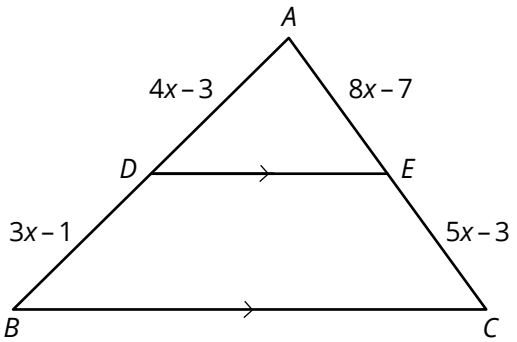


5.

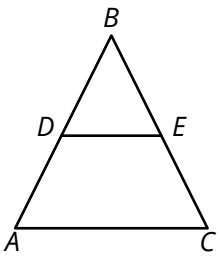


6.



D. Use the diagram and given information to write two statements that can be justified using the triangle midsegment theorem.

1.

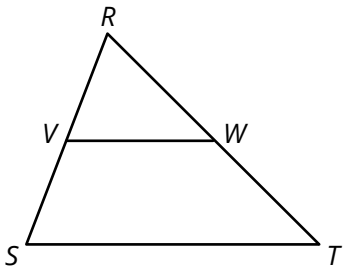


Given: ABC is a triangle.

D is the midpoint of $\overline{BA}$.

E is the midpoint of $\overline{BC}$.

2.

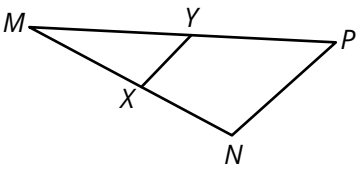


Given: RST is a triangle.

V is the midpoint of $\overline{RS}$.

W is the midpoint of $\overline{RT}$.

3.

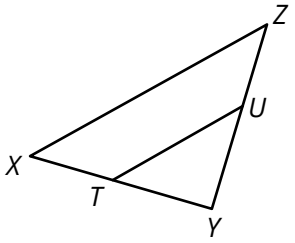


Given: MNP is a triangle.

X is the midpoint of $\overline{MN}$.

Y is the midpoint of $\overline{MP}$.

4.

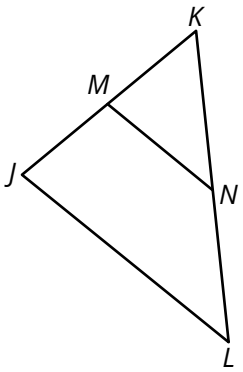


Given: XYZ is a triangle.

T is the midpoint of $\overline{YX}$.

U is the midpoint of $\overline{YZ}$.

5.

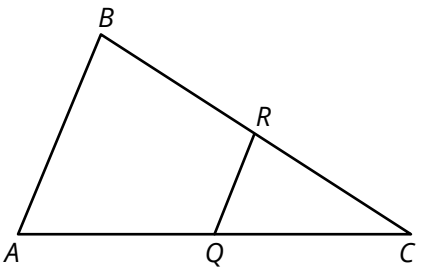


Given: *JKL* is a triangle.

M is the midpoint of $\overline{KJ}$.

N is the midpoint of $\overline{KL}$.

6.



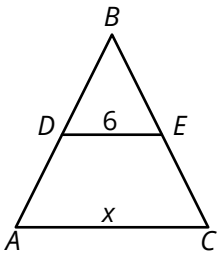
Given: *ABC* is a triangle.

Q is the midpoint of $\overline{AC}$.

R is the midpoint of $\overline{BC}$.

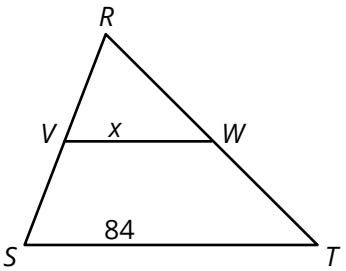
E. Use the diagram and the triangle midsegment theorem to determine the value of *x*.

1.



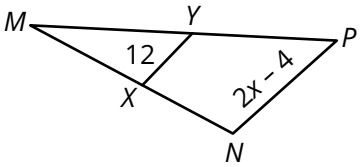
Given: $\overline{DE}$ is a midsegment of triangle *ABC*.

2.



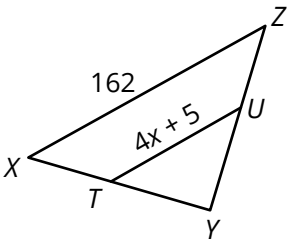
Given: $\overline{VW}$ is a midsegment of triangle *RST*.

3.



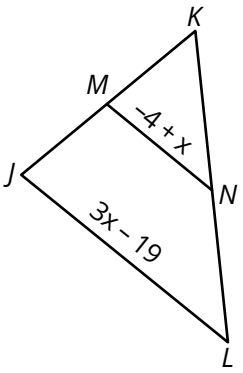
Given: $\overline{XY}$ is a midsegment of triangle *MNP*.

4.



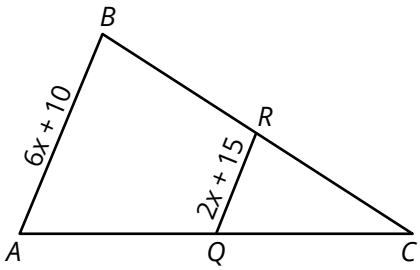
Given: $\overline{TU}$ is a midsegment of triangle *XYZ*.

5.



Given: $\overline{MN}$ is a midsegment of triangle *JKL*.

6.



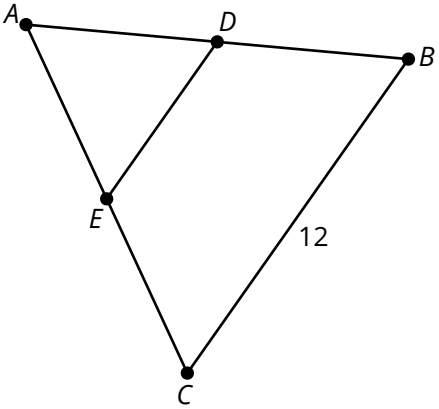
Given: $\overline{QR}$ is a midsegment of triangle *ABC*.

F. Use the triangle midsegment theorem and the trapezoid midsegment theorem to solve problems.

1. In triangle *ABC*, *D* is the midpoint of $\overline{AB}$ and *E* is the midpoint of $\overline{AC}$.

   Given: $\overline{DE} \parallel \overline{BC}$, $BC = 12$ *units*

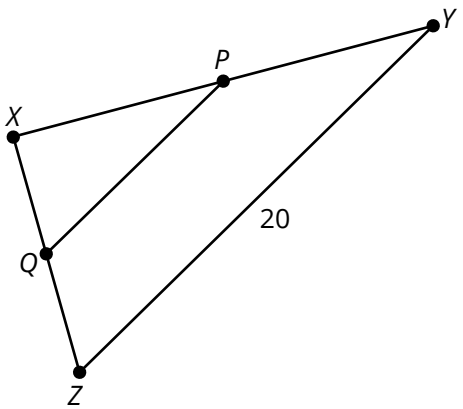
   Determine the length of segment *DE*.



2. In triangle *XYZ*, *P* is the midpoint of $\overline{XY}$ and *Q* is the midpoint of $\overline{XZ}$.

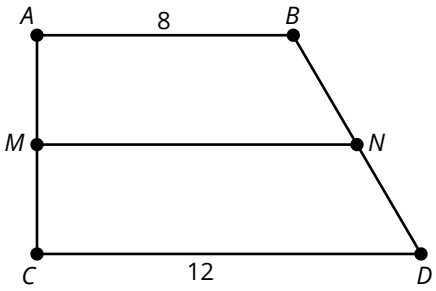
   Given: $YZ = 20$ *units*

   Determine the length of segment *PQ*.



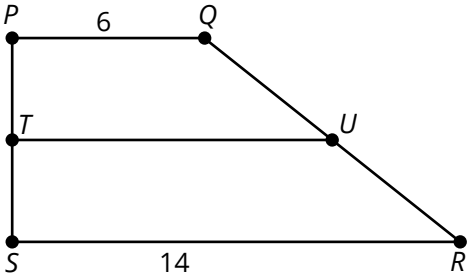
3. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $AB = 8$ *units*, and $CD = 12$ *units*.

   Determine the length of the midsegment $\overline{MN}$ of the trapezoid.



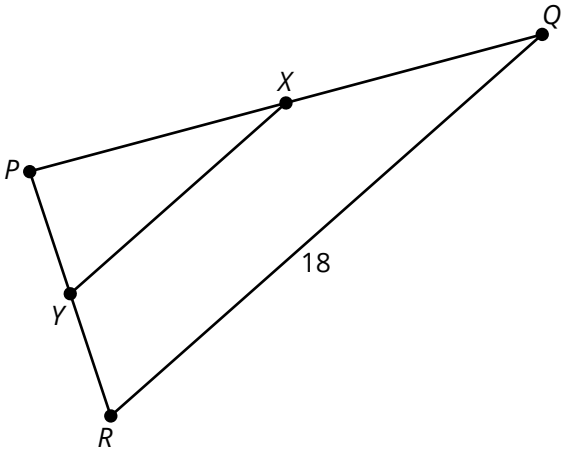
4. In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{SR}$, $PQ = 6$ *units*, and $SR = 14$ *units*.

   Determine the length of the midsegment $\overline{TU}$ of the trapezoid.



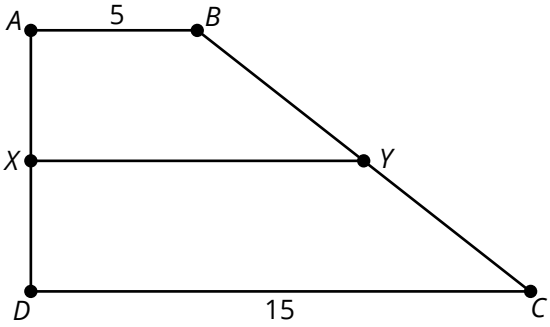
5. In triangle PQR, $\overline{XY}$ joins the midpoints of $\overline{PQ}$ and $\overline{PR}$, and the length of side QR is 18 units.

   Determine the length of $\overline{XY}$.

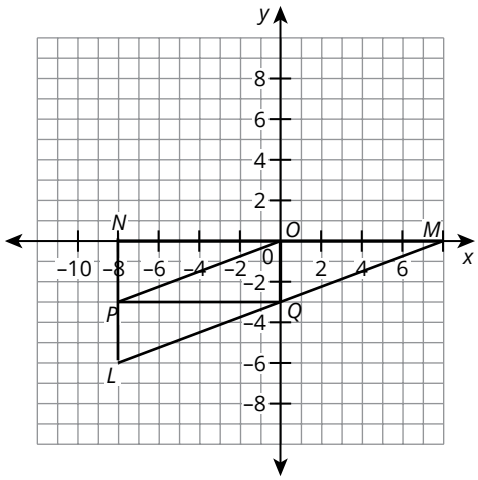


6. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $AB = 5$ *units* and $CD = 15$ *units*.

   Determine the length of the midsegment $\overline{XY}$ of the trapezoid.



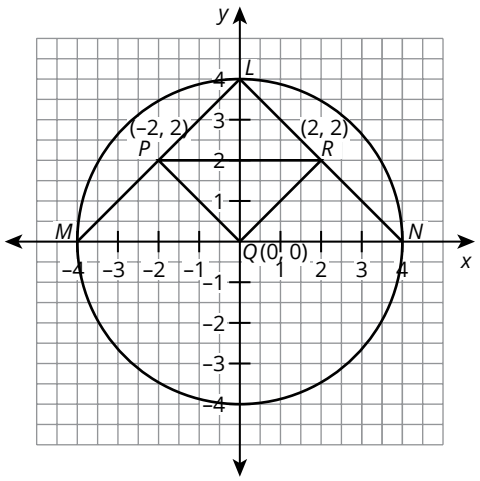
G. Given each diagram, compare the measures described. Simplify your answers, but do not evaluate any radicals.



1. The sides of triangle *LMN* have midpoints *O*(0, 0), *P*(−8, −3), and *Q*(0, −3). Compare the length of $\overline{OP}$ to the length of $\overline{LM}$.

2. Determine and compare the slopes of segment *OP* and segment *LM*.

3. Which theorem did you verify using the diagram?



4. The sides of triangle *LMN* have midpoints *P*(–2, 2,) *Q*(0, 0), and *R*(2, 2).

   Compare the length of $\overline{QP}$ to the length of $\overline{LN}$.

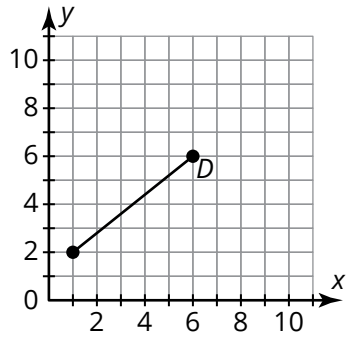
5. Determine and compare the slopes of segment *QP* and segment *LN*.

6. Which theorem did you verify using the diagram?

H. Given the coordinates for a triangle on a coordinate plane, verify whether segments are medians of the triangle, using the midpoint formula.

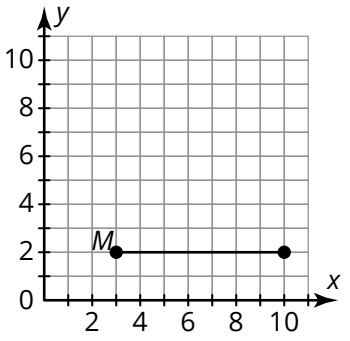
1. $A(1, 2)$ $B(5, 4)$ $C(7, 8)$

Determine whether $\overline{AD}$ is a median, where D is the midpoint of $\overline{BC}$.



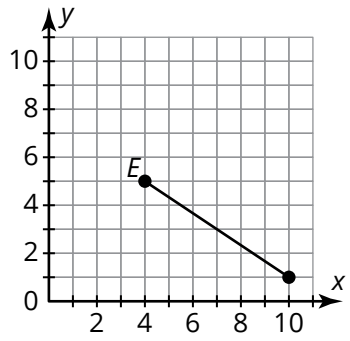
2. $A(0, 0)$ $B(6, 4)$ $C(10, 2)$

Determine whether $\overline{CM}$ is a median, where M is the midpoint of $\overline{AB}$.



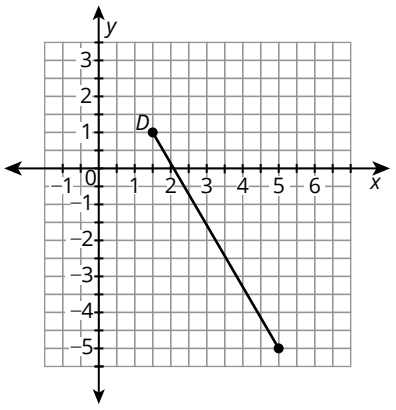
3. $A(2, 3)$ $B(6, 7)$ $C(10, 1)$

Determine whether $\overline{CE}$ is a median, where E is the midpoint of $\overline{AB}$.



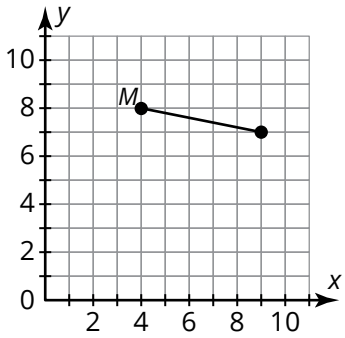
4. $A(-1, -1)$ $B(4, 3)$ $C(5, -5)$

Determine whether $\overline{CD}$ is a median, where D is the midpoint of $\overline{AB}$.



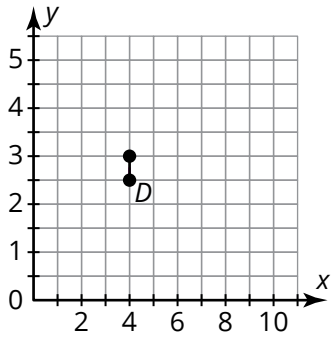
5. $A(3, 5)$ $B(9, 7)$ $C(5, 11)$

Determine whether $\overline{BM}$ is a median, where M is the midpoint of $\overline{AC}$.



6. $A(0, 0)$ $B(4, 3)$ $C(8, 5)$

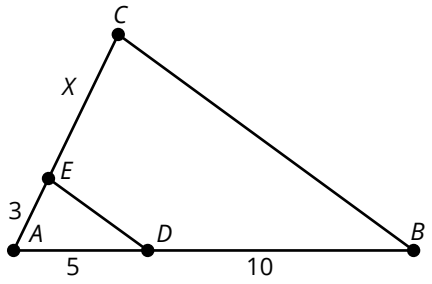
Determine whether $\overline{BD}$ is a median, where D is the midpoint of $\overline{AC}$.



I. Determine the value of x having to decide whether to use the Triangle Proportionality theorem or the triangle midsegment theorem.

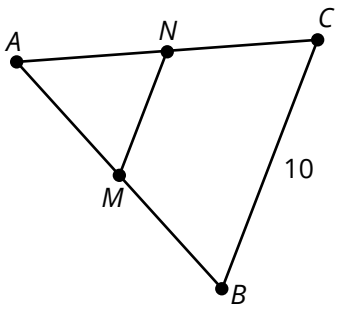
1. In triangle ABC, a point D lies on $\overline{AB}$, and a point E lies on $\overline{AC}$. Line segment DE is parallel to line segment BC and $AD = 5$, $DB = 10$, $AE = 3$, $EC = x$.

Determine the value of x.



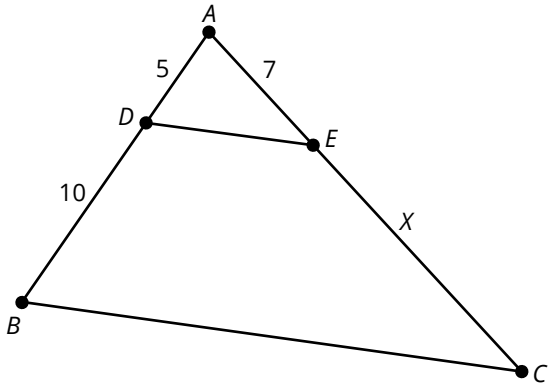
2. In triangle ABC, M is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{AC}$, $\overline{MN} \parallel \overline{BC}$, and the length of $\overline{BC}$ is 10.

   Determine the length of $\overline{MN}$.



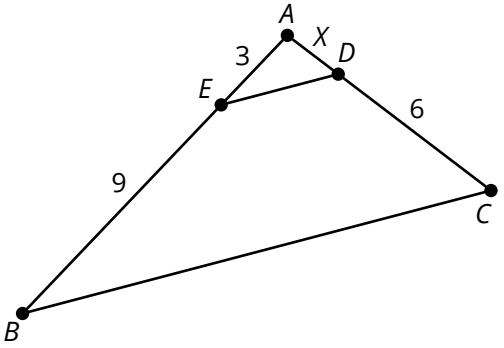
3. In triangle ABC, $\overline{DE}$ is drawn parallel to $\overline{BC}$. The lengths of the segments are $AD = 5$, $DB = 10$, $AE = 7$, and $EC = x$.

   Determine the value of x.



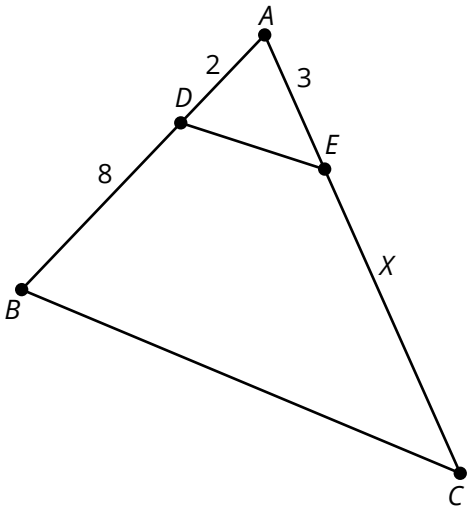
4. In $\triangle ABC$, D is a point on $\overline{AC}$, and E is a point on $\overline{AB}$. The line segment DE divides $\overline{AB}$ into two segments of lengths 3 and 9, and $\overline{AC}$ into two segments of lengths x and 6. The segment $\overline{DE} \parallel \overline{BC}$.

   Determine the value for x.

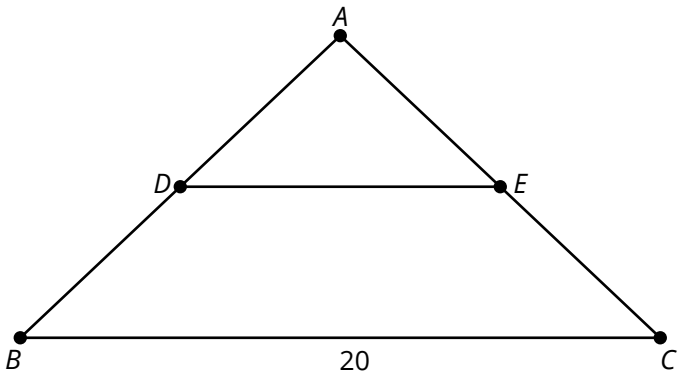


5. In $\triangle ABC$, $\overline{DE}$ intersects $\overline{AB}$ at D and $\overline{AC}$ at E. $\overline{DE} \parallel \overline{BC}$, and the lengths are given as: $AD = 2$, $DB = 8$, $AE = 3$, $EC = x$

   Determine the value of x.



6. In $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$. $\overline{DE} \parallel \overline{BC}$, and the length of $\overline{BC}$ is 20. Determine the length of $\overline{DE}$.



Extension

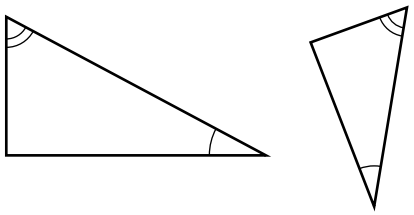
In $\triangle ABC$, D is the midpoint of $\overline{BC}$, and M is the centroid of $\triangle ABC$. The length of the median $\overline{AD}$ is 18 units, and $AM = x$ *units*. The centroid divides the median $\overline{AD}$ into two segments such that $AM = 2(MD)$.

Determine the value of x.

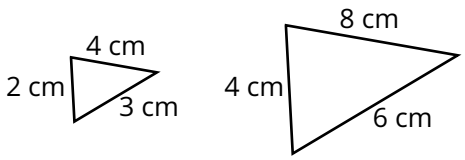
Spaced Practice

1. Determine whether the triangles in each pair are similar. Explain your reasoning.

   a.



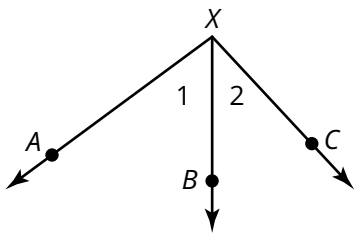
   b.



2. Gio is designing a ring for a friend. His friend wants to know the shape of the stone, and Gio tells her that it is a quadrilateral and the diagonals of the shape bisect at right angles. Is this enough information for Gio's friend to determine the shape of the stone? Explain your reasoning.

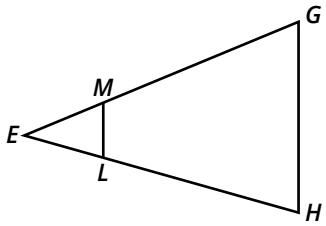
3. Given: $\angle 1$ and $\angle 2$ are complementary.

   Prove: $\overrightarrow{XA} \perp \overrightarrow{XC}$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary. 2. $m\angle 1 + m\angle 2 = 90°$ 3. $m\angle AXC = m\angle 1 + m\angle 2$ 4. $m\angle AXC = 90°$ 5. $\angle AXC$ is a right angle. 6. $\overrightarrow{XA} \perp \overrightarrow{XC}$	

4. Use the diagram and given statement to prove the Triangle Proportionality theorem.



Given: $\overline{ML} \parallel \overline{GH}$

Prove: $\frac{GM}{ME} = \frac{HL}{LE}$

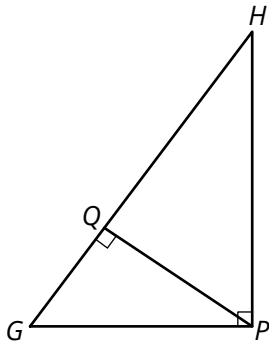
Statements	Reasons
1. $\overline{ML} \parallel \overline{GH}$ 2. $\angle EML \cong \angle G$ 3. $\angle ELM \cong \angle H$ 4. $\triangle EML \sim \triangle EGH$ 5. $\frac{GE}{ME} = \frac{HE}{LE}$ 6. $GE = GM + ME$ *and* $HE = HL + LE$ 7. $\frac{GM + ME}{ME} = \frac{HL + LE}{LE}$ 8. $\frac{GE}{ME} = \frac{HE}{LE}$	

IV. More Similar Triangles

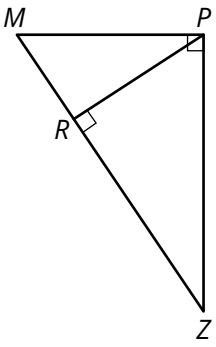
Topic Practice

A. Use the right triangle/altitude similarity theorem to write three similarity statements involving the triangles in each diagram.

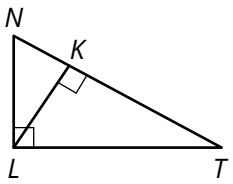
1.



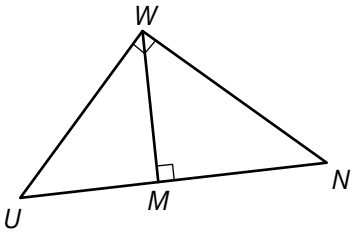
2.



3.



4.



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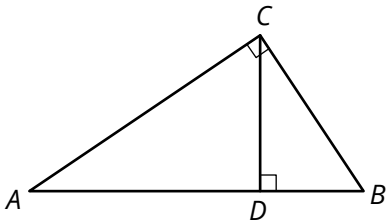

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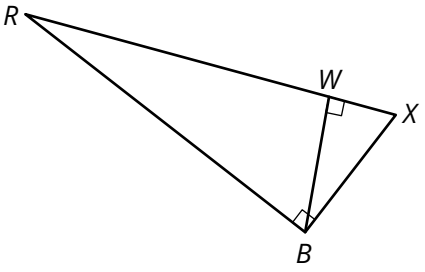
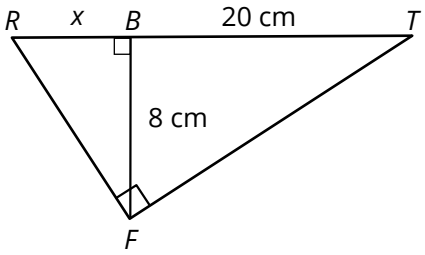
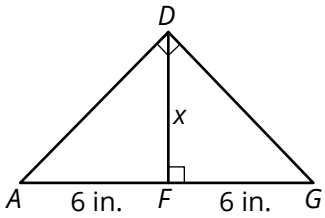
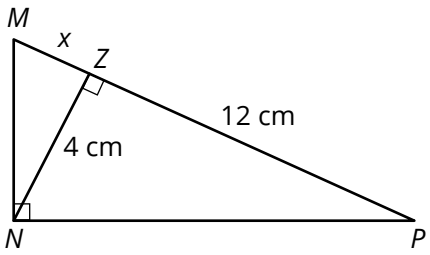
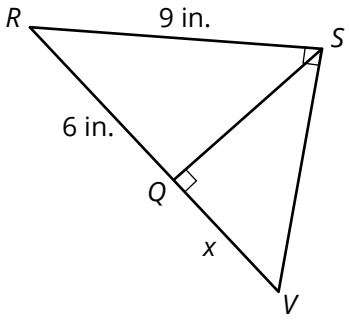


B. Determine the missing value(s). When necessary, leave answers in simplest radical form. For Questions 1-6, write which theorem you used to solve.

1. 

2. 

3. 

4. 

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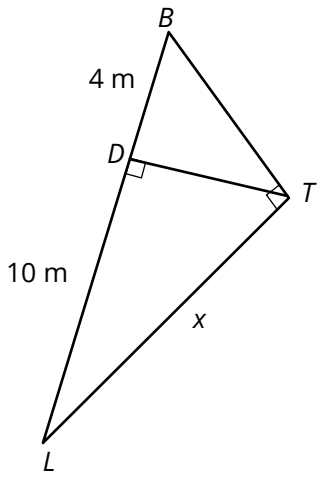

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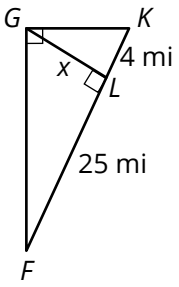

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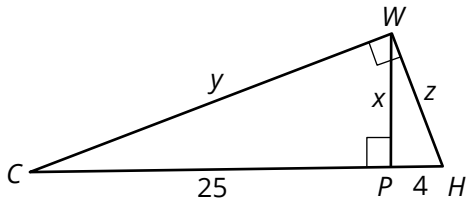

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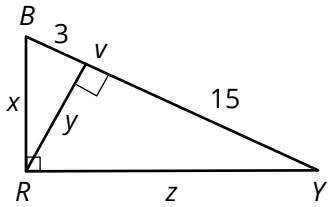

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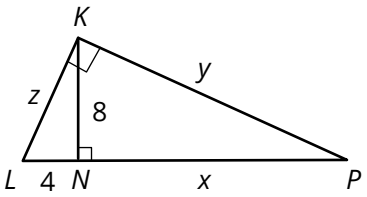

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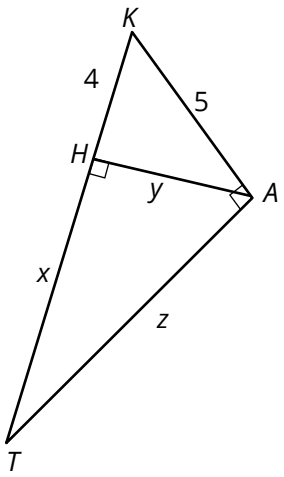
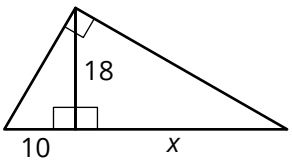
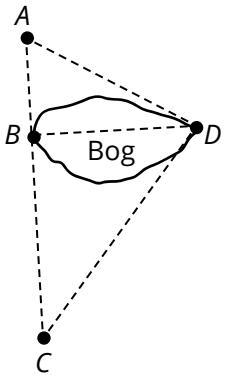


C. Use the given information to answer each question.

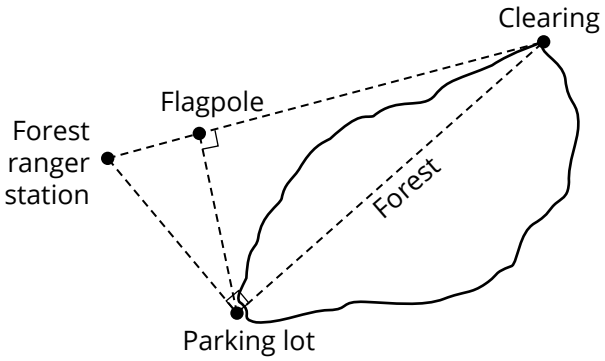
1. What is the value of x?



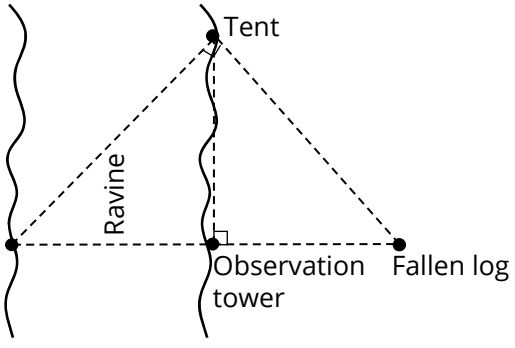
2. You are standing at point *D* in the diagram looking across a bog at point *B*. Point *B* is 84 yards from point *A* and 189 yards from point *C*. How wide across is the bog?



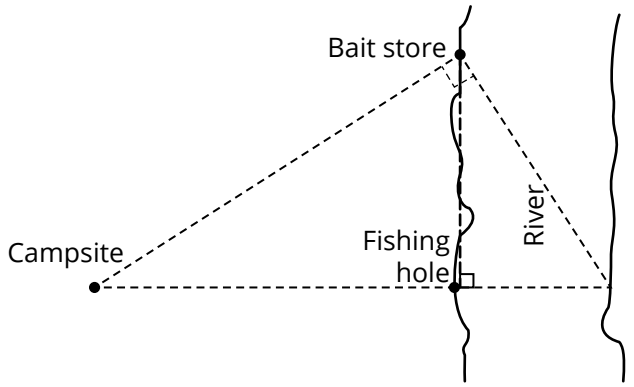
3. Linh wants to walk from the parking lot through the forest to the clearing, as shown in the diagram. She knows that the forest ranger station is 308 feet from the flagpole and the flagpole is 700 feet from the clearing. How far is the parking lot from the clearing?



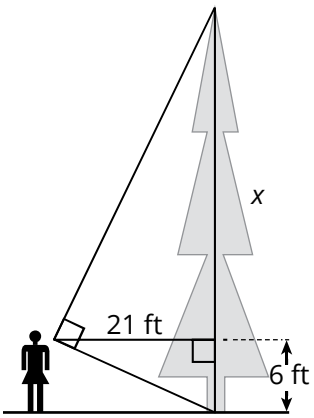
4. James is camping with his uncle at one edge of a ravine. The diagram shows the location of their tent. The tent is 1.2 miles from the fallen log and the fallen log is 0.75 miles from the observation tower. How wide is the ravine?



5. You are on a fishing trip with your friends. The diagram shows the location of the river, fishing hole, campsite, and bait store. The campsite is located 400 feet from the fishing hole. The bait store is located 220 feet from the fishing hole. How wide is the river?



6. You are standing 21 feet from a tree. Your line of sight to the top of the tree and to the bottom of the tree forms a 90-degree angle as shown in the diagram. The distance between your line of sight and the ground is 6 feet. Estimate the height of the tree.

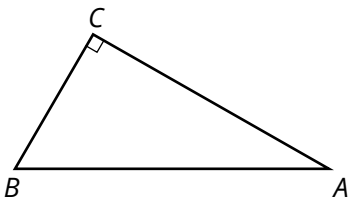


D. Use your knowledge of the Pythagorean Theorem to write the Given and Prove statements or complete each proof.

1. Write the Given and Prove statements that should be used to prove the Pythagorean Theorem.

   Given:

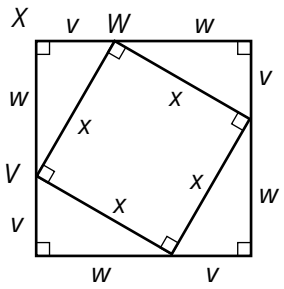
   Prove:



2. Write the Given and Prove statements that should be used to prove the Pythagorean Theorem.

   Given:

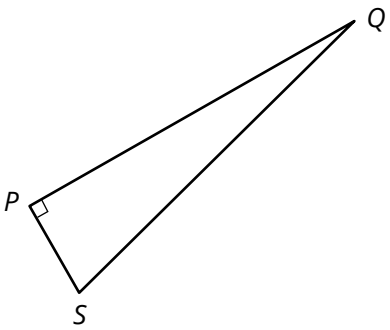
   Prove:



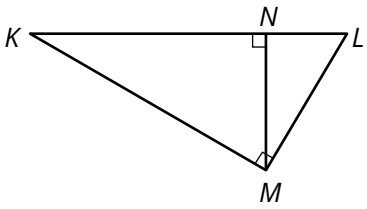
3. Write the Given and Prove statements that should be used to prove the Pythagorean Theorem.

   Given:

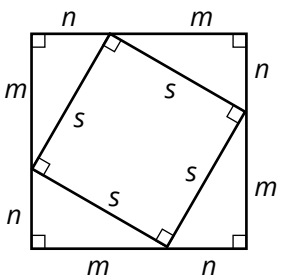
   Prove:



4. Prove the Pythagorean Theorem using similar triangles.



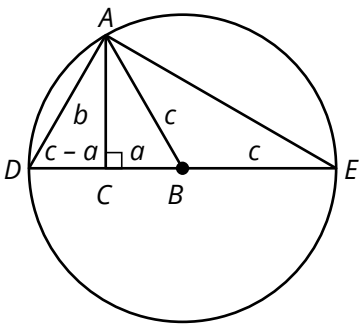
5. Prove the Pythagorean Theorem using algebraic reasoning.



