

Civil Engineering FE CBT Exam

Knowledge: Fluid Mechanics with 10 Example Problems

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1 Fluid Properties & Basic Relationships

1.1 Density, Specific Weight, and Specific Gravity

Fluid mechanics study begins with the thermodynamic and physical definitions of fluid mass and weight per unit volume. The NCEES FE Reference Handbook specifies three fundamental property relationships:

Fundamental Property Definitions

1. **Mass Density (ρ):** Mass per unit volume of fluid.

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V} \quad [\text{kg/m}^3 \text{ or } \text{lbm/ft}^3]$$

2. **Specific Weight (γ):** Weight (force) per unit volume of fluid.

$$\gamma = \lim_{\Delta V \rightarrow 0} \frac{\Delta W}{\Delta V} = \frac{\rho g}{g_c} \quad [\text{N/m}^3, \text{ kN/m}^3, \text{ or } \text{lbf/ft}^3]$$

3. **Specific Gravity (SG):** Dimensionless ratio of fluid density (or specific weight) relative to pure water at standard conditions (4°C).

$$SG = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w}$$

1.1.1 Standard Reference Properties of Pure Water

Examinees must know standard reference values for water at standard conditions (4°C/39.2°F):

- **SI Metric Units:**

$$\rho_w = 1,000 \text{ kg/m}^3, \quad \gamma_w = 9,810 \text{ N/m}^3 = 9.81 \text{ kN/m}^3, \quad g = 9.81 \text{ m/s}^2$$

- **US Customary Units (USCS):**

$$\rho_w = 62.4 \text{ lbm/ft}^3, \quad \gamma_w = 62.4 \text{ lbf/ft}^3, \quad g = 32.2 \text{ ft/s}^2$$

FE Exam Unit Conversion Trap: Gravitational Mass Factor g_c

In USCS units, mass is expressed in pounds-mass (lbm) or slugs, while force is in pounds-force (lbf). The conversion factor g_c relates mass and force:

$$g_c = 32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$$

When using density ρ in lbm/ft^3 , specific weight is calculated as $\gamma = \rho g / g_c$. On Earth where $g \approx 32.2 \text{ ft/s}^2$, numerically $\gamma \approx \rho$ ($62.4 \text{ lbf/ft}^3 \approx 62.4 \text{ lbm/ft}^3$).

1.2 Viscosity and Shear Stress

Viscosity represents a fluid's internal resistance to flow under applied shear forces.

Dynamic and Kinematic Viscosity

1. Dynamic (Absolute) Viscosity (μ): Measure of fluid internal friction.

$$\text{SI Units: } \text{Pa} \cdot \text{s} = \frac{\text{N} \cdot \text{s}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}} \quad \text{USCS Units: } \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} \text{ or } \frac{\text{lbm}}{\text{ft} \cdot \text{s}}$$

2. Kinematic Viscosity: $\nu = \mu/\rho$ in coherent units (SI or USCS with ρ in slugs/ft³). With μ in lbf·s/ft² and ρ in lbm/ft³, use $\nu = g_c \mu / \rho$.

1.2.1 Newton's Law of Viscosity

For Newtonian fluids, shear stress τ is directly proportional to the velocity gradient dv/dy :

$$\tau = \mu \frac{dv}{dy}$$

For thin fluid films of thickness δ bounded by a plate moving at constant velocity v , the velocity profile is linear ($v(y) = v y / \delta$):

$$\tau = \mu \frac{v}{\delta} \implies F_{\text{shear}} = \tau A = \mu A \frac{v}{\delta}$$

Non-Newtonian Power-Law Fluids

For non-Newtonian fluids, shear stress follows a power-law relationship:

$$\tau_t = K \left(\frac{dv}{dy} \right)^n$$

Where K is the consistency index and n is the power-law index:

- **Pseudoplastic (Shear-Thinning):** $n < 1$ (e.g., polymer solutions, blood, paint). Viscosity decreases with increasing shear rate.
- **Dilatant (Shear-Thickening):** $n > 1$ (e.g., cornstarch suspensions, wet sand). Viscosity increases with increasing shear rate.
- **Newtonian Fluid:** $n = 1$, where $K = \mu$.

