



Geometry

Course Guide

Welcome to the Course Guide for Secondary Mathematics, Geometry

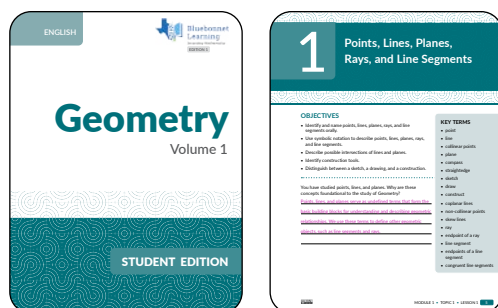
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INSTRUCTIONAL DESIGN

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On **Learning Together** days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

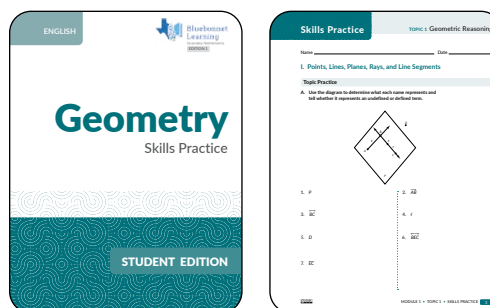
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.



SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

INSTRUCTIONAL DESIGN

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.

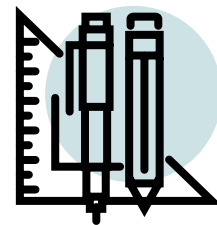
Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Mathematical Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.



What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1

Constructing a Coordinate Plane

1 OBJECTIVES

- Construct a perpendicular line segment and bisector using paper folding and using a compass and straightedge.
- Use construction tools to duplicate a line segment and construct a square.
- Use constructions and rigid motions to create a coordinate plane.
- Identify rigid motions that can be used to create shapes on a coordinate plane.
- Identify the coordinates of vertices of shapes on a coordinate plane orally and in writing.

You have recalled and investigated two-dimensional shapes on a plane. How can you construct a coordinate plane to help you analyze geometric objects?

2

3 KEY TERMS

- arc
- midpoint
- segment bisector
- perpendicular bisector
- transformation
- rigid motion

LESSON STRUCTURE

1 Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3 Key Terms

The mathematical terms for each lesson are identified to help you connect your everyday and mathematical language.



4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

.....
A compass is a tool used to create arcs and circles.
A straightedge is a ruler with no numbers.
An **arc** consists of two points on a circle and the curved line that connects the two points.
.....

4 Getting Started

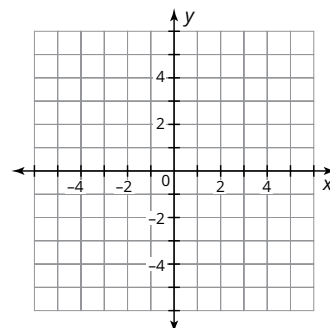
Constructing a Perpendicular Line Using Patty Paper

Remember, when you construct geometric figures, you create exact figures without measurements, using paper folding or a *compass* and a *straightedge*.

A coordinate plane is composed of two intersecting lines—the *x*-axis and *y*-axis.

You can also think of a coordinate plane as being composed of squares.

Each square is 1 unit long and 1 unit wide, with the definition of “1 unit” up to the person who constructs the coordinate plane.



In this lesson, you will consider how a coordinate plane is constructed using squares. To construct a square, you will first need to be able to construct perpendicular lines.

Let's start by experimenting with patty paper to construct a line perpendicular to a given line segment.

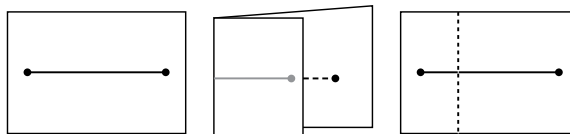
.....
You can represent a plane with a piece of paper since paper is a flat surface. Remember a geometric plane extends infinitely in all directions, unlike a physical piece of paper which has boundaries.
.....

WORKED EXAMPLE

Draw a line segment on a piece of patty paper.

Fold the line segment so that it lies on top of itself.

Open the patty paper. The crease represents a line perpendicular to the given line segment.



