

# Transformação em produto

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Matemática

# Fórmulas de Prostaferese

$$\text{sen}(a+b) = \text{sen}(a).\text{cos}(b) + \text{sen}(b).\text{cos}(a)$$

$$\text{Cos}(a+b) = \text{cos}(a).\text{cos}(b) - \text{sen}(a).\text{sen}(b)$$

$$\text{Tg}(a+b) = \frac{\text{tg}(a) + \text{tg}(b)}{1 - \text{tg}(a).\text{tg}(b)}$$

## Transformação em produto

$$\text{sen}(a+b) = \text{sen}(a).\text{cos}(b) + \text{sen}(b).\text{cos}(a)$$

$$\text{sen}(a-b) = \text{sen}(a).\text{cos}(b) - \text{sen}(b).\text{cos}(a)$$



$$\text{sen}(a+b) + \text{sen}(a-b) = 2.\text{sen}(a).\text{cos}(b)$$

$$\begin{cases} a + b = p \\ a - b = q \end{cases} \rightarrow a = \frac{p+q}{2} \rightarrow b = \frac{p-q}{2}$$

$$\text{Sen}(p) + \text{sen}(q) = 2.\text{sen}\left(\frac{p+q}{2}\right).\text{cos}\left(\frac{p-q}{2}\right)$$

## Transformação em produto

$$\text{sen}(a+b) = \text{sen}(a).\text{cos}(b) + \text{sen}(b).\text{cos}(a)$$

$$\text{sen}(a-b) = \text{sen}(a).\text{cos}(b) - \text{sen}(b).\text{cos}(a)$$

$$\text{sen}(a+b) - \text{sen}(a-b) = 2.\text{sen}(b).\text{cos}(a)$$

$$\begin{cases} a + b = p \\ a - b = q \end{cases} \rightarrow a = \frac{p+q}{2} \rightarrow b = \frac{p-q}{2}$$

$$\text{Sen}(p) - \text{sen}(q) = 2.\text{sen}\left(\frac{p-q}{2}\right).\text{cos}\left(\frac{p+q}{2}\right)$$

## Transformação em produto

$$\cos(a+b) = \cos(a).\cos(b) + \text{sen}(b).\text{sen}(a)$$

$$\cos(a-b) = \cos(a).\cos(b) - \text{sen}(b).\text{sen}(a)$$



$$\cos(a+b) - \cos(a-b) = 2.\cos(a).\cos(b)$$

$$\begin{cases} a + b = p \\ a - b = q \end{cases} \rightarrow a = \frac{p+q}{2} \rightarrow b = \frac{p-q}{2}$$

$$\cos(p) + \cos(q) = 2.\cos\left(\frac{p+q}{2}\right).\cos\left(\frac{p-q}{2}\right)$$

## Transformação em produto

$$\cos(a+b) = \cos(a).\cos(b) - \text{sen}(b).\text{sen}(a)$$

$$\cos(a-b) = \cos(a).\cos(b) + \text{sen}(b).\text{sen}(a)$$

$$\cos(a+b) - \cos(a-b) = -2.\text{sen}(a).\text{sen}(b)$$

$$\begin{cases} a + b = p \\ a - b = q \end{cases}$$

$$\rightarrow a = \frac{p+q}{2} \quad \rightarrow b = \frac{p-q}{2}$$

$$\cos(p) - \cos(q) = -2.\text{sen}\left(\frac{p+q}{2}\right).\text{sen}\left(\frac{p-q}{2}\right)$$

## Transformação em produto

$$\text{Sen}(p) + \text{sen}(q) = 2 \cdot \text{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

$$\text{Sen}(p) - \text{sen}(q) = 2 \cdot \text{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right)$$

$$\cos(p) + \cos(q) = 2 \cdot \cos\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

$$\cos(p) - \cos(q) = -2 \cdot \text{sen}\left(\frac{p+q}{2}\right) \cdot \text{sen}\left(\frac{p-q}{2}\right)$$

