Números Binomiais

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Números Binomiais

Número Binomial

$$\binom{n}{p} = \frac{n!}{p!.(n-p)!} = C_n^p$$

$n \geq p,\ n \in N, p \in N$

Exemplo

$$\binom{6}{2} = \frac{6!}{2!.(6-2)!}$$

$$\binom{6}{2} = \frac{6.5.4!}{2.1.4!} = 15$$

Exemplos

1. $\binom{7}{2} = \frac{7!}{2!.5!} = 21$
2. $\binom{6}{5} = \frac{6!}{5!.1!} = 6$
3. $\binom{7}{5} = \frac{7!}{5!.2!} = 21$
4. $\binom{8}{0} = \frac{8!}{0!.8!} = 1$

Consequências

1. $\binom{n}{0} = 1$
2. $\binom{n}{n} = 1$
3. $\binom{n}{1} = n$
4. $\binom{n}{n-1} = n$

Binomiais Complementares

$$\binom{7}{2} = \binom{7}{5} \qquad \binom{6}{4} = \binom{6}{2}$$

$$\binom{8}{3} = \binom{8}{5} \qquad \binom{5}{1} = \binom{5}{4}$$

$$x = 4 \qquad \binom{9}{x} = \binom{9}{4} \qquad x + 4 = 9$$

$$x = 5$$

$$S = \{4,5\}$$

Resolva a equação:

$$\binom{11}{2x-11} = \binom{11}{x+1}$$

$2x - 11 = x + 1$

~~$x = 12$~~

$2x - 11 + x + 1 = 11$

$3x = 21$

$x = 7$ ✓

$S = \{7\}$

Números Binomiais

Relação de Stifel

1. $\binom{7}{5}+\binom{7}{6}=\binom{8}{6}$

2. $\binom{10}{2}+\binom{10}{3}=\binom{11}{3}$

3. $\binom{5}{4}+\binom{5}{5}=\binom{6}{5}$

4. $\binom{81}{44}+\binom{81}{45}=\binom{82}{45}$

Relação de Stifel

$$\binom{n}{p}+\binom{n}{p+1}=\binom{n+1}{p+1}$$

Exemplo

$$\binom{7}{2}+\binom{7}{3}+\binom{8}{4}=\binom{9}{5}$$ (V)

$$\binom{8}{3}+\binom{8}{4}=\binom{9}{4}$$

Triangulo de Pascal

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

1 ➡ 2^0

1 1 ➡ 2^1

1 2 1 ➡ 2^2

1 3 3 1 ➡ 2^3

1 4 6 4 1 ➡ 2^4

Números Binomiais

1					
1	1				
1	2	1			
		+			
1	3	3	1		
		+			
1	4	6	4	1	
		=			
1	5	10	10	5	1

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

Números Binomiais

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$$1 + 3 + 6 + 10 = 20$$

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

De quantas formas podemos deixar um castelo aberto, sendo que ele possui 6 portas de entrada e ainda pode-se deixar de uma a todas as portas abertas?

a) 64
b) 63
c) 248
d) 254
e) 256

$$C_6^1 + C_6^2 + C_6^3 + \ldots + C_6^6$$

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \ldots + \binom{6}{6}$$

$$2^6 - \binom{6}{0}$$

$$64 - 1$$

$$63$$

Obrigado

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