6. Prove the Pythagorean Theorem using similar triangles.

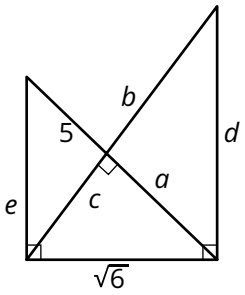
   Given: ABC with right angle C

   Place side a along the diameter of a circle of radius c so that B is at the center of the circle.

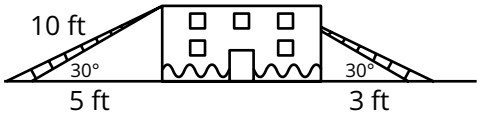


Extension

1. Calculate the value of each variable.



2. Two ladders are leaning against a building as shown.

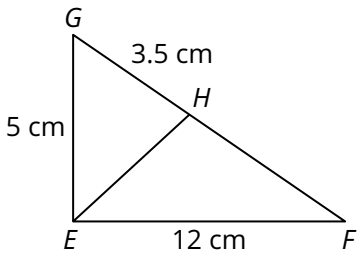


   a. Each ladder forms a triangle with the ground and the wall. Are the two triangles similar? Explain your reasoning.

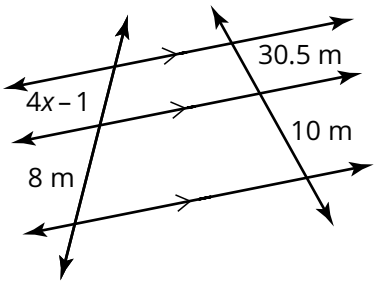
   b. Determine the length of the smaller ladder. Explain your reasoning.

Spaced Practice

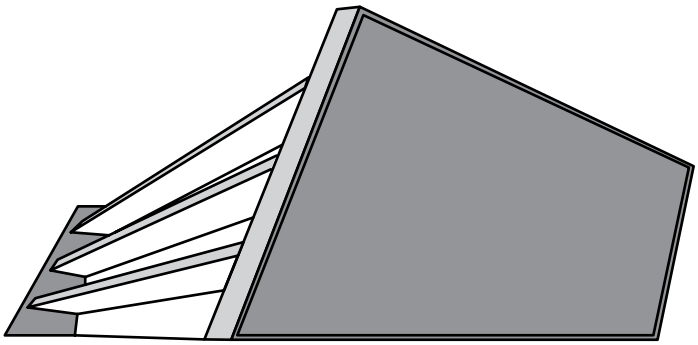
1. $\overline{EH}$ bisects $\angle E$. Calculate HF.



2. Use the proportional segments theorem to determine the value of x.



3. Consider the building shown. The side of the building is a quadrilateral. Classify the quadrilateral based only on each piece of given information.



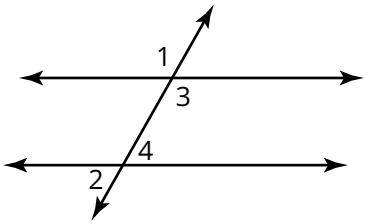
   a. The four angles of the quadrilateral are congruent.

   b. The diagonals of the quadrilateral are congruent.

4. Provide the statements to complete the proof.

   Given: $\angle 1$ and $\angle 2$ are supplementary.

   Prove: $\angle 3$ and $\angle 4$ are supplementary.



Statements	Reasons
	1. Given 2. Definition of supplementary angles 3. Vertical angles are congruent 4. Definition of congruent angles 5. Substitution property of equality 6. Definition of supplementary angles

CC BY NC

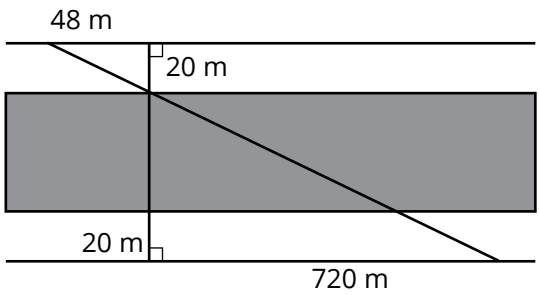
V. Application of Similar Triangles

Topic Practice

A. Use indirect measurement to calculate the unknown distance.

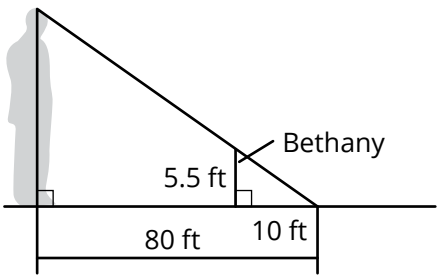
1. Lauren and Nakota are on opposite sides of a canyon that runs east to west, according to the graphic. They want to know how wide the canyon is. Each person stands 20 meters from the edge. Then, Lauren walks 48 meters west, and Nakota walks 360 meters east.

   What is the width of the canyon?



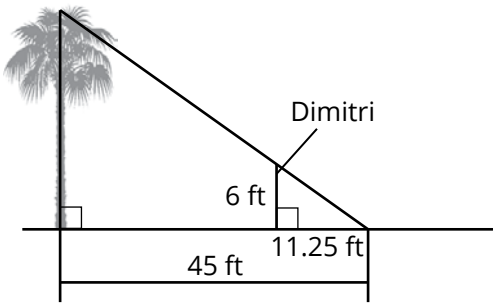
2. Alejandra wanted to measure the height of a statue. She lined herself up with the statue's shadow so that the tip of her shadow met the tip of the statue's shadow. She marked the spot where she was standing. Then, she measured the distance from where she was standing to the tip of the shadow, and from the statue to the tip of the shadow.

   What is the height of the statue?



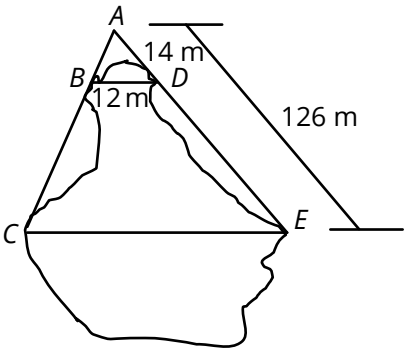
3. Joey wants to measure the height of a palm tree. He lines himself up with the palm tree's shadow so that the tip of his shadow meets the tip of the palm tree's shadow. Then, he asks a friend to measure the distance from where he was standing to the tip of his shadow and the distance from the palm tree to the tip of its shadow.

   What is the height of the palm tree?



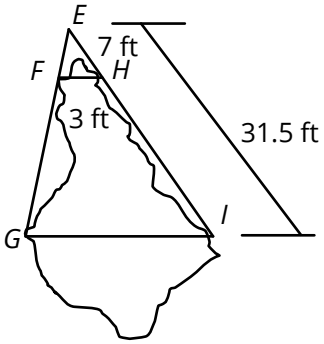
4. NIcky is making a map of a state park. He finds a small bog, and he wants to measure the distance across the widest part. He first marks the points *A*, *C*, and *E*. Andre measures the distances shown on the image. Andre also marks point *B* along $\overline{AC}$ and point *D* along $\overline{AE}$, such that $\overline{BD}$ is parallel to $\overline{CE}$.

   What is the width of the bog at the widest point?

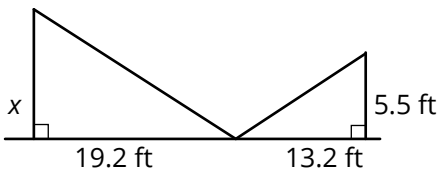


5. Hailey finds a tidal pool while walking on the beach. She wants to know the maximum width of the tidal pool. Using indirect measurement, she begins by marking the points *E*, G, and *I*. Hailey measures the distances shown on the image. Next, Hailey marks point *F* along $\overline{EG}$ and point *H* along $\overline{EI}$, such that $\overline{FH}$ is parallel to $\overline{GI}$.

   What is the distance across the tidal pool at its widest point?

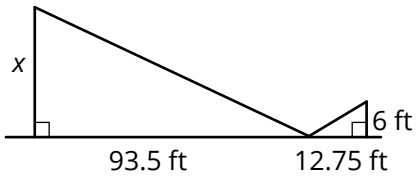


6. Brianna is visiting a museum. She wants to know the height of one of the sculptures. She places a small mirror on the ground between herself and the sculpture, then she backs up until she can see the top of the sculpture in the mirror.



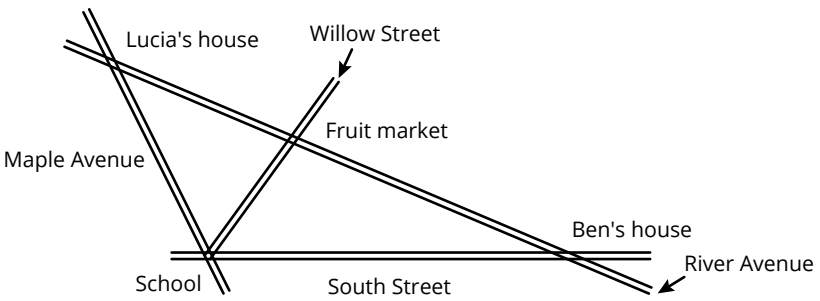
What is the height of the sculpture?

7. Miguel wants to know the height of his school. He places a small mirror on the ground between himself and the school, then he backs up until he can see the highest point of the school in the mirror.



What is the height of Miguel's school?

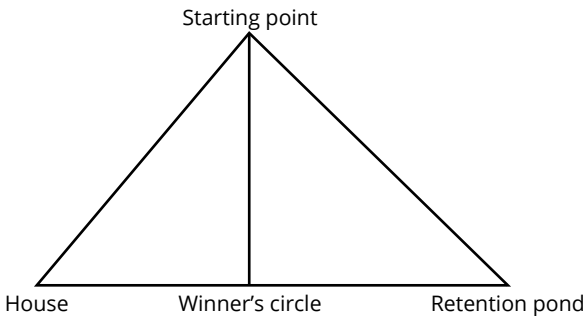
8. On the map shown, Willow Street bisects the angle formed by Maple Avenue and South Street. Lucia's house is 5 kilometers from the school and 4 kilometers from the fruit market. Ben's house is 6 kilometers from the fruit market.



How far is Ben's house from the school?

9. Xavier is hitting a golf ball towards the hole. The line from Xavier to the hole bisects the angle formed by the lines from Xavier to the oak tree and from Xavier to the sand trap. The oak tree is 200 yards from Xavier, the sand trap is 320 yards from Xavier, and the hole is 250 yards from the sand trap. How far is the hole from the oak tree?

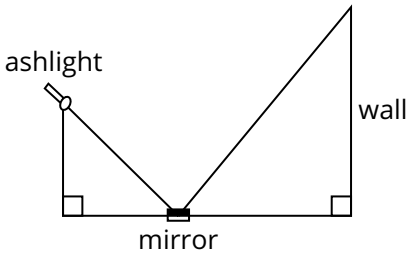
10. Antonio is racing a remote control car from the starting point to the winner's circle. That path bisects the angle formed by the lines from the starting point to the house and from the starting point to the retention pond. The house and the retention pond are each 500 meters from the starting point. The house is 720 meters from the retention pond.



How far is the winner's circle from the retention pond?

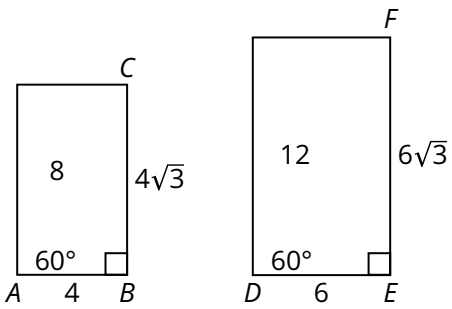
Extension

1. Brecken places a mirror on the ground $7\frac{1}{5}$ ft from a wall. She stands $4\frac{1}{10}$ ft away from the mirror. She is going to shine the flashlight on the mirror so that it reflects onto the wall. Determine the height at which Brecken should hold the flashlight if she wants it to shine on the wall $3\frac{2}{7}$ ft higher than the height of the flashlight.



2. Triangles *ABC* and *DEF* are similar.

   a. Compare the ratios of the length $\overline{AB}$ to the length of $\overline{AC}$ and the length of $\overline{DE}$ to the length of $\overline{DF}$.



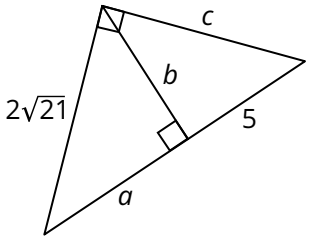
   b. Compare the ratios of the length of $\overline{BC}$ to the length of $\overline{AC}$ and the length of $\overline{EF}$ to the length of $\overline{DF}$.

   c. Comment on the similarities and differences between the ratios in parts (a) and (b).

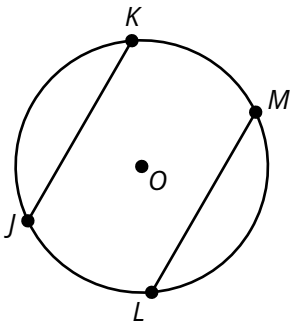
Spaced Practice

1. The geometric mean of two numbers is $4\sqrt{7}$. One of the numbers is 14. What is the other number?

2. Solve for a, b, and c.



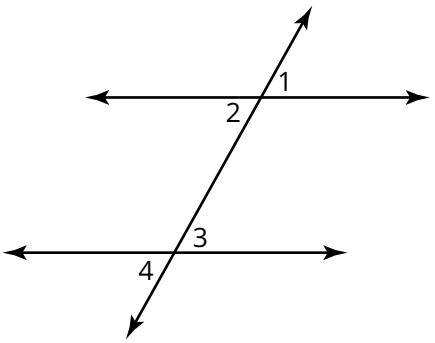
3. Consider the circle with its center at point O. Determine the information that is needed to show that $\overline{KJ}$ and $\overline{ML}$ are equidistant from point O.



4. Provide reasons to complete the proof.

   Given: $\angle 2 \cong \angle 3$

   Prove: $\angle 1 \cong \angle 4$



Statements	Reasons
1. $\angle 2 \cong \angle 3$	
2. $\angle 1 \cong \angle 2$	
3. $\angle 3 \cong \angle 4$	
4. $\angle 1 \cong \angle 4$	

VI. Partitioning Segments in Given Ratios

Topic Practice

A. Determine the midpoint of a line segment with each set of given endpoints.

1. (8, 0) and (4, 6)

2. (3, 8) and (9, 10)

3. (−7, 2.5) and (3, 6)

4. (6, −3) and (−4, 5)

5. (−10, −1) and (0, 4)

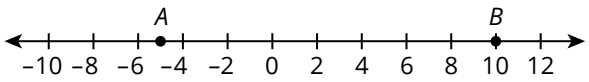
6. $\left(-2, \frac{7}{2}\right)$ and (−8, −9)

B. Determine the coordinates of the point that divides each directed segment *AB* into the given ratio.

1. Point $A = -5$

   Point $B = 10$

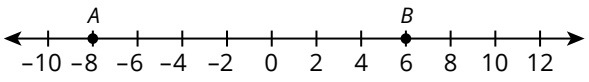
   Ratio: 1 : 4



2. Point $A = -8$

   Point $B = 6$

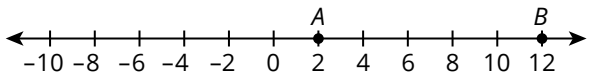
   Ratio: 2 : 5



3. Point $A = -2$

   Point $B = 12$

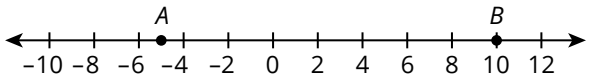
   Ratio: 3 : 2



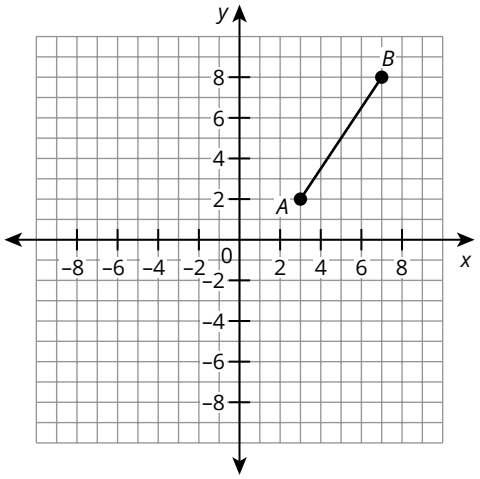
4. Point $A = -5$

   Point $B = 10$

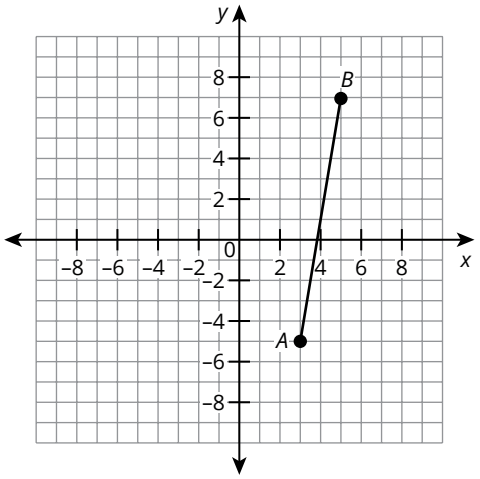
   Ratio: 4 : 1



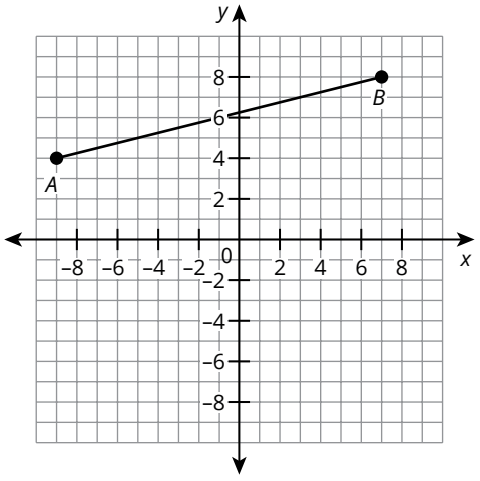
5. 1 : 3



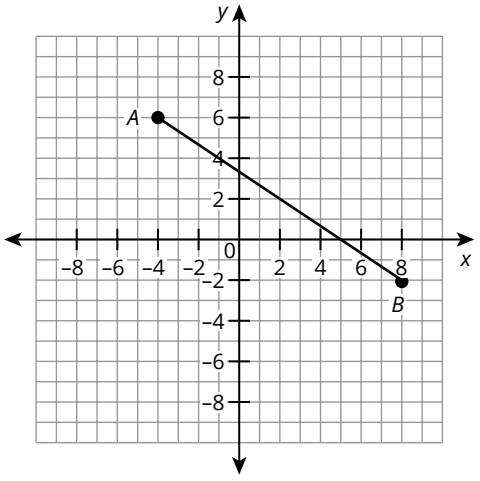
6. 1 : 1



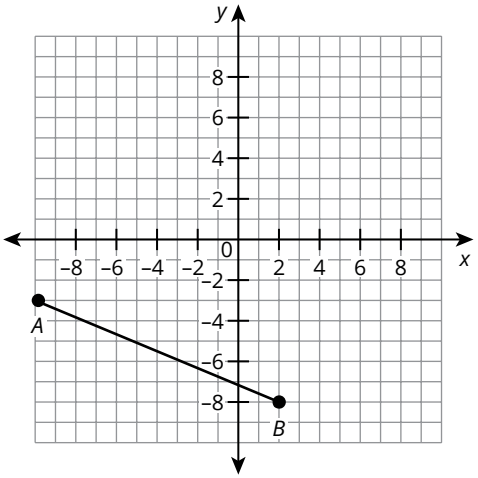
7. 3 : 1



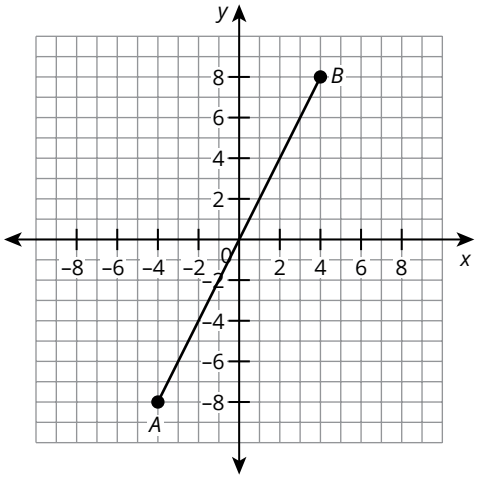
8. 1 : 2



9. 2 : 1

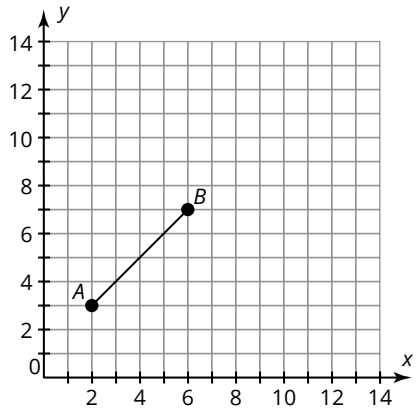


10. 1 : 1

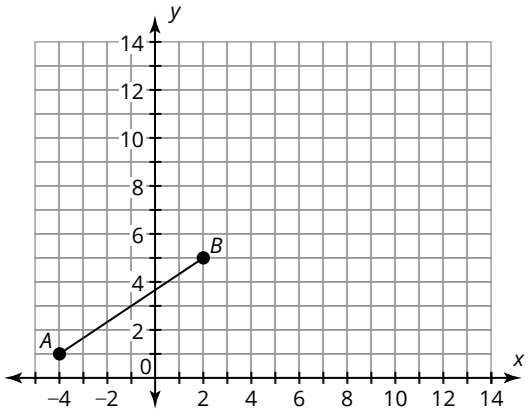


C. Partition Segments both on and off the coordinate plane.

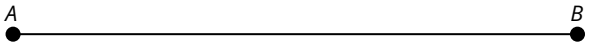
1. Given the segment with endpoints $A(2, 3)$ and $B(6, 7)$ partition the segment into two parts such that the ratio of the lengths of the two parts is 1 : 2. Determine the coordinate of point P that divides the segment.



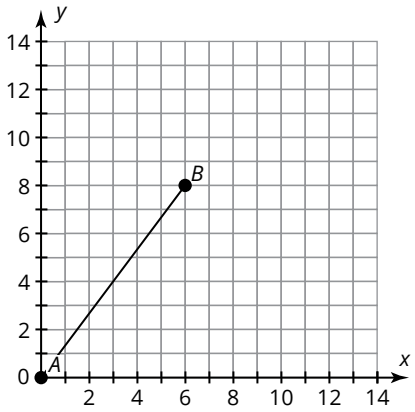
2. The points $A(-4, 1)$ and $B(2, 5)$ are the endpoints of a segment. Determine the coordinate of point P that divides the segment in a 3 : 1 ratio, where P is closer to A.



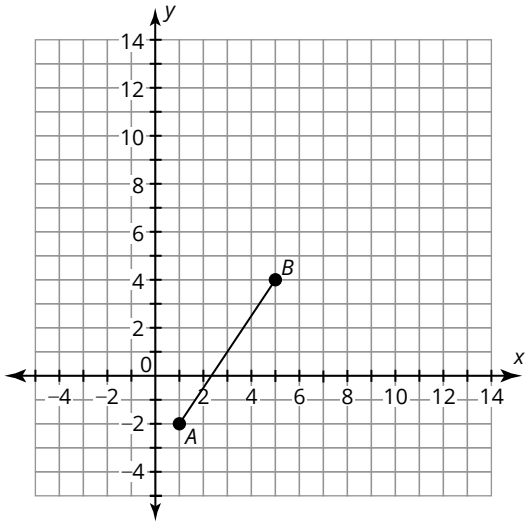
3. Consider the line segment from A to B with a length of 10 units. If P divides the segment in a 2 : 3 ratio, determine the lengths of the two parts of the segment.



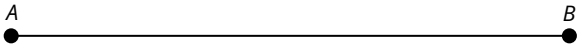
4. The endpoints of a segment are $A(0, 0)$ and $B(6, 8)$. Determine the point P that divides the segment into two equal parts.



5. The points $A(1, -2)$ and $B(5, 4)$ are the endpoints of a segment. Determine the point P that divides the segment in a 4 : 5 ratio, with P closer to A.



6. A segment has a length of 12 units. If a point P divides the segment into two parts such that the ratio is 1 : 4, determine the lengths of the two parts of the segment.



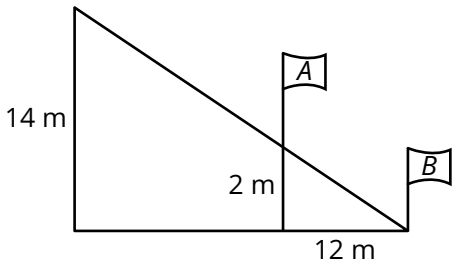
Extension

1. A point that divides a line segment into a 3 : 1 ratio is located at $\left(5\frac{3}{4}, 8\frac{1}{2}\right)$. The endpoints of the line segment have x-coordinates of -1 and 8 and their y-coordinates are different. Identify possible y-coordinates for the line segment's endpoints.

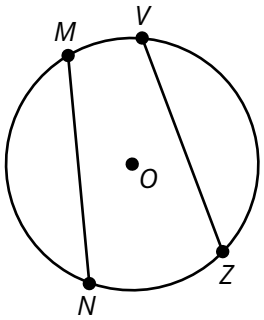
Spaced Practice

1. A wall with a height of 75 feet casts a shadow that is 120 feet long. A nearby pole casts a shadow that is 600 feet long. Draw a diagram to represent the situation. Then, determine the height of the pole.

2. Two friends are taking photos together to submit to an inline skating magazine. Brian is standing on top of a ramp that is 14 meters high. There are two flags on the ramp, one labeled A and one labeled B. Brian's friend, Neville, is going to try and get pictures of Brian when he is passing flag A and when he is passing flag B. Flag A is located 2 meters above the ground. The distance from the point on the ground under flag A and flag B is 12 meters. Determine the horizontal distance across the bottom of the ramp.



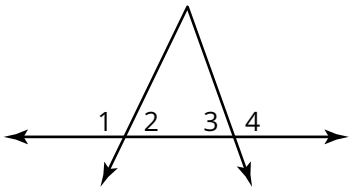
3. Consider the circle with its center at point O. Determine the information that is needed to show that $\overline{MN} \cong \overline{VZ}$.



4. Provide the reasons to complete the proof.

Given: $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 4$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
4. $\angle 2 \cong \angle 3$	
5. $m\angle 2 = m\angle 3$	
6. $m\angle 1 = m\angle 4$	
7. $\angle 1 \cong \angle 4$	

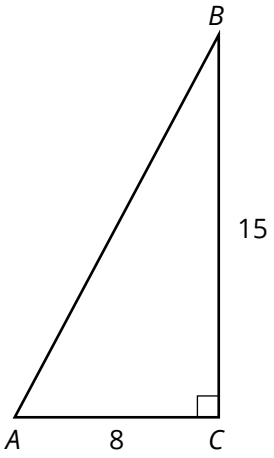
Name _____ Date _____

I. Introduction to Trigonometry

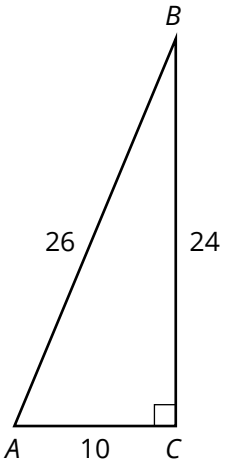
Topic Practice

A. Determine the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ using the indicated reference angle in each triangle. Write your answers as fractions in simplest form.

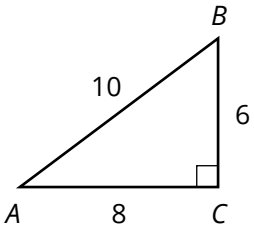
1. Reference angle: $\angle A$



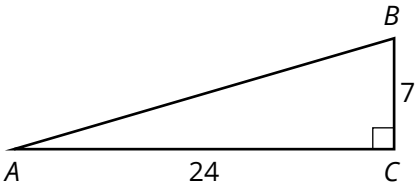
2. Reference angle: $\angle B$



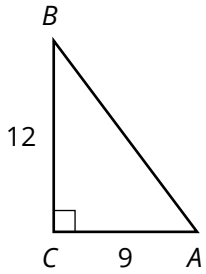
3. Reference angle: $\angle B$



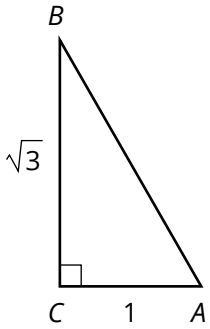
4. Reference angle: $\angle A$



5. Reference angle: $\angle A$

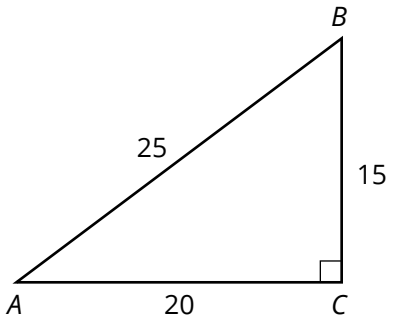


6. Reference angle: $\angle A$

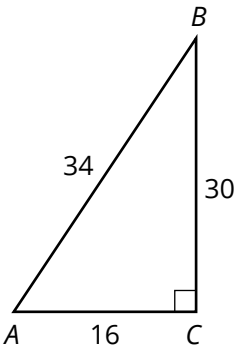


B. Determine the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ using the indicated reference angle in each triangle. Write your answers as fractions in simplest form.

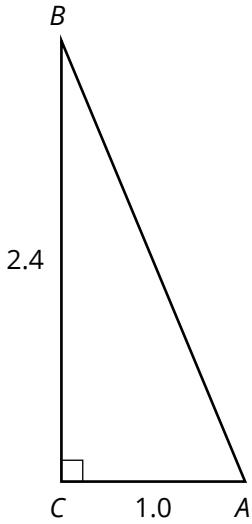
1. Reference angle: $\angle B$



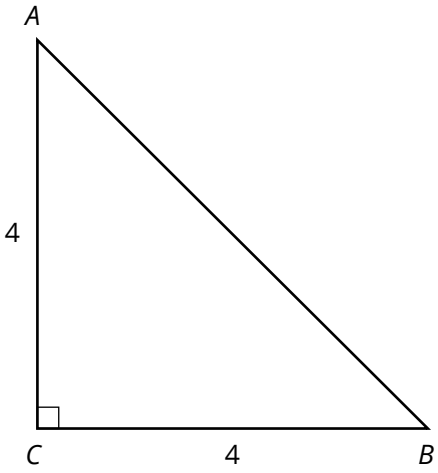
2. Reference angle: $\angle A$



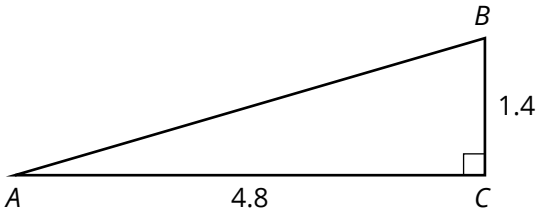
3. Reference angle: $\angle A$



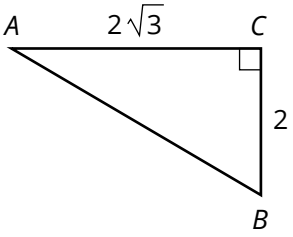
4. Reference angle: $\angle B$



5. Reference angle: $\angle A$

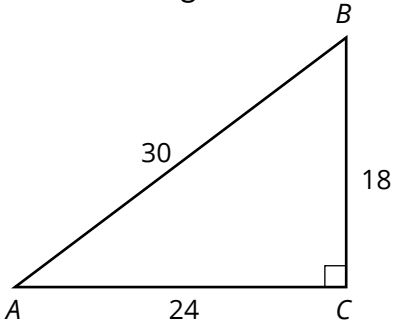


6. Reference angle: $\angle A$



C. Determine the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ using the indicated reference angle in each triangle. Write your answers as fractions in simplest form.

1. Reference angle: $\angle A$



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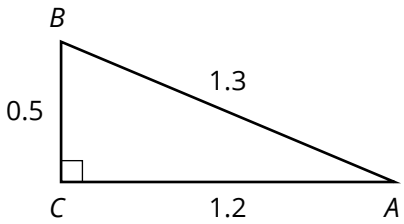
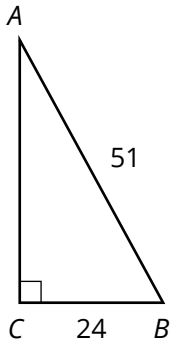
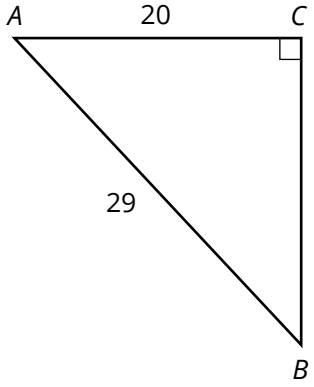
2. Reference angle: $\angle A$



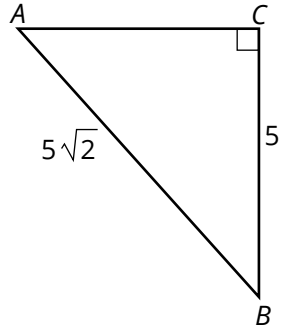
3. Reference angle: $\angle B$



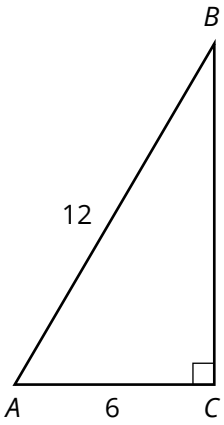
4. Reference angle: $\angle B$



5. Reference angle: $\angle A$

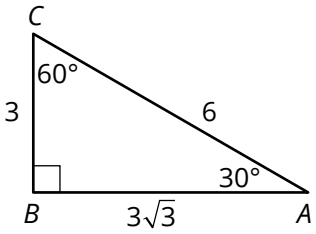


6. Reference angle: $\angle A$

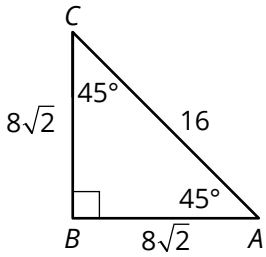


D. Calculate the indicated ratio.

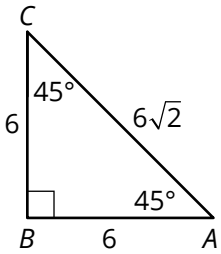
1. $\frac{\text{opposite}}{\text{hypotenuse}}$ for reference angle A



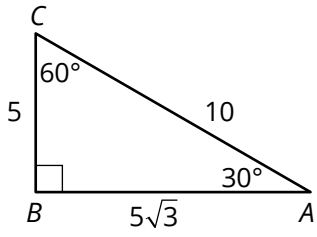
2. $\frac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle C



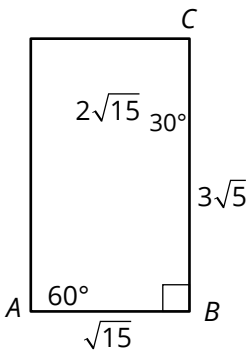
3. $\frac{\text{opposite}}{\text{hypotenuse}}$ for reference angle C



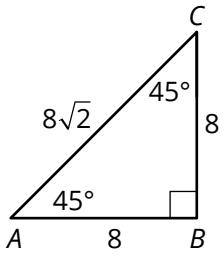
4. $\frac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle A



5. $\frac{\text{opposite}}{\text{adjacent}}$ for reference angle A

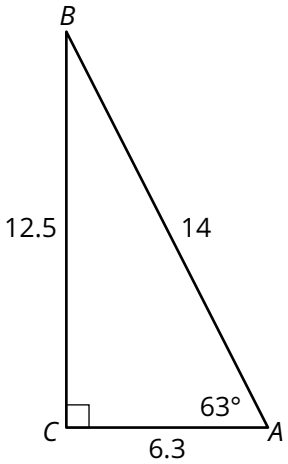


6. $\frac{\text{opposite}}{\text{adjacent}}$ for reference angle C



Extension

1. Consider $\triangle ABC$ shown in the figure.



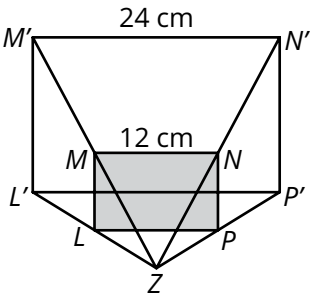
   a. Determine the side length ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ using $\angle A$ as the reference angle. Write each ratio as a decimal rounded to the hundredths.

   b. Trigonometric functions of angles include three important functions called the sine function, the cosine function, and the tangent function. These values can be determined with a graphing calculator. Use a graphing calculator to determine the sine (SIN), cosine (COS), and tangent (TAN) of 63°.

