

Civil Engineering FE CBT Exam

Knowledge: Mathematics & Statistics with 20 Example Problems

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1 Analytic Geometry

Analytic geometry applies algebra to lines, conics, and coordinates in two or three dimensions. Subtopic question counts are study-planning estimates; the exam specification gives a broader Mathematics and Statistics allocation.

1.1 Straight Lines in 2D Cartesian Coordinates

A straight line in two dimensions is defined by its slope m and position relative to the axes.

Fundamental Straight Line Equations

Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (1)$$

Slope-Intercept Form:

$$y = mx + b \quad \text{where } b \text{ is the } y\text{-intercept} \quad (2)$$

Point-Slope Form:

$$y - y_1 = m(x - x_1) \quad (3)$$

General Linear Form:

$$Ax + By + C = 0 \quad \text{where } m = -\frac{A}{B}, \quad b = -\frac{C}{B} \quad (4)$$

Parallel and Perpendicular Lines:

$$\text{Parallel Lines: } m_1 = m_2 \quad (5)$$

$$\text{Perpendicular Lines: } m_1 \cdot m_2 = -1 \quad \implies \quad m_2 = -\frac{1}{m_1} \quad (6)$$

1.1.1 Distance and Midpoint Formulas

For two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in a 2D Cartesian plane:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (7)$$

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (8)$$

In 3D space, for points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (9)$$

1.2 Conic Sections Classification

A general second-degree quadratic equation in two variables has the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad (10)$$

The geometry of the curve is identified using the ****discriminant**** $\Delta = B^2 - 4AC$:

Discriminant Classification Rules for Conic Sections

For a **real, nondegenerate** conic, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$:

- **Parabola:** $B^2 - 4AC = 0$ (one variable is quadratic, the other is linear when $B = 0$).
- **Ellipse:** $B^2 - 4AC < 0$ (and $A \neq C$ or $B \neq 0$). If $A = C$ and $B = 0$, it represents a **Circle**.
- **Hyperbola:** $B^2 - 4AC > 0$.

1.3 Parabolas

A parabola is the locus of points equidistant from a fixed point (the focus) and a fixed line (the directrix).

Standard Equations of a Parabola**Vertical Axis of Symmetry (Opens Up/Down):**

$$(x - h)^2 = 4p(y - k) \quad (11)$$

- Vertex: (h, k)
- Focus: $(h, k + p)$
- Directrix: $y = k - p$
- Axis of Symmetry: $x = h$
- If $p > 0$, parabola opens upwards; if $p < 0$, parabola opens downwards.

Horizontal Axis of Symmetry (Opens Left/Right):

$$(y - k)^2 = 4p(x - h) \quad (12)$$

- Vertex: (h, k)
- Focus: $(h + p, k)$
- Directrix: $x = h - p$
- If $p > 0$, parabola opens to the right; if $p < 0$, parabola opens to the left.

1.4 Ellipses

An ellipse is the set of points where the sum of distances to two fixed foci F_1 and F_2 is constant ($2a$).

Standard Equations of an Ellipse

Center at (h, k) , semi-major axis a , semi-minor axis b (where $a > b$):

Horizontal Major Axis:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \quad (13)$$

- Vertices: $(h \pm a, k)$
- Foci: $(h \pm c, k)$, where $c^2 = a^2 - b^2$
- Eccentricity: $e = \frac{c}{a} < 1$

Vertical Major Axis:

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1 \quad (14)$$

- Vertices: $(h, k \pm a)$
- Foci: $(h, k \pm c)$, where $c^2 = a^2 - b^2$

1.5 Hyperbolas

A hyperbola is the set of points where the absolute difference of distances to two foci is constant ($2a$).

Standard Equations of a Hyperbola

Center at (h, k) :

Horizontal Transverse Axis:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \quad (15)$$

- Foci: $(h \pm c, k)$, where $c^2 = a^2 + b^2$
- Asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

Vertical Transverse Axis:

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \quad (16)$$

- Foci: $(h, k \pm c)$, where $c^2 = a^2 + b^2$
- Asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

1.6 Circles and Spheres**1.6.1 Standard Equation of a Circle in 2D**

$$(x - h)^2 + (y - k)^2 = r^2 \quad (17)$$

where (h, k) is the center and r is the radius.

General equation of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0 \quad (18)$$

To convert to standard form, use **completing the square**:

$$\left(x + \frac{D}{2}\right)^2 + \left(y + \frac{E}{2}\right)^2 = \left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F \quad (19)$$

1.6.2 Standard Equation of a Sphere in 3D

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2 \quad (20)$$

where (h, k, l) is the 3D center and r is the radius.