# Transformação em produto

1. Simplificar a expressão:  $E = \frac{\text{sen}40^\circ + \text{sen}10^\circ}{\text{cos}15^\circ}$

$$\frac{2.\text{sen}\left(\frac{\quad + \quad}{2}\right).\text{cos}\left(\frac{\quad - \quad}{2}\right)}{\text{cos}15^\circ}$$

$$\frac{2.\text{sen}(25^\circ).\text{cos}(15^\circ)}{\text{cos}(15^\circ)}$$

$$E = 2.\text{sen}(25^\circ)$$



## Transformação em produto

(ITA) Simplifique o valor de  $y = \frac{\text{sen}3x + \text{sen}x}{4.\text{sen}x.\text{cos}x}$

$$\frac{2.\text{sen}\left(\frac{\quad}{\quad}\right).\text{cos}\left(\frac{\quad}{\quad}\right)}{4.\text{sen}x.\text{cos}x}$$

$$\frac{\cancel{2.2}.\cancel{\text{sen}x}.\cancel{\text{cos}x}.\text{cos}(x)}{\cancel{4.\text{sen}x}.\cancel{\text{cos}x}}$$

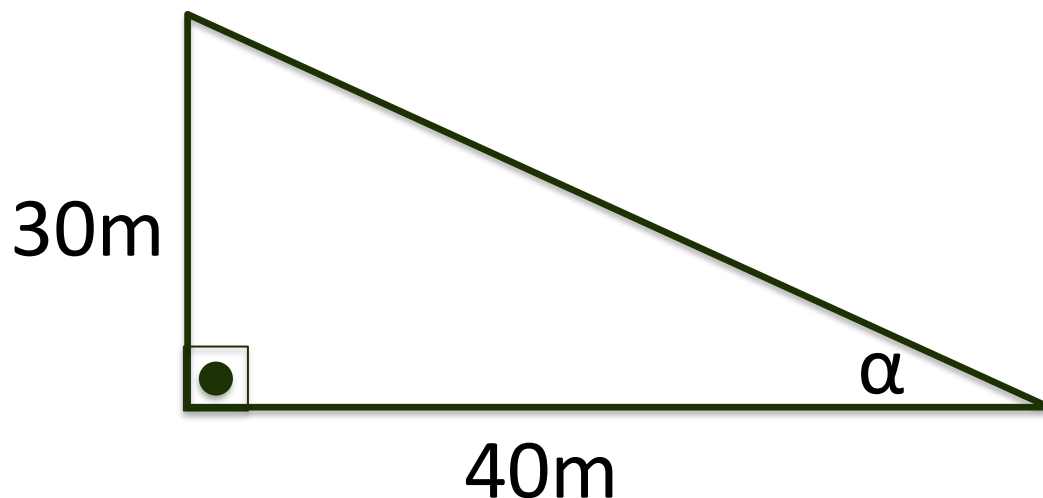
$$\frac{2.\text{sen}(2x).\text{cos}(x)}{4.\text{sen}x.\text{cos}x}$$

$$y = \text{cos}x$$

## Transformação em produto

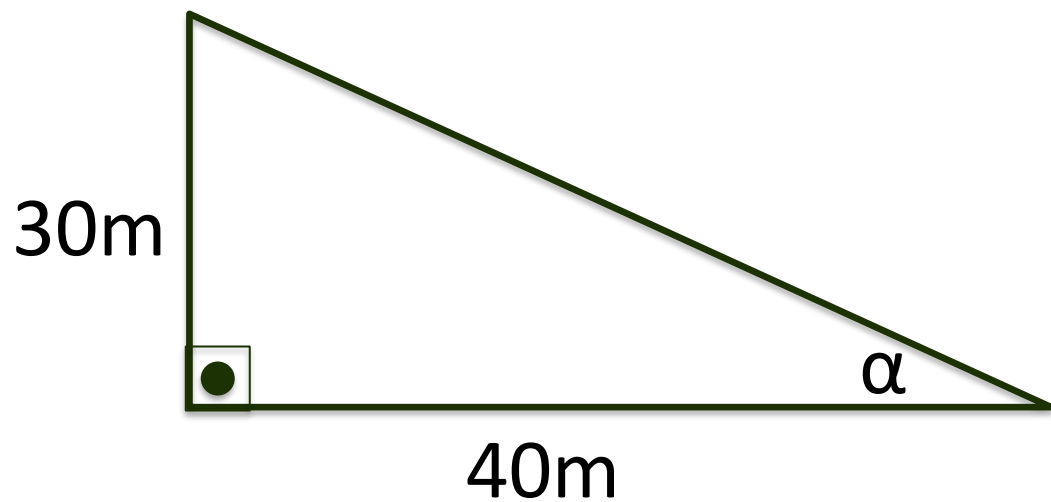
### Função circular inversa

João colocou um instrumento chamado teodolito para medir ângulos a 40m de um prédio de 30m de altura. Qual o ângulo  $\alpha$  medido por João?