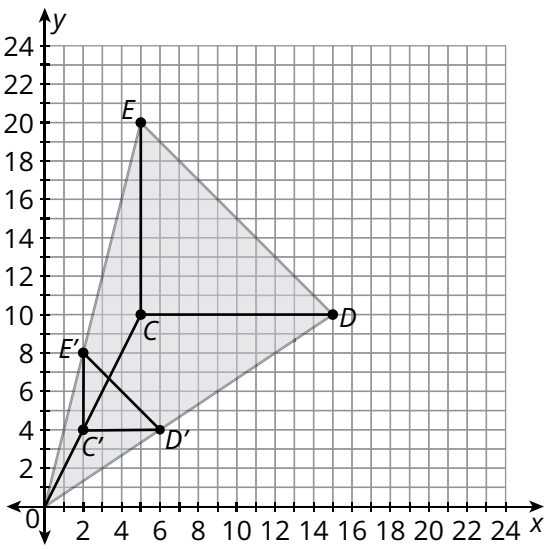
c. Compare the values from part (a) and part (b). What conclusion can you make about the sine, cosine, and tangent of an angle in a right triangle?

Spaced Practice

1. Given the pre-image and image, determine the scale factor.

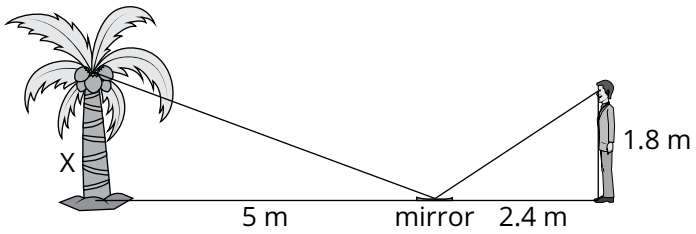


2. Given the pre-image and image, determine the scale factor. Then, describe the transformation using coordinate notation.



3. Determine each unknown height.

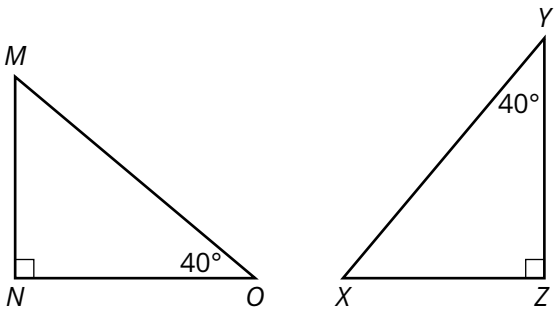
a.



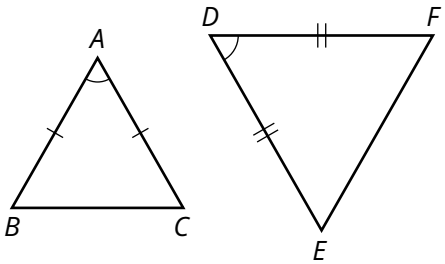
b. Pearl put a mirror 45 feet from the base of a building. She can see the top of the building in the mirror when she stands 12 feet from the mirror. If Pearl is 5 feet, 9 inches tall, what is the height of the building?

4. Determine whether the triangles are similar. When the triangles are similar, write a similarity statement. Explain your reasoning.

a.



b.

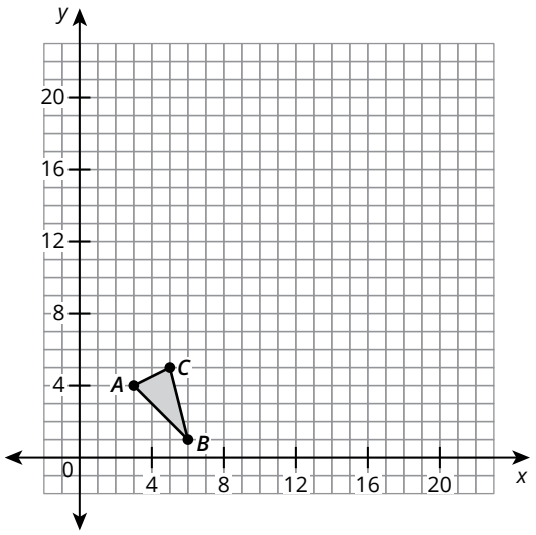


5. Suppose two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are on the coordinate plane. Derive the formula for the slope of the line passing through these two points.

   a. Define the change in the y and the change in x.

   b. Derive the slope formula using the ratio of the vertical change to the horizontal change.

6. A triangle ABC has the following vertices: $A(3, 4)$, $B(6, 1)$, and $C(5, 5)$. Translate the triangle 2 units left and 3 units up, then dilate by a scale factor of 2 with respect to the origin. Determine the image coordinates for triangle $A''B''C''$.

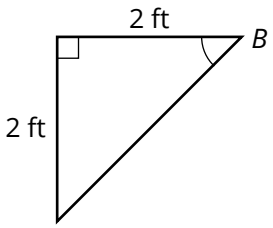


II. Tangent Ratio and Inverse Tangent

Topic Practice

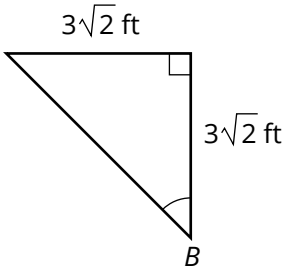
A. Calculate the tangent of the indicated angle in each triangle. Write your answers in simplest form.

1.



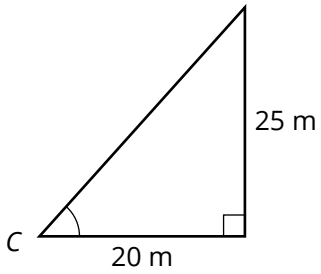
tan B =

2.



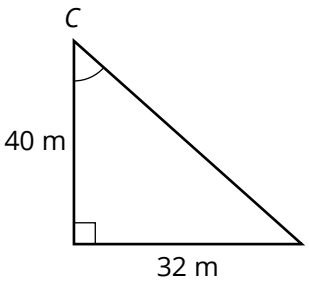
tan B =

3.



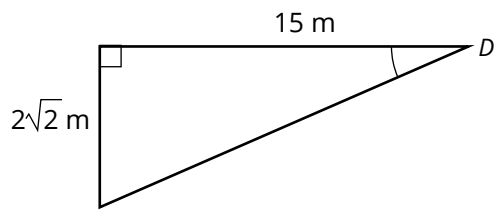
tan C =

4.



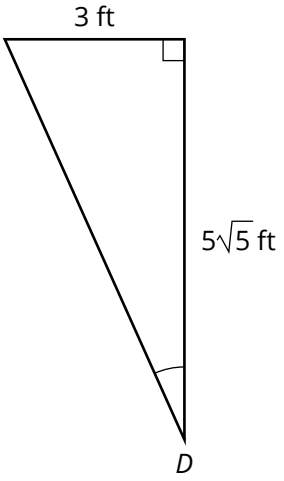
tan C =

5.



tan D =

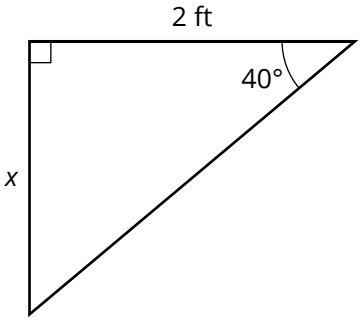
6.



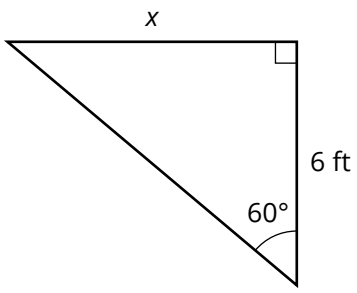
tan D =

B. Calculate the unknown length of each triangle. Round your answers to the nearest hundredth.

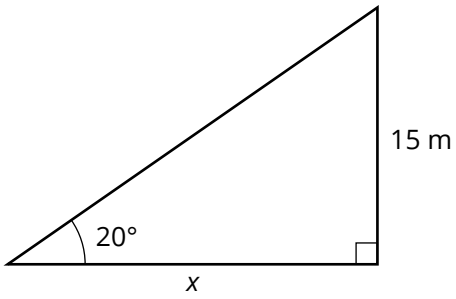
1.



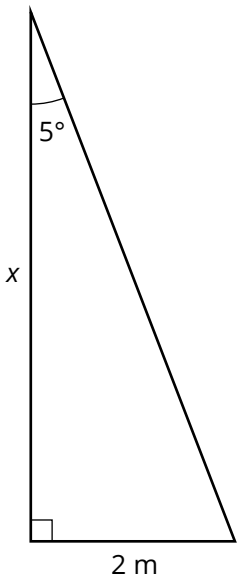
2.



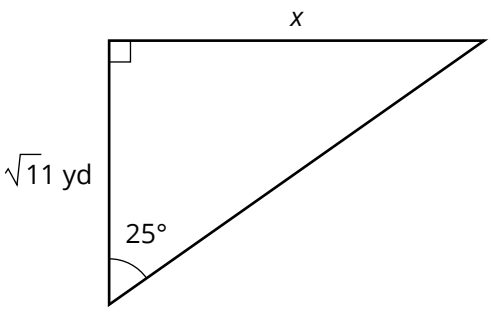
3.



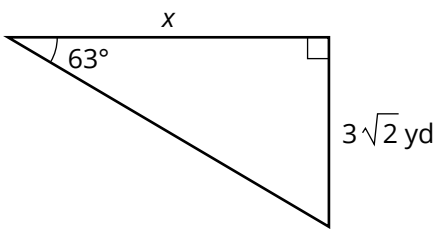
4.



5.

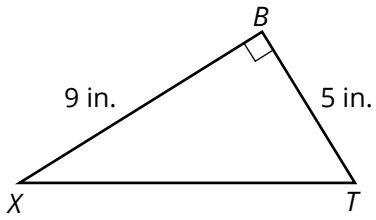


6.

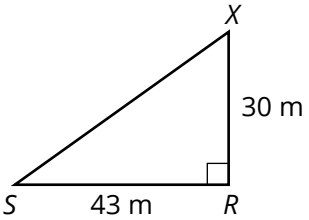


C. Calculate the measure of angle *X* for each triangle. Round your answers to the nearest hundredth.

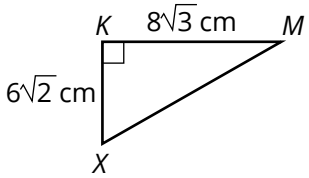
1.



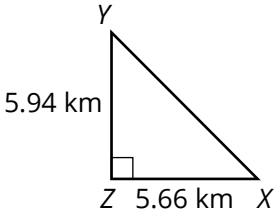
2.



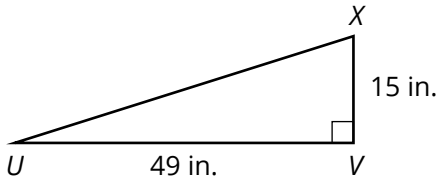
3.



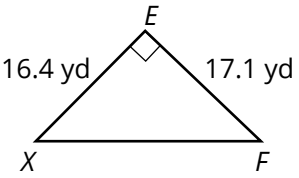
4.



5.

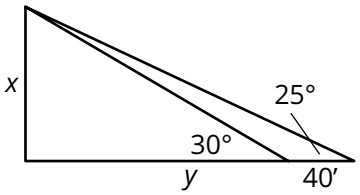


6.



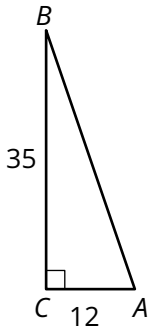
Extension

Bobby is standing near a lighthouse. He measured the angle formed from where he stood to the top of the lighthouse as 30°. Then, he backed up 40 feet and measured the angle again as 25°. Solve for the height of the lighthouse.

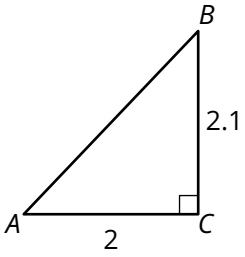


Spaced Practice

1. Determine the side length ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ using $\angle A$ as the reference angle. Write your answer as a fraction in simplest form.

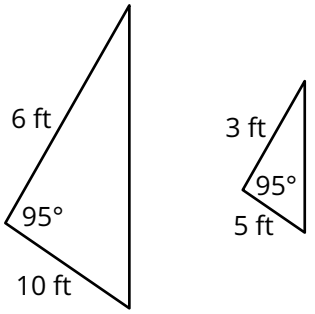


2. Determine the measure of angle A. Round your answer to the nearest tenth.

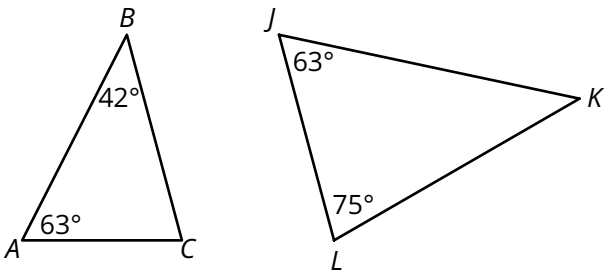


3. Determine whether the triangles in each pair are similar. Explain your reasoning.

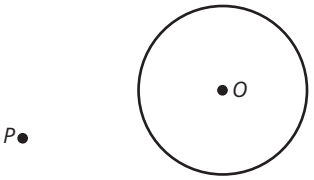
a.



b.



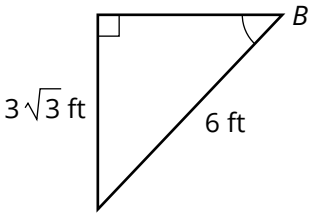
4. Construct the lines tangent to Circle *O* from point *P*.



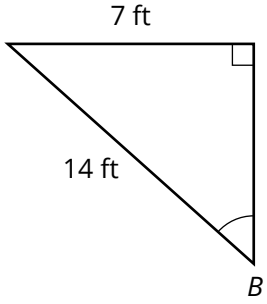
III. Exploring Sine and Cosine Ratios

Topic Practice

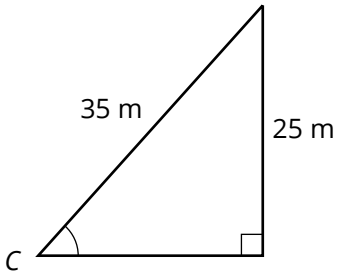
A. Calculate the sine of the indicated angle in each triangle. Write your answers in simplest form.

1. 

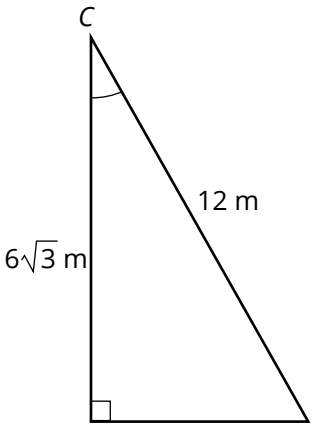
sin B =

2. 

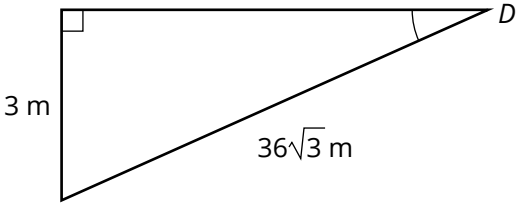
sin B =

3. 

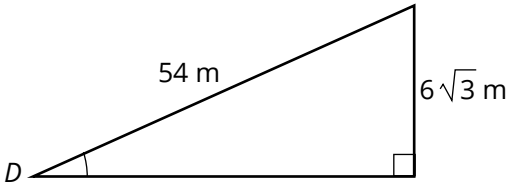
sin C =

4. 

sin C =

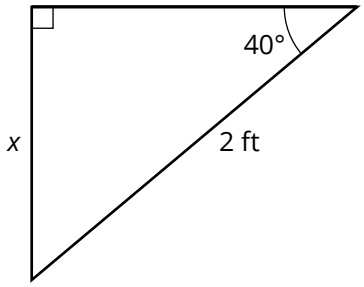
5. 

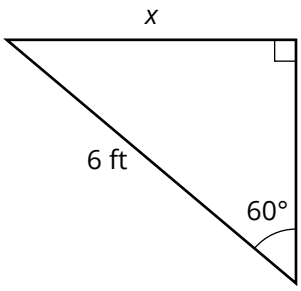
sin D =

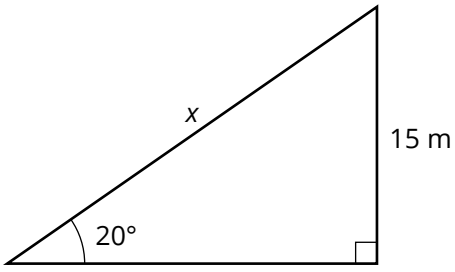
6. 

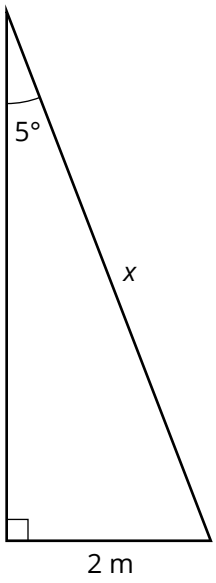
sin D =

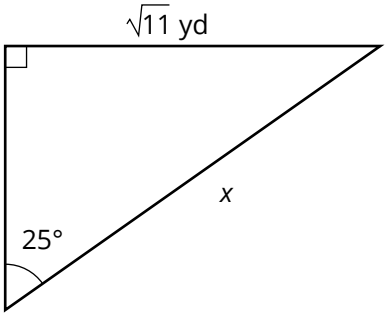
B. Calculate the unknown length of each triangle. Round your answers to the nearest hundredth.

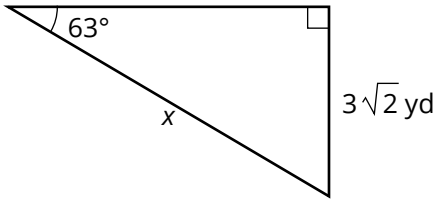
1. 

2. 

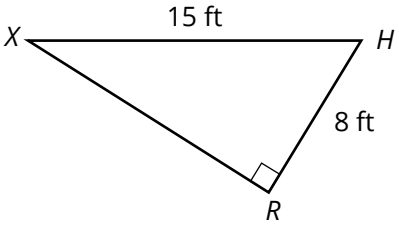
3. 

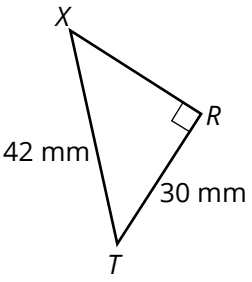
4. 

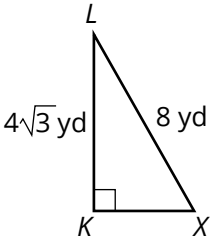
5. 

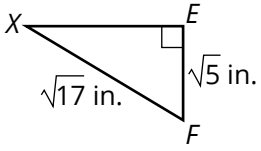
6. 

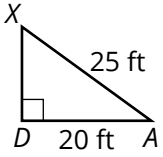
C. Calculate the measure of angle *X* for each triangle. Round your answers to the nearest hundredth.

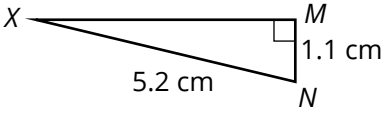
1. 

2. 

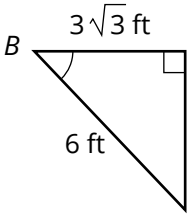
3. 

4. 

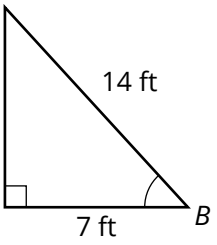
5. 

6. 

D. Calculate the cosine of the indicated angle in each triangle. Write your answers in simplest form.

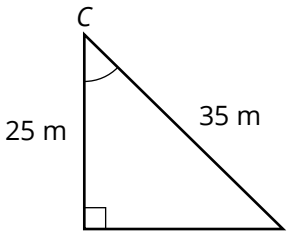
1. 

cos *B* =

2. 

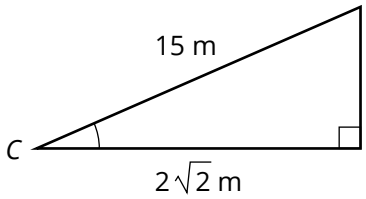
cos *B* =

3.



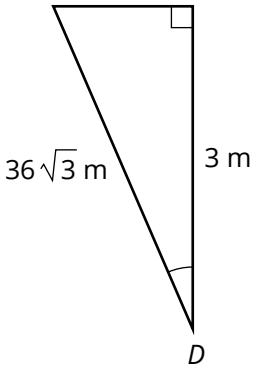
cos C =

4.



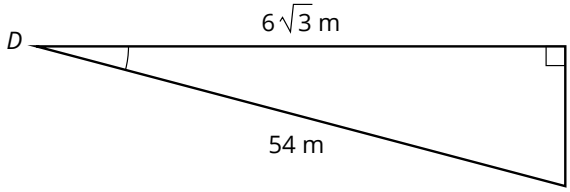
cos C =

5.



cos D =

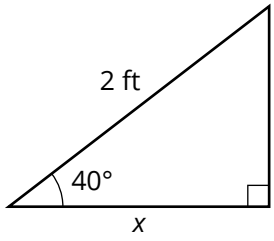
6.



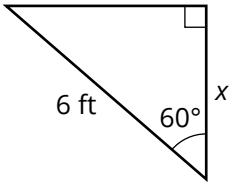
cos D =

E. Calculate the unknown length of each triangle. Round your answers to the nearest hundredth.

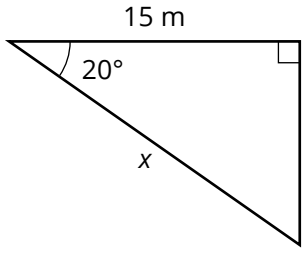
1.



2.



3.



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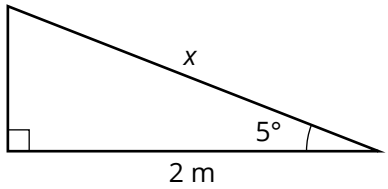

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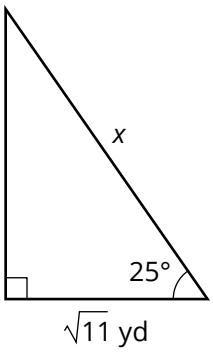

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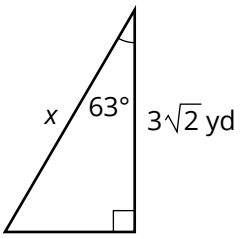
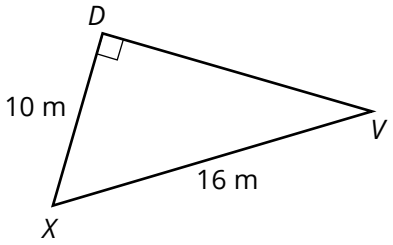
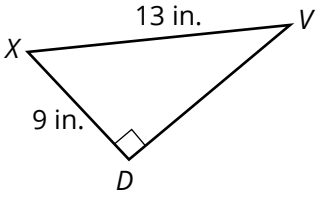


F. Calculate the measure of angle *X* for each triangle. Round your answers to the nearest hundredth.

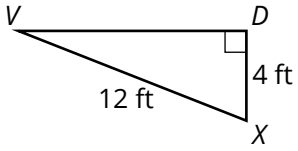
1.



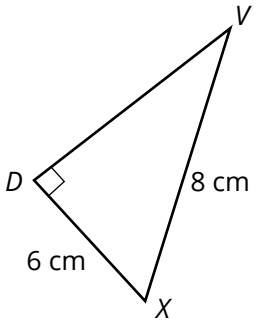
2.



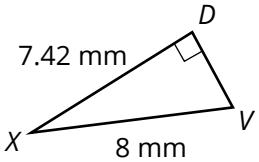
3.



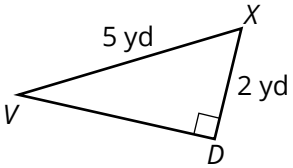
4.



5.

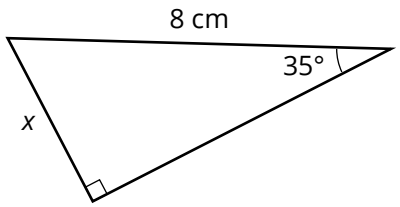


6.

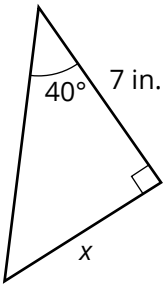


G. Determine the trigonometric ratio that you would use to solve for *x* in each triangle. Explain your reasoning. You do not need to solve for *x*.

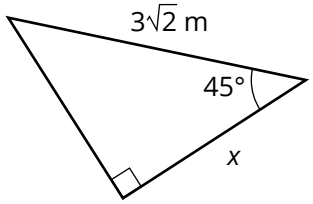
1.



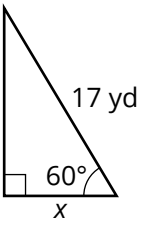
2.



3.



4.



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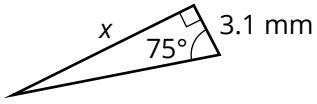
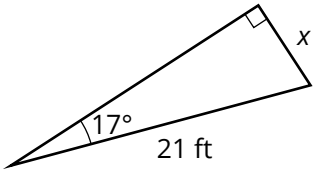
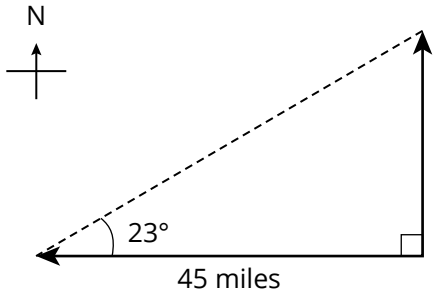


6.

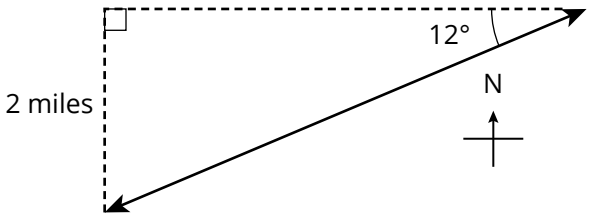


H. Solve each problem. Round your answers to the nearest hundredth.

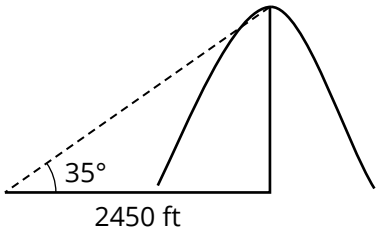
1. A boat travels in the following path. How far north did it travel?



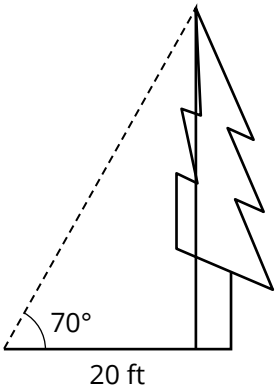
2. During a group hike, a park ranger makes the following path. How far west did they travel?



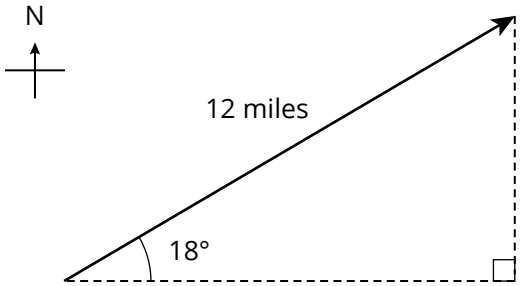
3. A surveyor makes the following diagram of a hill. What is the height of the hill?



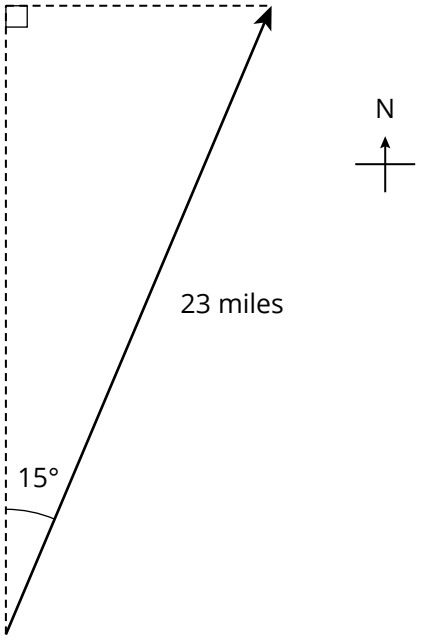
4. To determine the height of a tree, a botanist makes the following diagram. What is the height of the tree?



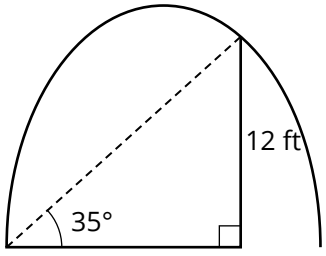
5. A scout troop traveled 12 miles from camp, as shown on the map below. How far north did they travel?



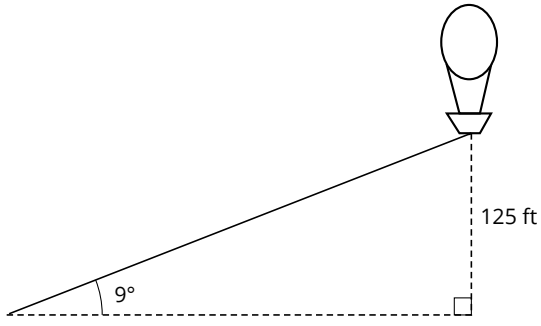
6. An ornithologist tracked a Cooper's hawk that traveled 23 miles. How far east did the bird travel?



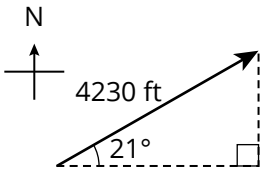
7. An architect needs to use a diagonal support in an arch. Her company drew the following diagram. How long does the diagonal support have to be?



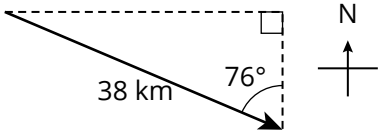
8. A hot air balloon lifts 125 feet into the air. The diagram below shows that the hot air balloon was blown to the side. How long is the piece of rope that connects the balloon to the ground?



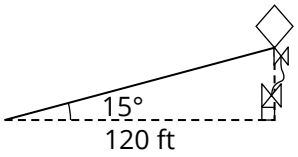
9. The path of a model rocket is shown below. How far east did the rocket travel?



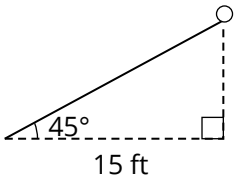
10. An ichthyologist tags a shark and charts its path. Examine his chart below. How far south did the shark travel?



11. A kite is flying 120 feet away from the base of its string, as shown below. How much string is let out?

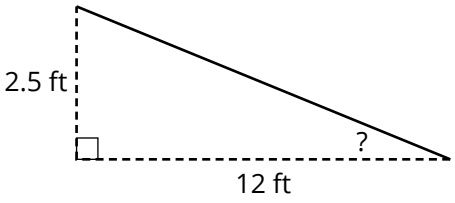


12. A pole has a rope tied to its top and to a stake 15 feet from the base. What is the length of the rope?

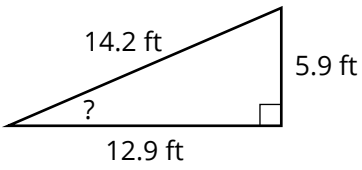


I. Solve each problem. Round your answers to the nearest hundredth.

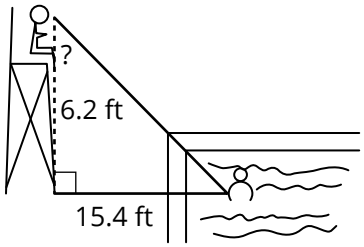
1. A moving truck is equipped with a ramp that extends from the back of the truck to the ground. When the ramp is fully extended, it touches the ground 12 feet from the back of the truck. The height of the ramp is 2.5 feet. Calculate the measure of the angle formed by the ramp and the ground.



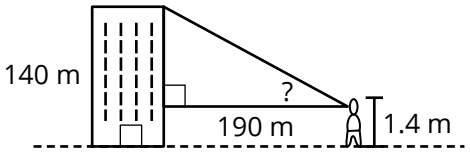
2. A park has a skateboard ramp with a length of 14.2 feet and a length along the ground of 12.9 feet. The height is 5.9 feet. Calculate the measure of the angle formed by the ramp and the ground.



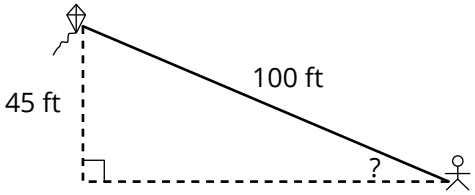
3. A lifeguard is sitting on an observation chair at a pool. The lifeguard's eye level is 6.2 feet from the ground. The chair is 15.4 feet from a swimmer. Calculate the measure of the angle formed when the lifeguard looks down at the swimmer.



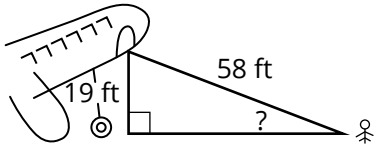
4. A surveyor is looking up at the top of a building that is 140 meters tall. His eye level is 1.4 meters above the ground, and he is standing 190 meters from the building. Calculate the measure of the angle from his eyes to the top of the building.



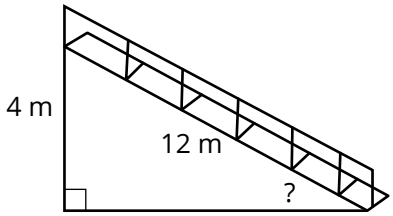
5. Jerome is flying a kite on the beach. The kite is attached to a 100-foot string and is flying 45 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the string and the ground.



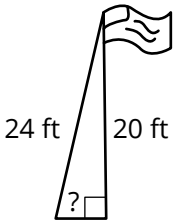
6. An airplane ramp is 58 feet long and reaches the cockpit entrance 19 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the ramp and the ground.



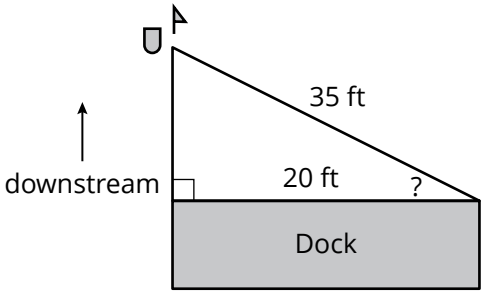
7. Bleachers in a stadium are 4 meters tall and have a length of 12 meters, as shown in the diagram. Calculate the measure of the angle formed by the bleachers and the ground.



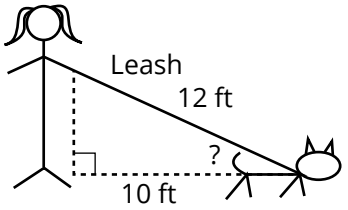
8. A 20-foot flagpole is raised by a 24-foot rope, as shown in the diagram. Calculate the measure of the angle formed by the rope and the ground.



9. You park your boat at the end of a 20-foot dock. You tie the boat to the opposite end of the dock with a 35-foot rope. The boat drifts downstream until the rope is extended as far as it will go, as shown in the diagram. What is the angle formed by the rope and the dock?

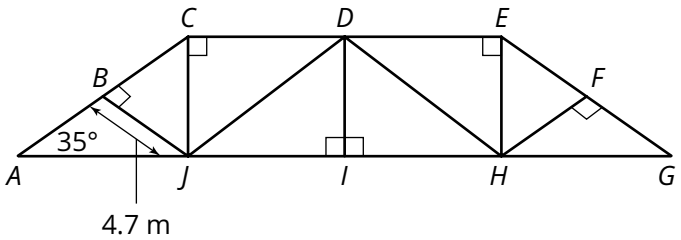


10. Rennie is walking her dog. The dog's leash is 12 feet long and is attached to the dog 10 feet horizontally from Rennie's hand, as shown in the diagram. What is the angle formed by the leash and the horizontal at the dog's collar?



Extension

1. Consider the bridge shown. Use the figure and the fact that $\triangle ACJ$ is congruent to $\triangle JDI$, $\triangle HDI$, and $\triangle GEH$ to complete parts (a) through (e). Round each answer to the nearest tenth.



   a. Determine the width AG of the bridge.