1.7 Polar Coordinates

A point P in a 2D plane can be represented by Cartesian coordinates (x, y) or polar coordinates (r, θ) .

Polar-Cartesian Transformations

Cartesian to Polar:

$$r = \sqrt{x^2 + y^2} \quad (21)$$

$$\theta = \arctan\left(\frac{y}{x}\right) \quad (\text{Adjust for quadrant of } (x, y)) \quad (22)$$

Polar to Cartesian:

$$x = r \cos \theta \quad (23)$$

$$y = r \sin \theta \quad (24)$$

2 Single-Variable Calculus

Calculus forms the mathematical backbone of engineering analysis. The NCEES FE Civil exam tests limits, differentiation, tangent/normal lines, optimization, partial derivatives, and integration.

2.1 Limits and Continuity

The limit of a function $f(x)$ as x approaches a is written $\lim_{x \rightarrow a} f(x) = L$.

2.1.1 L'Hôpital's Rule

When evaluating a limit results in an indeterminate form ($\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$), apply L'Hôpital's Rule:

L'Hôpital's Rule

If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$ (or both approach $\pm\infty$), then:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (25)$$

provided the limit of the derivatives exists.

2.2 Rules of Differentiation

Let $u(x)$ and $v(x)$ be differentiable functions, and let c be a constant.

Core Differentiation Formulas

$$\text{Power Rule: } \frac{d}{dx} (x^n) = nx^{n-1} \quad (26)$$

$$\text{Constant Multiple: } \frac{d}{dx} [c \cdot u] = c \cdot u' \quad (27)$$

$$\text{Sum/Difference: } \frac{d}{dx} [u \pm v] = u' \pm v' \quad (28)$$

$$\text{Product Rule: } \frac{d}{dx} [u \cdot v] = u'v + uv' \quad (29)$$

$$\text{Quotient Rule: } \frac{d}{dx} \left[\frac{u}{v} \right] = \frac{u'v - uv'}{v^2} \quad (30)$$

$$\text{Chain Rule: } \frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x) \quad (31)$$

$$\text{General Power Rule: } \frac{d}{dx} [u^n] = nu^{n-1} \cdot u' \quad (32)$$

2.2.1 Derivatives of Common Transcendental Functions

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0) \qquad (33)$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad (34)$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad (35)$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(a^x) = a^x \ln a \qquad (36)$$

2.3 Tangent Lines and Normal Lines

For a curve $y = f(x)$ at a specific point $P(x_0, y_0)$:

- Slope of the tangent line: $m_t = f'(x_0)$
- Equation of the tangent line: $y - y_0 = m_t(x - x_0)$
- Slope of the normal line: $m_n = -\frac{1}{m_t} = -\frac{1}{f'(x_0)}$
- Equation of the normal line: $y - y_0 = m_n(x - x_0)$

2.4 Optimization and Curve Sketching

Optimization involves finding relative (local) or absolute extrema of a function $f(x)$.

Procedure for Optimization

1. Find the first derivative $f'(x)$ and solve $f'(x) = 0$ or identify where $f'(x)$ is undefined. These x -values are ****critical points****.
2. At a differentiable stationary point, $f'(x_c) = 0$, use the second-derivative test:
 - If $f''(x_c) > 0$, $f(x_c)$ is a ****local minimum**** (concave upward).
 - If $f''(x_c) < 0$, $f(x_c)$ is a ****local maximum**** (concave downward).
 - $f''(x_c) = 0$: inconclusive. Check signs and interval endpoints for absolute extrema.

2.5 Partial Derivatives

For a multivariable function $z = f(x, y)$:

- Partial derivative with respect to x : $\frac{\partial f}{\partial x}$ or f_x is computed by treating y as a constant and differentiating with respect to x .
- Partial derivative with respect to y : $\frac{\partial f}{\partial y}$ or f_y is computed by treating x as a constant and differentiating with respect to y .

2.6 Numerical Root Finding: Newton-Raphson Method

To find a real root of $f(x) = 0$ iteratively starting from an initial guess x_0 :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (37)$$

3 Integral Calculus & Applications

Integral calculus is widely used in civil engineering to calculate areas, centroids, moments of inertia, structural work, and volumes.

3.1 Indefinite Integrals and Anti-derivatives

The indefinite integral $\int f(x)dx = F(x) + C$, where $F'(x) = f(x)$ and C is the constant of integration.