1.3 Surface Tension, Capillarity, and Vapor Pressure

1.3.1 Surface Tension (σ)

Surface tension is the tensile force per unit length acting across a liquid interface:

$$\sigma = \frac{F}{L} \quad [\text{N/m or lbf/ft}]$$

Capillary Rise Equation

The equilibrium height h of a liquid column in a vertical tube of diameter d :

$$h = \frac{4\sigma \cos \beta}{\gamma d} = \frac{2\sigma \cos \beta}{\gamma r}$$

Where β is the contact angle ($\beta < 90^\circ$ for wetting liquids like water on glass; $\beta > 90^\circ$ for mercury).

1.3.2 Vapor Pressure (P_v) and Cavitation

Vapor pressure P_v is the pressure at which a liquid vaporizes at a given temperature. If local fluid pressure drops to or below P_v , vapor bubbles form and collapse violently (cavitation), eroding pumps and pipes.

Reference Table: Physical Properties of Water (SI Units)

Temp (°C)	Specific Weight γ (kN/m ³)	Density ρ (kg/m ³)	Dynamic Visc. μ (Pa·s)	Kinematic Visc. ν (m ² /s)
0	9.805	999.8	0.001781	1.785×10^{-6}
10	9.804	999.7	0.001307	1.307×10^{-6}
20	9.789	998.2	0.001002	1.004×10^{-6}
30	9.764	995.7	0.000798	0.801×10^{-6}
50	9.689	988.0	0.000547	0.554×10^{-6}
100	9.399	958.4	0.000282	0.294×10^{-6}

2 Fluid Statics & Pressure Fields

2.1 Hydrostatic Pressure Distribution

In a static fluid (zero velocity gradients and zero shear stresses), pressure varies linearly with vertical depth h .

Fundamental Hydrostatic Equation

In coherent units, $\gamma = \rho g$; for lbm and lbf units, $\gamma = \rho g/g_c$.

$$\frac{dP}{dz} = -\gamma = -\rho g \implies P_2 - P_1 = -\gamma(z_2 - z_1) = \gamma h$$

Where $h = (z_1 - z_2)$ is the vertical fluid depth below a reference point.

2.2 Gauge, Absolute, and Vacuum Pressures

Pressure measurements are defined relative to datum pressure:

Gage vs. Absolute Pressure Relationships

1. Absolute Pressure (P_{abs}): Measured relative to absolute zero vacuum (0 Pa).

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gage}}$$

2. Vacuum Pressure (P_{vac}): Pressure below local atmospheric pressure.

$$P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}}$$

Standard Atmospheric Pressure Equivalents (1 atm at Sea Level)

Unit System	Standard Atmospheric Value
SI Metric (kPa, Pa, bar)	101.325 kPa = 101,325 N/m ² = 1.01325 bar
USCS Imperial (psi, psf)	14.696 psi $\approx$ 14.7 lbf/in ² = 2,116 lbf/ft ²
Liquid Columns (Hg, Water)	760 mm Hg = 29.92 in. Hg = 10.33 m H ₂ O = 33.9 ft H ₂ O

2.3 Barometers and Atmospheric Pressure

A barometer measures atmospheric pressure using a column of mercury ($SG_{\text{Hg}} = 13.6$).

Barometric Equation

Neglecting mercury vapor pressure in the sealed upper tube ($P_v \approx 0$):

$$P_{\text{atm}} = P_v + \gamma_{\text{Hg}} h_{\text{Hg}} \approx \gamma_{\text{Hg}} h_{\text{Hg}} = (SG_{\text{Hg}} \gamma_w) h_{\text{Hg}}$$

2.4 Manometry and Multi-Fluid Systems

Manometers measure pressure differentials using fluid columns in vertical or inclined U-tubes.

Step-by-Step Rules for Manometer Solving

To compute pressure differentials across multi-fluid manometers:

1. **Start at a known pressure point** (e.g., Pipe A pressure P_A or open atmosphere $P_{\text{atm}} = 0$ gage).
2. **Move along the manometer tube fluid path:**
 - Moving **DOWN** a fluid column: **ADD** pressure $(+\gamma h)$.
 - Moving **UP** a fluid column: **SUBTRACT** pressure $(-\gamma h)$.
3. **Jump horizontally** across continuous columns of the *same fluid* at identical elevations ($P_{\text{left}} = P_{\text{right}}$).
4. **Equate the final sum** to the pressure at the ending point (P_B).