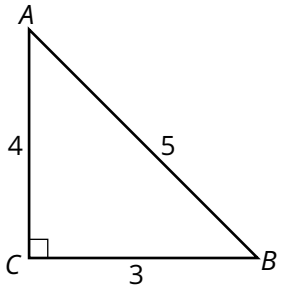
   b. Determine the height CJ of the bridge.

   c. Determine the length of the support beam JD.

   d. Determine the measure of $\angle JDH$.

   e. Determine the distance CE across the top of the bridge.

2. Consider $\triangle ABC$.



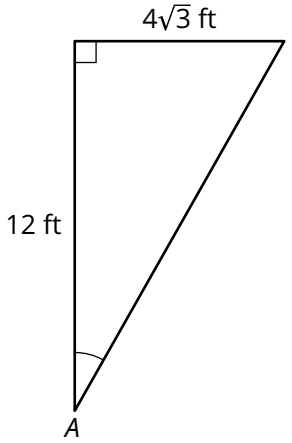
   a. Calculate the sine of $\angle B$. Determine what measure of $\angle A$ would have the equivalent ratio.

   b. Calculate the cosine of $\angle B$. Determine what measure of $\angle A$ would have the equivalent ratio.

   c. Calculate the tangent of $\angle B$.

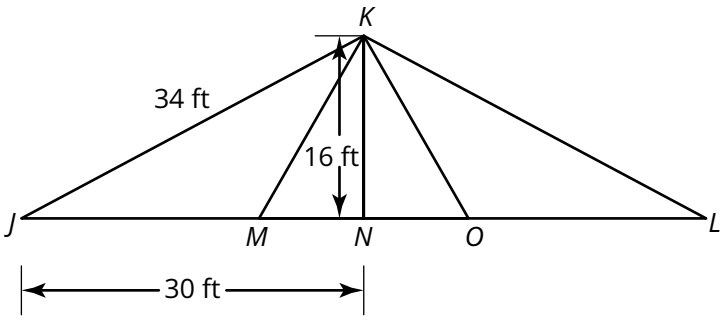
Spaced Practice

1. Calculate the tangent of the indicated angle in the triangle. Write your answer in simplest form.

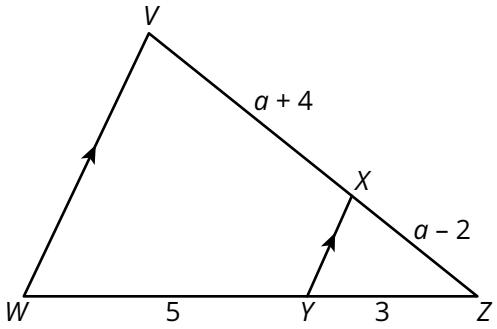


2. Use a calculator to approximate tan 50°. Round your answer to the nearest hundredth.

3. The figure shows a truss on a bridge. Segment *KM* bisects ∠*JKN*. Use this information to calculate *MN* and *MJ*.

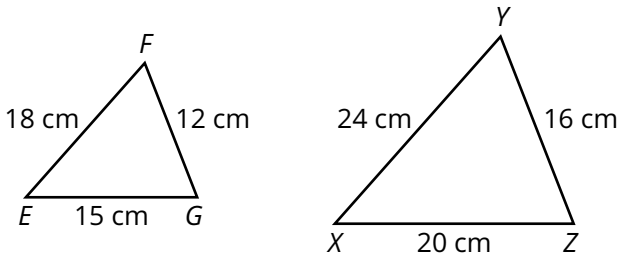


4. Use the Triangle Proportionality theorem and the proportional segments theorem to determine the unknown value.

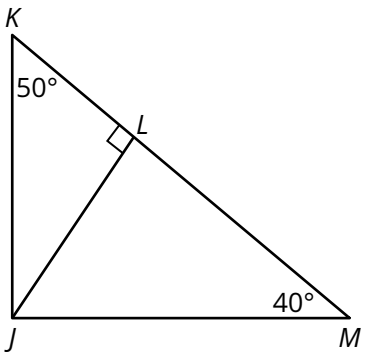


5. Determine whether the triangles are similar. When the triangles are similar, write a similarity statement. Explain your reasoning.

a.

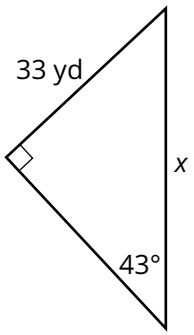


b.

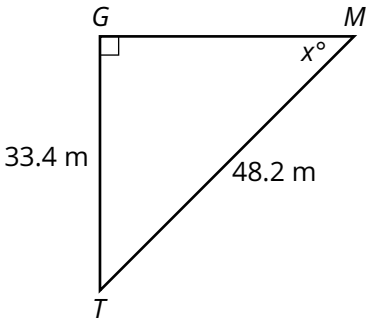


6. Use the sine ratio or the inverse sine to solve for x. Round to the nearest tenth.

a.

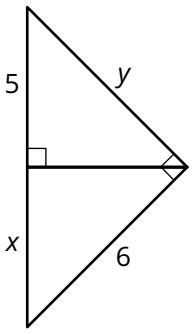


b.



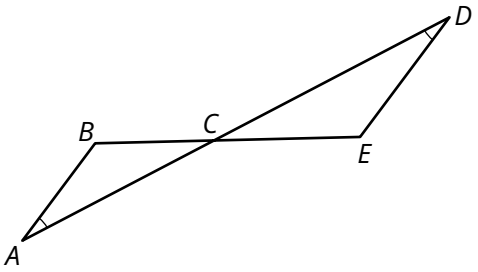
7. The geometric mean of two numbers is $4\sqrt{3}$. One of the numbers is 15. What is the other number?

8. Solve for the value of the variables.

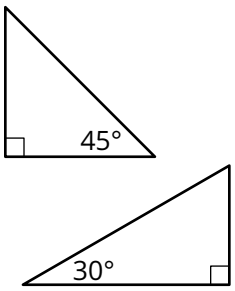


9. Determine whether the triangles are similar. When they are similar, write a similarity statement. Explain your reasoning.

a.

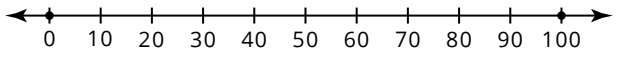


b.



Consider a situation where the line segment is located on a number line

10. An organization is designing a 100-kilometer walking path for the community park. They are required to locate signs and water fountains along the walking path. They are using this number line as a model of the walking path. Use the number line to determine the locations for the signs along the walking path. The start of the path is at 0 and the finish is at 100 on the number line.



a. Water fountains must be located along the path $\frac{1}{4}$ of the way and $\frac{3}{4}$ of the way between the path's start and finish. Describe how to locate and then plot points *A* and *B* on the number line. Point *A* is the location of the first water fountain. Point *B* is the location of the second water fountain.

b. The ratio 1 : 4 describes the location of a dog watering station. Describe how to locate and then plot point D on the number line which represents the location of the dog watering station.

c. A map must be located at the halfway point on the path. Describe how to locate and then plot point *M* on the number line which represents the location of the map.

11. James buried a time capsule $\frac{1}{3}$ of the way between his front porch and his shed. His porch is located at $P(3, 4)$ and his shed is located at $S(18, 13)$. Determine the location of James' time capsule. In what ratio does the location of the time capsule divide the directed line segment *PS*? In what ratio does it divide the line segment *SP*?

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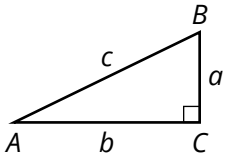
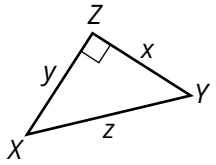
IV. Complement Angle Relationships

Topic Practice

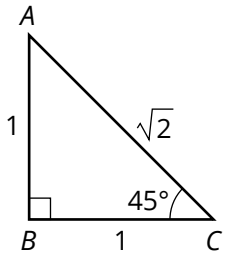
A. For each right triangle, name the given ratio in two different ways.

1. 

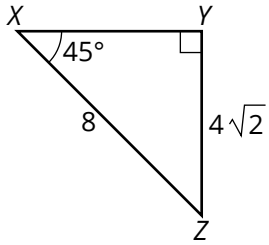
$\frac{a}{c}$

2. 

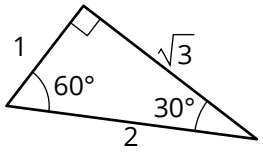
$\frac{y}{z}$

3. 

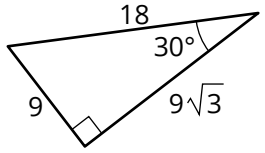
$\frac{\sqrt{2}}{2}$

4. 

$\frac{\sqrt{2}}{2}$

5. 

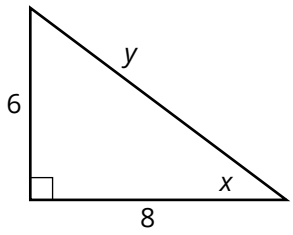
$\frac{\sqrt{3}}{2}$

6. 

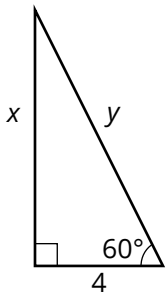
$\frac{1}{2}$

B. Solve each problem. Round your answers to the nearest hundredth.

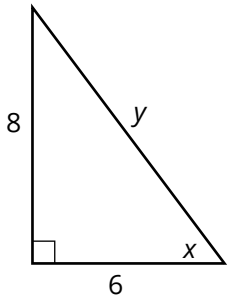
1. A right triangle has legs of length 6 and 8. Determine the hypotenuse and the value of x.



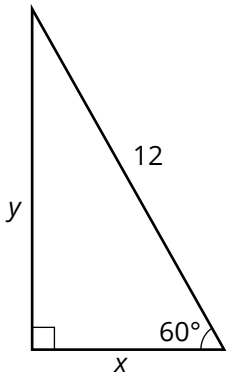
2. In a 30°-60°-90° triangle, the shorter leg is 4 units. Determine the lengths of the longer leg and the hypotenuse.



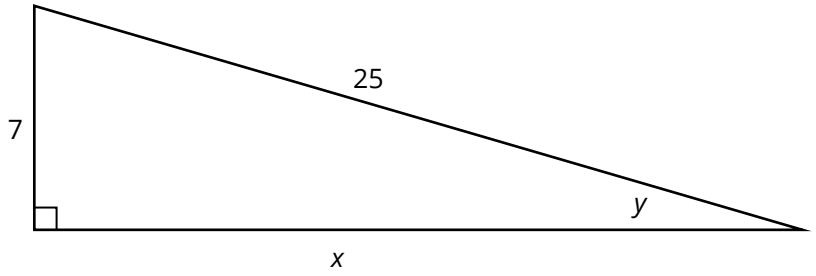
3. A ladder is leaning against a wall. The bottom of the ladder is 6 feet away from the wall, and ladder reaches a height of 8 feet on the wall. Determine the length of the ladder and the angle the ladder makes with the ground.



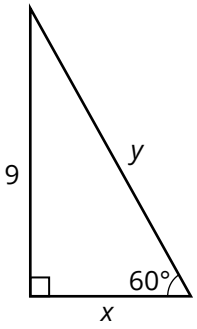
4. In a 30°-60°-90° triangle, the hypotenuse is 12 units. Determine the lengths of the shorter and longer legs.



5. In a right triangle, one leg is 7 units long, and the hypotenuse is 25 units. Determine the length of the other leg and the value of y.



6. In a 30°-60°-90° triangle, the longer leg is 9 units. Determine the lengths of the shorter leg and hypotenuse.



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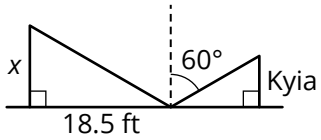
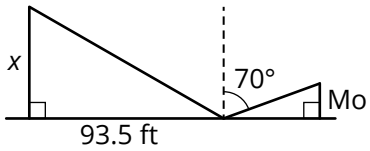
C. Solve each problem. Round your answers to the nearest hundredth.

1. Kyia is visiting a museum. She wants to know the height of one of the sculptures. She places a small mirror on the ground between herself and the sculpture. When Kyia looks in the mirror, the angle of incidence that allows her to see the sculpture is 60°.



What is the height of the sculpture?

2. Mo wants to know the height of his school. He places a small mirror on the ground between himself and the school, then he backs up until he can see the highest point of the school in the mirror. When Mo looks in the mirror, the angle of incidence that allows him to see the top of the school is 70°.



What is the height of the Mo's school?

3. You are standing 40 feet away from a building. The angle of elevation from the ground to the top of the building is 57°. What is the height of the building?

4. The angle of elevation from a ship to a 135-foot-tall lighthouse is 2°. How far is the ship from the lighthouse?

5. The angle of elevation from the top of a person's shadow on the ground to the top of the person is 45°. The top of the shadow is 50 inches away from the person. How tall is the person?

6. A plane is spotted above a hill that is 12,152 feet away. The angle of elevation to the plane is 28.47°. How high is the plane?

7. During the construction of a house, a 6-foot-long board is used to support a wall. The board has an angle of elevation from the ground to the wall of 67°. How far is the base of the wall from the board?

8. Museums use metal rods to position the bones of dinosaurs. If an angled rod needs to be placed 1.3 meters away from a bone, with an angle of elevation from the ground of 51°, what must the length of the rod be?

9. The angle of depression from the top of a building to a telephone line is 34°. If the building is 25 feet tall, how far from the building does the telephone line reach the ground?

10. An airplane flying 3500 feet from the ground sees an airport at an angle of depression of 77°. What is the horizontal distance between the airplane and the airport?

11. A factory conveyor has an angle of depression of 18° and drops 10 feet. How long is the conveyor?

12. A bicycle race organizer needs to put up barriers along a hill. The hill is 300 feet tall and from the top makes an angle of depression of 26°. How long does the barrier need to be?

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Extension

1. A hot air balloon leaves the ground at 8 AM. People on the ground watching the balloon are 250 meters from the take-off point.

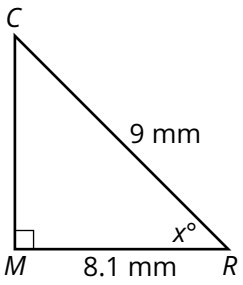
   a. The people in the balloon wave when the angle of elevation of the balloon is 25°. How high up is the balloon? Round the distance to the nearest tenth.

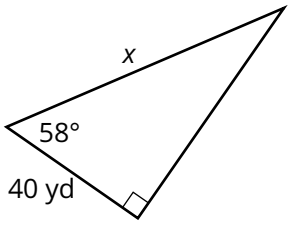
   b. The angle of elevation increases by 6°. How much further up is the balloon? Round the distance to the nearest tenth.

   c. The balloon ascends another 24.9 meters. By how much did the angle of elevation increase?

Spaced Practice

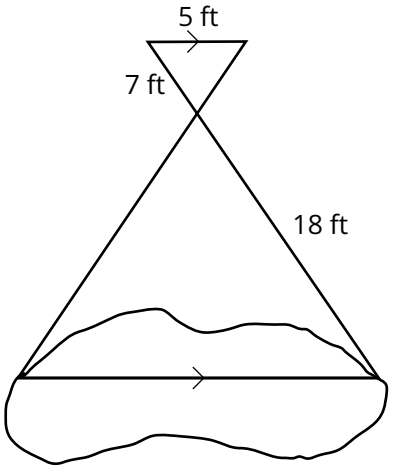
1. Use the cosine ratio, or the inverse cosine, to solve for x. Round to the nearest tenth.

   a. 

   b. 

2. Imani places a mirror 35 feet from the base of an elm tree. When she stands at a distance of 4 feet from the mirror, she can see the top of the tree in the reflection. Imani is 5 feet 4 inches tall. Draw a diagram to represent the situation. Then, determine the height of the tree.

3. Millie wants to know the distance between two points on a pond. She sets up triangles as shown in the figure. Determine the distance across the pond.



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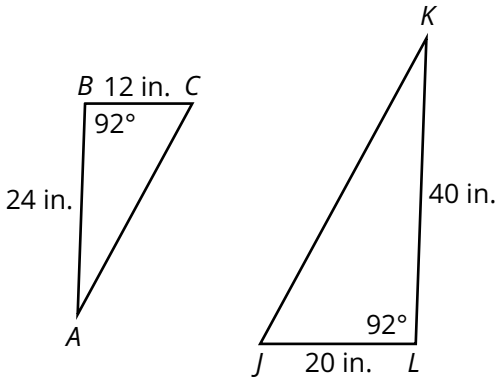
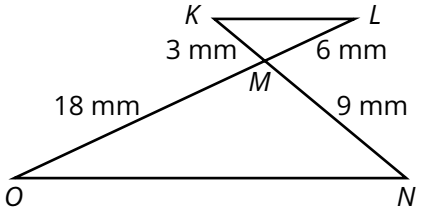
4. Determine whether the triangles are similar. When they are similar, write a similarity statement. Explain your reasoning.

a.



b.



MODULE
4

Connecting Geometric and Algebraic Descriptions

TOPIC 1: Circles

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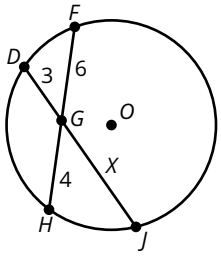
Name _____ Date _____

I. Tangents, Secants, and More Chords

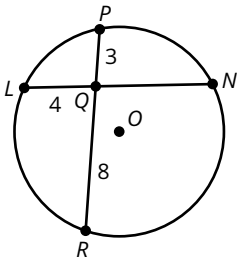
Topic Practice

A. Use each diagram and the segment chord theorem to write an equation involving the segments of the chords. Then, determine each unknown segment of a chord length.

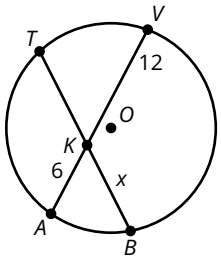
1.



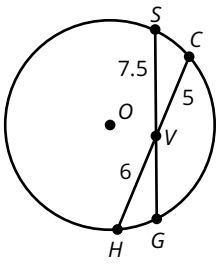
2. $LN = x$. Solve for x and then determine QN.



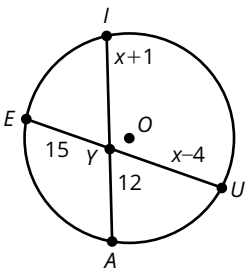
3. *TK* is 1 more unit than *KB*.



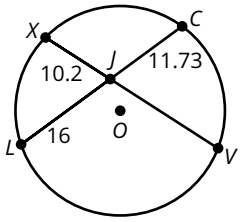
4. *SG* = *x*. Solve for *x* and then determine *VG*.



5.

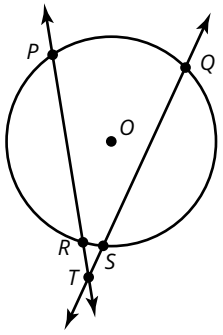


6.

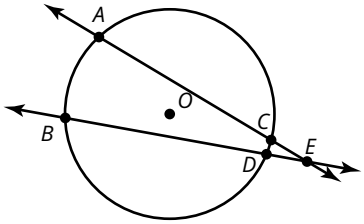


B. Name two secant segments and two external secant segments for circle *O*.

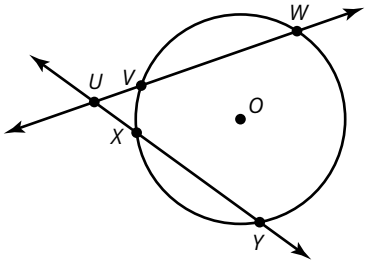
1.



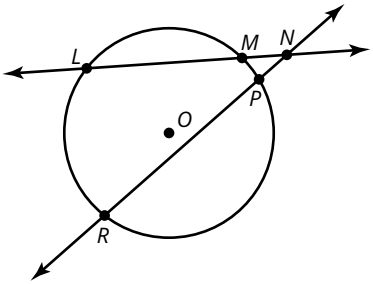
2.



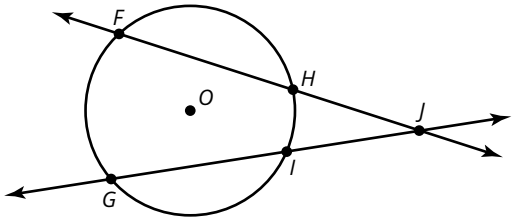
3.



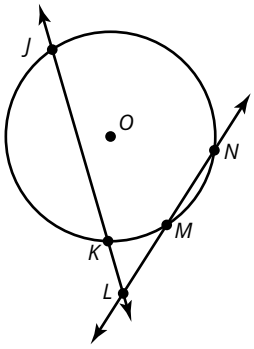
4.



5.

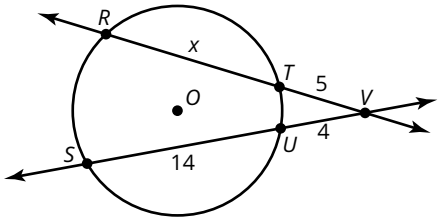


6.

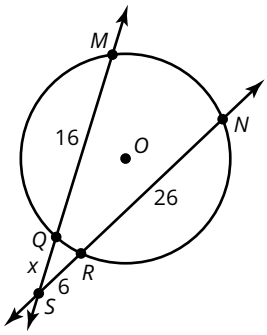


C. Use each diagram and the secant segment theorem to write an equation involving the secant segments. Then, determine the unknown secant segment length.

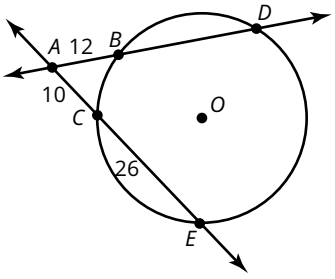
1.



2.



3. $AD = x$



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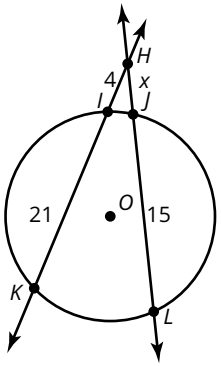

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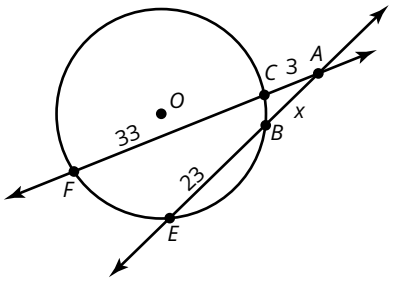

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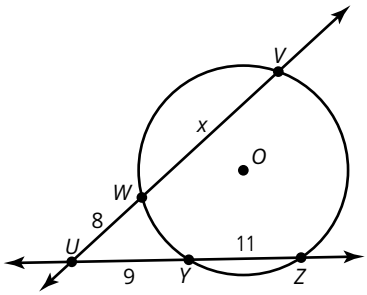
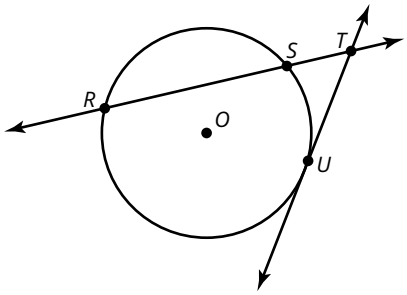


D. Name a tangent segment, a secant segment, and an external secant segment for circle *O*.

1.



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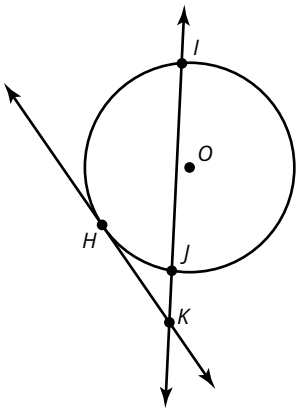

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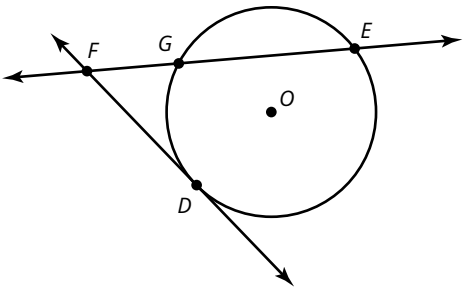

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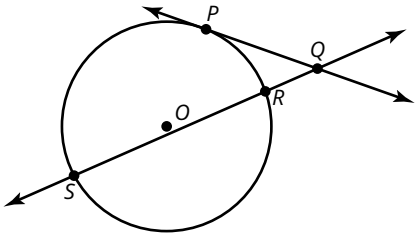

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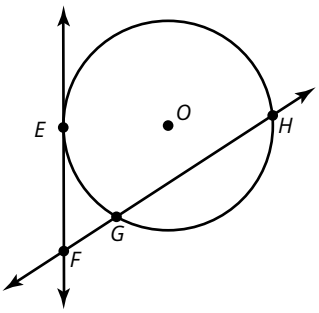

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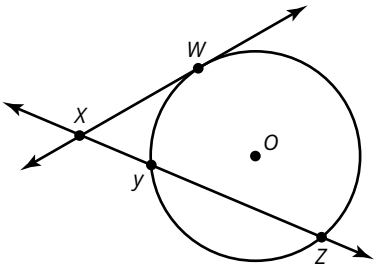
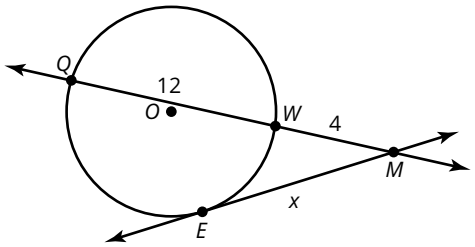


E. Use each diagram and the secant tangent theorem to write an equation involving the secant and tangent segments. Then, determine the unknown length. When necessary, leave your answer in simplest radical form.

1.



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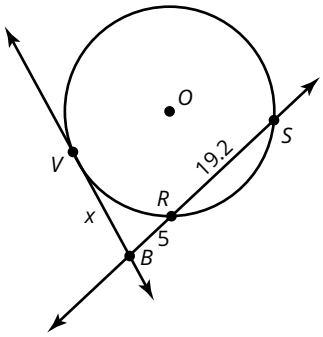

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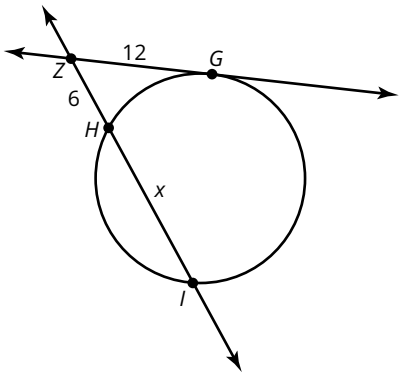

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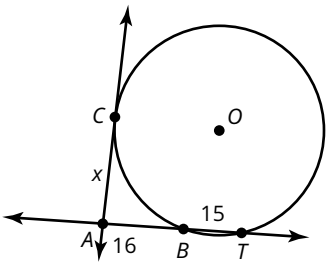

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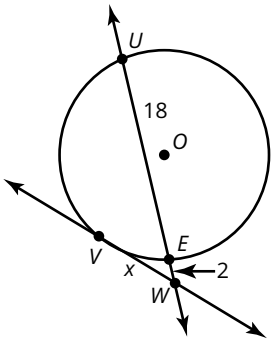

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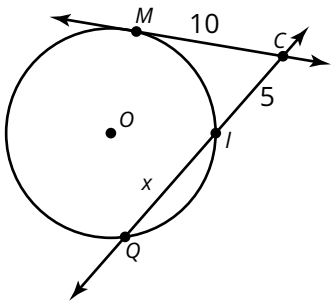
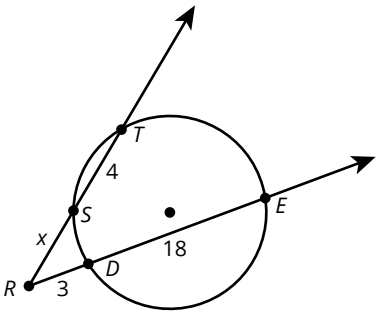


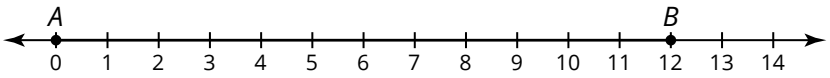
Extension

1. What is *RS*?

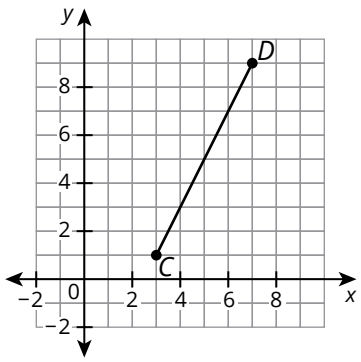


Spaced Practice

1. The number line shows line segment *AB* which starts at 0 and ends at 12. The ratio 1 : 5 describes the location of point *C* on the line segment *AB*. Describe how to locate and then plot point *C* on the number line.



2. The directed line segment *CD* is shown on the coordinate plane. Point *E* is located $\frac{3}{4}$ of the way between points *C* and *D*. Determine the coordinates of point *E* and the ratio that point *E* partitions line segment *CD*. Show your work.



3. $\triangle ABC$ has the vertex coordinates *A* at (−1, 2), *B* at (2, 5), and *C* at (1, −3). $\triangle ABC$ is translated 4 units down, reflected across the *y*-axis, and then dilated by a factor of 4 to create $\triangle DEF$. Represent this composition of transformations using coordinate notation and then determine the vertex coordinates of $\triangle DEF$.

4. Follow these steps to construct a chord in a circle and its perpendicular bisector.

   Step 1: Use a compass to draw a circle with any center *O*.

   Step 2: Choose two points *A* and *B* on the circle. Use a straightedge to draw the chord *AB*.

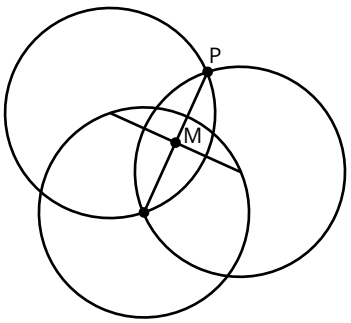
   Step 3: Place the compass at point *A* and draw an arc above and below the chord. Without changing the compass width, repeat from point *B* to intersect the first two arcs. Label the intersection points as *P* and *Q*.

   Step 4: Use a straightedge to connect *P* and *Q*, forming the perpendicular bisector of *AB*. Label the midpoint of *AB* as *M*, where the bisector crosses *AB*.

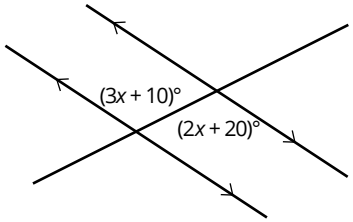
   Step 5: Continue the perpendicular bisector in both directions until it intersects the circle at a second point.

   Step 6: Label the point where the bisector meets the circle opposite *M* as *O* (the center of the circle).

   Extend this bisector to see where it meets the circle. Identify a pattern you observe and make a conjecture about the perpendicular bisector of any chord in a circle.



5. Solve for *x*.



II. Similarity Relationships in Circles

Topic Practice

A. Calculate the ratio of the length of each arc to the circle's circumference.

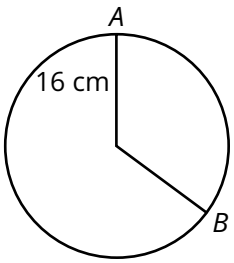
1. The measure of $\overset{\frown}{AB}$ is 40°.
2. The measure of $\overset{\frown}{CD}$ is 90°.
3. The measure of $\overset{\frown}{EF}$ is 120°.
4. The measure of $\overset{\frown}{GH}$ is 150°.
5. $m\overset{\frown}{IJ} = 105°$.
6. $m\overset{\frown}{KL} = 75°$.

B. Calculate each arc length. Express your answer in terms of π.

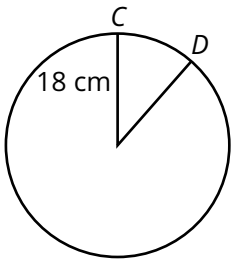
1. When the measure of $\overset{\frown}{AB}$ is 45° and the radius is 12 meters, what is the length of $\overset{\frown}{AB}$?
2. When the measure of $\overset{\frown}{CD}$ is 120° and the radius is 15 centimeters, what is the length of $\overset{\frown}{CD}$?
3. When the measure of $\overset{\frown}{EF}$ is 60° and the radius is 8 inches, what is the length of $\overset{\frown}{EF}$?
4. When the measure of $\overset{\frown}{GH}$ is 30° and the radius is 6 meters, what is the length of $\overset{\frown}{GH}$?
5. When the measure of $\overset{\frown}{IJ}$ is 80° and the diameter is 10 centimeters, what is the length of $\overset{\frown}{IJ}$?
6. When the measure of $\overset{\frown}{KL}$ is 15° and the diameter is 18 feet, what is the length of $\overset{\frown}{KL}$?
7. When the measure of $\overset{\frown}{MN}$ is 75° and the diameter is 20 millimeters, what is the length of $\overset{\frown}{MN}$?
8. When the measure of $\overset{\frown}{OP}$ is 165° and the diameter is 21 meters, what is the length of $\overset{\frown}{OP}$?

C. Calculate each arc length. Express your answer in terms of π and in decimal form rounded to the nearest hundredth.

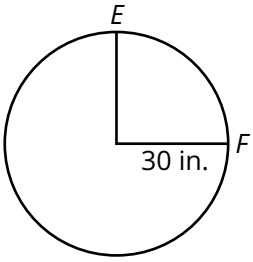
1. When the measure of $\overset{\frown}{AB}$ is 135°, what is the length of $\overset{\frown}{AB}$?



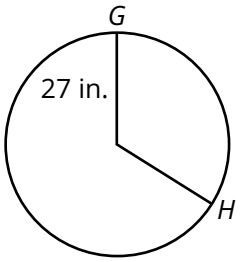
2. When the measure of $\overset{\frown}{CD}$ is 45°, what is the length of $\overset{\frown}{CD}$?



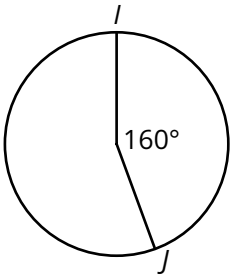
3. When the measure of $\overset{\frown}{EF}$ is 90°, what is the length of $\overset{\frown}{EF}$?



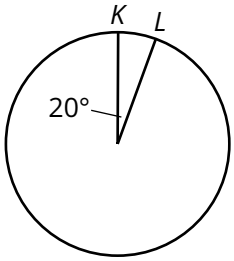
4. When the measure of $\overset{\frown}{GH}$ is 120°, what is the length of $\overset{\frown}{GH}$?



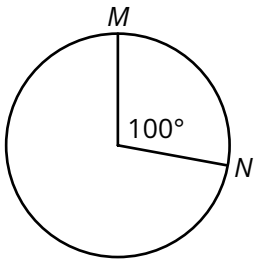
5. When the length of the diameter is 8 centimeters, what is the length of $\overset{\frown}{IJ}$?



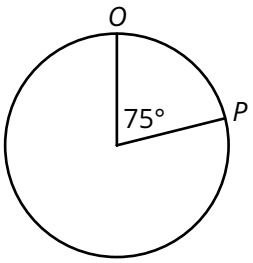
6. When the length of the radius is 7 feet, what is the length of $\overset{\frown}{KL}$?



7. When the length of the diameter is 22 centimeters, what is the length of $\overset{\frown}{MN}$?



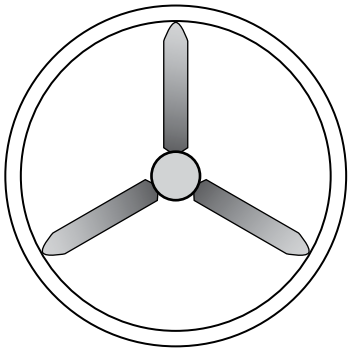
8. When the length of the radius is 17 centimeters, what is the length of $\overset{\frown}{OP}$?



D. Use the given information to answer each question.

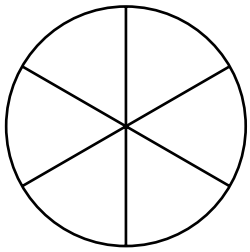
1. A fan has three thin blades that spin to produce a breeze. The diameter of the fan is 16 inches.

   Determine the length of an arc between two consecutive fan blades. Express your answer in terms of π.

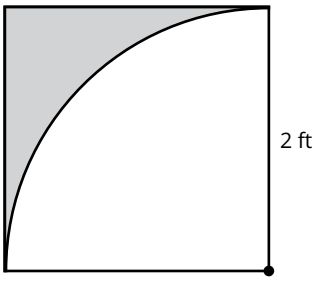


2. Jaylen's bakery has a spinning stand for each cake. Each cake has six equal slices and each slice has a radius of 6 inches.