Standard Integration Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \quad (38)$$

$$\int \frac{1}{x} dx = \ln |x| + C \quad (39)$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C \quad (40)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C \quad (41)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C \quad (42)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + C \quad (43)$$

$$\int \frac{1}{1+x^2} dx = \arctan(x) + C \quad (44)$$

3.2 Definite Integrals and Bounded Area

By the Fundamental Theorem of Calculus:

$$\int_a^b f(x) dx = F(b) - F(a) \quad (45)$$

3.2.1 Area Bounded by Curves

- Area between $y = f(x)$ and the x -axis from $x = a$ to $x = b$:

$$A = \int_a^b |f(x)| dx \quad (46)$$

- Area between two curves $y = f_T(x)$ (upper) and $y = f_B(x)$ (lower):

$$A = \int_a^b [f_T(x) - f_B(x)] dx \quad (47)$$

3.3 First Moments of Area and Centroids

For a planar region bounded by $y = f(x)$, $y = 0$, $x = a$, and $x = b$:

Centroid Formulas for Planar Regions**Total Area:**

$$A = \int_a^b y \, dx = \int_a^b f(x) \, dx \quad (48)$$

First Moment of Area about y -axis (M_y):

$$M_y = \int_a^b x \cdot y \, dx = \int_a^b x \cdot f(x) \, dx \quad (49)$$

First Moment of Area about x -axis (M_x):

$$M_x = \int_a^b \frac{y}{2} \cdot y \, dx = \frac{1}{2} \int_a^b [f(x)]^2 \, dx \quad (50)$$

Centroid Coordinates $(\bar{x}, \bar{y})$:

$$\bar{x} = \frac{M_y}{A} = \frac{\int_a^b x f(x) \, dx}{\int_a^b f(x) \, dx}, \quad \bar{y} = \frac{M_x}{A} = \frac{\frac{1}{2} \int_a^b [f(x)]^2 \, dx}{\int_a^b f(x) \, dx} \quad (51)$$

3.4 Volumes of Revolution

Revolving a planar curve $y = f(x)$ about a coordinate axis generates a 3D solid of revolution.

Volumes of Revolution Formulas**Rotation about x -axis (Disk Method):**

$$V = \pi \int_a^b [f(x)]^2 \, dx \quad (52)$$

Rotation about x -axis (Washer Method for region between $f_T(x)$ and $f_B(x)$):

$$V = \pi \int_a^b ([f_T(x)]^2 - [f_B(x)]^2) \, dx \quad (53)$$

Rotation about y -axis (Cylindrical Shell Method):

$$V = 2\pi \int_a^b x \cdot f(x) \, dx \quad (54)$$

4 Linear Algebra, Vectors, & Discrete Math

Linear algebra and vector analysis are required for structural analysis, fluid mechanics, and surveying computations.

4.1 Matrices and Determinants

A matrix $[A]$ of order $m \times n$ contains m rows and n columns.

4.1.1 Determinant of 2×2 and 3×3 Matrices

For a 2×2 matrix:

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc \quad (55)$$

For a 3×3 matrix:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) \quad (56)$$

4.2 Vector Algebra

A 3D vector $\mathbf{A}$ is written $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} = \langle A_x, A_y, A_z \rangle$.

Core Vector Operations

Magnitude:

$$\|\mathbf{A}\| = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad (57)$$

Unit Vector:

$$\mathbf{u}_A = \frac{\mathbf{A}}{\|\mathbf{A}\|} \quad (58)$$

Dot Product (Scalar Product):

$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z = \|\mathbf{A}\|\|\mathbf{B}\|\cos\theta \quad (59)$$

- Angle between vectors: $\theta = \arccos\left(\frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|}\right)$
- Orthogonality test: $\mathbf{A} \perp \mathbf{B} \iff \mathbf{A} \cdot \mathbf{B} = 0$

Cross Product (Vector Product):

$$\mathbf{A} \times \mathbf{B} = \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} \quad (60)$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k} \quad (61)$$

- Magnitude: $\|\mathbf{A} \times \mathbf{B}\| = \|\mathbf{A}\|\|\mathbf{B}\|\sin\theta$ (Equal to the area of the parallelogram formed by $\mathbf{A}$ and $\mathbf{B}$).