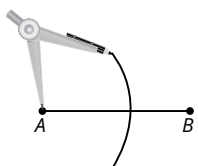
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ACTIVITY
1.1Constructing a Perpendicular Line
Using a Compass and a Straightedge

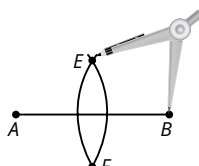
You can also construct segment bisectors using a compass and straightedge. To do this construction, you will draw arcs, making use of the fact that all the radii of a circle have an equal length.

WORKED EXAMPLE

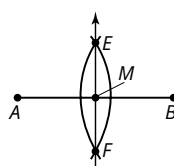
You can construct the segment bisector of $\overline{AB}$.

**Construct an Arc**

Open the radius of the compass to more than half the length of $\overline{AB}$. Use endpoint A as the center and construct an arc.

**Construct Another Arc**

Keep the compass radius and use point B as the center as you construct an arc. Label the points formed by the intersection of the arcs point E and point F.

**Construct a Line**

Connect points E and F. Line segment $\overline{EF}$ is the segment bisector of $\overline{AB}$. The point M represents the midpoint of $\overline{AB}$.

Line $\overline{EF}$ bisects $\overline{AB}$. Point M is the midpoint of $\overline{AB}$.

.....
Though the statement "all perpendicular bisectors are segment bisectors, but not all segment bisectors are perpendicular bisectors is true," the construction for a segment bisector is the same.
.....

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.



5

Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

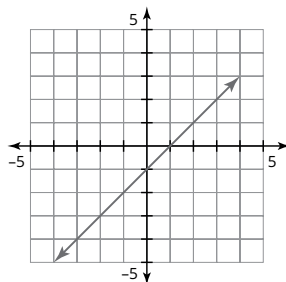
- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

... Talk the Talk 6

Walking on a Thin Line

Consider the line shown on the coordinate plane.



1. Suppose the line was constructed using rigid motions, starting with any line segment constructed in one square. Describe one possible sequence of translations that could produce the line.
2. How is the sequence of translations related to the slope of the line?
3. What is the equation for the line in slope-intercept form?

Remember ...

The equation for a line can be written in slope-intercept form, $y = mx + b$, where m represents the slope of the line, and b represents the y-intercept.

Lesson 1 Assignment

Write 7

Describe the similarities and differences of a segment bisector and a perpendicular bisector.

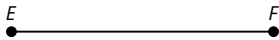
Remember 8

A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

A translation “slides” a figure up, down, left, or right. A reflection “flips” a figure across a line. A rotation “spins” a figure about a point.

Practice 9

1. Locate the midpoint of the line segment using construction tools and label it point M . Then, explain how you know that point M is the midpoint of $\overline{EF}$.



ASSIGNMENT

7 Write

Reflect on your work and clarify your thinking.

8 Remember

Take note of the key concepts from the lesson.

9 Practice

Use the concepts learned in the lesson to solve problems.

Lesson 1 Assignment

ASSIGNMENT

10 Prepare

Get ready for the next lesson.

Prepare 10

Determine the reciprocal of each value.

1. 3
2. -10
3. $\frac{1}{5}$
4. $-c$
5. $\frac{a}{b}, b \neq 0$

Research-Based Strategies

WORKED EXAMPLE

You can use prime factorization to rewrite radicals in simplest radical form.

Rewrite $\sqrt{60}$ in simplest radical form.

Write the prime factorization of 60.

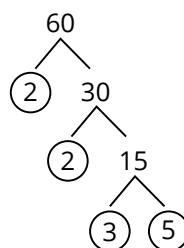
$$60 = 2^2 \cdot 3 \cdot 5$$

Extract the perfect squares

$$\sqrt{60} = 2 \cdot \sqrt{3 \cdot 5}$$

Rewrite the radicand.

$$\sqrt{60} = 2 \cdot \sqrt{15}$$



WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself ...

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine whether the work is correct or incorrect.

Sebastian and Mason described the probability of a heads up result for the first coin or a tails up result for the second coin.

