

Reasoning with Shapes

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Chess is closely related to deductive reasoning because it requires players to analyze a set of rules and use logical thinking to anticipate and counter their opponent's moves.

Geometry Reasoning

INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources **IL-1**

LESSON 1 Points, Lines, Planes, Rays, and Line Segments **5**

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INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Describe and apply the problem-solving model through written explanations.

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In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Getting Started

You Already Know A Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

Ask Yourself . . .

How do your strategies change based on what you are learning and what you already know about it?

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Think About . . .

Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

Be prepared to share your list of learning strategies with the class.

ACTIVITY 1.1

Learning Resources

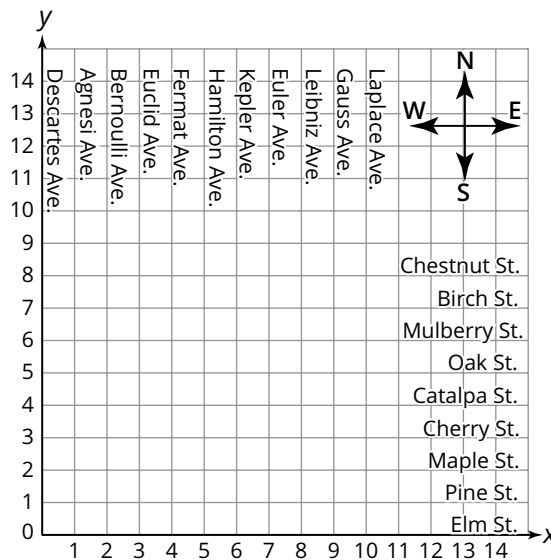
In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

Ethan, Isabella, and Samuel live in a town where the streets are laid out in a grid system.

Ethan lives 3 blocks east of Descartes Avenue and 8 blocks north of Elm Street. Isabella lives 11 blocks east of Descartes Avenue and 2 blocks north of Elm Street.

When Isabella walks directly to Ethan's house how far does she walk?
Explain your reasoning.



The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

PROBLEM SOLVING



.....

The Academic Glossary provides definitions of terms you will see throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of new key terms in each lesson.

.....

1. Locate the phrase **Explain your reasoning** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you determine how far Isabella must walk to get to Ethan's house?
2. What is a related word or phrase for **explain your reasoning**?

The problem-solving model provides a structure to help you become a better problem-solver.



Notice and Wonder

The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the questions to ask yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about the distance between Isabella and Ethan's houses?
4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?

Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.



Read through the questions to ask yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy to determine the distance Isabella must walk to get to Ethan's house.

6. Why do you think the second step of the problem-solving model process is important? How can you use the questions to ask yourself to help develop a strategy to solve the problem?

Predict and Analyze

The third step in the modeling process involves carrying out your plan. As you carry out the plan you will complete operations, analyze mathematical results, make predictions, and extend patterns.



Read through the questions to ask yourself for the third step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

7. Use your strategy to determine how far Isabella must walk to get to Ethan's house.

8. How can the questions to ask for the third step help you communicate your mathematical thinking?



Test and Interpret

The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the questions to ask yourself for the fourth step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

9. Samuel lives 13 blocks west of Descartes Avenue and 6 blocks north of Elm street. Can you apply the same reasoning to how far Isabella walks when she walks directly to Samuel's house?

10. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?



Report

The final step in the modeling process is to share your results.

11. How does listening to others help you learn mathematics?

Locate the TEKS mathematical process standards in the Course Guide.

12. Which TEKS mathematical process standard(s) did you use to determine the distance Isabella walks to get to Ethan's house?

Talk the Talk

You will see this symbol throughout the course to remind you to use the problem-solving model.

The Problem-Solving Model

In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.

Summarize what is involved in each phase of this problem-solving model.



Notice and Wonder

Organize and Mathematize

Predict and Analyze

Test and Interpret

Report

1

Points, Lines, Planes, Rays, and Line Segments

OBJECTIVES

- Identify and name points, lines, planes, rays, and line segments orally.
- Use symbolic notation to describe points, lines, planes, rays, and line segments.
- Describe possible intersections of lines and planes.
- Identify construction tools.
- Distinguish between a sketch, a drawing, and a construction.

.....

You have studied points, lines, and planes. Why are these concepts foundational to the study of Geometry?

KEY TERMS

- point
- line
- collinear points
- plane
- compass
- straightedge
- sketch
- draw
- construct
- coplanar lines
- non-collinear points
- skew lines
- ray
- endpoint of a ray
- line segment
- endpoints of a line segment
- congruent line segments

Getting Started

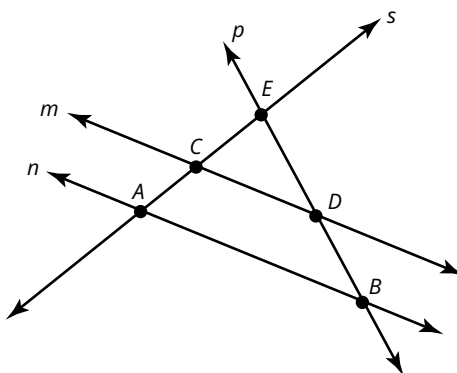
Points and Lines

There are three essential building blocks of geometry—the point, the line, and the plane. These three terms are called undefined terms; we can only describe and create mathematical models to represent them.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter.

A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase italic letter positioned next to the arrowhead. You can also name a line by any two points on the line.

A mathematical model of several points and lines is shown.



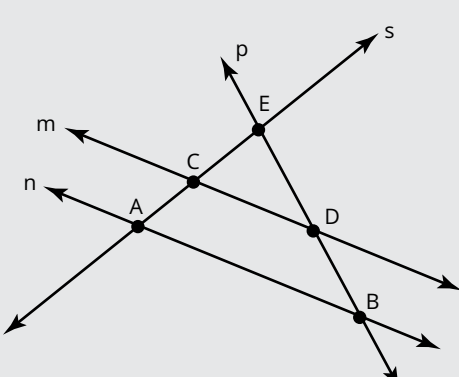
1. Does the name “line C” describe a unique line? Explain why or why not.
2. Does the name “line CD” describe a unique line? Explain why or why not.

3. Does the name “line s ” describe a unique line? Explain why or why not.
4. How many points are needed to name a specific line?
5. What is another name for line AB ?

Line AB can be written using symbols as $\overleftrightarrow{AB}$ and is read as “line AB .”

6. Analyze each model and explanation.

Liam



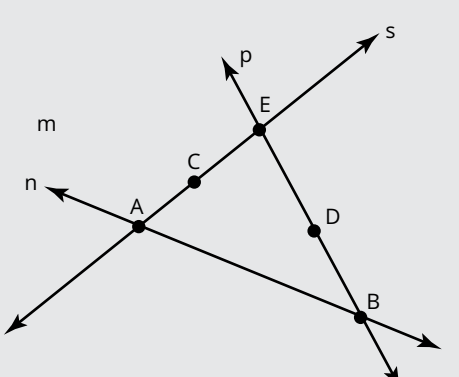
I drew point T on line s . Since only one line goes through T , point T describes just one line.

.....

Lines have names just like people. Many people may have the same first name. Many lines may pass through the same point.

.....

Victoria



I drew line v through points C and D . So, line CD describes two lines.

Describe the inaccuracy in each students' reasoning.

7. How many lines can be drawn through a single point?

.....

In Geometry, the three undefined terms are the foundation for building definitions. For example, the undefined term point is used to build the definition of collinear points.

.....

Collinear points are points that are located on the same line.

8. Use the diagram shown prior to Question 1.

a. Name three points that are collinear.

b. Name three points that are not collinear.

ACTIVITY 1.1

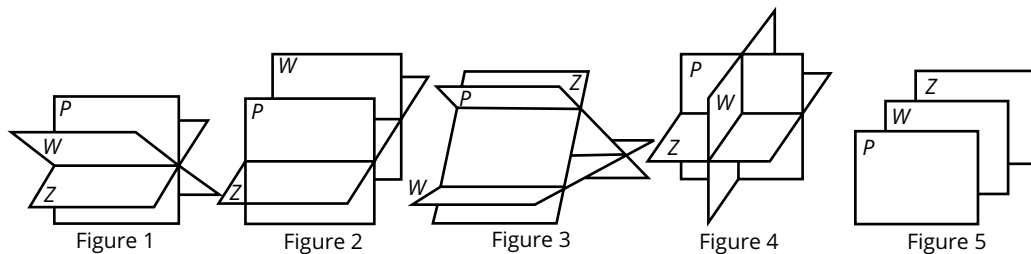
Planes

You investigated points and lines in the Getting Started.
Let's investigate the third undefined term, a *plane*.

A **plane** is described as a flat surface. A plane has an infinite length and width, but no depth, and extends infinitely in all directions. One real-world model of a plane is the surface of a still body of water. Three non-collinear points describe a unique plane, but planes are usually named using one capitalized italic letter located near a corner of the plane as drawn.

.....
non-collinear points are points that do not all lie on the same line.
.....

Three planes can intersect in a variety of ways or may not intersect at all.



1. Describe the intersection of planes *P*, *W*, and *Z* in each figure.

a. Figure 1

b. Figure 2

c. Figure 3

d. Figure 4

e. Figure 5

2. List all of the possible ways that three planes can intersect.

You can use many tools to create geometric figures. Some tools, such as a ruler or a protractor, are classified as measuring tools. A **compass** is a tool used to create circles and portions of circles.

A **straightedge** is a ruler with no numbers. It is important to know when to use each tool.

- When you **sketch** a geometric figure, the figure is created without the use of tools.
- When you **draw** a geometric figure, the figure is created with the use of tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.
- When you **construct** a geometric figure, the figure is created using only a compass and a straightedge.

3. Sketch and describe all possible ways that a line and a plane can intersect.

Coplanar lines are two or more lines that are located in the same plane.

Skew lines are two or more lines that do not intersect and are not parallel. Skew lines do not lie in the same plane.

4. Draw and label three coplanar lines.

5. Look around your classroom. Describe the location of two skew lines.

.....

The undefined terms
line and plane are used
to build the definitions
coplanar lines and
skew lines.

.....

ACTIVITY 1.2

Using Undefined Terms to Define New Terms

Think About...

The endpoint is where the ray *begins*? Why isn't it called the "begin-point" instead?

You used undefined terms to define collinear points, non-collinear points, coplanar lines, and skew lines. You will continue to build definitions in this lesson and throughout the course using the three undefined terms point, line, and plane.

A **ray** is a part of a line that begins with a single point and extends infinitely in one direction. The **endpoint of a ray** is the single point where the ray begins.

A ray is named using two capital letters, the first representing the endpoint and the second representing any other point on the ray. Ray AB can be written using symbols as $\overrightarrow{AB}$, which is read as "ray AB ."

1. Sketch and label $\overrightarrow{AB}$.

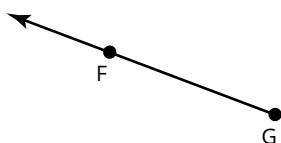


2. Sketch and label $\overrightarrow{BA}$.



3. Are $\overrightarrow{AB}$ and $\overrightarrow{BA}$ names for the same ray? Explain why or why not.

4. Use symbols to name the geometric figure shown.



A **line segment** is a part of a line that includes two points and all of the collinear points between the two points. The **endpoints of a line segment** are the points where the line segment begins and ends.

A line segment is named using two capital letters representing the two endpoints of the line segment. Line segment AB can be written using symbols as $\overline{AB}$, which is read as "line segment AB ."

5. Draw and label $\overline{AB}$.



6. Draw and label $\overline{BA}$.



Use a ruler or straightedge to create the segment

7. Are $\overline{AB}$ and $\overline{BA}$ names for the same line segment? Explain why or why not.

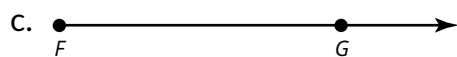
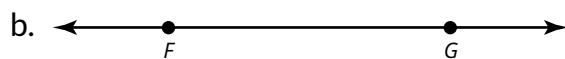
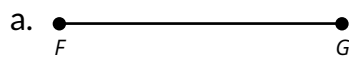
8. Use a ruler to measure $\overline{AB}$ in Question 5.

9. Complete the statement to express the measure of $\overline{AB}$.

a. " $AB = \underline{\hspace{1cm}}$ inches" is read as "the distance from point A to point B is equal to $\underline{\hspace{1cm}}$ inches."

b. How do you read " $SP = 8$ inches"?

10. Use symbols to name each geometric figure.



If two line segments have equal measure, then the line segments have the same length. **Congruent line segments** are two or more line segments of equal measure.

If $AB = CD$, then line segment AB is congruent to line segment CD by the definition of congruent line segments. This statement can be written using symbols as $\overline{AB} \cong \overline{CD}$ and is read as "line segment AB is congruent to line segment CD ."

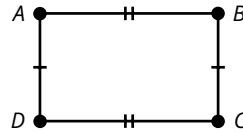
Use the congruence symbol, $\cong$, between references to congruent geometric figures; and the equal symbol, $=$, between references to equal lengths or distances.

.....
Don't forget the unit of
measure!
.....

WORKED EXAMPLE

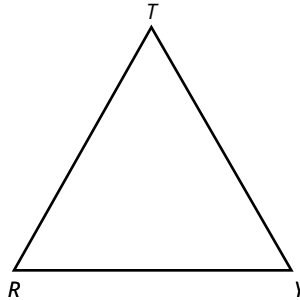
Markers are used to indicate congruent segments in geometric figures. If a diagram has more than one set of congruent segments then you use sets of markers.

The figure shows $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$.



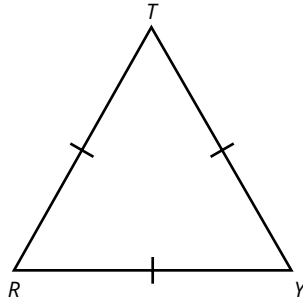
11. Draw and label two congruent line segments. Use markers to show congruency. Then, write a congruency statement that describes their relationship.

12. Ms. Flores drew the triangle shown and asked her students to classify it.



- a. Kai says the triangle is an equilateral triangle. Is she correct?

- b. Ms. Flores then drew markers and asked her students to classify the triangle.



Kai says the triangle is an isosceles triangle. Noah says the triangle is an equilateral triangle.

Who is correct?

- c. Ms. Flores also asked her students to write a statement that best describes the congruency of the line segments forming the triangle.



Kai

$$\overline{TR} = \overline{RY} = \overline{YR}$$

Noah

$$\overline{TR} \cong \overline{RY} \cong \overline{YR}$$

Who is correct?

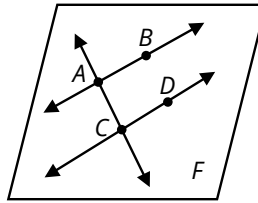
13. Use symbols to write two valid conclusions based on the figure shown. How do you read each conclusion?





Talk the Talk

Making a Valid Statement



1. Use symbols to name all points, lines, and planes shown.
2. Explain when it is appropriate to use the statement $JK = MN$ and when it is appropriate to use the statement $\overline{JK} \cong \overline{MN}$.
3. Use symbols to name all lines, rays, or segments shown.



Lesson 1 Assignment

Write

Describe the undefined terms point, line, and plane.

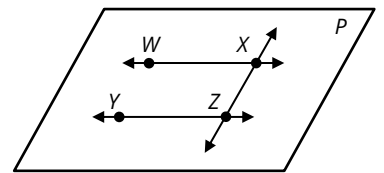
Remember

Points, lines, and planes are the building blocks of studying geometry. You can use these undefined terms to define other geometric objects.

Practice

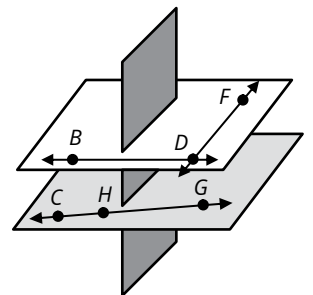
1. Identify each of the following in the figure shown.

- Name all points.
- Name all lines.
- Name all planes.



2. Identify each of the following in the figure shown.

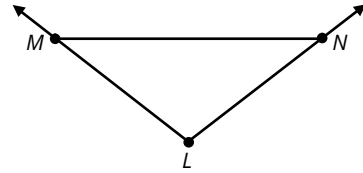
- Name all collinear points.
- Name all coplanar lines.
- Name all skew lines.



Lesson 1 Assignment

3. Identify each of the following in the figure shown.

a. Name all rays and identify each endpoint.



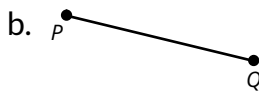
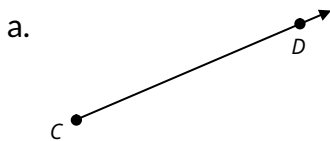
b. Name all line segments and identify the endpoints.

4. Explain the differences between sketching a geometric figure, drawing a geometric figure, and constructing a geometric figure.

5. Sketch two planes whose intersection is a line.

Lesson 1 Assignment

6. Sketch three planes whose intersection is a point.
7. Draw and label three collinear points X , Y , and Z such that point Y is between points X and Z and the distance between points X and Y is one half the distance between points Y and Z .
8. Use a symbol to represent the name of each geometric figure.



Lesson 1 Assignment

Prepare

Decide whether each statement is true or false and explain your reasoning.

1. All rectangles are quadrilaterals.
2. All rectangles are squares.
3. If it rains today, then it will not rain tomorrow.

2

Formal Reasoning in Euclidean Geometry

OBJECTIVES

- Identify the hypothesis and conclusion of a conditional statement.
- Explore the validity of conditional statements.
- Orally differentiate between postulates and theorems.
- Differentiate between Euclidean and spherical geometry.

.....

You have investigated many mathematical patterns and made observations.

How can you reason more formally in Euclidean geometry with postulates and theorems?

KEY TERMS

- conditional statement
- counterexample
- hypothesis
- conclusion
- postulate
- theorem
- validity
- Euclidean geometry
- proof
- great circle
- spherical geometry

Counterexamples

In this lesson you will start to use a more exact kind of reasoning commonly used in formal mathematics. When using formal mathematical reasoning, it is important to be able to identify false conclusions.

There are two reasons why a conclusion may be false. Either the assumed information is false, or the conclusion does not necessarily follow from the hypothesis.

Think About . . .

If it is daytime, then the sun must be shining.

This is an invalid argument. Both the assumed information and conclusion might be true, but the conclusion does not necessarily follow from the given.

1. Elijah tells his little brother that it will not rain for the next 30 days because he “knows everything.” Why is this conclusion false?
2. The sum of the interior angles of a figure is 360° , so the figure must be a square. Why is this conclusion false?
3. Write an example of a conclusion that is false because the assumed information is false.

4. Write an example of a conclusion that is false because the argument is not valid.

To show that a statement is false, you can provide a *counterexample*.

A **counterexample** is a specific example that shows that a general statement is not true.

5. Provide a counterexample for each statement to demonstrate that they are not true.
 - a. All prime numbers are odd.
 - b. The sum of the measures of two acute angles is always greater than 90° .

Conditional Statements

In geometry, *conditional statements* are the foundation of mathematical reasoning.

A **conditional statement** is a statement that can be written in the form “If p , then q .” It can be written using symbols as $p \rightarrow q$, which is read as “ p implies q .” In a conditional statement, the *hypothesis* is represented by the variable p . A **hypothesis** is the part of the “if-then” statement that follows “if.” In a conditional statement, the *conclusion* is represented by the variable q . A **conclusion** is the part of the “if-then” statement that follows the “then.”

The **validity** of a conditional statement is whether the statement is true or false. To evaluate whether a conditional statement is true, you must consider the relationship between the hypothesis and the conclusion of the statement. If a conditional statement could be true, then the statement is considered true. A conditional statement is either true or false, but not both.

WORKED EXAMPLE

You can identify the hypothesis and conclusion from a conditional statement.

Conditional Statement

If $x^2 = 36$, then $x = 6$ or $x = -6$.

Hypothesis

Conclusion

1. Consider the conditional statement.
If the measure of an angle is 32° , then the angle is acute.
 - a. Identify the hypothesis p .
 - b. Identify the conclusion q .

Use the given conditional statement in Question 1 to answer Questions 2 through 5.

2. If p is true and q is true, then the conditional statement is true.

- a. What does the phrase “If p is true” mean in terms of the conditional statement?
- b. What does the phrase “If q is true” mean in terms of the conditional statement?
- c. Explain why the conditional statement is true if both p and q are true.

3. If p is true and q is false, then the conditional statement is false.

- a. What does the phrase “If p is true” mean in terms of the conditional statement?
- b. What does the phrase “If q is false” mean in terms of the conditional statement?
- c. Explain why the conditional statement is false if p is true and q is false.

.....
When p is true and q is false, the conclusion does not follow from the hypothesis.
.....

.....
Continue to use this conditional statement to respond to Questions 3 through 5.
If the measure of an angle is 32° , then the angle is acute.
.....

.....
When p is true and q
is false, the conclusion
does not follow from
the hypothesis.
.....

4. If p is false and q is true, then the conditional statement is true.

a. What does the phrase “If p is false” mean in terms of the conditional statement?

b. What does the phrase “If q is true” mean in terms of the conditional statement?

.....
Think About . . .
If p is false and q is
true, the statement
is always true. Can
you think of other
examples that show
this?
.....

c. Explain why the conditional statement is true if p is false and q is true.

5. If p is false and q is false, then the validity of the conditional statement is true.

a. What does the phrase “If p is false” mean in terms of the conditional statement?

b. What does the phrase “If q is false” mean in terms of the conditional statement?

c. Explain why the conditional statement is true if both p and q are false.

6. Consider the conditional statement.

If $AB = 6$ inches and $BC = 6$ inches, then $\overline{AB} \cong \overline{BC}$.

- a. What is the hypothesis p ?
- b. What is the conclusion q ?
- c. If both p and q are true, what does that mean? Is the conditional statement true if both p and q are true?
- d. If p is true and q is false, what does that mean? Is the conditional statement true if p is true and q is false?
- e. If p is false and q is true, what does that mean? Is the conditional statement true if p is false and q is true?
- f. If both p and q are false, what does that mean? Is the conditional statement true if both p and q are false?



7. Mr. Davis wrote the statement shown on the board.

If $\overline{AC} \cong \overline{BC}$, then point C is the midpoint of $\overline{AB}$.

He asked his students to discuss the truth of this conditional statement.

Aaliyah said she believed the statement to be true in all situations. Mateo disagreed with Aaliyah and said that the statement was not true all of the time.

What is Mateo thinking and who is correct?

ACTIVITY 2.2

Postulates and Theorems

A **postulate** is a statement that is taken to be true without proof. A **theorem** is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.

The Elements is a book written by the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems. As a result, Euclid was able to develop a complete system we now know as **Euclidean geometry**.

Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one endpoint as its center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

Euclid used only the first four postulates to prove the first 28 propositions or theorems of *The Elements*, but was forced to use the fifth postulate, the parallel postulate, to prove the 29th theorem.

The Elements also includes five “common notions.”

1. Things that equal the same thing also equal one another.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things that coincide with one another equal one another.
5. The whole is greater than the part.



Greek mathematician Euclid is sometimes referred to as the Father of Geometry.

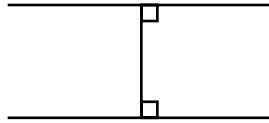
.....
A **proof** is a logical series of statements and reasons that starts with a hypothesis and arrives at a conclusion.
.....

.....
During President Abraham Lincoln's time as a lawyer in Illinois, he studied Euclid's works and learned to prove his theorems.
.....

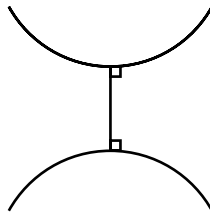
It is important to note that Euclidean geometry is not the only system of geometry. Examples of non-Euclidean geometries include hyperbolic, elliptic, and spherical geometry. The essential difference between Euclidean and non-Euclidean geometry is the nature of parallel lines.

Another way to describe the differences between these geometries is to consider two lines in a plane that are both perpendicular to a third line.

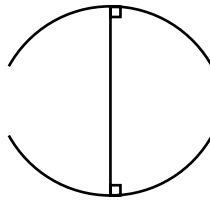
- In Euclidean geometry, the lines remain at a constant distance from each other and are known as parallels.



- In hyperbolic geometry, the lines “curve away” from each other.



- In elliptic geometry, the lines “curve toward” each other and eventually intersect.

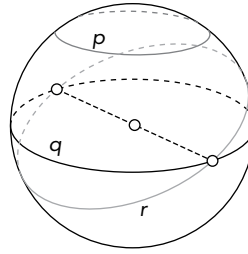


.....
A **great circle** is the intersection of a sphere and a plane passing through the center of the sphere.
.....

Spherical geometry, like its name implies, is a geometry that substitutes a sphere for a plane, which makes it different from Euclidean geometry in significant ways. Due to spherical geometry existing on a sphere or curved surface, some topics that you have learned about, such as parallel lines and the sum of the interior angles of a triangle, are very different.

In spherical geometry, lines are defined as **great circles** of a sphere, which divide the sphere into two congruent hemispheres.

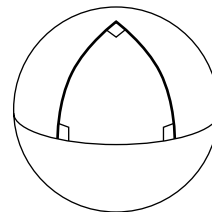
1. Which of the figures in the diagram shown— p , q , or r —appear to be spherical lines? Explain your reasoning.



2. Describe some similarities and differences between lines in Euclidean geometry and lines in spherical geometry, given this diagram.
3. Do you think that parallel lines can exist in spherical geometry? Explain your reasoning.
4. At how many points do 2 different spherical lines intersect? Explain your reasoning.

In spherical geometry, triangles have different properties as well. Consider the spherical triangle shown. It has 3 right angles, so the sum of its interior angle measures is 270° .

The sides of a spherical triangle are always arcs of great circles.



.....
Investigate the
properties of lines on
a sphere and spherical
triangles by drawing
on a ball, such as a
basketball or beach
ball.
.....

5. Describe what you think are some similarities and differences between triangles in Euclidean geometry and spherical triangles, given this diagram.
6. What do you think is the greatest sum of the interior angle measures of a spherical triangle? What is the least sum? Explain your reasoning.
7. What do you think would be the properties of a right spherical triangle? Explain your reasoning.



Talk the Talk

While You Were Away . . .

1. Write a short note to a friend explaining conditional statements and validity. Include definitions of all terms and examples that are easy to understand.

Lesson 2 Assignment

Write

Define each term in your own words.

1. counterexample
2. conditional statement
3. validity

Remember

A conditional statement is a statement that can be written in the form “if p , then q .” Written as $p \rightarrow q$, it is read “ p implies q .” The variable p represents the hypothesis and the variable q represents the conclusion.

A postulate is a statement that is accepted without proof. A theorem is a statement that can be proven.

Practice

1. Identify the hypothesis and conclusion of each conditional statement.
 - a. If it is raining, then I will carry an umbrella.
 - b. If a number is divisible by 3, then the sum of its digits is divisible by 3.
2. Give a counterexample to prove that each statement is not true.
 - a. If a number is divisible by 4, then it is divisible by 8.
 - b. If someone is a teacher, then they are always at school during the day.

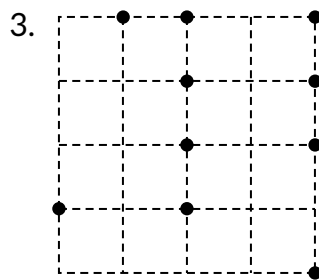
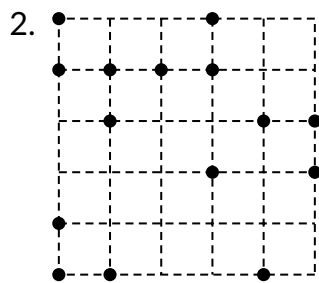
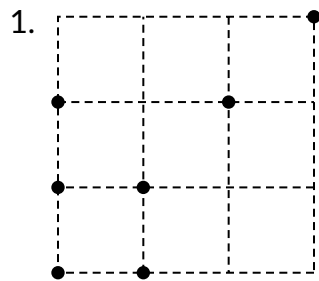
Lesson 2 Assignment

3. Consider the conditional statement.
“If the measure of an angle is 105° , then the angle is an obtuse angle.”
 - a. Identify the hypothesis and conclusion.
 - b. What is the validity of the conditional statement?
4. What is the difference between a postulate and a theorem?
5. What are some differences between Euclidean and spherical geometry?

Lesson 2 Assignment

Prepare

Identify and connect the vertices that form a square in each grid.



3

Making Conjectures and Deductive Reasoning

OBJECTIVES

- Recall properties of geometric figures.
- Understand that the results from measuring tools can be useful in composing a conjecture; however, they are not an acceptable form of mathematical reasoning to validate a conjecture.
- Distinguish between inductive and deductive reasoning through discussions and reading.
- Distinguish between undefined terms, definitions, postulates, conjectures, and theorems.
- Make a geometric conjecture and use deductive reasoning to validate it.

KEY TERMS

- inductive reasoning
- deductive reasoning
- conjecture
- diagonal
- angle
- vertex of an angle
- linear pair postulate
- segment addition postulate
- angle addition postulate
- auxiliary line

.....

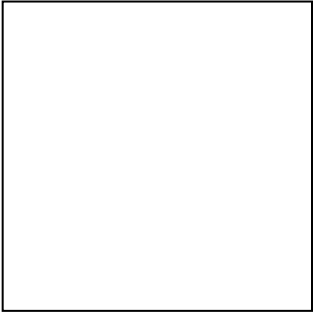
You have reasoned about lines and shapes in earlier grades and courses.

How can you apply formal geometric reasoning to what you know?

Getting Started

The Perfect Square

Can you sketch a “perfect” square freehand?



1. Try to sketch a “perfect” square, like the one shown, without tracing or using tools in the blank space below.
2. Explain how you could decide whether one square is closer to “perfect” than another. Use your criteria to judge your and your classmates’ best squares.
3. List some properties of squares that you know.

ACTIVITY 3.1

Mathematical Reasoning

The ability to use information to reason and make conclusions is very important in life and in mathematics. There are two common methods of reasoning. You can construct the name for each method of reasoning using your knowledge of prefixes, root words, and suffixes.

Word Fragment	Prefix, Root Word, or Suffix	Meaning
<i>in-</i>	Prefix	<i>toward or up to</i>
<i>de-</i>	Prefix	<i>down from</i>
<i>-duc-</i>	Root Word	<i>to lead and often to think, from the Latin word <i>duco</i></i>
<i>-tion</i>	Suffix	<i>the act of</i>

1. Form a word that means “the act of thinking down from.”
2. Form a word that means “the act of thinking toward or up to.”

Inductive reasoning is a way of thinking that uses induction. **Inductive reasoning** is a process of arriving at a conclusion by generalizing a statement from observing patterns. For example, you may notice that when you don’t study for a test, your grade is lower than when you do study for a test. You apply what you learned from these observations to the next test you take. Sometimes a conjecture made this way seems true, but one counterexample proves it to be false.

Deductive reasoning is way of thinking that uses deduction. **Deductive reasoning** is a process of using statements and ideas that you know to be true in order to prove new statements and ideas. These statements and ideas could include facts, definitions, theorems, postulates, or properties. For example, you may learn the rule for which direction to turn a screwdriver: “righty tighty, lefty loosey.” If you want to remove a screw, you apply the rule and turn the screwdriver counterclockwise.

Remember ...

a prefix is at the beginning of a word and a suffix is at the end.

Reasoning is the process of using existing knowledge to draw conclusions, make predictions, or construct explanations.

A **conjecture** is a mathematical statement that appears to be true, but has not been formally proved.

3. Chloe considered the following statements

- $4^2 = 4 \cdot 4$
- Nine cubed is equal to nine times nine times nine
- 10 to the fourth power is equal to four factors of 10 multiplied together

Chloe concluded that raising a number to a power is the same as multiplying the number as many times as indicated by the exponent. How did Chloe reach this conclusion?

4. Juan read that raising a number to a power is the same as multiplying that number as many times as indicated by the exponent. He had to determine seven to the fourth power using a calculator. So, he entered $7 \cdot 7 \cdot 7 \cdot 7$. How did Juan reach this conclusion?

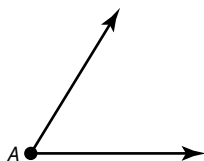
5. Identify the type of reasoning Chloe and Juan used to come to their conclusions. Explain your reasoning.

ACTIVITY
3.2

Creating Mathematical Ideas

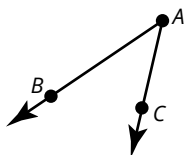
You have already used the three undefined terms *point*, *line*, and *plane* to define related terms such as *line segment* and *ray*. You can use definitions to define additional terms. An **angle** is formed by two rays that share a common endpoint.

When there is only a single angle in a diagram, you can name an angle by using the symbol for angle ($\angle$) and its vertex. For example, $\angle A$.

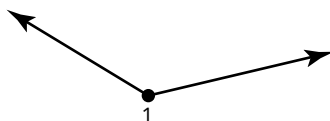


.....
The **vertex of an angle** is the common endpoint that the two rays of the angle share.
.....

You can also name an angle by three points, one point on one ray of an angle, then the vertex point, and the final point on the other ray of the angle. For example, $\angle BAC$.

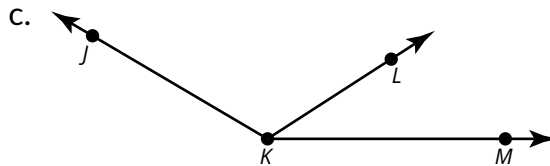
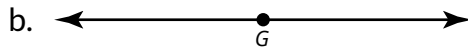
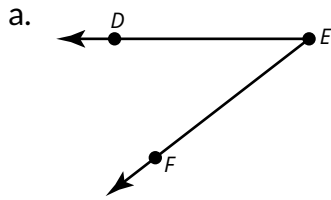


Finally, you can name an angle using a number. For example, $\angle 1$.



1. How is a definition different from an undefined term?

2. Name each angle.



Now, consider these three postulates.

- The linear pair postulate
- The segment addition postulate
- The angle addition postulate

The **linear pair postulate** states: “If two angles form a linear pair, then the angles are supplementary.”

3. Use the linear pair postulate to complete each representation.

a. Sketch and label a linear pair.

b. Use your labeled sketch and the linear pair postulate to write the hypothesis.

- c. Use your labeled sketch and the linear pair postulate to write the conclusion.
- d. Use your conclusion and the definition of supplementary angles to write a statement about the measures of the angles in your figure.

The **segment addition postulate** states: "If point B is on $\overline{AC}$ and between points A and C , then $AB + BC = AC$."

- 4. Use the segment addition postulate to complete each representation.
 - a. Sketch and label collinear points D , E , and F with point E between points D and F .
 - b. Use your labeled sketch and the segment addition postulate to write the hypothesis.
 - c. Use your labeled sketch and the segment addition postulate to write the conclusion.

The **angle addition postulate** states: "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

- 5. Use the angle addition postulate to complete each representation.
 - a. Sketch and label $\angle DEF$ with $\overrightarrow{EG}$ drawn in the interior of $\angle DEF$.

b. Use your labeled sketch and the angle addition postulate to write the hypothesis.

c. Use your labeled sketch and the angle addition postulate to write the conclusion.

You will use these postulates to make various conjectures.

You previously learned that a *conjecture* is a statement that appears to be true in many cases has yet to be proven true in all cases.

For example, you might conjecture that if point B is on $\overline{AC}$ and between points A and C , then $AB = AC - BC$.

If you can prove your conjecture, then the conjecture will become a theorem.

Theorems can then be used to make even more conjectures, which may also become theorems. Mathematicians use this process to create new mathematical ideas.

6. How is a theorem different from a conjecture or a postulate?

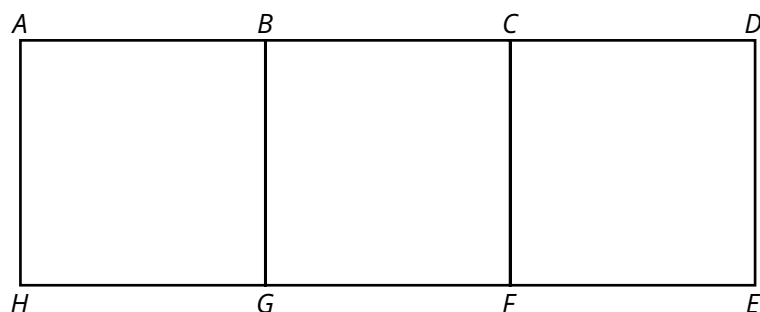
7. How is a postulate similar to an undefined term?

ACTIVITY 3.3

Analyzing a Diagram

In a way, mathematical reasoning is not different from scientific reasoning. In mathematics, you come up with educated guesses and test them to see if they're correct. You can experiment with different patterns and consider arguments about mathematical statements. And, like other scientists, mathematicians gather evidence and become more and more confident about a statement when they obtain more evidence for it.

Consider the diagram composed of three adjacent squares.



1. Draw $\overline{AG}$, $\overline{AF}$, $\overline{AE}$. Then label $\angle AGH$ using the letter a , label $\angle AFG$ using the letter b , and label $\angle AEF$ using the letter c .
2. Use a protractor to measure $\angle a$, $\angle b$, and $\angle c$. List the angle measures.
3. Compare your measurements with your classmates' measurements. What do you notice?

Remember ...

When you sketch a geometric figure, you create the figure without tools. Accuracy is not important. When you draw geometric figures, you can use tools such as rulers, protractors, or a coordinate plane to draw exact lengths and areas.

PROBLEM SOLVING



ACTIVITY

3.4

Making Conjectures

In the previous activity, you may have noticed that the sum of the measures of $\angle a$, $\angle b$, and $\angle c$ is close to or equal to 90° . Mia made a conjecture about the sum of the angle measures.

Mia

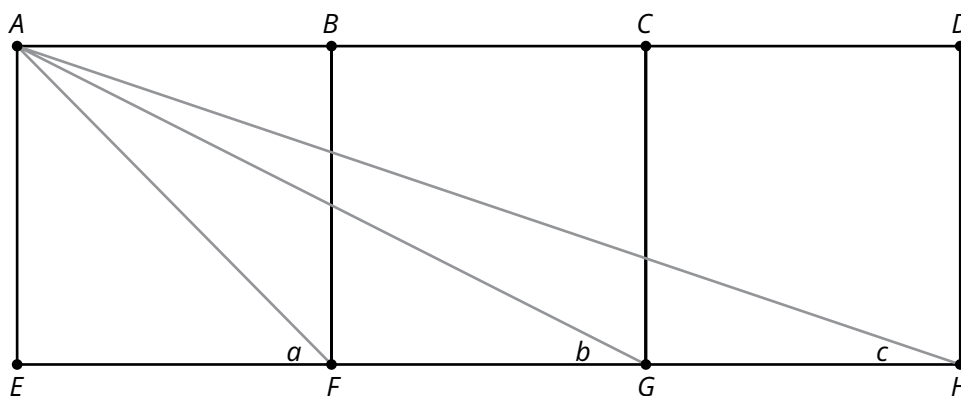


The size of the squares doesn't matter. Given any three adjacent and congruent squares, if the diagonals are drawn in the same way, the sum of the angle measures will always be 90° .

You can write Mia's conjecture as a conditional statement.

A **diagonal** is a line segment joining two vertices of a polygon but is not a side of the polygon.

Let's consider a diagram of three differently-sized adjacent and congruent squares. The same lines are drawn and triangles formed.



- Without measuring, do you think the size of the squares will affect the sum of the measures of $\angle a$, $\angle b$, and $\angle c$? Explain your thinking.

2. Use a protractor to test your prediction on these differently-sized squares. Record your results.

Ask Yourself . . .

Do your results support Mia's conjecture?

There are many different ways to verify that the sum of the angle measures could be 90° . Experimenting with different methods and visualizing are important tools that mathematicians use to approach problems in effective ways and gain confidence in their conclusions.

3. Copy each of the angles a , b , and c from the diagram onto three separate pieces of patty paper. How can you manipulate the three angles to show that their sum is 90° ?

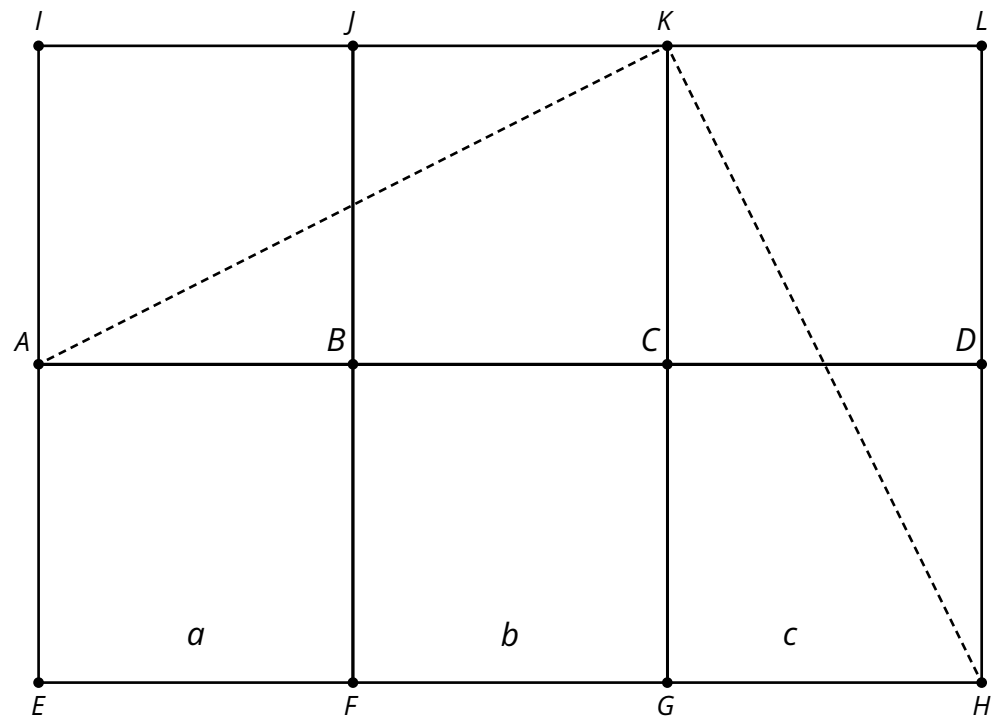
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An **auxiliary line** is a line or line segment added to a diagram to help in solving or proving a concept. Typically auxiliary lines are drawn with dashed lines.

.....

Making arguments about statements in mathematics may seem like a rigid process at times, but it can also involve a lot of creativity. You can, for example, draw extra lines, called *auxiliary lines*, and perform rigid motions like translations, reflections, and rotations when you are reasoning geometrically.

Let's consider a new diagram that is the result of translating the original three squares up and drawing two auxiliary line segments.



1. What other angle measures or sides lengths can you determine using the added figures? List the concepts and facts you are use to demonstrate your deductive reasoning.



Talk the Talk

Proving Yourself

In this course, you will move from making conjectures and creating informal arguments to proving, for good, that certain mathematical statements must be true. You will learn to use properties and definitions to prove or disprove many conjectures.

1. Complete the graphic organizer to distinguish between undefined terms, definitions, postulates, theorems, and conjectures and provide an example of each.

Term	Description	Example
Undefined term		
Definition		
Postulate		
Theorem		
Conjecture		

Lesson 3 Assignment

Write

Define each term in your own words.

1. inductive reasoning
2. deductive reasoning
3. conjecture
4. auxiliary line

Remember

Mathematicians make conjectures, test predictions, experiment with patterns, and consider arguments and different perspectives.

In mathematics, a statement is not true or false until it is proved to be true or false.

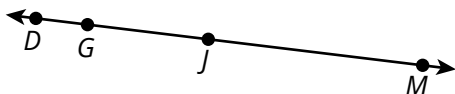
Formal and rigorous mathematical reasoning can involve creative thinking.

Practice

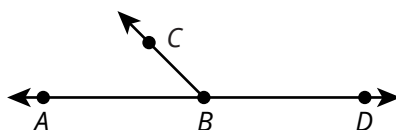
1. Determine whether inductive reasoning or deductive reasoning is used in each situation. Explain your reasoning.
 - a. Jackson reads a journal article that states that yogurt with live cultures greatly helps digestion and prevents problems associated with lactose intolerance. He notices that his mother has problems with digestion and is lactose intolerant. He suggests that she try eating yogurt because he thinks it may help her feel better.
 - b. Catilina is looking through books at a bookstore with her friend Miguel. She comes across a book she has not read by an author she enjoys. Miguel knows that Catilina has five books at home by this author and that she likes all of them. He concludes that she will probably like any book made by this author.

Lesson 3 Assignment

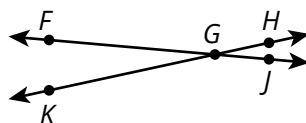
2. Use the segment addition postulate to write four different statements about the figure shown.



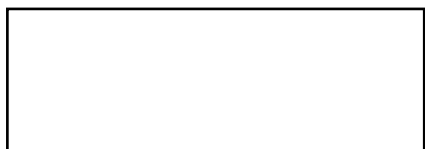
3. Use the linear pair postulate to write a symbolic statement about the figure.



4. Use the angle addition postulate to write a symbolic statement about the figure.

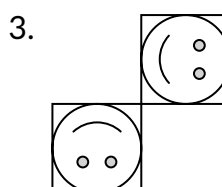
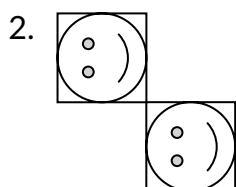


5. Measure the lengths of the diagonal of each rectangle using a ruler. Make a conjecture about the diagonals of a rectangle.



Prepare

Identify whether the transformation shown is a translation, reflection, or rotation. Justify your answer.



TOPIC 1 SELF-REFLECTION

Name: _____

Geometry Reasoning

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Geometry Reasoning* topic by:

TOPIC 1: <i>Geometry Reasoning</i>	Beginning of Topic	Middle of Topic	End of Topic
distinguishing between undefined terms, definitions, conjectures, postulate, theorems, and proofs.			
explaining the difference between the terms draw, sketch, and construct.			
verifying a conjecture is false using a counterexample.			
identifying the hypothesis and conclusion given a conditional statement.			
determining the validity of a conditional statement.			
recognizing a linear, exponential, or quadratic function by its equation or graph.			
explaining the difference between Euclidean geometry and spherical geometry.			
making a conjecture and using deductive reasoning to validate it.			

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Geometry Reasoning* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Geometry Reasoning

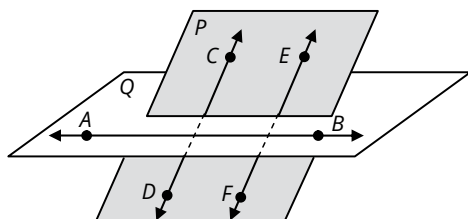
LESSON

1

Points, Lines, Planes, Rays, and Line Segments

There are three essential building blocks of geometry- the *point*, the *line*, and the *plane*. These three terms are called undefined terms; we can only describe and create mathematical models to represent them. A **point** is a location in space that has no size or shape. A **line** is a straight continuous arrangement of an infinite number of points. A **plane** is a flat surface that has an infinite length and width, but no depth. **Collinear points** are points that are located on the same line. **Coplanar** lines are two or more lines that are located in the same plane. **Non-collinear points** are points that do not lie on the same line. **Skew lines** are two or more lines that are not in the same plane.

Example



Points A and B lie on $\overline{AB}$, points C and D lie on $\overline{CD}$, and points E and F lie on $\overline{EF}$.

Line AB lies in plane Q . Lines CD and EF lie in plane P .

Points A and B are collinear. Points C and D are collinear. Points E and F are collinear.

Lines CD and EF are coplanar.

Lines AB and CD are skew. Lines AB and EF are skew.

Planes P and Q intersect.

KEY TERMS

- point [punto]
- line
- collinear points [puntos colineales]
- plane [plano]
- noncollinear points [puntos no colineales]
- compass [compás]
- straightedge
- sketch
- draw
- construct [construir]
- coplanar lines
- skew lines
- ray [rayo]
- endpoint of a ray
- line segment
- endpoints of a line segment
- congruent line segments
- conditional statement
- biconditional statement
- counterexample [contrajemplo]
- hypothesis [hipótesis]
- conclusion [conclusión]

- postulate [postulado]
- theorem [teorema]
- proof [prueba]
- validity [validez]
- Euclidean geometry [geometría euclidiana]
- great circle
- spherical geometry [geometría esférica]
- conjecture [conjetura]
- inductive reasoning [razonamiento inductivo]
- deductive reasoning [razonamiento deductivo]
- diagonal [diagonal]
- angle [ángulo]
- vertex of an angle [vértice de un ángulo]
- linear pair postulate [postulado del par lineal]
- segment addition postulate
- angle addition postulate
- auxiliary line

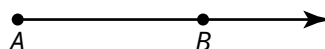
You can use many tools to create geometric figures. Some tools, such as a ruler or a protractor, are classified as measuring tools.

A **compass** is a tool used to create circles and portions of circles.

A **straightedge** is a ruler without numbers.

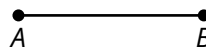
- When you **sketch** a geometric figure, the figure is created without using tools.
- When you **draw** a geometric figure, the figure is created using tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.
- When you **construct** a geometric figure, the figure is created using only a compass and a straightedge.

A **ray** is a part of a line that begins with a single point and extends infinitely in one direction. The **endpoint of a ray** is the single point where the ray begins.



A ray is named using two capital letters, the first representing the endpoint and the second representing any other point on the ray. You can write ray AB can be written using symbols as $\overrightarrow{AB}$.

A **line segment** is a part of a line that includes two points and all of the collinear points between the two points. The **endpoints** of a line segment are the points where the line segment begins and ends.



A line segment is named using two capital letters representing the two endpoints of the line segment Line segment AB can be written using symbols as $\overline{AB}$.

If two line segments have equal measure, then the line segments have the same length. **Congruent line segments** are two or more line segments of equal measure.

LESSON

2

Formal Reasoning in Euclidean Geometry

When reasoning in mathematics, it is important to be able to identify false conclusions. There are two reasons why a conclusion may be false. Either the assumed information is false, or the conclusion does not necessarily follow from the hypothesis.

For example, consider the statement, “If you are reading this sentence, then your first language must be English.” This is an invalid argument. Both the hypothesis and conclusion might be true, but the conclusion does not necessarily follow from the hypothesis.

To show that a statement is false, you can provide a counterexample.

A **counterexample** is a specific example that shows that a general statement is not true.

The **validity** of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the validity of the statement is considered true. The validity of a conditional statement is either true or false, but not both.

You can identify the **hypothesis** and **conclusion** from a **conditional statement**. For example:

conditional statement

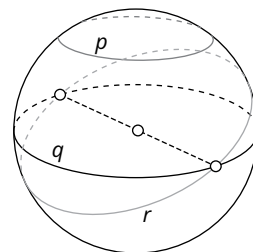
$$\text{If } \underbrace{x^2 = 36}_{\text{Hypothesis}}, \text{ then } \underbrace{x = 6 \text{ or } x = -6}_{\text{Conclusion}}.$$

A **proof** is a logical series of statements and reasons that starts with a hypothesis and arrives at a conclusion. A **postulate** is a statement taken to be true without proof. A **theorem** is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.

Euclid was a Greek mathematician who used a small number of undefined terms and postulates to systematically prove many theorems. As a result, Euclid was able to develop a complete system we now know as **Euclidean geometry**.

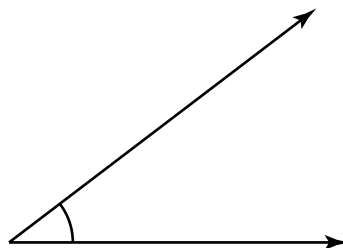
Spherical geometry, like its name implies, is a geometry that substitutes a sphere for a plane, which makes it different from Euclidean geometry in significant ways. In spherical geometry, some topics that you have learned about, such as parallel lines and the sum of the interior angles of a triangle are very different.

In spherical geometry, lines are defined as *great circles* of a sphere, which divide the sphere into two congruent hemispheres. A **great circle** is the intersection of a sphere and a plane passing through the center of the sphere.



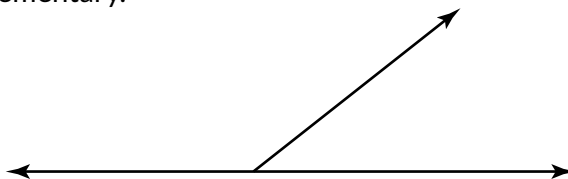
The ability to use information to reason and make conclusions is very important in life and in mathematics. There are two common methods of reasoning. **Inductive Reasoning** is a process of arriving at a conclusion by generalizing a statement from observing patterns. **Deductive Reasoning** is a process of using statements and ideas that you know to be true in order to prove new statements and ideas. These statements and ideas could include facts, definitions, theorems, postulates, or properties.

An **angle** is formed by two rays that share a common endpoint, called the **vertex of the angle**.

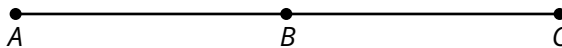


Three fundamental postulates—the *linear pair postulate*, the *segment addition postulate*, and the *angle addition postulate*—can be used to make various conjectures. If the conjectures are proven, then they will become theorems.