2.4.1 General Differential Manometer Equation

For a U-tube differential manometer connecting Pipe A and Pipe B:

$$P_A + \sum_{i=\text{down}} \gamma_i h_i - \sum_{j=\text{up}} \gamma_j h_j = P_B$$

FE Reference Diagram: U-Tube Differential Manometer

Consider Pipe A (water, γ_1) connected to Pipe B (oil, γ_3) via a U-tube containing heavy gage liquid 2 (γ_2 , mercury):

$$\text{Starting at } P_A : \quad P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Manometer Unit Consistency Checklist

Examinees often make unit errors in manometer problems. Follow these checks:

- Convert all heights h to meters (m) or feet (ft).
- Specific weight of water: $\gamma_w = 9.81 \text{ kN/m}^3 = 9,810 \text{ N/m}^3$.
- Specific weight of fluid i : $\gamma_i = SG_i \times \gamma_w$.
- Multiplying $\text{kN/m}^3 \times \text{m}$ directly yields pressure in kPa (kN/m^2).

3 Hydrostatic Forces on Submerged Surfaces

3.1 Submerged Plane Surfaces

When a planar gate A is submerged in a static fluid at an incline angle θ relative to the free water surface, hydrostatic pressure acts perpendicular to the surface.

Resultant Hydrostatic Force on Plane Surfaces

For zero free-surface gauge pressure (and the corresponding net pressure difference), the resultant is centroid pressure multiplied by surface area:

$$F_R = P_c A = \gamma h_c A = \gamma (y_c \sin \theta) A$$

Where:

- h_c = vertical depth from liquid surface to the centroid of area A (m or ft)
- y_c = slant distance along the plane slope from surface to centroid ($y_c = h_c / \sin \theta$)
- θ = inclination angle of the surface with the horizontal
- A = total area of the submerged planar gate (m^2 or ft^2)

3.2 Center of Pressure

Because pressure increases linearly with depth, the resultant force F_R acts at the Center of Pressure (CP), located below the centroid C .

Location of the Center of Pressure

The slant depth y_{cp} from the liquid surface to the Center of Pressure:

$$y_{cp} = y_c + \frac{I_{xc}}{y_c A}$$

The vertical depth to the center of pressure is:

$$h_{cp} = y_{cp} \sin \theta = h_c + \frac{I_{xc} \sin^2 \theta}{h_c A}$$

Where I_{xc} is the area moment of inertia of surface A about its horizontal centroidal axis.

3.2.1 Key Centroidal Moments of Inertia (I_{xc})

Geometric Formulas for Common Gate Shapes

Shape	Dimensions	Area (A)	Centroid (y_c from base)	Moment of Inertia (I_{xc})
Rectangle	Width b , Height h	bh	$h/2$	$bh^3/12$
Triangle	Base b , Height h	$\frac{1}{2}bh$	$h/3$	$bh^3/36$
Circle	Radius R , Diameter D	$\pi R^2 = \pi D^2/4$	R at center	$\pi R^4/4 = \pi D^4/64$

Crucial Property: Center of Pressure is Below Centroid

For a nonhorizontal plane with varying hydrostatic pressure, CP lies below the centroid:

$$y_{cp} > y_c \quad \text{and} \quad h_{cp} > h_c$$

For a horizontal plane, pressure is uniform and CP coincides with the centroid.

3.3 Submerged Curved Surfaces

Hydrostatic forces on curved walls are analyzed by resolving total force into horizontal (F_x) and vertical (F_y) components.

Force Components on Curved Surfaces

1. Horizontal component: Equal to the hydrostatic force on the actual vertical projection of the curved surface (not necessarily a rectangle).

$$F_H = \gamma h_{c,proj} A_{proj}$$

Its line of action passes through that projection's center of pressure.

2. Vertical component: Find magnitude and direction from equilibrium of the real or imaginary fluid volume bounded by the surface and free surface. For the usual one-sided hydrostatic loading, $|F_V| = \gamma V$. Liquid above can give a downward force; liquid below can give an upward force. The line of action passes through the volume centroid.

3. Resultant: $F_R = \sqrt{F_H^2 + F_V^2}$; find its direction with the signs of both components (atan2).

Plane-Gate Moment Equilibrium Procedure

For a plane gate under one-sided hydrostatic loading with zero net pressure at the free surface:

1. $F_R = \gamma h_c A$. 2. $y_{cp} = y_c + I_{xc}/(y_c A)$.

3. Sum moments about the hinge using the perpendicular lever arm. For a curved gate, use the separate horizontal and vertical force lines of action.

4 Buoyancy & Stability of Floating Bodies

4.1 Archimedes' Principle

Any body submerged in a fluid experiences an upward buoyant force equal to the weight of the displaced fluid.