Sebastian



I circled 2 outcomes and drew a rectangle around 2 outcomes, and there are 4 possible outcomes. So, the probability of flipping a heads on the first coin or a tails on the second is

$$\frac{2+2}{4} = \frac{4}{4} = 1.$$

Mason



I marked 4 outcomes, and there are 4 possible outcomes. But, I marked 1 of the outcomes twice. So, I have to subtract one of the outcomes and the probability of a heads up result for the first coin or a tails up result for the second coin is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} - \frac{1}{4} = \frac{3}{4}.$$

Ask Yourself ...

- Why is this method correct?
- Have I used this method before?

Ask Yourself ...

- Where is the error?
- Why is it an error?
- How can I correct it?



14. Calculate the distance between the points $(-1, -2)$ and $(-3, -7)$. Notice the similarity between these points and Question 13, part (a).

Samuel says that the solution must be the opposite of the solution of part (a). Harper disagrees and says that the solution will be the same as the solution of part (a). Who is correct? Explain your reasoning and state the correct solution.

Ask Yourself ...

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Thought Bubbles and Margin Notes

Look for these margin notes as you journey through this course. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are here to help and guide your learning.

Ask Yourself ...

Can distance be negative?

Remember ...

When you sketch a geometric figure, you create the figure without tools. Accuracy is not important. When you draw geometric figures, you can use tools such as rulers, protractors, or a coordinate plane to draw exact lengths and areas.

Think About ...

The endpoint is where the ray *begins*? Why isn't it called the "begin-point" instead?

TEKS Mathematical Process Standards

TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I Can” expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I CAN:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



Understanding the Problem-Solving Model

Productive mathematical thinkers are problem solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Using the Problem-Solving Model



Notice | Wonder

Understand the situation by asking these questions.

- What do I notice?
- What do I wonder?
- How do I analyze the given information to identify what is important?
- Do I have enough information to formulate a plan and determine a solution?



Organize | Mathematize

Devise a plan for your mathematical approach by asking these questions.

- What mathematical relationships exist between this problem and similar problems I have solved?
- What plan or strategy can I use to solve this problem?
- How can I efficiently solve this problem?
- How can I organize, record, and communicate my mathematics?



Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I display my work using multiple representations?
- Did I explain my reasoning in terms of the problem situation?
- Did I communicate the strategy used to determine the solution?

- Did I justify my mathematical argument clearly using precise mathematical language?
- Can I use my mathematical reasoning to make any predictions?



Test | Interpret

Look back at your work and ask these questions.

- Does my solution clearly and completely answer the original question/problem?
- Is my solution reasonable?
- Does my solution make sense in terms of the problem situation?
- Can I solve the problem using a different strategy? Would another strategy be more efficient?
- Can I justify my solution?



Report

As you share your mathematical reasoning with others, ask these questions.

- Did you use multiple representations to represent your mathematics?
- Did you justify your mathematical reasoning?
- Can others understand my process and solution?

The Problem-Solving Model Graphic Organizer



NOTICE

Understand the Problem



ORGANIZE

Devise a Plan



PREDICT

Carry Out the Plan



INTERPRET

Look Back



REPORT

Report

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Resources for Students and Families

Topic Summary

Each topic includes a Topic Summary. The Topic Summary contains a list of all key terms addressed in the topic and a summary of each lesson, including worked examples and key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.

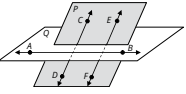
TOPIC 1 SUMMARY

Geometry Reasoning

LESSON 1 Points, Lines, Planes, Rays, and Line Segments

There are three essential building blocks of geometry—points, lines, and planes. These three terms are called undefined terms. They are used to describe and create mathematical models to represent objects in the world. A **point** is a location in space that has no size or shape. A **line** is a straight path that extends infinitely in both directions. A **plane** is a flat surface that extends infinitely in all directions. **Collinear** points are points that lie on the same line. **Coplanar** points are points that lie on the same plane. **Non-collinear** points are points that do not lie on the same line. **Skew** lines are two lines that do not intersect and are not coplanar.