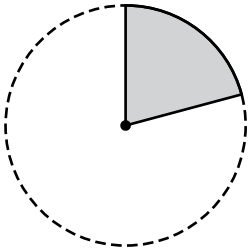
   Determine the length of an arc at the edge of each slice. Express your answer in terms of π.



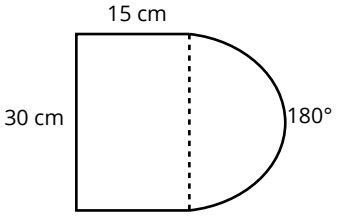
3. In a factory, a square sheet of metal has $\frac{1}{4}$ of a circle cut from it, as shown. The center of the circle is on one vertex of the square. What is the perimeter of the shaded region? Express your answer as a decimal rounded to the nearest hundredth.



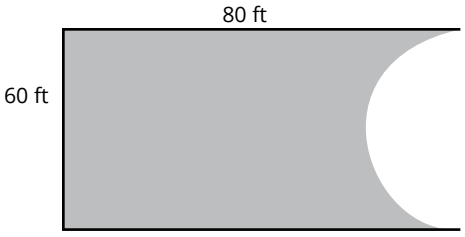
4. Part of an ancient coin was discovered. It was determined the piece was an 80° section of the entire coin. If the coin had a radius of 0.9 inches, what was the perimeter of this sector of the coin? Express your answer in terms of π.



5. Determine the perimeter of the figure below. Express your answer as a decimal rounded to the nearest tenth.



6. A semicircular cut was taken from the rectangle shown. Determine the perimeter of the shaded region. Express your answer as a decimal rounded to the nearest tenth.



E. Convert each radian measure to degrees.

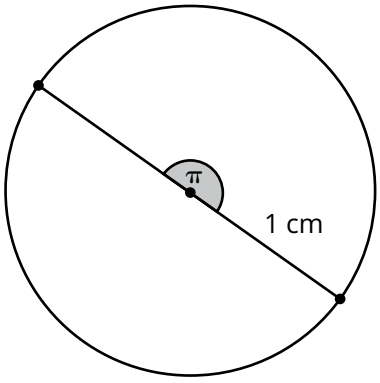
1. $\frac{\pi}{3}$
2. $\frac{3\pi}{2}$
3. $\frac{\pi}{4}$
4. 18π
5. 0.1π
6. $\frac{4\pi}{5}$

F. Convert each degree measure to radians.

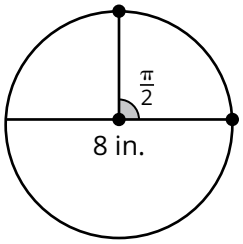
1. 100°
2. 180°
3. 30°
4. 540°
5. 1°
6. 345°

G. Determine each arc length.

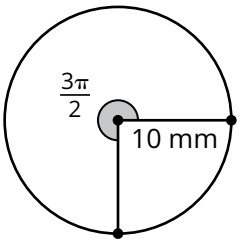
1. The radius of a circle is 1 centimeter. What is the length of an arc intercepted by an angle of π radians?



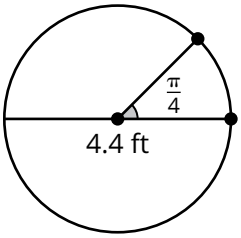
2. The diameter of a circle is 8 inches. What is the length of an arc intercepted by an angle of $\frac{\pi}{2}$ radians?



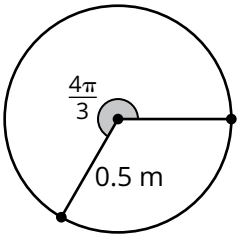
3. The radius of a circle is 10 millimeters. What is the length of an arc intercepted by an angle of $\frac{3\pi}{2}$ radians?



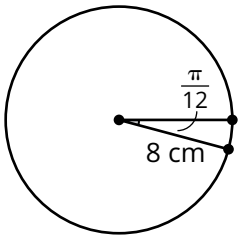
4. The diameter of a circle is 4.4 feet. What is the length of an arc intercepted by an angle of $\frac{\pi}{4}$ radians?



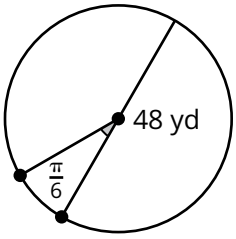
5. The radius of a circle is 0.5 meters. What is the length of an arc intercepted by an angle of $\frac{4\pi}{3}$ radians?



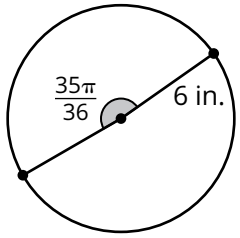
6. The radius of a circle is 8 centimeters. What is the length of an arc intercepted by an angle of $\frac{\pi}{12}$ radians?



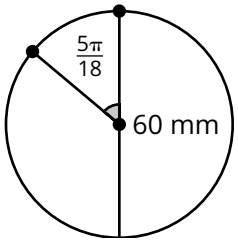
7. The diameter of a circle is 48 yards. What is the length of an arc intercepted by an angle of $\frac{\pi}{6}$ radians?



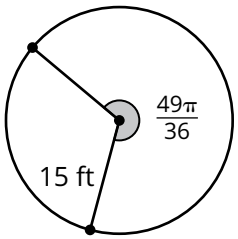
8. The radius of a circle is 6 inches. What is the length of an arc intercepted by an angle of $\frac{35\pi}{36}$ radians?



9. The diameter of a circle is 60 millimeters. What is the length of an arc intercepted by an angle of $\frac{5\pi}{18}$ radians?

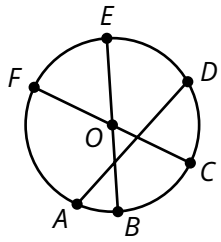


10. The radius of a circle is 15 feet. What is the length of an arc intercepted by an angle of $\frac{49\pi}{36}$ radians?



Extension

1. In circle O, the radius is 8 millimeters, $m\overset{\frown}{BC}$ is 60°, and $m\overset{\frown}{AB}$ is 35°.

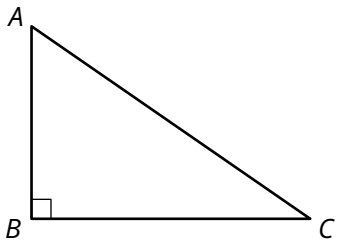


   a. Determine the length of $\overset{\frown}{FA}$ in millimeters.

   b. Determine the measure of $\overset{\frown}{FA}$ in radians.

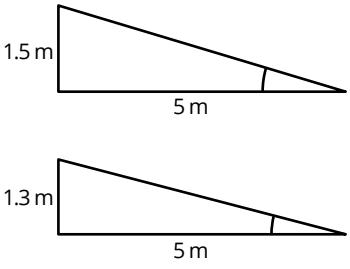
Spaced Practice

1. Complete the table of ratios given $\triangle ABC$.

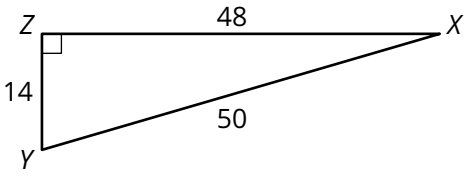


Reference Angle	sin	cos	tan
A			$\frac{5}{12}$
C			

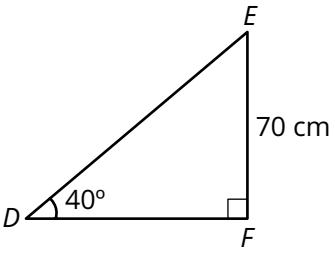
2. Paola is installing a new roof on her backyard shed. The shed is 5 meters wide. Paola is considering roof heights between 1.3 and 1.5 meters. Determine the minimum and maximum angle of elevation of the roof beams. Round the measures to the nearest tenth.



3. Determine the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ using $\angle X$ as the reference angle. Write your answers as fractions.



4. Determine side length *DF*. Round your answer to the nearest hundredth.



5. Consider the statement written below.

   If a quadrilateral has 2 pairs of parallel sides, the quadrilateral is a parallelogram.

   Write the contrapositive and inverse of the statement and then determine both statements' validity. Explain your reasoning.

   Contrapositive:

   Inverse:

6. Consider the statement written below.

   If a triangle has a right angle, then the triangle only has one acute angle.

   Write the contrapositive and inverse of the statement and then determine both statements' validity. Explain your reasoning.

   Contrapositive:

   Inverse:

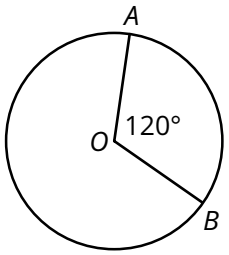
CC BY NC

III. Sectors and Segments of a Circle

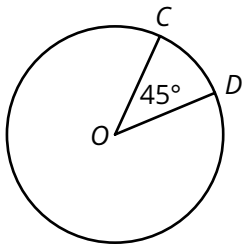
Topic Practice

A. Calculate the area of each sector. Express your answer in terms of π.

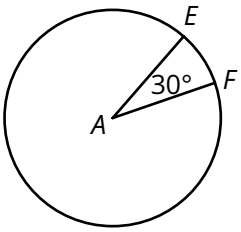
1. When the radius of the circle is 9 centimeters, what is the area of sector *AOB*?



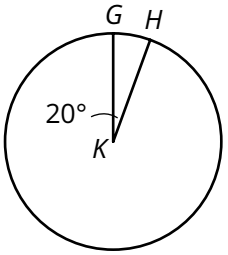
2. When the diameter of *O* is 32 meters, what is the area of sector *COD*?



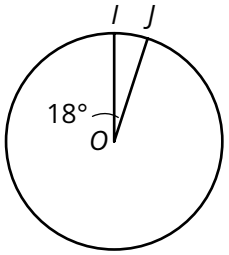
3. When the diameter of the circle is 30 feet, what is the area of sector *EAF*?



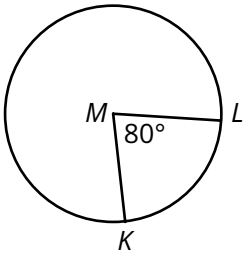
4. When the radius of *K* is 10 inches, what is the area of sector *GKH*?



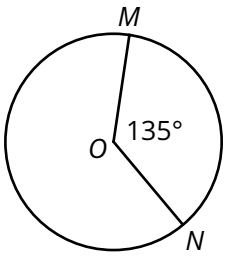
5. When the diameter of the circle is 64 centimeters, what is the area of sector *IOJ*?



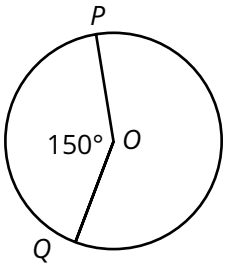
6. When the radius of *M* is 20 millimeters, what is the area of sector *KML*?



7. When the radius of the circle is 24 centimeters, what is the area of sector *MON*?

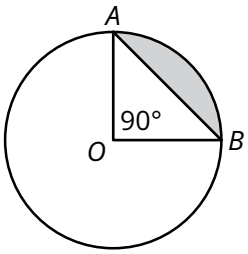


8. When the diameter of *O* is 42 meters, what is the area of sector *POQ*?

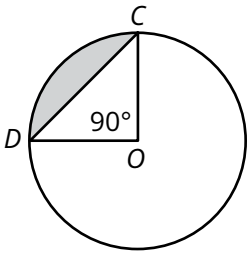


B. Calculate the area of each segment. Round your answer to the nearest tenth, when necessary.

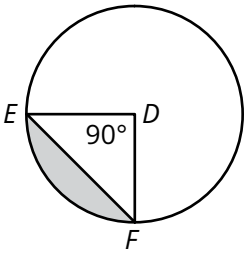
1. When the radius of the circle is 6 centimeters, what is the area of the shaded segment?



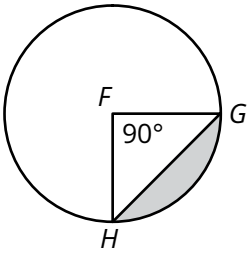
2. When the diameter of *O* is 28 inches, what is the area of the shaded segment?



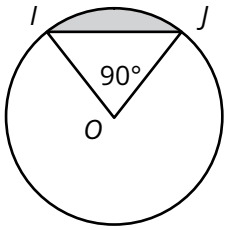
3. When the diameter of the circle is 34 feet, what is the area of the shaded segment?



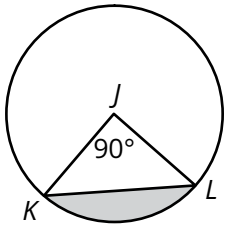
4. When the radius of *F* is 22 centimeters, what is the area of the shaded segment?



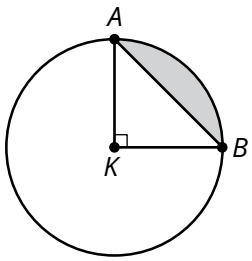
5. When the diameter of the circle is 50 meters, what is the area of the shaded segment?



6. When the radius of J is 30 centimeters, what is the area of the shaded segment?



C. In K, $m\overset{\frown}{AB} = 90°$. Use the given information to determine the exact length of the radius or diameter of K.



1. When the area of the shaded segment is $16\pi - 32$ square feet, what is the exact length of the radius of K?

2. When the area of the shaded segment is $25\pi - 50$ square inches, what is the exact length of the radius of K?

3. When the area of the shaded segment is $\pi - 2$ square meters, what is the exact length of the diameter of K?

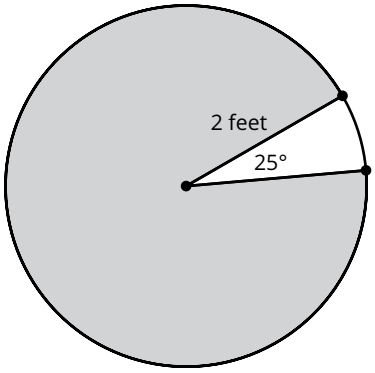
4. When the area of the shaded segment is $56.25\pi - 112.5$ square yards, what is the exact length of the radius K?

5. When the area of the shaded segment is $121\pi - 242$ square feet, what is the exact length of the diameter of K?

6. When the area of the shaded segment is $90.25\pi - 180.5$ square millimeters, what is the exact length of the radius of K?

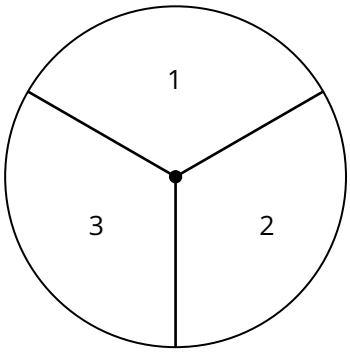
D. Determine the solution for each given situation. Express your answer in terms of π unless otherwise indicated.

1. Andrew paints the figure shown on the sign of his new restaurant. What is the area of the painted part of the figure, shown here as the shaded part?

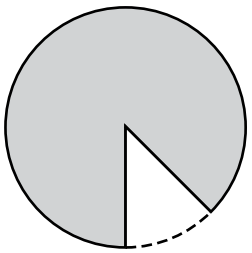


2. A spinner is divided into three equal sectors. The diameter of the spinner is 16 centimeters.

   What is the area of each sector?

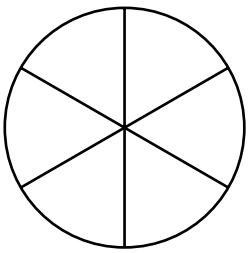


3. The wheel of cheese shown has a radius of 15 inches. However, a wedge has been removed with a central angle of 45°. What is the area of the shaded portion, which represents the amount of the cheese wheel that remains?



4. Eduardo's bakery has a spinning stand for each cake. Each cake has six equal slices and each slice has a radius of 6 inches.

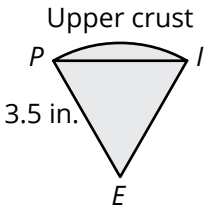
   To determine the area of the top of each slice of cake, calculate the area of one sector.



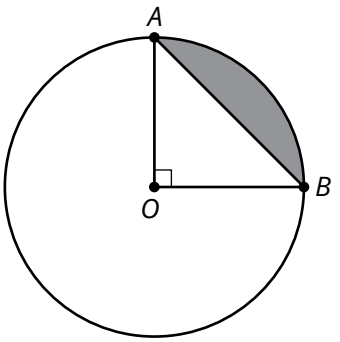
5. Jorge covers the entire top of his piece of pie with whipped cream, but he doesn't like to eat the upper portion of the pie crust where it is thickest.

   $\overline{EP}$ and $\overline{EI}$ are radii of a circle.
   IP = 3.5 inches
   $m\angle PEI = 60°$

   Determine the area of the upper crust. Round your answer to the nearest hundredth.



6. The length of the radius is 8 inches. Calculate the area of the shaded region of circle *O*.

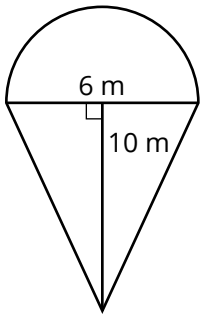


E. Determine the area and perimeter of each figure. Round each area and perimeter to the nearest tenth.

1. The diameter of the semi circle is 6 meters.

   Area:

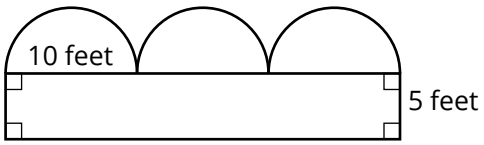
   Perimeter:



2. Each semi circle has a diameter of 10 feet.

   Area:

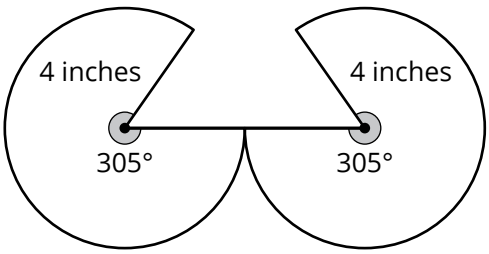
   Perimeter:



3. Each partial circle has a radius of 4 inches.

   Area:

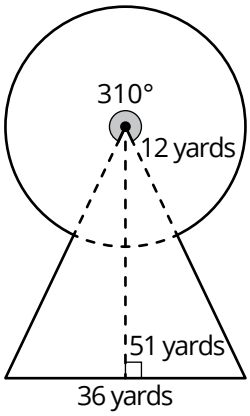
   Perimeter:



4. The partial circle has a radius of 12 yards.

   Area:

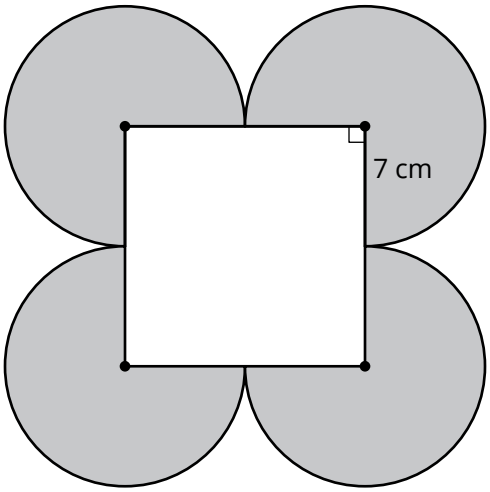
   Perimeter:



5. The four partial circles are congruent to each other.

   Area:

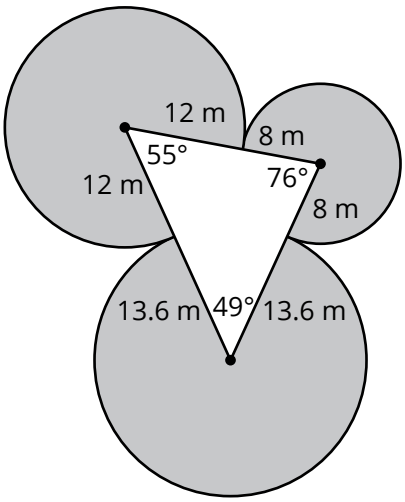
   Perimeter:



6. The diagram shows 3 partial circles and a triangle created by connecting the center of each circle to the center of the other circles. Determine the area and the perimeter of the 3 partial circles.

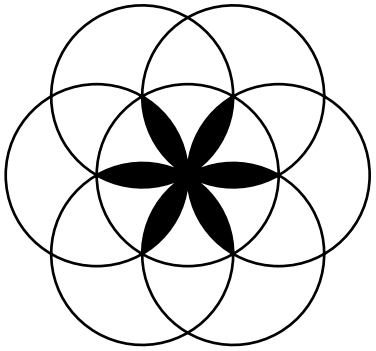
   Area:

   Perimeter:



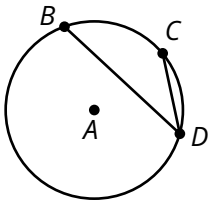
Extension

Each circle in the figure shown has a radius of 1 inch. The intersections of all the outer circles are at the center of the inner circle. The area of the shaded region.



Spaced Practice

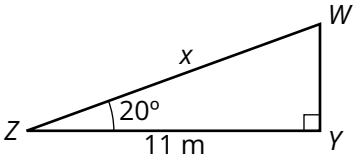
1. In circle A shown, the radius of the circle is 20 millimeters and $m\angle BDC$ is 32°. Determine the length of $\overset{\frown}{BC}$. Round your answer to the nearest hundredth.



2. If $\theta = \frac{2\pi}{3}$ and $r = 8$, what is the length of the intercepted arc?

3. Andrew is flying a kite. The angle of elevation of the kite varies between 56° and 60°. Andrew has let out 15 m of string on the kite. Draw a picture to model this situation. Then, determine the minimum and maximum heights of the kite. Round the heights to the nearest tenth.

4. Determine side length ZW to the nearest tenth of a meter.

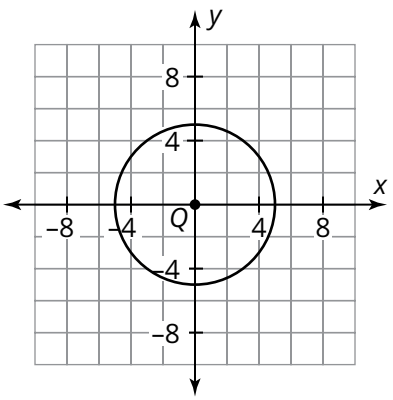


IV. Deriving the Equation of a Circle

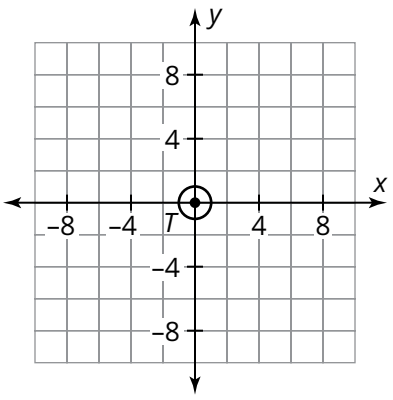
Topic Practice

A. Write an equation for each circle.

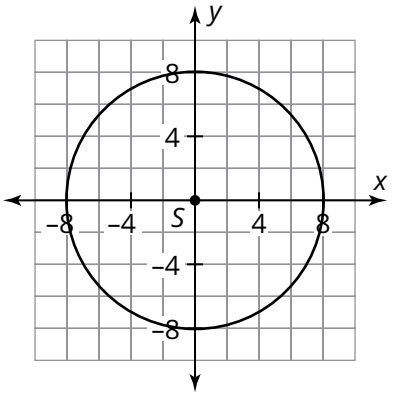
1.



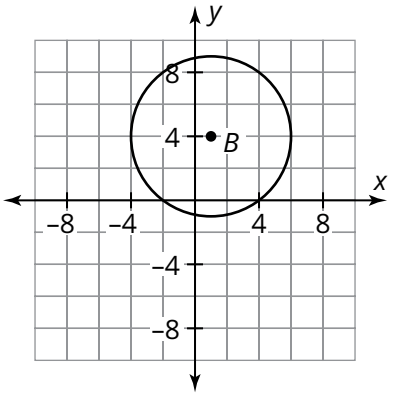
2.



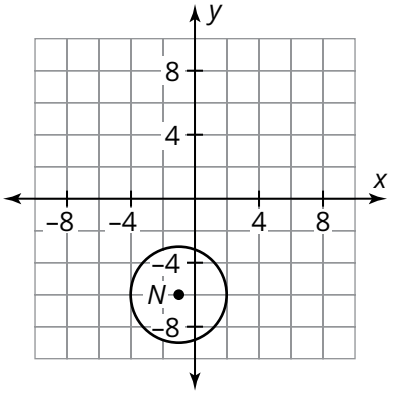
3.



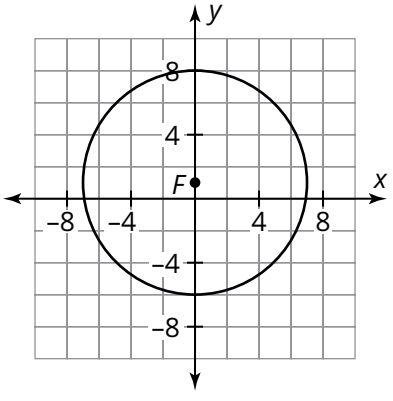
4.



5.



6.



B. Write an equation in standard form for each circle.

1. a circle with center point at the origin when $r = 4$

2. a circle with center point at the origin when $r = \frac{2}{3}$

3. a circle with center point (6, 5) when $r = 1$

4. a circle with center point (−8, −12) when $r = 7$

5. a circle with center point (3, −2) when $r = 0.5$

6. a circle with center point (2.5, −3.5) when $r = 0.8$

C. Write the equation of each circle in standard form. Then, identify the center point and radius of the circle.

1. $x^2 + y^2 + 6x - 2y + 1 = 0$

2. $x^2 + y^2 - 14x + 4y + 49 = 0$

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3. $x^2 + y^2 - 2x - 2y + 1 = 0$

4. $81x^2 + 81y^2 + 36x - 324y + 327 = 0$

5. $9x^2 + 9y^2 + 72x - 12y + 147 = 0$

6. $36x^2 + 36y^2 - 36x + 72y + 29 = 0$

D. Determine whether each equation represents a circle. If so, describe the location of the center and the radius of the circle.

1. $x^2 + y^2 = 324$

2. $(x + 7)^2 + (y + 8)^2 = 64$

3. $(x - 3)^2 + (y - 3)^2 = 8$

4. $x^2 + 24x + y^2 + 10y + 160 = 0$

5. $2x^2 + y^2 + 2x - 6y + 6 = 0$

6. $x^2 + 4y^2 - 3x + 3y - 9 = 0$

7. $x^2 - 6x + y^2 - 32y = -264$

8. $-6x + x^2 = 97 + 10y - y^2$

E. Determine the equation of a circle that has the same center as circle *P*, but with the given new circumference or area.

1. Circle *P*: $(x - 1)^2 + (y + 2)^2 = 49$

   New circumference: twice the circumference of circle *P*

2. Circle *P*: $(x + 3)^2 + y - 4)^2 = 100$

   New area: twice the area of circle *P*

3. Circle *P*: $x^2 + (y + 1)^2 = 14$

   New circumference: five times the circumference of circle *P*

4. Circle *P*: $(x - 10)^2 + y^2 = 1$

   New area: three times the area of circle *P*

5. Circle *P*: $(x + 7\sqrt{2})^2 + (y - \sqrt{3})^2 = 5\sqrt{2}$

   New area: six times the area of circle *P*

6. Circle *P*: $(x - \sqrt{3})^2 + (y - 2\sqrt{3})^2 = 8\sqrt{3}$

   New circumference: three times the circumference of circle *P*

Extension

1. Determine whether the point (1, 1) is inside, outside, or on the circle that is represented by the equation $x^2 + y^2 + 4x - 8y = 5$. Explain your reasoning.

2. Determine the radius of the circle given the location of the center of the circle and a point on the circle. Then, write the equation of the circle in standard form.

   a. A circle with a center at (0, 0) and the point (0, 3) on the circle

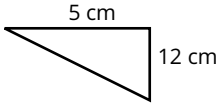
   b. A circle with a center at (1, 1) and the point (1, 5) on the circle

   c. A circle with a center at (–2, 5) and the point (–2, 7) on the circle

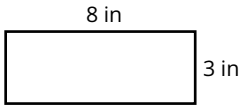
Spaced Practice

1. Determine how changing the dimensions affect the perimeter and the area of each figure.

   a. Doubling both dimensions.



b. Doubling the width and tripling the height.



2. If a triangle has two acute angles, then the third angle is a right angle. Is the conclusion of this statement true? Justify for your response.

3. Compare the number of right angles a triangle in Euclidean geometry can have to the number of right angles a triangle in spherical geometry can have. Justify your reasoning.

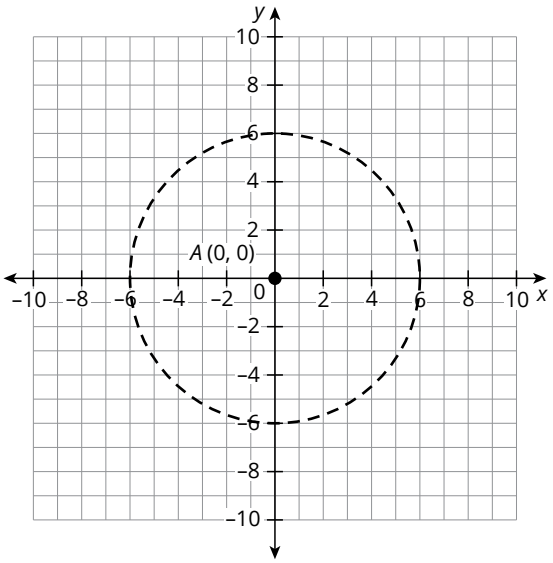
4. A community is building a compost bin that will be used as soil for the community garden. The bin will have three walls that will form a triangle. Two of the walls are 10 feet and 14 feet. What is the range of lengths that the third wall can be? Justify your response.

V. Determining Points on a Circle

Topic Practice

A. For each circle *A*, determine whether the given point *P* lies on the circle. Explain your reasoning. Note: All radii are integers.

1.



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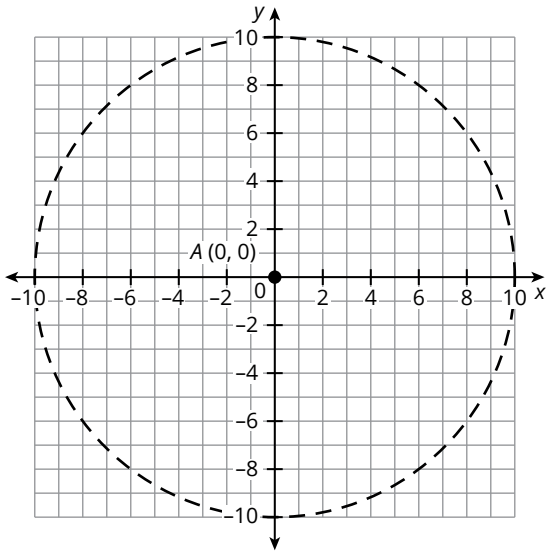

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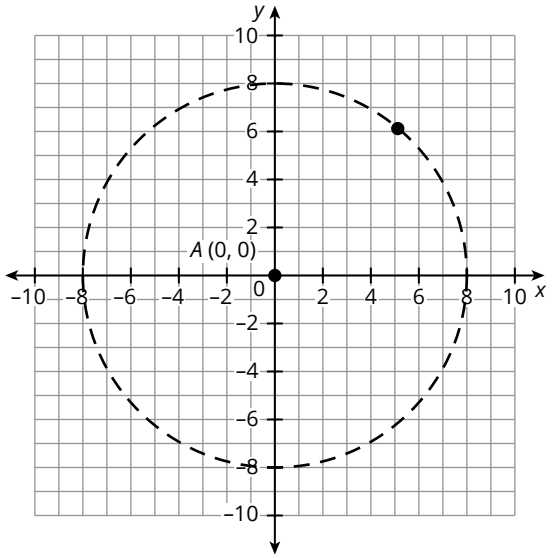
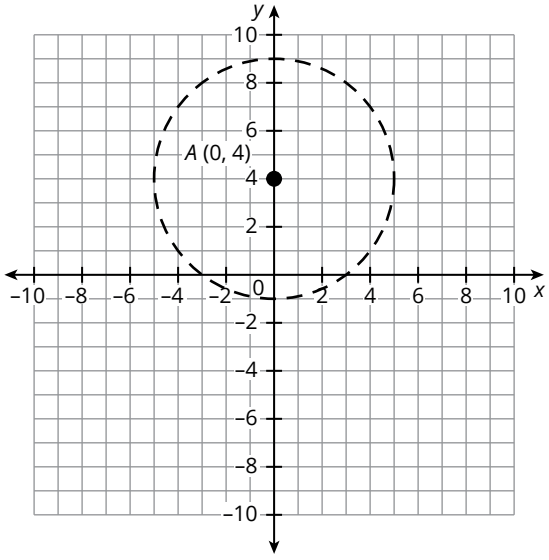

y
x
A (0, 0)
10
8
6
4
2
0
−2
−4
−6
−8
−10
−10 −8 −6 −4 −2 2 4 6 8 10

4.


y
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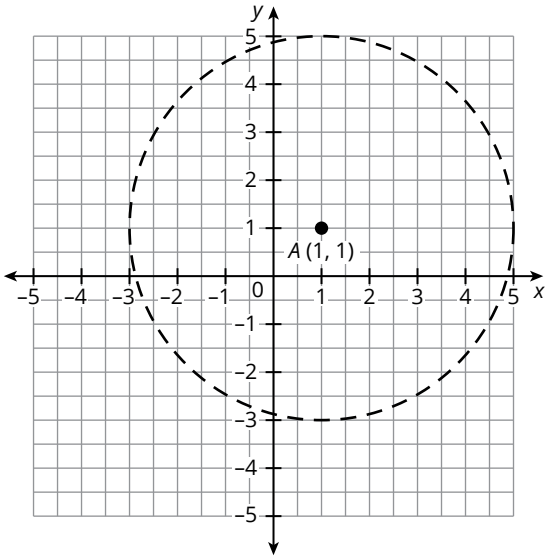

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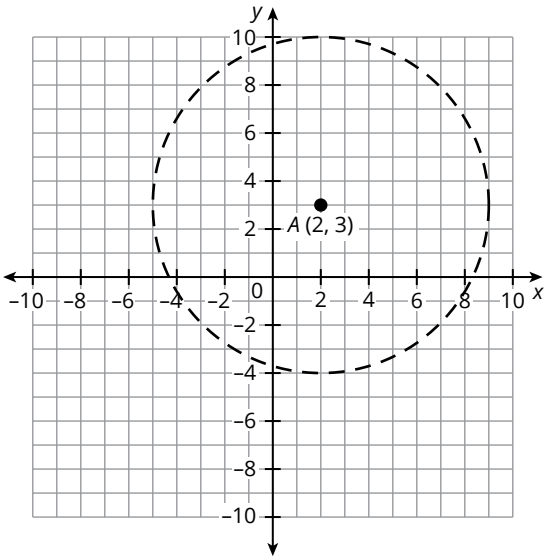

B. Write the equation of the circle. Then, determine whether each point lies on the circle.

1. Circle K has its center at (0, 0) and a radius of 4.

   Point N is at $(3, \sqrt{7})$.

2. Circle K has its center at (0, 0) and a radius of $\sqrt{65}$.

   Point N is at (4, 7).

3. Circle K has its center at (0, 0) and a radius of $\sqrt{34}$.

   Point N is at $(-3, -\sqrt{26})$.

4. Circle K has its center at (1, 2) and a radius of 5.

   Point N is at (4, –2).

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5. Circle K has its center at (3, 0) and a radius of 2.

   Point N is at $(4, -\sqrt{3})$.

6. Circle K has its center at (2, −5) and a radius of 6.

   Point N is at (8, −4).

7. Circle K has its center at (−1, 2) and a radius of $\sqrt{130}$.

   Point N is at (−10, −5)

8. Circle K has its center at (4, −3)and a radius of 5.

   Point N is at (2, 1).

9. Circle K has its center at $(0, -5)$ and a radius of $\sqrt{97}$.

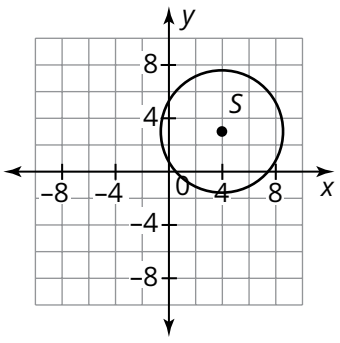
   Point N is at $(-4, 4)$.

10. Circle K has its center at $(0, 0)$ and a radius of $2\sqrt{2}$.

    Point N is at $(1, \sqrt{7})$.