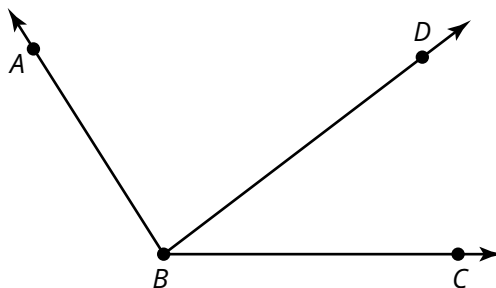
The **linear pair postulate** states, “If two angles form a linear pair, then the angles are supplementary.”



The **segment addition postulate** states, “If point B is on $\overline{AC}$ and between points A and C , then $AB + BC = AC$.”

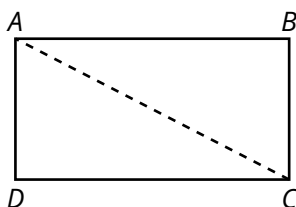


The **angle addition postulate** states, “If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.”

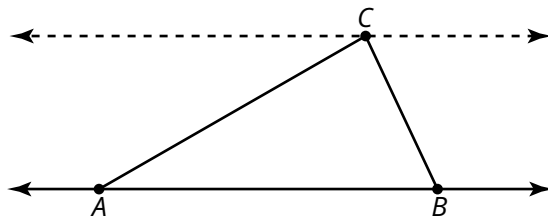


A **diagonal** is a line segment joining two vertices of a polygon but is not the side of the polygon.

For example, $\overline{AC}$ is a diagonal of rectangle $ABCD$.



An **auxiliary line** is a line or line segment added to a diagram to help in solving or proving a concept. For example, the dashed line drawn parallel to $\overleftrightarrow{AB}$ through point C is an auxiliary line that can be used to reason geometrically about the sum of the measures of the interior angles of a triangle.



The coordinate plane that you know about is made up of an infinite number of congruent, adjacent squares, like the tiling on a floor or wall.

Using a Rectangular Coordinate System

LESSON 1	Constructing a Coordinate Plane	65
LESSON 2	Parallel and Perpendicular Lines	87
LESSON 3	Classifying Quadrilaterals on the Coordinate Plane	109
LESSON 4	Classifying Triangles on the Coordinate Plane	139
LESSON 5	Area and Perimeter on the Coordinate Plane	157



1

Constructing a Coordinate Plane

OBJECTIVES

- Construct a perpendicular line segment and bisector using paper folding and using a compass and straightedge.
- Use construction tools to duplicate a line segment and construct a square.
- Use constructions and rigid motions to create a coordinate plane.
- Identify rigid motions that can be used to create shapes on a coordinate plane.
- Identify the coordinates of vertices of shapes on a coordinate plane orally and in writing.

KEY TERMS

- arc
- midpoint
- segment bisector
- perpendicular bisector
- transformation
- rigid motion

.....

You have recalled and investigated two-dimensional shapes on a plane. How can you construct a coordinate plane to help you analyze geometric objects?

Getting Started

Constructing a Perpendicular Line Using Patty Paper

.....
A compass is a tool used to create arcs and circles.

A straightedge is a ruler with no numbers.

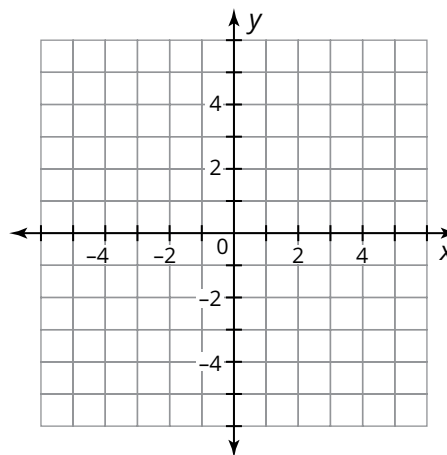
An **arc** consists of two points on a circle and the curved line that connects the two points.
.....

Remember, when you construct geometric figures, you create exact figures without measurements, using paper folding or a *compass* and a *straightedge*.

A coordinate plane is composed of two intersecting lines—the x-axis and y-axis.

You can also think of a coordinate plane as being composed of squares.

Each square is 1 unit long and 1 unit wide, with the definition of “1 unit” up to the person who constructs the coordinate plane.



In this lesson, you will consider how a coordinate plane is constructed using squares. To construct a square, you will first need to be able to construct perpendicular lines.

Let's start by experimenting with patty paper to construct a line perpendicular to a given line segment.

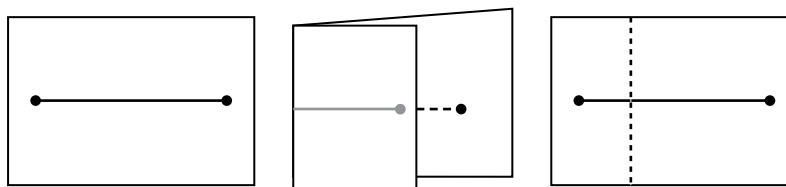
.....
You can represent a plane with a piece of paper since paper is a flat surface. Remember a geometric plane extends infinitely in all directions, unlike a physical piece of paper which has boundaries.
.....

WORKED EXAMPLE

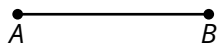
Draw a line segment on a piece of patty paper.

Fold the line segment so that it lies on top of itself.

Open the patty paper. The crease represents a line perpendicular to the given line segment.



1. Consider $\overline{AB}$.

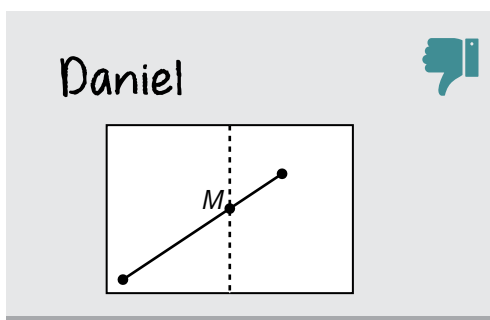


- a. Use patty paper to construct 2 different perpendicular lines through $\overline{AB}$.

- b. How can you fold the patty paper so that the perpendicular crease intersects the midpoint of $\overline{AB}$?

.....
 A **midpoint** of a line segment is the point that divides the line segment into two congruent segments.

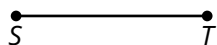
2. Daniel determined the midpoint of a line segment incorrectly. Explain what he did.



A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

.....
Think about...
 How can you be sure that you correctly bisected the line segment?

3. Use patty paper to bisect $\overline{ST}$.

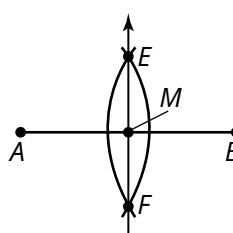
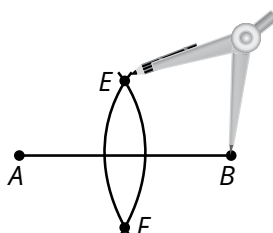
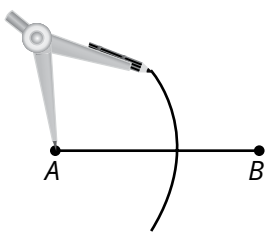


Constructing a Perpendicular Line Using a Compass and a Straightedge

You can also construct segment bisectors using a compass and straightedge. To do this construction, you will draw arcs, making use of the fact that all the radii of a circle have an equal length.

WORKED EXAMPLE

You can construct the segment bisector of $\overline{AB}$.



Construct an Arc

Open the radius of the compass to more than half the length of $\overline{AB}$. Use endpoint A as the center and construct an arc.



Construct Another Arc

Keep the compass radius and use point B as the center as you construct an arc. Label the points formed by the intersection of the arcs point E and point F.



Construct a Line

Connect points E and F. Line segment $\overline{EF}$ is the segment bisector of $\overline{AB}$. The point M represents the midpoint of $\overline{AB}$.

Line $\overline{EF}$ bisects $\overline{AB}$. Point M is the midpoint of $\overline{AB}$.

.....
Though the statement “all perpendicular bisectors are segment bisectors, but not all segment bisectors are perpendicular bisectors is true,” the construction for a segment bisector is the same.
.....

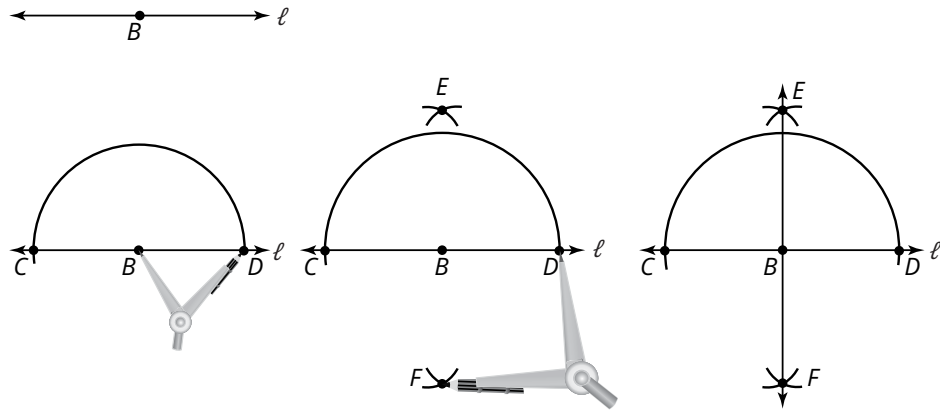
A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

1. All perpendicular bisectors are segment bisectors, but not all segment bisectors are perpendicular bisectors. Explain why this statement is true. Draw an example of both a segment bisector and perpendicular bisector to support your explanation.

You can construct a perpendicular line through a point on the line using a compass and straightedge. For this construction, you will continue to draw arc and make use of the fact that all radii of a circle have an equal length.

WORKED EXAMPLE

You can construct a line perpendicular to line ℓ through point B on the line.



Construct an Arc

Use B as the center and construct an arc. Label the intersection points C and D .

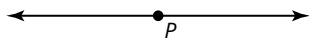
Construct Other Arcs

Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F .

Construct a Line

Use a straightedge to connect points E and F . Line EF is perpendicular to $\overleftrightarrow{CD}$.

2. Construct a line perpendicular to the given line through point P .

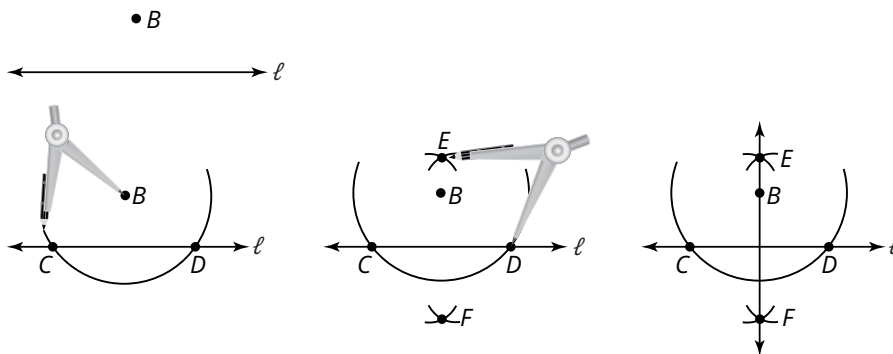


3. How are constructing a segment bisector and constructing a perpendicular line through a point on a line different?

You can also construct a perpendicular line through a point that is not on a line.

WORKED EXAMPLE

You can construct a line perpendicular to line ℓ through point B not on the line.



Construct an Arc

Use B as the center and construct an arc. Label the intersection points C and D .

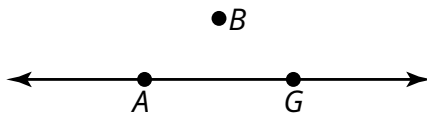
Construct Other Arcs

Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F .

Construct a Line

Use a straightedge to connect points E and F . Line EF is perpendicular to $\overleftrightarrow{CD}$ and passes through point B .

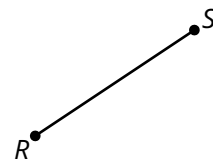
4. Construct a line perpendicular to line ℓ through point B



5. Diego is constructing the perpendicular bisector of $\overline{RS}$.

His work is shown.

Diego says that because the arcs do not intersect, this line segment does not have a midpoint. Mia disagrees and tells him he drew his arcs incorrectly and that he must redraw his arcs to determine the midpoint. Who is correct? Explain your reasoning.



6. Use construction tools to locate the perpendicular bisector of each given line segment. Label each midpoint as M .

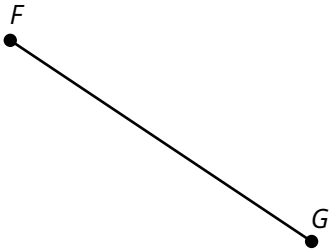
a.



b.



c.



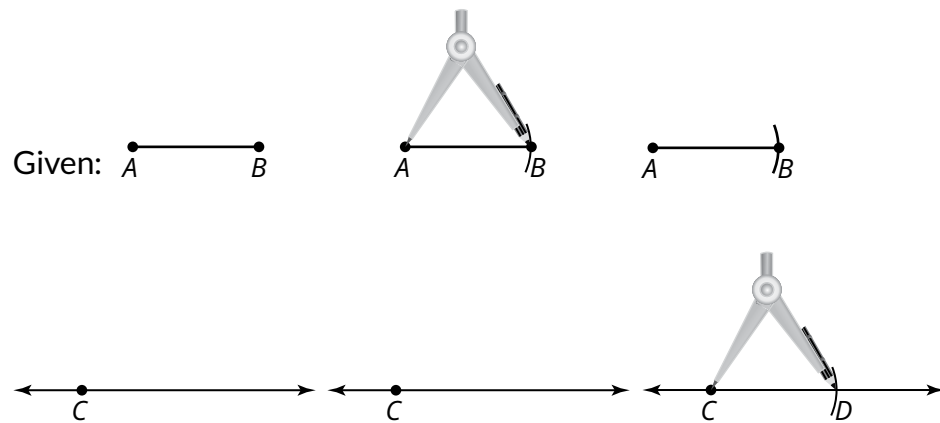
7. Choose a point on the perpendicular bisector of one of the line segments in Question 6 and label it X. Measure the distances from point X to each of the segment's endpoints. Choose another point on the perpendicular bisector and label it Y. Measure the distances from point Y to each of the segment's endpoints. What do you notice?
8. Make a conjecture about the distance from any point on a perpendicular bisector to the endpoints of the original segment.

Along with constructing perpendicular lines, you will need to be able to duplicate line segments in order to construct a square.

There are different ways to use construction tools to duplicate a line segment.

WORKED EXAMPLE

You can duplicate a line segment by constructing an exact copy of the original line segment.



Construct a Starter Line

Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point C on the line.

Measure Length

Set your compass at the length AB .

Copy Length

Place the compass at C. Mark point D on the new segment.

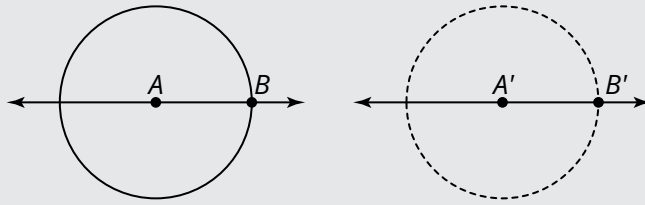
$$\overline{CD} \cong \overline{AB}.$$

1. Construct a line segment that is twice the length of $\overline{AB}$.



2. Harper and Sofia are duplicating $\overline{AB}$. Their methods are shown.

Harper

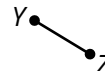
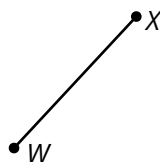
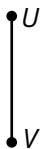


Sophia



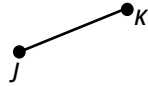
Which method is correct? Explain your reasoning.

3. Use a compass and a straightedge to duplicate each line segment.



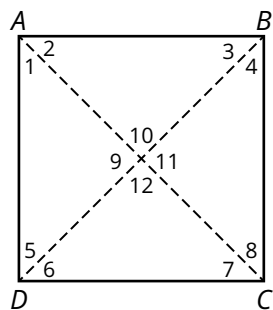
Mathematically, to duplicate a line segment means to create a congruent line segment that has an equal measure to the original line segment.

4. Use what you know about duplicating segments and constructing perpendicular lines to construct a square with the same side length as $\overline{JK}$.



5. Use patty paper to verify that the figure you constructed is a square.

6. Draw the diagonals of your square and label the angles as shown. What do you notice about the segments and angles? Use patty paper to justify any conjectures.



ACTIVITY
1.3

Rigid Motions

To complete the construction of a coordinate plane, you can perform rigid motions of a constructed square.

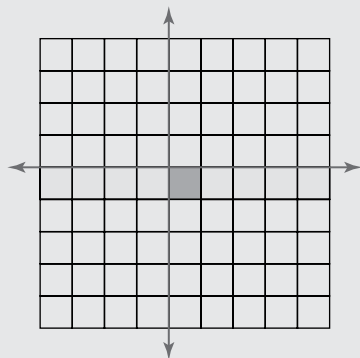
Recall that a **transformation** is the mapping, or movement, of the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure.

Jaden used translations to create a coordinate plane.

Jaden



I can translate a square to the right and to the left an infinite number of times. Then, I can translate that entire row of squares up and down an infinite number of times to create a coordinate plane.



.....
Remember . . .

A translation “slides” a figure up, down, left, or right.

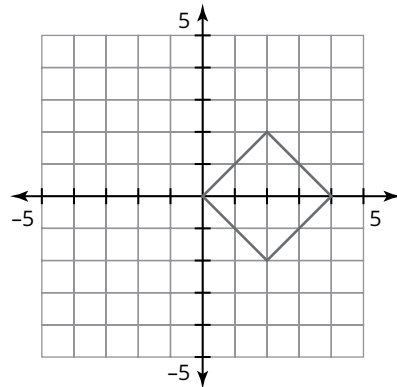
A reflection “flips” a figure across a line.

A rotation “spins” a figure about a point.
.....

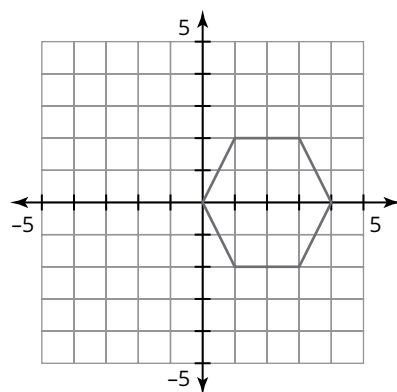
1. What other sequences of rigid motions of a square can you use to create a coordinate plane? Show your work and explain your reasoning.

2. The figures shown were each constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape. Then, give the coordinates of the vertices of the shape.

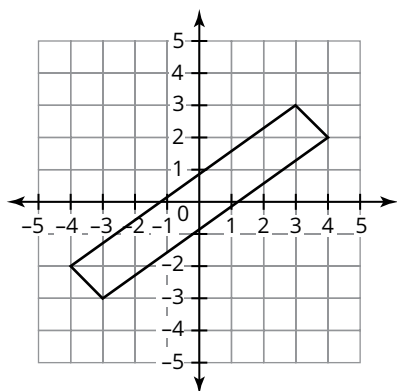
a.



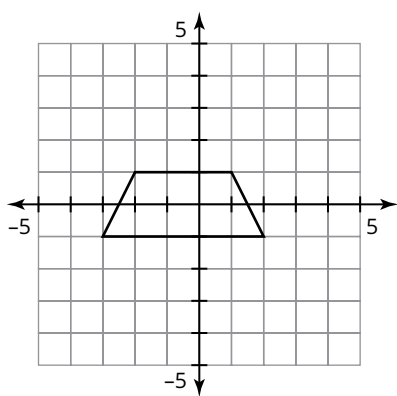
b.



c.



d.

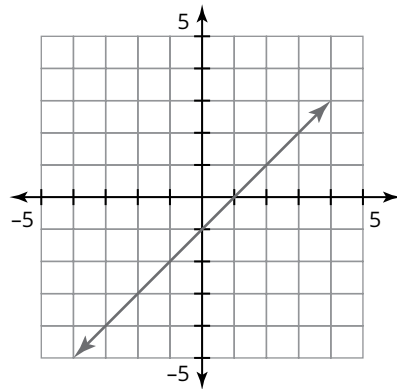




Talk the Talk

Walking on a Thin Line

Consider the line shown on the coordinate plane.



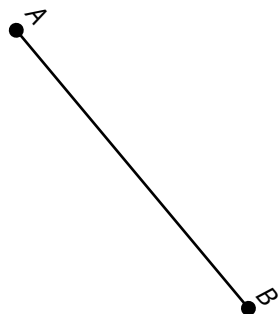
1. Suppose the line was constructed using rigid motions, starting with any line segment constructed in one square. Describe one possible sequence of translations that could produce the line.
2. How is the sequence of translations related to the slope of the line?

Remember ...

The equation for a line can be written in slope-intercept form, $y = mx + b$, where m represents the slope of the line, and b represents the y-intercept.

3. What is the equation for the line in slope-intercept form?

4. Construct the perpendicular bisector of the given segment.
Label the midpoint as M .



5. Use a compass and a straight edge to duplicate $\overline{EF}$.



Lesson 1 Assignment

Write

Describe the similarities and differences of a segment bisector and a perpendicular bisector.

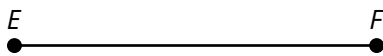
Remember

A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

A translation “slides” a figure up, down, left, or right. A reflection “flips” a figure across a line. A rotation “spins” a figure about a point.

Practice

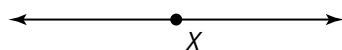
1. Locate the midpoint of the line segment using construction tools and label it point M . Then, explain how you know that point M is the midpoint of $\overline{EF}$.



Lesson 1 Assignment

2. Construct a line perpendicular to each given line and through the given point. Then, explain how you know the constructed line is perpendicular to the given line.

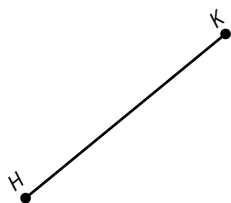
a.



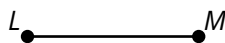
b.



3. Construct the perpendicular bisector of the given segment. Label the midpoint as M .

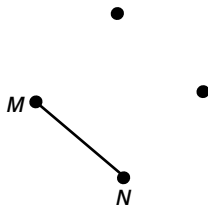


4. Use a compass and a straight edge to duplicate $\overline{LM}$.

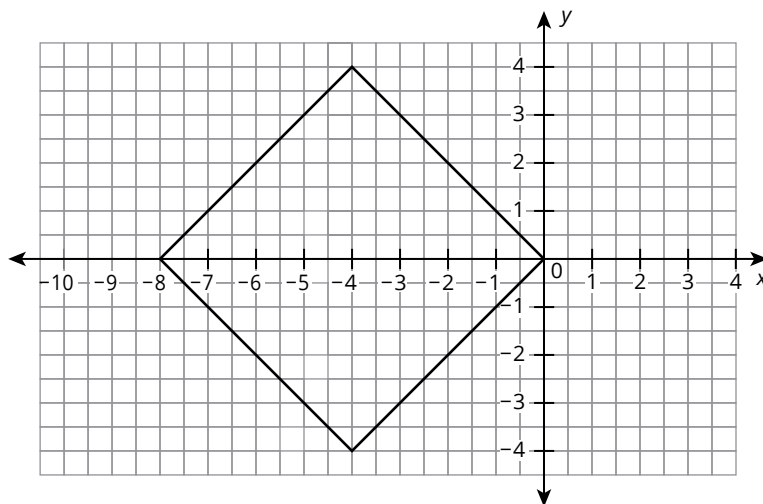


Lesson 1 Assignment

5. Use what you know about duplicating segments and constructing perpendicular lines to construct a square with the same side length as $\overline{MN}$.



6. The figure shown was constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape. Then, give the coordinates of the vertices of the shape.



Lesson 1 Assignment

Prepare

Determine the reciprocal of each value.

1. 3

2. -10

3. $\frac{1}{5}$

4. $-c$

5. $\frac{a}{b}, b \neq 0$

2

Parallel and Perpendicular Lines

OBJECTIVES

- Construct parallel lines.
- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of parallel lines, including horizontal and vertical lines.

.....

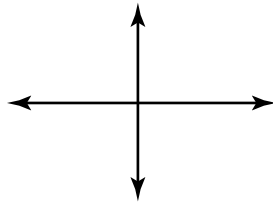
You have constructed line segments, perpendicular lines, squares, and a coordinate plane.

How can the coordinate plane be used to justify parallel and perpendicular line relationships?

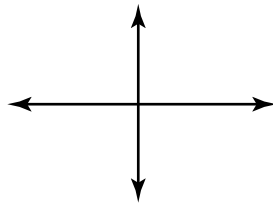
Getting Started

Which Way Is Up?

1. Sketch lines with a given slope.
 - a. Sketch two lines that have positive slopes. Then, compare sketches with your classmates and explain any similarities and differences.



- b. Sketch two lines that have negative slopes. Then, compare sketches with your classmates and explain any similarities and differences.



ACTIVITY 2.1

Deriving the Slope Formula

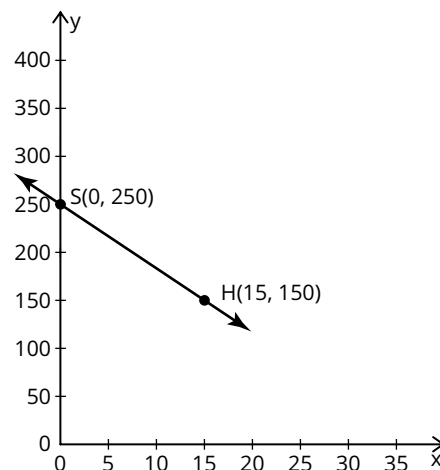
Recall that slope is a measure of the steepness of a line. A line with a positive slope progresses in an upward direction from left to right. A line with a negative slope progresses in a downward direction from left to right.

Let's derive the slope of a specific line.

1. Derive the slope of line SH .

a. Sketch a vertical segment from point S to a point that has the same y -coordinate of point H . Label the endpoint of your segment as point E .

b. Sketch a horizontal segment from point E to point H .



c. Determine the length of $\overline{SE}$ and explain how SE can be calculated using the y -coordinates of points S and H .

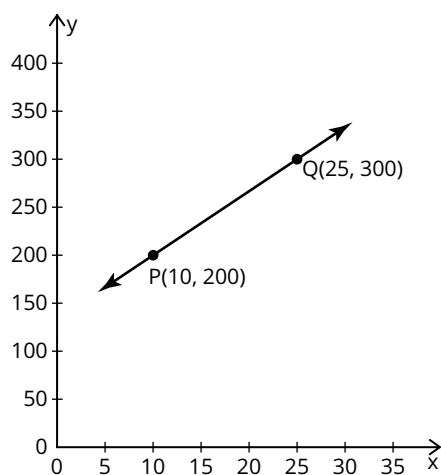
.....
The length of $\overline{SE}$ represents the rise of $\overrightarrow{SH}$, and the length of $\overline{EH}$ represents the run.
.....

d. Determine the length of $\overline{EH}$ and explain how EH can be calculated using the x -coordinates of points E and H .

e. Is the slope of line SH positive or negative? Explain your reasoning.

f. Write the ratio of the rise to the run for $\overleftrightarrow{SH}$.

2. Derive the slope of $\overleftrightarrow{PQ}$.



.....
The ratio of a line's
rise to its run, $\frac{\text{rise}}{\text{run}}$, is
the slope of a line.
.....

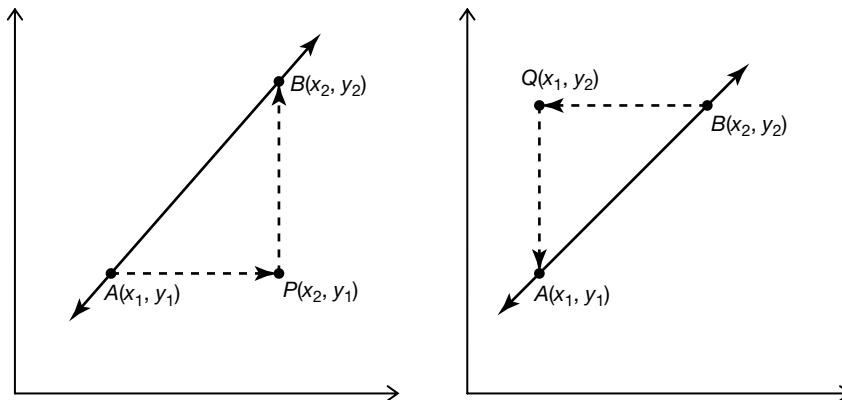
a. Is the slope of $\overleftrightarrow{PQ}$ positive or negative? Explain your reasoning.

b. Write the ratio of the rise to the run for $\overleftrightarrow{PQ}$. Explain your reasoning.

Now that you have derived the slopes of specific lines, let's derive the general slope formula.

WORKED EXAMPLE

Follow the steps to determine the slope of a line.



Step 1: Choose two points on the line. Choose points with coordinate values that do not require estimation. The two points are shown as A and B.

Step 2: Subtract the y-coordinates of points A and B to determine the rise—the vertical change from the first point to the second point.

$$\text{Vertical change: } y_2 - y_1$$

Step 3: Continue from the point where you left off on the graph to determine the run—the horizontal change from the first point to the second point. Subtract the x-coordinates of points A and B to determine the run.

$$\text{Horizontal change: } x_2 - x_1$$

Step 4: Write the slope as the ratio of the rise to the run, $\frac{\text{rise}}{\text{run}}$.

$$\text{From point A to B, the slope is } \frac{y_2 - y_1}{x_2 - x_1}.$$

$$\text{From point B to A, the slope is } \frac{y_1 - y_2}{x_1 - x_2}.$$

Remember ...

The symbol $\perp$ means *is perpendicular to*.

To calculate slope accurately, it is important to pay close attention to the subscripts of the x- and y-coordinates, and subtract them in the same order.

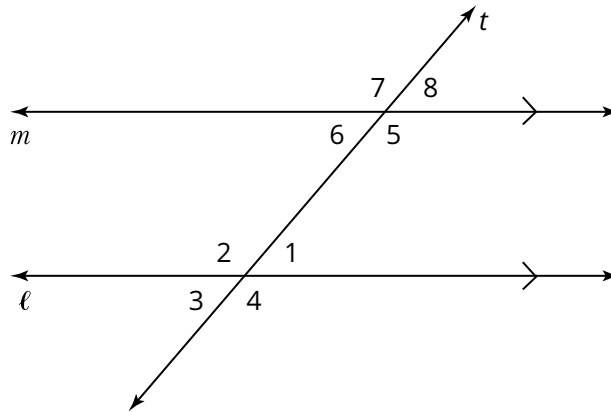
Constructing Parallel Lines

In the previous lesson, you constructed perpendicular lines. You can also construct parallel lines.

One strategy used to construct parallel lines is translation. The image of a line that has been translated is either the same line or a parallel line.

If line m is a translation of line ℓ , then the two lines are parallel. Recall that when parallel lines are cut by a transversal, several pairs of congruent angles are formed.

When you draw arrows on multiple lines in a diagram, it means that those lines are parallel. The arrows let you know that the lines are always the same distance apart and are in the same plane.



Corresponding angles:

$$\angle 8 \cong \angle 1$$

Alternate interior angles:

$$\angle 1 \cong \angle 6$$

Alternate exterior angles:

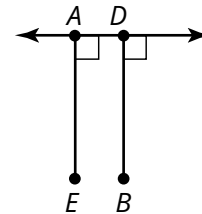
$$\angle 4 \cong \angle 7$$

Remember ...

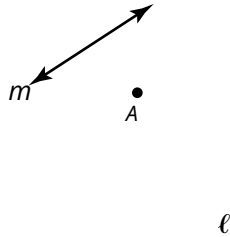
the symbol $\perp$ means *perpendicular to*.

Consider $\overline{AE} \perp \overleftrightarrow{AD}$ and $\overline{DB} \perp \overleftrightarrow{AD}$.

1. Is there enough information to conclude that the two segments are parallel? Explain your reasoning.



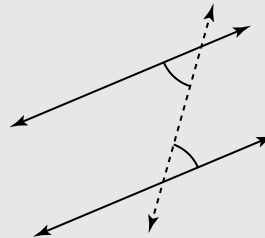
2. Use your reasoning from Question 1 and patty paper to construct a line parallel to line m through point A . Describe your process.



3. Explain why Samuel is correct.

Samuel

You can duplicate any angle measure, not just right angles, to construct parallel lines.



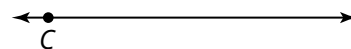
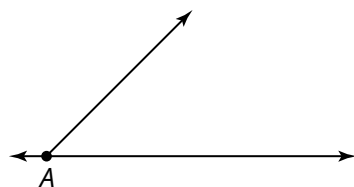
To use Samuel's method, you need to know how to use construction tools to duplicate an angle.

WORKED EXAMPLE

Construct a Starter Line

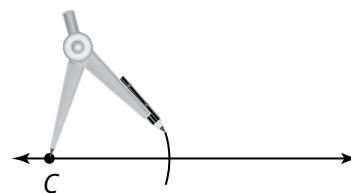
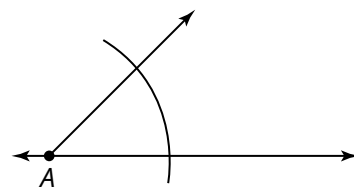
Use a straightedge to construct a starter line. Label point C on the new line.

Given:



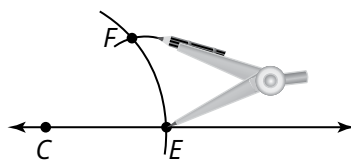
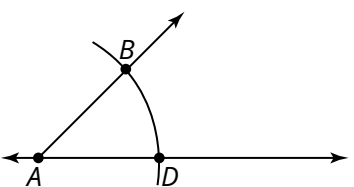
Construct an Arc

Construct an arc with center A . Using the same radius, construct an arc with center C .



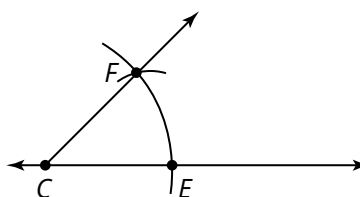
Construct Another Arc

Label points B , D , and E . Construct an arc with a radius the length of BD and center E . Label the intersection F .



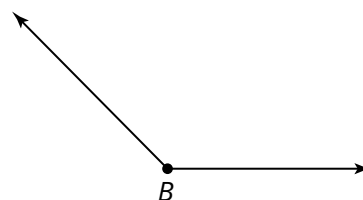
Construct a Ray

Construct ray CF .
 $\angle BAD > \angle FCE$



Mathematically when you duplicate an angle you create a congruent angle that has an equal measure to the original angle.

4. Duplicate angle B . Verify with patty paper.



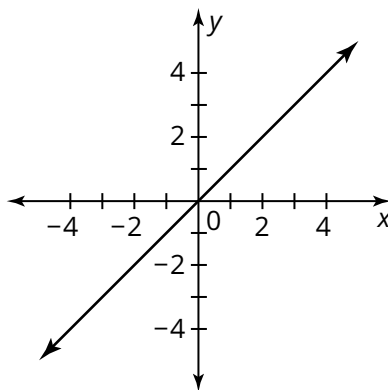
5. Now that you know how to duplicate an angle, use Samuel's method to construct a line parallel to line q .



6. Can you use Samuel's method with other angle pair relationships, like alternate exterior and corresponding angles? If so, explain your reasoning or justify it with constructions.

7. Consider the line $y = x$ on the coordinate plane shown.

- a. Translate the line $y = x$ to create a parallel line.



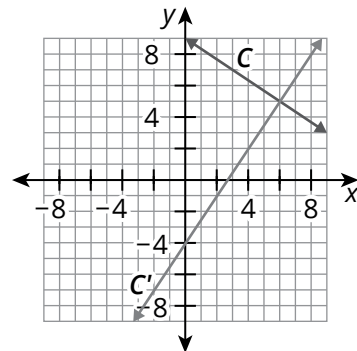
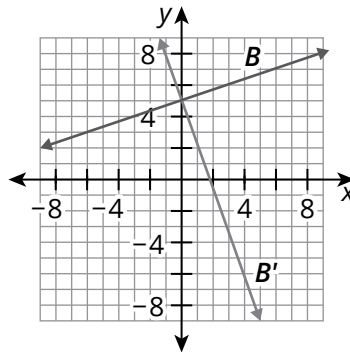
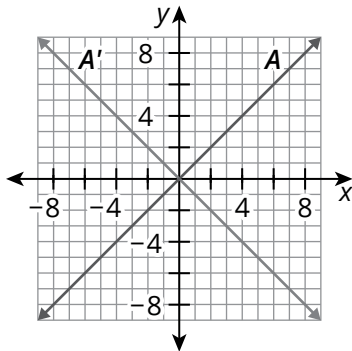
- b. Write the equation of your line.

- c. Describe the slopes of parallel lines on the coordinate plane.

Slopes of Perpendicular Lines

Recall that perpendicular lines or line segments form a right angle at the point of intersection.

Consider the three graphs shown. Each shows a line and its rotation 90° about a point, which is also the point of intersection.



Remember ...

The reciprocal of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. Because the product of a number and its reciprocal is one, reciprocal numbers are also known as multiplicative inverses.

1. Are the lines in each graph perpendicular? Explain your reasoning.

2. Write the equation for each line and its transformation. Make a conjecture about the slopes of perpendicular lines.

We made a conjecture that if two lines are perpendicular, the slopes are opposite reciprocals. Let's prove this conjecture using deductive reasoning.

WORKED EXAMPLE

If two lines are perpendicular, then their slopes are negative reciprocals.

The graph shown can be used to analyze the validity of this statement.

Assumption: $p \perp q$

Let m_1 = slope of line p and
let m_2 = slope of line q .

Point R lies on line p .

Conclusion: $m_1 = -\frac{1}{m_2}$

Perform a 90° counterclockwise rotation of point R using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q due to a 90° rotation.

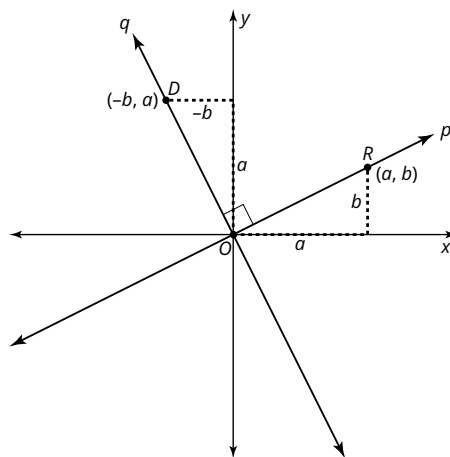
Since this rotation maps the positive x -axis to the positive y -axis, and the positive y -axis to the negative x -axis, then the coordinates of $R(a, b)$ are transformed into the coordinates of $D(-b, a)$. Graphically, you can follow the movement of lengths a and b under the rotation.

Using the graph, you can identify the slope of line p as $m_1 = \frac{b}{a}$, and the slope of line q as $m_2 = \frac{a}{-b}$.

Using these slopes, you can demonstrate that $m_1 = -\frac{1}{m_2}$.

$$\begin{aligned}\frac{b}{a} &= -\frac{1}{\frac{a}{-b}} \\ &= -1 \cdot \frac{-b}{a} \\ &= \frac{b}{a}\end{aligned}$$

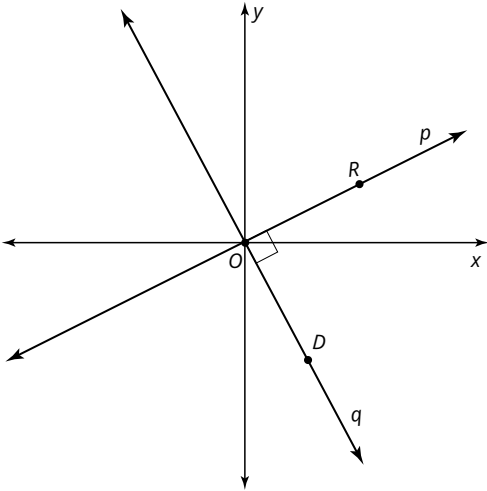
The slope of line q is the negative reciprocal of the slope of line p .



.....
The product of
the slopes of
perpendicular lines
is -1 .
.....

Now that you have proven the conjecture true, the statement is a theorem. There is often more than one way to prove a theorem. Suppose that point R is rotated 90° clockwise using point O as the center of rotation.

3. Rewrite the assumption and conclusion using the clockwise rotation of point R .



4. Line j and line k are perpendicular. Given each slope of line j , determine the slope of line k .

a. $m = \frac{2}{3}$

b. $m = -\frac{4}{5}$

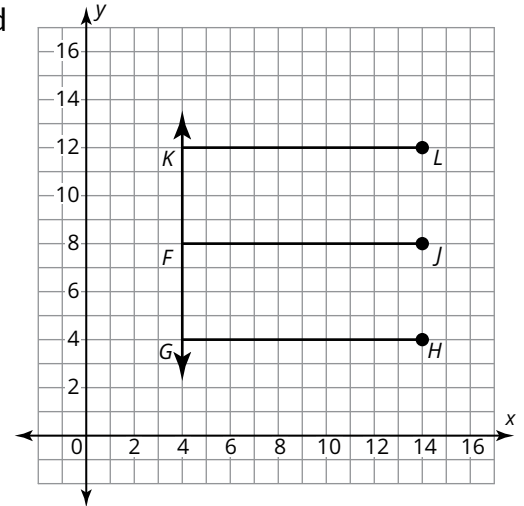
c. $m = -3$

ACTIVITY
2.4

Writing Equations of Parallel and Perpendicular Lines

Consider the graph shown.

1. Use a straightedge to extend $\overline{GK}$ to create line p , extend $\overline{GH}$ to create line q , extend $\overline{FJ}$ to create line r , and extend $\overline{KL}$ to create line s .



2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if x increases by one unit, by how many units does y change?
3. Describe the slope of any horizontal line. Explain your reasoning.

4. Consider the vertical line you drew in Question 1. Suppose that y increases by one unit. By how many units does x change?
5. Describe the slope of any vertical line. Explain your reasoning.
6. Write an equation for a line that is perpendicular to the line given by $x = 5$ and passes through the point $(1, 0)$.
7. Write an equation for a line that is perpendicular to the line given by $y = -2$ and passes through the point $(5, 6)$.
8. Write an equation for a line that is parallel to the line $y = 4$ and passes through the point $(-1, 2)$.
9. Write an equation for a line that is parallel to the line $x = -3$ and passes through the point $(7, 4)$.

You can write the equation of a parallel or perpendicular line using what you know about the slope of that line and any point on that line.

10. Write the equation of a line that is parallel to $y = -3x - 1$ and passes through the point $(-1, 5)$.

.....
Remember...

You can use point-slope form to write an equation for any line if you know its slope and one point on that line.

$$y - y_1 = m(x - x_1)$$

.....

11. Write the equation of the line perpendicular to $y = -\frac{3}{4}x$ that passes through the point $(3, -8)$.

12. Write the equation of a line that passes through the point $(-2, 7)$ and is perpendicular to a line that passes through the points $(-6, 1)$ and $(0, 4)$.

13. Write the equation of a line that passes through the point $(4, -6)$ and is parallel to a line that passes through the points $(-2, 3.5)$ and $(4, 5)$.

14. A pair of perpendicular lines intersect at the point $(5, 9)$. Write the equation of the line that is perpendicular to the line that also passes through the point $(-4, 4)$.



Talk the Talk

Parallels the Lesson

Previously, you analyzed a Worked Example that demonstrates, “If two lines are perpendicular, then their slopes are negative reciprocals of each other.”

1. Using deductive reasoning, write an explanation to justify the conjecture: “If two lines are parallel, then their slopes are equal.” Include a sketch.

Assumption: $p \parallel q$

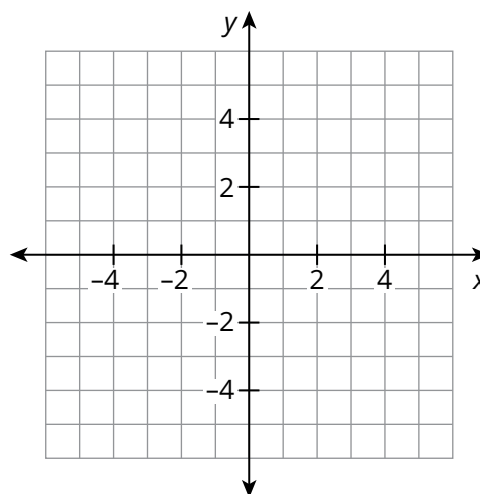
Let the slope of p be m_1 .

Let the slope of q be m_2 .

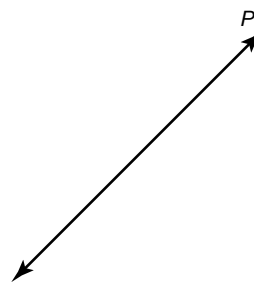
Conclusion: $m_1 = m_2$

Remember...

The symbol $\parallel$ means
parallel to.



2. Construct a line parallel to line p .



Lesson 2 Assignment

Write

Explain in your own words why the slope of a vertical line is undefined.

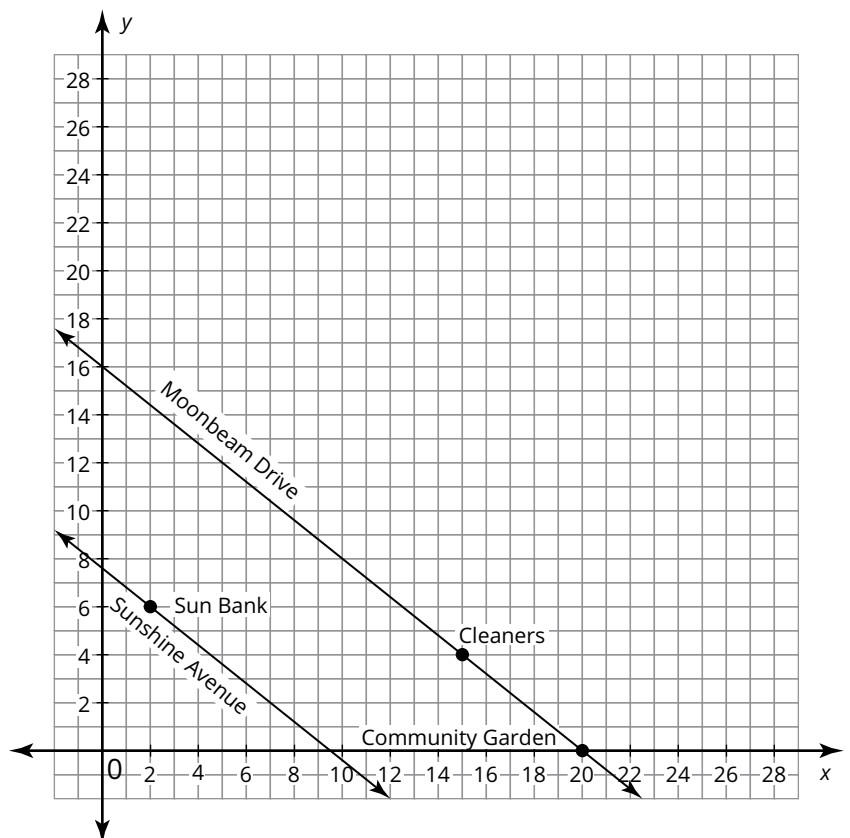
Remember

The slopes of perpendicular lines are negative reciprocals. Any vertical line is perpendicular to any horizontal line.

Practice

Christopher is a developer and plans to build a new development. Use the grid to help Christopher create a map for his development. Each gridline represents one block.

1. There are two main roads that pass through the development, Moonbeam Drive and Sunshine Avenue. Are these two roads parallel to each other? Explain your reasoning.



Lesson 2 Assignment

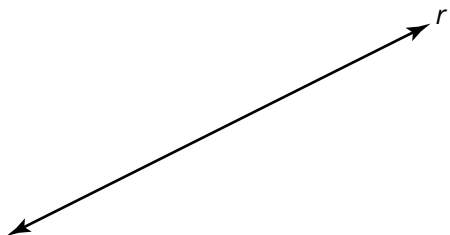
2. Christopher wants to build a road named Stargazer Boulevard that will be parallel to Moonbeam Drive. On this road, he will build a diner located 7 blocks north of the community garden. Determine the equation of the line that represents Stargazer Boulevard. Show your work. Then draw and label Stargazer Boulevard on the coordinate plane.
3. Christopher wants to build a road named Rocket Drive that connects Sun Bank to Moonbeam Drive. He wants this road to be as short as possible. Determine the equation of the line that represents Rocket Drive. Show your work. Then draw and label Rocket Drive on the coordinate plane.

Lesson 2 Assignment

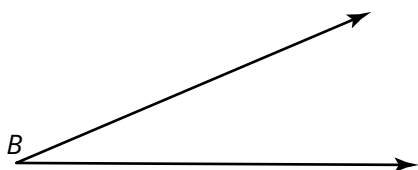
- Two office buildings are to be located at the points $(8, 4)$ and $(12, 10)$. Would the shortest road between the two office buildings be a line that is perpendicular to Moonbeam Drive? Explain your reasoning.
- A straight road named Planet Drive is planned that will connect the diner and the community garden. What is the equation of the line that represents Planet Drive? Show your work. Draw and label Planet Drive on the coordinate plane.
- Christopher decides that another road to be named Saturn Avenue is needed that will go past the cleaners and be perpendicular to Planet Drive. Determine the equation of the line that represents Saturn Avenue. Show your work. Draw and label Saturn Avenue on the coordinate plane.

Lesson 2 Assignment

7. Construct a line parallel to line r .



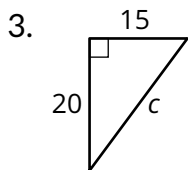
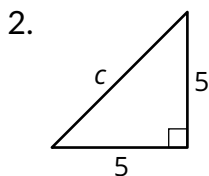
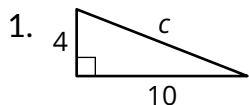
8. Duplicate $\angle B$.



Lesson 2 Assignment

Prepare

Determine the length of each hypotenuse. Round your answer to the nearest tenth, if necessary.



3

Classifying Quadrilaterals on the Coordinate Plane

OBJECTIVES

- Use the Pythagorean Theorem to derive the distance formula.
- Apply the distance formula on the coordinate plane.
- Classify a quadrilateral given the locations of its vertices on a coordinate plane through oral conversations and discussions.
- Apply the midpoint formula on the coordinate plane.

KEY TERMS

- distance formula
- extracting perfect squares
- radical
- radicand
- midpoint formula

.....

You know the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals. You also know how to determine the length of the hypotenuse of a right triangle. How can you use what you know to classify polygons that lie on a coordinate plane?

Getting Started

You Better Shape Up

Remember ...

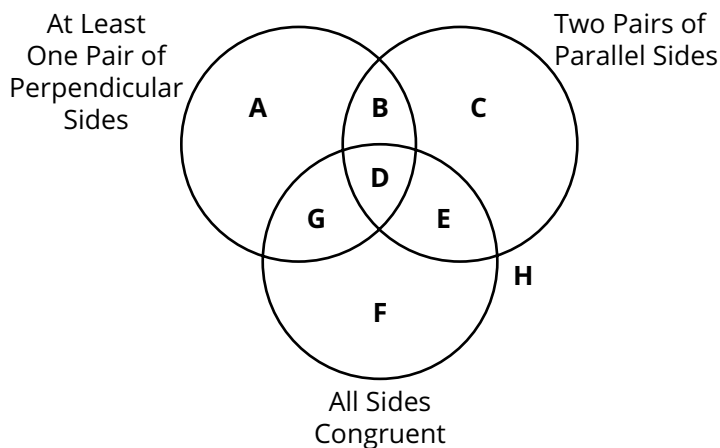
Congruent means to have the same size, shape, and measure.

Ask Yourself ...

Why is *H* not in a circle?

Polygons are often classified by properties, such as the lengths of their sides, the relationships between their sides, and the measures of their angles.

The Venn diagram contains three circles each representing a different property. Letters A through H represent any polygon that has the property described by every circle in which it appears.



A letter may be used more than once or not at all!

1. Match each polygon to one of the lettered regions of the Venn diagram. Write the corresponding letter next to each polygon.

trapezoid

parallelogram

rectangle

square

rhombus

right triangle

isosceles triangle

equilateral triangle

scalene triangle

2. Is there a region that cannot be matched to one of the polygons?
Explain your reasoning.

3. Use the Venn diagram to compare the properties of each pair of polygons.

a. parallelogram and rhombus

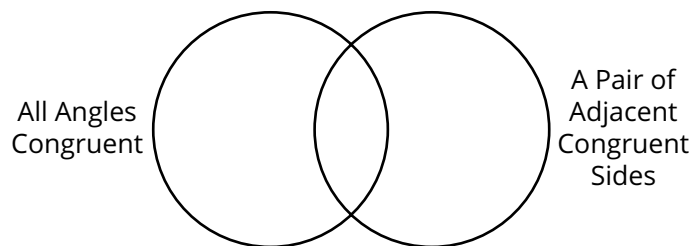
b. rectangle and square

.....
Remember ...

A trapezoid has exactly one pair of parallel sides. An isosceles triangle has two congruent sides. An equilateral triangle has three congruent sides.
.....



4. Isabella and Ethan analyze the Venn diagram shown.



Isabella says that the overlapping region describes a rhombus. Ethan says the overlapping region describes a square. Who's correct? Explain your reasoning.

.....
Remember ...