Example



Points A and B lie on $\overleftrightarrow{AB}$, points C and D lie on $\overleftrightarrow{CD}$, and points E and F lie on $\overleftrightarrow{EF}$.
Line $\overleftrightarrow{AB}$ lies in plane Q. Lines $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ lie in plane R.
Points A and B are collinear. Points C and D are collinear.
Lines $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ are coplanar.
Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are skew. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ are coplanar.
Planes P and Q intersect.

KEY TERMS

- postulate [postulado]
- theorem [teorema]
- proof [prueba]
- validity [validez]
- Euclidean geometry [geometría euclidiana]
- great circle
- spherical geometry [geometría esférica]
- conjecture [conjetura]
- inductive reasoning [razonamiento inductivo]
- deductive reasoning [razonamiento deductivo]
- diagonal [diagonal]
- angle [ángulo]
- vertex of an angle [vértice de un ángulo]
- linear pair postulate [postulado del par lineal]
- segment addition postulate
- auxiliary line

You can use many tools such as a ruler or a protractor to draw a line. A **compass** is a tool used to draw a circle. A **straightedge** is a tool used to draw a line. When you **sketch** a geometric figure, you are drawing it. When you **draw** a geometric figure, you are using a tool to create it. A drawing is more accurate than a sketch. When you **construct** a geometric figure, you are using only a compass and a straightedge to create it. A **ray** is a part of a line that starts at a point and extends infinitely in one direction. The point where the ray begins is called the **endpoint**. A ray is named using two points: the endpoint and another point on the ray. You can write ray $\overrightarrow{AB}$ or $\overrightarrow{BA}$. A **line segment** is a part of a line that has two endpoints. The endpoints of a line segment are the points where the segment begins and ends. A line segment is named using two endpoints of the line segment, such as $\overline{AB}$. If two line segments have the same length, they are called **congruent** line segments. Two line segments of equal length are congruent.

LESSON 2 Formal Geometry

When reasoning in mathematics, you must be careful not to draw false conclusions. There are many assumptions that are not necessarily true. For example, consider the statement: "If a number is even, then it is divisible by 2." This statement is true. However, the converse statement, "If a number is divisible by 2, then it is even," is also true. Both the hypothesis and the conclusion are true.

To show that a statement is false, you can provide a counterexample. A **counterexample** is a specific example that shows that a general statement is not true.

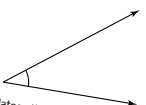
The **validity** of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the validity of the statement is considered true. The validity of a conditional statement is either true, false, or not both.

A **conclusion** from a **conditional** statement is the part of the statement that follows the "if" clause.

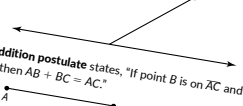
LESSON 3 Conjectures and Deductive Reasoning

The ability to use information to reason and make conclusions is very important in life and in mathematics. There are two common methods of reasoning. **Inductive Reasoning** is a process of arriving at a conclusion by generalizing a statement from observing patterns. **Deductive Reasoning** is a process of using statements and ideas that you know to be true in order to prove new statements and ideas. These statements and ideas could include facts, definitions, theorems, postulates, or properties.

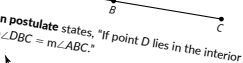
An **angle** is formed by two rays that share a common endpoint, called the **vertex of the angle**.



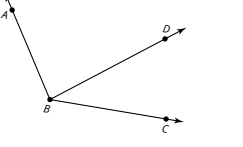
Three fundamental postulates—the **linear pair postulate**, the **segment addition postulate**, and the **angle addition postulate**—can be used to make various conjectures. If the conjectures are proven, then they will become theorems. The **linear pair postulate** states, "If two angles form a linear pair, then the angles are supplementary."



The **segment addition postulate** states, "If point B is on $\overline{AC}$ and between points A and C, then $AB + BC = AC$."



The **angle addition postulate** states, "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.