4.3 Progressions and Series

4.3.1 Arithmetic Progression

A sequence where consecutive terms differ by a constant d :

$$n\text{-th Term: } a_n = a_1 + (n - 1)d \quad (62)$$

$$\text{Sum of } n \text{ Terms: } S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n - 1)d] \quad (63)$$

4.3.2 Geometric Progression

A sequence where consecutive terms have a constant ratio r :

$$n\text{-th Term: } a_n = a_1 r^{n-1} \quad (64)$$

$$\text{Sum of } n \text{ Terms: } S_n = \frac{a_1(1 - r^n)}{1 - r} \quad (r \neq 1) \quad (65)$$

$$\text{Infinite Sum } (|r| < 1): S_\infty = \frac{a_1}{1 - r} \quad (66)$$

5 Differential Equations & Laplace Transforms

Differential equations model dynamic physical engineering processes such as fluid dynamics, heat transfer, structural vibration, and contaminant transport.

5.1 First-Order Linear Differential Equations

Standard form:

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (67)$$

Integrating Factor Method

Step 1: Calculate the Integrating Factor $IF(x)$:

$$IF(x) = e^{\int P(x) dx} \quad (68)$$

Step 2: The general solution is:

$$y \cdot IF(x) = \int Q(x) \cdot IF(x) dx + C \quad (69)$$

$$y(x) = e^{-\int P(x) dx} \left(\int Q(x) e^{\int P(x) dx} dx + C \right) \quad (70)$$

5.2 Second-Order Linear Homogeneous DEs with Constant Coefficients

Standard form:

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0 \quad (71)$$

Assume $y = e^{rx}$. Substituting yields the **characteristic equation**:

$$ar^2 + br + c = 0 \quad (72)$$

Roots $r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

General Solutions for Second-Order Homogeneous DEs

Case 1: Real, distinct roots ($b^2 - 4ac > 0$):

$$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

Case 2: Real, repeated root ($b^2 - 4ac = 0$), $r = -b/(2a)$:

$$y(x) = (C_1 + C_2 x) e^{rx}$$

Case 3: Complex conjugate roots ($b^2 - 4ac < 0$):

$r = \alpha \pm i\beta$, $\alpha = -b/(2a)$, $\beta = \sqrt{4ac - b^2}/(2|a|)$.

$$y(x) = e^{\alpha x} [C_1 \cos(\beta x) + C_2 \sin(\beta x)]$$

For a physical oscillator with $x = \text{time}$, $a > 0$, $c > 0$ and $b \geq 0$, these correspond to overdamped, critically damped and underdamped cases. At $b = 0$ the oscillator is undamped. Root type alone does not imply dissipative stability.

5.3 Laplace Transforms

The Laplace transform converts a time-domain function $f(t)$ into a complex frequency-domain function $F(s)$:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t) dt \quad (76)$$

Common Laplace Transform Pairs

$$\mathcal{L}\{1\} = \frac{1}{s} \quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad (77)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad \mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2} \quad (78)$$

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2} \quad \mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \quad (79)$$

Transform of Derivatives:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) \quad (80)$$

$$\mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0) \quad (81)$$

Final Value: requires an existing limit; rational $sF(s)$ poles must have $\Re(s) < 0$.

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) \quad (82)$$

6 Probability & Statistics

Engineers utilize statistics for quality control, material testing, hydrological risk, and traffic forecasting.

6.1 Descriptive Statistics

Given a sample dataset $\{x_1, x_2, \dots, x_n\}$:

Sample Metrics Formulas

Sample Mean ($\bar{x}$):

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (83)$$

Weighted Mean ($\bar{x}_w$):

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i} \quad (84)$$

Sample Variance (s^2):

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} \quad (85)$$

Sample Standard Deviation (s):

$$s = \sqrt{s^2} \quad (86)$$

Coefficient of Variation (CV):

$$CV = \frac{s}{\bar{x}} \times 100\% \quad (87)$$

6.2 Probability Rules, Permutations, and Combinations

- **Permutations** (order matters):

$$P(n, r) = \frac{n!}{(n-r)!} \quad (88)$$

- **Combinations** (order does NOT matter):

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (89)$$

- **Addition Law of Probability:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (90)$$

- **Conditional Probability:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (91)$$

If events A and B are independent, $P(A \cap B) = P(A) \cdot P(B)$.

6.3 Common Probability Distributions

6.3.1 Binomial Distribution

Models k successes in n independent Bernoulli trials with success probability p :

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (92)$$

$$\text{Mean: } \mu = np, \quad \text{Variance: } \sigma^2 = np(1 - p) \quad (93)$$

6.3.2 Standard Normal Distribution (Z-distribution)

A continuous normal distribution with mean μ and standard deviation σ :

$$Z = \frac{X - \mu}{\sigma} \quad (94)$$

The probability $P(X \leq x) = P(Z \leq z) = \Phi(z)$ is obtained from standard normal tables.