Adjacent sides are sides that share a vertex.
.....

5. Determine whether each statement is always true, sometimes true, or never true. Explain your reasoning.

a. A rectangle is a parallelogram.

b. A rhombus is a square.

c. A scalene triangle is a right triangle.

d. A parallelogram is a trapezoid.

e. A right triangle is an equilateral triangle.

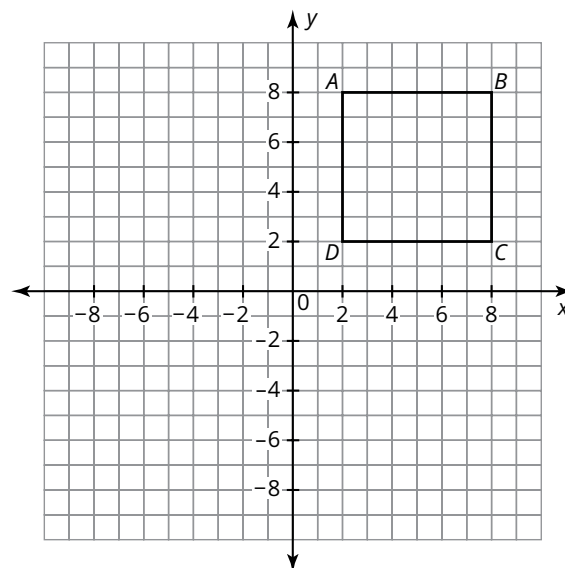
ACTIVITY
3.1

Calculating Distance on the Coordinate Plane

Let's analyze quadrilaterals that lie on a coordinate plane and classify them by their properties.

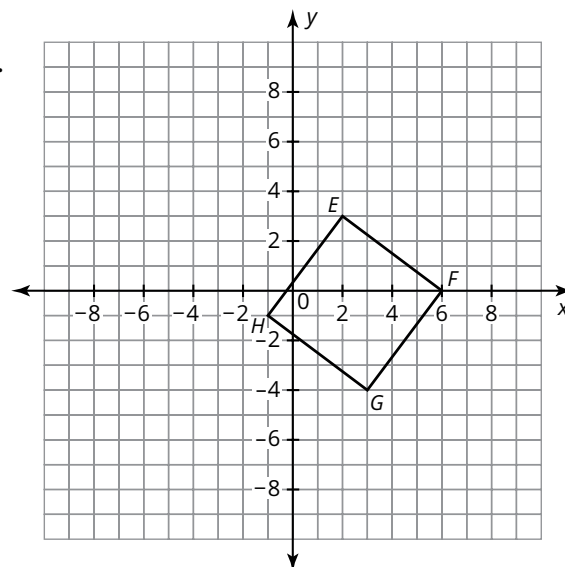
Consider quadrilateral $ABCD$ shown.

1. Classify the quadrilateral. Justify your reasoning.



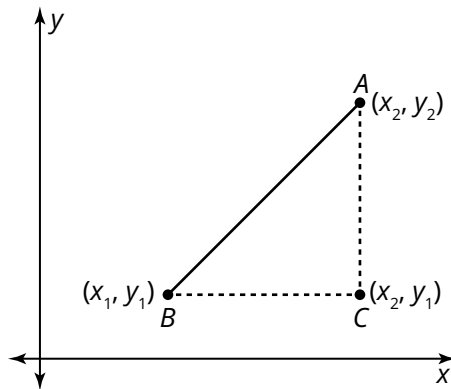
Now consider quadrilateral $EFGH$ shown.

2. Determine whether quadrilateral $EFGH$ can be classified as a parallelogram. Justify your reasoning.
3. Determine whether quadrilateral $EFGH$ can be classified as a rectangle. Justify your reasoning.



4. What information do you need to classify quadrilateral $EFGH$ as a square?
5. On quadrilateral $EFGH$, draw a right triangle EFR such that $\overline{EF}$ is the hypotenuse. Use the coordinate of point E and point R to write and simplify an absolute value expression that represents the distance between the two points.
6. Use the coordinates of point F and point R to write and simplify an absolute value expression that represents the distance between the two points.
7. Use the Pythagorean Theorem to determine the length of $\overline{EF}$.

You used the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. Your method can be written as the *distance formula*. You can derive the *distance formula* using any two points on a coordinate plane. Consider points A and B on the coordinate plane.



.....
Use absolute value expressions since the difference represents a distance.
.....

8. Use the points B and C to write an absolute value expression that represents the horizontal distance between the points A and B.

9. Use the points A and C to write an absolute value expression that represents the vertical distance between the points A and B.

10. Use the Pythagorean Theorem to determine the distance AB.

.....
Think About ...

Do you need the absolute value signs when using the Pythagorean Theorem?
.....

You just derived the **distance formula**, which states that if (x_1, y_1) and (x_2, y_2) are points on the coordinate plane, then distance between (x_1, y_1) and (x_2, y_2) is calculated using the formula given:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ask Yourself . . .

Can distance be negative?

11. When you use the distance formula, does it matter which point you identify as (x_1, y_1) and which point you identify as (x_2, y_2) ? Explain your reasoning.

12. Can quadrilateral $EFGH$ be classified as a square? Justify your reasoning.

.....
Remember . . .

Prime factorization is when you express a number as a product of only prime factors.

13. Use the distance formula to calculate the distance between each pair of points. Round your answer to the nearest tenth, if necessary. Show all your work.

a. $(1, 2)$ and $(3, 7)$

b. $(-6, 4)$ and $(2, -8)$

c. $(-5, 2)$ and $(-6, 10)$



14. Calculate the distance between the points $(-1, -2)$ and $(-3, -7)$. Notice the similarity between these points and and Question 13, part (a).

Samuel says that the solution must be the opposite of the solution of part (a). Harper disagrees and says that the solution will be the same as the solution of part (a). Who is correct? Explain your reasoning and state the correct solution.

WORKED EXAMPLE

You can use prime factorization to rewrite radicals in simplest radical form.

Rewrite $\sqrt{60}$ in simplest radical form.

Write the prime factorization of 60.

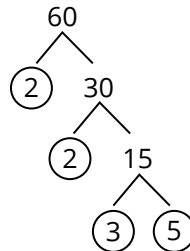
$$60 = 2^2 \cdot 3 \cdot 5$$

Extract the perfect squares

$$\sqrt{60} = 2 \cdot \sqrt{3 \cdot 5}$$

Rewrite the radicand.

$$\sqrt{60} = 2 \cdot \sqrt{15}$$



.....

The symbol $\sqrt{}$ is called a **radical**.

The **radicand** is the quantity under a radical.

radical radicand

$\sqrt{25}$

The process of removing perfect square numbers from under a radical symbol is **extracting perfect squares**.

.....

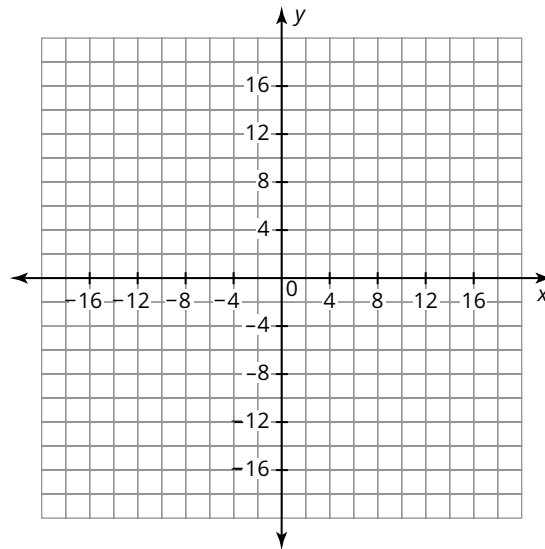
15. Rewrite the distance from Question 13, part (b) in simplest radical form.

ACTIVITY
3.2

Using the Distance Formula to Verify the Congruence of Segments

You can use the distance formula to verify that two segments are congruent.

1. Plot the point $(6, 10)$ on the coordinate plane and label the point P .
2. Plot the point $(2, 3)$ on the coordinate plane and label the point J .
3. Draw a line segment connecting the two points to create line segment PJ .



.....
Length is the same
as distance on the
coordinate plane!
.....

4. Determine the length of line segment PJ .

5. Line segment PJ is horizontally translated 10 units to the left.

a. Graph the image of pre-image $\overline{PJ}$. Label the new points P' and J' .

b. Identify the coordinates of P' and J' .

c. Use the distance formula to calculate the length of segment $P'J'$.

d. Compare the length of segment $P'J'$ with the length of segment PJ and describe what effect the translation had on the length of the segment.

6. Line segment $P'J'$ is vertically translated 14 units down.

a. Graph the image of pre-image $\overline{P'J'}$. Label the new points P'' and J'' .

b. Identify the coordinates of P'' and J'' .

.....
The prime symbol, like on P' or P'' , indicates that this point is related to the original point P . P' is read as "P prime" and P'' is read as "P double prime."
.....

.....
Remember ...

When you translate a figure, the original figure is called the pre-image. The new figure created from the translation is called the image.
.....

c. Use the distance formula to calculate the length of segment $P''J''$.

d. Compare the length of segment $P''J''$ with the length of segment $P'J'$ and describe what effect the translation had on the length of the segment.

7. Line segment $P''J''$ is horizontally translated 10 units to the right.

a. Without graphing, predict the coordinates of P''' and J''' .

b. Graph the image of pre-image $\overline{P''J''}$. Label the new points P''' and J''' .

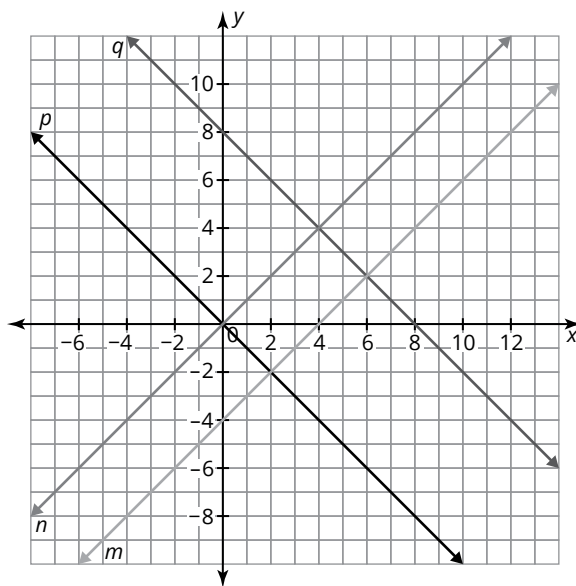
c. Use the distance formula to calculate the length of segment $P'''J'''$.

d. Compare the length of segment $P'''J'''$ with the length of segment $P''J''$ and describe what effect the translation had on the length of the segment.

ACTIVITY
3.3

Using the Distance Formula to Verify Parallelism

The distance between two parallel lines is the length of the perpendicular segment between the two lines.



1. Consider the graph.
 - a. Write the equation of line m .
 - b. Write the equation of line n .
 - c. Write the equation of line p .
 - d. Write the equation of line q .

2. Describe the relationship between each set of lines. Justify your reasoning.
- a. line m and line n
 - b. line m and line p
 - c. line m and line q
 - d. line n and line p
 - e. line n and line q
 - f. line p and line q
3. Determine the coordinates of the point of intersection of line n and p . Label the point of intersection as point A .
4. Determine the coordinates of the point of intersection of line m and p . Label the point of intersection as point B .
5. Use the distance formula to calculate the length AB . Write your answer in simplest radical form.

6. Determine the coordinates of the point of intersection of line n and q . Label the point of intersections as set C .
7. Determine the coordinates of the point of intersection of line m and q . Label the point of intersection as point D .
8. Use the distance formula to calculate the length between the two points. Write your answer in simplest radical form.

AB is a perpendicular line segment between lines m and n .

CD is also a perpendicular line segment between lines m and n .

The length of every perpendicular line segment between two parallel lines is the same because the lines never intersect.

9. Verify that the opposite sides of rectangle $ABCD$ are parallel using the distance formula.

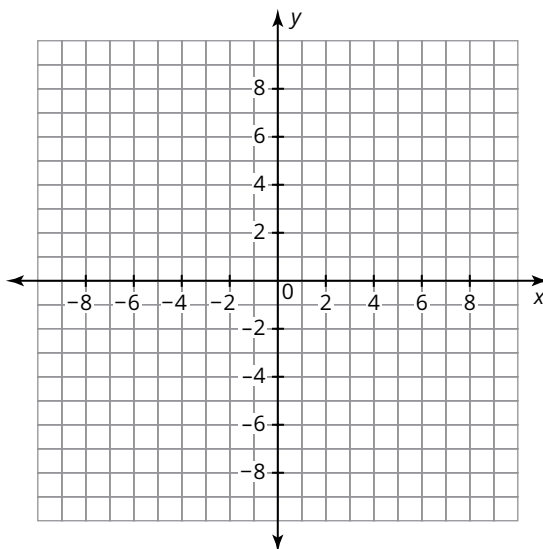


ACTIVITY
3.4

Classifying a Quadrilateral on the Coordinate Plane

In this activity, you will classify quadrilaterals by examining the lengths and relationships of their sides.

1. Graph quadrilateral $ABCD$ using points $A(-5, 6)$, $B(-8, 2)$, $C(-5, -2)$, and $D(-2, 2)$.

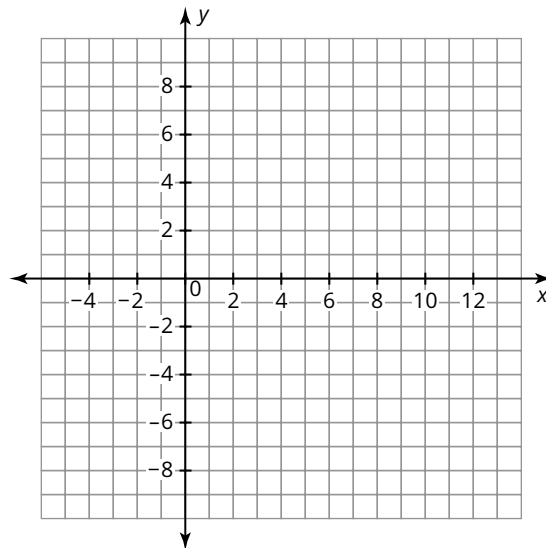


2. Consider the sides of quadrilateral $ABCD$.
 - a. Determine each side length of quadrilateral $ABCD$. Can you classify quadrilateral $ABCD$ from its side lengths? If so, identify the type of figure. If not, explain why not.

- b. Determine the slope of each line segment in the quadrilateral. Describe the relationship between the slopes. Can you identify the figure? If so, identify the type of figure. If not, explain why not.

Review the definition of **describe** in the Academic Glossary.

3. Graph quadrilateral $ABCD$ using points $A(8, 8)$, $B(3, -7)$, $C(10, -6)$, and $D(13, 3)$. Classify this quadrilateral as a trapezoid, a rhombus, a rectangle, a square, or none of these. Explain your reasoning.



Think About...

What is the difference between a square and a rhombus?

Think About...

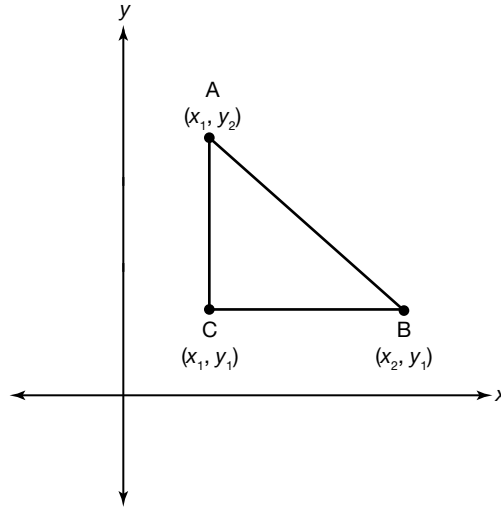
Which types of figures can you eliminate as you determine information about the figure?

ACTIVITY
3.5

Classifying a Quadrilateral Formed by Midpoints

You have used the distance formula to determine the distance between two points. You can also determine the midpoint between two points on a coordinate plane.

Consider points A, B, and C on the coordinate plane.



.....

Remember ...

A midpoint is the point that is exactly halfway between two given points.

.....

1. Use the diagram to describe each distance algebraically.
 - a. the vertical distance from the x-axis to point C
 - b. the distance from point C to point A
 - c. half the distance from point C to point A
 - d. the vertical distance from the x-axis to point C plus have the distance to point A

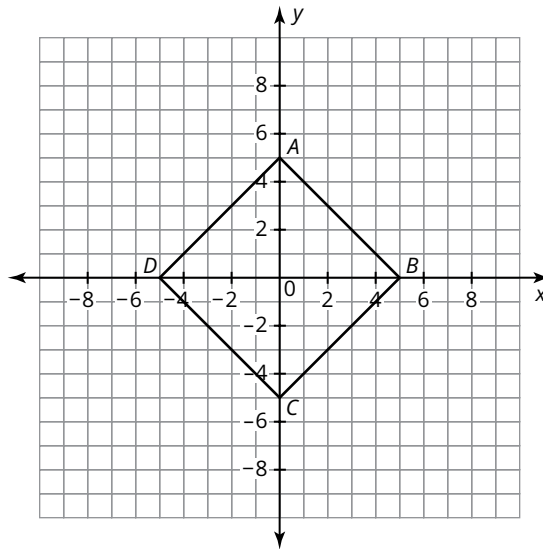
2. Simplify your expression from Question 1, part(d).
3. Use the diagram to describe each distance algebraically.
 - a. the horizontal distance from the y -axis to point C .
 - b. the distance from point C to point B .
 - c. half the distance from point C to point B
 - d. the horizontal distance from the y -axis to point C plus half the distance from point C to point B
4. Simplify your expression from Question 4, part (d).

The coordinates of the points that you determined in Questions 2 and 4 are midpoints. You derived the *midpoint formula*.

The **midpoint formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

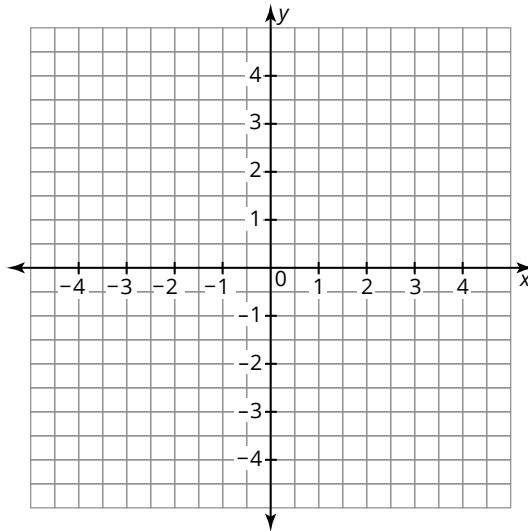
Use the midpoint formula to determine the midpoints of each side of the given figures.

5. Given square $ABCD$.



- Determine and label the midpoint of each side of the square.
- Determine the polygon formed by connecting the consecutive midpoints of each side of a square and justify your conclusion.
- If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.

6. Sketch any rhombus that is not a square. Label the midpoint of each side of the rhombus.
- a. Determine the polygon formed by connecting the consecutive midpoints of each side of a rhombus and justify your conclusion.



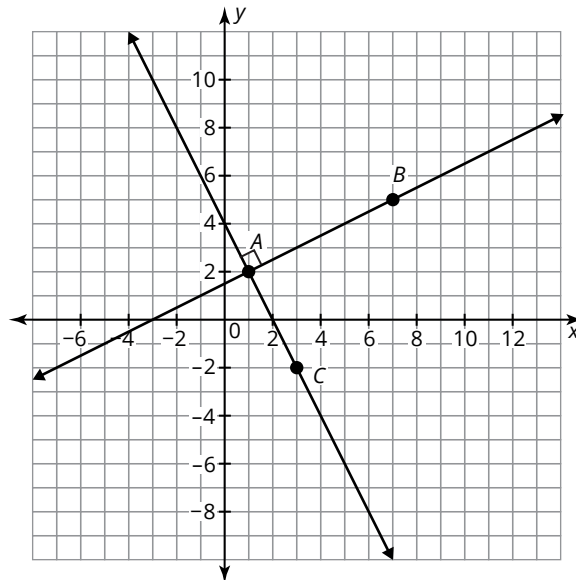
- b. If the same process was repeated one more time by connecting the consecutive midpoints of each side of the polygon determined in part (a), describe the polygon that would result.



Talk the Talk

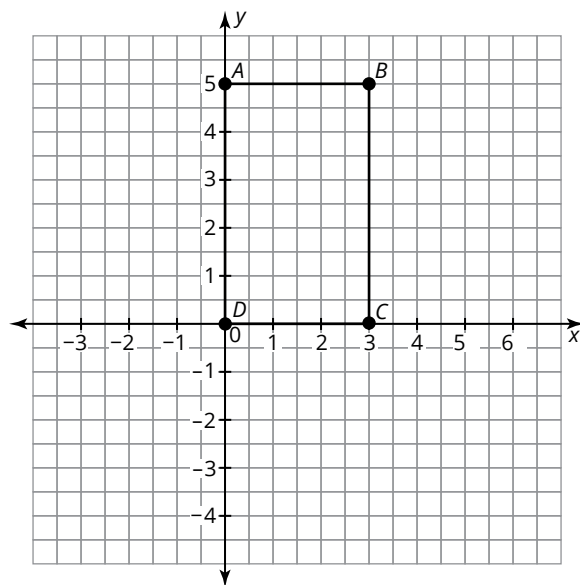
Look, Ma! No Gridlines!

1. Line AC is perpendicular to line AB at A .
 - a. Determine the coordinates of point D such that line CD is parallel to line AB and line BD is perpendicular to line AB at B .



- b. Determine the perpendicular distance between lines AB and CD . Explain your reasoning.

2. Points A, B, C, and D form a rectangle. If point B is to be moved to point (5,3), in which coordinate should point C be moved so that the quadrilateral formed is a parallelogram? Explain your reasoning.



Lesson 3 Assignment

Write

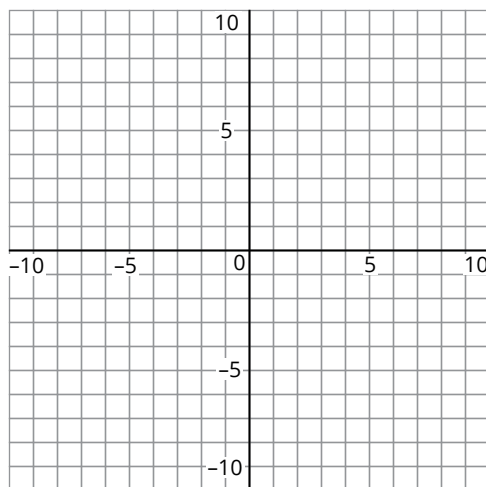
Describe how the distance formula, the slope formula, and the midpoint formula can be used to classify quadrilaterals on the coordinate plane.

Remember

The distance formula states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between the points is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Practice

1. An engineer is planning to place four lamp posts along two streets, ensuring the streets to be parallel on a coordinate grid. The four posts A , B , C , and D will be positioned at the following coordinates: $A(-2, 6)$, $B(4, 9)$, $C(0, 2)$, and $D(6, 5)$.
 - a. Plot and label the points on the coordinate plane.



- b. If posts A and B are placed on one street and posts C and D on another street, determine whether the streets are parallel.

Lesson 3 Assignment

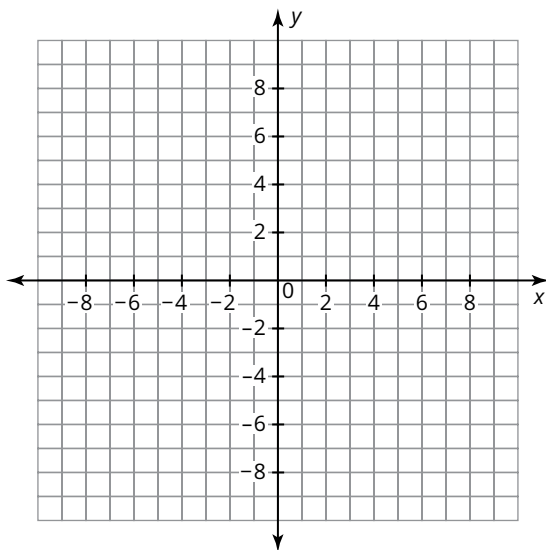
- c. Determine the perpendicular distance between the two streets.
Explain your reasoning.

Lesson 3 Assignment

- d. The engineer rechecked his plan to ensure the lamp posts are the same distance apart on each street. How can he verify this? Explain your reasoning.

Lesson 3 Assignment

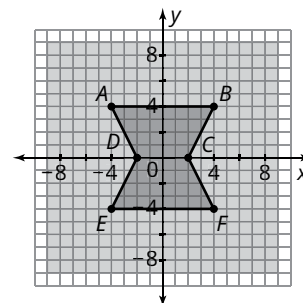
2. Sofia is an interior floor designer. When designing a new floor, she uses a coordinate grid to represent the room. The client wants a rectangular tile insert to be placed in the floor of the room. The coordinates for 3 of the corners of the insert are $A(-7, -4)$, $B(1, 6)$, and $C(6, 2)$.
- a. Plot and label the points on a coordinate plane, then determine the coordinates of the fourth point of the rectangular tile insert. Plot this as point D and connect the points to form the rectangle.



- b. To prove the figure you drew is a rectangle, verify that the length of opposite sides are equal.

Lesson 3 Assignment

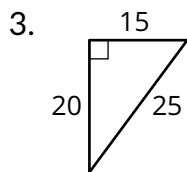
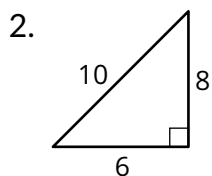
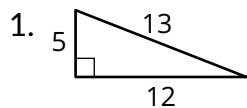
3. A client of Sofia's has asked her to create a new wood floor for his living room. She creates the design by laying wood strips in different directions, as shown on the coordinate grid. Determine whether Quadrilateral $ABCD$ is a trapezoid, a rhombus, a rectangle, or a square. Explain your reasoning.



Lesson 3 Assignment

Prepare

Determine the perimeter and area of each triangle.



4

Classifying Triangles on the Coordinate Plane

OBJECTIVES

- Apply the distance formula on the coordinate plane orally and in writing.
- Classify a triangle given the locations of its vertices on a coordinate plane.

KEY TERMS

- biconditional statement
- converse
- Pythagorean triple

.....

You know the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals. You also know how to determine the length of the hypotenuse of a right triangle. How can you use what you know to classify triangles that lie on a coordinate plane?

The Converse of a Conditional Statement

The **converse** of a conditional statement written in the form “If p , then q ” is the statement written in the form “If q , then p .” The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are interchanged.

WORKED EXAMPLE

Consider the conditional statement:

If a number is divisible by 2, then it is divisible by 4.

Hypothesis (p): a number is divisible by 2

Conclusion (q): it is divisible by 4

The converse of this conditional statement is: If a number is divisible by 4, then it is divisible by 2.

1. Use the conditional statement to answer each question.
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.
 - a. Identify the hypothesis and conclusion of the conditional statement.
 - b. Write the converse of the conditional statement.

c. Is the converse of the conditional statement true?

2. Use the conditional statement to answer each question.

If a quadrilateral has four equal sides, then it is a square.

a. Identify the hypothesis and conclusion of the conditional statement.

b. Write the converse of the conditional statement.

c. Is the converse of the conditional statement true?

Ask Yourself . . .

Can the conditional statement be shown false with a counterexample?

The Pythagorean Theorem states “If a triangle is a right triangle with a hypotenuse length c and leg lengths a and b , then $a^2 + b^2 = c^2$ ”.

1. Write the converse of the Pythagorean Theorem.

When a conditional statement and its converse are both true, this is called a *biconditional statement*. A **biconditional statement** is a statement written in the form “if and only if p , then q ”. It is a combination of both a conditional statement and the converse of that conditional statement.

2. Write the Pythagorean Theorem and its converse as a biconditional statement.

.....

Think about ...

which measures
represent legs of
the right triangle
and which measure
represents the
hypotenuse.

.....

3. Determine whether the triangle with the given side lengths is a right triangle.

a. 9, 12, 15

b. 8, 8, 11

ACTIVITY 4.2

Pythagorean Triples

You may have noticed that each of the right triangles in the Getting Started had side lengths that were integers. Any set of three positive integers a , b , and c that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple**. For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

- Complete the table to identify more Pythagorean triples.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple	3	4	5	$9 + 16 = 25$
Multiply by 2				
Multiply by 3				
Multiply by 5				

Ask Yourself . . .

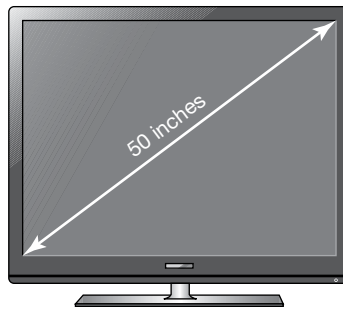
Would the side lengths 3, 4, and 5 form a right triangle if each was multiplied by 2.2?

- Determine a new Pythagorean triple not used in Question 2, and complete the table.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean triple				
Multiply by 2				
Multiply by 3				
Multiply by 5				

- Record other Pythagorean triples that your classmates determined.

4. The size of a television is identified by the diagonal measurement of its screen. A television has a 50-inch screen and a height of 30 inches. What is the length of the base of the television screen?

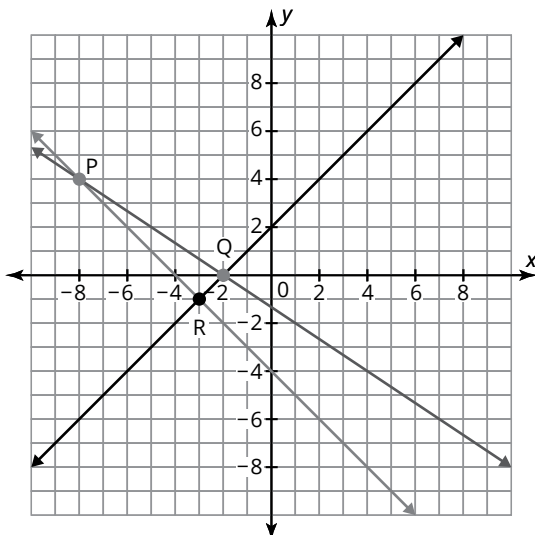


5. Some artists include the measure of the diagonal when listing the dimensions of a rectangular painting. What is the length of the base of a painting with a diagonal that measures 100cm and a height of 60 cm?

ACTIVITY
4.3

Using the Distance Formula to Verify Perpendicularity

You can use the distance formula to determine whether two lines are perpendicular.



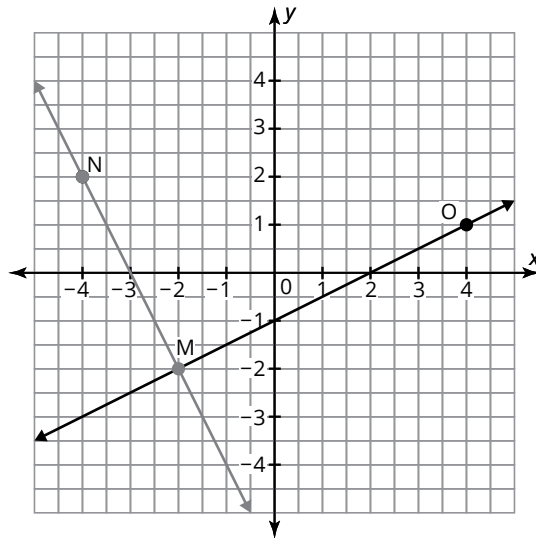
1. Use the distance formula to verify that $\overline{PR} \perp \overline{QR}$.
 - a. Use the distance formula to determine the distance between point R and Q. Write your answer in simplest radical form.

PROBLEM SOLVING



- b. Use the distance formula to determine the distance between point P and R . Write your answer in simplest radical form.
- c. Use a straight edge to draw $\overline{PQ}$, then use the distance formula to determine the distance between point P and Q . Write your answer in simplest radical form.
- d. Use the Converse of the Pythagorean Theorem to demonstrate that the triangle is a right triangle.

2. Use the distance formula to verify that $\overline{MN} \perp \overline{MO}$.

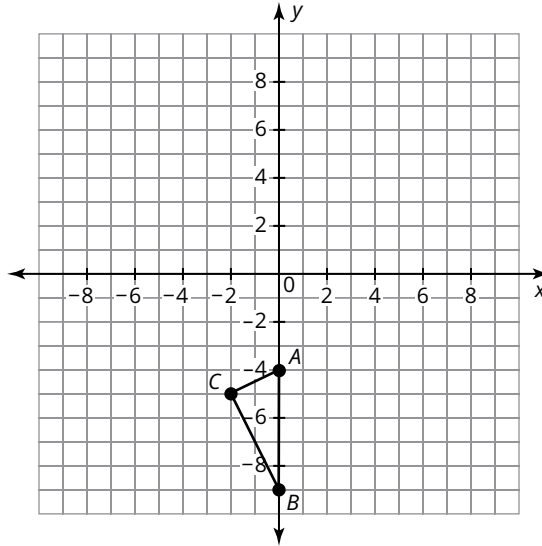


ACTIVITY
4.4

Classifying Triangles on the Coordinate Plane

Let's analyze triangles that lie on a coordinate plane and classify them by their properties.

Consider $\triangle ABC$.



Ask Yourself . . .

How can you determine the lengths of the sides of this triangle?

1. Classify $\triangle ABC$.
 - a. Consider the sides of $\triangle ABC$ to describe it as scalene, isosceles, or equilateral. Explain your reasoning.
 - b. Consider the slope of each side to determine whether $\triangle ABC$ is a right triangle. Justify your conclusion.

- c. Noah used the Pythagorean Theorem to determine whether $\triangle ABC$ was a right triangle.

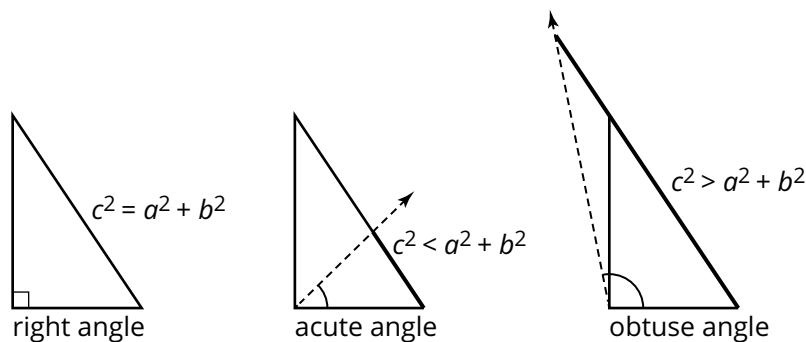
Noah

$a^2 + b^2 = c^2$
 $(\sqrt{5})^2 + (\sqrt{20})^2 = 5^2$
 $5 + 20 = 25$
 $25 = 25$



Describe why Noah's reasoning is correct.

You can also use the relationship among the sides of a triangle to determine whether the triangle is acute or obtuse. Given a , b , and c are the sides of a triangle with c as the longest side, when $c^2 < a^2 + b^2$, the triangle is acute, and when $c^2 > a^2 + b^2$, the triangle is obtuse.



2. Determine whether each set of side lengths creates an acute, right, or obtuse triangle. Identify whether the side lengths form a Pythagorean triple.

- a. 42 cm, 36 cm, 15 cm

b. 10 cm, 6 cm, 8 cm

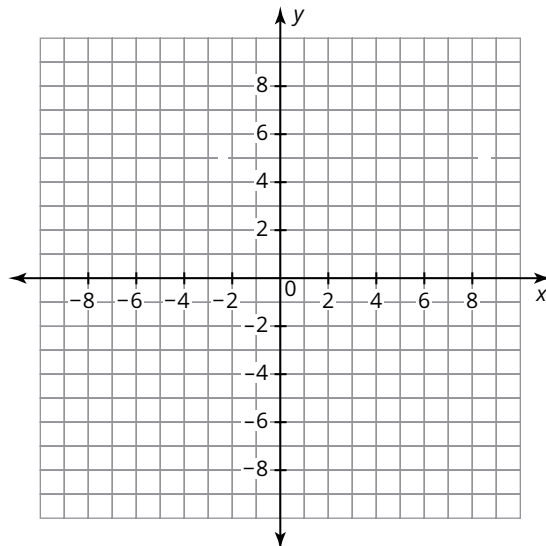
c. 18.5 m, 11 m, 15 m

d. 4 ft, $\sqrt{65}$ ft, 7 ft

.....
Remember ...

Are you using a
straight edge to draw
the triangle?

3. Graph $\triangle JKL$ using points $J(-2, 4)$, $K(8, 4)$, and $L(6, -2)$.



4. Classify $\triangle JKL$.

a. Consider the sides of $\triangle JKL$. Describe the triangle as scalene, isosceles, or equilateral. Explain your reasoning.

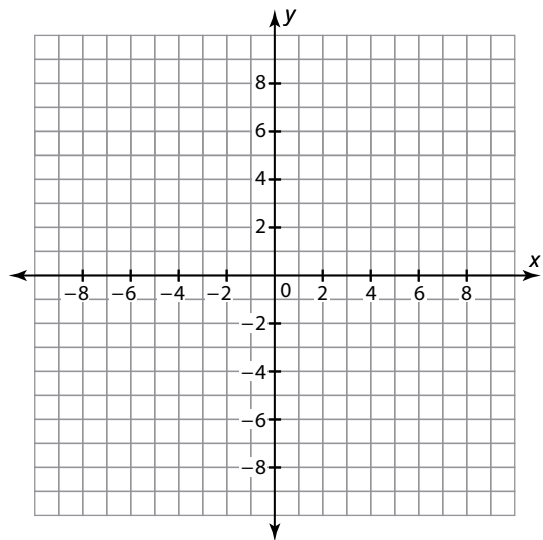
b. Consider the angles of $\triangle JKL$. Describe the triangle as acute, obtuse, or right. Explain your reasoning.



Talk the Talk

Tri- Tri- Again

1. A rectangular sign has a diagonal measuring 65 inches and a height of 39 inches. What is the length of the base of the sign?
2. The vertices of a triangle are $A(1, 3)$, $B(5, 3)$, and $C(1, 7)$. Classify the triangle by its sides and angles.



Lesson 4 Assignment

Write

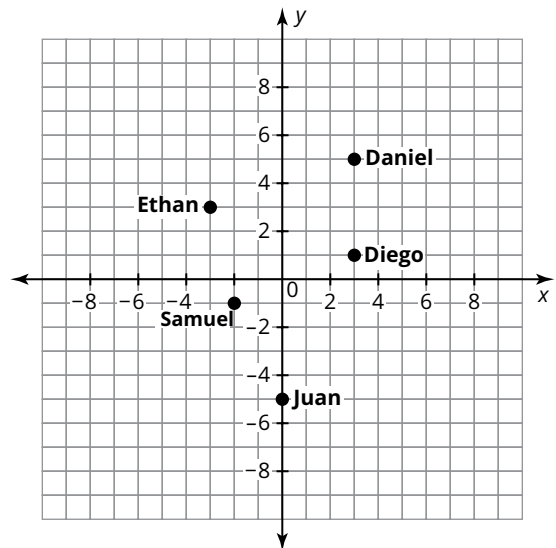
Describe how you can use the distance formula and the slope formula to classify triangles on the coordinate plane.

Remember

You can generate a new Pythagorean triple from a known triple using a scale factor.

Practice

1. The grid represents a map of Ethan's neighborhood. It shows the locations of his house as well as the houses of four friends.
 - a. Draw a triangle between the houses of Ethan, Samuel, and Diego. Use the distance formula to determine whether this triangle is a scalene, isosceles, or equilateral triangle.



Lesson 4 Assignment

- b. Determine whether the triangle is a right triangle. Explain your reasoning. If it is not a right triangle, determine whether it is acute or obtuse.
- c. Ethan, Daniel, and Diego are meeting for band rehearsal. Daniel claims that the distance from Ethan's house to his house is the same as the distance from Ethan's house to Diego's house. Is his claim correct? Explain your answer. What kind of triangle is formed if you connect their houses?
- d. A new boy, Liam, moved into the neighborhood at the location $(-3, -5)$. Plot and label Liam's house on the grid. Then, determine whether the triangle formed by connecting his house, Ethan's house, and Juan's house is a right triangle.

Lesson 4 Assignment

Prepare

Determine the distance between each set of points. Round your answer to the nearest tenth, if necessary.

1. $(2, -3)$ and $(-4, 1)$

2. $(-4.75, -8.5)$ and $(3.25, 5.5)$

3. $\left(\frac{5}{4}, \frac{9}{4}\right)$ and $(0, 10)$

5

Area and Perimeter on the Coordinate Plane

OBJECTIVES

- Determine the perimeter and area of rectangles and triangles on the coordinate plane.
- In writing, explain how to use transformations to discover efficient strategies for determining the perimeter and area of rectangles and triangles.
- Determine the perimeter and the area of composite figures on a coordinate plane.
- Use the distance formula to solve real-world problems involving perimeters of parallelograms, trapezoids, and hexagons.
- Decompose polygons—including trapezoids and hexagons—to solve real-world problems involving area.
- Determine the area of a regular polygon.

KEY TERMS

- composite figure
- regular polygon
- apothem

.....

You have used the distance formula and the slope formula to classify geometric figures on the coordinate plane.

How can you use these same formulas to determine the perimeter and area of polygons on the coordinate plane?

Getting Started

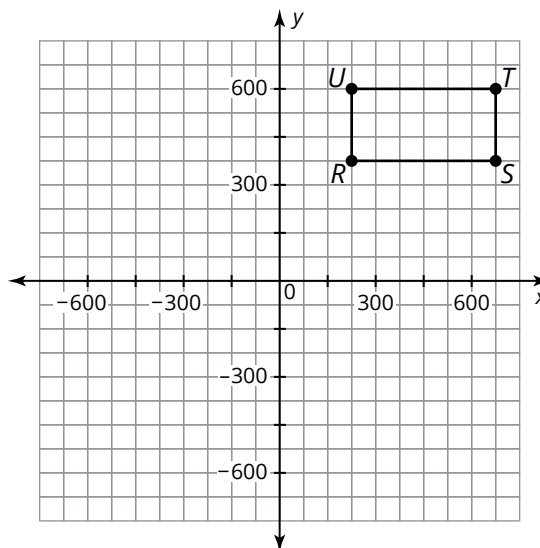
Perimeter and Area of Figures on the Coordinate Plane

Remember ...

The perimeter of a geometric figure is calculated by adding the side lengths.

When a rectangle is graphed along gridlines, you can determine the perimeter and area by simply counting units or square units on the coordinate plane. This is true if all coordinates are integers. If they are fractions or decimals, it presents a challenge.

Analyze rectangle $RSTU$ on the coordinate plane shown.



Remember ...

The formula for area of a rectangle is $A = bh$, where A represents the area, b represents the base, and h represents the height.

1. Calculate the perimeter and area of rectangle $RSTU$.
2. How would doubling the height of the rectangle affect the perimeter and the area?
3. How would doubling the length of the base of the rectangle affect the perimeter and the area?

Victoria



If I translate rectangle $RSTU$ to have at least one point of image $R'S'T'U'$ on the origin, it is easier to calculate the perimeter and area of rectangle $RSTU$ because one of the points will have coordinates $(0, 0)$.

Think About...

Notice the intervals along the axes.

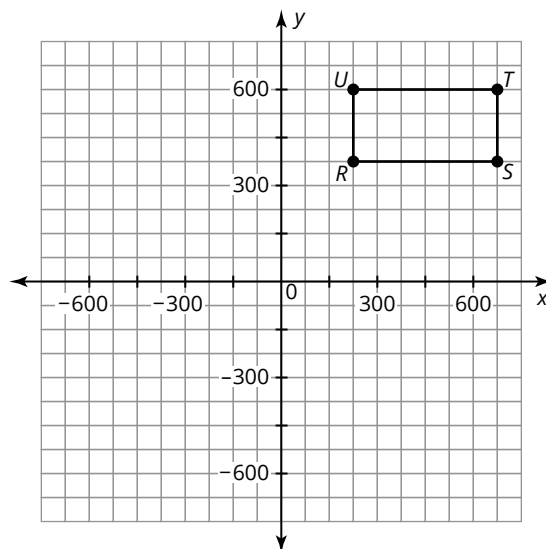
Victoria used another strategy to determine the perimeter and area of rectangle $RSTU$.

4. How do you know a translation of rectangle $RSTU$ will have the same area and perimeter as the pre-image $RSTU$? Explain your reasoning.

5. Explain why Victoria's rationale is correct.

6. Translate rectangle $RSTU$ so that point R is located at the origin.

a. List the coordinates of rectangle $R'S'T'U'$.



b. Determine the perimeter and area of $R'S'T'U'$. What do you notice?

ACTIVITY
5.1

Using the Distance Formula to Determine Perimeter and Area on the Coordinate Plane

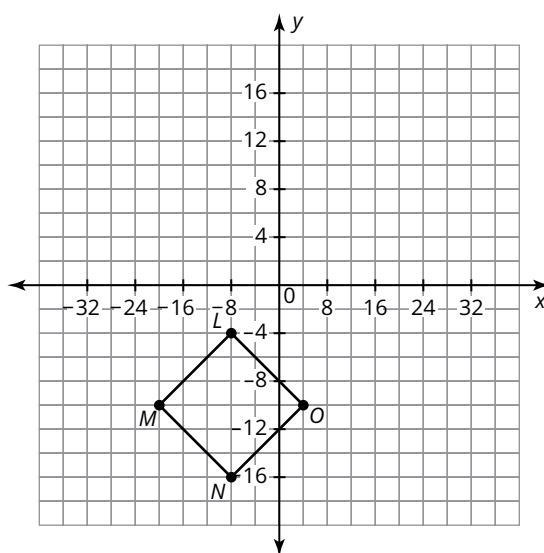
When the sides of a rectangle do not lie on the gridlines of the coordinate plane, you can use the distance formula to determine the lengths of the sides.

1. Consider quadrilateral $LMNO$.

.....
Think About...

Is the quadrilateral a square, a rectangle, or a rhombus?
.....

- a. Determine the perimeter and area of quadrilateral $LMNO$. Round your answer to the nearest hundredth, if necessary.

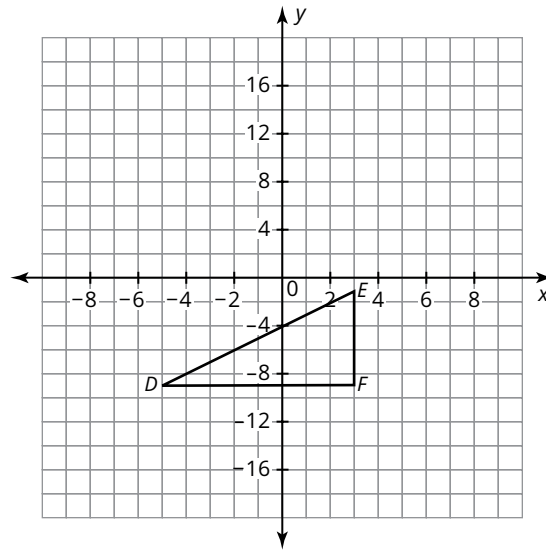


- b. Double the side lengths of quadrilateral $LMNO$ keeping the center of the polygon at $(-8, -10)$. How does this affect the perimeter and the area? What are the new coordinates?

- c. Describe how you could translate quadrilateral $LMNO$ to make the perimeter and area calculations more efficient.

2. Consider $\triangle DEF$ with vertices $D(-5, -9)$, $E(3, -1)$, and $F(3, -9)$.

- a. Determine the perimeter and area of $\triangle DEF$. Round your answer to the nearest hundredth, if necessary.



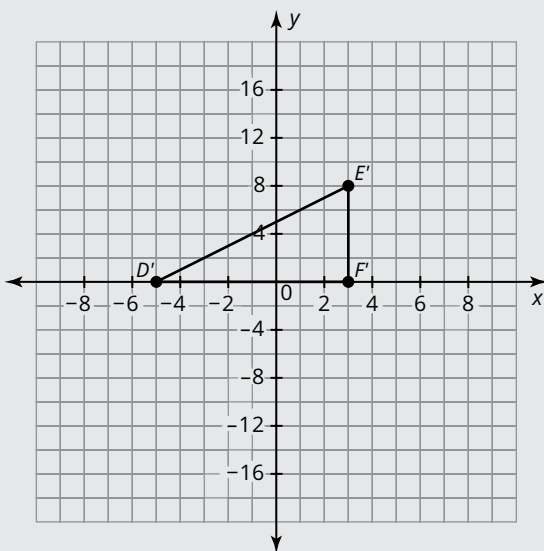
- b. Double the height. What are the coordinates of the new triangle? Determine the perimeter and area of the new triangle and describe the change.

c. Double the length of the base. What are the coordinates of the new triangle? Determine the perimeter and area of the new triangle and describe the change.

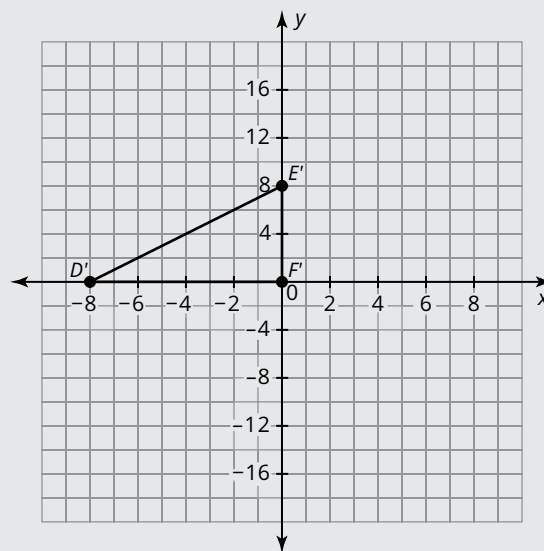
d. Double the length of both the base and the height. Determine the perimeter and area of the new triangle and describe the change.

3. Mr. Nguyen gives his class $\triangle DEF$ and asks them to determine the area and perimeter. Four of his students decide to first transform the figure and then determine the perimeter and area. Their transformations are shown.

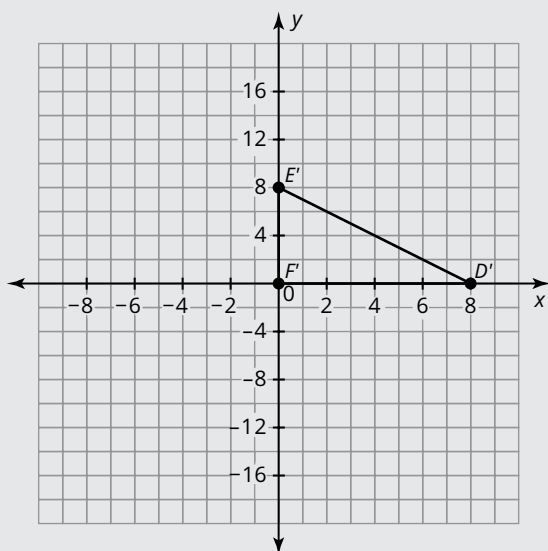
Liam



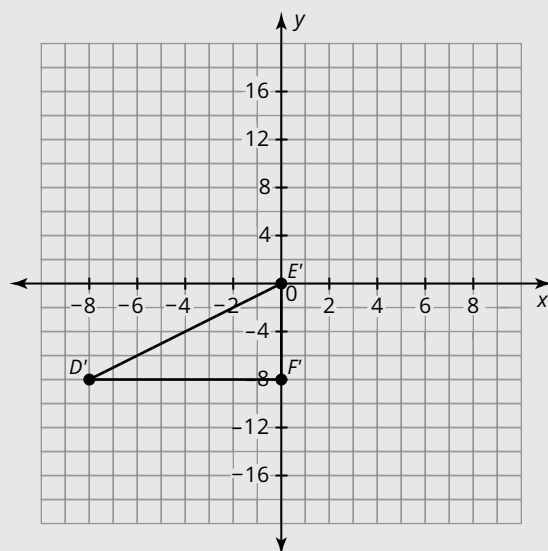
Aaliyah



Elijah



Kai

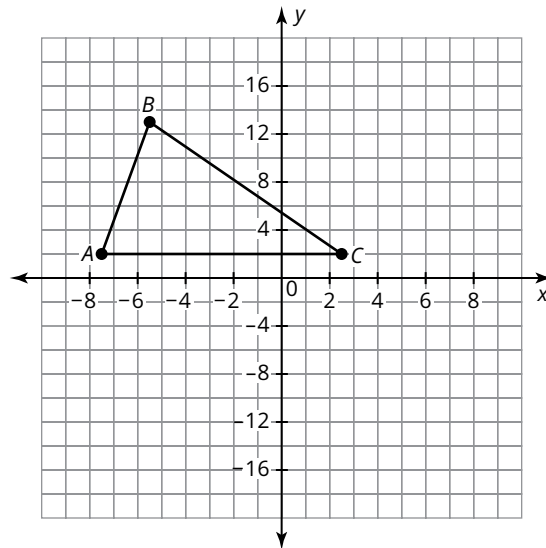


a. Describe the transformation(s) each student made to $\triangle DEF$.

b. Whose method do you think is most efficient? Explain your reasoning.

c. What do you know about the perimeter and area of all the triangles? Explain your reasoning.