# Transformação em produto

Função circular inversa



$$\text{Tg}\alpha = \frac{30}{40}$$

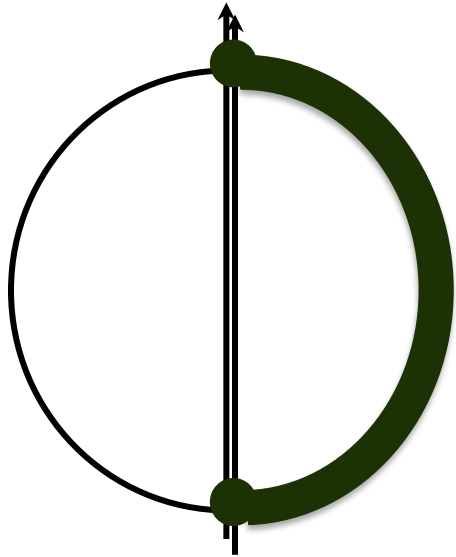
$$\text{Tg}\alpha = \frac{3}{4}$$

$$\alpha = \text{arctg } 3/4$$

# Transformação em produto

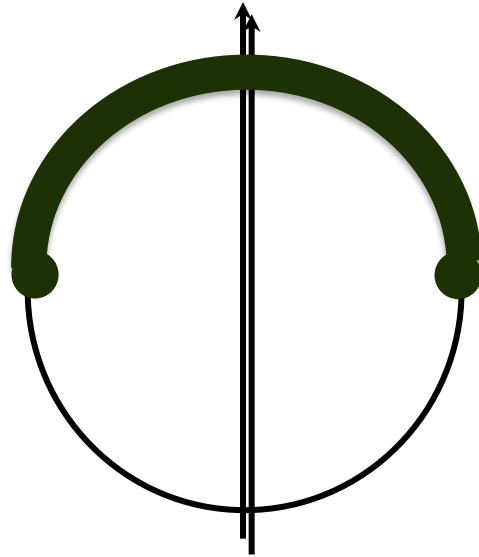
## Função circular inversa

$$y = \arcsen x$$



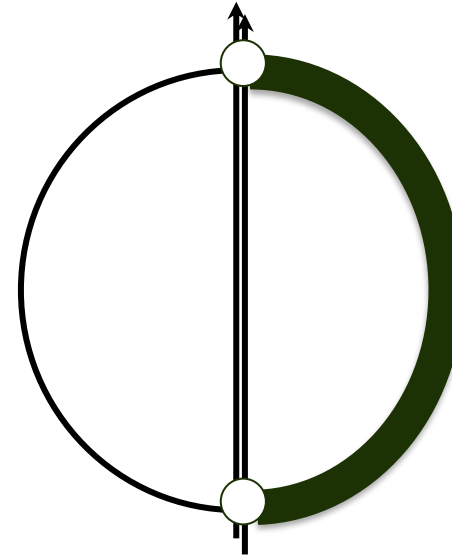
$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq y \leq \pi$$

$$y = \arctg x$$



$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$

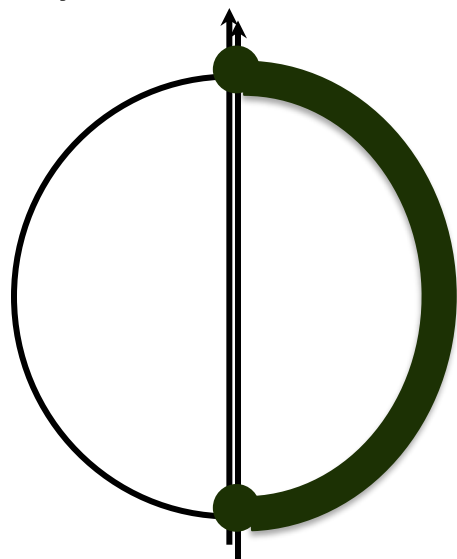
Calcule x nos seguintes exemplos:

a)  $x = \arcsen(1/2)$     $30^\circ$

# Transformação em produto

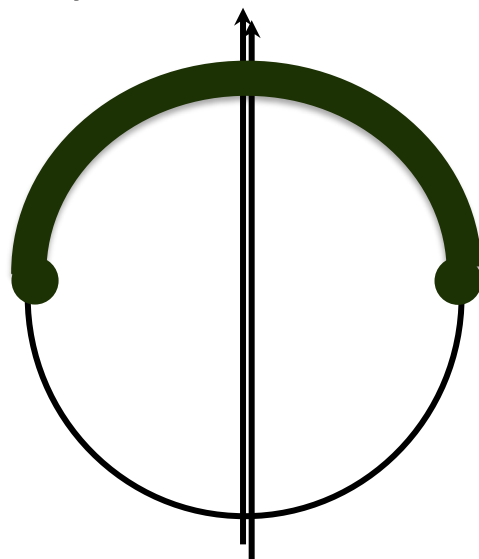
## Função circular inversa

$$y = \arcsen x$$



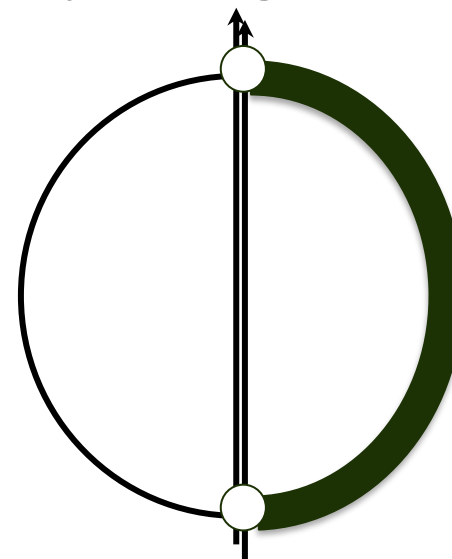
$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq x \leq \pi$$

$$y = \arctg x$$



$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

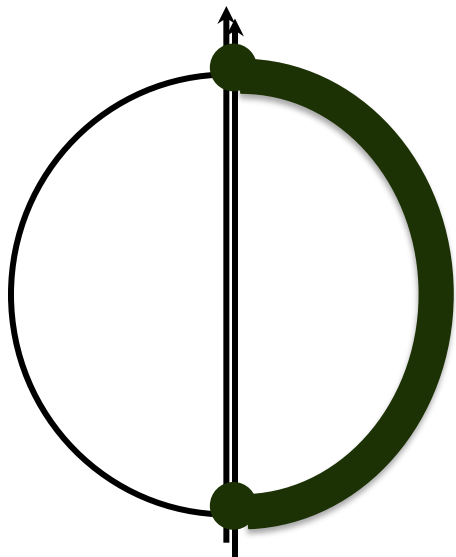
Calcule x nos seguintes exemplos:

b)  $x = \arcsen(-1/2) - 30^\circ$

# Transformação em produto

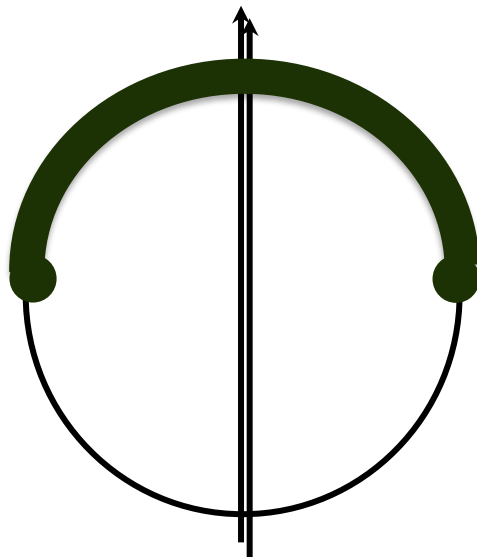
## Função circular inversa

$$y = \arcsen x$$



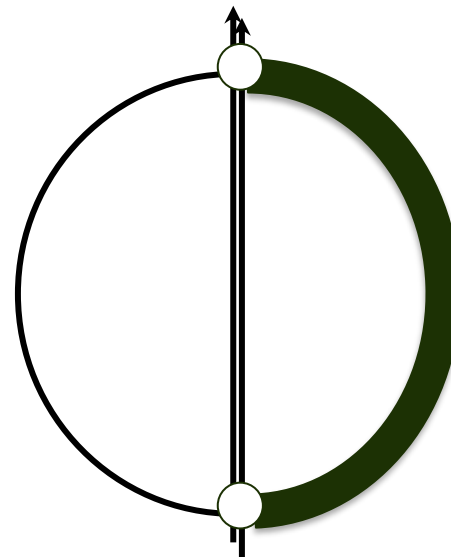
$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq x \leq \pi$$