Archimedes' Buoyant Force Equation

$$F_B = \gamma_{\text{fluid}} V_{\text{displaced}} = \frac{\rho_{\text{fluid}} g}{g_c} V_{\text{displaced}}$$

Where F_B acts upward through the **Center of Buoyancy** (B), located at the centroid of the displaced fluid volume.

4.2 Equilibrium of Floating Bodies

For a body floating freely in static equilibrium:

$$\sum F_y = 0 \implies F_B = W_{\text{body}}$$

$$\gamma_{\text{fluid}} V_{\text{displaced}} = \gamma_{\text{body}} V_{\text{body, total}}$$

Submerged Volume Fraction Formula

The fraction of total body volume submerged below the liquid surface:

$$\frac{V_{\text{displaced}}}{V_{\text{total}}} = \frac{\gamma_{\text{body}}}{\gamma_{\text{fluid}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} = \frac{SG_{\text{body}}}{SG_{\text{fluid}}}$$

4.3 Timber Raft Maximum Cargo Calculations

A classic FE Civil problem involves determining the maximum cargo weight a timber raft can hold prior to complete submergence.

Maximum Cargo Capacity Equation

At threshold submergence ($V_{\text{displaced}} = V_{\text{raft, total}}$):

$$W_{\text{cargo, max}} + W_{\text{raft}} = F_{B, \text{max}} = \gamma_{\text{water}} V_{\text{raft, total}}$$

$$W_{\text{cargo, max}} = (\gamma_{\text{water}} - \gamma_{\text{timber}}) V_{\text{raft, total}}$$

4.4 Stability of Submerged and Floating Bodies

Stability indicates whether a restoring moment develops when a submerged or floating body undergoes a small angular tilt $\Delta\theta$.

Submerged Body Stability Criteria

For a fully submerged body (e.g., submarine, diving vessel):

- **Stable Equilibrium:** Center of Buoyancy B is located **ABOVE** Center of Gravity G ($z_B > z_G$).
- **Unstable Equilibrium:** B is located **BELOW** G ($z_B < z_G$).
- **Neutral Equilibrium:** B and G coincide at the exact same location.

4.5 Metacentric Height for Floating Vessels

When a floating vessel tilts through angle $\Delta\theta$, the Center of Buoyancy shifts from B to B' . A vertical line through B' intersects the vessel centerline at the Metacenter (M).

Metacentric Height (GM) Equation

The metacentric height GM determines rotational stability:

$$GM = MB - GB = \frac{I_o}{V_{\text{sub}}} - GB$$

Where:

- I_o = area moment of inertia of the waterline cross-section about its tilt axis
- V_{sub} = total submerged volume of the body
- $GB = z_G - z_B$ is signed; a center of gravity below B gives $GB < 0$.

Stability Criteria:

- **Stable ($GM > 0$):** Metacenter M is above Center of Gravity G .
- **Unstable ($GM < 0$):** Metacenter M is below Center of Gravity G .

5 Fluid Kinematics, Energy & Bernoulli Equation

5.1 One-Dimensional Continuity Equation

Continuity expresses conservation of mass for steady flow through a control volume.

Continuity Flow Formulas

1. Mass Flow Rate ($\dot{m}$):

$$\dot{m} = \rho Q = \rho A v \quad [\text{kg/s or lbm/s}]$$

2. Volumetric Flow Rate (Q):

$$Q = A v \quad [\text{m}^3/\text{s}, \text{ L/s, or cfs (ft}^3/\text{s)}]$$

3. Incompressible Continuity ($\rho = \text{const}$):

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_2 = v_1 \left(\frac{D_1}{D_2} \right)^2$$

5.2 The Bernoulli Equation

Bernoulli's equation represents mechanical energy conservation along a streamline.

Bernoulli Equation (Head and Pressure Forms)

Required Assumptions: Incompressible ($\rho = \text{const}$), steady, inviscid ($h_f = 0$), no pumps/turbines, along a streamline.

Head Form (units of length: meters or feet):

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

Pressure Form (units of pressure: Pa or psf):

$$P_1 + \gamma z_1 + \gamma \frac{v_1^2}{2g} = P_2 + \gamma z_2 + \gamma \frac{v_2^2}{2g}$$

Where P/γ = Pressure Head, z = Elevation Head, and $v^2/(2g)$ = Velocity Head.

5.3 General Energy Equation with Friction Losses & Pumps

In real piping systems, friction causes head loss (h_f), while pumps add head (h_p) and turbines extract head (h_t).