Consider $\triangle ABC$ with vertices $A(-7.5, 2)$, $B(-5.5, 13)$, and $C(2.5, 2)$.



4. Determine the perimeter of $\triangle ABC$. Round your answer to the nearest hundredth, if necessary.
5. Consider how to determine the area of $\triangle ABC$.
 - a. What information is needed about $\triangle ABC$ to determine its area?



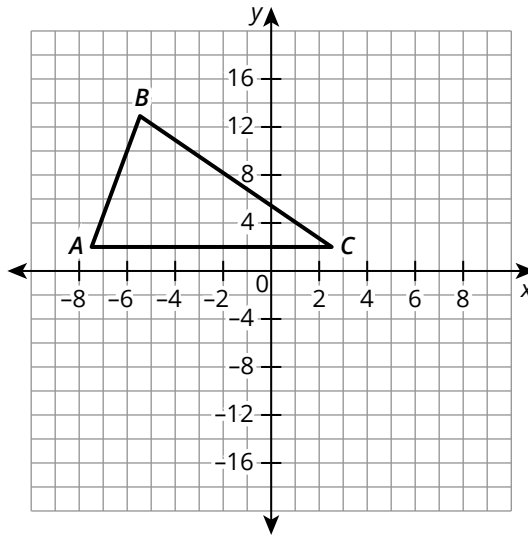
- b. Noah says that $\overline{AB}$ can be used as the height. Ava disagrees and says that $\overline{BC}$ can be used as the height. Ethan disagrees with both of them and says that none of the line segments currently on the triangle can be used as the height. Who is correct? Explain your reasoning.

- c. Draw and label $\overline{BD}$ to represent the height of $\triangle ABC$. Then, determine the height of $\triangle ABC$.

d. Determine the area of $\triangle ABC$.

6. Consider a more efficient way to determine the area and perimeter of $\triangle ABC$.

a. Transform $\triangle ABC$ on the coordinate plane. Label the image $A'B'C'$. Describe the transformation(s) completed and explain your reasoning.



b. Determine the perimeter and area of $\triangle A'B'C'$. Round your answer to the nearest hundredth, if necessary.

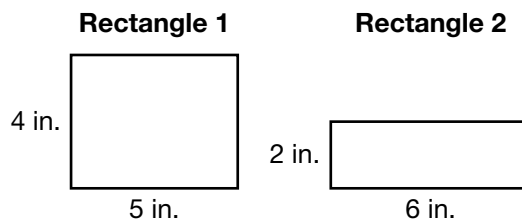
c. Compare these calculations to your previous calculations. How did the translation change your calculations?

ACTIVITY
5.2

Proportional and Non-Proportional Dimensions Changes

In the previous activities, you investigated how doubling one or both of the dimensions of a figure affected its area and perimeter. Now let's investigate how both proportional and non-proportional changes in a figure's dimensions affect its perimeter and area.

1. Consider the following rectangles with the dimensions shown.



Complete the table to determine how doubling or tripling each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.

		Original Rectangle	Rectangle Formed by Doubling Dimensions	Rectangle Formed by Tripling Dimensions
Rectangle 1	Linear Dimensions	$b = 5$ in. $h = 4$ in.	$b = 10$ in. $h = 8$ in.	$b = 15$ in. $h = 12$ in.
	Perimeter (in.)	$2(5 + 4) = 18$	$2(10 + 8) = 36$	$2(15 + 12) = 54$
	Area (in. ²)	$5(4) = 20$	$10(8) = 80$	$15(12) = 180$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in. ²)			

2. Describe how a proportional change in the linear dimensions of a rectangle affects its perimeter.

a. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 2?

b. What happens to the perimeter of a rectangle when its dimensions increase by a factor of 3?

c. What would happen to the perimeter of a rectangle when its dimensions increase by a factor of 4?

d. Describe how you think the perimeter of the resulting rectangle would compare to the perimeter of a 4×10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the perimeter of the resulting rectangle.

e. In terms of k , can you generalize change in the perimeter of a rectangle with base b and height h , given that its original dimensions are multiplied by a factor k ?

.....
Remember...

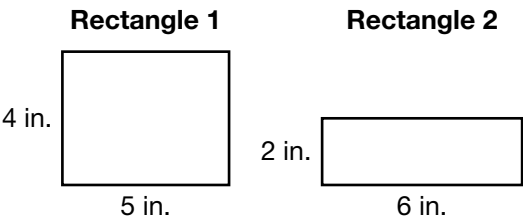
Multiplying side lengths by a number is a proportional change.
.....

3. Describe how a proportional change in the linear dimensions of a rectangle affects its area.
- a. What happens to the area of a rectangle when its dimensions increase by a factor of 2?
 - b. What happens to the area of a rectangle when its dimensions increase by a factor of 3?
 - c. What would happen to the area of a rectangle when its dimensions increase by a factor of 4?
 - d. Describe how you think the area of the resulting rectangle would compare to the area of a 4×10 rectangle if the dimensions of the original rectangle were reduced by a factor of $\frac{1}{2}$. Then, determine the area of the resulting rectangle.
 - e. In terms of k , can you generalize change in the area of a rectangle with base b and height h , given that its original dimensions are multiplied by a factor k ?

A non-proportional change to a two dimensional figure occurs when the side lengths are changed in a way that their relationship to each other no longer stays the same.

4. Do you think a non-proportional change in the linear dimensions of a two-dimensional figure will have the same effect on perimeter and area as proportional change? Explain your reasoning.

5. Consider the following rectangles with the dimensions shown.



.....

When only one dimension of the rectangle is doubled or tripled, the other dimension remains the same.

.....

Complete the table to determine how doubling or tripling one dimension of the rectangle affects its perimeter and area. The information for Rectangle 1 has been done for you.

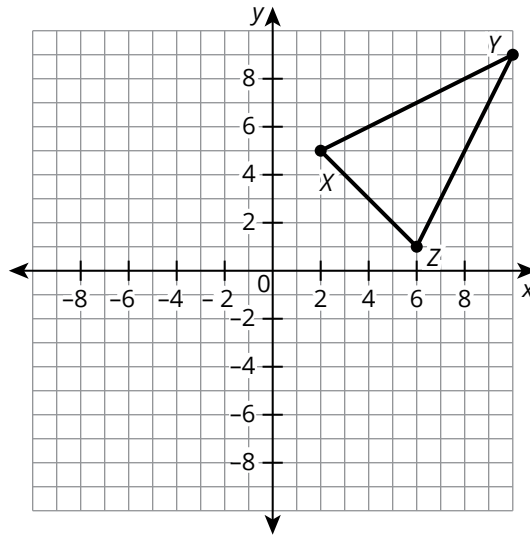
		Original Rectangle	Rectangle Formed by Doubling the Height	Rectangle Formed by Tripling the Base
Rectangle 1	Linear Dimensions	$b = 5 \text{ in.}$ $h = 4 \text{ in.}$	$b = 5 \text{ in.}$ $h = 8 \text{ in.}$	$b = 15 \text{ in.}$ $h = 4 \text{ in.}$
	Perimeter (in.)	$2(5 + 4) = 18$	$2(5 + 8) = 26$	$2(15 + 4) = 38$
	Area (in. ²)	$5(4) = 20$	$5(8) = 40$	$15(4) = 60$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in. ²)			

6. Describe how a non-proportional change in the linear dimensions of a rectangle affects its perimeter.
 - a. What happened to the perimeters of each rectangle when you only doubled the height?
 - b. What happened to the perimeters of each rectangle when you only tripled the base?
 - c. What happens to the perimeter of each rectangle if the height is doubled and the base is tripled?
 - d. What happens to the perimeter when the dimensions of a rectangle change non-proportionally?

7. Describe how a non-proportional change in the linear dimensions of a rectangle affects its area.
- a. What happens to the area of a rectangle when only the height doubles?
 - b. What happens to the area of a rectangle when only its base is tripled?
 - c. What would happen to the area of a rectangle when one dimension is increased by a factor of 4 and one dimension is increased by a factor of 3.
 - d. What happens to the area when the dimensions of a rectangle change non-proportionally by different scaling factors?
 - e. Summarize the change in a rectangle's area caused by a non-proportional change in its linear dimensions and the change in a rectangle's area caused by a proportional change in its linear dimensions.

Calculating Heights of Triangles

Consider $\triangle XYZ$ with vertices $X(2, 5)$, $Y(10, 9)$, and $Z(6, 1)$.

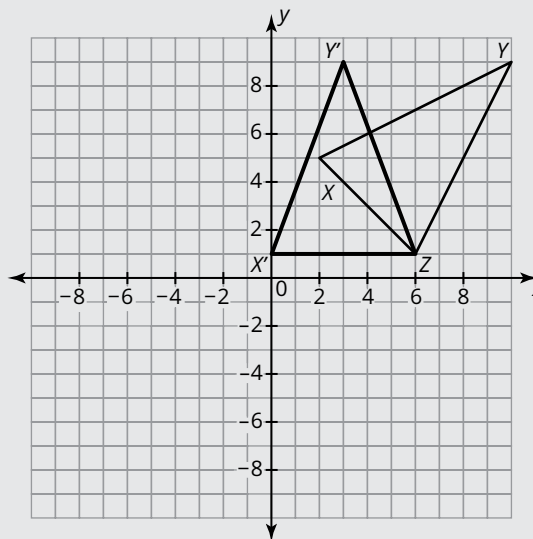


1. Determine the perimeter of $\triangle XYZ$. Round your answer to the nearest hundredth, if necessary.
2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangles in previous activities?

3. Samuel wanted to transform the triangle to make the calculations easier. How can you determine that Samuel did not transform the triangle correctly?

Samuel

I can rotate the triangle so the base is on a gridline to determine its height. The triangle has a height of 8 units.



Remember...

The altitude, or height, of a triangle is the perpendicular distance from a vertex to the line containing the opposite side.

.....

Remember...

The slopes of perpendicular lines are negative reciprocals.

.....

Let's use $\overline{XY}$ as the base of $\triangle XYZ$. You can draw $\overline{ZW}$ to represent the height. Remember that the height is perpendicular to the base. To determine the length of the height, you need to locate point W , which is located at the intersection of $\overline{XY}$ and $\overline{ZW}$.

WORKED EXAMPLE

Calculate the slope of the base, $\overleftrightarrow{XY}$. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{10 - 2} = \frac{4}{8} = \frac{1}{2}$

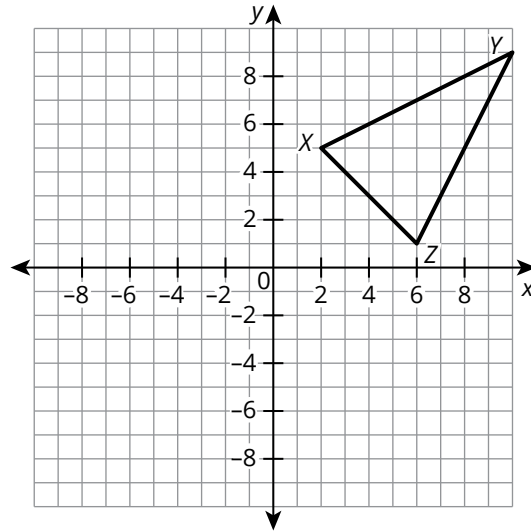
Calculate the slope of the height, $\overleftrightarrow{ZW}$. $m = -2$

You can write equations for $\overleftrightarrow{XY}$ and $\overleftrightarrow{ZW}$ and solve the system to determine where the two lines intersect.

Determine the equations of the lines containing the base and the height.	Base $\overleftrightarrow{XY}$	Height $\overleftrightarrow{ZW}$
	$X(2, 5), m = \frac{1}{2}$	$Z(6, 1), m = -2$
	$y - y_1 = m(x - x_1)$	$y - y_1 = m(x - x_1)$
	$y - 5 = \frac{1}{2}(x - 2)$	$y - 1 = -2(x - 6)$
	$y = \frac{1}{2}x + 4$	$y = -2x + 13$

Solve the system of equations to determine the coordinates of the point of intersection.	$\frac{1}{2}x + 4 = -2x + 13$	$y = -2x + 13$
	$\frac{5}{2}x = 9$	$y = -2\left(\frac{18}{5}\right) + 13$
	$x = \frac{18}{5}$	$y = \frac{29}{5}$

4. Identify the coordinates of the point of intersection. Plot this point on the coordinate plane and label it point W . Draw $\overleftrightarrow{ZW}$ to represent the height.



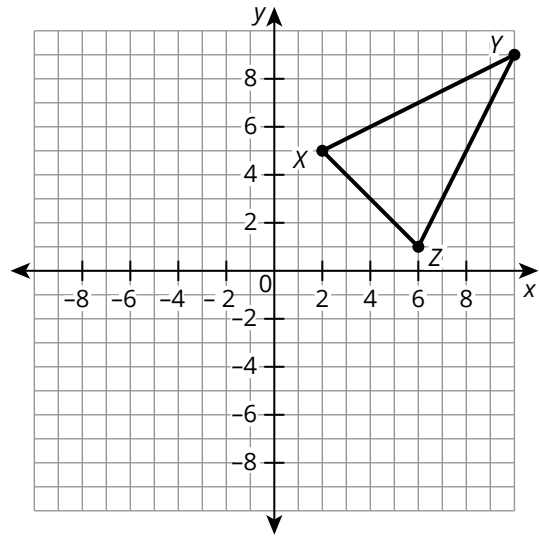
5. Determine the area of $\triangle XYZ$.
- Determine the height of the triangle.
 - Determine the base of the triangle.
 - Determine the area of the triangle.

You know that any side of a triangle can be thought of as the base of the triangle.

6. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

7. Triangle XYZ is graphed on the coordinate plane. This time consider side $\overline{XZ}$ as the base.



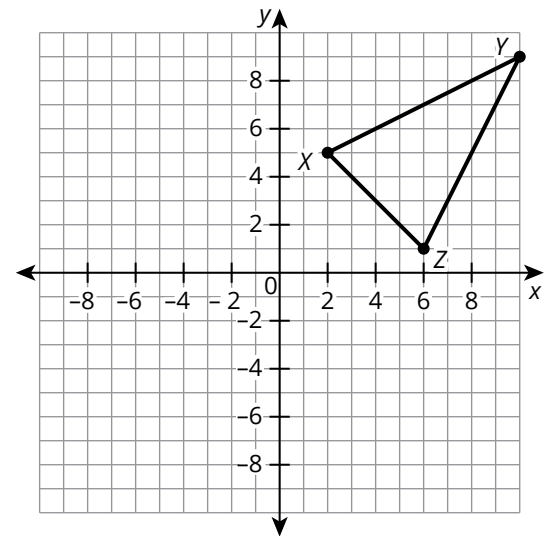
- a. Let point V represent the intersection point of the height, $\overline{YV}$, and the base. Determine the coordinates of point V.

- b. Determine the height of $\triangle XYZ$.

- c. Determine the base of $\triangle XYZ$.

- d. Determine the area of $\triangle XYZ$.

8. Triangle XYZ is graphed on the coordinate plane. Determine the area of $\triangle XYZ$ using side $\overline{YZ}$ as the base.



9. Compare the three areas you determined for $\triangle XYZ$.
Was your prediction in Question 6 correct?

ACTIVITY
5.4

Perimeter and Area of a Composite Figure

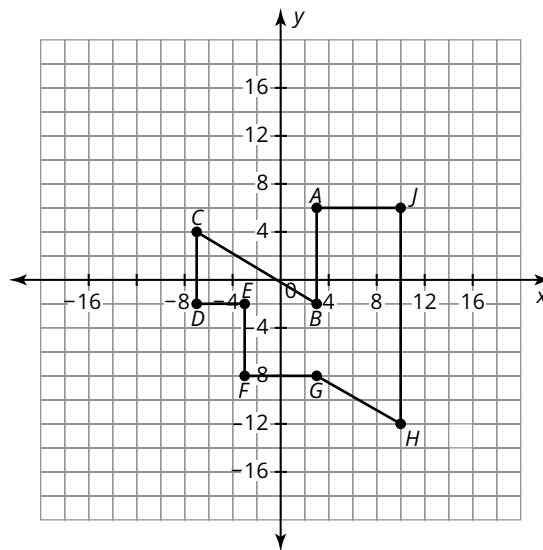
You can use a similar process to determine the perimeter of a rectangle or triangle with any polygon. You can use the distance formula to calculate the distance between any set of vertices and then add the lengths of all the sides.

You can determine the area of a *composite figure* by dividing the figure into a combination of different shapes, like rectangles and triangles. A **composite figure** is a figure that is formed by combining different shapes.

Diego has an irregular backyard because it backs onto the foothill of a mountain and is very rocky. The composite figure graphed on the coordinate plane represents the flat area of Diego's backyard. Each interval of the coordinate plane represents two yards.

Ask Yourself . . .

What shapes can you visualize in this composite figure?



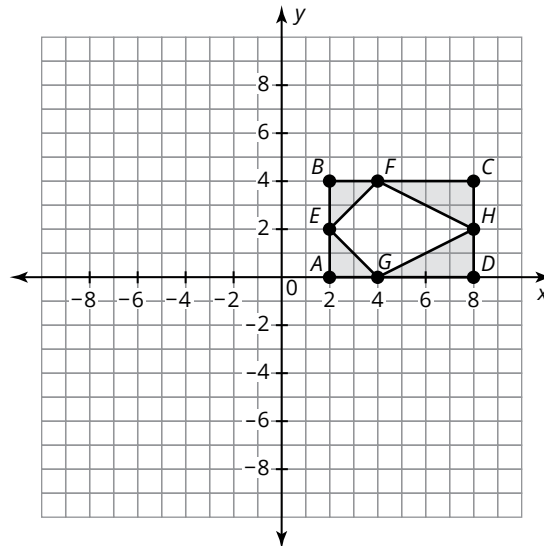
1. Diego will install fencing all around the flat area of his backyard. Determine the amount of fencing he needs to the nearest whole yard.

- Diego wants to lay grass sod in the flat area of his backyard. Determine the total area of sod he needs.
- Compare the method you used to determine the area of sod Diego needs to your classmates' methods. If you had a different way of dividing up the composite figure, did your answers differ? Explain why or why not.
- Fencing costs \$5.45 per foot and sod costs \$0.62 per square foot. To allow for measurement error, Diego plans to buy an extra 10% of both materials. How much will it cost Diego to purchase these materials?

ACTIVITY
5.5

Solving Problems with Perimeter and Area

A pattern for a quilt patch is drawn on a coordinate plane. The patch consists of a kite in the center of a rectangle.



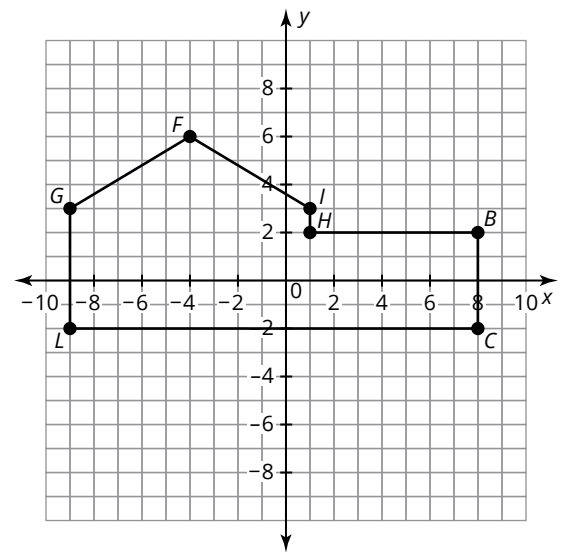
Ask Yourself . . .

Did you make a plan to solve the problem?

1. Daniel is in charge of buying the red fabric that will cover the shaded portion of the quilt patch. How many square inches of red fabric are needed for each patch?
2. Daniel must also purchase the blue fabric that will cover the non-shaded portion of the quilt patch in the figure. How many square inches of blue fabric are needed for each patch?

Harper's living room has a vaulted ceiling. She wants to paint the wall in the room. The wall is represented by the figure on the coordinate plane. Each interval on the coordinate plane represents one foot.

Harper and Juan are going to paint the wall and want to place painter's tape along the edge of each of the walls so the paint does not touch any other wall, the ceiling, or the floor.



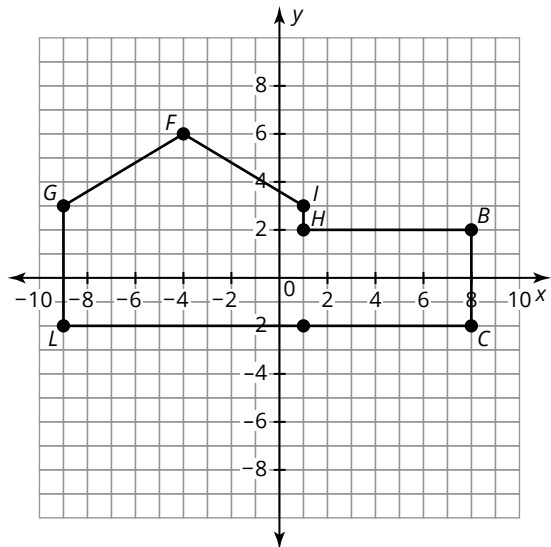
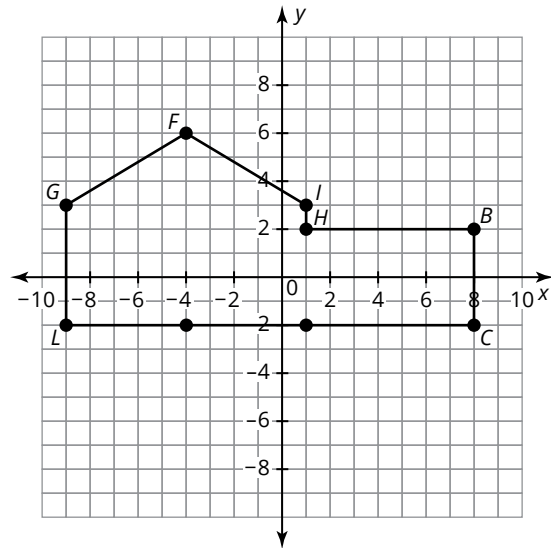
- What is the length of painter's tape (to the nearest whole foot) that Harper and Juan need to cover the edges of the wall?

Ask Yourself . . .

How can you use a transformation of the composite figure on the coordinate plane as part of your strategy?



4. Juan says he can draw two vertical line segments and a horizontal line segment to divide the composite figure into two right triangles, two squares, and a rectangle. Harper says she can draw a horizontal line segment and a vertical line segment to divide the figure into an isosceles triangle and two rectangles. Who's correct? Explain your reasoning.

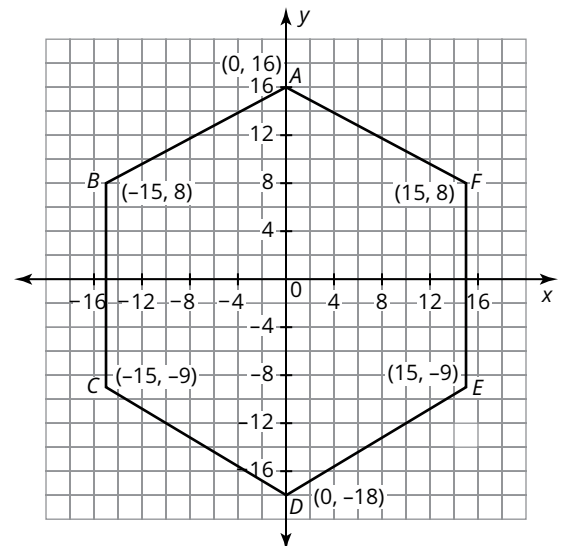


5. One gallon of paint covers approximately 400 square feet. Harper estimates she has about one fourth of a gallon of paint remaining of the color she wants to use. Does she have enough paint for the wall? Explain your reasoning.

Ask Yourself . . .

Did you justify your mathematical reasoning?

Sofia and Jayden are designing a gazebo for the local park. The polygon shown on the coordinate plane represents the base of the gazebo. Each interval on the coordinate plane represents two feet.



6. The base of the gazebo needs to be built with lengths of lumber around the outside to support the floorboards. What is the length of lumber needed for the outside of the base?

Ask Yourself . . .

What tools or strategies can you use to solve this problem?

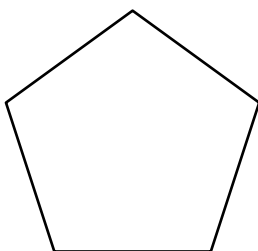
7. How many square feet of floorboards are needed for the base of the gazebo? Describe how you determined your answer and show your work.

ACTIVITY
5.6

Areas of Regular Polygons

A **regular polygon** is a polygon with all sides congruent and all angles congruent. The area of a regular polygon can be thought of as the area of a composite shape.

1. A regular pentagon is shown.
 - a. Locate and place a point at the center of the regular pentagon. From the center point, draw line segments to connect the point with each vertex of the pentagon.



- b. Describe the new polygons formed by adding these line segments.
 - c. What information do you need to calculate the area of each new polygon?
 - d. What formula is used to calculate the area of each new polygon?
 - e. Describe a strategy to determine the area of the entire regular pentagon.

2. How can you use this strategy to determine the area of other regular polygons?

To determine the area of a regular polygon you can use the formula $A = n\left(\frac{1}{2}\right)bh$, where n is the number of congruent triangles you can draw in the regular polygon (which is the same as the number of sides of the polygon), b is the base of the triangle, and h is the height of the triangle.

The **apothem** of a regular polygon is the perpendicular segment from the center of a regular polygon to one of its sides. The height of each of the congruent triangles you drew is the apothem of the regular polygon.

The base of each of the congruent triangles is the same as the side of the regular polygon.

The perimeter of the regular polygon is the same as $n \cdot b$, since all sides of a regular polygon are the same.

Using the commutative property of multiplication

$$A = n\left(\frac{1}{2}\right)bh = \left(\frac{1}{2}\right)(h)(nb)$$

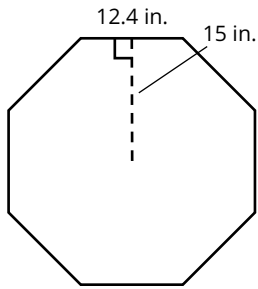
By substitution,

$$A = \frac{1}{2}aP$$

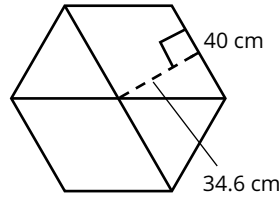
where P represents the perimeter of the regular polygon and a represents the apothem.

For each regular polygon, draw the congruent triangles and use the triangles to calculate the area. Then, calculate the area using the formula $A = \frac{1}{2}aP$.

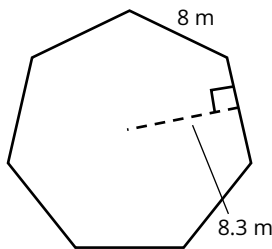
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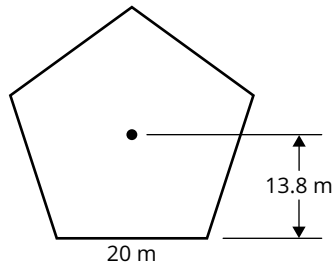
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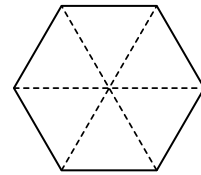


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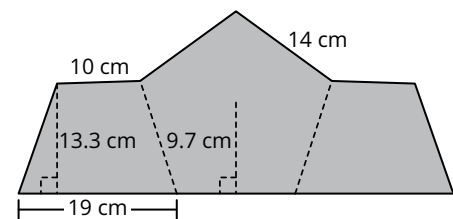


Solve each problem involving regular polygons.

7. A regular hexagon has an area of 540 cm^2 , and its side lengths are each 15 cm. Calculate the apothem of the hexagon.



8. Calculate the area of the figure composed of a regular pentagon and 2 congruent trapezoids.

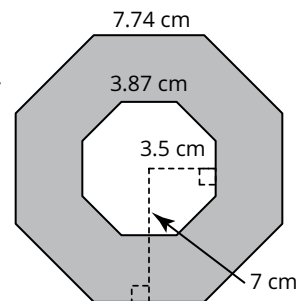




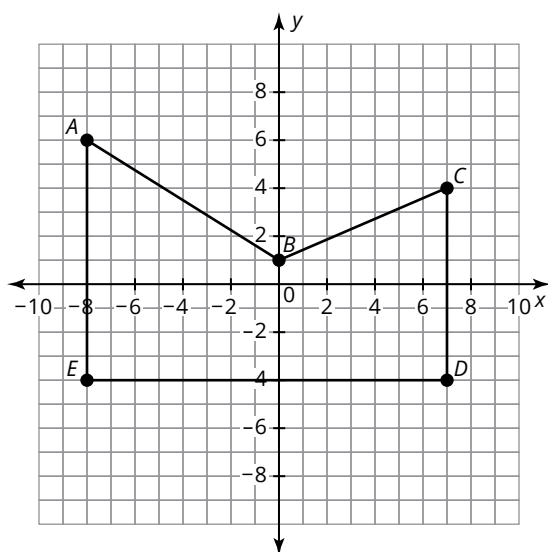
Talk the Talk

Shape It Up

1. A regular octagon has an octagonal hole as shown. Calculate the area of the shaded region.



2. Chloe, an interior designer, is tasked with redesigning a school classroom. She proposes a design represented by a composite figure graphed on the coordinate plane. Each interval of the coordinate plane represents 1 meter. The school requires the design to have a total perimeter of less than 60 meters and a total area of less than 200 square meters. Based on these criteria, should the school accept Emma's proposal? Explain your reasoning.



3. Mia and Mateo are designing a poster for a school event. The original poster board is a rectangle with dimensions 24 inches by 36 inches. They decide to resize the poster so that both the width and height are reduced to half their original sizes. Mia says, "If we reduce the dimensions by half, the new perimeter will also be half of the original, and the new area will be half of the original area." Mateo says, "If we reduce the dimensions by half, the new perimeter will be half the original, but the new area will be one-fourth of the original area." Who's correct? Explain your reasoning.

Lesson 5 Assignment

Write

Describe how you can determine the area of a composite figure.

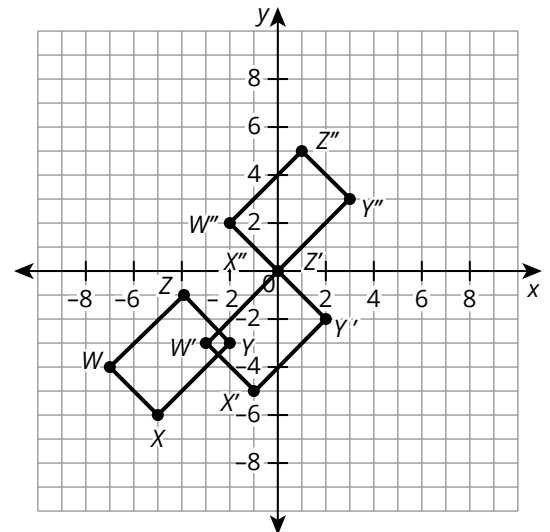
Remember

Rigid transformations can make calculating the perimeter and area of figures on the coordinate plane more efficient.

Any side of a triangle can be considered its base, and the height of the triangle is the perpendicular distance from the base to the opposite vertex.

Practice

- Minh translates rectangle $WXYZ$ vertically up 1 unit and horizontally to the right 4 units to produce the image $W'X'Y'Z'$. Liam translates the rectangle vertically up 6 units and horizontally to the right 5 units to produce the image $W''X''Y''Z''$.
 - Would you prefer to use Minh's translation or Liam's translation to determine the perimeter and area of the rectangle? Explain your reasoning.
 - Calculate the perimeter and area of the rectangle. Show your work.

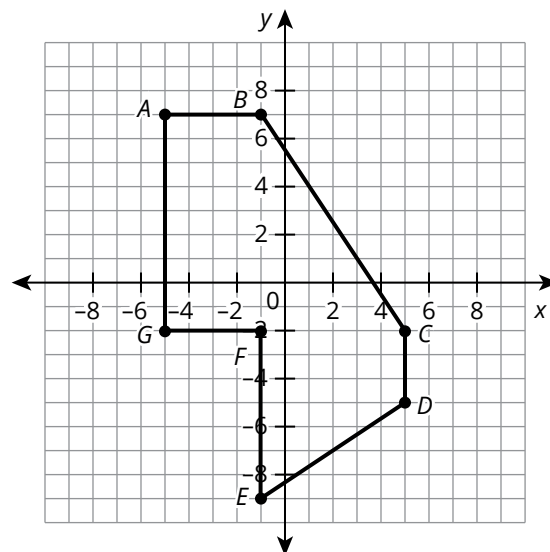


Lesson 5 Assignment

- c. The dimensions of rectangle $WXYZ$ are multiplied by a factor of 4. How do the perimeter and area of the resulting rectangle relate to the perimeter and area of the original rectangle?
- d. The height of rectangle $WXYZ$ is multiplied by a factor of 3. Victoria says that the area of the resulting rectangle is 9 times the area of the original rectangle. Is she correct? Explain your reasoning.

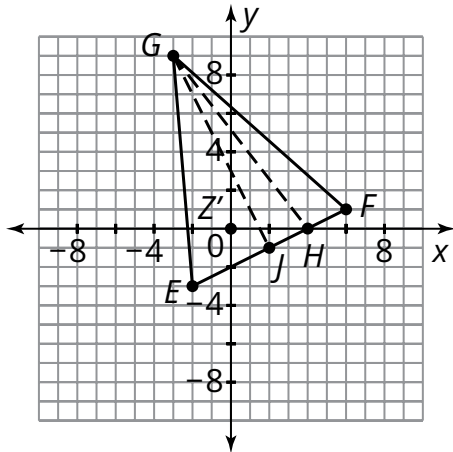
2. Composite figure $ABCDEFGG$ is given.

- a. Determine the perimeter of figure $ABCDEFGG$.
- b. Determine the area of figure $ABCDEFGG$.

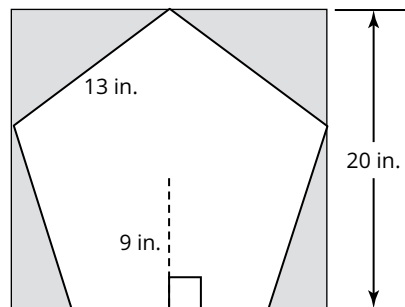


Lesson 5 Assignment

3. Elijah claims that $\overline{GH}$ is the height of $\triangle EFG$, and Aaliyah claims that $\overline{GJ}$ is the height of $\triangle EFG$.

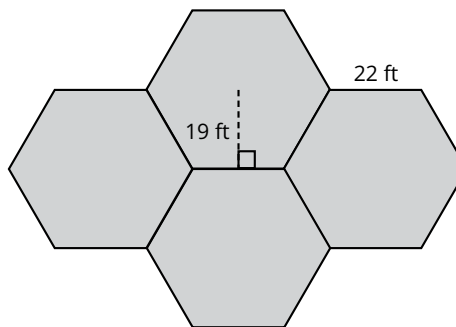


- Who is correct? Justify your response.
 - Calculate the area of $\triangle EFG$. Show your work.
4. The figure is made from a square and a regular pentagon. Calculate the area of the shaded region.



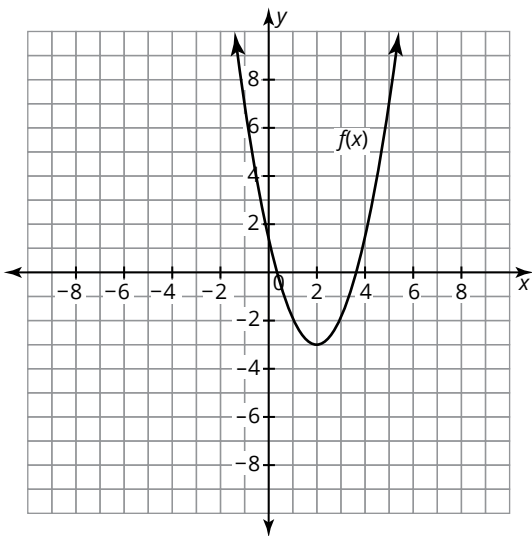
Lesson 5 Assignment

5. Use the formula $A = \frac{1}{2}aP$ to find the area of the shaded region.



Prepare

Consider the graph of $f(x)$



Complete the table

$f(x)$	$-f(x)$	$f(-x)$
$(-1, 6)$		
$(0, 1)$		
$(1, -2)$		

Using a Rectangular Coordinate System

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

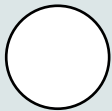
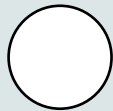
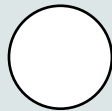
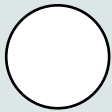
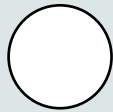
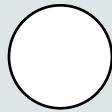
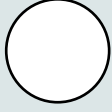
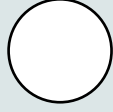
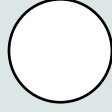
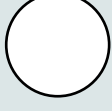
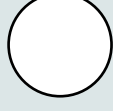
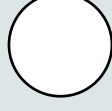
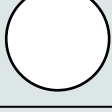
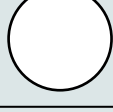
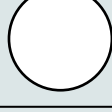
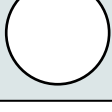
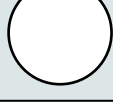
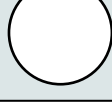
Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Using a Rectangular Coordinate System* topic by:

TOPIC 2: <i>Using a Rectangular Coordinate System</i>	Beginning of Topic	Middle of Topic	End of Topic
constructing a segment bisector, a perpendicular bisector, and a perpendicular line through a point not on a line.			
determining the midpoint of a segment.			
constructing a congruent line segment by duplicating a given segment.			
recognize that slope of perpendicular lines are negative reciprocals.			
identifying and writing equations of parallel lines, including horizontal and vertical lines.			
determining the equation of a line parallel or perpendicular to a given line that passes through a given point.			
deriving and using the slope formula to solve problems.			

continued on the next page

TOPIC 2 SELF-REFLECTION
continued

TOPIC 2: <i>Using a Rectangular Coordinate System</i>	Beginning of Topic	Middle of Topic	End of Topic
deriving and using the distance formula to solve problems.			
using the slope criteria for parallel and perpendicular lines to determine the relationship between sides of a figure.			
deriving and using the midpoint formula to solve problems.			
calculating the perimeter of a figure on the coordinate plane using the distance formula.			
computing the area of a figure on the coordinate plane using the coordinates of the figure, the distance formula, and the appropriate area formula.			
decomposing a given composite figure into a set of non-overlapping triangles and rectangles and determining the total area of the figure.			

At the end of the topic, take some time to reflect on your learning and answer the following questions.

- Describe a new strategy you learned in the *Using a Rectangular Coordinate System* topic.

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Using a Rectangular Coordinate System Summary

LESSON

1

Constructing a Coordinate Plane

When you construct geometric figures, you create exact figures without measurements, using only a compass and a straightedge. A compass is a tool used to create *arcs* and circles. An **arc** consists of two points on a circle, and the curved line that connects the two points. A straightedge is a ruler with no numbers.

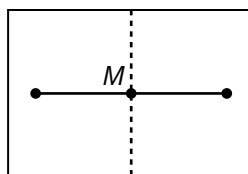
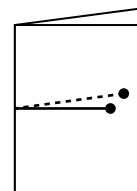
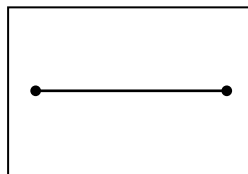
The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

You can use patty paper to bisect a line segment.

Draw a line on the paper.

Fold the paper so the endpoints of the line segment lie on top of each other.

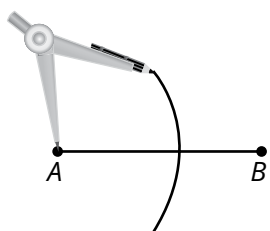
Open the paper. The crease represents the segment bisector, and the midpoint is located where the crease intersects the line segment.



KEY TERMS

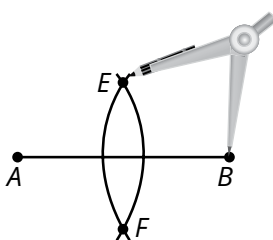
- arc [arco]
- midpoint [punto medio]
- segment bisector [bisectriz de segmento]
- perpendicular bisector [bisectriz perpendicular]
- transformation [transformación]
- rigid motion [movimiento rígido]
- distance formula [fórmula de la distancia]
- extracting perfect squares
- radical [radical]
- radicand [radicando]
- midpoint formula [fórmula del punto medio]
- composite figure [figura compuesta]
- regular polygon [polígono regular]
- apothem [apotema]

To construct a segment bisector using only a compass and straightedge, you make use of the fact that all the radii of a circle have an equal length.



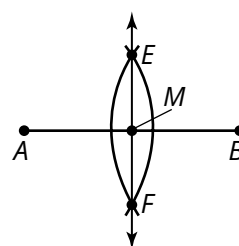
Construct an Arc

Open the radius of the compass to more than half the length of $\overline{AB}$. Use endpoint A as the center and construct an arc.



Construct Another Arc

Keep the compass radius and use point B as the center as you construct an arc. Label the points formed by the intersection of the arcs point E and point F.



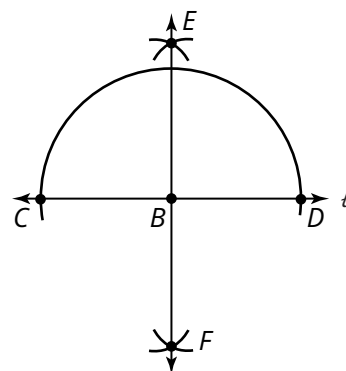
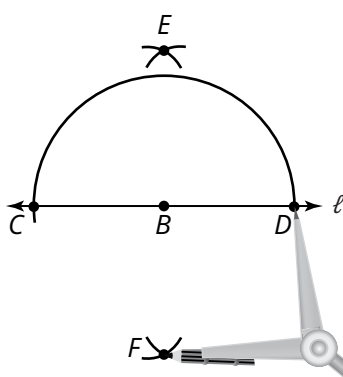
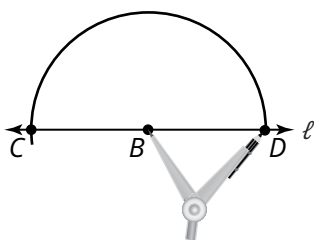
Construct a Line

Connect points E and F. Line segment EF is the segment bisector of $\overline{AB}$. The point M represents the midpoint of $\overline{AB}$.

Line EF bisects $\overline{AB}$. Point M is the midpoint of $\overline{AB}$.

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

You can use a compass and straightedge to create a perpendicular bisector.



Construct an Arc

Use B as the center and construct an arc. Label the intersections points C and D.

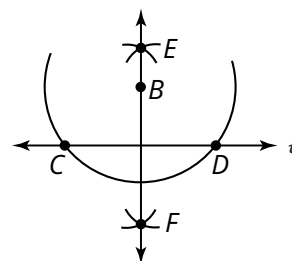
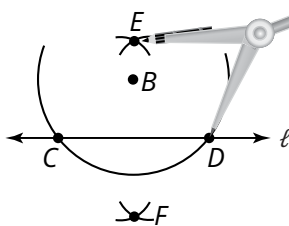
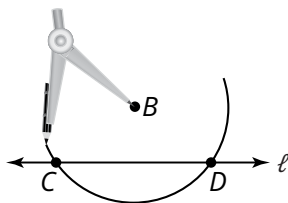
Construct other Arcs

Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F.

Construct a Line

Use a straightedge to connect points E and F. Line EF is perpendicular to $\overleftrightarrow{CD}$.

You can also construct a perpendicular line through a point not on a line.



Construct an Arc

Use B as the center and construct an arc. Label the intersection points C and D .

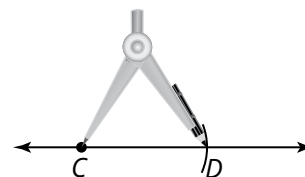
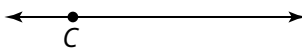
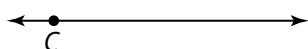
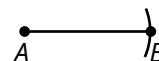
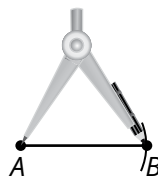
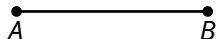
Construct other Arcs

Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F .

Construct a Line

Use a straightedge to connect points E and F . Line EF is perpendicular to $\overleftrightarrow{CD}$.

You can construct a congruent line segment by duplicating the original line segment.



Construct a Starter Line

Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point C on the line.

Measure Length

Set your compass to the length of $\overline{AB}$.

Copy Length

Place the compass to C and mark point D on the new segment.

$$\overline{CD} \cong \overline{AB}$$

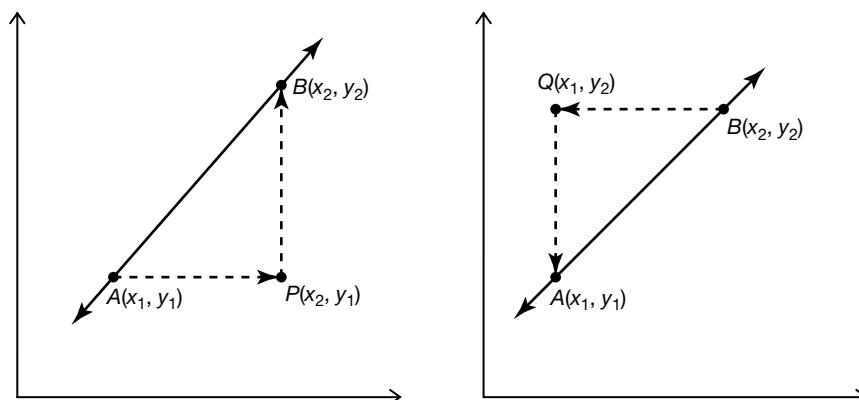
A **transformation** is the mapping, or movement, of the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure. Three types of rigid motion transformations are translations, reflections, and rotations.

LESSON

2

Parallel and Perpendicular Lines

You can derive the formula for the slope of a line.



Step 1: Choose two points on the line. Choose points with coordinate values that do not require estimation. The two points are shown as A and B .

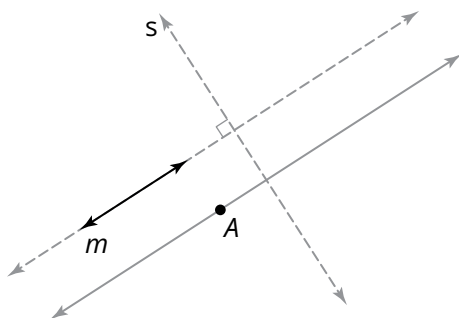
Step 2: Subtract the y -coordinates of points A and B to determine the rise—the vertical change from the first point to the second point.

Step 3: Continue from the point where you left off on the graph to determine the run—the horizontal change from the first point to the second point. Subtract the x -coordinates of points A and B to determine the run.

Step 4: Write the slope, $\frac{\text{rise}}{\text{run}}$.

From point A to B , the slope is $\frac{y_2 - y_1}{x_2 - x_1}$.

From point B to A , the slope is $\frac{y_1 - y_2}{x_1 - x_2}$.



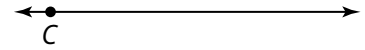
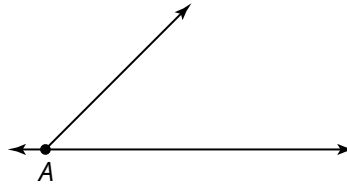
You can construct parallel lines.

For example, to construct a line parallel to line m through point A , you can construct a line s perpendicular to line m . Then, construct a line perpendicular to m through point A . If two lines are perpendicular to the same line, then they are parallel to each other.

You can also construct parallel lines by duplicating an angle formed by a transversal and a line. You can duplicate an angle using constructions.

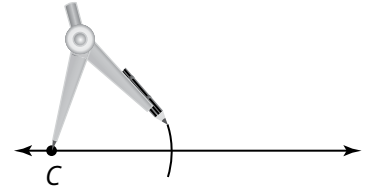
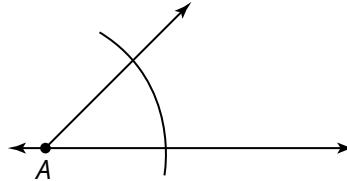
Construct a Starter Line

Use a straightedge to construct a starter line.
Label point C on the new line.



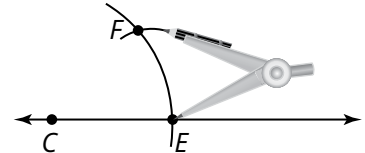
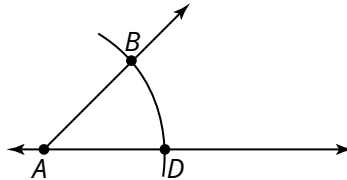
Construct an Arc

Construct an arc with center A. Using the same radius, construct an arc with center C.



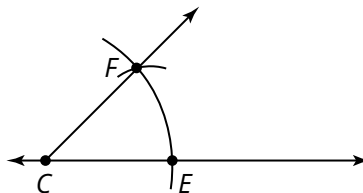
Construct Another Arc

Label points B, D, and E.
Construct an arc with radius BD and center E.
Label the intersection F.



Construct a Ray

Construct ray CF.
 $\angle BAD \cong \angle FCE$.

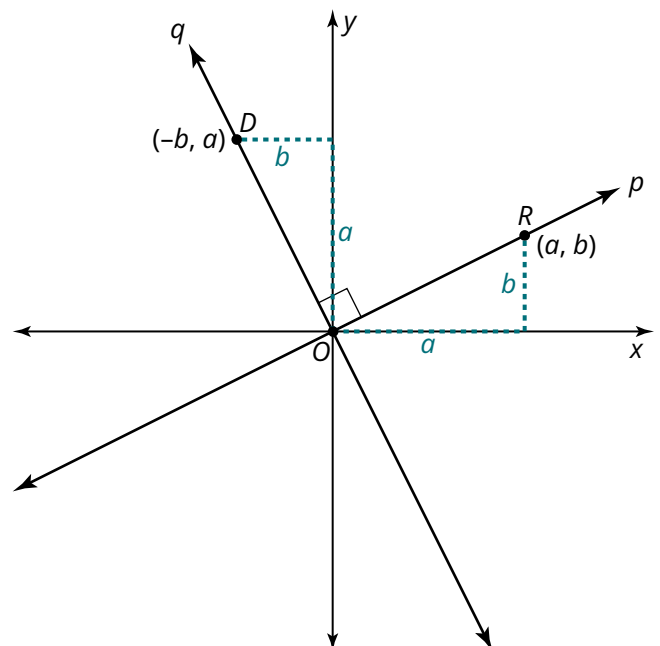


Perpendicular lines or line segments form a right angle at the point of intersection. You can think of perpendicular lines as a line and its rotation 90° about a point, which is also the point of intersection.

The product of the slopes of perpendicular lines is -1 .

For example, consider the graph.

The slope of line q is the negative reciprocal of the slope of line p because $\frac{b}{a} \cdot \frac{a}{-b} = -1$.



The slope of any horizontal line is 0 since no matter the change for x , there is 0 change for y . The slope of any vertical line is undefined since no matter the change for y , there is 0 change for x .

All horizontal lines are parallel to each other since their slopes are equal and all vertical lines are parallel since their slopes are equal. A horizontal and a vertical line are always perpendicular to each other.

To write an equation for a line that passes through the point $(-4, -2)$ and is perpendicular to the line $y = 3$, first determine that the line given by $y = 3$ is a horizontal line. Therefore, a line that is perpendicular to $y = 3$ is a vertical line. A vertical line that passes through the point $(-4, -2)$ has the equation $x = -4$.

You can use what you know about the slopes of perpendicular lines and point-slope form to write the equation of a perpendicular line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line perpendicular to $y = 4x - 1$ must have a slope of $-\frac{1}{4}$, since $4 \cdot -\frac{1}{4} = -1$.

Using the given point and point-slope form, you can set up an equation to solve for b , the y -intercept.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - 2 &= -\frac{1}{4}(x - (-4)) \\y - 2 &= -\frac{1}{4}x - 1 \\y &= -\frac{1}{4}x + 1\end{aligned}$$

Therefore, the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$ is $y = -\frac{1}{4}x + 1$.

Since a line and its translation are parallel to each other, the slopes of parallel lines are equal.

You can use what you know about the slopes of parallel lines and point-slope form to write the equation of a parallel line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point $(-4, 2)$ and is parallel to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line parallel to $y = 4x - 1$ must also have a slope of 4.

Using the given point and point-slope form, you can set up an equation to solve for b , the y -intercept.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - 2 &= 4(x - (-4)) \\y - 2 &= 4(x + 4) \\y - 2 &= 4x + 16 \\y &= 4x + 18\end{aligned}$$

Therefore, the equation of the line that passes through the point $(-4, 2)$ and is parallel to $y = 4x - 1$ is $y = 4x + 18$.

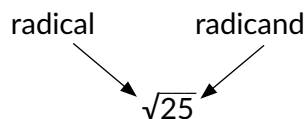
Classifying Quadrilaterals on the Coordinate Plane

You can use the Pythagorean Theorem to derive the formula for the distance between two points on the coordinate plane. This method can be written as the distance formula. The **distance formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Note that absolute value symbols are used because length is always positive.

You can use prime factorization to rewrite radicals in simplest form. The process of removing perfect square number from under the radical symbol is called **extracting perfect squares**.