$$y = \arctg x$$



$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

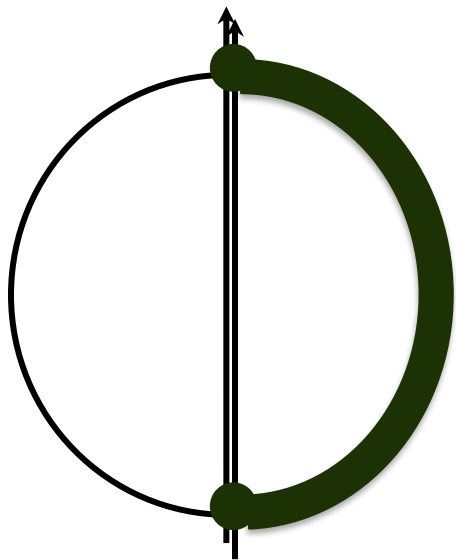
Calcule x nos seguintes exemplos:

c)  $x = \arccos(1/2) \quad 60^\circ$

# Transformação em produto

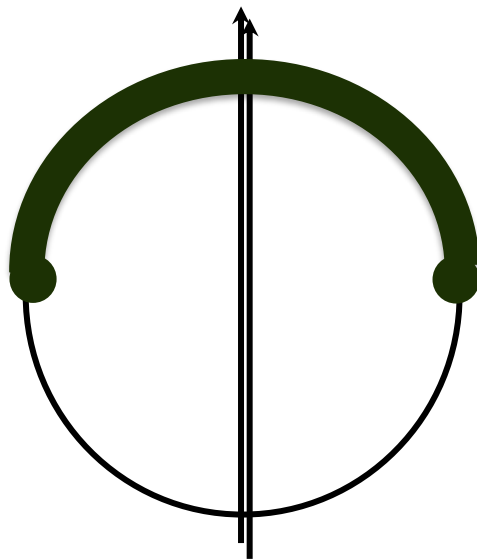
## Função circular inversa

$$y = \arcsen x$$



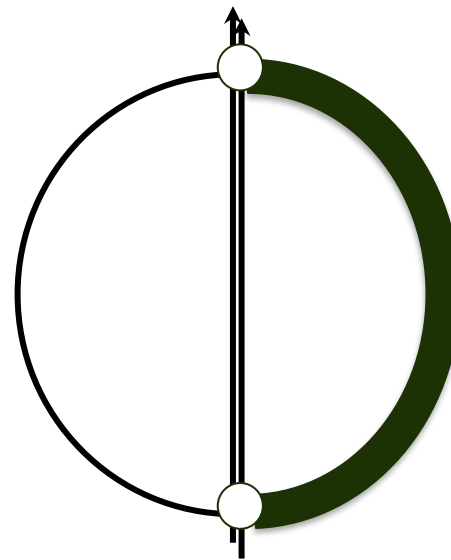
$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq x \leq \pi$$

$$y = \arctg x$$



$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

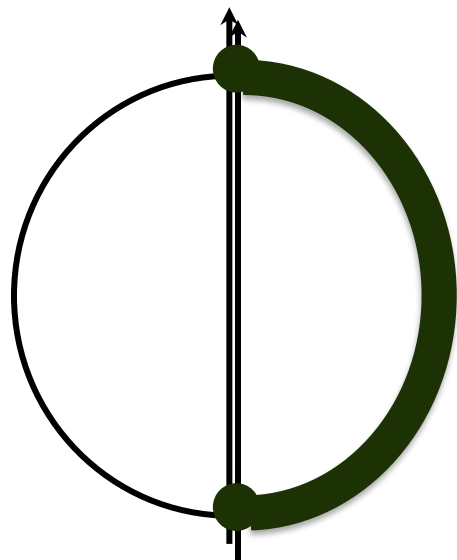
Calcule x nos seguintes exemplos:

d)  $x = \arccos(-1/2)$   $120^\circ$

# Transformação em produto

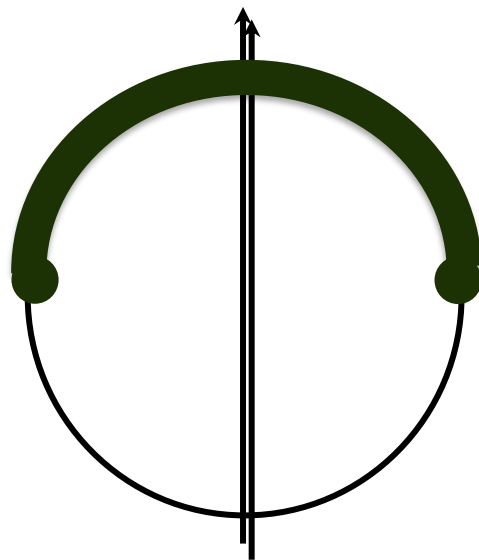
## Função circular inversa

$$y = \arcsen x$$



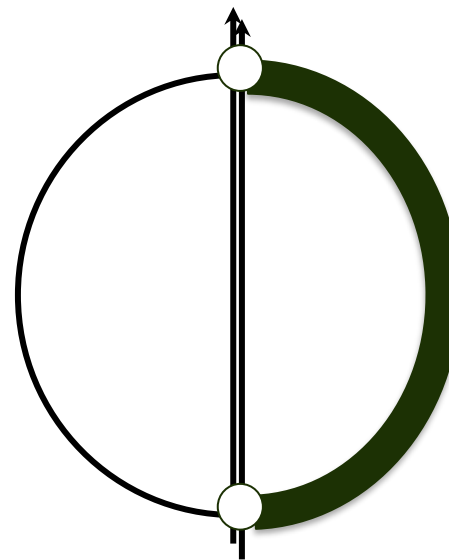
$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq x \leq \pi$$

$$y = \text{arctg} x$$



$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

Calcule  $x$  nos seguintes exemplos:

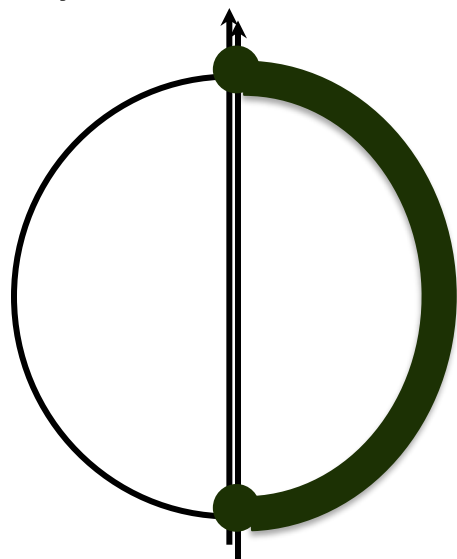
e)  $x = \text{arctg}(1) \quad 45^\circ$



# Transformação em produto

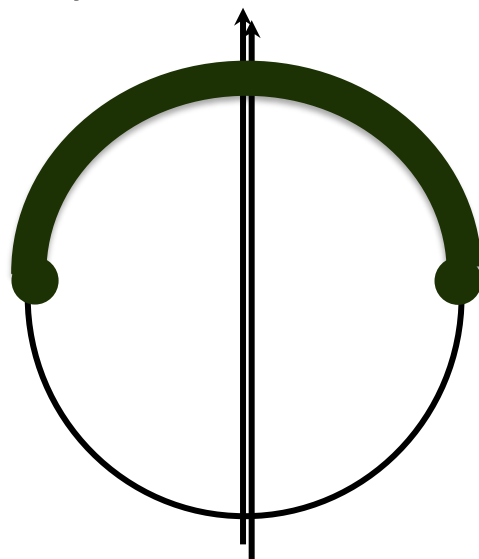
## Função circular inversa

$$y = \arcsen x$$



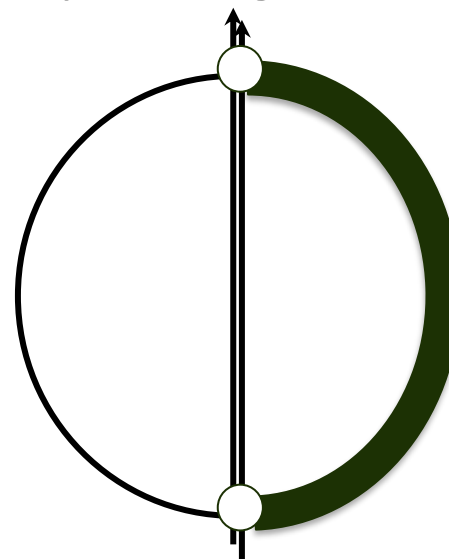
$$\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$y = \arccos x$$



$$0 \leq x \leq \pi$$

$$y = \text{arctg} x$$



$$\frac{-\pi}{2} < x < \frac{\pi}{2}$$

Calcule x nos seguintes exemplos:

f)  $x = \text{arctg}(-1) \quad -45^\circ$

# Transformação em produto

## Função circular inversa

Qual o valor de  $x = \text{sen}(\text{arctg}3/4)$

$$\text{arctg}3/4 = a$$

$$\text{Tg } a = 3/4$$

$$x = \text{sen}(a)$$

$$x = \frac{3}{5}$$