General Energy Equation

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_p = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_t + h_f$$

Where h_f is total head loss ($h_f = h_{\text{major}} + h_{\text{minor}}$).

5.3.1 Pump Power Equation

Pump Power Formulas

1. Fluid Power Added ($\dot{W}_{\text{fluid}}$):

$$\dot{W}_{\text{fluid}} = \gamma Q h_p = \frac{\rho g}{g_c} Q h_p \quad [\text{W or ft} \cdot \text{lb}_f/\text{s}]$$

2. Total Purchased Electrical Power ($\dot{W}_{\text{purchased}}$):

$$\dot{W}_{\text{purchased}} = \frac{\gamma Q h_p}{\eta_{\text{total}}} = \frac{\gamma Q h_p}{\eta_{\text{pump}} \eta_{\text{motor}}}$$

Horsepower Conversion: 1 hp = 550 ft·lb_f/sec = 745.7 Watts = 0.746 kW.

5.4 Pitot Tubes and Stagnation Pressure

A Pitot tube measures fluid velocity by converting kinetic head to pressure head at a stagnation point ($v = 0$).

Pitot Tube Velocity Formula

Applying Bernoulli between static stream s and stagnation point 0:

$$v = \sqrt{\frac{2g_c(P_0 - P_s)}{\rho}} = \sqrt{2g \frac{P_0 - P_s}{\gamma}} = \sqrt{2g \Delta h}$$

6 Flow Measurement Devices & Closed Conduits

6.1 Venturi Meter, Orifices, and Discharge Coefficients

Free Discharge Orifice

For an open tank discharging through orifice A_0 at vertical depth h :

$$Q = C A_0 \sqrt{2gh}$$

Where $C = C_c C_v$ ($C_c \approx 0.62$, $C_v \approx 0.98$, $C \approx 0.61$ for sharp edges).

Submerged Orifice between Two Tanks

When an orifice connects two tanks with liquid levels h_1 and h_2 :

$$Q = C A_0 \sqrt{2g(h_1 - h_2)} = C A_0 \sqrt{2g\Delta h}$$

Tank Drawdown Time Equation

Time Δt to lower tank water level from h_1 to h_2 through orifice A_0 :

$$\Delta t = \frac{2A_t}{C A_0 \sqrt{2g}} \left(\sqrt{h_1} - \sqrt{h_2} \right)$$

Incompressible Venturi Meter Equation

Flow through a horizontal Venturi meter with entrance A_1 and throat A_2 :

$$Q = \frac{C_v A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g \left(\frac{P_1 - P_2}{\gamma} + z_1 - z_2 \right)}$$

6.2 Flow Regimes and Pipe Friction Losses

Flow in closed conduits is governed by the dimensionless Reynolds Number (Re):

Reynolds Number (Re) and Flow Regimes

$$Re = \frac{\rho v D}{\mu} = \frac{v D}{\nu}$$

Laminar Flow: $Re < 2,100$

Transitional Flow: $2,100 \leq Re \leq 4,000$

Turbulent Flow: $Re > 4,000$

6.2.1 Darcy-Weisbach Friction Loss Equation

Major head loss due to pipe wall friction along length L :

$$h_f = f \frac{L}{D} \frac{v^2}{2g}$$

Friction Factor f Equations

1. Laminar Flow ($Re < 2100$): Independent of wall roughness:

$$f = \frac{64}{Re}, \quad \Delta P = \frac{128 \mu L Q}{\pi D^4}, \quad h_f = \frac{\Delta P}{\gamma}$$

2. Turbulent Flow ($Re > 4000$): Determined via Moody Chart or Colebrook Equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

6.2.2 Minor Loss Coefficients (K_L)

Minor head losses from valves, fittings, and entrances: $h_m = K_L v^2 / (2g)$.

Representative Minor Loss Coefficients (K_L)

Fitting / Component	Loss Coeff. K_L	Fitting / Component	Loss Coeff. K_L
Sharp-Edged Pipe Entrance	0.50	90° Standard Elbow	0.90
Well-Rounded Entrance	0.04	45° Standard Elbow	0.42
Re-entrant Entrance (Protruding)	0.80	Fully Open Gate Valve	0.15
Submerged Pipe Exit (All types)	1.00	Fully Open Globe Valve	10.00

7 Impulse-Momentum, Piping Forces & Open Channels

7.1 The Impulse-Momentum Principle

Newton's second law applied to a control volume states that net external force equals net momentum flux outflow minus inflow.