Extension

1. Consider circle S with its center point located at $(4, 3)$ and a radius of $2\sqrt{5}$ units, as shown. If the x-coordinate of a point on the circle is 6, what are the possible values for the y-coordinate?



Spaced Practice

1. Write an equation in standard form for each description.

   a. A circle with a center at M (1, −4) and a radius of 7

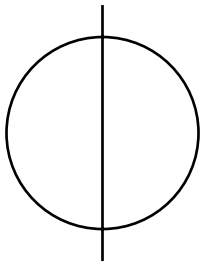
   b. A circle with the same center as circle M, but whose circumference is 10 times that of circle M

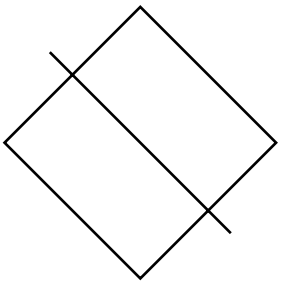
2. Determine whether $x^2 + y^2 + 6x + 8y + 9 = 0$ represents a circle. If so, write the equation in standard form and describe the location of the center and radius.

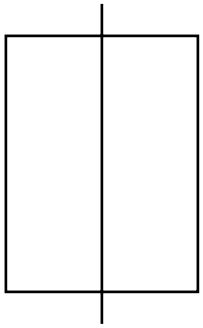
Name _____ Date _____

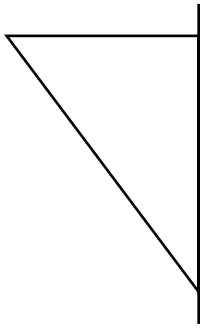
I. Building Three-Dimensional Figures

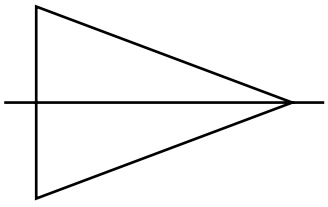
A. Write the name of the solid figure that would result from rotating the plane figure shown around the indicated axis.

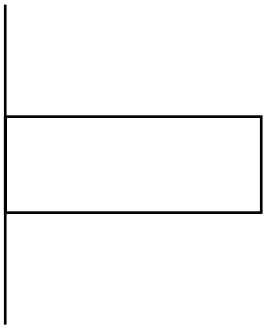
1. 

2. 

3. 

4. 

5. 

6. 

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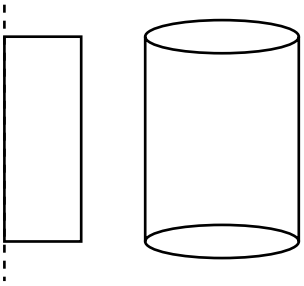
B. Relate the dimensions of the plane figure to the solid figure that results from its rotation around the given axis.

1.



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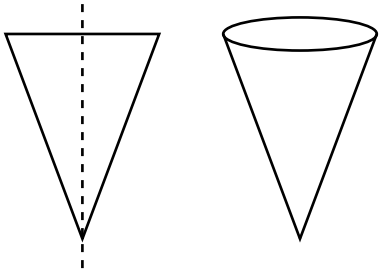

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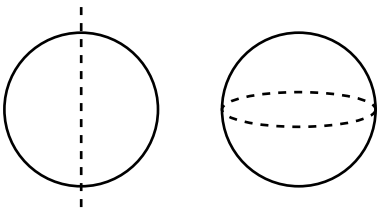

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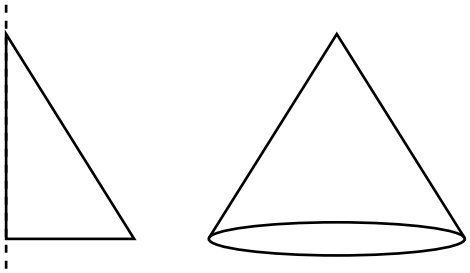

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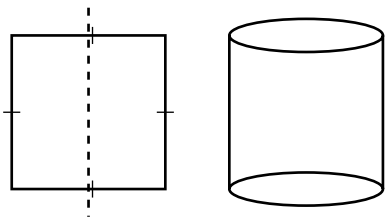

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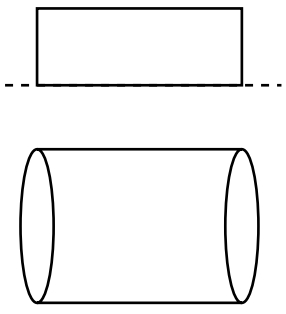
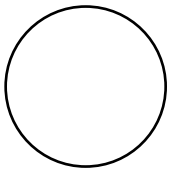
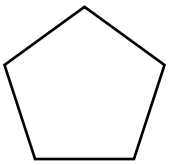
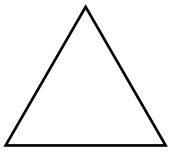
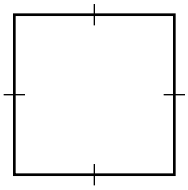


C. Name the solid formed by stacking 1000 of the congruent shapes shown.

1. 

2. 

3. 

4. 

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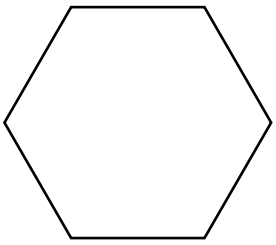
6. 

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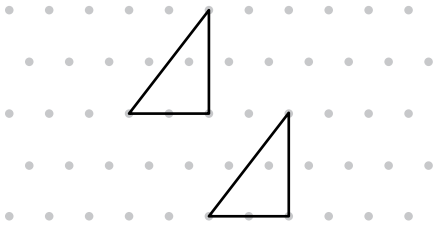
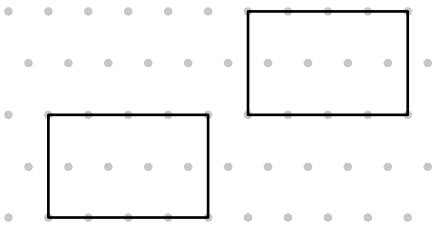
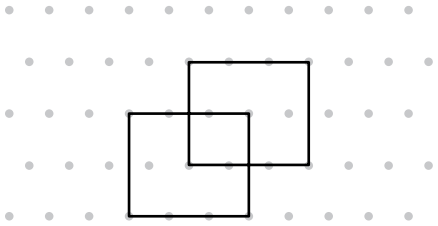
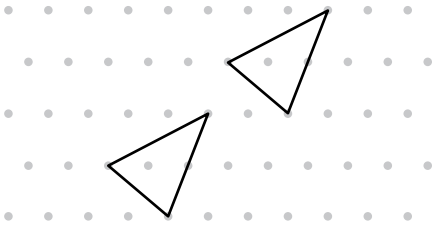
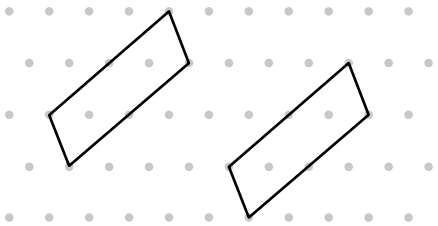
D. Connect the corresponding vertices of the figure and the translated figure. Name the shape that was translated and name the resulting solid figure.

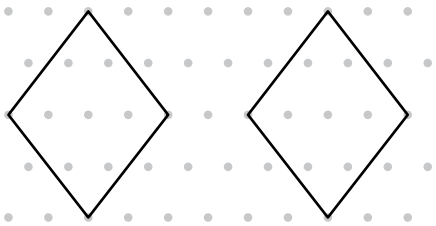
1. 

2. 

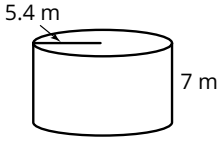
3. 

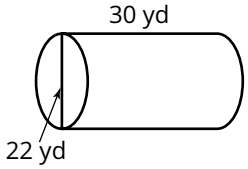
4. 

5. 

6. 

E. Calculate the volume of each cylinder. Round decimals to the nearest hundredth.

1. 

2. 

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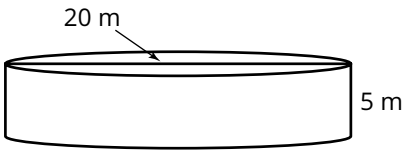

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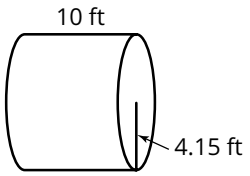

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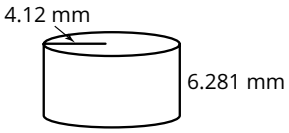

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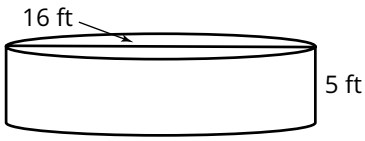

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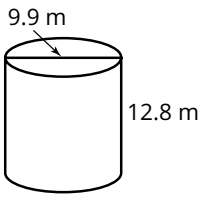

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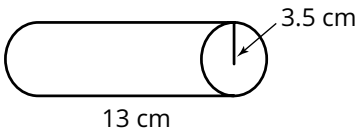


Extension

1. The penny is a commonly used coin in the U.S. monetary system. A penny has a diameter of 19 millimeters and a thickness of 1.27 millimeters. The volume of a penny is 360 cubic millimeters. Suppose you stack 10 pennies on top of each other to form a cylinder.

   a. What is the height of the stack of pennies?

b. What is the volume of the stack of pennies?

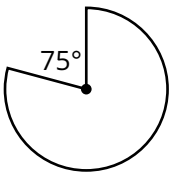
c. Determine the area of the face of a penny. Use 3.14 for π. Round your answer to the nearest hundredth.

d. Multiply the area of a penny by the height of the stack of pennies. Compare your product to the answer from part (b). How do the two answers compare? What does this tell you about calculating the volume of a cylinder?

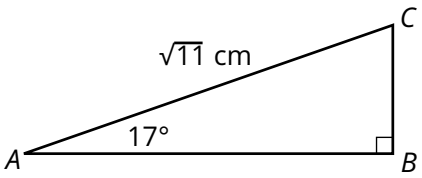
Spaced Practice

1. The Elizabeth Tower in London holds a clock with a bell known as Big Ben. The clock face measures 7 meters in diameter. Determine the area of the sector formed by the minute hand and the hour hand on the clock when the time is 11:00.

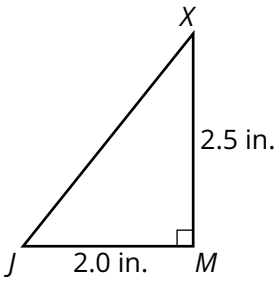
2. A pizza shop needs a new building sign. The owner wants a circular sign with a 5-foot diameter. He wants to remove a portion so that the shape looks like a pizza with a slice taken out of it, and then he is going to paint the sign. The diagram shows what the sign will look like. How much surface area of the sign will be left for the owner to paint? Use 3.14 for π. Round your answer to the nearest hundredth when necessary.



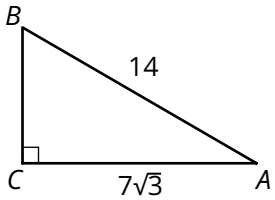
3. Determine side length BC. Round your answer to the nearest tenth.



4. Calculate the measure of $\angle X$. Round your answer to the nearest tenth.



5. Determine the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$, and $\frac{\text{opposite}}{\text{adjacent}}$ using $\angle A$ as the reference angle. Write your answers as fractions.



II. Cross-Sections

A. Describe how a plane can intersect each solid to form the given cross-section.

1. line segment, cylinder

2. point, cone

3. point, sphere

4. line segment, pyramid

5. line segment, cube

6. point, pyramid

B. Describe the shape of the cross-section created by each intersection described.

1. A plane intersects a cube such that the plane is parallel to a face of the cube.

2. A plane intersects a cube along the corresponding diagonals of opposite faces of the cube, so that the plane divides the cube into congruent triangular prisms.

3. A plane intersects a sphere such that the intersection is not a point and the plane does not go through the center of the sphere.

4. A plane intersects a sphere such that the plane goes through the center of the sphere.

5. A plane intersects a cone such that the plane is parallel to the base of the cone and the intersection is not at the vertex of the cone.

6. A plane intersects a cone such that the plane is perpendicular to the base of the cone and goes through the vertex of the cone.

7. A plane intersects a cylinder such that the plane is perpendicular to the bases of the cylinder and goes through the centers of both bases.

8. A plane intersects a cylinder such that the plane is parallel to the bases of the cylinder.

9. A plane intersects a rectangular pyramid such that the plane is parallel to the base of the pyramid and the intersection is not at the vertex of the pyramid.

10. A plane intersects a rectangular pyramid such that the plane is perpendicular to the base of the pyramid and the plane goes through the vertex of the pyramid.

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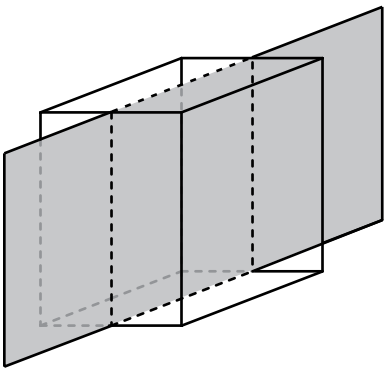
11. A plane intersects a cube such that one of the corners of the cube is cut off.

12. A plane intersects a cylinder such that the plane is neither parallel nor perpendicular to the bases of the cylinder, and the intersection is not a point.

C. Describe the shape of each cross-section shown.

1.



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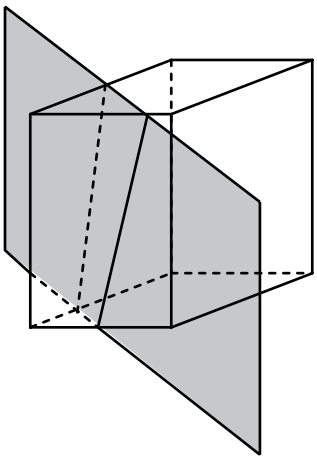

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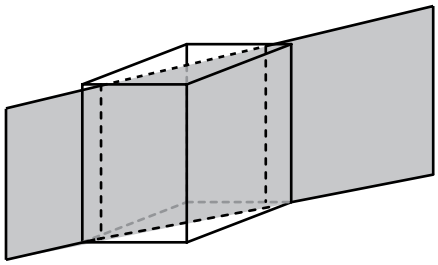

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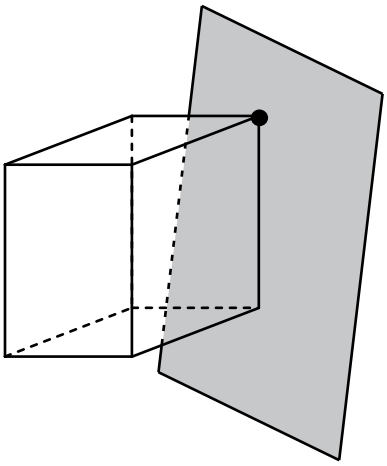

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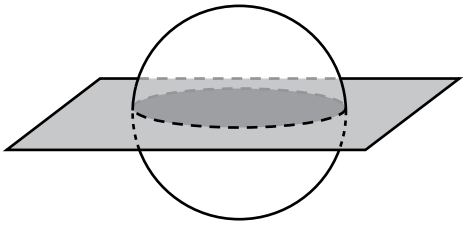

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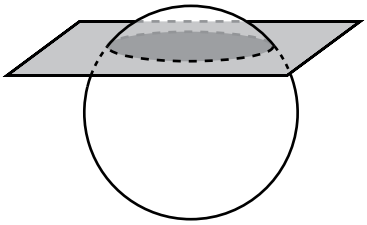

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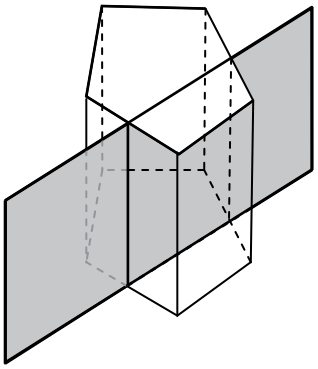

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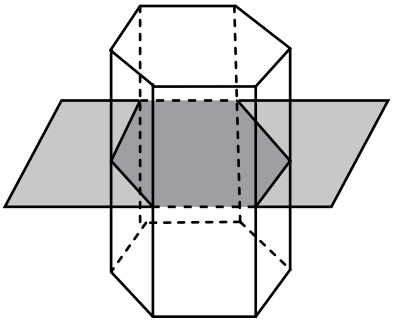

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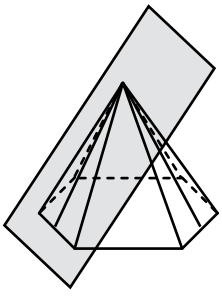

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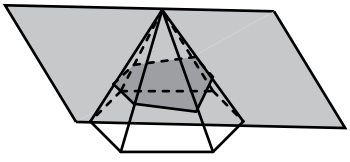


D. Use the given information to sketch and describe all possible cross-sections formed when a plane intersects each figure.

1. cylinder

2. rectangular prism

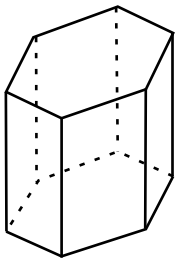
3. pentagonal prism

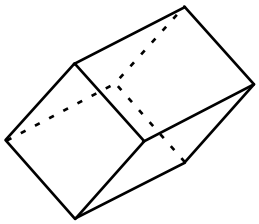
4. triangular prism

5. triangular pyramid

6. hexagonal pyramid

E. Consider two cross-sections of the given solid. One cross-section is parallel to the base of the solid, and the other cross-section is perpendicular to the base of the solid. Determine the shape of each of these cross-sections.

1. 

2. 

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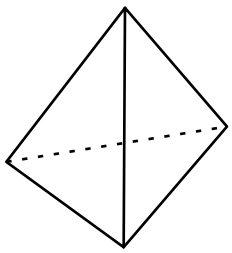

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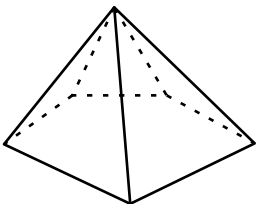

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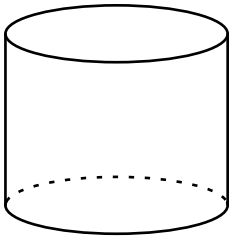

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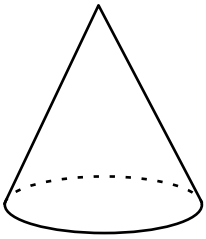
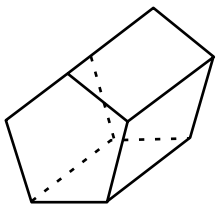
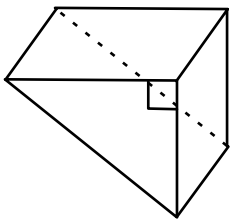
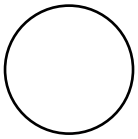


7. 

8. 

F. Draw a solid that could have each cross-section described.

1. cross-section parallel to the base



2. cross-section perpendicular to the base

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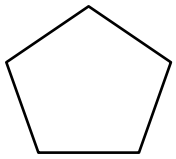
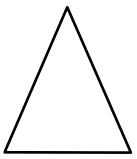
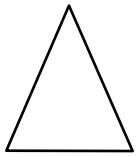
3. cross-section parallel to the base



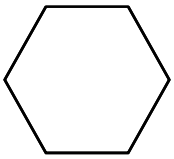
4. cross-section parallel to the base



5. cross-section perpendicular to the base



6. cross-section parallel to the base

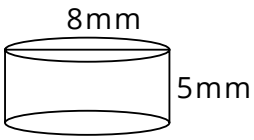


Extension

Determine whether there is a way that a plane can intersect a rectangular prism so that the resulting cross-section is a hexagon. When there is a way, sketch the plane intersecting the rectangular prism so that a hexagon shape results.

Spaced Practice

1. Determine the volume of the figure. Round to the nearest whole number.



2. Determine the length of each intercepted arc. Give the exact length and an approximation rounding to the nearest hundredth.

   a. The measure of the central angle is 75°. The length of the radius is 16 mm.

   b. The measure of the central angle is 140°. The length of the radius is 30 ft.

3. When $r = 7$ and the intercepted arc length is 6, what is the measure of the central angle in radians?

4. Convert 72° to radians.

5. Convert $\frac{\pi}{8}$ to degrees.

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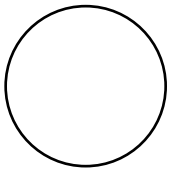
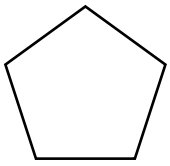
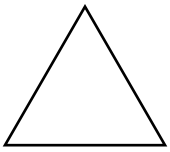
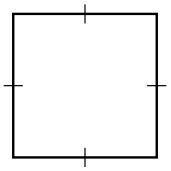
III. Building Volume Formulas for Pyramids, Cones, and Spheres

A. Name the solid formed by stacking similar shapes so that each layer of the stack is composed of a slightly smaller shape than the previous layer.

1. 

2. 

3. 

4. 

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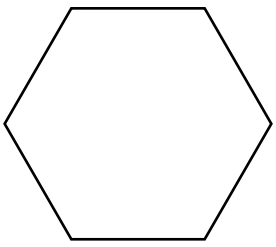
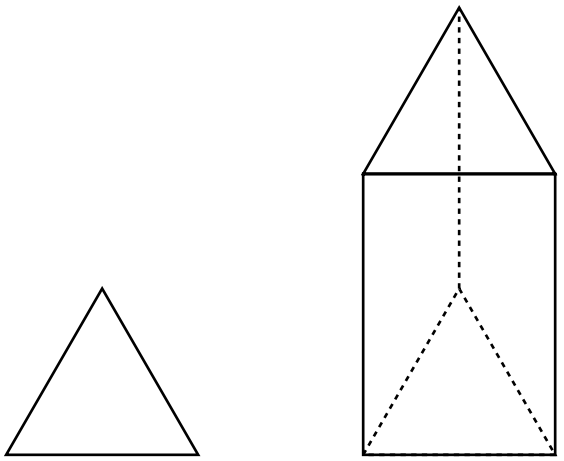
6. 

B. Relate the dimensions of the given plane shape to the related solid figure. Tell whether the shape was made by stacking congruent or similar shapes.

1.



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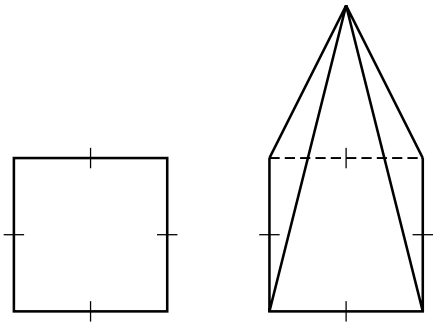

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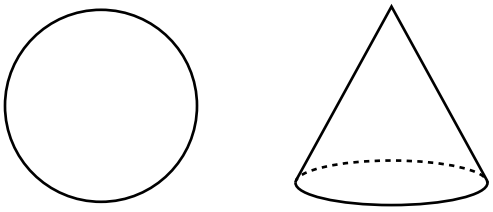

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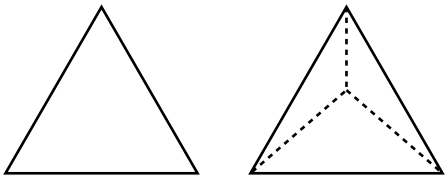

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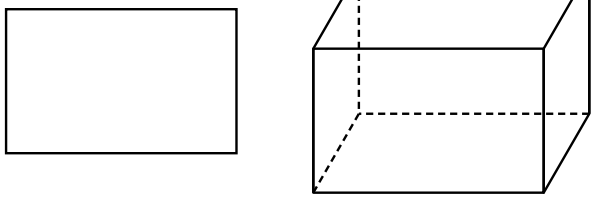

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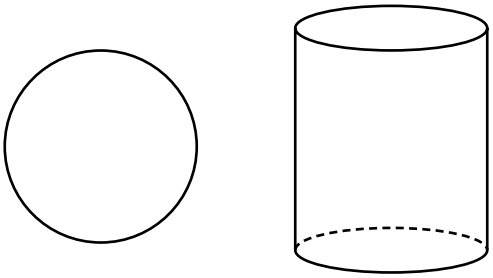
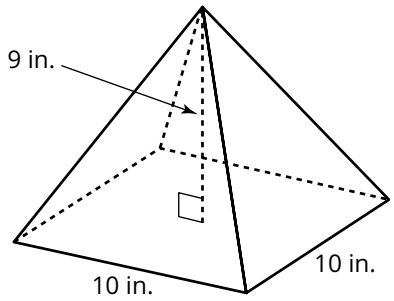


C. Calculate the volume of each square pyramid. Round answers to the nearest hundredth, when necessary.

1.



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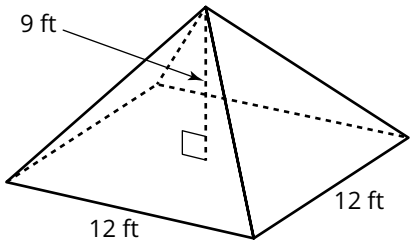

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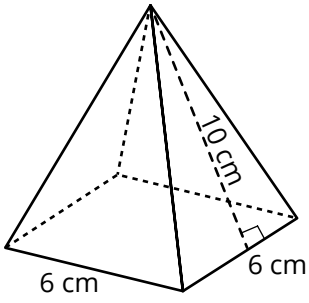

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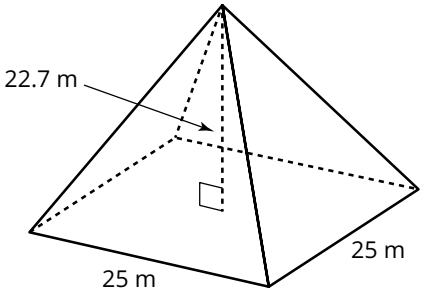

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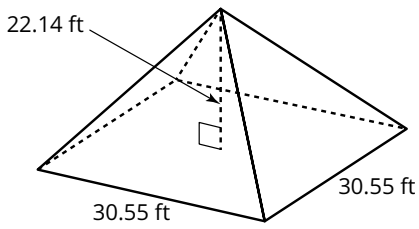

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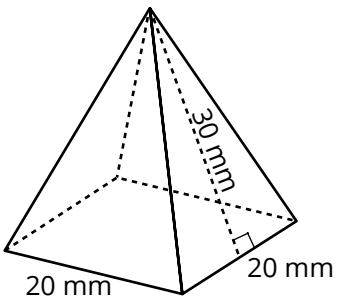
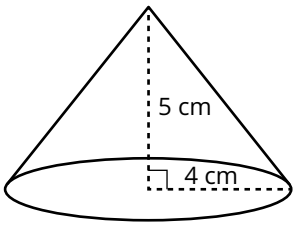


D. Calculate the volume of each cone. Express your answer in terms of π and decimal form rounded to the nearest hundredth.

1.



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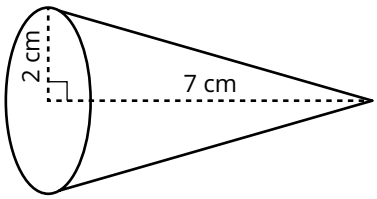

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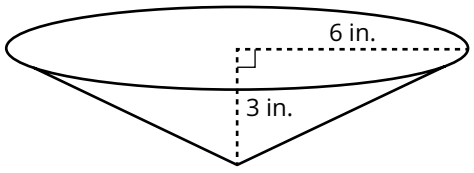

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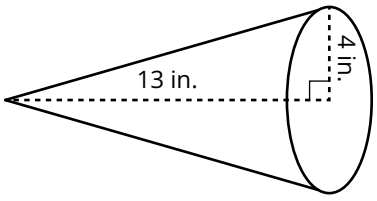

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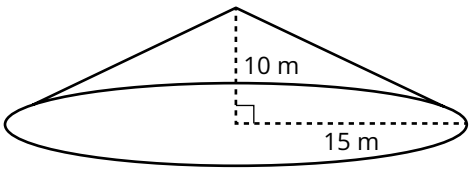

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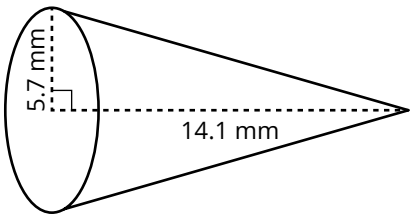

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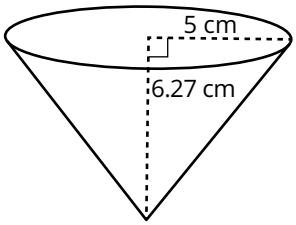

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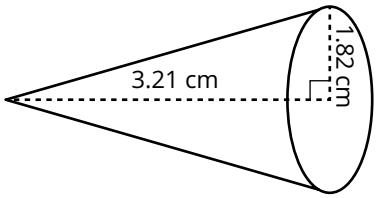
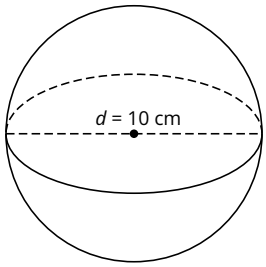


E. Determine the volume for each sphere or hemisphere. Express your answer in terms of π and in decimal form. Round your answer to the nearest hundredth, when needed.

1.



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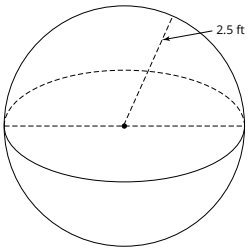

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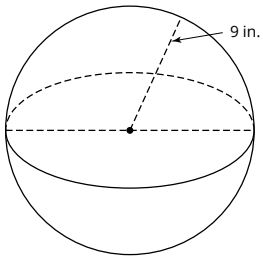

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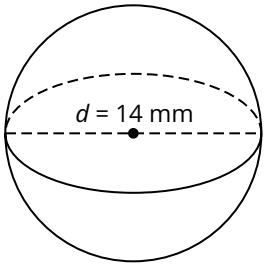

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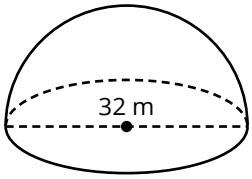

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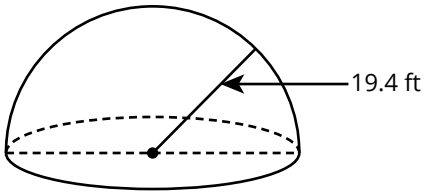


F. Determine each volume rounding to the nearest tenth and describe each effect on the volume when a dimension is changed.

1. Calculate the volume of a sphere with a radius of 8 meters.

2. Calculate the volume of the sphere in Question 1 when the radius is tripled.

CC BY NC

3. Describe how tripling the radius of a sphere impacts the volume of the sphere.

4. Calculate the volume of the sphere in Question 1 when the radius is halved.

5. Describe how halving the radius of a sphere impacts the volume of the sphere.

6. Calculate the volume of a cylinder with a radius of 6 feet and a height of 10 feet.

7. Calculate the volume of the cylinder in Question 6 when the radius is tripled.

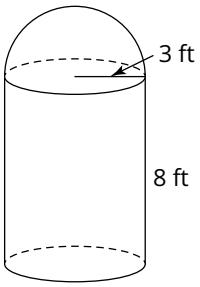
8. Describe how tripling the radius of a cylinder impacts the volume of the cylinder.

9. Calculate the volume of the cylinder in Question 6 when the radius is halved.

10. Describe how halving the radius of a cylinder impacts the volume of the cylinder.

G. Determine the volume of each composite figure. Round your answer to the nearest hundredth.

1.



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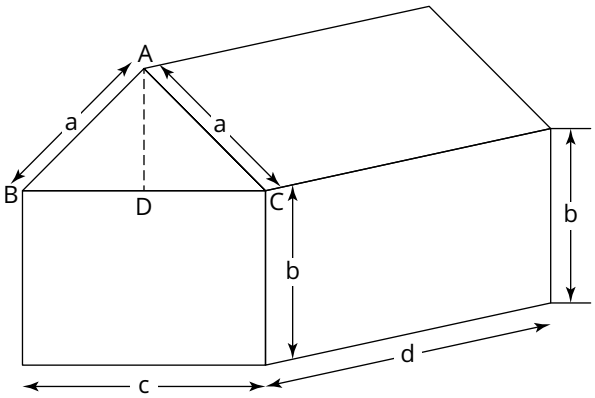
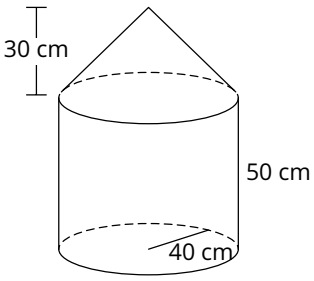


$a = 10$ ft, $b = 12$ ft, $c = 16$ ft, $d = 20$ ft, $AD = 6$ ft

3.



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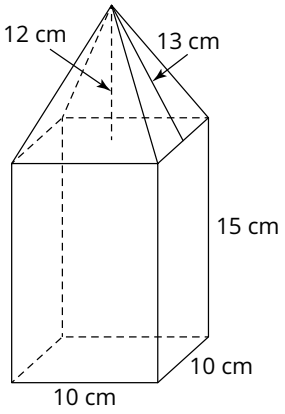
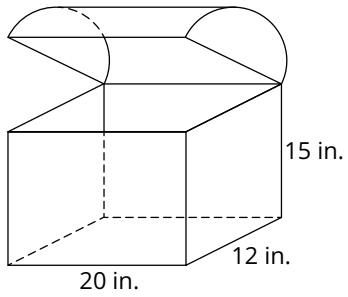
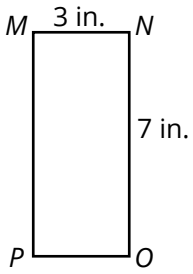


5. 

6. A sphere is placed in a right cylinder so that exactly half the sphere extends above the cylinder. The height of the cylinder is 9.1 cm and radius of the cylinder is 5.2 cm. Determine the volume of the figure.

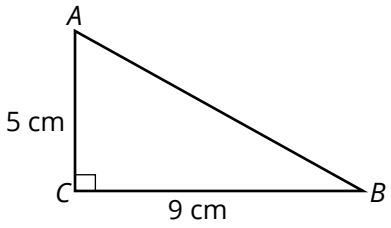
Extension

1. Rotate rectangle *MNOP* around line segment *NO*. Determine the volume of the solid that is formed. Round to the nearest hundredth.



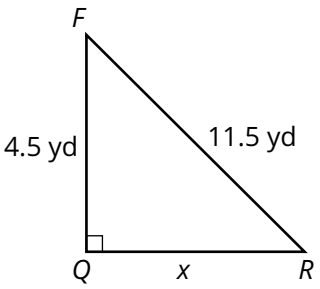
Spaced Practice

1. Rotate $\triangle ABC$ around line segment CB. What solid is formed and what are its dimensions?

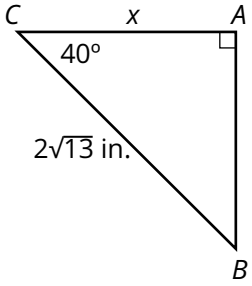


2. Solve for x in each triangle.

a.

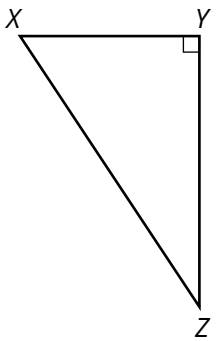


b.



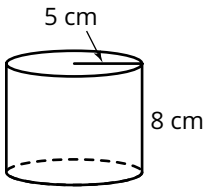
3. Complete the table when m∠Z is 30°.

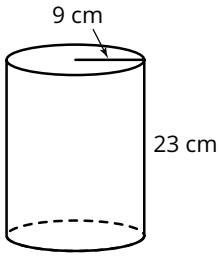
Reference Angle	sin	cos	tan
X	$\frac{\sqrt{3}}{2}$		
Z			

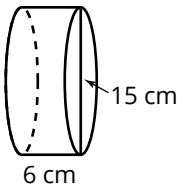


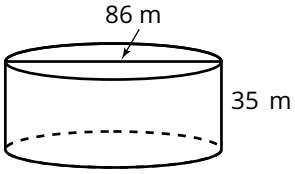
IV. Building Lateral and Surface Area Formulas for Pyramids, Cones, and Spheres

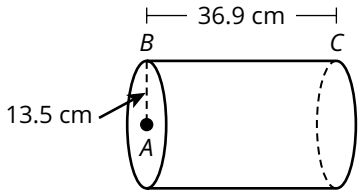
A. Determine lateral and total surface area of each right cylinder. Express your answer in terms of π and decimal rounded to the nearest hundredth.