The symbol $\sqrt{\quad}$ is called a **radical**. The **radicand** is the quantity under a radical.



You can apply the distance formula to determine the lengths of segments on a coordinate plane, to verify that line segments are congruent, and determine whether two lines are parallel or perpendicular.

For example, rewrite $\sqrt{60}$ in simplest radical form.

Write the prime factorization of 60.

$$60 = 2^2 \cdot 3 \cdot 5$$

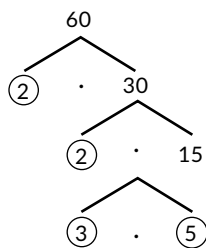
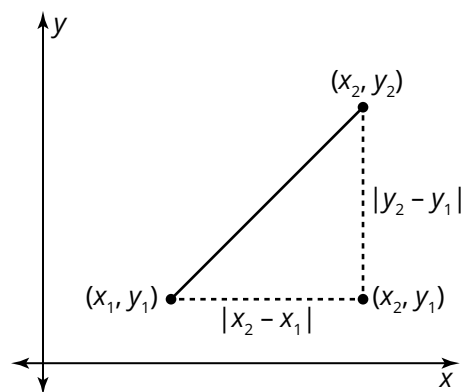
Extract the perfect squares

$$\sqrt{60} = 2 \cdot \sqrt{3 \cdot 5}$$

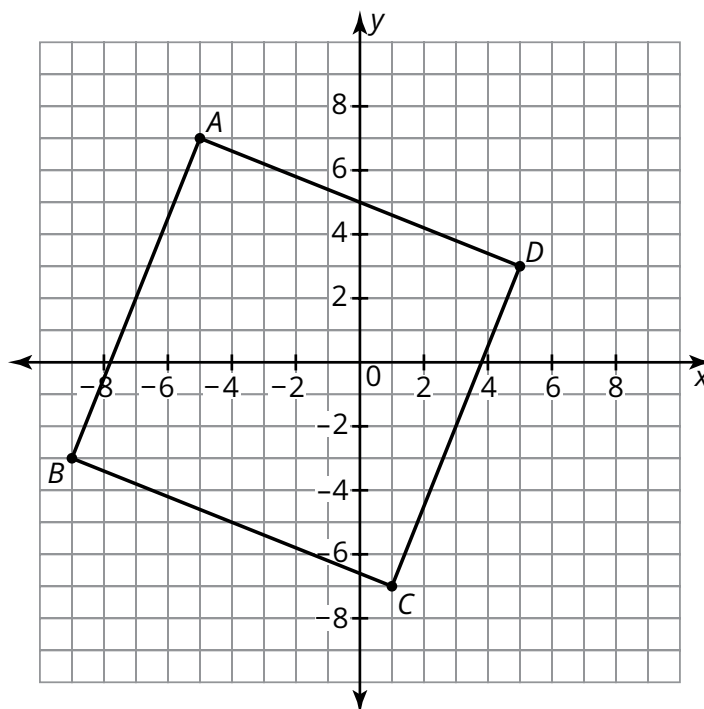
Rewrite the radicand.

$$\sqrt{60} = 2 \cdot \sqrt{15}$$

To determine the coordinates of the midpoint of a segment on the coordinate plane, you can use the midpoint formula. The **midpoint formula** states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Given quadrilateral $ABCD$, determine the midpoint of $\overline{AD}$.



Use the midpoint formula, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Point A is located at $(-5, 7)$ and point D is located at $(5, 3)$.

The midpoint of $\overline{AD}$ is located at $\left(\frac{-5 + 5}{2}, \frac{7 + 3}{2}\right)$ or $(0, 5)$.

LESSON 4

Classifying Triangles on the Coordinate Plane

The **converse** of a conditional statement written in the form “If p , then q ” is the statement written in the form “If q , then p .” The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are interchanged.

For example, consider the statement, “If a number is even, it is divisible by 2.

The hypothesis, p , is that a number is even. The conclusion, q , is that it is divisible by 2.

The converse of the statement is “If a number is divisible by 2, then it is even.

The converse of a statement is not always true. However, when the statement and its converse are true, it is called a *biconditional statement*. A **biconditional statement** is written as “ p if and only if q .” It is a combination of both a conditional statement and the converse of that conditional statement.

For example, remember the statement, “If a number is even, it is divisible by 2”. Both the statement and its converse are true. Therefore, you can rewrite the statement as the biconditional statement: A number is even if and only if, it is divisible by 2.

The Pythagorean Theorem states that if a triangle is a right triangle with a hypotenuse length c and leg lengths a and b , then $a^2 + b^2 = c^2$.

A **Pythagorean triple** is a set of integers a , b , and c that satisfy the Pythagorean Theorem. For example, the integers 3, 4, and 5 for a Pythagorean triple because $3^2 + 4^2 = 5^2$.

You can classify triangles given on the coordinate plane.

For example, classify $\triangle ABC$ as scalene, isosceles, or equilateral by determining AC , CB , and AB . Since points A and B have the same x -value, determine the length of AB by determining the absolute value of the difference in the y -values.

$$AB = |-4 - (-9)|$$

$$AB = 5$$

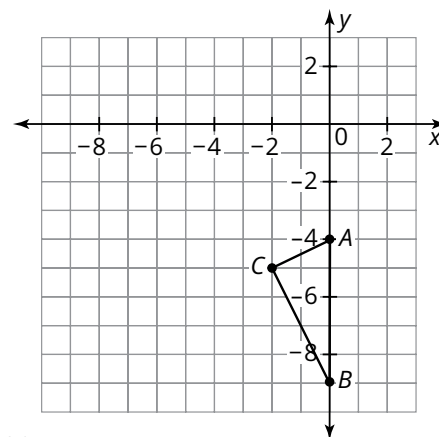
Then, use the distance formula to determine AC and CB .

$$d = \sqrt{(0 - (-2))^2 + (-4 - (-5))^2} \quad d = \sqrt{(0 - (-2))^2 + (-9 - (-5))^2}$$

$$d = \sqrt{(2)^2 + (1)^2} \quad d = \sqrt{(2)^2 + (-4)^2}$$

$$d = \sqrt{5} \quad d = \sqrt{20}$$

$$AC = \sqrt{5} \quad CB = \sqrt{20} = 2\sqrt{5}$$



Since all sides of the triangle are different lengths, the triangle is scalene.

The slopes of adjacent sides of a polygon can also be used to determine if the sides form a right angle since the slopes of perpendicular lines are negative reciprocals of each other. For example, in $\triangle ABC$, $\overline{AC}$ has a slope of $\frac{1}{2}$ and $\overline{BC}$ has a slope of -2 . Since $\frac{1}{2}$ and -2 are negative reciprocals, the line segments form a right angle and $\triangle ABC$ can be classified as a right scalene triangle.

LESSON

5

Area and Perimeter on the Coordinate Plane

You can use distance formula to determine the perimeter and area of rectangles and triangles on the coordinate plane. The perimeter of a geometric figure is calculated by adding the side lengths. The formula for the area of a rectangle is $A = bh$, and the formula for the area of triangle is $A = \frac{1}{2}bh$, where A represents area, b represents the base, and h represents height.

To determine the area of a triangle, first determine the height of the triangle. The altitude, or height, of a triangle is the perpendicular distance from a vertex to the line containing the opposite side, represented by a line segment.

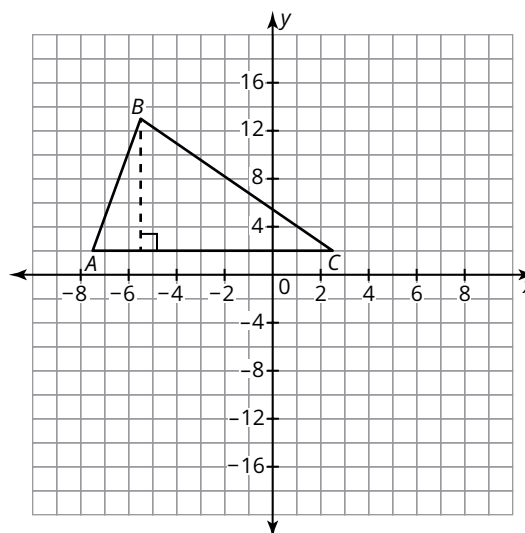
For example, again consider $\triangle ABC$ with vertices $A(-7.5, 2)$, $B(-5.5, 13)$, and $C(2.5, 2)$. The height of the triangle can be created using a line segment from vertex B to base $\overline{AC}$.

The height is $13 - 2 = 11$ units.

The length of the base, $\overline{AC}$, is $|-7.5 - 2.5| = 10$ units.

$$A = \frac{1}{2}(11)(10) = 55$$

The area of the triangle is 55 square units.

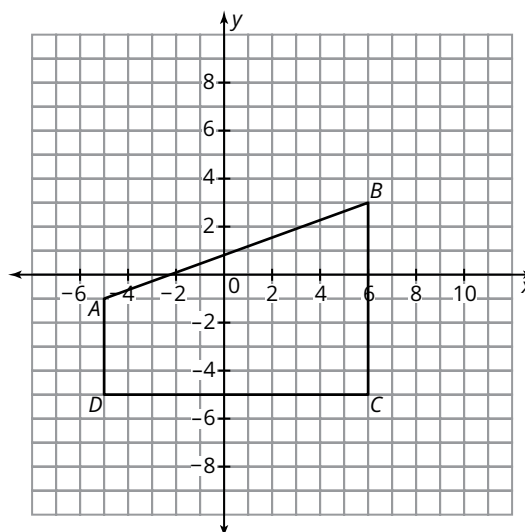


When the base of a triangle is not horizontal, the points that can be used to create a height perpendicular to the base must be identified algebraically by determining the equation of the line containing the base and the equation of the line perpendicular to the base that passes through the opposite vertex, and then solving the system of equations.

You can translate shapes on the coordinate plane to make the process of determining the perimeter and area more efficient. Translations are rigid motions that preserve the size and shape of a figure. The pre-image and the image are congruent because in a translation, all vertices must be rigidly moved from one location to another location.

You can determine the area of a **composite figure** by dividing the figure into a combination of rectangles and triangles. A composite figure is a figure that is formed by combining different shapes.

For example, suppose Aida has a bedroom that is on the top floor of her house. The roof slants downward, creating two congruent trapezoid shaped walls. One of the walls in her room is represented on the coordinate plane by quadrilateral $ABCD$. Each interval on the coordinate plane represents one foot.



You can determine the area of quadrilateral $ABCD$ by breaking it into a triangle and a rectangle and determining the area of each.

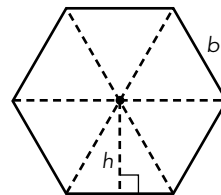
The area of the triangle with vertices at $(-5, -1)$, $(6, 3)$, and $(6, -1)$, is 22 square feet. The area of the rectangle with vertices at $(-5, -1)$, $(6, -1)$, $(6, -5)$, and $(-5, -5)$ is 44 square feet.

The area of the wall is 22 square feet + 44 square feet = 66 square feet.

A **regular polygon** is a polygon with all sides congruent and all angles congruent. The area of a regular polygon can be thought of as the area of a composite shape. To calculate the area of a regular polygon, decompose the polygon into congruent triangles, calculate the area of one triangle, and multiply the area of the triangle by the number of congruent triangles that comprise the polygon.

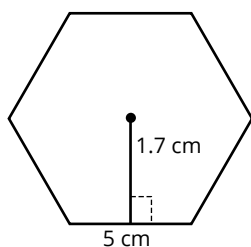
The area of a regular hexagon can be determined if you know the side length of the hexagon and the height of one of the six congruent triangles you decompose the shape into.

$$A = \frac{1}{2}bh(6) = 3bh$$



You can also use the formula $A = \frac{1}{2}aP$, where a is the length of the *apothem* and P is the perimeter of the polygon. The **apothem** is a line segment from the center of a regular polygon to the midpoint of its sides.

For example, determine the area of the regular hexagon.



The perimeter of the hexagon is $5 \cdot 6 = 30$ cm.

The length of the apothem is 1.7 cm.

$$A = \frac{1}{2}aP$$

$$A = \frac{1}{2} \cdot 1.7 \cdot 30$$

$$A = 25.5$$

The area of the regular hexagon is 25.5 cm².



A whirligig is all about rotation, though some of the more elaborate ones may illustrate translations and reflections as well. Whirligigs go by many names: pinwheels, spinners, buzzers, gee-haws, whirlyjigs, whirlybirds, or whirlies.

Sequences of Rigid Motions

LESSON 1	Translations On and Off the Plane	213
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1

Translations On and Off the Coordinate Plane

OBJECTIVES

- Represent points and geometric figures on and off the coordinate plane.
- Verbally describe translations as functions that take points on the plane as inputs and produce translated points as outputs.
- Compare transformations that preserve distance and angles, called rigid motion transformations, to transformations which are non-rigid.

KEY TERMS

- coordinate notation
- congruent angles
- congruent triangles
- translation

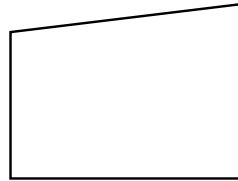
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You have learned how to represent translations of linear, quadratic, and exponential functions. How can you describe geometric translations of figures on and off the coordinate plane?

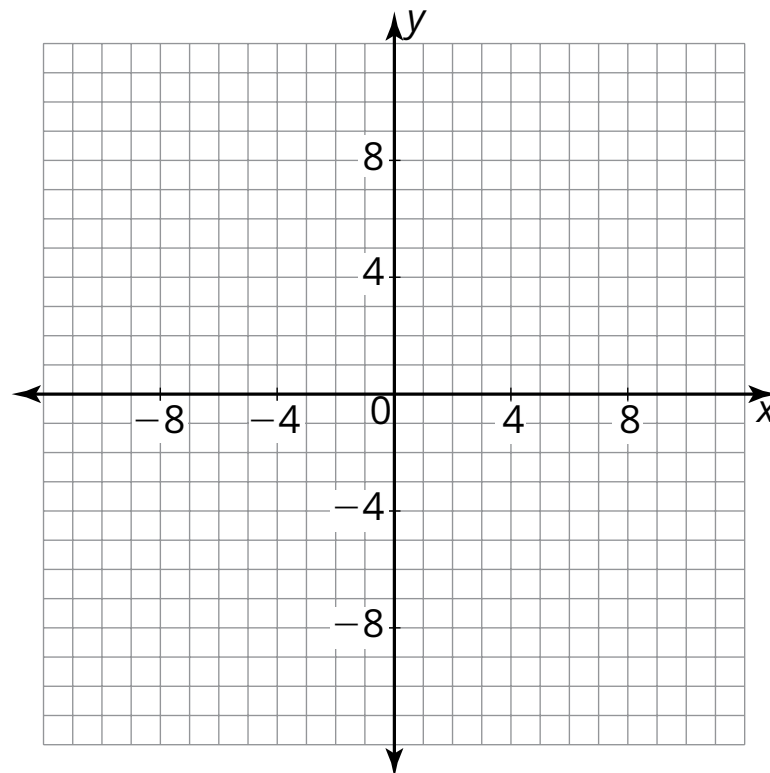
Getting Started

Translating Geometric Figures on the Coordinate Plane

Trace the figure shown onto patty paper and then cut out the figure.



1. Graph trapezoid $ABCD$ by plotting the points $A(3, 9)$, $B(3, 4)$, $C(11, 4)$, and $D(11, 10)$.



2. Translate trapezoid $ABCD$ on the coordinate plane. Graph the image and record the vertex coordinates in the table.
 - a. Translate trapezoid $ABCD$ 15 units to the left to form trapezoid $A'B'C'D'$.
 - b. Translate trapezoid $ABCD$ 12 units down to form trapezoid $A''B''C''D''$.

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (3, 9)$		
$B (3, 4)$		
$C (11, 4)$		
$D (11, 10)$		

Coordinate notation describes how the original coordinates of a point are changed into new coordinates after one or more transformations. The coordinate notation is written in the form $(x, y) \rightarrow (\text{new expression in terms of } x, \text{ new expression in terms of } y)$.

The arrow tells you that the point (x, y) is moving to a new location.

You read the coordinate notation $(x, y) \rightarrow (x + 3, y - 2)$ as (x, y) is mapped to $(x + 3, y - 2)$.

Let's consider translations without graphing.

3. Write the rule for the translation of trapezoid $ABCD$ to $A'B'C'D'$ in coordinate notation.

4. Write the rule for the translation of trapezoid $ABCD$ to $A''B''C''D''$ in coordinate notation.

5. The vertices of parallelogram $DEFG$ are $D(-9, 7)$, $E(-12, 2)$, $F(-3, 2)$, and $G(0, 7)$.
- Determine the vertex coordinates of image $D'E'F'G'$ if parallelogram $DEFG$ is translated 14 units down.
 - How did you determine the image coordinates without graphing?
 - Write a rule for how you determine the image coordinates without graphing.
 - Determine the vertex coordinates of image $D''E''F''G''$ if parallelogram $DEFG$ is translated 8 units to the right.
 - How did you determine the image coordinates without graphing?
 - Write a rule for how you determine the image coordinates without graphing.

Translation Functions

Transformations are used frequently in web design and game animation and are often described as functions, which take points, distances, and angles as inputs. The functions output a new set of points after applying a transformation. These transformations move objects around on the screen.

Suppose you are designing a website banner for a new restaurant. The banner will show three congruent triangles animated from left to right, and then the name will fade in.



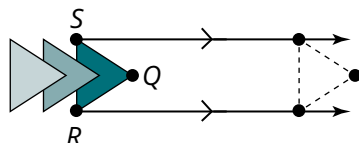
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Think About...

A translation moves a set of points a specified distance in a specified direction along parallel lines.

.....

1. Consider the translation of the first triangle, $\triangle SQR$.



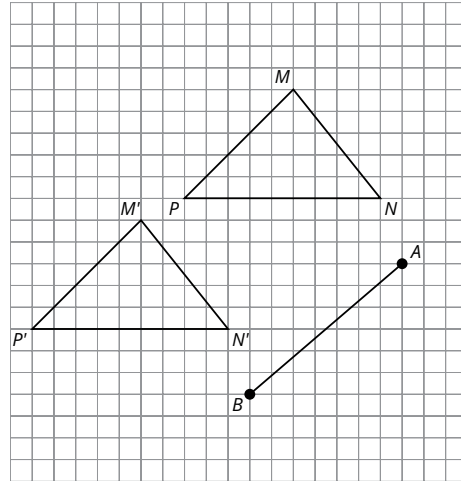
- Label the points of the image, $\triangle S'Q'R'$.
- What relationship is there between $\overline{SS'}$ and $\overline{RR'}$?
- Measure the lengths of the two line segments used in the translation. What do you notice?
- What do you know about the distance QQ' ? What do you know about the line containing $\overline{QQ'}$?

WORKED EXAMPLE

A translation can be measured as a directed line segment.

$\triangle MNP$ was translated to produce $\triangle M'N'P'$. The triangle was translated a distance equal to the distance between points A and B . It was translated in the direction from point A to point B .

So, $\overrightarrow{AB}$ is the directed line segment used to measure this translation.



2. Suppose each grid square is 1 unit $\times$ 1 unit.
 - a. What is the distance from point A to point B ?
 - b. Compare the distance AB with the distances MM' , NN' , and PP' . What do you notice?
 - c. Can you draw another directed line segment on the grid which defines the translation of $\triangle MNP$ to $\triangle M'N'P'$? If so, draw the segment on the grid. Explain your thinking.

Remember ...

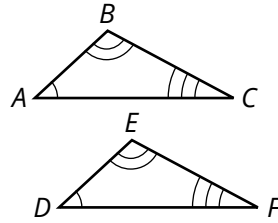
Use the $\cong$ between references to congruent geometric figures; and the equal symbol, $=$, between references to equal lengths, measures, or distances.

Congruent angles are two or more angles of equal measure. When $m\angle ABC = m\angle DEF$, angle ABC is congruent to angle DEF by the definition of congruent angles. This statement can be written using symbols as $\angle ABC \cong \angle DEF$ and is read as angle ABC is congruent to angle DEF .

Arc markers are used to indicate congruent angles in geometric figures. When a diagram has more than one set of congruent angles, sets of arc markers can be used.

WORKED EXAMPLE

Congruent triangles are two triangles that have the same shape and size. In congruent triangles, corresponding sides and angle measures are congruent. To write a triangle congruency statement, you must write the corresponding vertices in the same order. For example, when $\triangle ABC$ is mapped onto $\triangle DEF$ by a rigid motion transformation, the $\triangle ABC \cong \triangle DEF$.



This figure shows that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

- Write equality and congruence statements to compare the corresponding sides and angles of the pre-image $\triangle MNP$ and the image $\triangle M'N'P'$.
- Write a triangle congruency statement for triangles MNP and $M'N'P'$.

A **translation** is a function, which takes as its input a set of pre-image points and outputs a set of image points. The pre-image points are translated a distance of AB in the direction AB . A translation is a rigid motion transformation that preserves size and shape.

- Consider the translation of the website banner from Question 1. Describe the translation of $\triangle SQR$.

.....

Remember ...

A function is a rule that assigns exactly one output to each input. Inputs to functions do not have to be numbers. They can be points, too.

.....

.....

A transformation preserving size and shape is also called an isometry. A translation is an example of an isometry.

.....



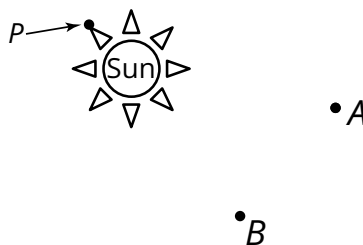
6. Aaliyah says that the exact same function describes every triangle in the animated web banner. Is Aaliyah correct? Explain your thinking, and then draw the translations to justify your answer.

Ask Yourself . . .

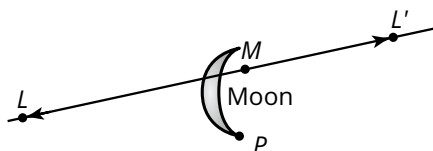
Did you justify your mathematical reasoning?

7. Complete each translation using the given function description.

- a. Every point on the sun moves a distance equal to $\overline{AB}$ in the direction of $\overline{AB}$.



- b. Every point on the moon moves a distance equal to $\overline{L'M}$ in the direction of $\overline{ML}$.



8. Explain why you can use parallel lines when describing translations.

ACTIVITY 1.2

Determining Congruence Using Translations

When users click on the menu of a restaurant, copies of 3 triangles will move from the corners and top of the web page to the center to form the background behind the word “Menu” as shown.



1. Describe and draw the translation functions to show how each triangle will move on the page.

Ask Yourself . . .

What tools or strategies can you use to solve this problem?

2. Are the triangles all congruent? Explain why or why not.

The owner of the restaurant is thinking about using smaller triangles on the sides of the menu web page. She still wants the triangles to move and merge to form the triangle background behind the word “Menu.”



3. Would these transformations be rigid motion transformations? Demonstrate why or why not and explain your process.

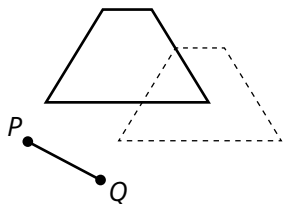


Talk the Talk

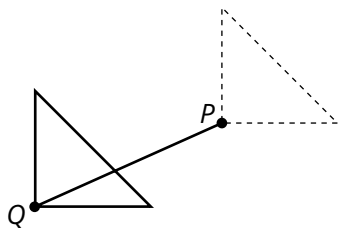
Rigid Motions on the Menu

1. Describe the function that translates each pre-image to its image. Each solid figure is a pre-image, and each dashed figure is an image.

a.



b.



2. What similarities and differences are there between a geometric translation function and an algebraic equation which shows the translation of a function?

Lesson 1 Assignment

Write

Determine whether each statement is true or false. When the statement is false, rewrite it so that it becomes a true statement.

1. A translation function along parallel lines is a type of geometric transformation that is non-rigid.
2. Rigid motion transformations on the coordinate plane are just like rigid motion transformations off the coordinate plane because size and shape are preserved.
3. Translate square $ABCD$ 6 units to the right to form square $A'B'C'D'$. The coordinate notation $(x, y) \rightarrow (x, y + 6)$ represents the translation mapping square $ABCD$ to square $A'B'C'D'$.

Remember

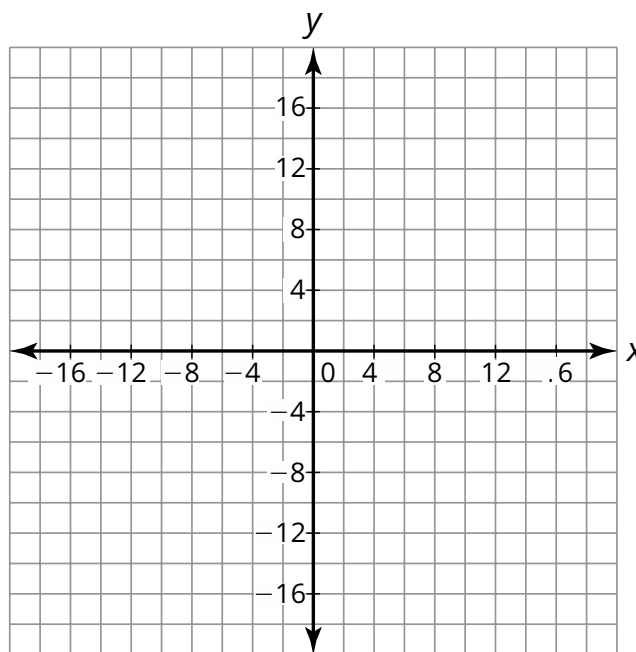
Coordinate notation may be used to represent translations on the coordinate plane.

A translation is a function which takes as its input a set of pre-image points and outputs a set of image points. The pre-image points are translated a distance of AB in the direction AB .

A translation is a rigid motion that preserves size and shape.

Practice

1. Graph rectangle $JKLM$ by plotting the points $J(5, 6)$, $K(15, 6)$, $L(15, 11)$, and $M(5, 11)$.

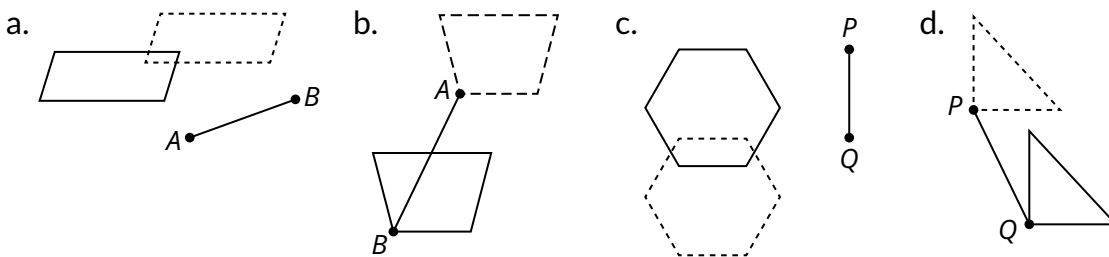


Lesson 1 Assignment

- a. Translate rectangle $JKLM$ 18 units left on the coordinate plane to form rectangle $J'K'L'M'$ and then write the rule for this translation mapping rectangle $JKLM$ to $J'K'L'M'$ in coordinate notation.

- b. Translate rectangle $JKLM$ 4 units down on the coordinate plane to form rectangle $J''K''L''M''$ and then write the rule for this translation mapping rectangle $JKLM$ to $J''K''L''M''$ in coordinate notation.

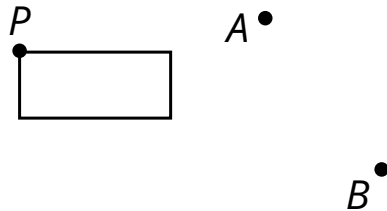
2. Describe the translation function for each translation. Each solid figure is a pre-image, and each dashed figure is an image.



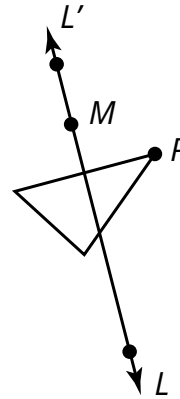
Lesson 1 Assignment

3. Complete each translation given the function description.

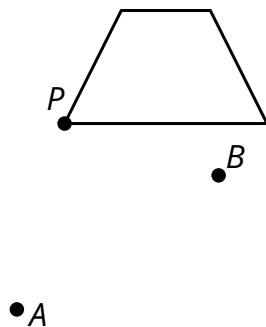
- a. Every point on the rectangle moves a distance equal to AB in the direction of $\overrightarrow{AB}$.



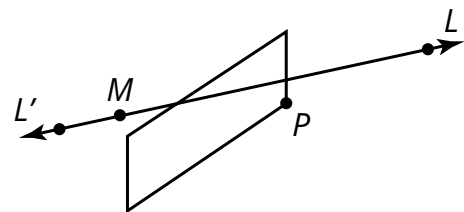
- b. Every point on the triangle moves a distance equal to $L'M$ in the direction of $\overrightarrow{L'M}$.



- c. Every point on the trapezoid moves a distance equal to BA in the direction of $\overrightarrow{BA}$.



- d. Every point on the parallelogram moves a distance equal to $L'M$ in the direction of $\overrightarrow{L'M}$.

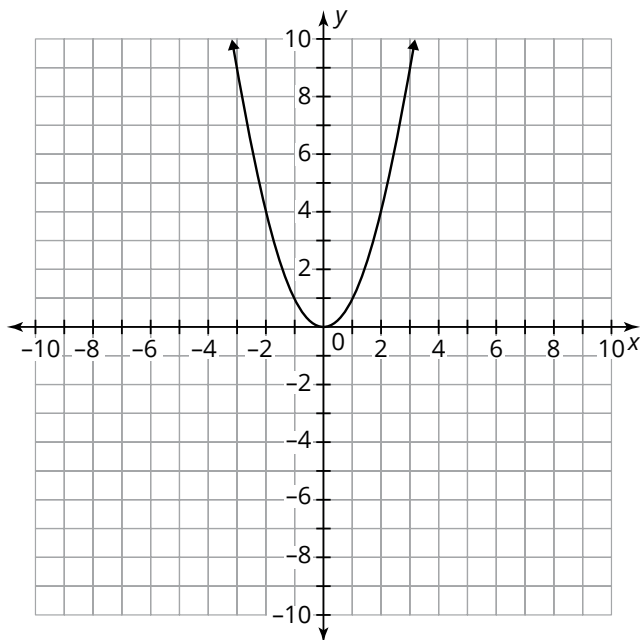


Lesson 1 Assignment

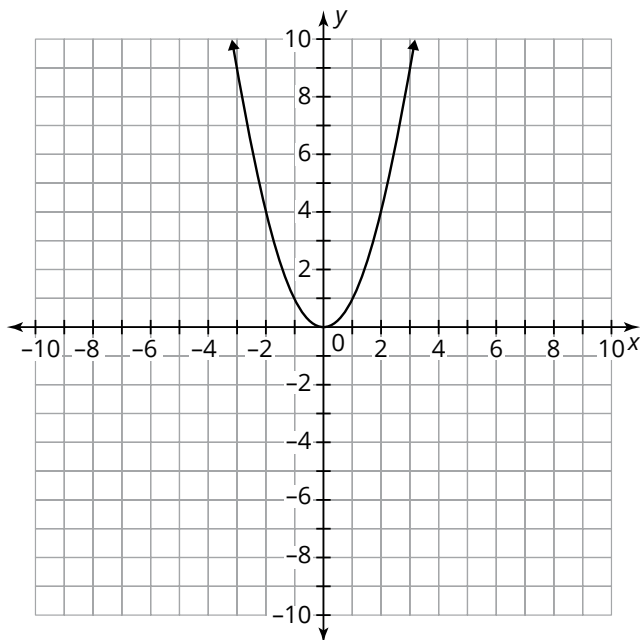
Prepare

The coordinate plane shows the graph of $f(x)$. Sketch the graph of $g(x)$.

1. $g(x) = f(x) - 2$



2. $g(x) = f(x - 3)$



2

Sequences of Multiple Translations

OBJECTIVES

- Represent the translation of a figure along a directed line segment with a sequence of translations involving a horizontal translation and a vertical translation.
- Use the Pythagorean Theorem to relate the distance of a directed line segment used in a translation to the horizontal and vertical translations used to map the same translation as the directed line segment.
- Verbally describe a sequence of translations on the coordinate plane using coordinate notation.
- Determine congruence statements between the pre-image and image formed by a sequence of translations.
- Compare a sequence of translations on the coordinate plane to transformations off the coordinate plane.

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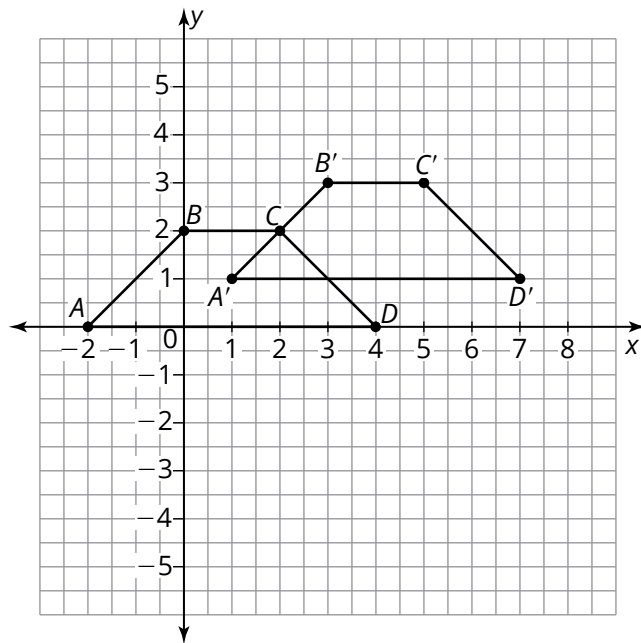
You have described single translations.

How can you describe a translation by a directed line segment on and off the coordinate plane using a sequence of transformations?

Getting Started

Multiple Translations on the Coordinate Plane

You have described translations on and off the coordinate plane and have described translations along a directed line segment. You can use a sequence of horizontal and vertical translations to describe the translation of a figure along a directed line segment.

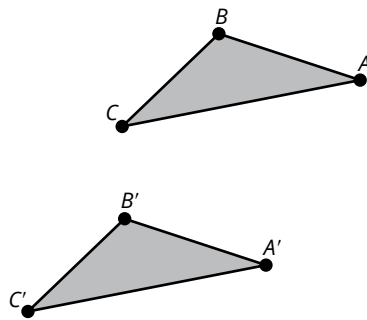


1. Describe a single translation along a directed line segment that will map trapezoid $ABCD$ to trapezoid $A'B'C'D'$.
2. Use the distance formula to determine the lengths of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and $\overline{DD'}$.
3. Describe a sequence of a horizontal and a vertical translation that will map trapezoid $ABCD$ to trapezoid $A'B'C'D'$.

4. Use the Pythagorean Theorem to determine the lengths of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and $\overline{DD'}$.

5. How is the length of the directed line segment using the Pythagorean Theorem related to the sequence of a vertical and horizontal translation that maps trapezoid $ABCD$ to trapezoid $A'B'C'D'$.

6. Does it matter whether you translate horizontally or vertically first?



7. Describe a sequence of translations that maps $\triangle ABC$ to $\triangle A'B'C'D'$.

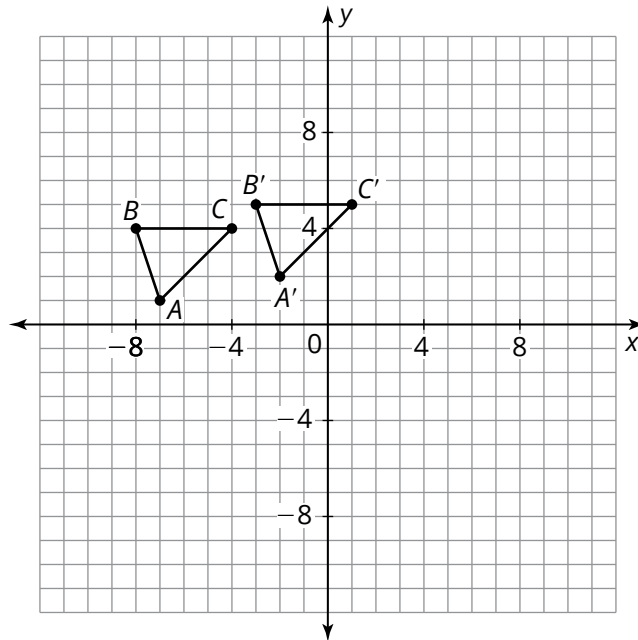
You can also describe a transformation off the coordinate plane using a sequence of transformations.

ACTIVITY 2.1

Sequences of Translations Using Coordinate Notation

When a sequence of rigid transformations is performed, it can also be called a *composition of rigid transformations*.

You have used coordinate notation to describe a translation of a figure on the coordinate plane. You can also use coordinate notation to describe a sequence of translations on the coordinate plane.



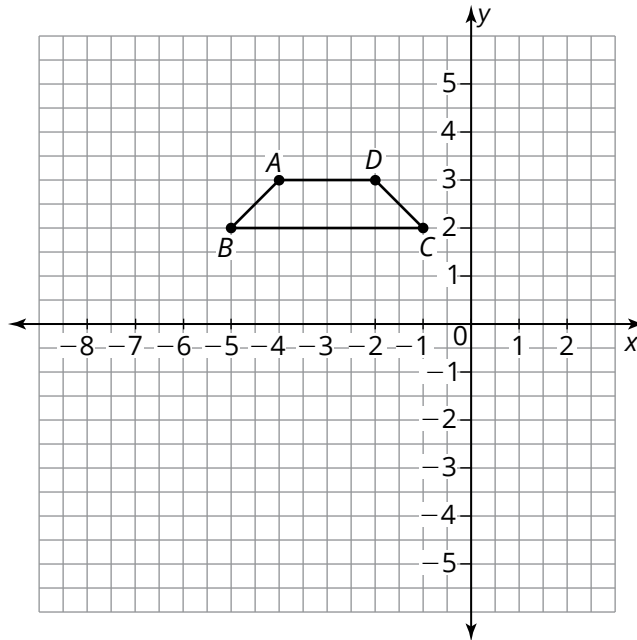
1. Record the vertex coordinates in the table.

Coordinates of $\triangle ABC$	Coordinates of $\triangle A'B'C'$

2. Describe a sequence of translations that maps $\triangle ABC$ to $\triangle A'B'C'$.
3. Write the rule for the sequence of translations that maps $\triangle ABC$ to $\triangle A'B'C'$ using coordinate notation.

4. Explain why the two triangles are congruent? Then, write a triangle congruency statement for the two triangles.

Consider the trapezoid on the coordinate plane.



5. Translate the trapezoid to the left 2 units, then up 1 unit. Draw the image trapezoid $A'B'C'D'$ on the coordinate plane.
6. Record the vertex coordinates in the table.

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$

7. Write the rule for the sequence of translations that maps trapezoid $ABCD$ to trapezoid $A'B'C'D'$ using coordinate notation.

You can also determine the pre-image or image coordinates of a figure undergoing a sequence of translations without graphing the translations on the coordinate plane.

8. Triangle HON is translated 18 units to the left and 10 units up. The coordinates of the pre-image $\triangle HON$ are $H(-2, 4)$, $O(1, 5)$, and $N(-1, 7)$. Determine the coordinates of the image $\triangle H'O'N'$.
9. How did you determine the coordinates of the image without performing the translations on the graph.
10. Write a rule for the sequence of translations that maps $\triangle HON$ to its image $\triangle H'O'N'$.

Since translations are rigid motion transformations, a sequence of translations preserves the size and shape of the pre-image.

11. Write equality and congruence statements to compare the corresponding sides and angles of the pre-image $\triangle HON$ to its image $\triangle H'O'N'$.
12. Write a triangle congruency statement for the two triangles.
13. Triangle DEF is translated 8 units to the right and 3 units down. The coordinates of the image $\triangle D'E'F'$ are $D'(3, -2)$, $E'(2, 6)$, $F(7, -1)$. Determine the coordinates of the pre-image.
14. How did you determine the coordinates of the pre-image without performing the translations on the graph.

15. How is determining the coordinates of the pre-image different from determining the coordinates of the image?
16. Write a rule for the sequence of translations that maps $\triangle DEF$ to $\triangle D'E'F'$.
17. Write congruence statements to compare the corresponding sides and angles of the pre-image $\triangle DEF$ and the image $\triangle D'E'F'$.



18. Emma

There are multiple different ways to write a triangle congruency statement for the two triangles.

Victoria

There is only one way to write a triangle congruency statement for the two triangles since the vertices must be in order.

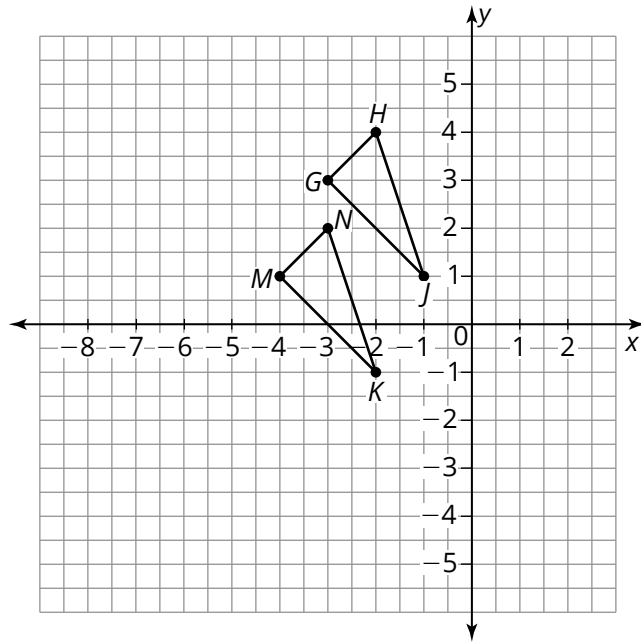
Who's correct? Explain your reasoning.



Talk the Talk

Off the Grid

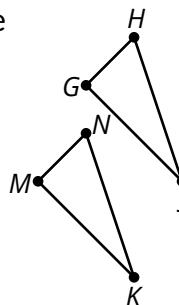
You performed sequences of translations on the coordinate plane. Let's compare those transformations with the same ones performed off the coordinate plane.



1. Describe the sequence of translations that maps the pre-image, $\triangle GHJ$ to the image, $\triangle MNK$.

2. Write the rule that describes the sequence of translations that maps $\triangle GHJ$ to $\triangle MNK$ using coordinate notation.

3. Describe the sequence of translations that maps the pre-image, $\triangle GHJ$ to the image, $\triangle MNK$



4. Describe the mapping of $\triangle GHJ$ to $\triangle MNK$ using a directed line segment.

The congruency statements are the same whether on or off the grid.

5. Write congruency statements to compare the corresponding congruent sides and figures of the pre-image and the image.

Lesson 2 Assignment

Write

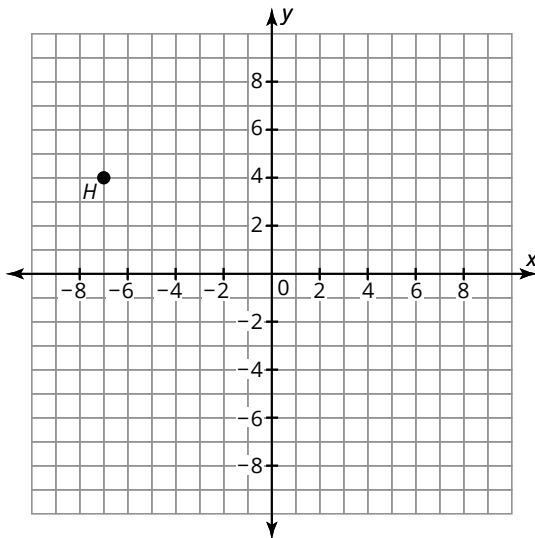
How can you find the length of the directed line segment on the coordinate plane that can be described as a sequence of translations involving a horizontal translation and then a vertical translation?

Remember

A horizontal and vertical translation can describe a single translation given by a directed line segment on and off the coordinate plane. The pre-image and image formed by a translation or a sequence of translations are congruent because shape and size are the same.

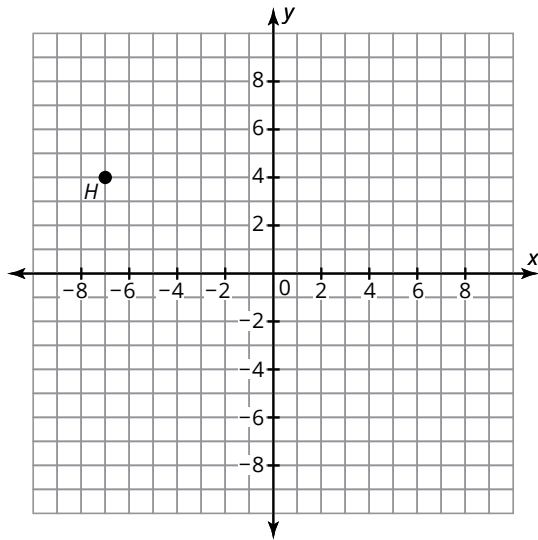
Practice

1. Graph parallelogram $ABCD$ by plotting points $A(2, -2)$, $B(7, -2)$, $C(9, -5)$, and $D(4, -5)$.



Lesson 2 Assignment

- a. Translate parallelogram $ABCD$ along the directed line segment DH to form parallelogram $EFGH$.



- b. Describe a sequence of translations that will map parallelogram $ABCD$ to parallelogram $EFGH$.
- c. What is the length to the nearest tenth of a directed line segment that maps parallelogram $ABCD$ to parallelogram $EFGH$.
- d. Write the rule for the sequence of translations that maps parallelogram $ABCD$ to parallelogram $EFGH$ using coordinate notation.

Lesson 2 Assignment

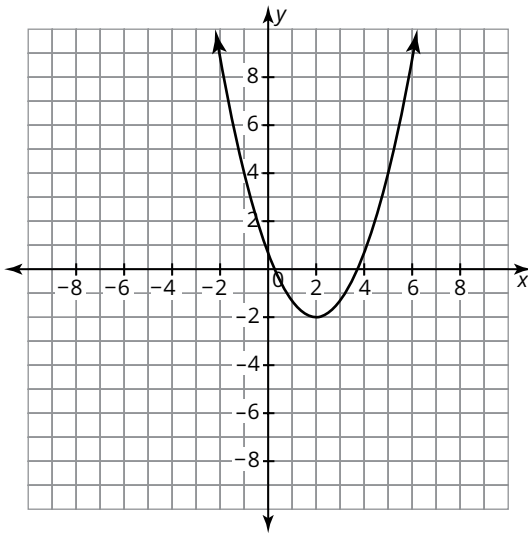
- e. Write 8 congruence statements between the corresponding sides and angles of the pre-image parallelogram $ABCD$ to its image parallelogram $EFGH$.
- f. Write a parallelogram congruency statement for the two parallelograms.

Lesson 2 Assignment

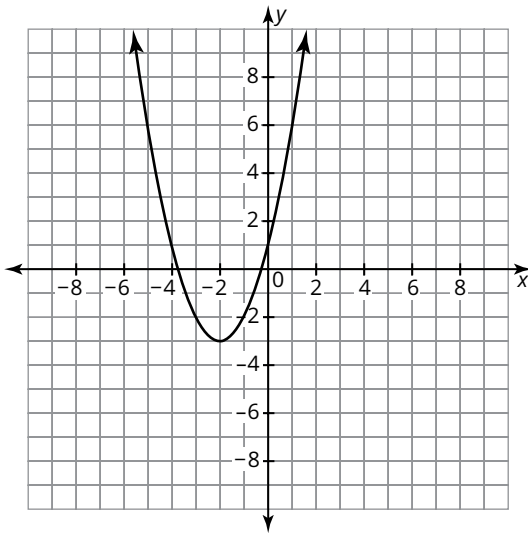
Prepare

Each coordinate plane shows the graph of $f(x)$. Sketch the graph of $g(x)$.

1. $g(x) = -f(x)$



2. $g(x) = f(-x)$



3

Reflections On and Off the Coordinate Plane

OBJECTIVES

- Reflect geometric figures on a coordinate plane.
- Represent reflections in the plane using patty paper and using a compass and straight edge.
- Verbally describe reflection transformations as functions that take points as inputs and output reflected points.
- Use construction tools to bisect an angle.
- Identify reflections as points equidistant to the perpendicular bisector of line segments connecting the pre-image and image points of the reflection.
- Specify a sequence of translations and reflections that will carry a figure onto a congruent figure.

KEY TERMS

- reflection
- angle bisector
- perpendicular bisector theorem

You have learned how to represent the horizontal or vertical translation of a function.

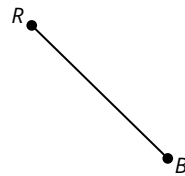
How can you describe geometric reflections of figures on the plane?

Getting Started

Reflecting on Bisecting

In previous lessons, you constructed the perpendicular bisector of line segments.

1. Construct a perpendicular bisector of $\overline{RB}$.



2. Use patty paper to trace the diagram you constructed. How can you use patty paper to map point R onto point B ? Explain the transformation you used.
3. Can you draw two points in the plane that cannot be mapped onto each other by the same transformation you used in Question 2? Justify your answer.

ACTIVITY 3.1

Reflecting Geometric Figures on the Coordinate Plane

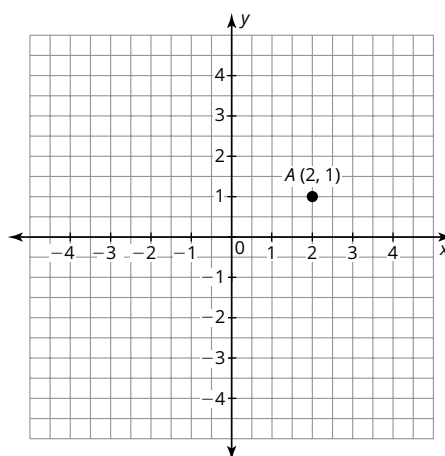
Recall that figures that are mirror images of each other are called reflections. A **reflection** is a rigid motion that reflects, or “flips,” a figure over a given line called a line of reflection. A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line.

Let’s reflect point A over the y-axis.

WORKED EXAMPLE

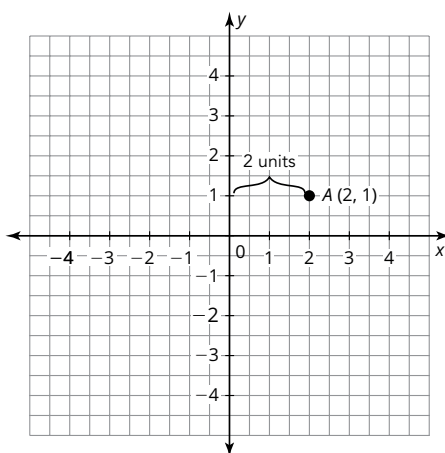
Step 1: Plot a point anywhere in the first quadrant, but *not* at the origin.

Point A is plotted at (2, 1)



Step 2: Next, count the number of x-units from point A to the y-axis.

Point A is 2 units from the y-axis.

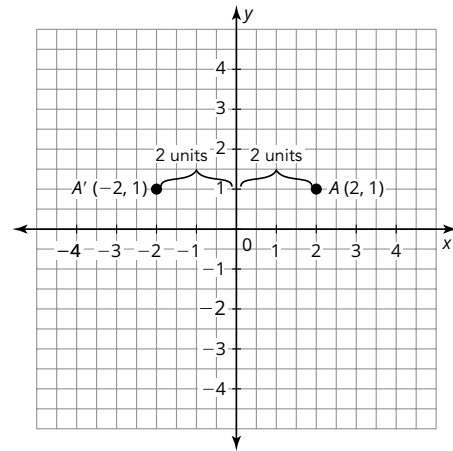


Note: The same cut out provided in Lesson 1 is provided again in this lesson. Students may use the trapezoid from the previous lesson or the cutout provided.

WORKED EXAMPLE

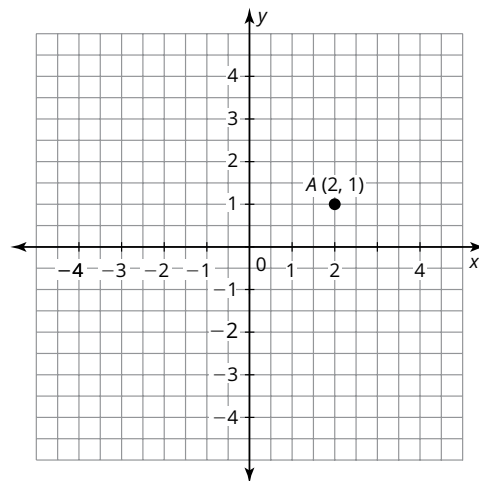
Step 3: Then, count the same number of x -units on the opposite side of the y -axis to locate the reflection of point A . Label the point A' .

Step 4: Point A' is located at $(-2, 1)$. Point A has been reflected across the y -axis.



1. What do you notice about the coordinates of point A and the coordinates of image A' ?
2. Predict the coordinates of A'' if point A is reflected over the x -axis. Explain your reasoning.

3. Reflect point A across the x-axis on the coordinate plane shown. Verify whether your prediction for the location of the image was correct. Graph the image and label it A''.