Control Volume Momentum Equations

$$\sum \mathbf{F} = \frac{1}{g_c} \left(\sum \dot{m}_{\text{out}} \mathbf{v}_{\text{out}} - \sum \dot{m}_{\text{in}} \mathbf{v}_{\text{in}} \right) = \frac{\rho Q}{g_c} (\mathbf{v}_2 - \mathbf{v}_1)$$

$$\sum F_x = \frac{\rho Q}{g_c} (v_{2x} - v_{1x}), \quad \sum F_y = \frac{\rho Q}{g_c} (v_{2y} - v_{1y})$$

7.2 Forces on Pipe Bends and Thrust Block Sizing

Momentum Equations for Horizontal Pipe Bend (α Angle)

For a horizontal bend, let R_x, R_y be forces of the pipe on the fluid:

$$R_x + P_1 A_1 - P_2 A_2 \cos \alpha = \frac{\rho Q}{g_c} (v_2 \cos \alpha - v_1)$$

$$R_y - P_2 A_2 \sin \alpha = \frac{\rho Q}{g_c} v_2 \sin \alpha$$

Thrust on the pipe is $\mathbf{F} = -\mathbf{R}$; fluid weight enters vertical equilibrium only.

$$\mathbf{F}_{\text{thrust}} = \sqrt{F_x^2 + F_y^2} \implies A_{\text{thrust block}} = \frac{F_{\text{thrust}}}{q_{\text{allowable soil bearing capacity}}}$$

7.3 Manning's Open Channel Flow Equations

Manning's Equation & Critical Depth

$$v = \frac{K}{n} R_H^{2/3} S^{1/2}, \quad Q = Av = \frac{K}{n} A R_H^{2/3} S^{1/2}$$

Where $K = 1.0$ (SI) or 1.486 (USCS), $R_H = A/P$. Critical Depth for rectangular channel: $y_c = \sqrt[3]{q^2/g}$. Froude Number $Fr = v/\sqrt{gy_h}$.

8 Example Problems and Solutions

Problem 1: Flow Measurement - Channel Roughness Correction Factor

Topic Category: Flow Measurement / Open Channel Flow · **Source:** FE Review Session

Question Statement:

A rectangular concrete diversion channel has a water depth of 1.25 ft and bottom width of 4.0 ft. Centerline mid-depth velocity is 1.5 ft/s. Applying a correction factor $C_{\text{corr}} = 0.90$ for channel boundary roughness and edge friction, the volumetric discharge Q (cfs) is most nearly:

- (A) 3.5 cfs
- (B) 4.5 cfs
- (C) 6.8 cfs
- (D) 7.5 cfs

Solution

1. Flow Area: $A = B \times y = 4.0 \text{ ft} \times 1.25 \text{ ft} = 5.0 \text{ ft}^2$. 2. Corrected Velocity:
 $v_{\text{avg}} = C_{\text{corr}} \times v_{\text{mid}} = 0.90 \times 1.5 \text{ ft/s} = 1.35 \text{ ft/s}$. 3. Discharge: $Q = A \times v_{\text{avg}} = 5.0 \text{ ft}^2 \times 1.35 \text{ ft/s} = 6.75 \text{ cfs} \approx 6.8 \text{ cfs}$.
Answer: (C)

Problem 2: Flow Measurement - Pipe Contraction Velocity Ratio

Topic Category: Flow Measurement / Continuity · **Source:** FE Reference Handbook p. 185

Question Statement:

An incompressible liquid flows through a concentric pipe reduction from upstream diameter $D_A = 22 \text{ cm}$ ($v_A = 2.0 \text{ m/s}$) to downstream diameter $D_B = 16 \text{ cm}$. Downstream velocity v_B is most nearly:

- (A) 1.5 m/s
- (B) 2.8 m/s
- (C) 3.8 m/s
- (D) 4.2 m/s

Solution

1. Incompressible Continuity: $A_A v_A = A_B v_B \Rightarrow v_B = v_A (D_A/D_B)^2$. 2. Calculation:
 $v_B = 2.0 \text{ m/s} \times (22/16)^2 = 2.0 \times 1.8906 = 3.781 \text{ m/s} \approx 3.8 \text{ m/s}$. **Answer: (C)**

Problem 3: Flow Measurement - Pitot Tube Stagnation Velocity & Discharge**Topic Category:** Flow Measurement / Pitot Tubes · **Source:** FE Reference Handbook p. 194**Question Statement:**

A Pitot probe in a 20-cm water pipe measures $\Delta h = 0.15$ m. Assume its measured speed equals the cross-sectional mean. Find Q in L/s.