TOPIC 1 SELF-REFLECTION

Name: _____

Geometry Reasoning

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Geometry Reasoning* topic by:

TOPIC 1: <i>Geometry Reasoning</i>	Beginning of Topic	Middle of Topic	End of Topic
distinguishing between undefined terms, definitions, conjectures, postulate, theorems, and proofs.	☐	☐	☐
explaining the difference between the terms draw, sketch, and construct.	☐	☐	☐
verifying a conjecture is false using a counterexample.	☐	☐	☐
identifying the hypothesis and conclusion given a conditional statement.	☐	☐	☐
determining the validity of a conditional statement.	☐	☐	☐
recognizing a linear, exponential, or quadratic function by its equation or graph.	☐	☐	☐
explaining the difference between Euclidean geometry and spherical geometry.	☐	☐	☐
making a conjecture and using deductive reasoning to validate it.	☐	☐	☐

continued on the next page

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MODULE 1 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SELF-REFLECTION

continued

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Geometry Reasoning* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

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MODULE 1 • TOPIC 1 • SELF-REFLECTION

Math Glossary

A course-specific Math Glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.

Math Glossary

A

addition property of equality

The addition property of equality states: If a , b , and c are real numbers and $a = b$, then $a + c = b + c$.

Example

If $x = 2$, then $x + 5 = 2 + 5$, or $x + 5 = 7$ is an example of the addition property of equality.

Addition Rule for probability

The Addition Rule for probability states: "The probability that Event A occurs or Event B occurs is the probability that Event A occurs plus the probability that Event B occurs minus the probability that both A and B occur."

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip or flipping a heads on the second flip.

Let A represent the event of flipping a heads on the first flip. Let B represent the event of flipping a heads on the second flip.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$

$$P(A \text{ or } B) = \frac{3}{4}$$

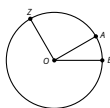
So, the probability of flipping a heads on the first flip or flipping a heads on the second flip is $\frac{3}{4}$.

adjacent arcs

Adjacent arcs are two arcs of the same circle sharing a common endpoint.

Example

Arcs ZA and AB are adjacent arcs.



adjacent side

The adjacent side is the adjacent side of the reference angle that is not the hypotenuse.

Example



MATH GLOSSARY G-1

Course Family Guide

The Course Family Guide provides you and your family an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

The purpose of the Course Family Guide is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



COURSE FAMILY GUIDE

Geometry

How to support your student as they learn

Geometry Mathematics

Read and share with your student.

Research-Based Instruction

Research-based strategies and best practices are woven into instructional materials.

Thorough explanation of key concepts are presented in a manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide provides information on where your student has been and where they are going when studying the mathematical content in this course.

Where have we been?

In Grade 8, students explored the Pythagorean Theorem by looking at patterns, making a conjecture, and then informally proving the theorem using models and diagrams. This topic starts to introduce students to the formal concept of proof by deriving conjectures, postulates, and theorems - terms they will use later when they write their own proofs.

Where are we going?

Proof is the corner stone of mathematics. Students will continue to develop their reasoning skills through the use of models and diagrams. They will use the formal concept of proof to derive conjectures, postulates, and theorems - terms they will use later when they write their own proofs.

The instructional materials balance conceptual understandings. In this course, students' progress from Concrete-Representational-Abstract (CRA) to conceptual understanding and build toward proficiency.

Engaging with Grade Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons and students are confident in sharing their learning.	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. The Learning Together Days target discrete skills that may be practiced to achieve proficiency.

Thumbs Up, Thumbs Down, and Who's Correct questions address your student's common misconceptions and provide opportunities for peer work analysis.

Thumbs Up, Thumbs Down

When you see a Thumbs Up:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

When you see a Thumbs Down:

- Take your time to read through the incorrect solution.
- Think about what went wrong.

Ask Yourself:

- Where is the error?
- Why is it an error?
- Have I seen this error before?

Who's Correct

When you see a Who's Correct item:

- Take your time to read through the solution.
- Question the strategy or reason given.
- Determine if correct or incorrect.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.

Skills Practice

1. Identify the given information.

2. Identify the goal.

3. Plan a strategy.

4. Execute the plan.

5. Check the solution.

Extension Opportunities

1. Identify the given information.

2. Identify the goal.

3. Plan a strategy.

4. Execute the plan.

5. Check the solution.

KEY TERMS

- arc [arco]
- midpoint [punto medio]
- segment bisector [bisectriz de segmento]
- perpendicular bisector [bisectriz perpendicular]
- transformation [transformación]
- rigid motion [movimiento rígido]
- distance formula [fórmula de la distancia]
- extracting perfect squares
- radical [radical]
- radicand [radicando]
- midpoint formula [fórmula del punto medio]
- composite figure [figura compuesta]
- regular polygon [polígono regular]
- apothem [apotema]

Refer to the Math Glossary for definitions of Key Terms.