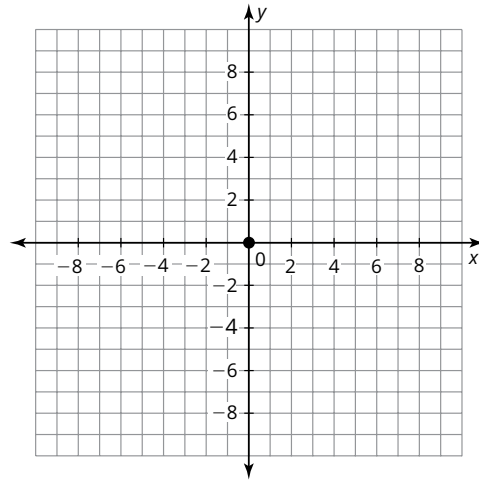
4. What do you notice about the coordinates of A and A''?

You can use coordinate notation to describe the reflection of a point across either the x-axis or the y-axis.

5. a. Write the rule for the reflection of the point (x, y) across the x-axis in coordinate notation.
- b. Write the rule for the reflection of the point (x, y) across the y-axis in coordinate notation.

.....
Think About ...
Does this make sense when the line of reflection is not the x- or y-axis?
.....

6. Graph point J at $(5, 7)$ on the coordinate plane shown.



7. Reflect point J across the y -axis on the coordinate plane. Label the image J' . Determine the coordinates of J' . Then, describe how you determined the location of image J' .

8. Reflect point J across the x -axis on the coordinate plane. Label the image J'' . Determine the coordinates of J'' . Then, describe how you determined the location of image J'' .

You can also reflect polygons on the coordinate plane. You can model the reflection of a polygon across a line of reflection. Just as with rotating a polygon on a coordinate plane, you will first need to establish a pre-image.

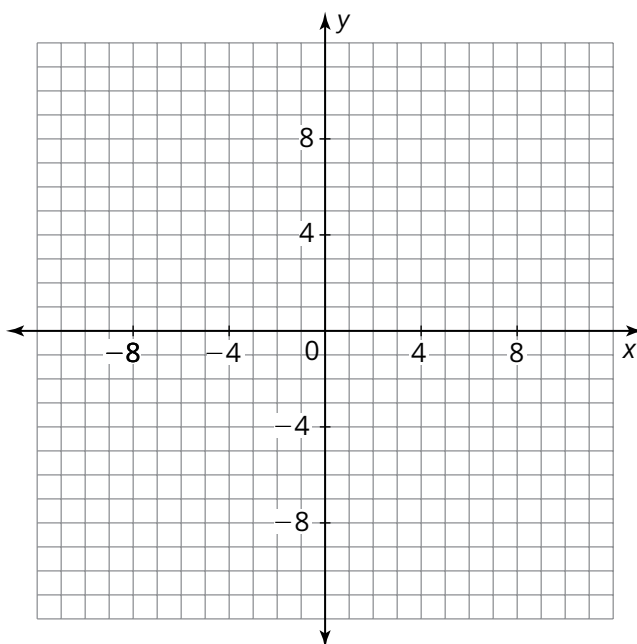
Cut out a copy of the figure shown.



9. Graph trapezoid $ABCD$ by plotting the points $A (3, 9)$, $B (3, 4)$, $C (11, 4)$, and $D (11, 10)$.

Now that you have graphed the pre-image, you are ready to model the reflection of the polygon on the coordinate plane. For this modeling, you will reflect the polygon across the y -axis.

- First, take your trapezoid that you cut out previously and set it on top of trapezoid $ABCD$ on the coordinate plane.
- Next, determine the number of units point A is from the y -axis.
- Then, count the same number of units on the opposite side of the y -axis to determine where to place the image in Quadrant II.
- Finally, physically flip the trapezoid across the y -axis like you are flipping a page in a book.



10. Reflect trapezoid $ABCD$ across each given line of reflection. Graph and label each image on the coordinate plane and record each image's coordinates in the table.

a. Reflect trapezoid $ABCD$ over the y -axis to form trapezoid $A'B'C'D'$.

b. Reflect trapezoid $ABCD$ over the x -axis to form trapezoid $A''B''C''D''$.

Coordinates of Trapezoid $(ABCD)$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (3, 9)$		
$B (3, 4)$		
$C (11, 4)$		
$D (11, 10)$		

c. Write the rule for the reflection mapping trapezoid $ABCD$ to $A'B'C'D'$ in coordinate notation.

d. Write the rule for the reflection mapping trapezoid $ABCD$ to $A''B''C''D''$ in coordinate notation.

11. What similarities do you notice between reflecting a single point across the x - or y -axis and reflecting a polygon across the x - or y -axis?

Let's consider reflections without graphing.

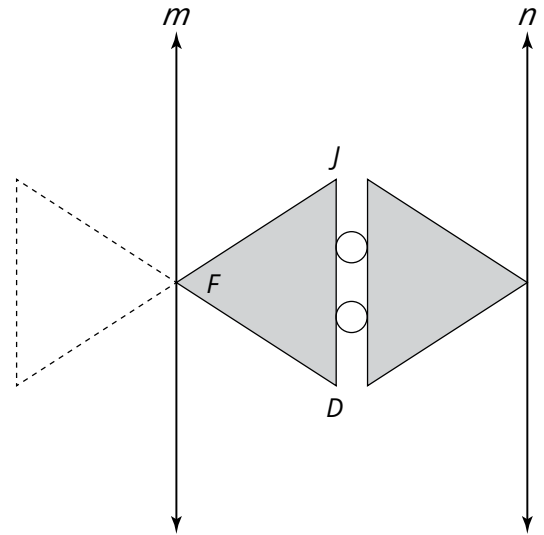
12. The vertices of parallelogram $DEFG$ are $D(-9, 7)$, $E(-12, 2)$, $F(-3, 2)$, and $G(0, 7)$.
- Determine the vertex coordinates of image $D'E'F'G'$ if parallelogram $DEFG$ is reflected across the x -axis.
 - How did you determine the image coordinates without graphing?
 - Write a rule for how you determine the image coordinates without graphing.
 - Determine the vertex coordinates of image $D''E''F''G''$ if parallelogram $DEFG$ is reflected across the y -axis.
 - How did you determine the image coordinates without graphing?
 - Write a rule for how you determine the image coordinates without graphing.

Reflection Functions

You saw in the previous lesson that translations can be used in web design and game animation, are often described as functions, which take points, distances, and angles as inputs and outputs a new set of points after applying a transformation. The same is true for reflections.

To reveal a message during a game, you have to make two triangles fly open.

1. Consider the reflection of the left triangle, $\triangle FJD$, across line m .
 - a. Label the points of the image, $\triangle F'J'D'$.
 - b. What relationship is there among $\overline{JJ'}$ and $\overline{DD'}$? Justify your answer.
 - c. What relationship is there among $\overline{JJ'}$, $\overline{DD'}$, and line m ? Justify your answer.
 - d. Reflect the other triangle across line n . Label the points of the pre-image and image. Compare this reflection with the reflection of the first triangle.



2. Compare the pre-image and the image. Then, compare the corresponding sides of the pre-image and the image.

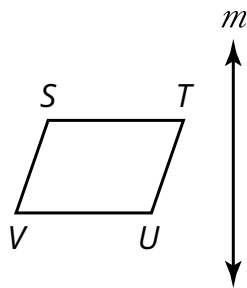
A reflection is a function which takes as its input the location of a point with respect to some line of reflection ℓ and outputs the opposite of the location of the point with respect to the line of reflection.

3. Consider the reflections from Question 1.
Describe the reflection of $\triangle FJD$.

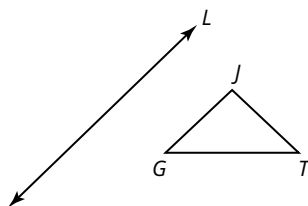
4. Describe the relationship between corresponding points of a reflection and the line of reflection.

5. Complete each reflection using the description of the reflection function.

- a. Reflect parallelogram $STUV$ across line m .

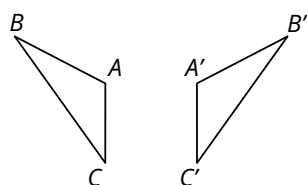


b. Reflect $\triangle JTG$ across line l

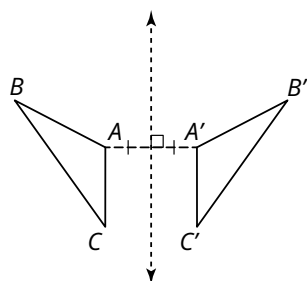


It is possible to construct the line of reflection when given two figures that are reflections of one another.

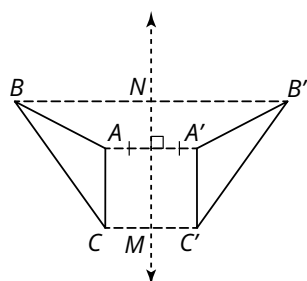
WORKED EXAMPLE



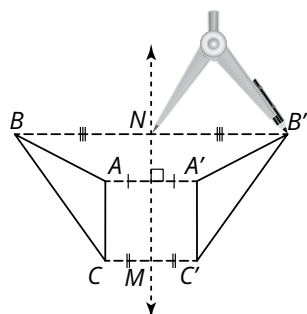
1. Label the vertices.



2. Connect two corresponding points. Construct the perpendicular bisector of the line segment connecting them.



3. Connect remaining corresponding points. Label the intersection of each line segment with the perpendicular bisector.



4. Use a compass to determine whether each intersection point is the midpoint of the line segment connecting corresponding vertices. If that is the case, the perpendicular bisector is the line of reflection. If not, the figures are not reflections of one another.

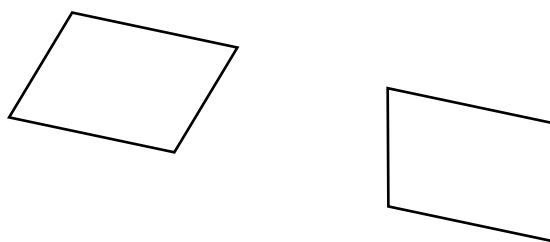
$$\overline{CM} \cong \overline{C'M}$$

$$\overline{BN} \cong \overline{B'N}$$

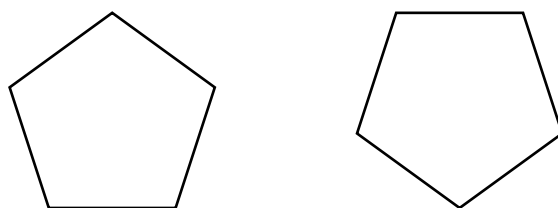
$\overleftrightarrow{MN}$ is the line of reflection.

6. Using constructions, determine whether the figures are reflections of one another. If so, identify the line of reflection.

a.



b.



c.



Ask Yourself . . .

What patterns do you notice?

7. How do you know whether two figures are not reflections of one another?

ACTIVITY
3.3

Bisecting an Angle

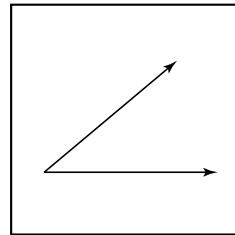
Just as line segments can be bisected, angles can be bisected too. If a ray is drawn through the vertex of an angle and divides the angle into two angles of equal measure, or two congruent angles, this ray is called an **angle bisector**. The construction used to create an angle bisector is called bisecting an angle.

.....
Lines and line segments can also bisect an angle.
.....

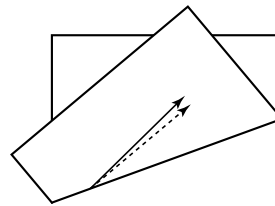
One way to bisect an angle is using patty paper.

WORKED EXAMPLE

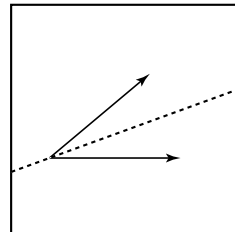
Draw an angle on the patty paper.



Fold the patty paper so that the rays lie on top of each other. The fold should pass through the vertex of the angle.



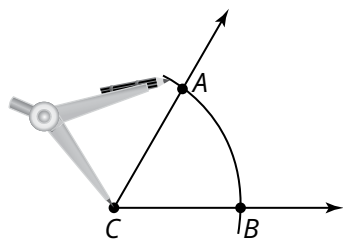
Open the patty paper. The crease represents the angle bisector.



1. Harper states that as long as the crease goes through the vertex, it is an angle bisector. Is she correct? Why or why not?

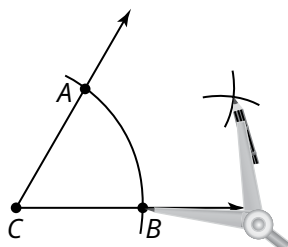
You can also bisect an angle using a compass and a straightedge.

WORKED EXAMPLE



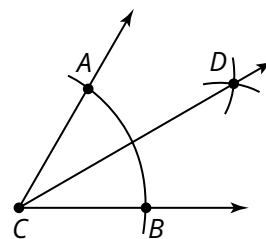
Construct an Arc

Place the compass at C . Construct an arc that intersects both sides of the angle. Label the intersections A and B .



Construct Another Arc

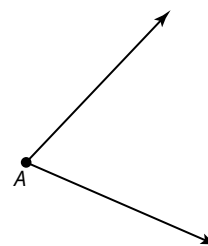
Place the compass at A . Construct an arc. Then, place the compass point at B . Using the same radius, construct another arc.



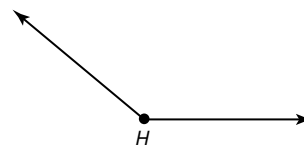
Construct a Ray

Label the intersection of the two arcs D . Use a straightedge to construct a ray through C and D . Ray CD bisects $\angle C$.

2. Construct the bisector of $\angle A$. Use patty paper to verify your construction.



3. Construct an angle that is one-fourth the measure of $\angle H$. Explain how you performed the construction.



4. Describe how to construct an angle that is one-eighth the measure of $\angle H$ from Question 3.

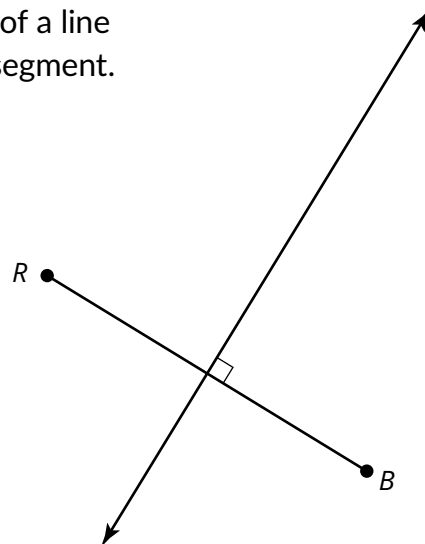
ACTIVITY
3.4

Perpendicular Bisector Theorem

In the Getting Started, you saw that the perpendicular bisector of a line segment is the line of reflection between the endpoints of the segment.

Consider $\overline{RB}$ with perpendicular bisector ℓ .

1. Label point P anywhere on ℓ . What do you notice about the relationship between point P and points R and B ?



You can prove that the endpoints of a line segment are equidistant from any point on the perpendicular bisector.

WORKED EXAMPLE

Given: Line ℓ is the perpendicular bisector of $\overline{RB}$.

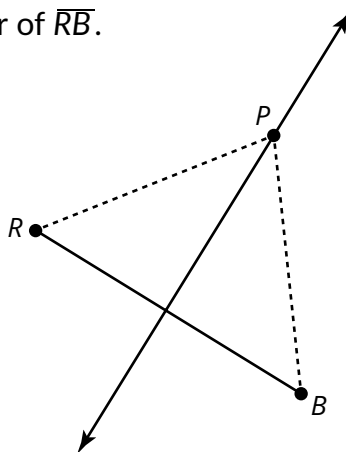
Point P is on ℓ .

Prove: $PR = PB$

The reflection of point P across line ℓ is point P by the definition of reflection.

Because line ℓ is the perpendicular bisector of $\overline{RB}$, the reflection of point R across line ℓ is point B .

Therefore, $PR = PB$.



2. Provide a reason why $PR = PB$.

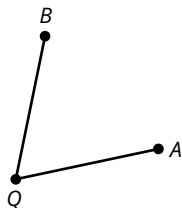
The **perpendicular bisector theorem** states: “Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment.” Remember that a theorem is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.

Let’s consider the relationship between endpoints of a line segment and any point on the perpendicular bisector from another perspective.

Suppose you have two points that are equidistant from a third point. You can show that the third point lies on the perpendicular bisector of the segment connecting the points.

3. Consider point Q , which is equidistant from points A and B .

- a. Draw $\overrightarrow{QP}$ so that it bisects $\angle BQA$. This makes $\angle BQP$ and $\angle PQA$ congruent angles. Label the intersection of $\overrightarrow{QP}$ and $\overline{AB}$ as point C .



- b. Describe the location of $\overline{QB'}$ if $\overline{QB}$ is reflected across $\overrightarrow{QP}$. What does this tell you about the distances BC and CA ? Explain your thinking.
- c. Explain how you know that $\overrightarrow{QP}$ is a perpendicular bisector of $\overline{AB}$.

ACTIVITY
3.5

Sequences of Rigid Motions Using Translations and Reflections

You have learned that reflections and translations are rigid motion transformations, which means that they preserve distances and angle measures.

1. Sofia made a conjecture about reflections. She said that you can always map one congruent line segment onto the other using at most two reflections.

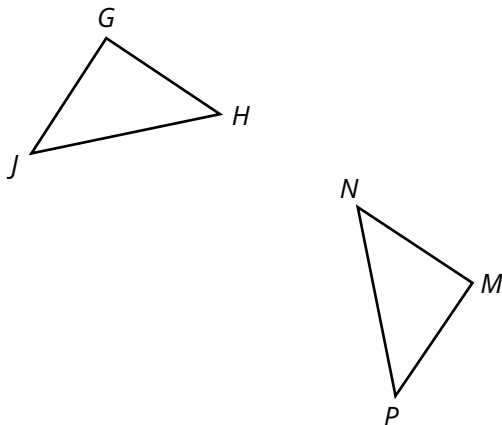
Do you think Sofia is correct? Draw examples or counterexamples to justify your reasoning.



Remember ...

Any point that is equidistant from two other points lies on the perpendicular bisector between the two points.

2. Triangle GHJ is a reflection of $\triangle MNP$. Describe the locations of the midpoints of $\overline{GM}$, $\overline{HN}$, and $\overline{JP}$.



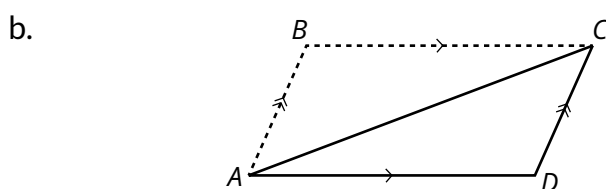
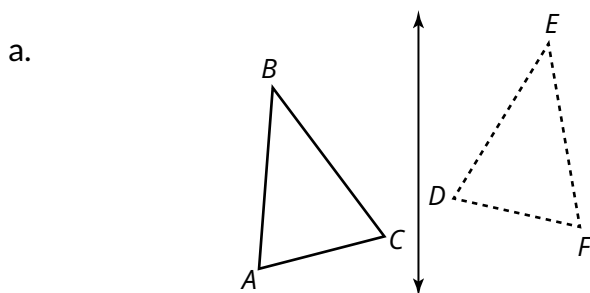
.....

Remember ...

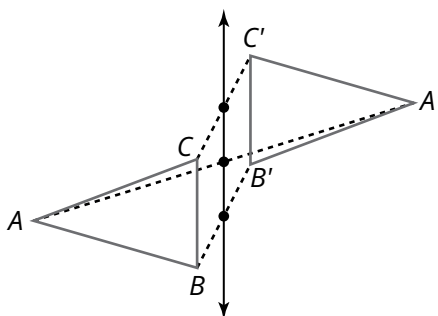
Sides that have the same number of arrowhead markings are parallel to one another.

.....

3. Describe and sketch the sequence of translations and reflections that shows that the two figures in each pair are congruent. Images are shown with dashed lines.



4. Ethan was investigating a transformation of $\triangle ABC$ to $\triangle A'B'C'$ and discovered that the midpoints of the segments connecting corresponding points were collinear, but the line was not a perpendicular bisector of each segment. He thought that this must not be a reflection.



Isabella disagreed. She said that you can translate first to show that the midpoints lie on a perpendicular bisector.
Who is correct? Explain your reasoning.

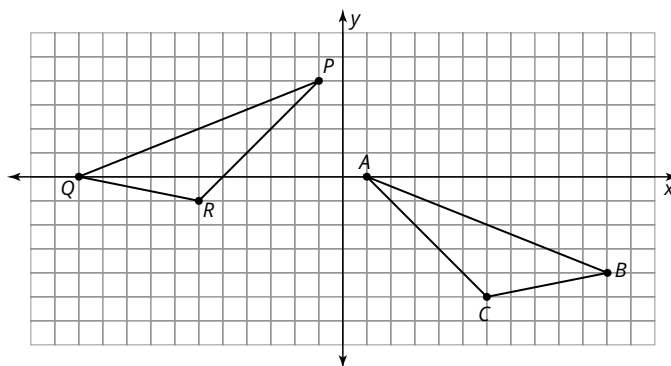


Talk the Talk

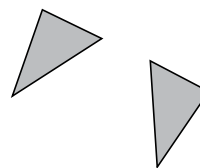
But We're Off the Grid

In this lesson, you translated and reflected figures in the plane. Let's compare those transformations with the same ones performed on a coordinate plane.

1. Describe the sequence of translations and reflections which maps the pre-image triangle, $\triangle PQR$, to the image triangle, $\triangle ABC$ on the coordinate plane.



2. How are translating and reflecting geometric figures off the coordinate plane different from performing these transformations on the coordinate plane? How are they the same?
3. Determine whether the figures are reflections of one another. If so, identify the line of reflection.



Lesson 3 Assignment

Write

Explain why a reflection of a figure is a rigid motion transformation.

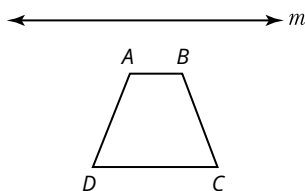
Remember

- A reflection is a function, R , which takes as its input, P , the location of a point with respect to some line of reflection, ℓ , and outputs the opposite of the location of P with respect to the line of reflection.
- An angle bisector is a line, line segment, or ray that is drawn through the vertex of an angle and divides the angle into two congruent angles.
- Any point that is equidistant from two other points lies on the perpendicular bisector between the two points.

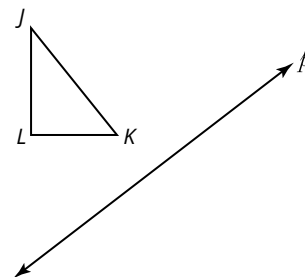
Practice

1. Complete each reflection, given the description.

a. Reflect the image $ABCD$ across line m .

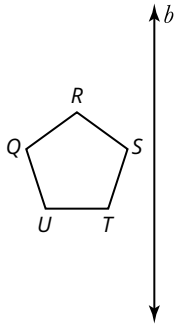


b. Reflect the image JKL across line p .

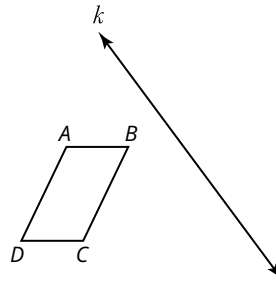


Lesson 3 Assignment

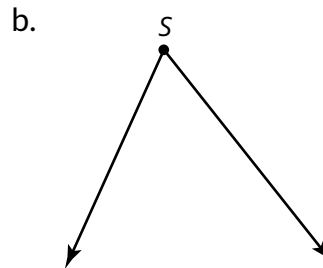
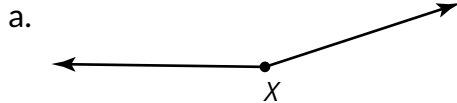
c. Reflect the image $QRSTU$ across line b .



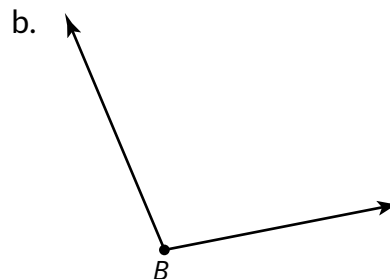
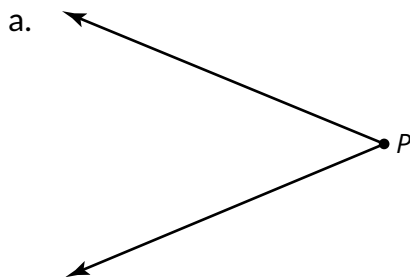
d. Reflect the image $ABCD$ across line k .



2. Duplicate each angle using construction tools.

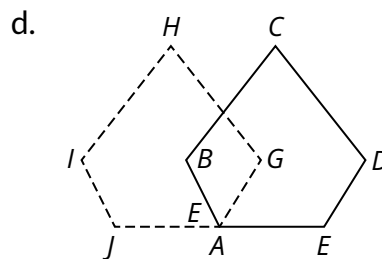
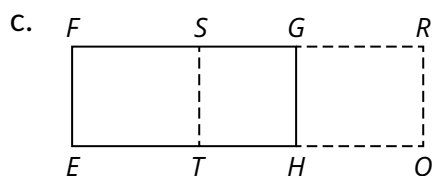
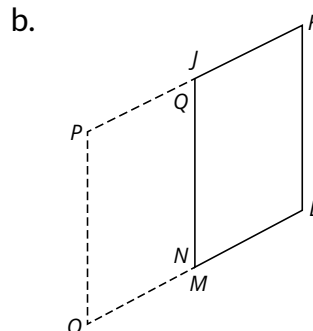
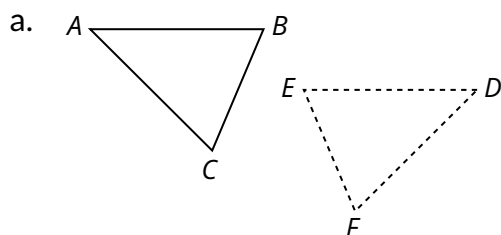


3. Construct the angle bisector of each given angle.



Lesson 3 Assignment

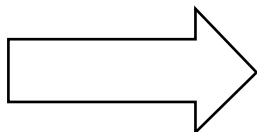
4. Describe the sequence of translations and reflections that show that the two figures in each pair are congruent. Images are shown with dashed lines.



Lesson 3 Assignment

Prepare

1. Redraw the figure shown as described.



- a. So that is turned 180° .
- b. So that is turned 90° counterclockwise.
- c. So that is turned 90° clockwise.

4

Rotations On and Off the Coordinate Plane

OBJECTIVES

- Rotate geometric figures on the coordinate plane.
- Represent rotations in the plane.
- Write descriptions of rotation transformations as functions that take points as inputs and output rotated points.
- Specify a sequence of translations, reflections, and rotations that will carry a figure onto a congruent figure.

KEY TERMS

- central angle
- concentric circles
- angle of rotation
- rotation

.....

You have studied how rigid motion transformations, such as rotations, reflections, and translations, preserve the size and shape of a figure.

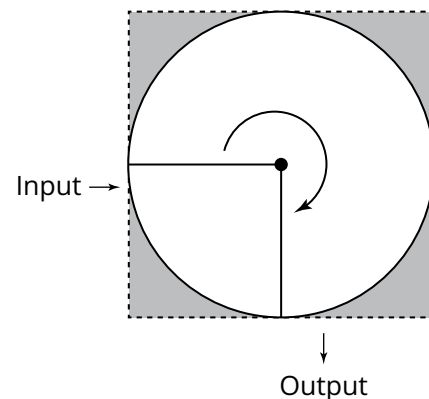
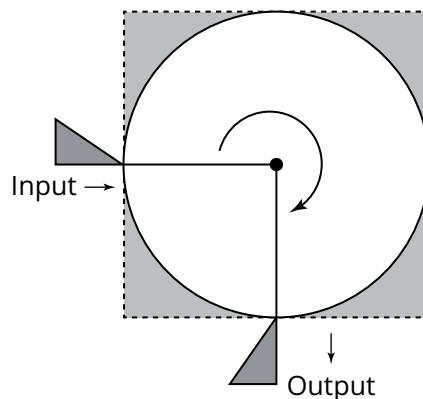
Given that $\triangle ABC$ has undergone a rigid motion transformation where A is mapped to A' , B is mapped to B' , and C is mapped to C' , write a congruency statement showing the original triangle and its image are congruent. Explain your reasoning.

Getting Started

Transformation Machine

You have learned that functions take a value as input and output another value. In this topic, you have described geometric rigid motions as functions. You can represent a geometric rigid motion with a transformation machine.

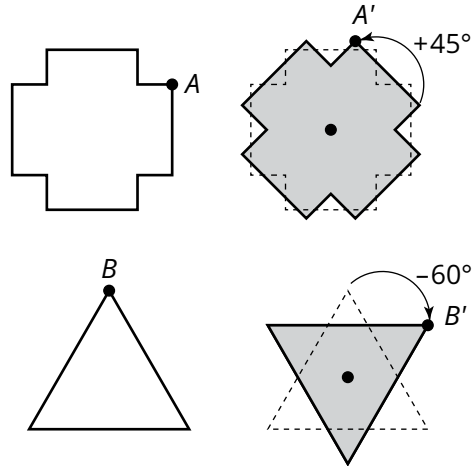
1. The machine on the left shows a rigid motion. For the transformation machine on the right, draw an input shape on patty paper. Then apply the transformation and draw the output shape.



2. Identify the rigid motion represented by the transformation machine.

3. Describe the transformation function. Explain how the input shape is “carried” by geometric objects in the transformation machine to result in the output shape.

An **angle of rotation** is a directed angle based on a circle. Positive angles of rotation turn counterclockwise, and negative angles of rotation turn clockwise.



.....
The measure of the angle itself is not negative, but the negative indicates the direction of rotation.
.....

4. Identify the angle of rotation that was used in the transformation machine.

ACTIVITY
4.1**Rotating Geometric Figures on the Coordinate Plane**

Recall that a rotation is a rigid motion that turns a figure about a fixed point, called the point of rotation. The angle of rotation is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise.

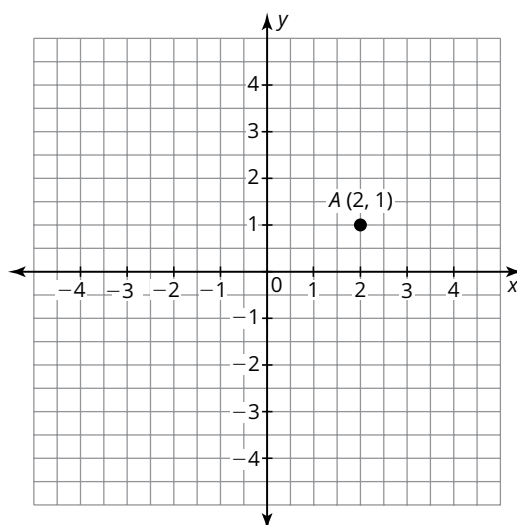
Let's rotate point A about the origin. The origin will be the point of rotation, and you will rotate point A 90° , 180° , and 270° .

First, let's rotate point A 90° counterclockwise about the origin.

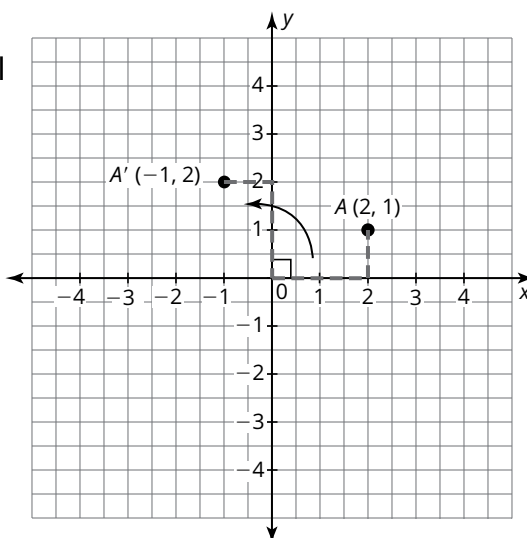
WORKED EXAMPLE

Step 1: Plot a point anywhere in the first quadrant, but *not* at the origin.

Point A is plotted at (2, 1).



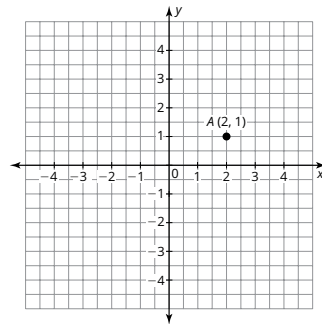
Step 2: Next, draw a “hook” from the origin to point A, using the coordinates and horizontal and vertical line segments as shown.



Step 3: Rotate the “hook” 90° counterclockwise as shown.

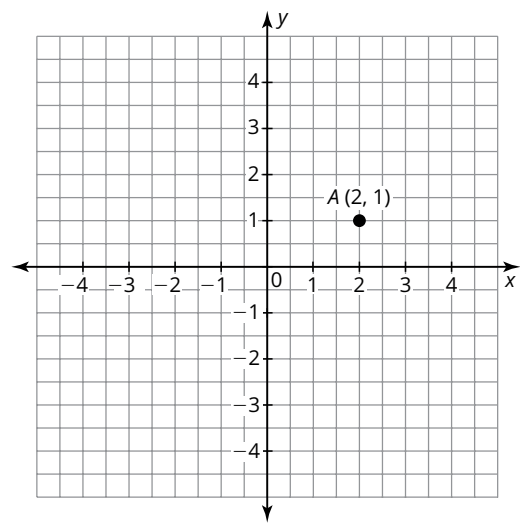
Point A' is located at (-1, 2). Point A has been rotated 90° counterclockwise about the origin.

1. What do you notice about the coordinates of point A and the coordinates of point A'?
2. Predict what the coordinates of point A'' will be if you rotate point A' 90° counterclockwise about the origin.
3. Graph and label point A'. Rotate point A' about the origin 90° counterclockwise on the coordinate plane shown. Label the point A''.



- a. What are the coordinates of point A''? Was your prediction for the coordinates of point A'' correct?
- b. What do you notice about the coordinates of points A and A''? How are the two points related?

4. Rotate point A'' about the origin 90° counterclockwise on the coordinate plane shown. Label the point A''' .



- a. What are the coordinates of point A''' ?
- b. What do you notice about the coordinates of point A and point A''' ? How are the two points related?

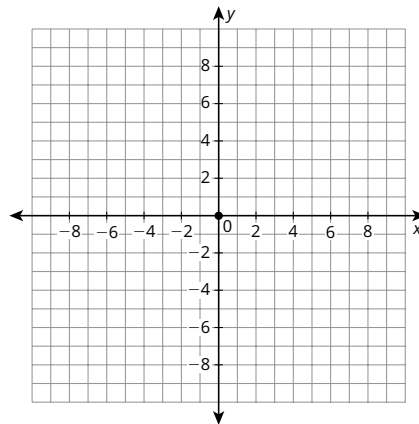
You may have noticed that the values of the x- and y-coordinates seem to switch places for every 90° rotation about the origin. You may have also noticed that the rotation from point A to A'' is a 180° counterclockwise rotation about the origin, and that the rotation from point A to A''' is a 270° counterclockwise rotation about the origin.

5. Determine the coordinates of point (x, y) after rotations of $90^\circ, 180^\circ, 270^\circ$, and 360° .

Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(x, y)				

You can verify that the information in the table is correct by using a test point. Plot test point Q on the coordinate plane and rotate the point 90° , 180° , 270° , and 360° counterclockwise about the origin.

6. Graph and label point Q at (5, 7) on the coordinate plane.



Remember ...

that the table shows values for the coordinates, but coordinates for a plotted point are always in the form (x, y) !

7. Use the origin (0, 0) as the point of rotation.

- Rotate the pre-image Q 90° counterclockwise about the origin. Label the image Q' . Determine the coordinates of image Q' , then describe how you determined the location of point Q' .
- Rotate the pre-image Q 180° counterclockwise about the origin. Label the image Q'' . Determine the coordinates of image Q'' , then describe how you determined the location of image Q'' .
- Rotate the pre-image Q 270° counterclockwise about the origin. Label the image Q''' . Determine the coordinates of point Q''' , then describe how you determined the location of image Q''' .

- d. Rotate the pre-image Q 360° counterclockwise about the origin. Label the image Q'''' . Determine the coordinates of point Q'''' , then describe how you determined the location of image Q'''' .

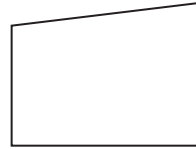
You have been rotating points about the origin on a coordinate plane. However, do you think polygons can also be rotated on the coordinate plane?

You can use models to help show that you *can* rotate polygons on a coordinate plane. However, before we start modeling the rotation of a polygon on a coordinate plane, let's graph the trapezoid to establish the pre-image.

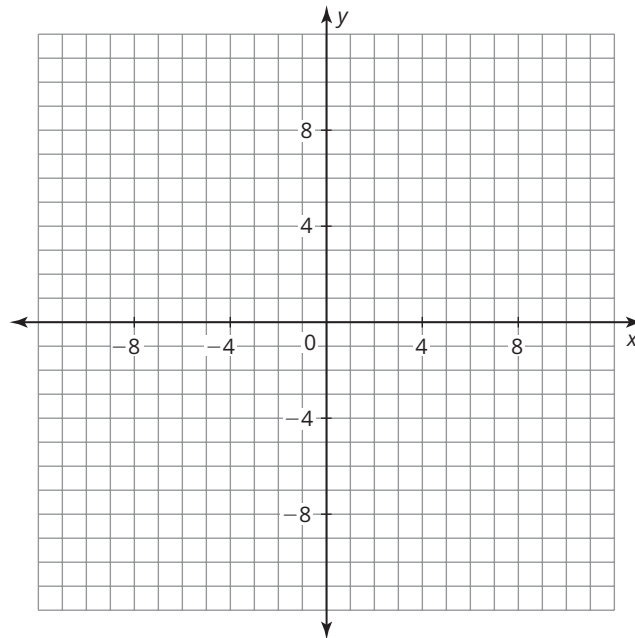
8. Graph trapezoid $ABCD$ by plotting the points $A (-12, 9)$, $B (-12, 4)$, $C (-4, 4)$, and $D (-4, 10)$.

Now that you have graphed the pre-image, you are ready to model the rotation of the polygon on the coordinate plane.

Cut out a copy of the figure shown.



- First, fold a piece of tape in half and tape it to both sides of the trapezoid you cut out.
- Then, take your trapezoid and set it on top of trapezoid $ABCD$ on the coordinate plane, making sure that the tape covers the origin $(0, 0)$.
- Finally, put a pin or your pencil point through the tape at the origin and rotate your model counterclockwise.



9. Rotate trapezoid $ABCD$ about the origin for each given angle of rotation. Graph and label each image on the coordinate plane and record the coordinates in the table.
- Rotate trapezoid $ABCD$ 90° counterclockwise about the origin to form trapezoid $A'B'C'D'$.
 - Rotate trapezoid $ABCD$ 180° counterclockwise about the origin to form trapezoid $A''B''C''D''$.

Coordinates of Trapezoid $ABCD$	Coordinates of Trapezoid $A'B'C'D'$	Coordinates of Trapezoid $A''B''C''D''$
$A (-12, 9)$		
$B (-12, 4)$		
$C (-4, 4)$		
$D (-4, 10)$		

- Write the rule for the rotation mapping trapezoid $ABCD$ to $A'B'C'D'$ in coordinate notation.
- Write the rule for the rotation mapping trapezoid $ABCD$ to

10. What similarities do you notice between rotating a single point about the origin and rotating a polygon about the origin?

Let's consider rotations without graphing.

Ask Yourself . . .

What patterns do you notice?

11. The vertices of parallelogram $DEFG$ are $D(-9, 7)$, $E(-12, 2)$, $F(-3, 2)$, and $G(0, 7)$.
- Determine the vertex coordinates of image $D'E'F'G'$ if parallelogram $DEFG$ is rotated 90° counterclockwise about the origin.
 - How did you determine the image coordinates without graphing?
 - Write a rule using coordinate notation for how you determine the image coordinates without graphing.
 - Determine the vertex coordinates of image $D''E''F''G''$ if parallelogram $DEFG$ is rotated 180° counterclockwise about the origin.
 - How did you determine the image coordinates without graphing?

- f. Write a rule using coordinate notation for how you determine the image coordinates without graphing.
- g. Determine the vertex coordinates of image $D'''E'''F'''G'''$ if parallelogram $DEFG$ is rotated 270° counterclockwise about the origin.
- h. How did you determine the image coordinates without graphing?
- i. Write a rule using coordinate notation for how you determine the image coordinates without graphing.

12. Ethan claims that if he is trying to determine the coordinates of an image that is rotated 180° about the origin, it does not matter which direction the rotation occurred. Samuel claims that the direction is important to know when determining the image coordinates. Who is correct? Explain why the correct student's rationale is correct.

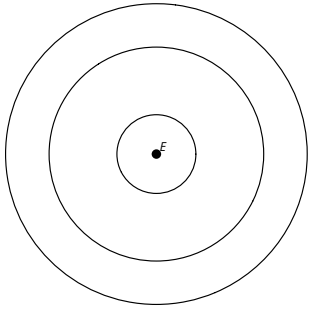


.....

Concentric circles

are circles with a
common center point.

.....



Circles are important for understanding rotations.

1. Consider three concentric circles with center at point E .
 - a. Draw one point on each circle so that you can connect the points to form $\triangle ABC$. Draw the sides of the triangle.
 - b. Choose a positive or negative rotation angle, t .
 - c. Draw A' as an endpoint of EA' such that $\angle AEA'$ has a measure of t degrees.
 - d. Repeat part (c) to draw points B' and C' . Connect the transformed points to form $\triangle A'B'C'$.
2. What do you notice about the pairs of line segments $\overline{EA}$ and $\overline{EA'}$, $\overline{EB}$ and $\overline{EB'}$, and $\overline{EC}$ and $\overline{EC'}$? Explain your observations.
3. Does the transformation you created preserve distances and angle measures? Explain your thinking.

A **rotation** is a function that maps its input, a point, to another location, by a specific rotation angle around a fixed point. This movement to a new location is described by a center of rotation, E , and a rotation angle, t .

Because the rotation is defined about a point E , the movement of a specific point traces an arc that is part of the circumference of a circle with center E . The arc can be labeled by the starting point, P , and the endpoint, P' , or as, $\widehat{PP'}$. The degree measure of this arc is equivalent to the degree rotation, t , that creates a *central angle* in Circle E .

A **central angle** of a circle is an angle whose vertex is at the center of the circle.

4. Consider the rotation you created in Question 1.
 - a. Describe the function that represents the rotation of $\triangle ABC$.
 - b. Do the arcs you created all have the same measure?
Explain your answer.

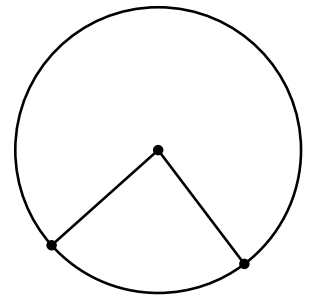
5. Compare the pre-image and the image. Then, compare the corresponding sides of the pre-image and the image.

.....
Remember ...

A circle is named by the point that is its center.
.....

.....
Think About ...

A circle is a rotation of a point around a given center 360 degrees. A circle is a set of points that are all a given distance from a center point.

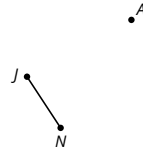


The central angle measures the distance the point is rotated.
.....

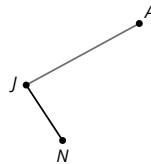
It is possible to apply the rotation function to a figure by using a protractor and ruler.

WORKED EXAMPLE

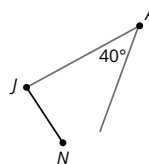
Rotate $\overline{JN}$ 40° , using point A as the center of rotation.



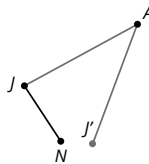
1. Draw a line segment from the center of rotation, A , to one endpoint of the line segment.



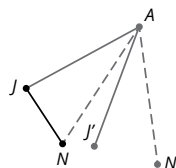
2. Using a protractor, draw a 40° angle. Use the center of rotation, A , as the vertex and the line segment drawn, $\overline{AJ}$, as one side of the angle. Since the angle measure is positive, place the angle in the counter-clockwise direction of the line segment drawn.



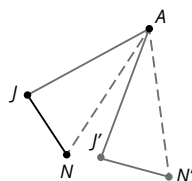
3. Use a ruler or compass to extend the side of the angle so that it is the same length as $\overline{AJ}$. Label the other endpoint J' .



4. Repeat steps 1, 2, and 3 using the other endpoint of the original line segment.



5. Connect endpoints J' and N' . Segment $\overline{J'N'}$ is the result of a 40° rotation of $\overline{JN}$ about point A .

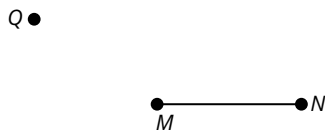


6. Complete each rotation using the given function.

- a. Rotate $\overline{VH}$ 75° , using point E as the center of rotation.



- b. Rotate $\overline{MN}$ -30° , using point Q as the center of rotation.



Remember ...

When you construct, you use only a compass and straightedge. Here you are drawing, so you can use those tools as well as measuring tools, such as a protractor and ruler.

PROBLEM SOLVING



ACTIVITY 4.3

Determining the Center of Rotation

You have seen that corresponding points on rotated figures are equidistant from the center of rotation.

1. Draw an example to explain why Harper is correct.

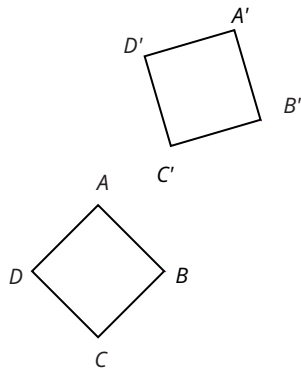
Harper



If two points Q and Q' are equidistant from the center, then the perpendicular bisector of $\overline{QQ'}$ passes through the center.

2. Use what you know to determine the center of rotation and rotation angle for the transformation of each figure.

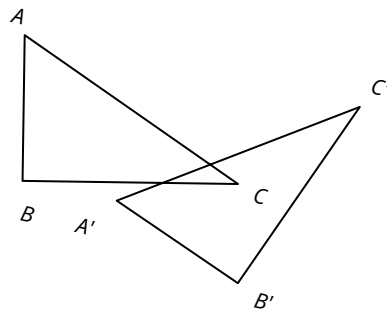
a.



Ask Yourself . . .

Did you make a plan to solve the problem?

b.



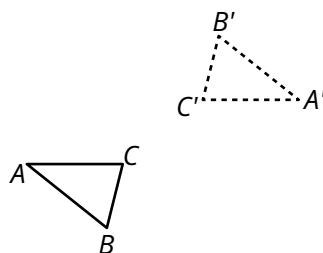
ACTIVITY
4.4

Sequences of Rigid Motion Transformations

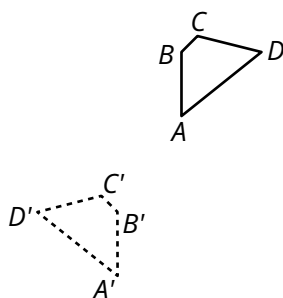
You have learned that translations, reflections, and rotations are rigid motion transformations, which means that they preserve distances and angle measures.

- Describe and sketch the sequence of rigid motion transformations that shows that the two figures in each pair are congruent. Images are shown with dashed lines.

a.



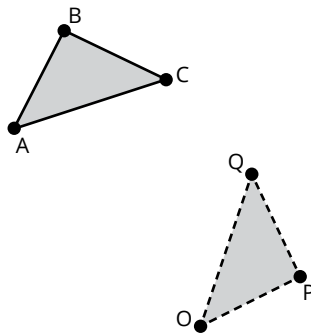
b.



Talk the Talk

That's What It's All About!

1. The vertices of $\triangle ABC$ are $(6, -1)$, $(4, -2)$, and $(3, 2)$. Determine the vertex coordinates of the image $\triangle A'B'C'$, if $\triangle ABC$ is rotated 270° counterclockwise about the origin.
2. Write a rule using coordinate notation for how you determine the image coordinates without graphing.
3. Describe and sketch the sequence of rigid motion transformations that shows that the two figures in the pair are congruent. Images are shown with dashed lines.



Lesson 4 Assignment

Write

Complete each definition.

1. A function that rotates points around a center point through an angle is called a _____.
2. Concentric circles are circles with a common _____.

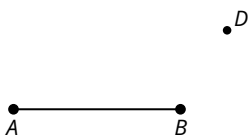
Remember

A rotation is a function that maps its input, a point, to another location. This movement to a new location is defined by a center of rotation, E , and a rotation angle, t .

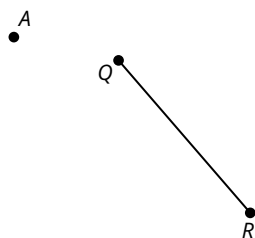
Practice

1. Complete each rotation.

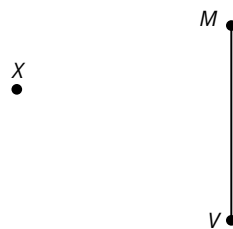
- a. $\overline{AB}$, 45° point D as center of rotation.



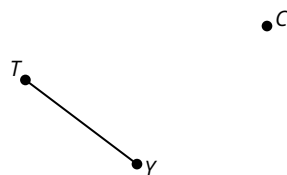
- c. $\overline{QR}$, 125° point A as center of rotation.



- b. $\overline{MV}$, 25° point X as center of rotation.



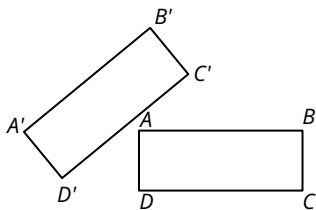
- d. $\overline{TY}$, 80° point C as center of rotation.



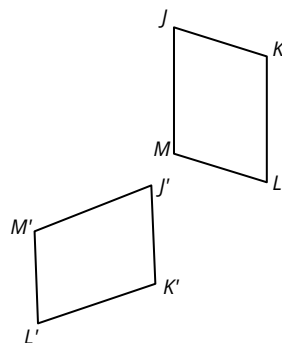
Lesson 4 Assignment

2. Use what you know to determine the center of rotation and rotation angle for the transformation of each figure.

a.



b.

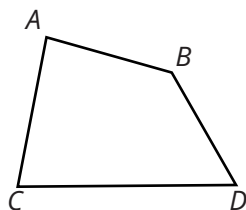


3. The vertices of parallelogram $PQRS$ are $P(4, -5)$, $Q(8, -1)$, $R(3, 6)$, and $S(-2, 1)$.

a. Determine the vertex coordinates of $P'Q'R'S'$ if parallelogram $PQRS$ is rotated 90° counterclockwise about the origin.

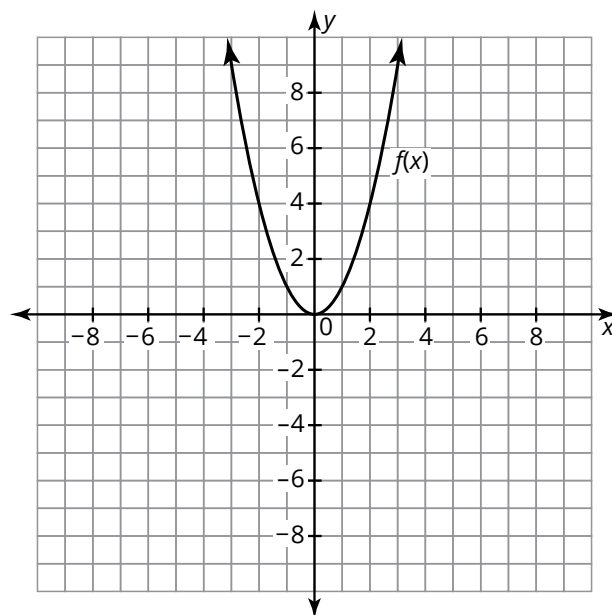
b. Determine the vertex coordinates of image $P''Q''R''S''$ if parallelogram $PQRS$ is rotated 180° counterclockwise about the origin.

4. Sketch the sequence of rigid motion transformations if quadrilateral $ABCD$ is reflected across a vertical line and then translated up which shows the two figures in each pair are congruent.



Lesson 4 Assignment

Prepare



Consider the graph of $f(x)$.
Complete the table.

$f(x)$	$-f(x) + 5$	$-f(x \cdot 1)$
$(-2, 4)$		
$(0, 0)$		
$(2, 4)$		

5

Compositions of Functions

OBJECTIVES

- Identify and verbally explain a sequence of rigid motion transformations that maps a pre-image to an image on the coordinate plane.
- Perform a composition of rigid motion transformations on the coordinate plane.
- Determine the vertex coordinates of the image that results after a sequence of rigid motion transformations without graphing when given the vertex coordinates of the pre-image.
- Write a rule for a composition of rigid motion transformations on the coordinate plane using coordinate notation.
- Understand that the image that results after a composition of rigid motion transformations is congruent to its pre-image.
- Write congruency statements between the image and pre-image of a composition of rigid motion transformations.

.....