- (A) 42.5 L/s
- (B) 53.9 L/s
- (C) 62.1 L/s
- (D) 74.8 L/s

Solution

1. Stagnation Velocity: $v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.81 \times 0.15} = 1.7155$ m/s. 2. Area: $A = \frac{\pi}{4}(0.20 \text{ m})^2 = 0.031416 \text{ m}^2$. 3. Discharge: $Q = 0.031416 \times 1.7155 = 0.053894 \text{ m}^3/\text{s} = 53.89 \text{ L/s} \approx 53.9 \text{ L/s}$.
Answer: (B)

Problem 4: Flow Measurement - Submerged Sharp-Notched Orifice**Topic Category:** Flow Measurement / Submerged Orifice · **Source:** FE Reference Handbook p. 194**Question Statement:**

A sharp-notched submerged orifice (12 cm high by 20 cm wide) connects two tanks. Tank A head is $h_1 = 2.7$ m; Tank B head is $h_2 = 0.8$ m. Orifice coefficient is $C = 0.60$. Water flow rate (L/s) is most nearly:

- (A) 60 L/s
- (B) 72 L/s
- (C) 88 L/s
- (D) 98 L/s

Solution

1. Net Head and Area: $\Delta h = 2.7 - 0.8 = 1.9$ m. $A_0 = 0.12 \times 0.20 = 0.024 \text{ m}^2$. 2. Calculation: $Q = CA_0\sqrt{2g\Delta h} = 0.60 \times 0.024 \times \sqrt{2 \times 9.81 \times 1.9} = 0.08792 \text{ m}^3/\text{s} = 87.92 \text{ L/s} \approx 88 \text{ L/s}$. **Answer: (C)**

Problem 5: Fluid Properties & Buoyancy - Timber Raft Cargo Capacity**Topic Category:** Fluid Properties / Buoyancy · **Source:** FE Reference Handbook p. 184**Question Statement:**

A raft (6.0 ft wide by 8.0 ft long) is built from 8×8 nominal timbers (actual dressed thickness 7.5 in). Wet timber unit weight is $\gamma_{\text{wood}} = 45.0 \text{ lbf/ft}^3$; water unit weight is $\gamma_w = 62.4 \text{ lbf/ft}^3$. Maximum cargo weight (lbs) before top surface submergence is most nearly:

- (A) 520 lbs
- (B) 650 lbs
- (C) 820 lbs
- (D) 1,350 lbs

Solution

1. Raft Volume: $V_{\text{raft}} = (7.5/12) \times 6.0 \times 8.0 = 30.0 \text{ ft}^3$. 2. Equilibrium: $W_{\text{cargo,max}} = (\gamma_w - \gamma_{\text{wood}})V_{\text{raft}} = (62.4 - 45.0) \times 30.0 = 17.4 \times 30.0 = 522 \text{ lbs} \approx 520 \text{ lbs}$. **Answer: (A)**

Problem 6: Fluid Properties - Barometer Height & Altitude Pressure**Topic Category:** Fluid Properties / Pressure · **Source:** FE Reference Handbook p. 182**Question Statement:**

A mercury barometer ($SG_{\text{Hg}} = 13.6$) reads column height $h_{\text{Hg}} = 725 \text{ mm}$ at a mountain station ($\rho_w = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$). Atmospheric pressure (kPa) is most nearly:

- (A) 96.7 kPa
- (B) 98.1 kPa
- (C) 100.1 kPa
- (D) 101.3 kPa

Solution

1. Mercury Specific Weight: $\gamma_{\text{Hg}} = 13.6 \times 1000 \times 9.81 = 133.416 \text{ kN/m}^3$. 2. Barometric Pressure: $P_{\text{atm}} = 133.416 \text{ kN/m}^3 \times 0.725 \text{ m} = 96.726 \text{ kPa} \approx 96.7 \text{ kPa}$. **Answer: (A)**

Problem 7: Fluid Statics - Water-Mercury-Oil U-Tube Manometer**Topic Category:** Fluid Statics / Manometry · **Source:** FE Reference Handbook p. 182**Question Statement:**

A mercury U-tube manometer ($SG_{\text{Hg}} = 13.6$) connects Pipe A (water, $P_A = 100$ kPa gage) to Pipe B (oil, $SG_{\text{oil}} = 0.80$). Following a pressure path from A to B, move down 1.0 m through water, then up 0.90 m through mercury, then up 0.60 m through oil. The gage pressure P_B is most nearly:

- (A) -15.0 kPa
- (B) 15.0 kPa
- (C) 81.9 kPa
- (D) 208.8 kPa

Solution

- Specific weights: $\gamma_w = 9.81$ kN/m³, $\gamma_{\text{oil}} = 7.848$ kN/m³, $\gamma_{\text{Hg}} = 133.416$ kN/m³.
- Pressure path: $P_B = P_A + \gamma_w(1.0) - \gamma_{\text{Hg}}(0.90) - \gamma_{\text{oil}}(0.60) = 100 + 9.81 - 120.074 - 4.709 = -14.97$ kPa, approximately -15.0 kPa. **Answer: (A)**

Problem 8: Fluid Statics - Submerged Inclined Gate Center of Pressure**Topic Category:** Fluid Statics / Submerged Surfaces · **Source:** FE Reference Handbook p. 183**Question Statement:**

A square submerged gate (16 cm × 16 cm) is mounted on a wall inclined at $\theta = 30^\circ$ to horizontal. Top edge slant depth is $y_{\text{top}} = 46$ cm. Slant depth to Center of Pressure (y_{cp}) is most nearly:

- (A) 50.00 cm
- (B) 50.05 cm
- (C) 54.10 cm
- (D) 54.40 cm

Solution

- Centroid Depth and Properties: $y_c = 46 + 8 = 54.0$ cm, $A = 256$ cm², $I_{xc} = 16(16^3)/12 = 5,461.33$ cm⁴.
- Center of Pressure: $y_{cp} = y_c + I_{xc}/(y_c A) = 54.0 + 5,461.33/(54.0 \times 256) = 54.0 + 0.395 = 54.40$ cm. **Answer: (D)**

Problem 9: Energy Equation - Constricted Nozzle Exit Velocity**Topic Category:** Energy & Bernoulli / Pipe Flow · **Source:** FE Reference Handbook p. 184**Question Statement:**

Water flows through a vertical reducing nozzle from Section A (10 cm diameter, $P_A = 100$ kPa gage) to Section B (4 cm diameter, $P_B = 0$ gage). Section B is 2.0 m below Section A. Assuming $h_f = 0$, exit velocity v_B is most nearly:

- (A) 6 m/s
- (B) 16 m/s
- (C) 21 m/s
- (D) 42 m/s

Solution

1. Continuity: $v_A = v_B(4/10)^2 = 0.16v_B$. 2. Bernoulli ($z_B = 0$, $z_A = 2.0$ m):

$$\frac{100,000}{9,810} + 2.0 + \frac{(0.16v_B)^2}{19.62} = \frac{v_B^2}{19.62} \implies 12.194 = \frac{0.9744v_B^2}{19.62} \implies v_B = 15.67 \text{ m/s} \approx 16 \text{ m/s}.$$

Answer: (B)**Problem 10: Pressure Thrust - Horizontal Pipe Elbow & Anchor Sizing****Topic Category:** Pipe Forces / Pressure Thrust · **Source:** FE Reference Handbook p. 186**Question Statement:**

A 12 in diameter horizontal pipe carries water at gage pressure $P = 50.0$ psi. A 30° horizontal elbow changes the pipe alignment. Neglect momentum-flux effects and calculate the static pressure thrust F_{thrust} and required thrust-block soil bearing area A_{block} for $q_{\text{allow}} = 2,000$ psf. The thrust force and bearing area are most nearly:

- (A) 1,450 lbs and 0.8 ft²
- (B) 2,930 lbs and 1.5 ft²
- (C) 5,655 lbs and 2.8 ft²
- (D) 8,480 lbs and 4.2 ft²

Solution

1. Pressure Force: $A = \frac{\pi}{4}(12)^2 = 113.10 \text{ in}^2 \Rightarrow F_1 = F_2 = 50.0 \times 113.10 = 5,655 \text{ lbs}$. 2. Thrust Components: $F_x = 5655(1 - \cos 30^\circ) = 757.8 \text{ lbs}$, $F_y = 5655 \sin 30^\circ = 2,827.5 \text{ lbs}$. 3. Resultant Thrust & Block Area: $F_{\text{thrust}} = \sqrt{(757.8)^2 + (2827.5)^2} = 2,927 \text{ lbs} \approx 2,930 \text{ lbs}$. $A_{\text{block}} = 2,927/2,000 = 1.46 \text{ ft}^2 \approx 1.5 \text{ ft}^2$.
Answer: (B)