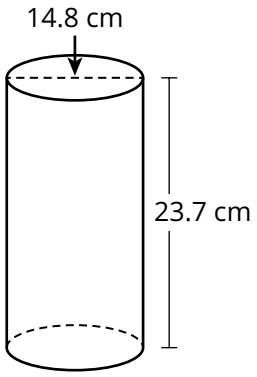
1. 

2. 

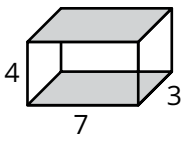
3. 

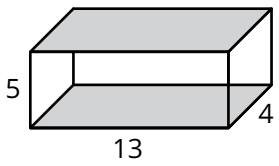
4. 

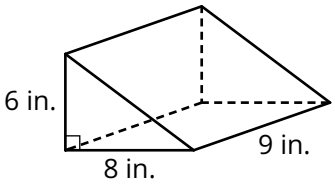
5. 

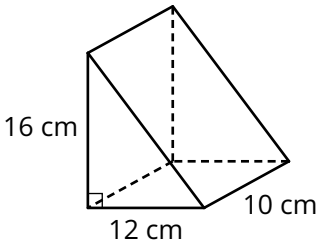
6. 

B. Determine the lateral surface area and total surface area of each right prism. When necessary, the prism's bases are shaded. Round your answer to the nearest hundredth, where appropriate.

1. 

2. 

3. 

4. 

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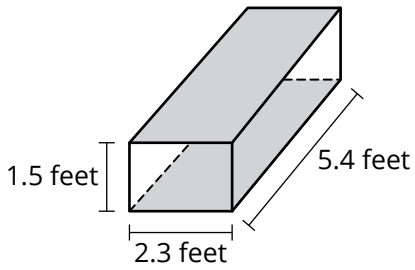

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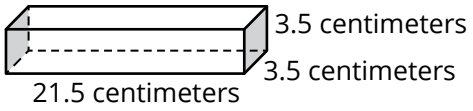
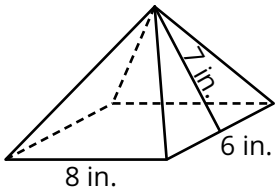


C. Determine the lateral surface area and total surface area of each right pyramid. Bases with a single label on one edge are regular polygons. Round your answer to the nearest hundredth, where appropriate.

1.



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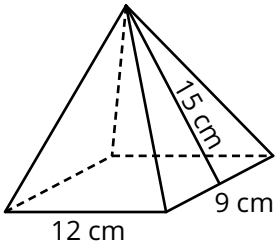

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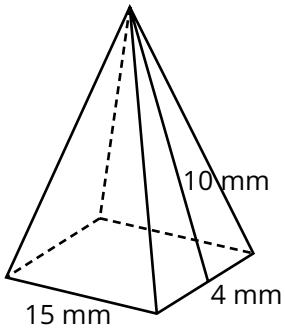

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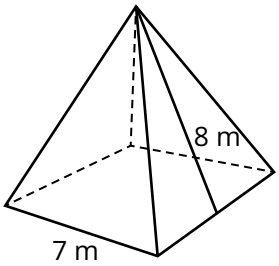

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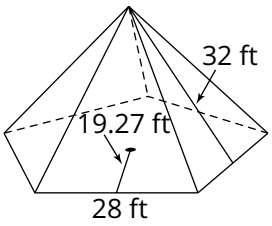

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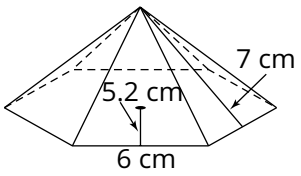
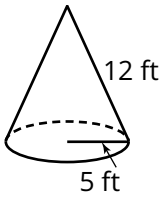


D. Determine the lateral surface area and total surface area of each right cone. Round your answer to the nearest hundredth.

1.



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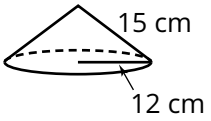

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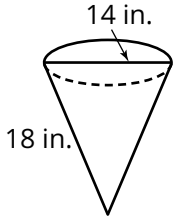

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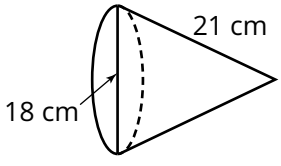

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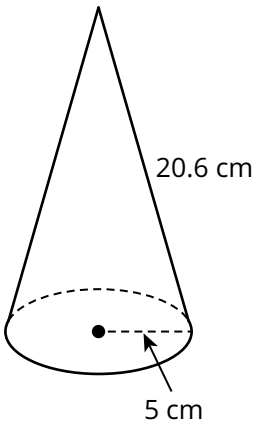

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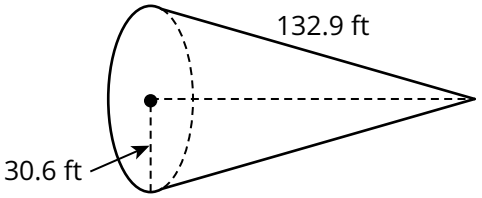
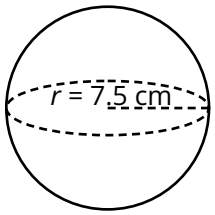


E. Determine the lateral and total surface area for each sphere or hemisphere. Express your answer in terms of π and in decimal form. Round your answer to the nearest hundredth, when needed.

1.



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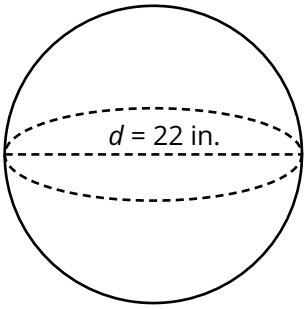

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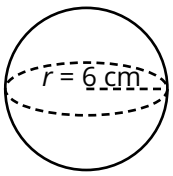

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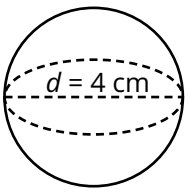

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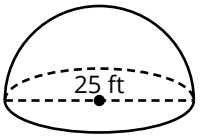

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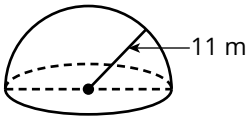


F. Determine each surface area rounding to the nearest tenth and describe each effect on the surface area when a dimension is changed.

1. Calculate the surface area of a sphere with a radius of 5 meters.

2. Calculate the surface area of the sphere in Question 1 when the radius is tripled.

3. Describe how tripling the radius of a sphere impacts the surface area of the sphere.

4. Calculate the surface area of the sphere in Question 1 when the radius is halved.

5. Describe how halving the radius of a sphere impacts the surface area of the sphere.

6. Calculate the surface area of a cylinder with a radius of 4 feet and a height with the same length as the radius.

7. Calculate the surface area of the cylinder in Question 6 when the radius is tripled.

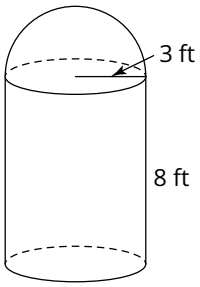
8. Describe how tripling the radius of the cylinder that had the same radius and height impacts the surface area of the cylinder.

9. Calculate the surface area of the cylinder in Question 6 when the radius is halved.

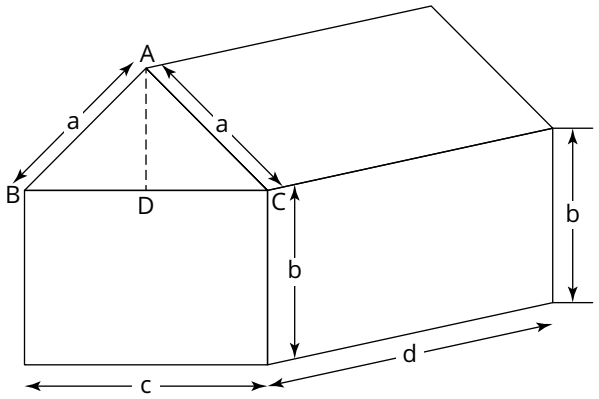
10. Describe how halving the radius of the cylinder that had the same radius and height impacts the surface area of the cylinder.

G. Determine the lateral surface area and total surface area of each composite figure. Round your answer to the nearest hundredth.

1.

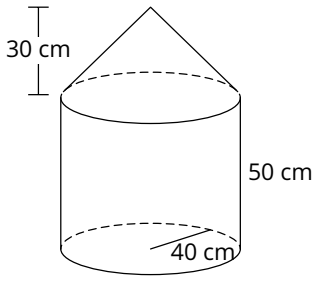


2.

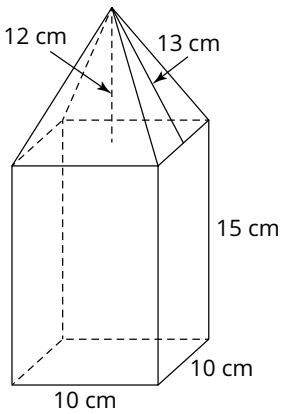


$a = 10$ ft, $b = 12$ ft, $c = 16$ ft, $d = 20$ ft, $AD = 6$ ft

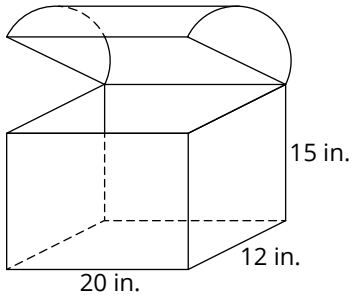
3.



4.



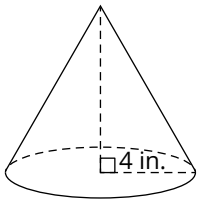
5.



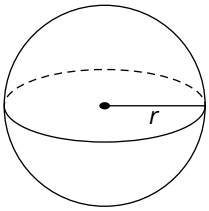
6. A sphere is placed in a right cylinder so that exactly half the sphere extends above the cylinder. The height of the cylinder is 9.1 cm and radius of the cylinder is 5.2 cm. Determine the total surface area of the figure.

H. Calculate the unknown dimension for each figure.

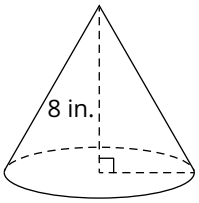
1. The surface area of the cone is 44π square inches. Determine its slant height.



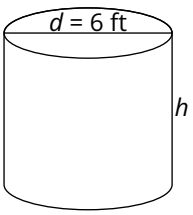
2. The volume of the sphere is 4500π cubic centimeters. Determine its radius.



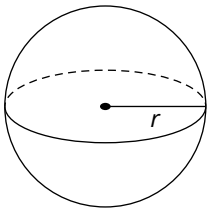
3. The volume of the cone is 96π cubic inches. Its height is 8 inches. Determine the radius of the cone.



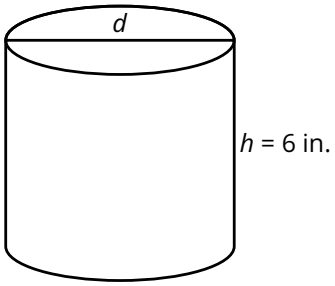
4. The surface area of the right cylinder is 60π square feet. Determine the height of the cylinder when its diameter is 6 feet.



5. The surface area of a sphere is approximately 113.04 square feet. Determine the radius of the sphere. Use 3.14 for π.



6. The volume of a cylinder is 301.44 cubic inches. Determine the diameter of the cylinder if the height is 6 inches. Use 3.14 for π.



I. Describe how the given changes to the dimensions of the figure will affect the indicated measure.

1. What happens to the area of a rectangle with length 12 inches and width 8 inches when its dimensions are increased by a factor of 2?

2. What happens to the volume of a sphere with a radius of 3 feet when the radius is decreased by a factor of $\frac{1}{3}$?

3. What happens to the perimeter of a regular pentagon with a side length of 17 centimeters when its dimensions are increased by 10 centimeters?

4. What happens to the surface area of a right rectangular prism with a length of 8 inches, a width of 6 inches, and a height of 5 inches when its dimensions are quadrupled?

5. What happens to the volume of a square pyramid with a height of 45 centimeters and a base side length of 25 when the dimensions are decreased by 5 centimeters?

6. What happens to the surface area of a cylinder with a height of 28 inches and a diameter of 16 centimeters when the dimensions are decreased by a factor of 4?

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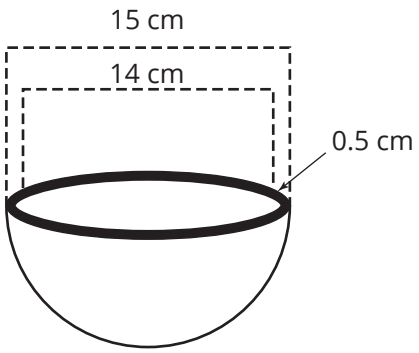
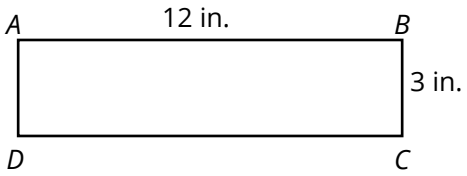
Extension

1. A kitchen bowl has an outside diameter of 15 cm and an inside diameter of 14 cm with a lip that is 0.5 cm wide. Determine the lateral surface area, the total surface area, and the bowl's capacity. Round your answers to the nearest whole number.



2. What is the lateral surface area and total surface area of the solid formed by rotating rectangle *ABCD* around line segment *BC*? When necessary round your answer to the nearest hundredth.



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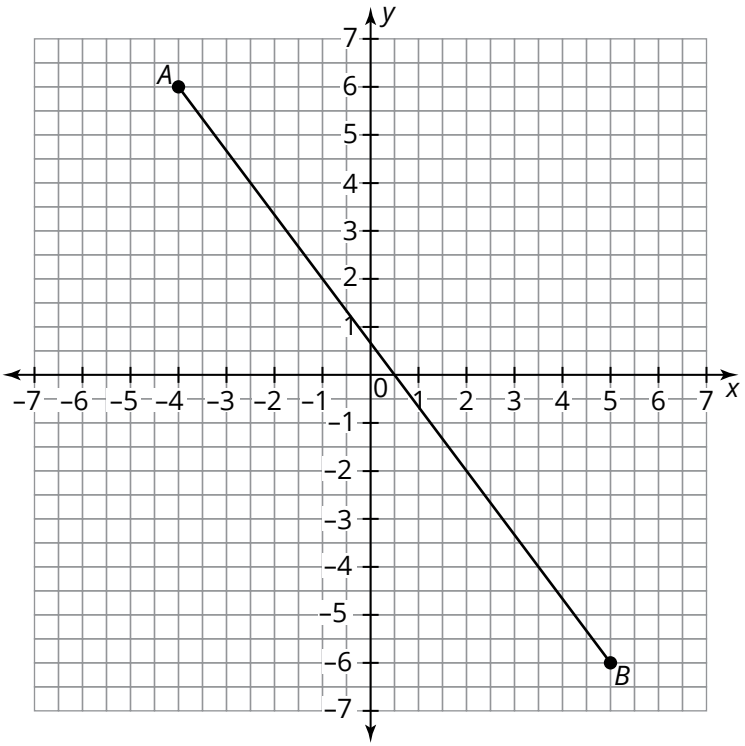
Spaced Practice

1. What are the coordinates of the point T that is $\frac{2}{3}$ of the way along the directed line segment AB shown on the coordinate plane?



2. $\triangle PQR$ has vertex coordinates P at (1, 2), Q at (5, 8), and R at (3, −4). What are the vertex coordinates of $\triangle PQR$ after it is translated down 4 units and reflected across the y-axis? Write the composition of transformation using coordinate notation.

3. If a quadrilateral has a right angle, then it is a rectangle. Determine the validity of the conjecture and justify your reasoning.

4. In Euclidean geometry, a line extends infinitely and part of that line is a line segment that can be measured. What shape corresponds to a line and line segment in spherical geometry? Is this shape infinite? What does a segment look like?

5. A company uses a triangle as their logo. The two sides of the triangular logo measure 5 mm and 9 mm. What are the possible lengths of the third side? Express as an inequality.

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Making Informed Decisions

TOPIC 1: Independence and Conditional Probability

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TOPIC 2: Computing Probabilities

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Name _____ Date _____

I. Compound Sample Spaces

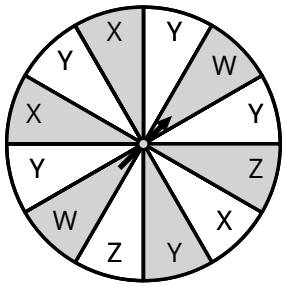
Topic Practice

A. Identify the sample space for each situation.

1. A number cube with sides labeled with 1 to 6 dots is rolled once.

2. An ice cream shop has a sale for its most popular ice cream flavors. Customers can have one scoop of ice cream in a cup or a cone, and the flavors on sale are chocolate, vanilla, and strawberry. It can be served with or without sprinkles.

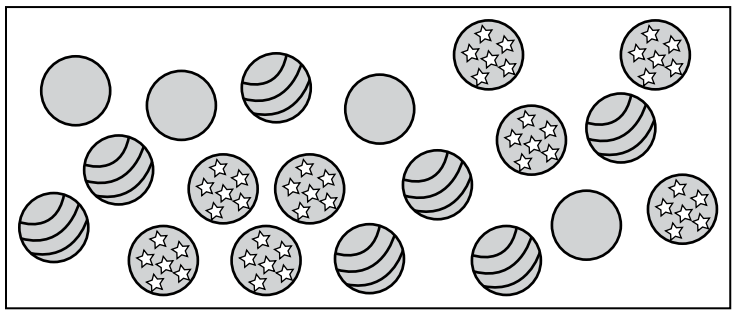
3. You spin the spinner one time.



4. A jar contains 3 red marbles, 4 blue marbles, 2 green marbles, and 1 yellow marble.

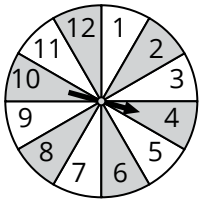
5. An even number between 1 and 15 is chosen at random.

6. A ball is chosen at random from the box.



B. Determine the probability of each event, $P(E)$, and its complement, $P(E^c)$.

1. You spin the spinner one time.



P(greater than 7) =

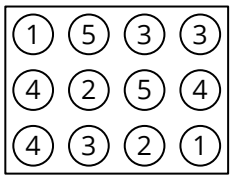
P(not greater than 7) =

2. You write the letters A to K on separate note cards. Then you choose a card at random.

P(vowel) =

P(not a vowel) =

3. You choose a ball at random from the box.



$P(5)$ =

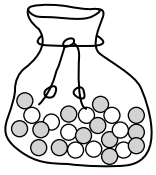
P(not a 5) =

4. You have 5 quarters, 2 dimes, 3 nickels, and 6 pennies. You choose a coin at random.

P(a coin worth more than 5 cents) =

P(not a coin worth more than 5 cents) =

5. There is a total of 30 balls, half of which are shaded. You choose a ball at random from the bag.



P(shaded) =

P(not shaded) =

6. Among the students in a class, 10 ride the bus, 3 walk, and 5 ride a car to school. A student is chosen at random.

P(walk) =

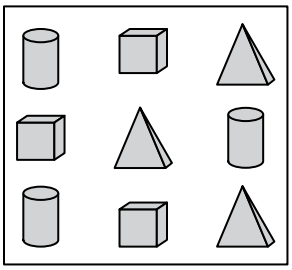
P(not walk) =

C. Construct a probability model for each situation. Then, state whether it is a uniform probability model or a non-uniform probability model.

1. A box contains 4 plain bagels, 2 blueberry bagels, 1 sesame seed bagel, and 5 cheese bagels. A bagel is chosen at random from the box.

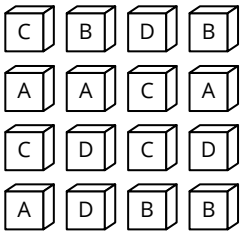
2. Jasmine has 3 pairs of blue socks, 2 pairs of white socks, 4 pairs of green socks, and 1 pair of brown socks. She chooses a pair of socks at random from a drawer.

3. A shape is chosen at random from the set.



4. There are 6 oranges, 4 apples, 3 kiwis, and 9 pears in your refrigerator. You randomly choose a piece of fruit to eat.

5. You randomly choose a block from the set.



6. A choral group consists of 5 sopranos, 3 altos, 4 tenors, and 3 basses. A group member is chosen at random to sing a solo at a concert.

D. Sketch a tree diagram, and write an organized list to represent each sample space.

1. Show all of the different 3-digit numbers using the numbers 4, 5, and 6, where each digit is used only once.

2. Lucas, Mason, Sebastian, and Gabriel race to the end of the field. Show all of the different ways of finishing in the top two spots.

3. Lunch includes a drink of your choice. The options are orange juice, apple juice, or cranberry juice. What are the possible outcomes for your choice of drink on two days?

4. What are the possible outcomes for flipping a coin 3 times?

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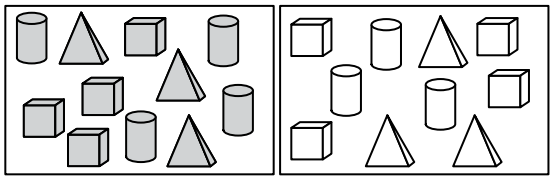
5. The pizza shop offers a weekly special that includes one free vegetable topping and one free meat topping with every large pizza. The vegetable toppings are peppers, mushrooms, onions, and olives. The meat toppings are sausage and pepperoni. Show all the possible combinations of a pizza with one vegetable topping and one meat topping.

6. You just make it to the ice cream store before closing. The only remaining frozen yogurt flavors are strawberry, peach, and lemon. You can choose one scoop in a cup or one scoop in a cone. Show all the possible combinations of one scoop of frozen yogurt in a cup or a cone.

E. For each situation, identify the following.

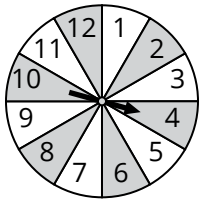
- What are the actions?
- What are the outcomes of each action?
- Do the outcomes of each action belong to disjoint sets or intersecting sets?
- What events are described?
- Are the events independent or dependent?

1. You randomly choose one shaded block and one unshaded block. The result is a shaded cylinder and an unshaded cube.



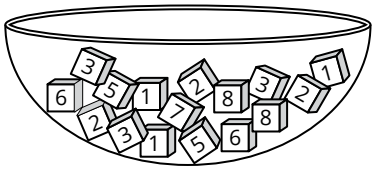
2. A teacher randomly chooses 2 students from a class, Diego and Luna, to solve a math problem on the board.

3. You spin the spinner and flip a coin, resulting in a 3 and tails up.



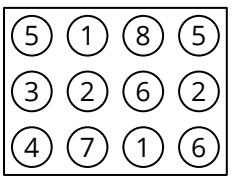
4. You randomly choose a number between 1 and 50. Your friend chooses a number between 51 and 100. Your choice is 6 and your friend's choice is 77.

5. A bowl contains numbered cubes. You randomly withdraw a cube from the bowl, and then your friend randomly withdraws a cube from the remaining ones. Your choice is a 3, and your friend's choice is a 5.



6. The school lunchroom offers a choice of 5 different vegetable wraps. You randomly choose a different one each day. On the first day of the week your choice was a mixed vegetable wrap, and on the second day your choice was a spinach and mushroom wrap.

7. You randomly choose one numbered ping pong ball and then choose another numbered ping pong ball. Your first choice is an even-numbered ping pong ball, and your second choice is an odd-numbered ping pong ball.



8. At the local deli, you can have your choice of bread and cheese on every sandwich. You randomly choose rye bread and Swiss cheese.

F. Use the Fundamental Counting Principle to determine the number of possible outcomes for each situation. Show your calculations.

1. There are 5 students scheduled to read their essays aloud in an English class one day. The teacher will randomly choose the order of the students. In how many different orders can the students read their essays?

2. A restaurant offers a special price for customers who order a sandwich, soup, and a drink for lunch. The diagram shows the restaurant's menu. How many different lunches are possible?

Lunch Menu		
Sandwiches	**Soup**	**Drinks**
Cheese Chicken Ham and Egg Turkey Club	Minestone Chicken Noodle Vegetable	Cola Tea Coffee

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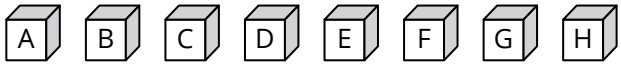
3. A website requires users to make up a password that consists of three letters (*A* to *Z*) followed by three digits (0 to 9). Neither letters nor digits can be repeated. How many different passwords are possible?

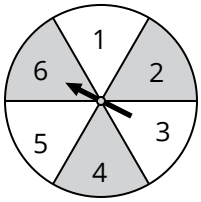
4. Letter blocks are arranged in a row from *A* to *H*, as shown.



How many different arrangements in a row could you make with these blocks?

5. Nahimana has 12 favorite songs. She sets her audio player to continuously play songs, randomly selecting a song each time. How many different possibilities are there for the first five songs that come up?

6. You spin the spinner shown in the diagram 5 times. How many different outcomes are possible?



7. A photographer arranges 12 members of a soccer team in a row to take a group picture. How many different arrangements are possible?

8. The travel lock shown in the figure requires users to move the spinners to a 4-digit code that will open the lock. Each spinner includes the digits 0 to 9. How many different codes are possible with the lock?

Extension

1. Kayla rolls two number cubes. Both number cubes are numbered from 1 to 6.

   a. What are the actions?

   b. What are the outcomes of each action?

   c. Determine the size of the sample space using the Fundamental Counting Principle. Show your calculation.

   d. Write an organized list that represents the sample space.

   e. What is the probability of rolling two dice with a sum greater than 9, P(sum greater than 9)?

2. Adriana rolls a number cube that is numbered 1 through 6. He then flips a coin with heads on one side and tails on the other side.

   a. Are the events in each outcome of the sample space independent or dependent? Explain.

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b. Write an organized list that represents the sample space.

c. What is the probability that the number on the cube is even and the result of the coin flip is tails, *P*(even number and tails)?

d. If just the cube is rolled, what is the probability that the number rolled on the cube is an even number, *P*(even number)?

e. If just the coin is flipped, what is the probability that it is tails, *P*(tails)?

f. How do the answers to parts (c), (d), and (e) relate to each other?

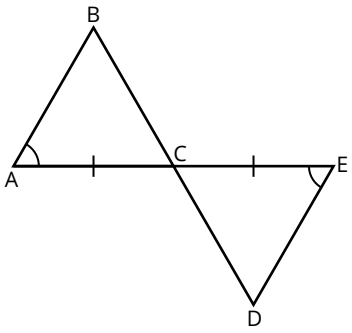
Spaced Practice

1. Write an equation in standard form for each.

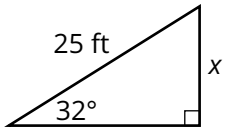
   a. A circle with a center at $M(-2, 15)$ and a radius of $\sqrt{11}$.

   b. A circle with the same center as circle *M*, but whose area is 3 times that of circle *M*.

2. Identify whether the triangles are congruent. Use a congruence theorem to justify your response and an explanation of why the triangles are congruent or not congruent.



3. Use a trigonometric ratio to solve for the value of x. Round your answer to the nearest tenth.

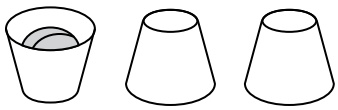


II. Compound Probability with *And*

Topic Practice

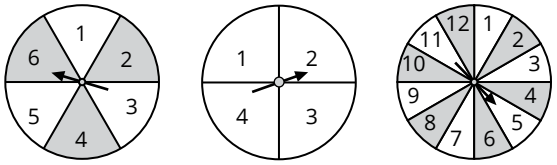
A. Determine the probability of each individual event. Then, determine the probability of each compound event. Show your calculations.

1. The "shell game" consists of placing three opaque cups, representing shells, upside down on a table and hiding a ball under one of the cups. A player, who has not seen where the ball is hidden, has to choose one of the cups. If the ball is hidden under it, the player wins. What is the probability that a player will win 5 times in a row?

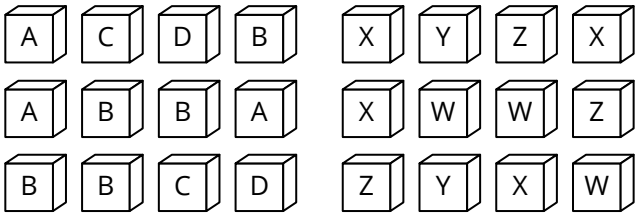


2. There are 24 students in a math class. Each day, the teacher randomly chooses 1 student to show a homework problem solution on the board. What is the probability that the same student will be chosen 5 days in a row?

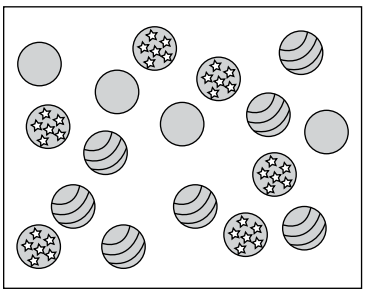
3. You spin each spinner in the diagram one time. What is the probability that the first two spinners land on 1?

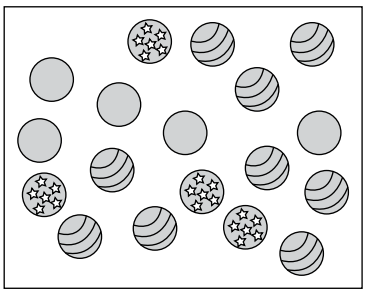


4. You randomly choose a block from each set below. What is the probability of choosing a block labeled W from the second set?



5. You randomly choose a marble from each set. What is the probability that both marbles have stripes?





6. A store is having a grand opening sale. To attract customers, the manager plans to randomly choose one of the first 50 customers each day for a prize. The prize giveaway will occur each day for 5 days. If you and a friend are among the first 50 customers each day, what is the probability that one or the other of you will win the prize every day?

B. Determine the probability that each event will occur. Then, determine the probability that both or all of the dependent events will occur. Show your calculations.

1. A common deck of playing cards includes 4 aces. Altogether there are 52 cards. When you randomly choose 4 cards from the deck, what is the probability of choosing 4 aces?

2. A bag contains 8 red ribbons, 7 green ribbons, and 3 yellow ribbons. When you randomly remove 3 of the ribbons from the bag, what is the probability that the first two ribbons will be yellow?

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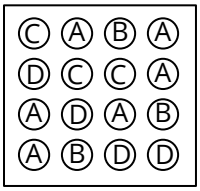
3. A box contains discs with letters on them, as shown in the diagram. You randomly remove four of the discs, one at a time, and set them in a row on a table. What is the probability that the discs you remove will be, in order, A B C D?



4. Logan has 6 quarters, 4 dimes, 3 nickels, and 8 pennies in his pocket. When he randomly removes 3 coins from his pocket, what is the probability of choosing a quarter first?

5. The table shows the birth months of students in a class. When 4 students in the class are chosen at random, what is the probability that they will all have birthdays in June, July, or August?

Month	January	February	March	April	May	June
Number of Students	2	3	1	0	3	2

Month	July	August	September	October	November	December
Number of Students	6	1	3	5	2	0

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6. Mei writes the numbers 1 to 45 on separate cards. She then randomly chooses three of the cards. What is the probability that the 1st card will not include the digit 9 in the number, but the 2nd and 3rd cards will include the digit 9 in the number?

Extension

1. A batch of 100 calculators are manufactured in a day on a factory line. There are 5 calculators that are defective in the batch. On another line, a batch of 5000 computers are manufactured. There are 50 computers that are defective in the batch. On the third line, a batch of 1000 fitness watches are manufactured. There are 25 watches that are defective in the batch.

   a. A quality control inspector randomly picks a calculator, a computer, and a fitness watch from the three batches. What is the probability all of them are defective?

b. A quality control inspector randomly picks 3 calculators from the batch without replacing any of them. What is the probability the first is defective and the second two are not defective?

c. A quality control inspector randomly picks 2 calculators, 1 computer, and 2 watches without replacing any of them. What is the probability none of the products are defective?

2. Parker has a bag of candy that contains 4 lollipops, 8 chocolate bars, and 5 pieces of taffy.

   a. When Parker randomly picks a piece of candy, what is the probability it is a lollipop?

   b. When Parker randomly picks a piece of candy, what is the probability it is a chocolate bar?

   c. When Parker randomly picks a piece of candy, what is the probability it is a lollipop or a chocolate bar?

   d. What is the mathematical relationship between the answers to parts (a), (b), and (c)?

Spaced Practice

1. Suppose you spin an evenly divided spinner once. The spinner is numbered 1 through 10.

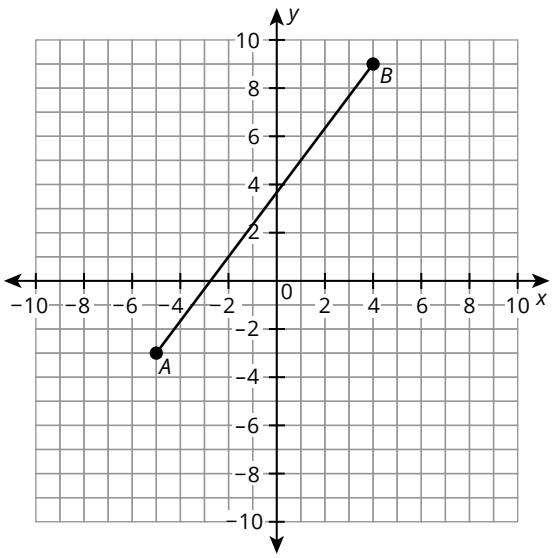
   a. Identify the sample space.

   b. Determine the probability of landing on an odd number, $P(\text{odd})$.

   c. Determine the probability of landing on a number that is not a multiple of 5, $P(\text{not a multiple of 5})$.

2. Determine whether $4x^2 + 4y^2 - 16x - 24y + 16 = 0$ represents a circle. When yes describe the location of the center and radius.

3. The directed line segment AB is shown on the coordinate plane. What are the coordinates of the point T located $\frac{1}{3}$ of the way between point A and point B?



4. Write the converse and inverse of this statement. Then, determine the validity of the original statement, its converse and its inverse.

   If a polygon has exactly three sides, then the polygon is a triangle.

5. Write the converse of this statement. Then, determine the validity of the original statement and its converse. When possible, write a corresponding biconditional statement.

   If a rectangle has four equal sides, then the rectangle is a square.

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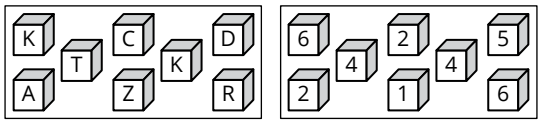
III. Compound Probability with *Or*

Topic Practice

A. Use the Addition Rule for probability to determine the probability that one or the other of the independent events described will occur.

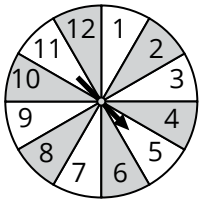
1. You randomly choose a block from each set in the diagram. What is the probability that you will choose a block labeled with a T or a block labeled with a 6?



2. The vegetable display at a market has exactly 48 apples and 36 oranges. Of these, 2 of the apples are rotten and 2 of the oranges are rotten. You randomly choose an apple and an orange from the display. What is the probability that the apple or the orange is rotten?

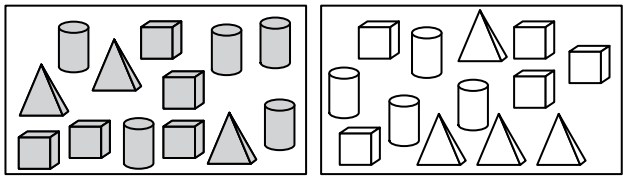
3. The sides of a 6-sided number cube are labeled from 1 to 6. You roll the cube 2 times. What is the probability that it will land with a 1 facing up the first roll or the second roll?

4. You spin the spinner 2 times. What is the probability that it will land on a number greater than 9 the first spin or a number less than 6 the second spin?



5. There are 28 students in a math class and 24 students in a history class. In each of the classes, 7 of the students are members of the school band. A student is chosen at random from each class. What is the probability that the student chosen in the math class or the student chosen in the history class is in the band?

6. You randomly choose a block from each set of shapes. What is the probability of choosing a pyramid from the shaded set or a cylinder from the unshaded set?



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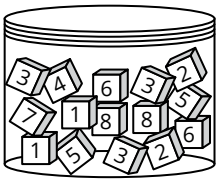
B. Use the Addition Rule for probability to determine the probability that one or the other of the dependent events will occur.

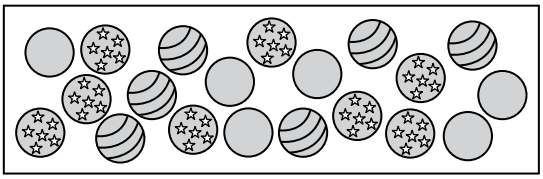
1. You have 6 blue socks, 8 white socks, 4 green socks, and 2 brown socks in a drawer. You randomly remove 2 socks from the drawer. What is the probability that the first sock will be blue or the second sock will be green?