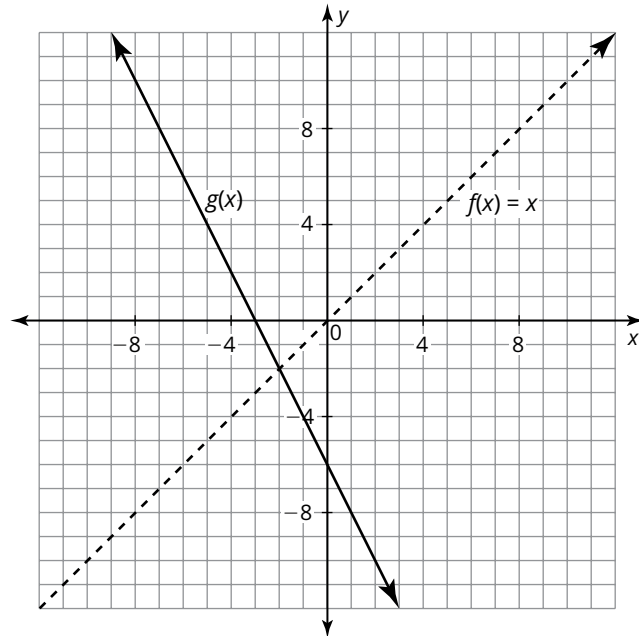
You have written rules for translations, rotations, and reflections on the coordinate plane.

How can you use what you learned about rules for translations, rotations, and reflections on the coordinate plane to help you write a rule for a composition of rigid motion transformations?

Getting Started

Basic Steps

Consider the parent function $f(x) = x$ and the transformed graph $g(x)$.



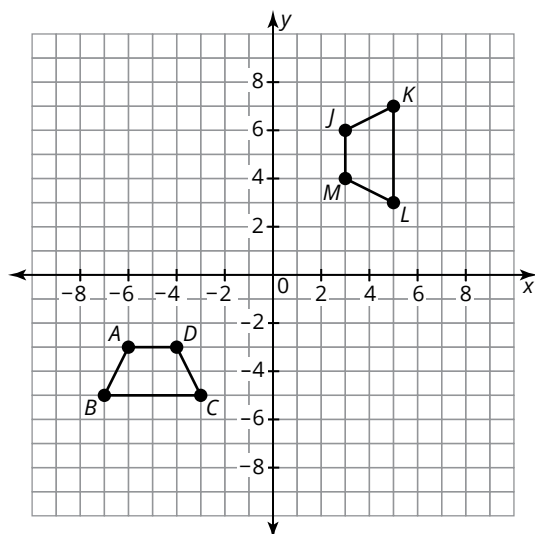
1. Describe a sequence of transformations from the graph of $f(x)$ to the graph of $g(x)$.
2. Write the function equation for $g(x)$ in terms of $f(x)$.
3. Complete the table with the corresponding coordinates.

$f(x)$	$g(x)$
$(-1, -1)$	
$(0, 0)$	
$(1, 1)$	
$(2, 2)$	

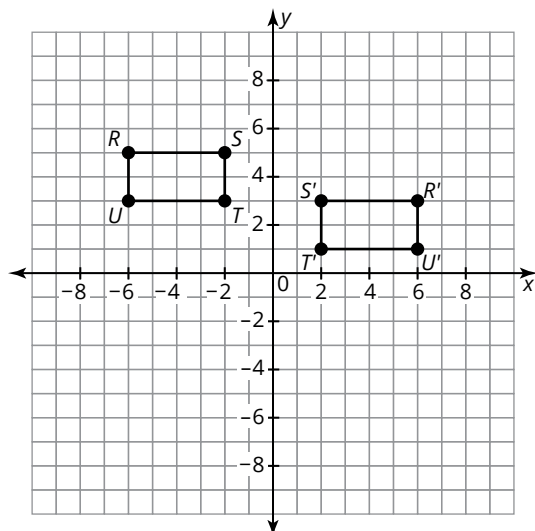
Multiple Transformations

You can identify the sequence of rigid motion transformations that maps a pre-image to an image on the coordinate plane.

1. The pre-image trapezoid $ABCD$ and image trapezoid $JKLM$ are shown on the coordinate plane. What is a sequence of transformations that maps trapezoid $ABCD$ to trapezoid $JKLM$?



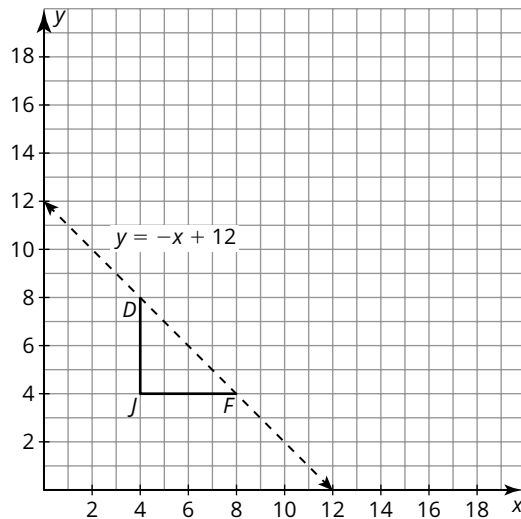
2. A sequence of rigid motion transformations maps rectangle $RSTU$ to rectangle $R'S'T'U'$.



- Describe a sequence of transformations that maps rectangle $RSTU$ to rectangle $R'S'T'U$.
- Describe a different sequence of transformations that maps rectangle $RSTU$ to rectangle $R'S'T'U$.

A composition of transformations involves performing multiple transformations on the coordinate plane, and when these transformations are carried out in a specific order, they form a sequence of transformations.

- Consider $\triangle JDF$ on the coordinate plane shown.



- Write the coordinates of the vertices of the triangle.
- Show a reflection of $\triangle JDF$ across the line $y = -x + 12$ shown. Label the new image as $\triangle J'D'F'$. Write the coordinates of the vertices of the image.

- c. Show a translation of the image to the right 10 units. Label the new image as $\triangle J''D''F''$. Write the coordinates of the vertices of the image.

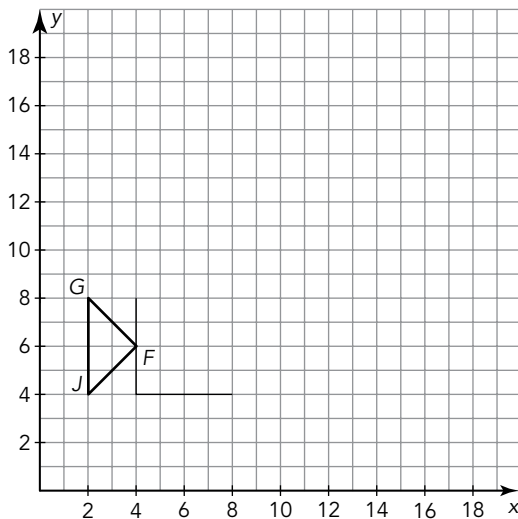


4. Ava says that there are many different ways to use multiple transformations to transform $\triangle JDF$ to create $\triangle J''D''F''$.

Is Ava correct? If so, identify a sequence of rigid motions that can transform $\triangle JDF$ to create $\triangle J''D''F''$.

5. Consider $\triangle JGF$ on the coordinate plane shown. Determine the coordinates of the vertices of the image, $\triangle J'G'F'$, under the composition of transformations given:

- a rotation of 90° clockwise about point F
- a translation 4 units down

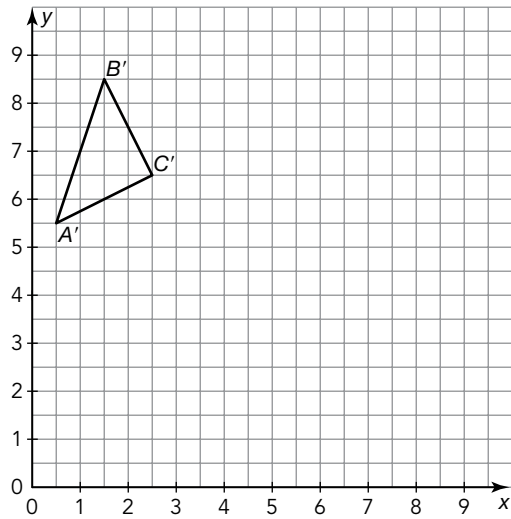


6. Describe a single rigid motion that could produce the same result as the composition of transformations in Question 4. Explain your reasoning.
7. Describe how you could determine the coordinates of the pre-image, $\triangle JDF$, if you were given the coordinates of the image, $\triangle J''D''F''$, and the sequence of rigid motions.

8. Triangle $A'B'C'$ is the result of this composition of rigid motions:

- a translation up 2 units
- a reflection across the line $x = 2\frac{1}{2}$

Determine the coordinates of the vertices of the pre-image, $\triangle ABC$.
Explain your reasoning.



ACTIVITY
5.2

More Compositions of Transformations

1. The vertices of a parallelogram $DEFG$ are $D(-10, -5)$, $E(-8, -8)$, $F(-3, -8)$, and $G(-5, -5)$. Determine the vertex coordinates of the image parallelogram $HJKL$ after a rotation 270° counterclockwise about the origin, a reflection across the y -axis, and a translation down 4 units on the pre-image parallelogram $DEFG$.
 - a. Determine the coordinates of the image without graphing.
 - b. How did you determine the image coordinates without graphing?
 - c. Write a rule using coordinate notation for how to determine the image coordinates without graphing.
 - d. Describe the relationship between the figures and their corresponding sides and angles under the composition of rigid motion transformations.

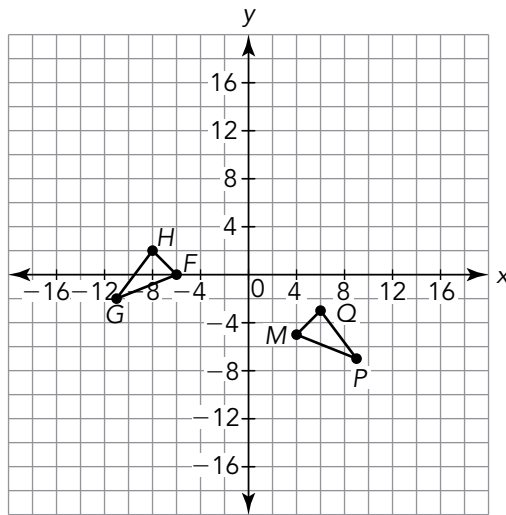
2. The vertices of a trapezoid $QRST$ are $Q(-6, 11)$, $R(-6, 8)$, $S(-2, 8)$, and $T(-4, 11)$. Determine the vertex coordinates of the image parallelogram $ABCD$ after a translation right 12 units, a rotation 90° counterclockwise about the origin, and a reflection across the x -axis on the pre-image parallelogram $QRST$.
- Determine the coordinates of the image without graphing.
 - How did you determine the image coordinates without graphing?
 - Write a rule using coordinate notation for how to determine the image coordinates without graphing.
 - Describe the relationship between the figures and their corresponding sides and angles under the composition of rigid motion transformations.



Talk the Talk

The Same But Different

1. The pre-image $\triangle MPQ$ and image $\triangle FGH$ are shown on the coordinate plane. What is a sequence of transformations that maps $\triangle MPQ$ to $\triangle FGH$?



2. The vertex coordinates of rectangle $BCDE$ are $B(7, 3)$, $C(7, -1)$, $D(12, -1)$, and $E(12, 3)$. Determine the vertex coordinates of the image rectangle $RSTU$ after this composition of rigid motion transformations:
 - a reflection across the y -axis
 - a rotation 180° counterclockwise about the origin
 - a translation down 9 units

Lesson 5 Assignment

Write

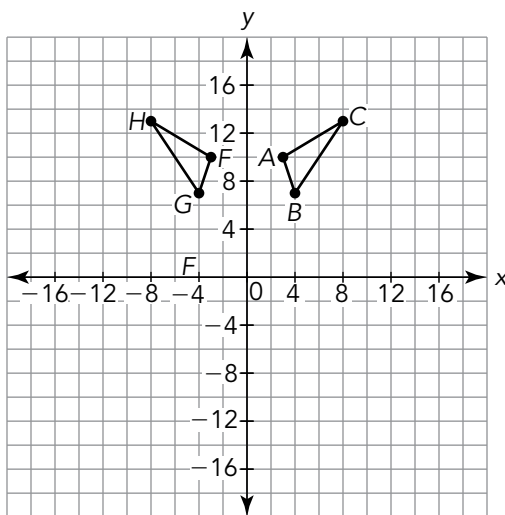
How can you check the rule you wrote using coordinate notation for a composition of rigid motion transformations is correct without graphing?

Remember

There could be more than one way to describe a sequence of rigid motion transformations on the coordinate plane. You can use the rules you learned for translations, reflections, and rotations on the coordinate plane to write a rule for a composition of rigid motion transformations.

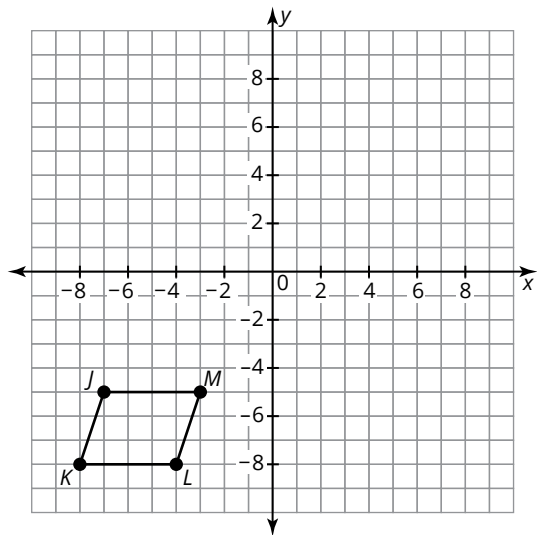
Practice

1. The pre-image $\triangle ABC$ and image $\triangle FGH$ are shown on the coordinate plane. What are two different sequences of transformations that map $\triangle ABC$ to $\triangle FGH$?



Lesson 5 Assignment

2. Consider parallelogram $JKLM$ on the coordinate plane shown.
- a. Reflect parallelogram $JKLM$ across line $x = -3$ and then translate right 6 units on the coordinate plane. Label the image parallelogram $PQRS$.



- b. Can you map parallelogram $JKLM$ to parallelogram $PQRS$ using a single transformation? Explain your reasoning.

3. The vertex coordinates of trapezoid $BCDE$ are $B(5, -6)$, $C(3, -9)$, $D(10, -9)$, and $E(10, -6)$. Determine the vertex coordinates of the image trapezoid $JKLM$ after a translation up 5 units, a reflection across the y -axis, and a rotation 180° counterclockwise about the origin.
- a. Determine the coordinates of the image without graphing.
- b. Write a rule for how to determine the image coordinates without graphing.
- c. Describe the relationship between the figures and their corresponding sides and angles under the composition of rigid motion transformations.

Lesson 5 Assignment

Prepare

Write a description of each geometric object and provide an example.

1. point
2. line
3. line segment
4. circle

Sequences of Rigid Motions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

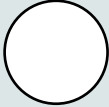
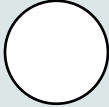
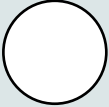
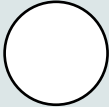
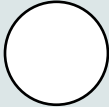
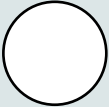
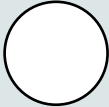
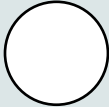
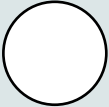
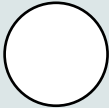
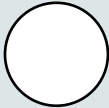
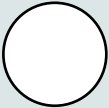
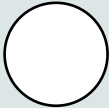
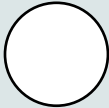
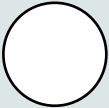
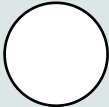
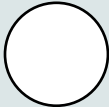
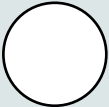
Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Sequences of Rigid Motions* topic by:

TOPIC 3: <i>Sequences of Rigid Motions</i>	Beginning of Topic	Middle of Topic	End of Topic
describing rigid motion transformations using coordinate notation.			
defining translations in terms of distances along parallel lines.			
understanding that a translation maps all points in the pre-image equal distances along parallel lines to corresponding points in the image.			
defining reflections in terms of distance across perpendicular lines.			
determining the line of reflection for a given reflection.			
understanding that the line of reflection is the perpendicular bisector of the line segments connecting each point in the pre-image to its corresponding point in the image.			
perform rigid motion transformations using patty paper or other construction tools.			

continued on the next page

TOPIC 3 SELF-REFLECTION
continued

TOPIC 3: <i>Sequences of Rigid Motions</i>	Beginning of Topic	Middle of Topic	End of Topic
recognize the difference between transformations that are rigid motions and those that are not.			
recognizing the difference between transformations that are rigid motions and those that are not.			
defining rotations in terms of angles around a circle.			
understanding that a rotation maps all points in the pre-image an equal angle around concentric circles to corresponding points in the image.			
determining the center of rotation for a given rotation.			
specifying a sequence of transformations that maps one figure onto another in the plane.			

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Sequences of Rigid Motions* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 3 SUMMARY

Sequences of Rigid Motions

LESSON 1

Translations On and Off the Coordinate Plane

A **translation** moves a set of points a specified distance in a specified direction along parallel lines. When an image is horizontally translated c units on the coordinate plane, the value of the x -coordinates change by c units. When an image is vertically translated c units on the coordinate plane, the value of the y -coordinate changes by c units. **Coordinate notation** describes how the original coordinates of a point are changed into new coordinates after one of more transformations. For example, the mapping $(x, y) \rightarrow (x - 3, y - 2)$ represents a translation left 3 units and down 2 units. The coordinates of an image after a translation are summarized in the table.

Original Point	Horizontal Translation to the Left	Horizontal Translation to the Right	Vertical Translation Up	Vertical Translation Down
(x, y)	$(x - c, y)$	$(x + c, y)$	$(x, y + c)$	$(x, y - c)$

For example, the coordinates of $\triangle ABC$ are $A(0, 2)$, $B(2, 6)$, and $C(3, 3)$.

When $(\triangle ABC)$ is translated down 8 units, the coordinates of the image are $A'(0, -6)$, $B'(2, -2)$, and $C'(3, -5)$.

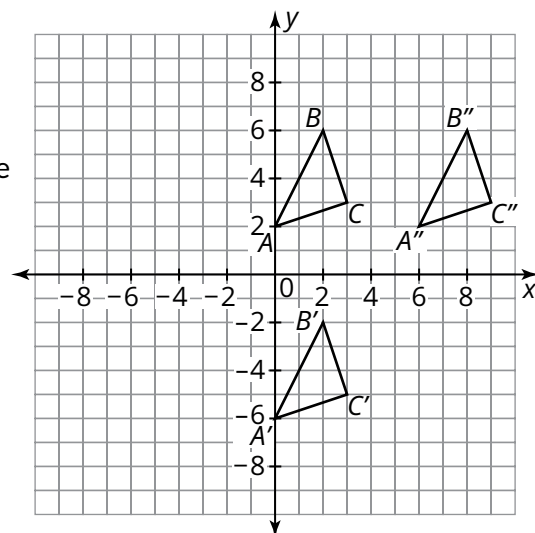
When $\triangle ABC$ is translated right 6 units, the coordinates of the image are $A''(6, 2)$, $B''(8, 6)$, and $C''(9, 3)$.

The translated triangles are *congruent triangles*. **Congruent triangles** are triangles that have the same shape and size. In congruent triangles corresponding side and angle measures are the same. Angles with the same angle measure are called congruent angles.

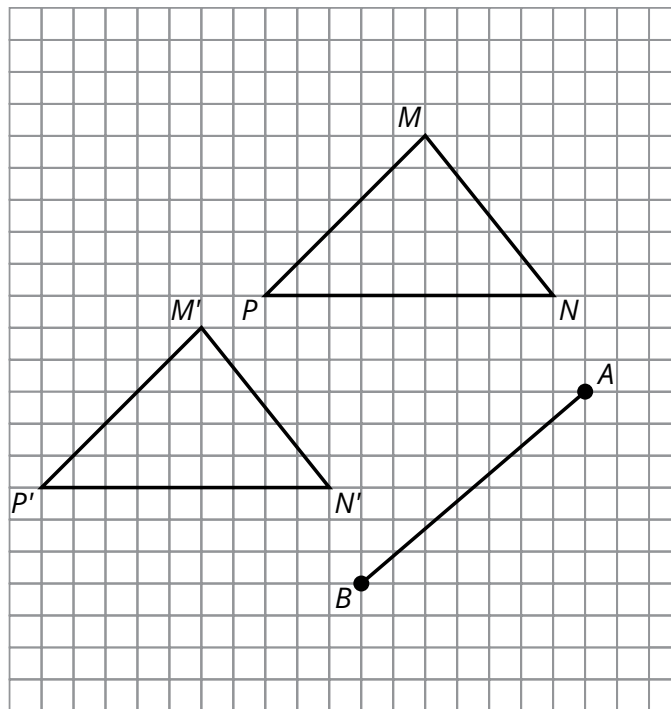
A translation can be measured as a directed line segment.

KEY TERMS

- coordinate notation [notación de coordenadas]
- congruent angles [ángulos congruentes]
- congruent triangles [triángulos congruentes]
- translation [traslación]
- angle bisector [bisectriz de un ángulo]
- reflection [reflexión]
- perpendicular bisector theorem [teorema de la bisectriz perpendicular]
- rotation [rotación]
- angle of rotation [ángulo de rotación]
- central angle [ángulo central]
- concentric circle [círculo concéntrico]



$\triangle MNP$ was translated to produce $\triangle M'N'P'$. The triangle was translated a distance equal to the distance between points A and B . It was translated in the direction from point A to point B . So, $\overline{AB}$ is the directed line segment used to measure this translation.

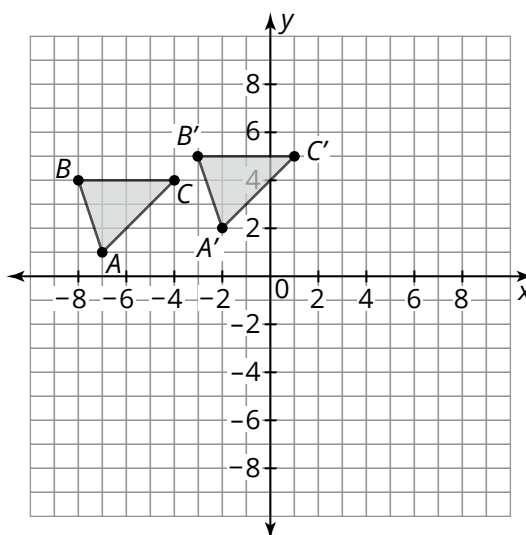


LESSON

2

Sequences of Multiple Translations

You can use a sequence of translations to describe multiple translations on and off the coordinate plane. For example, $\triangle ABC$ is translated right 5 units and up 1 unit to form $\triangle A'B'C'$. You can represent this sequence of translations using coordinate notation as $(x, y) \rightarrow (x + 5, y + 1)$.



The two triangles are congruent since translations are rigid motion transformations that preserve size and shape. This is also true for sequences of transformations off the coordinate plane.

Figures that are mirror images of each other are called *reflections*. A *reflection* is a rigid motion that reflects, or “flips,” a figure over a given line called a line of reflection. A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line. When an image on the coordinate plane is reflected across the y -axis, the value of the x -coordinate of the image is opposite the x -coordinate of the pre-image. When an image on the coordinate plane is reflected across the x -axis, the value of the y -coordinate of the image is opposite the y -coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

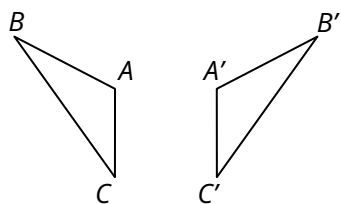
Original Point	Coordinates of Image After a Reflection Over the x -axis	Coordinates of Image After a Reflection Over the y -axis
(x, y)	$(x, -y)$	$(-x, y)$

For example the coordinates of Quadrilateral $ABCD$ are $A(3, 2)$, $B(2, 5)$, $C(5, 7)$, and $D(6, 1)$.

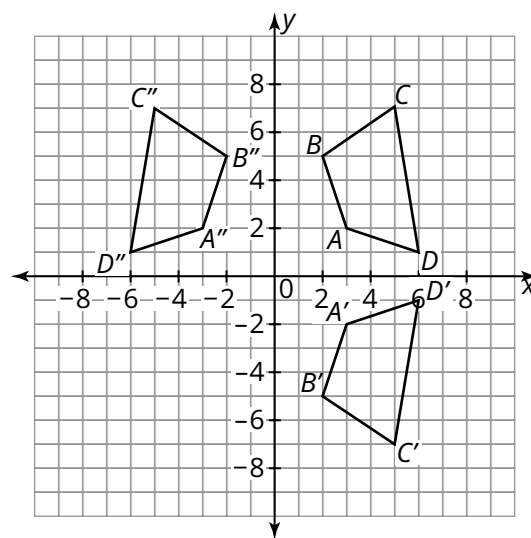
When Quadrilateral $ABCD$ is reflected across the x -axis, the coordinates of the image are $A'(3, -2)$, $B'(2, -5)$, $C'(5, -7)$, and $D'(6, -1)$.

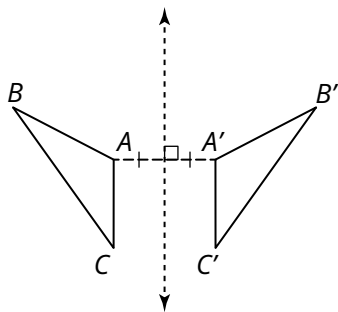
When Quadrilateral $ABCD$ is reflected across the y -axis, the coordinates of the image are $A''(-3, 2)$, $B''(-2, 5)$, $C''(-5, 7)$, and $D''(-6, 1)$.

It is possible to construct the line of reflection when given two figures that are reflections of one another.

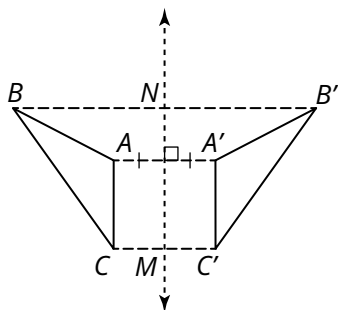


1. Label the vertices.

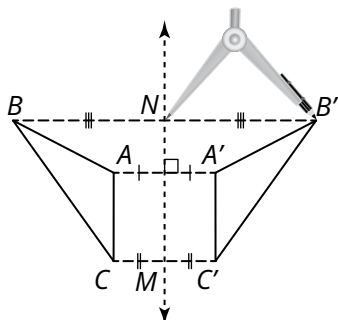




2. Connect two corresponding points. Construct the perpendicular bisector of the line segment connecting them.



3. Connect remaining corresponding points. Label the intersection of each line segment with the perpendicular bisector.



4. Use a compass to determine whether each intersection point is the midpoint of the line segment connecting corresponding vertices. If that is the case, the perpendicular bisector is the line of reflection. If not, the figures are not reflections of one another.

$$\overline{CM} \cong \overline{C'M}$$

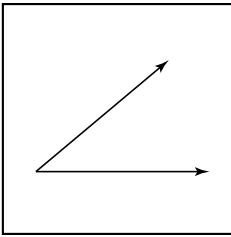
$$\overline{BN} \cong \overline{B'N}$$

$\overline{MN}$ is the line of reflection.

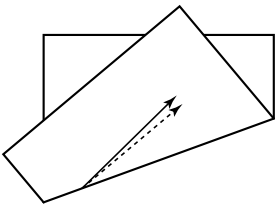
You know that the perpendicular bisector of a line segment is the line of reflection between the endpoints of the segment. The **perpendicular bisector theorem** states: "Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment."

Just as line segments can be bisected, angles can be bisected too. If a ray is drawn through the vertex of an angle and divides the angle into two angles of equal measure, or two congruent angles, this ray is called an **angle bisector**. The construction used to create an angle bisector is called bisecting an angle. One way to bisect an angle is using patty paper.

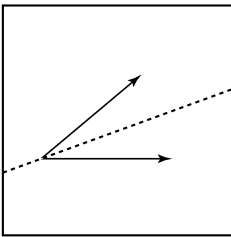
Draw an angle on the patty paper.



Fold the patty paper so that the rays lie on top of each other. The fold should pass through the vertex of the angle.



Open the patty paper. The crease represents the angle bisector.



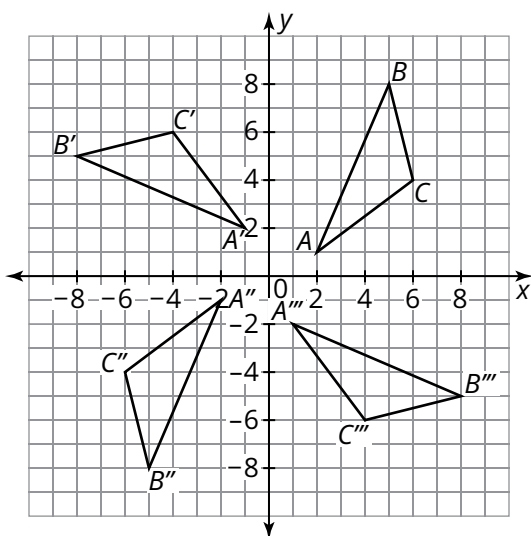
LESSON

4

Rotations On and Off the Coordinate Plane

A **rotation** is a rigid motion that turns a figure about a fixed point called the *point of rotation*. The figure is rotated in a given direction for a given angle called the *angle of rotation*. The **angle of rotation** is the measure of the amount the figure is rotated about the point of rotation. The direction of a rotation can either be clockwise or counterclockwise. When the center of rotation is at the origin $(0, 0)$, and the angle of rotation is 90° , 180° , 270° , or 360° , the coordinates of an image can be determined using the rules summarized in the table.

Original Point	Coordinates After a 90° Counterclockwise Rotation About the Origin	Coordinates After a 180° Counterclockwise Rotation About the Origin	Coordinates After a 270° Counterclockwise Rotation About the Origin	Coordinates After a 360° Counterclockwise Rotation About the Origin
(x, y)	$(-y, x)$	$(-x, -y)$	$(y, -x)$	(x, y)



For example, the coordinates of $\triangle ABC$ are $A(2, 1)$, $B(5, 8)$, and $C(6, 4)$.

When $\triangle ABC$ is rotated 90° counterclockwise about the origin, the coordinates of the image are $A'(-1, 2)$, $B'(-8, 5)$, and $C'(-4, 6)$.

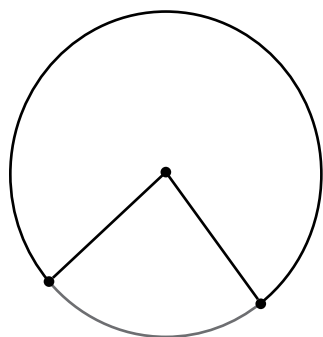
When $\triangle ABC$ is rotated 180° about the origin, the coordinates of the image are $A''(-2, -1)$, $B''(-5, -8)$, and $C''(-6, -4)$.

When $\triangle ABC$ is rotated 270° counterclockwise about the origin, the coordinates of the image are $A'''(1, -2)$, $B'''(8, -5)$, and $C'''(4, -6)$.

When $\triangle ABC$ is rotated 360° counterclockwise about the origin, the coordinates of the image and the pre-image have the same coordinate.

A rotation maps its input, a point, to another location, which is described by a center of rotation, E , and a rotation angle, t .

Because the rotation is defined about a point E , the movement of a specific point traces an arc that is part of the circumference of a circle with center E . The arc can be labeled by the starting point, P , and the endpoint, P' , or as $\widehat{PP'}$. The degree measure of this arc is equivalent to the degree rotation, t , that creates a central angle in Circle E .



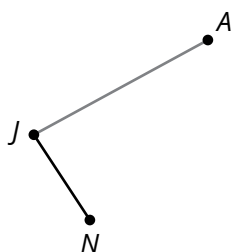
A **central angle** of a circle is an angle whose vertex is at the center of the circle.

A circle is a rotation of a point around a given center 360° . An angle is the measure of the distance the point is rotated as measured by the central angle.

It is possible to rotate a figure by using a protractor and a straightedge.

A

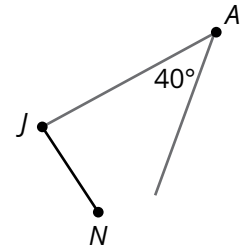
Rotate $\overline{JN}$ 40° , using point A as the center of rotation.



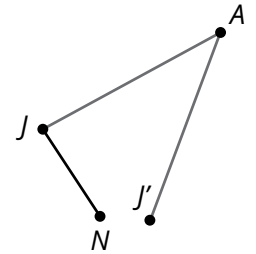
1. Draw a line segment from the center of rotation, A , to one endpoint of the line segment.

- Using a protractor, draw a 40° angle. Use the center of rotation, A , as the vertex and the line segment drawn, $\overline{AJ}$, as one side of the angle.

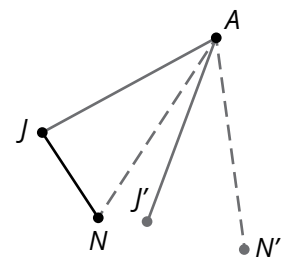
Since the angle measure is positive, place the angle in the counterclockwise direction of the line segment drawn.



- Use a ruler to extend the side of the angle so that it is the same length as $\overline{AJ}$. Label the other endpoint J' .

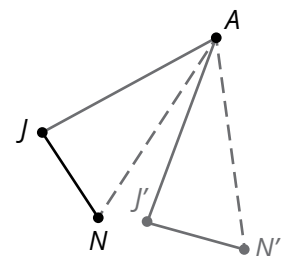


- Repeat steps 1, 2, and 3 using the other endpoint of the original line segment.

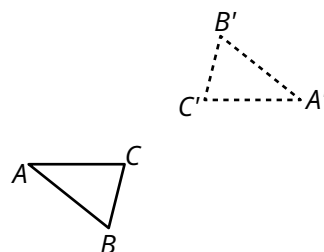


- Connect endpoints J' and N' .

Segment $\overline{J'N'}$ is the result of a 40° rotation of $\overline{JN}$ about point A .



You can use a sequence of rigid motion transformations to show that two figures are congruent. For example, to show $\triangle ABC$ is congruent to $\triangle A'B'C'$, Triangle ABC can be reflected across a horizontal line and then reflected across a vertical line.



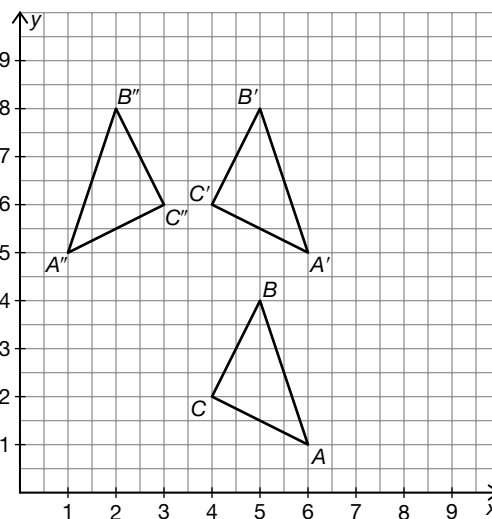
You can perform a composition of rigid motion transformations on the coordinate plane.

For example, $\triangle A''B''C''$ is the result of a composition of rigid motions on $\triangle ABC$.

- a translation up 4 units
- a reflection over the line $x = 3\frac{1}{2}$

The coordinates of $\triangle ABC$ are $A(6, 1)$, $B(5, 4)$, $C(4, 2)$.

- Translate $\triangle ABC$ up 4 units. The coordinates of $\triangle A'B'C'$ are $A'(6, 5)$, $B'(5, 8)$, $C'(4, 6)$.
- Reflect $\triangle A'B'C'$ over the line $x = 3\frac{1}{2}$. The coordinates of $\triangle A''B''C''$ are $A''(1, 5)$, $B''(2, 8)$, $C''(3, 6)$.



Because rigid motions maintain the size and shape of an image, you can use a composition of translations, reflections, and rotations to verify that two figures are congruent. There is often more than one composition of transformations that can be used to verify that two figures are congruent.



Congruent triangles have the same size and shape, while similar triangles have the same shape but may be different sizes. This skylight illustrates both.

Congruence Through Transformations

LESSON 1	Reflectional and Rotational Symmetry	321
LESSON 2	Proving Triangle Congruence Theorems	335
LESSON 3	Using Triangle Congruence to Solve Problems	361



1

Reflectional and Rotational Symmetry

OBJECTIVES

- Identify geometric figures with line symmetry and rotational symmetry.
- Identify lines of symmetry for different geometric figures.
- Orally describe rotations that carry a figure onto itself.
- Orally describe reflections that carry a figure onto itself.

KEY TERMS

- reflectional symmetry
- rotational symmetry

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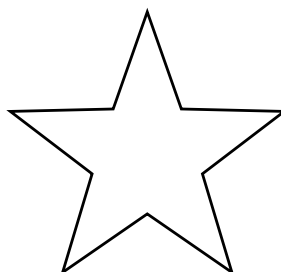
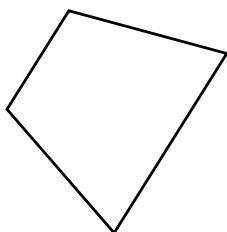
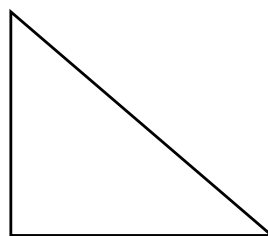
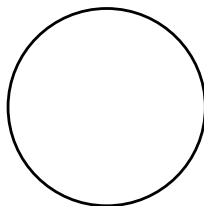
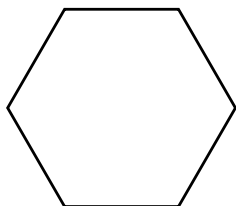
You have learned that pre-images are congruent to images after rigid motion transformations.

How can you use transformations to show that a figure can be carried onto itself?

Getting Started

WOW MOM

Consider the different shapes shown.



1. Copy each shape onto patty paper. For each shape:
 - a. Determine whether you can fold the shape so that half of the figure lies exactly on the other half of the figure. When possible, are there any other folds that will do this?
 - b. Determine whether you can rotate the figure so that it looks exactly like it did before the rotation. When possible, are there other rotations that will do this?

Reflectional and Rotational Symmetries

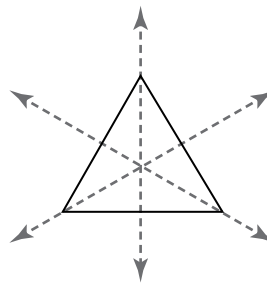
Consider the shapes from the Getting Started.

1. Name the shapes.

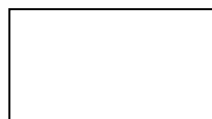
A plane figure has **reflectional symmetry** when you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side of the line. Recall that the line that you drew on each shape is called the *line of symmetry*. A figure may have more than one line of symmetry.

2. Which shapes have reflectional symmetry?

3. Consider the equilateral triangle shown. It has 3 lines of symmetry. Draw these lines of symmetry.



4. How many lines of symmetry does the rectangle shown have? Show the line(s) of symmetry.



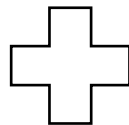
5. How many lines of symmetry are there in a square? Show the line(s) of symmetry.



A plane figure can also have **rotational symmetry** if you can rotate the figure more than 0° but less than 360° and the resulting figure is the same as the original figure in the original position.

6. Which shapes in the Getting Started have rotational symmetry?

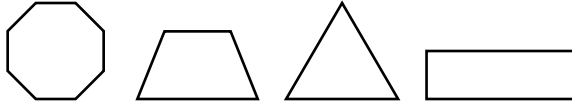
7. Do you think that the given shape has rotational symmetry? Why or why not?



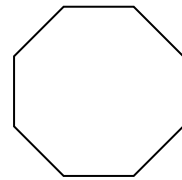
8. Can a shape have both reflectional and rotational symmetry? Explain your reasoning.

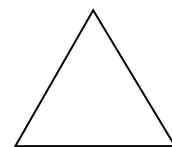
You have also identified transformations that carry a figure onto itself. Reflectional and rotational symmetry are properties of figures that can be carried onto themselves by reflections and rotations.

9. Consider the 4 shapes shown.



a. Describe the reflections and rotations that can carry each figure onto itself.





.....

Review the definition of **apply** in the Academic Glossary.

.....

- b. Ethan says that the horizontal line of symmetry in the rectangle means that a reflection across that line carries the figure onto itself. He also says that it means that a 180° rotation will carry the figure onto itself.

Is Ethan correct? Does his reasoning apply to other figures? Explain your thinking using the shapes from the Getting Started.

Identifying Symmetry

The standard alphabet has many letters that have a variety of symmetries, including reflectional and rotational symmetry. Some letters have a vertical line of symmetry. Other letters have a horizontal line of symmetry.

1. Which letter(s) have a horizontal but not a vertical line of symmetry?

A C H M

2. Which letter(s) have a vertical but not a horizontal line of symmetry?

M B H X

3. Which letter(s) have both a horizontal and a vertical line of symmetry?

Z E H M

4. Which letter(s) have rotational symmetry?

Z W K M



Talk the Talk

Check

OKEECHOBEE, is the name of a city, a county, and a lake in Florida. OKEECHOBEE, along with the title of the Getting Started activity, WOW MOM, and the title of this activity, CHECK, all have rotational and/or reflectional symmetry.

1. Identify the symmetries in each word. Explain your thinking.

Think About . . .

Can you think of a word that has reflectional symmetry using a vertical line of symmetry?

2. Consider the rotational symmetries of an equilateral triangle, square, and regular hexagon.
 - a. What relationship exists between the rotational symmetries of each figure and its interior angle measures?

- b. Test the pattern you noticed on a regular pentagon and regular hexagon. What do you notice?

Lesson 1 Assignment

Write

Explain the difference between reflectional symmetry and rotational symmetry in your own words.

Remember

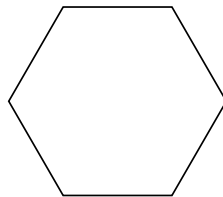
A plane figure has reflectional symmetry if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side. A plane figure has rotational symmetry if you can rotate the figure more than 0° and less than 360° and the resulting figure is the same as the original figure.

There is a sequence of reflections and/or rotations that can carry any plane figure onto itself.

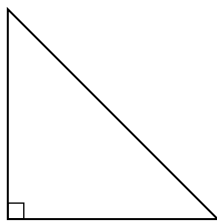
Practice

1. Determine how many lines of symmetry each figure has. Then draw all lines of symmetry for each figure.

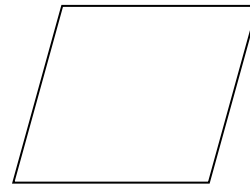
- a. regular hexagon



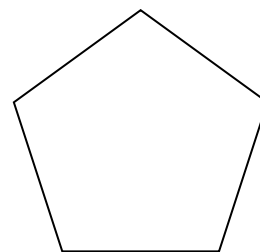
- b. isosceles right triangle



- c. rhombus



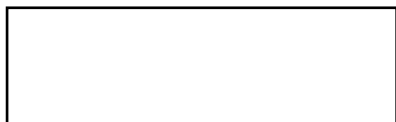
- d. regular pentagon



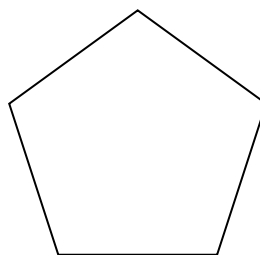
Lesson 1 Assignment

2. Describe the sequence of rotations and/or reflections that can carry each figure onto itself. Use the figure's reflectional and rotational symmetry, when possible, to justify your answer.

a.



b.

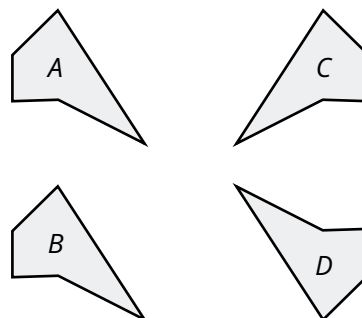


Lesson 1 Assignment

Prepare

Describe transformations that map Figure A onto the other figures shown.

1. Figure *B*



2. Figure *C*

3. Figure *D*

2

Proving Triangle Congruence Theorems

OBJECTIVES

- Use writing to explain the definition of congruence in terms of rigid motions and show that two triangles are congruent.
- Prove the Side-Side-Side congruence theorem using rigid motion transformations.
- Prove the Side-Angle-Side congruence theorem using rigid motion transformations.
- Prove the Angle-Side-Angle congruence theorem using rigid motion transformations.

KEY TERMS

- Side-Side-Side congruence theorem (SSS)
- corresponding parts of congruent triangles are congruent (CPCTC)
- Side-Angle-Side congruence theorem (SAS)
- included angle
- Angle-Side-Angle congruence theorem (ASA)
- included side
- Angle-Angle-Side congruence (AAS)
- Hypotenuse Angle congruence theorem (HA)

.....

You have defined the rigid motion transformations.
How can you use rigid motion transformations to prove congruence theorems?



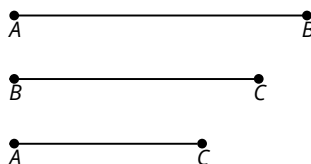
.....
 You need to construct
 congruent segments.

Getting Started

Can You Build It?

You can investigate patterns and make conjectures about geometric relationships.

1. Use the given line segments to construct triangle ABC . Then, write the steps you performed to construct the triangle.



2. Analyze the triangle you created.

- a. Classify $\triangle ABC$. Explain your reasoning.

b. Compare your triangle to your classmates' triangles. Are the triangles congruent? Why or why not?

c. Make a conjecture. How many different triangles can be formed given the lengths of three distinct sides?

3. What conjecture can this lead you to make about triangles' side lengths?

Ask Yourself . . .

What precise mathematical language do you need to communicate your mathematical reasoning?

ACTIVITY 2.1

Congruent Line Segments by Reflection

Remember congruent line segments and congruent angles are often denoted using special markers, rather than given measurements.

Think About...

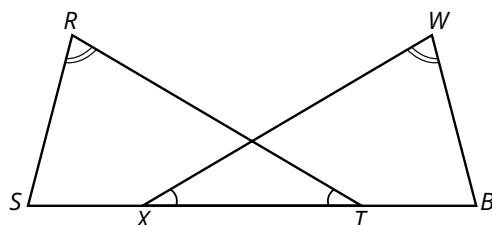
Make sure you are properly naming angles.

Think About...

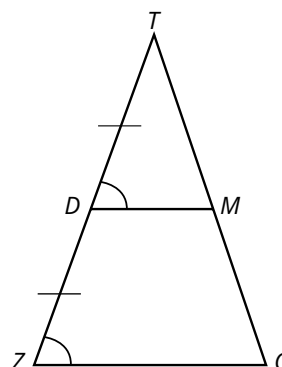
Although $\overline{DF}$ and $\overline{FD}$ represent the same line segment, when writing a congruence statement for segments that are part of a larger figure, think about how the sides and angles in the figure correspond to one another.

- Write the congruence statements represented by the markers in each diagram.

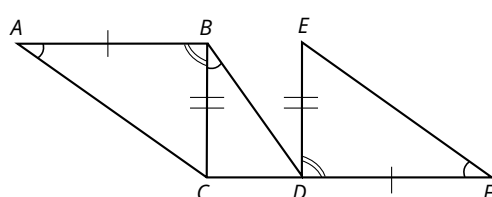
a.



b.



c.



In previous lessons, you learned that:

- Rigid motion transformations preserve distances and angle measures.
- Any point in the plane can be reflected across a line to map onto another point in the plane.
- A point is equidistant from two other points if and only if it lies on their perpendicular bisector.

You can use these facts to prove a conjecture that you have explored. Prove that you can always map one congruent line segment onto the other using at most two reflections.

WORKED EXAMPLE

Prove that a segment can be mapped onto itself in at most two reflections.

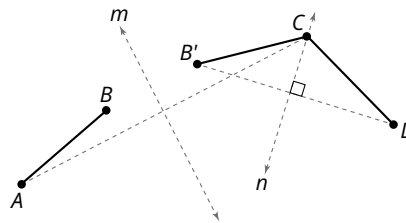
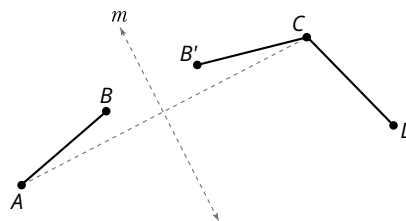
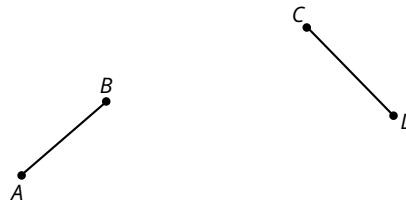
Suppose that $\overline{AB} \cong \overline{CD}$.

Since any point in the plane can be reflected across a line to map onto another point in the plane, you know that point C is a reflection of point A across line m . Reflect $\overline{AB}$ across line m , so that point A maps to point C .

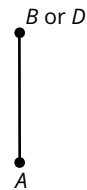
Reflections preserve distances, so you know that $\overline{CB'} \cong \overline{CD}$, because $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{CB'}$. Thus, point C is equidistant from point B' and from point D .

Since a point is equidistant from two other points if and only if it lies on their perpendicular bisector, this means that point C lies on the perpendicular bisector of $\overline{B'D}$ (line n).

Thus, one last reflection across the perpendicular bisector maps $\overline{CB'}$ onto $\overline{CD}$.



2. The proof in the Worked Example shows two reflections. Create 3 distinct examples in which $\overline{AB}$ maps onto $\overline{CD}$ in just one reflection. Explain your example.



3. Use the Worked Example to explain why you do not need more than two reflections to map a line segment onto a congruent line segment in the plane.

ACTIVITY
2.2

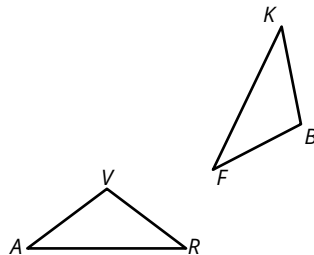
Side-Side-Side Congruence

You have used constructions to make a conjecture about the conditions necessary for triangle congruence. You will now prove the Side-Side-Side congruence theorem using rigid motion transformations.

Consider two triangles such that the three sides of one triangle are congruent to the three sides of the second triangle. You can prove that this criteria is sufficient to demonstrate the two triangles are congruent.

Given $\triangle VAR$ and $\triangle BKF$.

Suppose $VA = BK$, $VR = BF$, and $AR = KF$.



.....
You can use the fact that a segment can be mapped onto itself in at most two reflections to prove $\triangle VAR$ is congruent to $\triangle BKF$.
.....

1. Complete the steps to show the proof that $\triangle VAR \cong \triangle BKF$.
 - a. Draw the reflection of $\triangle VAR$ across a line that maps point V onto point B . Label the image as $\triangle V'A'R'$. Give the reason(s) you can create this reflection.
 - b. Draw the reflection of $\triangle V'A'R'$ which maps $\overline{V'A'}$ onto $\overline{BK}$. Label the image as $\triangle V''A''R''$. Give the reason(s) you can create this reflection.

2. Summarize the proof you completed in Question 1. Explain how knowing that the three corresponding sides of two triangles are congruent proves that the two triangles are congruent.

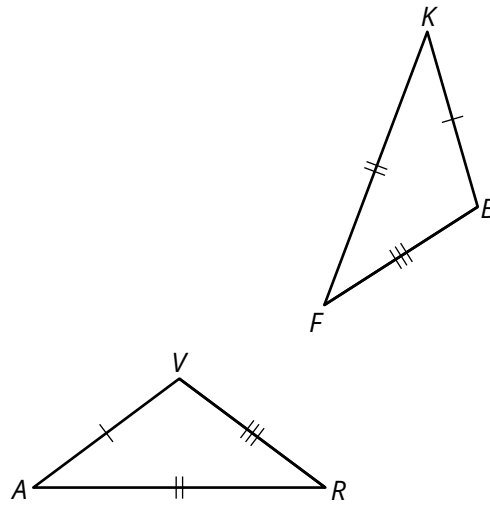
.....
Any theorem can be
used in the future as a
reason in other proofs.
.....

Because this relationship has been proved to be true, you may now refer to it as a theorem. The **Side-Side-Side congruence theorem (SSS)** states: “If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.”

If two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle. **Corresponding parts of congruent triangles are congruent**, abbreviated as **CPCTC**, is often used as a reason in proofs. CPCTC states that corresponding angles and sides in two congruent triangles are congruent. This reason can only be used after you have proven that the triangles are congruent.

3. Since you have proven $\triangle VAR \cong \triangle BKF$ using the conditions necessary for the SSS congruence theorem, apply CPCTC by
 - a. writing congruence statements for all corresponding side and angle relationships of the pre-image and image.

- b. Based on part a, use slash markers and arc markers to denote the remaining congruent sides and angles.



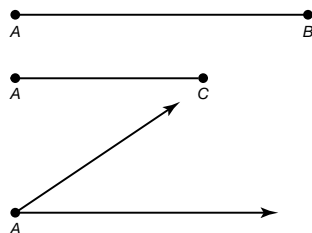
ACTIVITY
2.3

Side-Angle-Side Congruence Theorem

.....
An **included angle** is the angle formed by two sides of a triangle.
.....

You have determined that if the sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent. Let's investigate other conditions sufficient to prove triangle congruence.