FG-8 GEOMETRY • COURSE FAMILY GUIDE

FG-9 GEOMETRY • COURSE FAMILY GUIDE

Topic Family Guides

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, “Where have we been?” and “Where are we going?” Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of key terms, applying math in the real world, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.

Family Guide

MODULE 1 Reasoning with Shapes

Geometry

TOPIC 1 Geometry Reasoning

In this topic, students are introduced to the three essential elements of geometry - the point, the line, and the plane. They use these terms to define additional terms such as line segment, ray, and angle. Students differentiate between inductive and deductive reasoning to build the foundation of proof by exploring conditional statements, then learn about making conjectures and explore the difference between postulates and theorems.

Where have we been?

In Grade 8, students explored the Pythagorean Theorem by looking at patterns, making a conjecture, and then informally proving the theorem using models and diagrams. This topic starts to introduce students to the formal concept of proof by defining conjecture, postulate, and theorem - terms they will use later when they write their own proofs.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

People use deductive reasoning every day to make logical decisions, solve problems, and draw conclusions based on facts in school, work, and everyday life.

KEY TERMS

- point [punto]
- line
- collinear points [puntos colineales]
- plane [plano]
- noncollinear points [puntos no colineales]
- compass [compás]
- straightedge
- sketch
- draw
- construct [construir]
- coplanar lines
- skew lines
- ray [rayo]
- endpoint of a ray
- line segment
- endpoints of a line segment
- congruent line segments
- conditional statement
- biconditional statement
- counterexample [contraejemplo]
- hypothesis [hipótesis]
- conclusion [conclusión]
- postulate [postulado]
- theorem [teorema]
- proof [prueba]
- validity [validez]
- Euclidean geometry [geometría euclidiana]
- great circle

Where are we going?

A compass is a tool used to create arcs and circles.

A counterexample is a statement that is not true. For example, a friend says, "All numbers are even." A counterexample is the number 1, which is not even.

Spherical geometry is a type of geometry that uses a sphere to represent the surface of the Earth. It is different from Euclidean geometry because it uses great circles of a sphere instead of straight lines. A great circle is the intersection of a sphere and a plane that passes through the center of the sphere.

In Lesson 1: Points, Lines, and Planes, we introduced the three essential elements of geometry: the point, the line, and the plane. Although the terms point, line, and plane are simple, they are the building blocks of geometry. A point is described by its location, but it has no size or shape, but it is a location. A line is described as a straight path that extends infinitely in both directions. It has no width and is made up of an infinite number of points.

Math in the Real World

Deductive reasoning isn't just for solving angle puzzles or proving triangles congruent—it's your secret weapon for life! It's the power of starting with facts and building logical steps to reach a conclusion, whether you're cracking a geometry proof or figuring out why your friend is always late to meet up. This skill fuels everything from designing skyscrapers to debating who gets the last slice of pizza. It helps scientists develop theories, lawyers build cases, and detectives solve mysteries. So next time you're working through a proof, remember—you're not just learning math; you're training to be a problem-solving superhero in whatever field you choose!

The Linear Pair Postulate, Segment Addition Postulate, and Angle Addition Postulate

Three fundamental postulates—the linear pair postulate, the segment addition postulate, and the angle addition postulate—can be used to make various conjectures. If the conjectures are proven, then they will become theorems.

The linear pair postulate states, "If two angles form a linear pair, then the angles are supplementary."

The segment addition postulate states, "If point B is on $\overline{AC}$ and between points A and C, then $AB + BC = AC$."

The angle addition postulate states, "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

A mathematical model of several points and lines is shown.

- spherical geometry [geometría esférica]
- conjecture [conjetura]
- inductive reasoning [razonamiento inductivo]
- deductive reasoning [razonamiento deductivo]
- diagonal [diagonal]

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