2. The figure shows number cubes in a jar. Without looking, you randomly remove two cubes from the jar. What is the probability that the first cube you remove will have a 2 on it or the second cube you remove will have a 3 on it?



3. You and a friend decide to sign up for soccer tryouts. Altogether, there are 42 people trying out. What is the probability that you will be chosen to try out first or your friend will be chosen to try out second?

4. You choose two balls from the set in the figure and place both balls on a table. What is the probability that the first ball you choose will have stars on it or the second ball you choose will have stripes on it?



5. A standard deck of cards has four aces, four Kings, and four Queens. There are 52 cards altogether in the deck. One at a time, you randomly choose two cards from the deck and lay them on a table. What is the probability that the first card you choose is an ace or the second card you choose is a King?

6. You have 26 songs on your music player. Of these, 4 are your favorite songs. Your player is set to randomly play different songs until all 26 are played. If you listen to 2 songs, what is the probability that the first song played or the second song played will be one of your favorites?

7. You decide to randomly choose two days this week to go jogging. What is the probability that the first day you choose will be Monday or the second day you choose will be Tuesday?

8. You randomly choose two different numbers in the box below. What is the probability that the first number you choose will be in a shaded box or the second number you choose will be in a shaded box?

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

Extension

1. Two decks of cards are used for a game.

   a. A player chooses one card from Deck A and one card from Deck B. Write an organized list to represent the sample space in this situation.

Deck A

K	Q	J
3 cards	2 cards	1 card

Deck B

10	9
3 cards	2 cards

   b. Use the sample space to determine the probability that a Queen is selected from Deck A or a 10 is selected from Deck B.

c. Use the Addition Rule for Probability to determine the probability that a Queen is selected from Deck A or a 10 is selected from Deck B. Show your work.

2. A deck of cards contains only face cards. There are four kings, four queens, and four jacks.

   a. A card is randomly selected. What is the probability the card is a queen?

   b. Two cards are randomly selected. If the first card selected is a queen and the card is replaced in the deck, what is the probability the second card is a queen?

   c. Two cards are randomly selected. If the first card selected is a queen and the card is not replaced in the deck, what is the probability the second card is a queen?

   d. Two cards are randomly selected. If the first card selected is a queen and the card is replaced in the deck, what is the probability the second card is a king?

   e. Two cards are randomly selected. If the first card selected is a queen and the card is not replaced in the deck, what is the probability the second card is a king?

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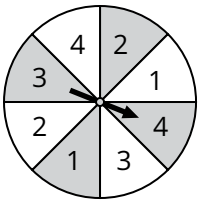
Spaced Practice

1. A board game includes a spinner and a deck of cards. A player spins the spinner once and then randomly chooses a card. What is the probability that the spinner will land on a 3 and the player will choose a card that says, "Go Back 1 Space"?



Pick a Card	Lose a Turn	Go Back 1 Space	Go Ahead 1 Space
6 cards	4 cards	3 cards	2 cards

2. A school district has two schools, School A and School B. The drawing shows the number of students in each class that had perfect attendance last year. The superintendent of the schools randomly chooses two students from School B. What is the probability that both students are from Class 6?

School A

Class 1	Class 2	Class 3
7 students	5 students	4 students

School B

Class 4	Class 5	Class 6	Class 7
8 students	6 students	3 students	2 students

3. Determine the validity of this statement. For a statement that is false, provide a counterexample that justifies your response.

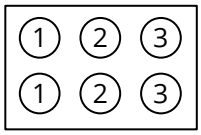
   If a quadrilateral has 2 pairs of parallel sides, then the quadrilateral is a rectangle.

IV. Calculating Compound Probability

Topic Practice

A. Determine the probability that each compound event will occur with replacement.

1. You randomly choose a number from the set, replace it, and then randomly choose another number. What is the probability of choosing a 2 first and a 3 second?



2. A box contains 25 marbles. There are 6 blue, 2 green, 8 red, 1 yellow, and 8 orange marbles. You randomly choose 3 marbles, one after the other. Each time, you place the marble back in the box before choosing the next one. What is the probability that the first marble is green, the second marble is red, and the third marble is blue?

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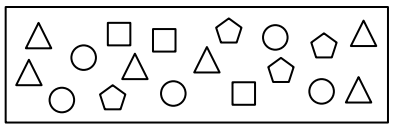
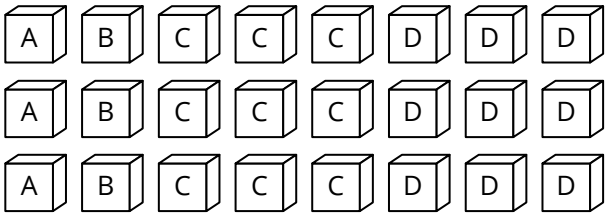
3. You choose a shape at random from the box, replace it, and then choose another shape at random. What is the probability that the first shape is a triangle or the second is a square?



4. You choose a block at random from the set, replace it, and then choose another block. What is the probability that you will choose an A block the first time or a D block the second time?

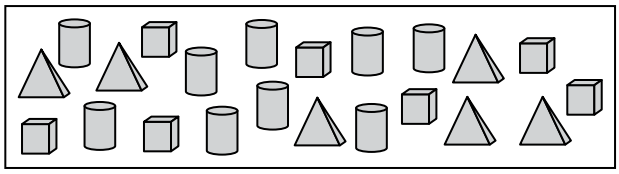


5. You have 4 quarters, 6 dimes, 3 nickels, and 9 pennies in your pocket. You randomly draw a coin out of your pocket, replace it, and then draw out another coin. What is the probability that the first coin is a quarter or the second coin is a dime?

6. A box contains 6 blue blocks, 4 green blocks, 8 orange blocks, 12 yellow blocks, and 14 red blocks. You randomly choose 3 blocks from the box. Each time you choose a block, you replace it before choosing the next one. What is the probability of choosing a green block first, a yellow block second, and a blue block third?

B. Determine the probability that each compound event will occur without replacement.

1. You randomly choose three shapes from the set, one after the other, without replacement. What is the probability that the first shape is a pyramid, the second shape is a cube, and the third shape is a cylinder?



2. A fruit bowl contains 6 apples, 2 pears, and 4 oranges. You randomly choose one fruit, and then without replacement, you choose another fruit. What is the probability that you choose a pear first or an orange second?

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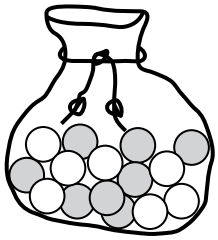
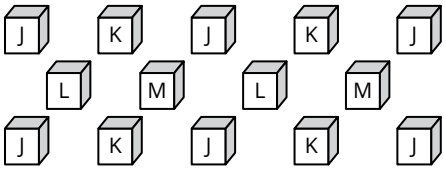
3. You randomly choose one ball from the bag without replacement, and then choose another ball. What is the probability that you will choose a white ball first or a shaded ball second?



4. A teacher is dividing the 24 members of a class into groups to work on different projects. The letter A, B, or C is written on each of 24 cards, and the cards are placed in a box. There are eight A cards, six B cards, and ten C cards. Each student randomly draws a card from the box, without replacement, to determine the student's group assignment. What is the probability that the first student will draw out an A or the second student will draw out a B?

5. You have 8 black socks, 6 blue socks, 2 green socks, and 4 white socks in a drawer. You randomly draw out two socks, one after the other, without replacement. What is the probability that you will draw out a black sock first and a black sock second?

6. You draw a block at random from the set. Then, without replacing it, you draw another block at random from the set. What is the probability that the first block has a J on it or the second block has a K on it?



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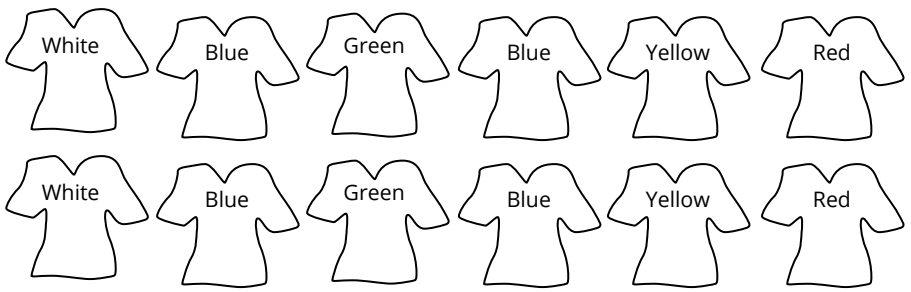
7. A standard deck of 52 playing cards is composed of four cards each of aces, Kings, Queens, and Jacks, as well as four cards of each number from 2 to 10. You randomly draw out a card and, without replacement, then draw out another card. What is the probability that the first card is a numbered card or the second card is a King?

8. The diagram shows the t-shirts that you have in a drawer. You randomly remove two t-shirts from the drawer, one after the other, without replacement. What is the probability that the first t-shirt will be blue and the second t-shirt will be blue?



Extension

1. A game includes a deck of cards with a color on each card. The table shows the numbers of each type of card. When a card is chosen, it is not replaced in the deck.

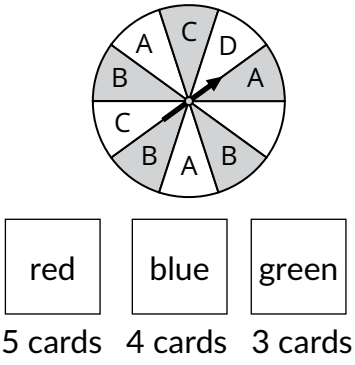
Number of Cards	Color on Card
15	red
12	yellow
10	green
14	blue
16	orange
15	purple
20	teal

   a. A player draws out ten cards. What is the probability that all the cards are red except the last card, which is teal?

   b. A player draws out five cards. What is the probability that none of the cards are orange?

Spaced Practice

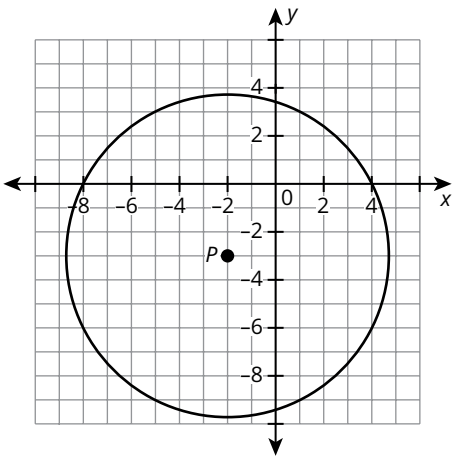
1. Consider the spinner and the set of cards shown in the figure. A player spins the spinner one time and then randomly chooses a card. What is the probability that the spinner will land on the letter B or the player will choose a blue card?



2. Two bags of marbles with the given contents are used for a game.
   Bag A: 6 red, 5 blue, 4 white
   Bag B: 6 green, 5 orange, 4 purple, 3 black. A player chooses two marbles from Bag B. What is the probability that the player will choose an orange marble first or a black marble second?

3. Consider circle P with its center point located at $(-2, -3)$ and a radius of $3\sqrt{5}$ units, as shown.

   Verify that point $H(-8, 0)$ lies on circle P.



4. A landscaping company is creating a triangular flowerbed. They are using a 6-foot board and an 11-foot board as the two sides of this triangular flowerbed. What are the possible lengths of the third board they could use to complete the triangular flowerbed? Write your response as a compound inequality.

Name _____ Date _____

I. Compound Probability for Data Displayed in Two-Way Tables

Topic Practice

A. The two-way frequency table shows the current inventory of hardwood that a lumberyard carries. Suppose a board is selected at random from the lumberyard's inventory. Use the table to calculate each probability. Round to the nearest tenth of a percent, if necessary.

Type of Hardwood \ Size	1 · 2	1 · 3	1 · 4	1 · 6	Total
Oak	20	13	17	12	62
Maple	14	28	9	19	70
Cherry	8	17	28	25	78
Total	42	58	54	56	210

1. $P(\text{oak})$

2. $P(\text{maple})$

3. $P(\text{cherry})$

4. $P(1 \cdot 2)$

5. $P(1 \cdot 3)$

6. $P(1 \cdot 4)$

7. $P(1 \cdot 6)$

8. $P(\text{maple and } 1 \cdot 3)$

9. $P(\text{oak and } 1 \cdot 2)$

10. $P(\text{maple or cherry})$

11. $P(\text{cherry or } 1 \cdot 4)$

12. $P(\text{maple or } 1 \cdot 6)$

B. Of the students in Jesse's homeroom, 10 have brown hair, 6 have black hair, 4 have auburn hair, 3 have blonde hair, and 2 have red hair. Calculate each relative frequency. Write each result as a fraction and as a decimal.

1. brown hair

2. black hair

3. auburn hair

4. blonde hair

5. red hair

6. brown or black hair

7. auburn or red hair

8. not brown hair

C. The two-way relative frequency table shows the results of a survey on the mayor's job approval. Suppose a member of the sample population is selected at random. Use the table of relative frequencies to calculate each probability. Express each probability as a decimal.

Do You Approve of the Mayor's Job Performance?	Party Affiliation: Republican	Democrat	Independent	Total
Approve	0.14	0.25	0.03	0.42
Disapprove	0.22	0.1	0.12	0.44
No Opinion	0.03	0.07	0.04	0.14
Total	0.39	0.42	0.19	1

1. *P*(approve)

2. *P*(disapprove)

3. *P*(no opinion)

4. *P*(Republican)

5. *P*(Democrat)

6. *P*(independent)

7. *P*(Republican and disapprove)

8. *P*(Democrat and no opinion)

9. *P*(independent and approve)

10. *P*(disapprove or no opinion)

11. *P*(Democrat or independent)

12. *P*(Democrat or disapprove)

13. *P*(Republican or approve)

14. *P*(independent or disapprove)

Extension

1. An amusement park randomly surveyed 130 guests about the type of ride they preferred between three types of rides: roller coaster, ferris wheel, and pirate ship. The guests were also asked their ages. The responses and totals are shown in the table.

		Type of Ride			Total
		Roller Coaster	Ferris Wheel	Pirate Ship	
Age	< 30	36	4	1	41
	30–39	25	6	8	39
	40–49	12	12	5	29
	50–59	4	10	7	21
Total		77	32	21	130

a. What is the probability that a randomly chosen guest prefers the ferris wheel?

b. What is the probability that a randomly chosen guest is aged 40–49 and prefers the roller coaster?

c. What number for the probabilities from parts (a) and (b) denotes the sample space?

d. Suppose a guest is randomly chosen and it is known that they preferred the pirate ship. How many guests would be included in the sample space? Explain your reasoning.

e. What is the probability that out of the guests who preferred the pirate ship, the guest was younger than 30 years old?

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Spaced Practice

1. A game includes a deck of cards with a shape on each card. The table shows the number of each type of card. Suppose each time a card is chosen, the card is replaced before another card is chosen. A player draws three cards. What is the probability the first card will have a square on it and the last card will have a star on it?

Number of Cards	Shape on Card
5	triangle
15	square
4	rectangle
6	circle
12	star

2. A school is giving out prizes at a pep rally to random students whose names are picked out of a hat. The table shows the number of students in each grade. Suppose each time a student's name is chosen, the student's name does not go back into the hat. The principal draws two names. What is the probability that the first student is in twelfth grade or the second student is in tenth grade?

Number of Students	Grade of Student
25	ninth
32	tenth
28	eleventh
25	twelfth

3. A bag contains a red marble, a blue marble, a yellow marble, a green marble, an orange marble, and a purple marble. Jeanine randomly picks a marble from the bag, without replacing it, and she then chooses another marble from the bag. Sketch a tree diagram that represents the sample space.

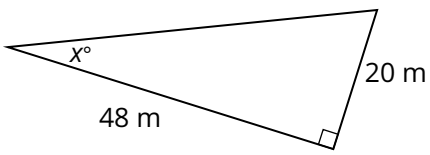
4. Kamau randomly chooses a card from a deck of eight cards without replacing it. The cards are numbered 1 through 8.

   a. Identify the sample space.

   b. Determine the probability of picking a card that has a number less than 6, $P(\text{less than } 6)$.

   c. Determine the probability of picking a card that has a number that is a prime number, $P(\text{prime number})$.

5. Use a trigonometric ratio to solve for the value of x. Round your answer the nearest tenth.



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II. Conditional Probability

Topic Practice

A. Determine each conditional probability.

1. Given $P(A) = 0.25$, $P(B) = 0.3$, and $P(A \text{ and } B) = 0.1$, determine $P(B|A)$.

2. Given $P(A) = 0.25$, $P(B) = 0.3$, and $P(A \text{ and } B) = 0.1$, determine $P(A|B)$.

3. Given $P(A) = \frac{2}{9}$, $P(B) = \frac{3}{10}$, and $P(A \text{ and } B) = \frac{2}{15}$, determine $P(A|B)$.

4. Given $P(A) = \frac{2}{9}$, $P(B) = \frac{3}{10}$, and $P(A \text{ and } B) = \frac{2}{15}$, determine $P(B|A)$.

5. Given $P(A) = \frac{5}{6}$, $P(B) = \frac{1}{2}$, and $P(A \text{ and } B) = \frac{1}{12}$, determine $P(B|A)$.

6. Given $P(A) = 0.12$, $P(B) = 0.2$, and $P(A \text{ and } B) = 0.05$, determine $P(A|B)$.

B. A five-sided letter die with faces labeled A, B, C, D, and E is rolled twice. The table shows the sample space of possible outcomes. Use the table to determine each probability. Express each answer as a fraction in simplest form.

First Roll \ Second Roll	A	B	C	D	E
A	A, A	A, B	A, C	A, D	A, E
B	B, A	B, B	B, C	B, D	B, E
C	C, A	C, B	C, C	C, D	C, E
D	D, A	D, B	D, C	D, D	D, E
E	E, A	E, B	E, C	E, D	E, E

1. *P*(B on the first roll)

2. *P*(vowel on the second roll)

3. *P*(consonant on the second roll)

4. *P*(A or B on the first roll)

5. *P*(A on the first roll and consonant on the second roll)

6. *P*(vowel on the first roll and vowel or D on the second roll)

7. *P*(B or D on the second roll, given that the first roll was a consonant)

8. *P*(consonant on the second roll, given that the first roll was a vowel)

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9. *P*(A on the second roll, given that the first roll was a vowel)

10. *P*(vowel on the second roll, given that the first roll was not an A)

C. The two-way frequency table shows the results of a survey in which sophomore and junior students were asked about their favorite type of music. Use the table to answer each question. Round each answer to the nearest tenth of a percent, if necessary.

		Favorite Type of Music			
Grade Level		Hip Hop	Country	Pop	Total
	Sophomore	21	10	4	35
	Junior	9	14	12	35
	Total	30	24	16	70

1. Determine $P(\text{hip hop}|\text{sophomore})$.

2. Use your answer from question 1 to determine whether choosing hip hop music and being a sophomore are independent or dependent events. Explain your reasoning.

3. Determine $P(\text{junior}|\text{pop})$.

4. Use your answer from question 3 to determine whether being a junior and choosing pop music are independent or dependent events. Explain your reasoning.

5. Determine *P*(sophomore|country).

6. Use your answer from question 5 to determine whether being a sophomore and choosing country music are independent or dependent events. Explain your reasoning.

7. Determine *P*(junior|hip hop).

8. Use your answer from question 7 to determine whether being a junior and choosing hip hop are independent or dependent events. Explain your reasoning.

D. The jar on Mrs. Wilson's desk contains 20 green paper clips, 30 red paper clips, 15 white paper clips, and 10 black paper clips. She selects a paper clip without looking, does not replace it, and selects another. Determine each probability. Round each answer to the nearest tenth of a percent, if necessary.

1. *P*(both paper clips are red)

2. *P*(both paper clips are white)

3. *P*(second paper clip is green|first paper clip is black)

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4. *P*(second paper clip is white|first paper clip is red)

5. *P*(second paper clip is green|first paper clip is not green)

6. *P*(second paper clip is not red|first paper clip is red)

Extension

1. A random sample of 100 people are surveyed about their shopping habits. 60 reported using credit cards, and out of those people, 22 of them are less than 30 years old. Out of the people who do not use credit cards, 15 are 30 years old or older. Suppose a person is randomly chosen. Use the conditional probability formula to determine the probability that the person chosen uses a credit card, given that the person is older than 30 years old.

Spaced Practice

1. A survey was taken of 60 customers in a pet store to compare the number of pets owned by the shopper and the age of the shopper. The responses are shown in the table.

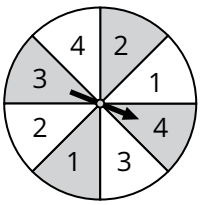
		Number of Pets			
		1	2	3	4
Age	20–39	6	7	4	2
	40–59	7	11	4	3
	60–79	8	5	2	1

Calculate each probability.

a. A randomly chosen customer has 3 pets.

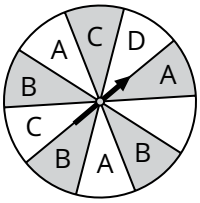
b. A randomly chosen customer has 2 pets and is aged 60–79.

2. A board game includes a spinner and a deck of cards. A player chooses a card from the deck and then chooses another card from the deck without replacing the first card. What is the probability that the first card chosen is "Go Ahead 1 Space" and the second card is "Pick a Card"?



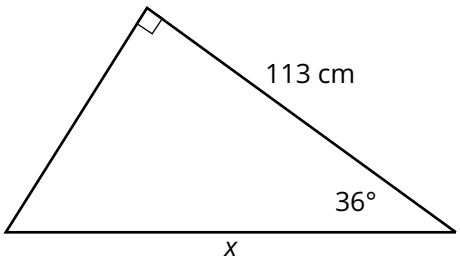
PICK A CARD	LOSE A TURN	GO BACK 1 SPACE	GO AHEAD 1 SPACE
6 Cards	4 Cards	3 Cards	2 Cards

3. Consider the spinner shown in the figure. A player spins the spinner two times. What is the probability that the spinner will land on the letter A the first time or on the letter C the second time?



4. Use a trigonometric ratio to solve for the value of x. Round your answer to the nearest tenth.

a.



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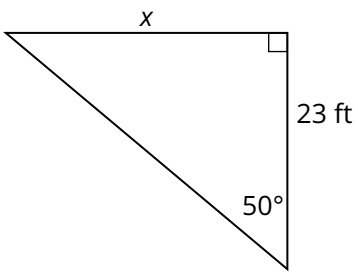

III. Permutations and Combinations

Topic Practice

A. Evaluate each expression.

1. ${}_8P_3$

2. ${}_5P_5$

3. ${}_{10}P_4$

4. ${}_7P_5$

5. ${}_6P_6$

6. ${}_9P_6$

B. Calculate the number of ways the letters of each word can be arranged.

1. SUNNY

2. FACTORIAL

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3. ARRANGE

4. PROBABILITY

5. PARALLEL

6. MISSISSIPPI

C. A website requires users to enter a 6-digit personal identification number (PIN) for security. A user's PIN must use the digits 1, 2, 3, 4, 5, and 6, and no digit can be used more than once. Determine each probability. Express each answer as a fraction in simplest form.

1. Suppose a user is unable to remember his PIN and enters the digits randomly. What is the probability that he will guess correctly?

2. Suppose a user randomly selects a PIN. What is the probability that the user's PIN is an even number?

3. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with 3 even digits and ends with 3 odd digits?

4. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with the digits 123?

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5. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with the digit 1 and ends with the digit 6?

6. Suppose a user randomly selects a PIN. What is the probability that the user's PIN begins with the digits 12 and ends with the digits 56?

D. Calculate the number of ways each arrangement can be made.

1. 12 flowers planted around the base of a tree

2. 7 dinner guests seated around a table

3. 10 candles arranged around the outside of a circular birthday cake

4. 3 baseball players standing in a circle under a fly ball

5. 6 teachers seated around a circular table at a conference

6. 5 kittens arranged around a ball of yarn

E. Evaluate each expression.

1. ${}_{11}C_4$

2. ${}_{6}C_5$

3. ${}_{5}C_3$

4. ${}_{12}C_{10}$

5. ${}_{8}C_4$

6. ${}_{7}C_2$

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F. A group of 6 seniors, 5 juniors, and 4 sophomores are running for student council. The council is made up of 6 members. Assume that each student has an equal chance of being elected to student council. Determine each probability, and express each answer as a fraction in simplest form.

1. What is the probability that 2 seniors, 2 juniors, and 2 sophomores are elected?

2. What is the probability that the student council is made up of all seniors?

3. What is the probability that 3 seniors, 2 juniors, and 1 sophomore are elected?

4. What is the probability that 3 juniors and 3 sophomores are elected?

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5. What is the probability that 2 seniors and 4 sophomores are elected?

6. What is the probability that 1 senior, 4 juniors, and 1 sophomore are elected?

G. Calculate the number of possible outcomes in each of the following situations.

1. A computer code uses 4 randomly selected letters of the alphabet. If no letters are repeated, how many possible codes are there?

2. Twelve students are competing in the finals of a spelling bee. The top 3 finishers are awarded a gold, silver, and bronze medal. In how many ways can the medals be won?

3. A summer camp offers 12 different afternoon activities. Caleb selects 2 of the activities to do today. How many possible outcomes are there if the order of the activities is important?

4. There are 15 different seminars at a teacher's convention. Mrs. Alvarez will choose 3 of the seminars to attend today. How many possible outcomes are there if the order of the seminars is important?

5. A committee of 4 students is to be formed from a homeroom of 25 students. How many different committees are possible?

6. A pizzeria offers 8 different toppings on their pizzas. If a customer wants to order a 3-topping pizza, how many possible options are there?

7. Seven friends are playing musical chairs. There are 5 chairs in the first round, so only 5 of the friends will move on to the second round. How many different groups of friends are possible for the second round of the game?

8. Fran has 4 pennies, 3 nickels, 5 dimes, and 2 quarters in her pocket. In how many ways can she pull 3 coins out of her pocket if the order of the coins is not important?

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Extension

A student is taking a three-question true-or-false quiz. The student only studied a little bit, so she will make educated guesses for each question. The probability she gets each question correct is $\frac{2}{3}$. List all the possible ways the student can answer the questions as well as the probabilities for each. Use *C* for correct and *I* for incorrect.

Spaced Practice

1. Giorgio is stopping at a coffee shop on the way to work. The probability that Giorgio has to wait in line and buys a doughnut is $\frac{5}{8}$. The probability that Giorgio has to wait in line is $\frac{7}{10}$. Giorgio buys a doughnut every 40 out of 55 times he goes to the coffee shop.

    a. What is the probability Giorgio buys a doughnut given that he waited in line?

b. Are waiting in line and buying a doughnut independent or dependent events? Explain your reasoning.

2. In a school, when a student is chosen at random, the probability that the student takes physics and chemistry is $\frac{3}{25}$. The probability that the student takes just chemistry is $\frac{3}{5}$, and the probability that the student takes just physics is $\frac{1}{5}$.

   a. What is the probability a randomly chosen student takes physics given that they are taking chemistry?

   b. Are taking physics and taking chemistry independent or dependent events? Explain your reasoning.

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3. A game includes a deck of cards with a shape on each card. The table shows the numbers of each type of card. Suppose each time a card is chosen, the card is replaced before another card is chosen. A player draws five cards. What is the probability they will all have a different shape on them?

Number of Cards	Shape on Card
5	triangle
15	square
4	rectangle
6	circle
12	star

4. A sphere has a volume of 6367.4 cubic centimeters. Determine the radius of the sphere. Use 3.14 for π and round your answer to the nearest tenth.

5. A cone that is 5.5 inches high has a volume of 87.56 cubic inches. Determine the radius of the cone. Use 3.14 for π and round your answer to the nearest tenth.

IV. Independent Trials

Topic Practice

A. Determine the probability in each situation. Express each answer as a fraction in simplest form.

1. On average, Malcolm makes a par on $\frac{2}{3}$ of the golf holes that he plays. What is the probability that he will make a par on each of the next 2 holes that he plays?

2. Clarence rolls 2 number cubes. What is the probability that he will roll a number greater than 4 on both rolls?

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3. For every 5 penalty kicks that Missy attempts, she scores on average 3 goals. What is the probability that Missy will score 2 goals in her next 2 penalty kick attempts?

4. On average, 3 out of 4 students order spaghetti at the cafeteria when it is offered. What is the probability that the first 2 students in line for lunch will have spaghetti if it is offered?

5. A spinner has 8 equal size spaces with the colors: red, green, yellow, red, blue, white, green, red. If the spinner is spun twice, what is the probability of spinning red on both spins?

6. On average, 4 out of 5 people get coffee when they walk by the campus coffee shop. What is the probability that the next two students walking by will buy coffee?

B. According to a recent survey, 80% of high school students have their own cell phone. Suppose 10 high school students are selected at random. Determine each probability. Round your answers to the nearest tenth of a percent, if necessary.

1. *P*(8 of the students have cell phones)
2. *P*(5 of the students have cell phones)
3. *P*(3 of the students do not have cell phones)
4. *P*(all of the students have cell phones)
5. *P*(9 of the students have cell phones)
6. *P*(none of the students have cell phones)

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C. Based on past results, a batter knows that the opposing pitcher throws a fastball 75% of the time and a curveball 25% of the time. Suppose the batter sees 8 pitches during a particular at-bat. Determine each probability. Round your answers to the nearest tenth of a percent, if necessary.

1. *P*(4 fastballs and 4 curveballs)

2. *P*(all fastballs)

3. *P*(5 fastballs and 3 curveballs)

4. *P*(5 curveballs and 3 fastballs)

5. *P*(7 fastballs and 1 curveball)

6. *P*(no fastballs)

Extension

1. A student is taking a multiple choice quiz. There are 10 questions on the quiz, and each question has 5 possible responses. The student has to guess on each question because they did not study for the quiz.

   a. What is the probability that the student gets at least a 70% on the quiz? Show your work.

   b. Is there a good chance the student will pass the quiz by guessing? Explain your reasoning.

Spaced Practice

1. Sylvia is at an ice cream parlor that has 20 different flavors of ice cream. She is going to have a bowl of ice cream with two scoops. How many different ways can the scoops be chosen? State whether the question uses permutations or combinations. Then, calculate the answer.

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2. How many different ways can the letters in the word MATHEMATICS be arranged?

3. A school is giving out prizes at a pep rally to random students whose names are picked out of a hat. The table shows the numbers of students in each grade. Suppose each time a student's name is chosen, the student's name does not go back into the hat. The principal draws three names. What is the probability that the first student is in eleventh grade and the third student is in eleventh grade?

Number of Students	Grade Level
25	ninth
32	tenth
28	eleventh
25	twelfth

4. Two bags of marbles are used for a game with the given content.

   Bag A: 6 red, 5 blue, 4 white

   Bag B: 6 green, 5 orange, 4 purple, 3 black

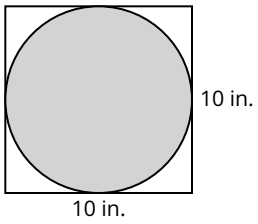
   A player chooses two marbles from Bag A. What is the probability that the player will choose a red marble first or a white marble second?

V. Expected Value

Topic Practice

A. Determine the probability that a dart that lands on a random part of each target will land in the shaded scoring section. Assume that all squares in a figure and all circles in a figure are congruent unless otherwise marked. Round each answer to the nearest tenth of a percent, if necessary.

1.



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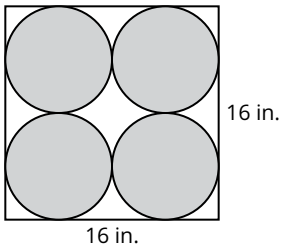

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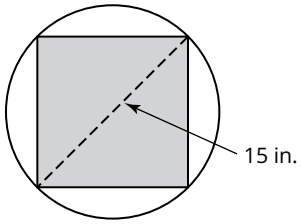

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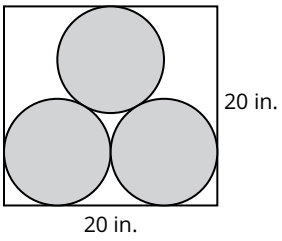

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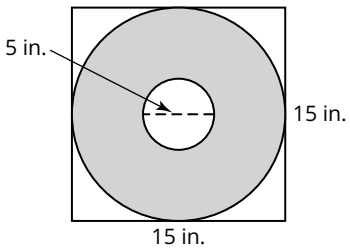

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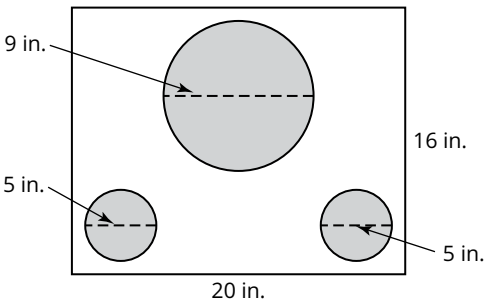

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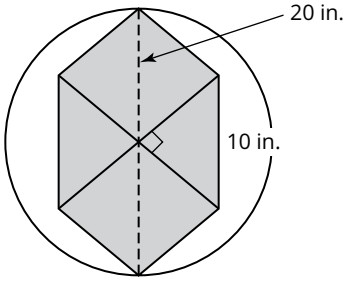

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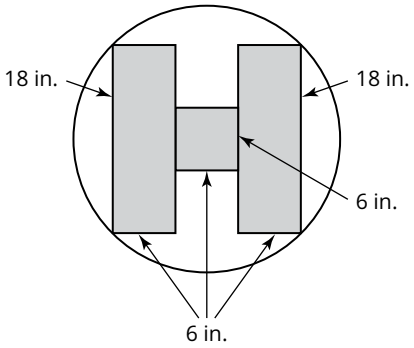
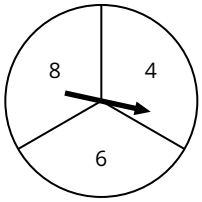


B. Calculate the expected value of spinning each spinner one time. Round to the nearest hundredth, if necessary.

1.



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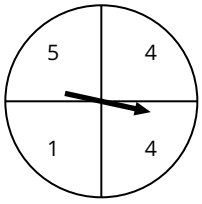

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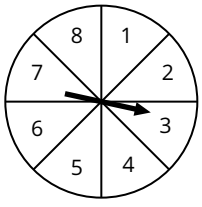

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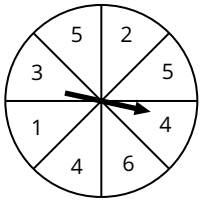

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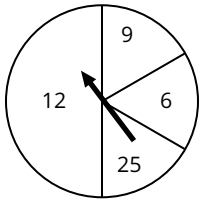

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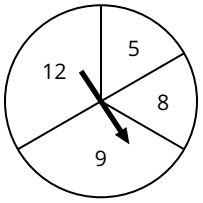
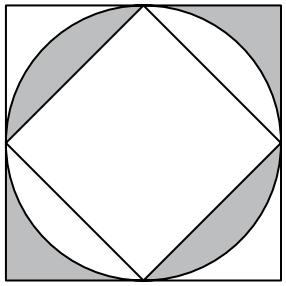


Extension

1. The board shown is a square inside a circle that is inside another square. The radius of the circle is $\sqrt{3}$ meters.



   a. A dart is thrown and lands on a random spot on the board. Determine the probability of hitting the shaded region. Write your answer as a percent rounded to the nearest tenth.

   b. If the dart lands on a shaded area inside the circle, the player is given 10 points. If the dart lands on a shaded area outside the circle, the player is given 20 points. What is the expected value for the number of points a player will get?

Spaced Review

1. In a restaurant, $\frac{3}{5}$ of the customers order meat dishes and $\frac{2}{5}$ of the customers order meatless dishes. If eight people sit down for a meal, what is the probability that 3 of them order meat dishes and 5 of them order meatless dishes?

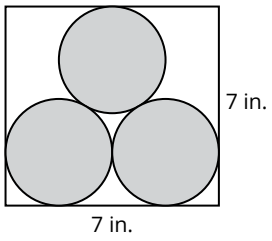
2. A spinner has 4 blue spaces and 2 yellow spaces. What is the probability of 3 blue outcomes and 1 yellow outcome when the spinner is spun 4 times?

3. While playing a board game, a player rolls a number cube once and then picks a card. The number cube is numbered 1 through 6. There are 3 cards, each with one animal on them including a pig, a goat, and a cow.

   a. Write an organized list that represents the sample space.

   b. Determine the size of the sample space using the Fundamental Counting Principle. Show your calculation.

4. Determine the probability that a dart that lands on a random part of the target will land in the shaded scoring section. Assume that all circles in a figure are congruent. Round each answer to the nearest tenth of a percent, when necessary.



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