1. Construct $\triangle ABC$ using the two line segments and included angle shown. Then, write the steps you performed to construct the triangle.



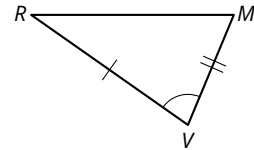
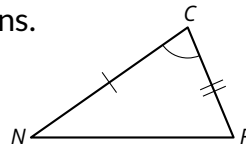
2. How does the length of side $\overline{BC}$ compare to the length of your classmates' side $\overline{BC}$?
3. Use a protractor to measure angle B and angle C in triangle ABC .
4. How do the measures of your corresponding angles compare to the measures of your classmates' corresponding angles?
5. Is your triangle congruent to your classmates' triangles? Why or why not?
6. What conjecture can this lead you to make about the conditions necessary to show triangle congruence?
7. If you were given one of the non-included angles, $\angle C$ or $\angle B$, instead of $\angle A$, do you think everyone in your class would have constructed an identical triangle? Explain your reasoning.

Consider two triangles that have two sides and an included angle congruent. Analyze the proof using rigid motion transformations to demonstrate that this criteria is sufficient to prove the two triangles are congruent.

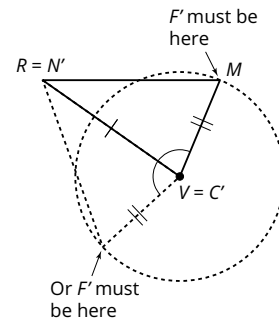
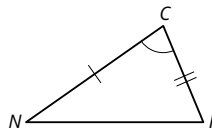
WORKED EXAMPLE

Given $\triangle CNF$ and $\triangle VRM$, $\overline{CN} \cong \overline{VR}$,
 $\overline{CF} \cong \overline{VM}$, and $\angle C \cong \angle V$.
 Prove $\triangle CNF \cong \triangle VRM$.

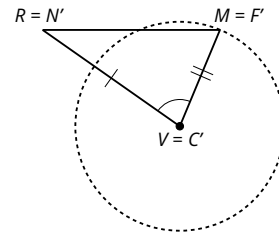
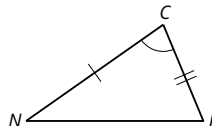
You know that you can map $\overline{CN}$ to $\overline{VR}$ in one or two reflections.



Since reflections preserve distance, this means that point F' must be on the circle centered at V with radius $\overline{VM}$, because $\overline{CF} \cong \overline{VM}$.



And since reflections preserve angle measures, there are only two possible locations for point F' to be on the circle. If point F' is not at M , then a reflection across $\overline{VR}$ will map point F' onto point M .



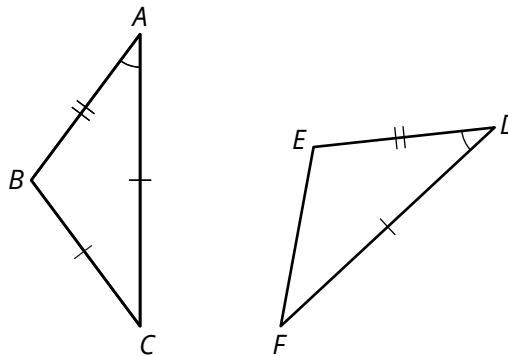
8. Explain the final steps in the Worked Example.
- Why are there only two locations that point F' could be in relation to points V and R ?
 - Why will a reflection across $\overline{VR}$ map point F' onto point M ?

Because this relationship has been proved to be true, you may now refer to it as a theorem. The **Side-Angle-Side congruence theorem (SAS)** states: “If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent.”

9. Consider the diagram shown where $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.
Prove $\triangle ABC \cong \triangle DEF$. Explain your steps.

10. Since you have proven $\triangle ABC \cong \triangle DEF$ using the conditions necessary for the SAS congruence theorem, apply CPCTC by
- writing congruence statements for all corresponding parts and angle relationships of the pre-image and image.

- Based on part a, use slash markers and arc markers to denote the remaining congruent sides and angles.

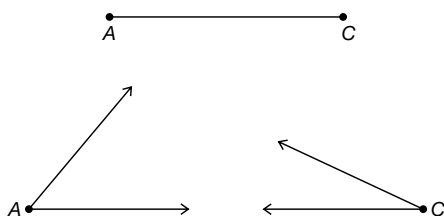


ACTIVITY
2.4

Angle-Side-Angle Congruence Theorem

Are there other theorems that prove triangles congruent?

1. Use the given two angles and included line segment to construct triangle ABC . Then, write the steps you performed to construct the triangle.



.....
An **included side** is the side between two angles of a triangle.
.....

2. Compare your triangle to your classmates' triangles. Are the triangles congruent? Why or why not?

3. What conjecture can this lead you to make about the conditions necessary to prove triangle congruence?



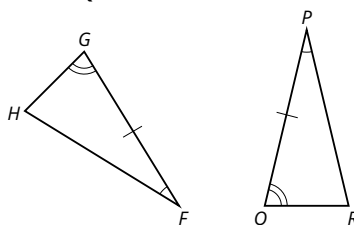
4. Diego says that if the line segment and angles in the construction had not been labeled, then all the triangles would not have been congruent. Mia disagrees, and says that there is only one way to put two angles and a side together to form a triangle, whether they are labeled or not. Who is correct? Explain your reasoning.

Ask Yourself . . .

Did you justify your mathematical reasoning?

Use reflections to prove the Angle-Side-Angle congruence theorem.

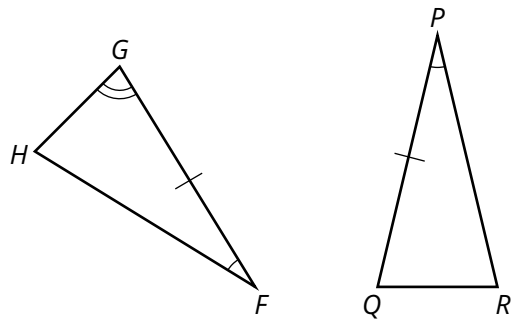
5. Consider $\triangle FGH$ and $\triangle PQR$ where $\overline{FG} \cong \overline{PQ}$, $\angle G \cong \angle Q$, and $\angle F \cong \angle P$.
Prove that $\triangle FGH \cong \triangle PQR$.



Because this relationship has been proved to be true, you may now refer to it as a theorem. The **Angle-Side-Angle congruence theorem (ASA)** states: “If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent.”

6. Since you have prove $\triangle FGH \cong \triangle PQR$ using the conditions necessary for the ASA congruence theorem, apply CPCTC by
- writing congruence statements for all corresponding parts and angle relationships of the pre-image and image.

- Based on part a, use slash markers and arc markers to denote the remaining congruent sides and angles.



ACTIVITY
2.5

More Triangle Congruence Theorems

Let's explore whether other combinations of side and angle measurements can be used to determine whether triangles are congruent.

.....
Remember . . .

The triangle sum theorem states that the sum of the measures of the interior angles of a triangle is 180 degrees.
.....

Let's conjecture that two right triangles can be determined to be congruent by demonstrating that the hypotenuse and acute angle of one triangle are congruent to the hypotenuse and acute angle of the other triangle.

Also, let's conjecture that two triangles can be determined to be congruent by demonstrating that two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle.

Daniel



I can prove both conjectures by using the triangle sum theorem and the ASA congruence theorem.

1. Explain why Daniel is correct. Draw examples to demonstrate your reasoning.

Because you have demonstrated these relationships are true, you can now refer to them as theorems. The **Hypotenuse-Angle (HA) congruence theorem** states: "If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two triangles are congruent." The **Angle-Angle-Side (AAS) congruence theorem** states: "If two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the two triangles are congruent."

ACTIVITY
2.6

Non-Examples of Congruence Theorems

Thus far, you have explored and demonstrated the conditions necessary to prove each of the triangle congruence theorems:

- Side-Side-Side congruence theorem (SSS)
 - Side-Angle-Side congruence theorem (SAS)
 - Angle-Side-Angle congruence theorem (ASA)
 - Angle-Angle-Side congruence theorem (AAS)
 - Hypotenuse Angle congruence theorem (HA)
1. Harper wondered why AAA isn't on the list of congruence theorems. Provide a counterexample to show Harper why Angle-Angle-Angle (AAA) *is not* considered a triangle congruence theorem.



2. Isabella and Ethan each made a statement about the validity of the conditions Side-Side-Angle (SSA) for triangle congruence. Who's correct? Explain your reasoning.

Isabella

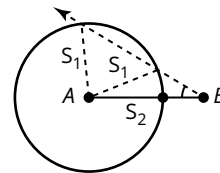
Side-Side-Angle is a valid congruence theorem. It is just like the SAS, ASA and AAS congruence theorems. You only need three parts to determine the triangles are congruent.

Ethan

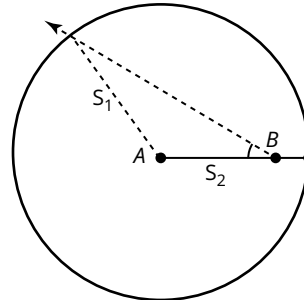
I can provide two different counterexamples to demonstrate that the conditions of Side-Side-Angle are not sufficient to show triangle congruence.

In my drawings line segment AB is the second 'S' in SSA, Point A is used as the center of a circle. The length of the circle radius is the first 'S' in SSA.

CASE 1:



CASE 2:



Ask Yourself

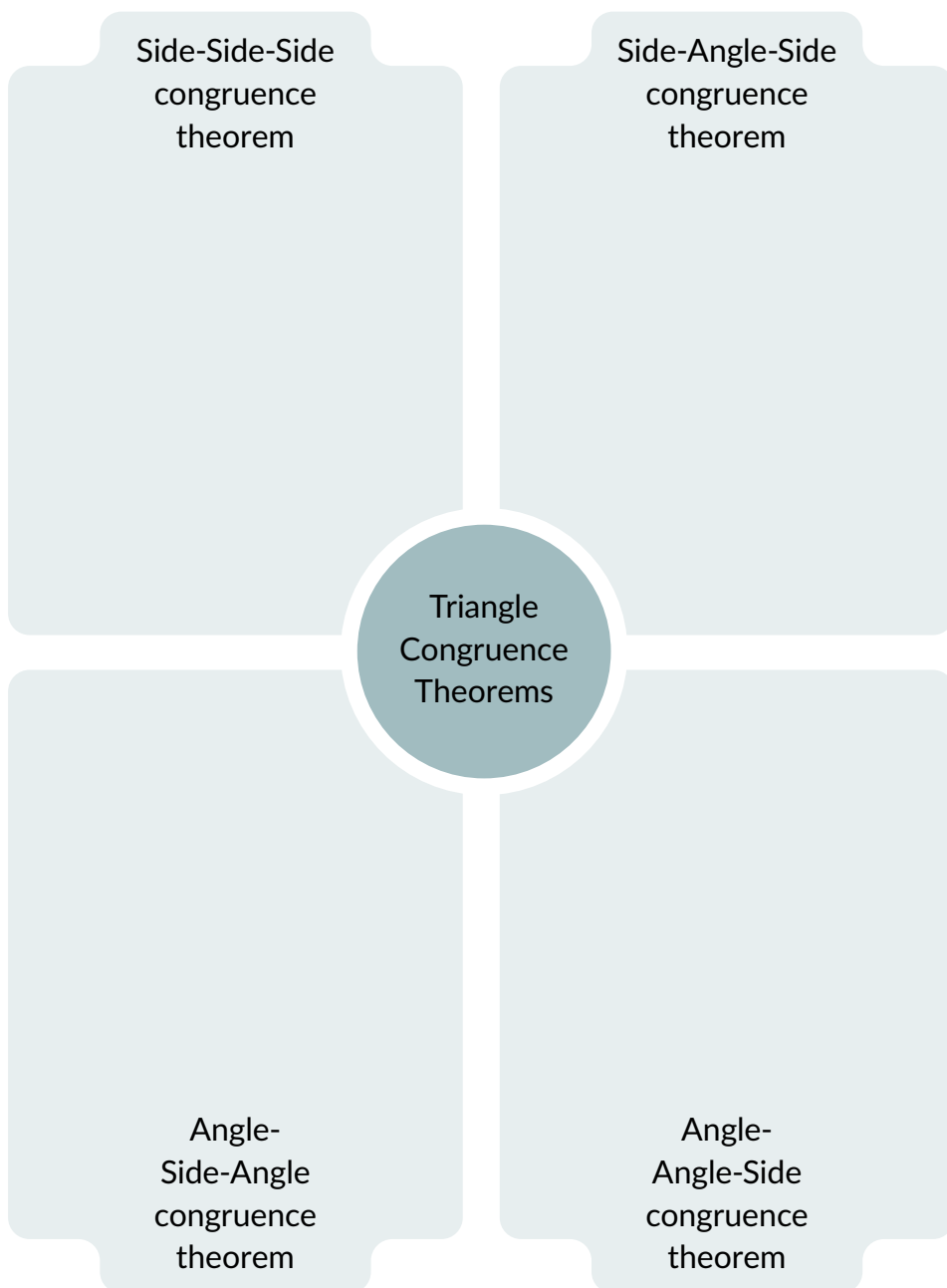
How can you organize and record your mathematical ideas?



Talk the Talk

The Right Combination

1. Complete the graphic organizer to summarize what you have learned about the triangle congruence theorems.



Hypotenuse
Angle congruence
theorem

Non-examples
of triangle
congruence
theorems

Triangle
Congruence
Theorems

Lesson 2 Assignment

Write

Explain how you can list the six pairs of corresponding parts of congruent triangles by using the triangle congruence statement rather than a diagram.

Remember

The SSS congruence theorem states that if three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

The SAS congruence theorem states that if two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

The ASA congruence theorem states that if two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

The HA congruence theorem states that if the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two triangles are congruent.

The AAS congruence theorem states that if two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the two triangles are congruent.

Lesson 2 Assignment

Practice

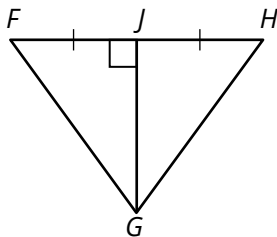
1. Draw two triangles that correspond with each congruence statement. Then list the six pairs of congruent corresponding parts.

a. $\triangle CGB \cong \triangle JMV$

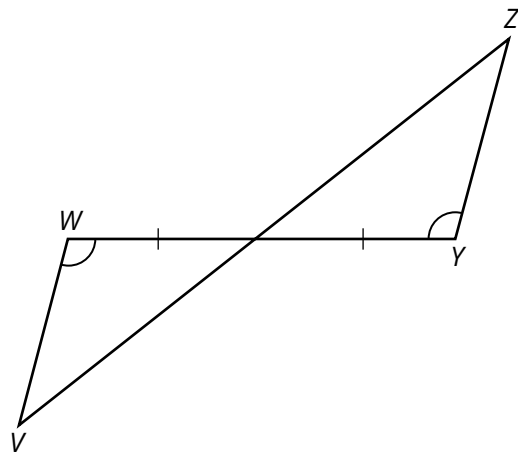
b. $\triangle LBR \cong \triangle MDS$

2. Determine which congruence theorem proves the triangles are congruent.

a. $\triangle FJG \cong \triangle HJG$

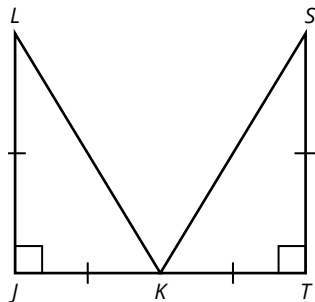


b. $\triangle VWX \cong \triangle ZYX$

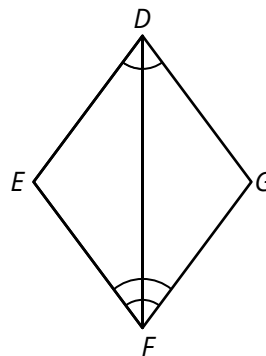


Lesson 2 Assignment

c. $\triangle KJL \cong \triangle KTS$



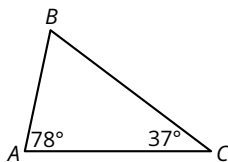
d. $\triangle DEF \cong \triangle DGF$



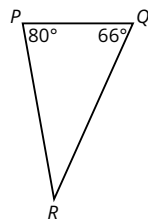
Prepare

Determine the measure of the unknown angle in each triangle.

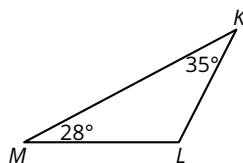
1.



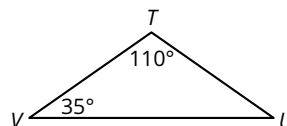
2.



3.



4.



3

Using Triangle Congruence to Solve Problems

OBJECTIVES

- Use triangle congruence theorems to identify congruent triangles.
- Identify the information from the text and discussions to determine if two triangles are congruent by SSS, SAS, or ASA.

KEY TERM

- reflexive property of congruence

.....

You have proven that Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle congruence theorems are valid criteria to determine triangle congruence.

How can you apply these theorems to problems?

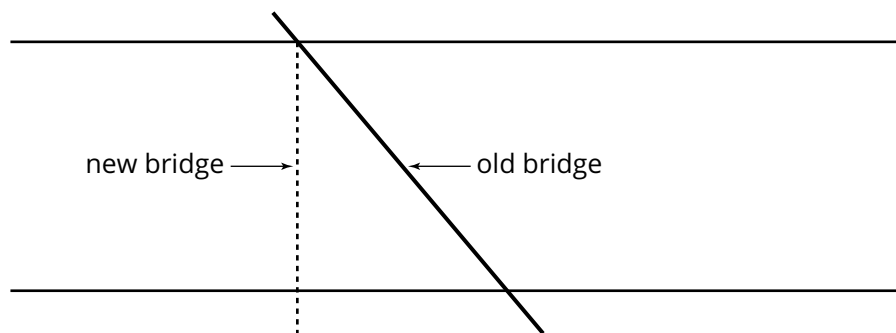
Getting Started

Think about...

You can't assume that an angle is a right angle or that two figures are congruent, even if it seems like it.

A Bridge Too Far

Suppose you are planning to build a new bridge across a river to replace the old bridge, which has grown unusable. You need to know exactly how long to make the bridge, but you can't measure the width of the river. The only thing you know is the length of the old bridge.



1. Explain how you could use congruent triangles to determine the exact length of the new bridge needed to cross the river. Explain your thinking.

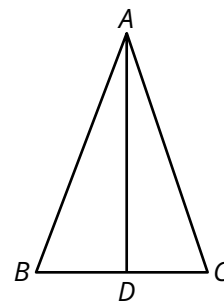
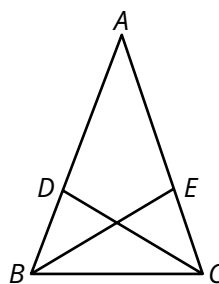
Using SSS, SAS, and ASA

When identifying congruent parts of triangles and other figures, you can use the *reflexive property of congruence*. The **reflexive property of congruence** states that all geometric figures or parts of a geometric figure are congruent to themselves.

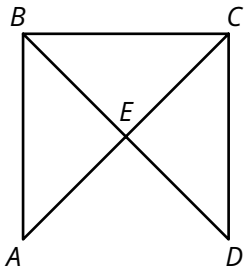
You know that the Side-Side-Side, Side-Angle-Side, and Angle-Side-Angle congruence theorems can be used as valid reasons to demonstrate triangles are congruent. Consider each theorem as you analyze the given statements and diagrams.

- Suppose $\angle DBC \cong \angle ECB$, and $\angle DCB \cong \angle EBC$.

Are there congruent triangles in this diagram?
Explain your reasoning.



Triangle ABD and $\triangle ACD$ share side $\overline{AD}$.
Therefore, $\overline{AD} \cong \overline{AD}$
by the reflexive
property of
congruence.
Triangle ABC and $\triangle ABD$ share $\angle B$.
Therefore, $\angle B \cong \angle B$
by the reflexive
property.



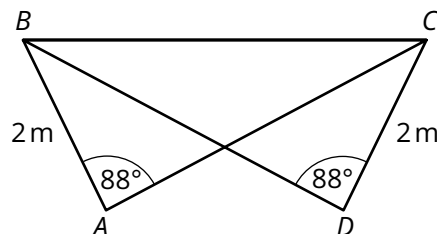
2. Suppose $\overline{AC} \cong \overline{DB}$, $\overline{AB} \perp \overline{BC}$, and $\overline{DC} \perp \overline{CB}$.

a. What information would you need to conclude $\triangle CAB$ is congruent to $\triangle BDC$ using the AAS congruence theorem?

b. What information would you need to conclude $\triangle ABE$ is congruent to $\triangle DCE$ using the SAS congruence theorem?

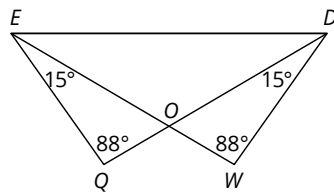


3. Ava says that since $\triangle ABC$ and $\triangle DCB$ have two pairs of congruent corresponding sides and congruent corresponding angles, then the triangles are congruent by SAS. Is Ava correct? Explain your reasoning.

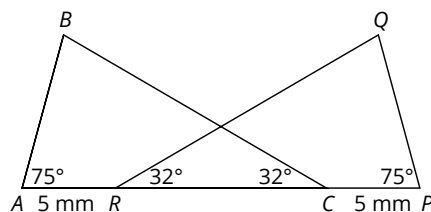


Determine whether there is enough information to prove that the two triangles are congruent. When there is enough information, write the congruence statements to justify your reasoning.

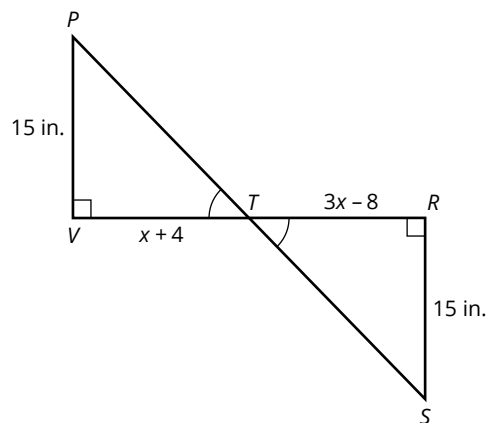
4. $\triangle EQD \stackrel{?}{\cong} \triangle DWE$



5. $\triangle ABC \stackrel{?}{\cong} \triangle PQR$



6. Based on the figure, can the value of x be determined? Explain your reasoning. If so, determine the value of x in the figure.



Remember ...

Corresponding parts of congruent triangles are congruent (CPCTC).

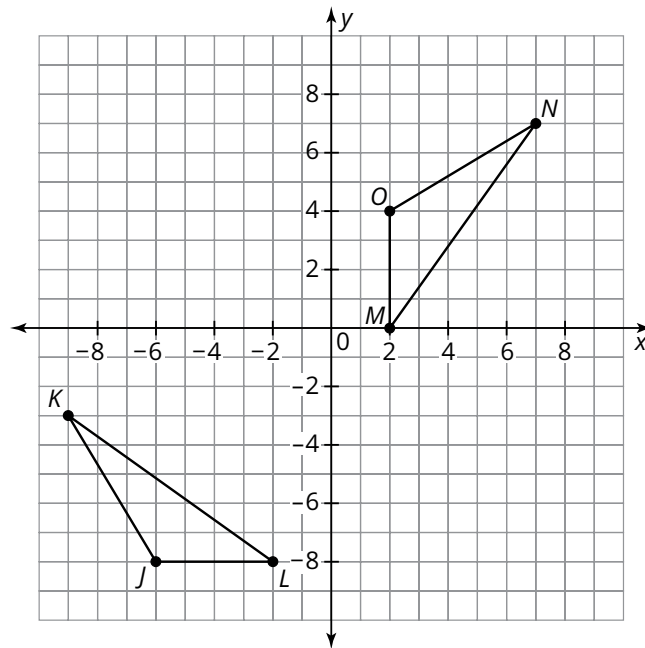
ACTIVITY
3.2

SSS, SAS, and ASA on the Coordinate Plane

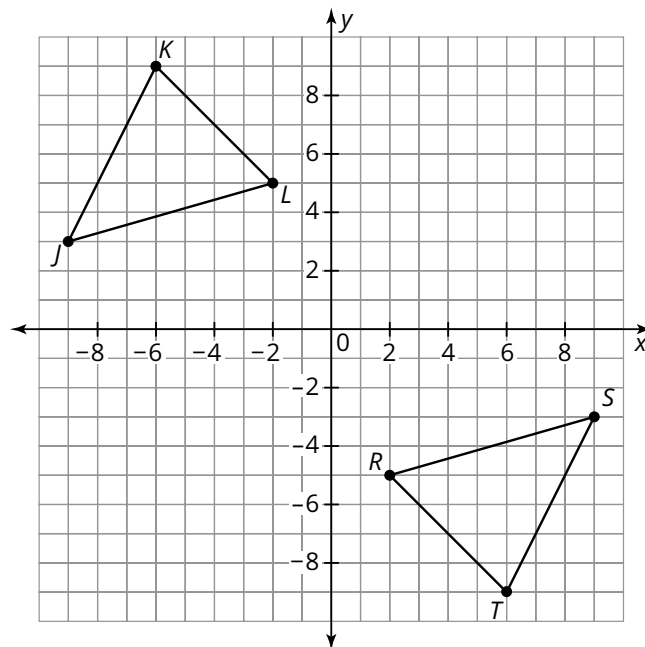
Think about ...

How does your reasoning change when figures are represented on a coordinate plane?

1. Use the distance formula and patty paper to show that the two triangles on the coordinate plane are congruent by SAS. Show your work.



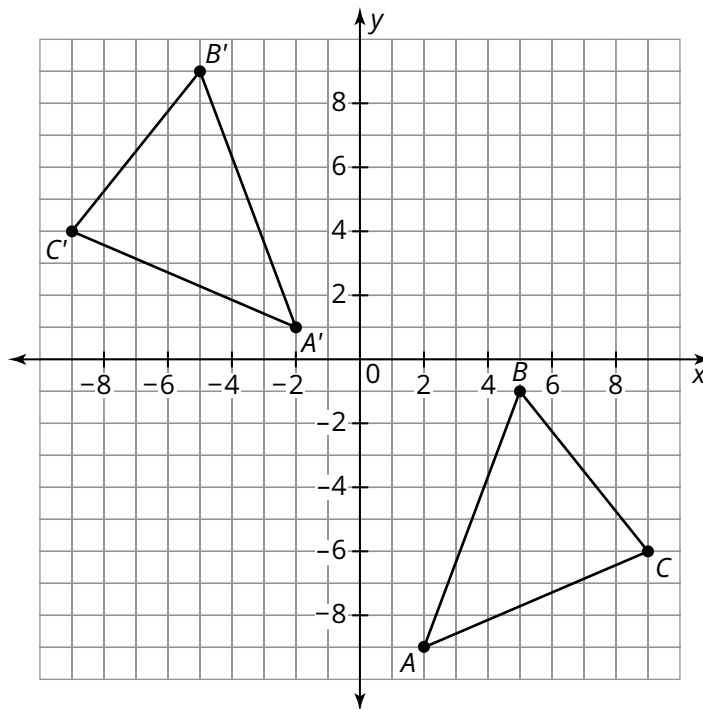
2. Use the distance formula and patty paper to show that the two triangles on the coordinate plane are congruent by SSS. Show your work.



Ask Yourself ...

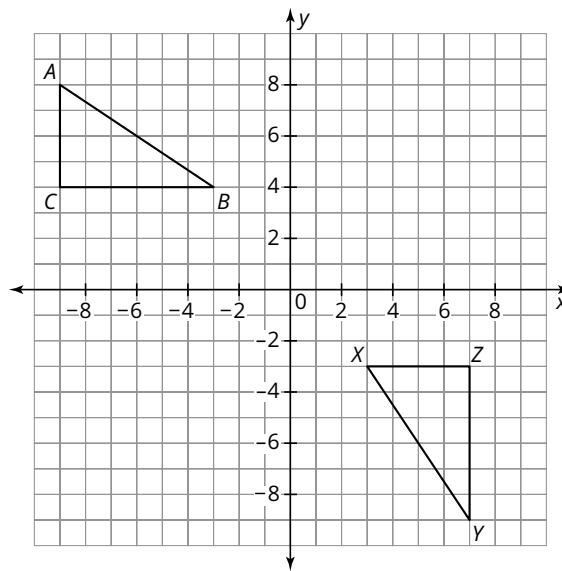
What patterns do you notice?

3. Emma wants to translate $\triangle ABC$ and then reflect it across the y -axis to form a new triangle in Quadrant II. She uses what she knows about transformations to determine the vertices of $\triangle A'B'C'$ before performing the transformations.

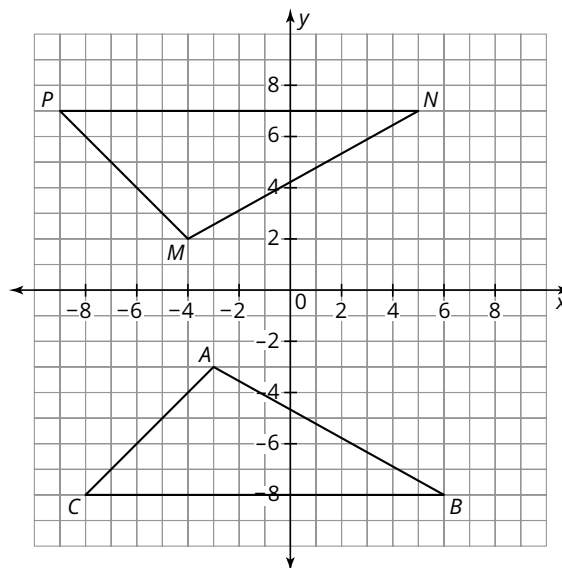


- a. Describe how Emma can use the ASA congruence theorem to determine whether or not she transformed $\triangle ABC$, such that the image is congruent to the pre-image.
- b. Did Emma perform the transformations on $\triangle ABC$ so that the image is congruent to the pre-image? Explain your reasoning.

4. Describe how to prove the given triangles are congruent. Use the key terms *included angle* and *Side-Angle-Side congruence theorem* in your answer.



5. Describe how to prove the given triangles are congruent. Use the key terms *side* and *Angle-Side-Angle congruence theorem* in your answer.





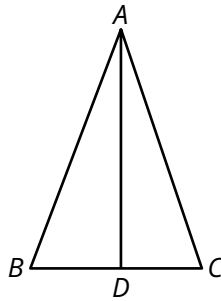
Talk the Talk

Camera Congruence

A security camera is positioned at the corner of a rectangular store, aiming diagonally across the room to monitor a specific shelf. The camera's field of view creates a triangular area of the floor that is visible, with the corner of the store and two points on the opposite wall as vertices of the triangle. A second store is built with the same layout.

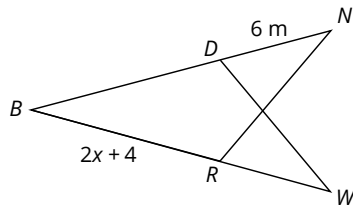
1. How can you use the properties of congruent triangles to determine whether the camera at the second store is placed in the same position as the camera at the first store?

2. A flagpole stands upright, perpendicular to the ground, at point D . Two support cables anchored to the ground stabilize it at points B and C .



What information would you need to conclude $\triangle ABD$ is congruent to $\triangle ACD$ using the SAS congruence theorem?

3. Determine the value of x , given $\triangle BDW \cong \triangle BRN$ and $BW = 16$ m.



Lesson 3 Assignment

Write

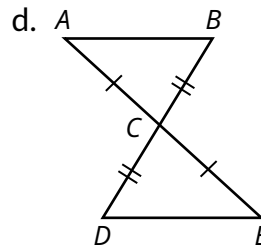
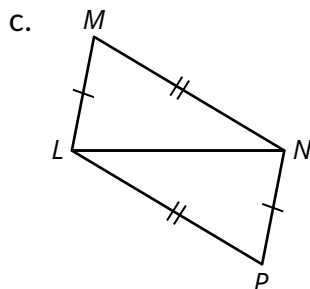
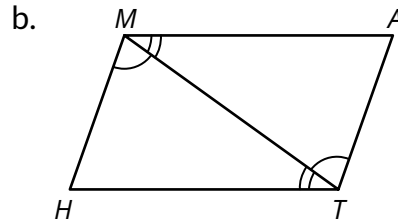
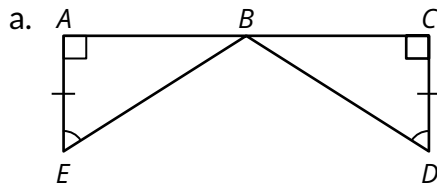
Explain in your own words how to determine which congruence theorem to use to identify congruent triangles.

Remember

The SSS, SAS, and ASA conditions for triangle congruence can be applied on and off the coordinate plane to solve real-world and mathematical problems.

Practice

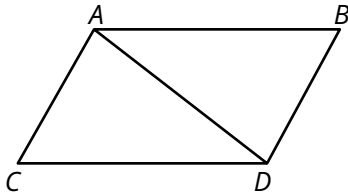
1. State the theorem that proves the triangles are congruent. Then write a congruence statement.



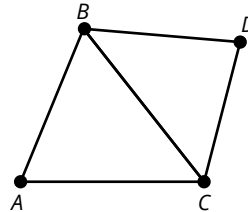
Lesson 3 Assignment

2. For each figure, determine whether there is enough information to conclude that the indicated triangles are congruent. If so, state the theorem you used.

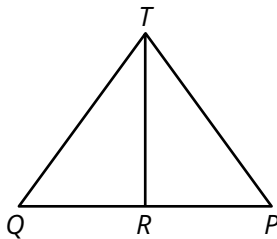
- a. Given $\overline{AC} \parallel \overline{BD}$, $\overline{AB} \parallel \overline{CD}$
Is $\triangle ABD \cong \triangle DCA$?



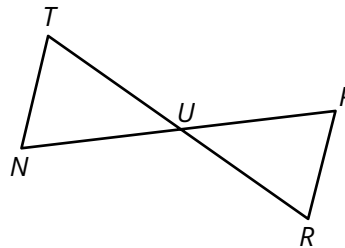
- b. Given: $\angle A \cong \angle D$ and $\angle ABC \cong \angle DBC$
Is $\triangle ABC \cong \triangle DBC$?



- c. Given: $\overline{TQ} \cong \overline{TP}$
Is $\triangle TRQ \cong \triangle TRP$?

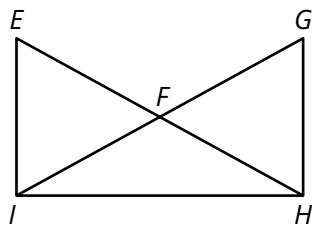


- d. Given: $\overline{TR}$ intersects $\overline{NP}$, $\overline{TU} \cong \overline{RU}$,
 $\overline{NU} \cong \overline{PU}$
Is $\triangle TUN \cong \triangle RUP$?

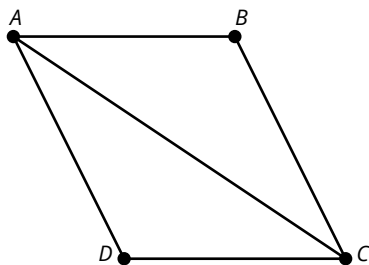


Lesson 3 Assignment

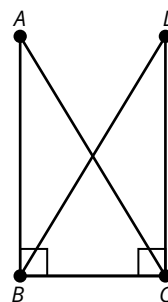
- e. Given $\overline{EI} \cong \overline{GH}$, $\overline{EH} \cong \overline{GI}$,
Is $\triangle EHI \cong \triangle GIH$?



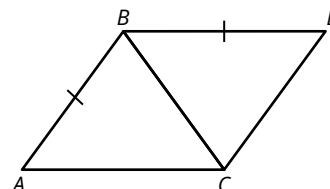
- g. Given: $\overline{AB} \parallel \overline{DC}$ and $\angle B \cong \angle D$
Is $\triangle ABC \cong \triangle ADC$?



- f. Given: $\overline{AC} \cong \overline{DB}$ and $\angle A \cong \angle D$
Is $\triangle ABC \cong \triangle DCB$?

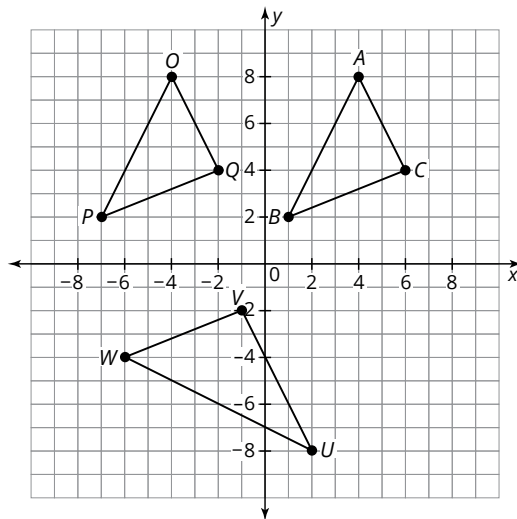


- h. Given: $\overline{AB} \cong \overline{BD}$
Is $\triangle ABC \cong \triangle DBC$?



Lesson 3 Assignment

3. Analyze the triangles shown.

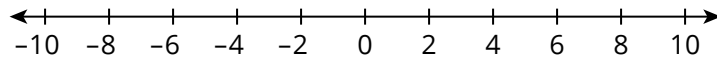


- Use the ASA congruence theorem and a protractor to determine whether $\triangle ABC$ is congruent to $\triangle OPQ$.
- Use the AAS congruence theorem and a protractor to determine whether $\triangle ABC$ is congruent to $\triangle UVW$.

Lesson 3 Assignment

Prepare

Identify the opposite of each number on the number line.



1. -8

2. $-|-2|$

3. 9

4. $-(-7)$

Congruence Through Transformations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Congruence Through Transformations* topic by:

TOPIC 4: <i>Congruence Through Transformations</i>	Beginning of Topic	Middle of Topic	End of Topic
calculating the number of lines of symmetry and the degree of rotational symmetry of regular polygons.			
identifying corresponding sides and corresponding angles of congruent triangles.			
explaining that in a pair of congruent triangles, corresponding sides are congruent and corresponding angles are congruent.			
defining congruent figures as figures that have the same shape and size and describing a sequence of rigid motions that maps one onto the other.			
listing the sufficient criteria to prove triangles are congruent.			
mapping a triangle with one of the sufficient criteria onto the original triangle and showing that corresponding sides and angles are congruent.			

continued on the next page

TOPIC 4 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Congruence Through Transformations* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 4 SUMMARY

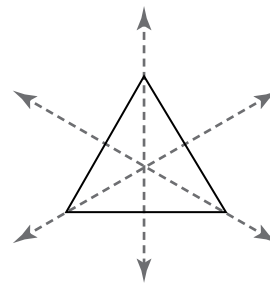
Congruence Through Transformations

LESSON

1

Reflectional and Rotational Symmetry

A plane figure has **reflectional symmetry** if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side of the line. The line that you can draw on each shape is called the line of symmetry. A figure can have more than one line of symmetry. The triangle shown has three lines of reflectional symmetry.



A plane figure can also have **rotational symmetry** if you can rotate the figure more than 0° but less than 360° and the resulting figure is the same as the original figure in the original position.

A shape, such as a square, can have both reflectional and rotational symmetry. The standard alphabet has many letters that have a variety of symmetries, including reflectional and rotational symmetry. Some letters have a vertical line of symmetry. Other letters have a horizontal line of symmetry.

LESSON

2

Proving Triangle Congruence Theorems

It cannot be assumed that angles are right angles or that two figures are congruent, even if they appear to be. Labels and symbols indicate right angles and congruent figures.

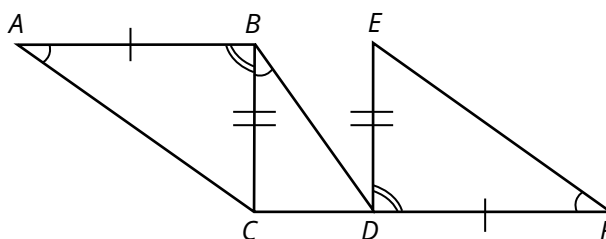
Congruent line segments and congruent angles are often denoted using special markers rather than given measurements. Slash markers indicate congruent line segments. When multiple line segments contain a single slash marker, this implies that all of the line segments are congruent. Double and triple slash markers also denote other line segment congruencies. Arc markers indicate congruent angles. When multiple angles contain a single arc marker, this implies that those angles are congruent. Double and triple arc markers denote other angle congruencies.

KEY TERMS

- reflectional symmetry [simetría de reflexión]
- rotational symmetry [simetría de rotación]
- Side-Side-Side congruence theorem (SSS)
- corresponding parts of congruent triangles are congruent (CPTTC) [las partes correspondientes de triángulos congruentes son congruentes]
- Side-Angle-Side congruence theorem (SAS)
- included angle [ángulo incluido]
- Angle-Side-Angle congruence theorem (ASA)
- included side
- Hypotenuse Angle congruence theorem (HA) [Teorema de congruencia del ángulo hipotenusa]
- Angle-Angle-Side congruence theorem (AAS)
- reflexive property of congruence [propiedad reflexiva de la congruencia]

The markers on the diagram indicate congruent line segments and angles.

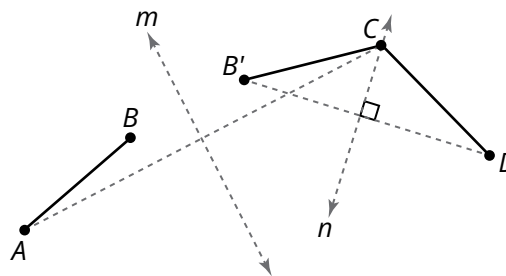
$$\begin{aligned}\overline{AB} &\cong \overline{DF} \text{ and } \overline{BC} \cong \overline{ED} \\ \angle BAC &\cong \angle CBD \cong \angle DFE \\ \angle ABC &\cong \angle FDE\end{aligned}$$



You can prove the conjecture that you can always map one congruent line segment onto the other using at most two reflections.

For example, suppose that $\overline{AB} \cong \overline{CD}$.

Since any point in the plane can be reflected across a line to map onto another point in the plane, point C is a reflection of point A across line m . Reflect $\overline{AB}$ across this line.



Reflections preserve distances, so $\overline{CB'} \cong \overline{CD}$, because $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \cong \overline{CB'}$. Thus, point C is equidistant from point B' and from point D.

Since a point is equidistant from two other points if and only if it lies on their perpendicular bisector, point C lies on the perpendicular bisector of $\overline{B'D}$ (line n). Thus, one last reflection across the perpendicular bisector maps $\overline{CB'}$ to $\overline{CD}$.

You can prove that triangles are congruent given different minimal criteria.

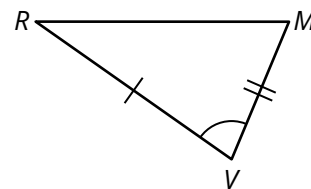
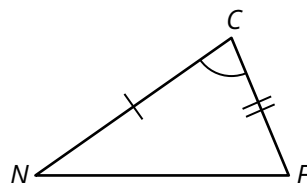
The **Side-Side-Side congruence theorem (SSS)** states, “If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.”

If two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle. **Corresponding parts of congruent triangles are congruent**, abbreviated as **CPCTC**, is often used as a reason in proofs. CPCTC states that corresponding angles or sides in two congruent triangles are congruent. This reason can only be used after you have proven that the triangles are congruent.

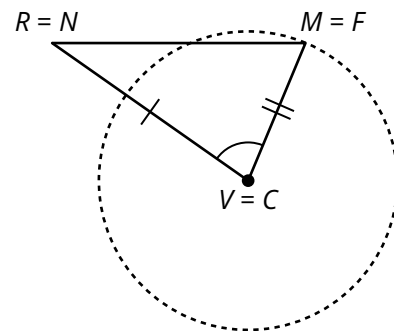
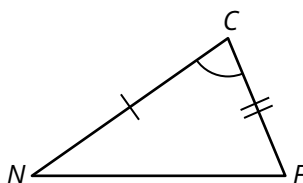
The **Side-Angle-Side congruence theorem (SAS)** states, “If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent.” An **included angle** is the angle formed by two sides of a triangle. Consider this proof of the Side-Angle-Side congruence theorem:

Given $\triangle CNF$ and $\triangle VRM$, $\overline{CN} \cong \overline{VR}$, $\overline{CF} \cong \overline{VM}$, and $\angle C \cong \angle V$.

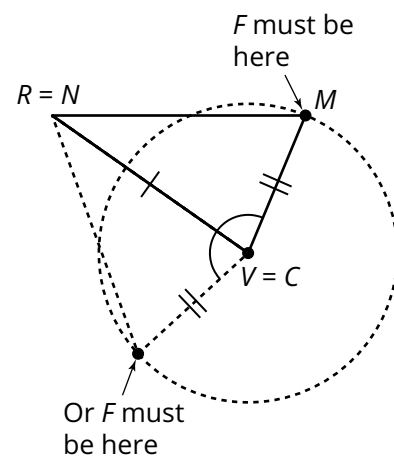
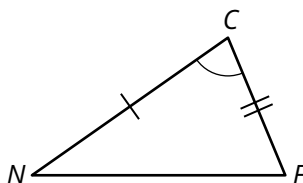
You can map $\overline{CN}$ to $\overline{VR}$ in one or two reflections.



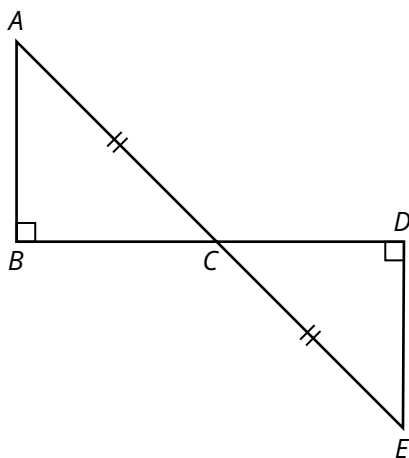
Since reflections preserve distance, this means that point F must be on the circle centered at V with radius CM , because $\overline{CF} \cong \overline{CM}$.



Since reflections preserve angle measures, there are only two possible locations for point F to be on the circle. If point F is not at M , then a reflection across $\overline{VR}$ will map point F to point M .

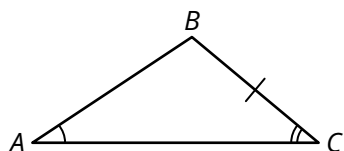


The **Angle-Side-Angle congruence theorem (ASA)** states, “If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent.” An **included side** is a line segment between two consecutive angles of a figure. The proof of this theorem is similar to how the Side-Angle-Side congruence theorem is proven.



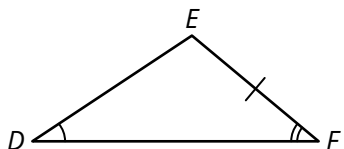
The **Hypotenuse-Angle (HA) congruence theorem** states: “If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two triangles are congruent.”

For example, in right triangle ABC , $\overline{AC}$ is the hypotenuse and in right triangle EDC , $\overline{EC}$ is the hypotenuse. Since $\overline{AC} \cong \overline{EC}$ and $\angle ACB \cong \angle ECD$ by the Vertical Angle Theorem, then $\triangle ABC \cong \triangle EDC$.



The **Angle-Angle-Side (AAS) congruence theorem** states: “If two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the two triangles are congruent.”

For example, $\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$, and $\angle C \cong \angle F$. Therefore, $\triangle ABC \cong \triangle DEF$ by AAS.



AAA and SSA are not triangle congruence theorems. Having these congruent parts of triangles does not mean that the triangles themselves are congruent.

LESSON 3

Using Triangle Congruence Theorems to Solve Problems

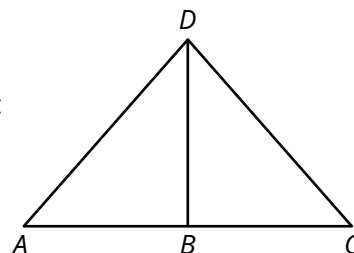
When identifying congruent parts of triangles and other figures, you can use the *reflexive property of congruence*. The **reflexive property of congruence** states that all geometric figures or parts of a geometric figure are congruent to themselves.

You can use the triangle congruence theorems to solve problems on and off the coordinate plane.

Given $\overline{AD} \cong \overline{DC}$ and $\overline{DB}$ bisects $\overline{AC}$.

You can use the Side-Side-Side congruence theorem to demonstrate that $\triangle ADB$ is congruent to $\triangle CDB$.

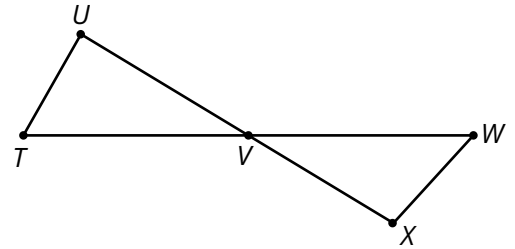
- Using the definition of a bisector, B is the midpoint of $\overline{AC}$ and therefore $\overline{AB} \cong \overline{BC}$.



- Since $\overline{DB}$ is the same side in each triangle, then $\overline{DB} \cong \overline{DB}$.
- Therefore, $\triangle ADB \cong \triangle CDB$ by SSS.

Given V is the midpoint of $\overline{UX}$ and V is the midpoint of $\overline{TW}$.
You can use the Side-Angle-Side congruence theorem to demonstrate that $\triangle UVT$ is congruent to $\triangle XVW$.

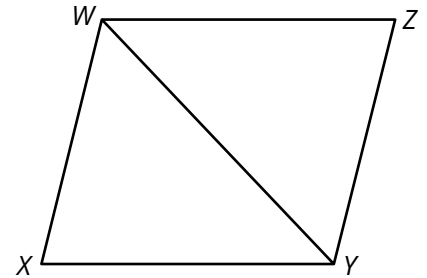
- By definition of a midpoint $\overline{UV} \cong \overline{XV}$ and $\overline{TV} \cong \overline{WV}$.
- Using the definition of vertical angles, $\angle UVT \cong \angle XVW$.
- Therefore, $\triangle UVT \cong \triangle XVW$ by SAS.



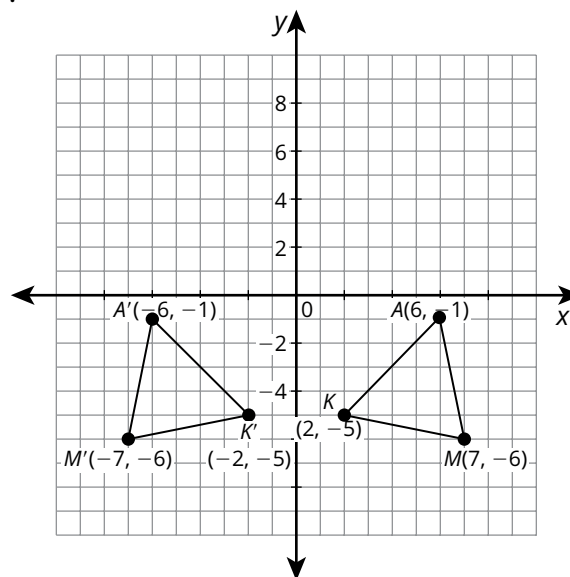
Given $\overline{WZ} \parallel \overline{XY}$ and $\overline{WX} \parallel \overline{ZY}$.

You can use the Angle-Side-Angle congruence theorem to demonstrate that $\triangle WXY$ is congruent to $\triangle YZW$.

- Using the definition of alternate interior angles, $\angle ZWY \cong \angle XYW$ and $\angle XWY \cong \angle ZYW$.
- Since $\overline{WY}$ is the same side in each triangle, then $\overline{WY} \cong \overline{WY}$.
- Therefore, $\triangle WXY \cong \triangle YZW$ by ASA.



For example, use the SAS congruence theorem to prove that $\triangle AMK$ is congruent to $\triangle A'M'K'$.



Use the distance formula to determine the length of each side.

$$\begin{aligned}KA &= \sqrt{(6 - 2)^2 + [-1 - (-5)]^2} \\&= \sqrt{4^2 + 4^2} \\&= \sqrt{16 + 16} \\&= \sqrt{32}\end{aligned}$$

$$\begin{aligned}K'A' &= \sqrt{[-5 - (-1)]^2 + [-2 - (-6)]^2} \\&= \sqrt{(-4)^2 + 4^2} \\&= \sqrt{16 + 16} \\&= \sqrt{32}\end{aligned}$$

$$\begin{aligned}KM &= \sqrt{(7 - 2)^2 + [-6 - (-5)]^2} \\&= \sqrt{5^2 + (-1)^2} \\&= \sqrt{25 + 1} \\&= \sqrt{26}\end{aligned}$$

$$\begin{aligned}K'M' &= \sqrt{[-7 - (-2)]^2 + [-6 - (-5)]^2} \\&= \sqrt{(-5)^2 + (-1)^2} \\&= \sqrt{25 + 1} \\&= \sqrt{26}\end{aligned}$$

Then, use a protractor to measure $\angle K$ and $\angle K'$.

$$m\angle K = 58^\circ$$

$$m\angle K' = 58^\circ$$

The lengths of the pairs of the corresponding sides and the measures of the pair of corresponding included angles are equal. Therefore, the triangles are congruent by the SAS congruence theorem.

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