6.4 Linear Regression and Correlation

Fits a straight line $y = a + bx$ to n data pairs (x_i, y_i) .

Least-Squares Regression Equations

Slope (b):

$$b = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2} \quad (95)$$

Intercept (a):

$$a = \bar{y} - b\bar{x} = \frac{\sum y_i - b \sum x_i}{n} \quad (96)$$

Sample Correlation Coefficient (r):

$$r = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{[n \sum x_i^2 - (\sum x_i)^2] [n \sum y_i^2 - (\sum y_i)^2]}} \quad (97)$$

7 Example Problems and Solutions

This section contains 20 exam-style problems covering the FE Civil CBT Mathematics and Statistics specifications. Each problem includes a clear problem statement, multiple-choice options, the correct answer, and a full step-by-step solution.

Problem 1: Distance and Slope of a Line Segment

A land surveying crew determines two boundary monuments located at Cartesian coordinates $A(-3, 4)$ and $B(5, -2)$. What is the length of line segment AB and the slope of a line perpendicular to AB ?

- A. Distance = 10 units; Perpendicular Slope = $-3/4$
- B. Distance = 10 units; Perpendicular Slope = $+4/3$
- C. Distance = 12 units; Perpendicular Slope = $+3/4$
- D. Distance = 14 units; Perpendicular Slope = $-4/3$

Solution

Correct Answer: B

Step 1: Calculate Distance AB

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-3))^2 + (-2 - 4)^2} \quad (98)$$

$$d = \sqrt{8^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ units} \quad (99)$$

Step 2: Calculate Slope m_{AB}

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{5 - (-3)} = \frac{-6}{8} = -\frac{3}{4} \quad (100)$$

Step 3: Calculate Perpendicular Slope $m_{\perp}$

$$m_{\perp} = -\frac{1}{m_{AB}} = -\frac{1}{-3/4} = +\frac{4}{3} \quad (101)$$

Problem 2: Conic Section Identification

Determine the geometric shape represented by the second-degree general equation:

$$3x^2 - 4xy + 2y^2 + 8x - 6y + 12 = 0 \quad (102)$$

- A. Parabola
- B. Circle
- C. No real curve (empty set)
- D. Hyperbola

Solution**Correct Answer: C****Step 1: Identify Coefficients from General Form** Comparing $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$:

$$A = 3, \quad B = -4, \quad C = 2 \quad (103)$$

Step 2: Evaluate the Discriminant

$$B^2 - 4AC = (-4)^2 - 4(3)(2) = 16 - 24 = -8 \quad (104)$$

Step 3: Check reality. Set $u = x + 1$, $v = y - \frac{1}{2}$. The equation becomes $3u^2 - 4uv + 2v^2 + \frac{13}{2} = 0$. The quadratic form is positive definite, so the left side is at least 6.5. There are **no real points** (C).**Problem 3: Standard Equation of a Parabola**

A parabolic drainage channel cross-section has its vertex at $(2, -1)$ and its directrix at $y = -4$. What is the standard equation of the parabola?

- A. $(x - 2)^2 = 12(y + 1)$
- B. $(x + 2)^2 = 12(y - 1)$
- C. $(y + 1)^2 = 12(x - 2)$
- D. $(x - 2)^2 = 3(y + 1)$

Solution**Correct Answer: A****Step 1: Determine Orientation and Parameter p** The vertex is $(h, k) = (2, -1)$ and the directrix is horizontal: $y = -4$. Since the directrix is horizontal, the axis of symmetry is vertical, so the equation has form $(x - h)^2 = 4p(y - k)$.The directrix formula is $y = k - p$:

$$-4 = -1 - p \implies p = 3 \quad (105)$$

Since $p = 3 > 0$, the parabola opens upwards.**Step 2: Substitute into Standard Form**

$$(x - 2)^2 = 4(3)(y - (-1)) \implies (x - 2)^2 = 12(y + 1) \quad (106)$$

Problem 4: Converting Sphere Equation to Standard Form

Find the center (h, k, l) and radius r of the sphere defined by:

$$x^2 + y^2 + z^2 - 6x + 8y - 4z + 13 = 0 \quad (107)$$

- A. Center $(3, -4, 2)$, Radius $r = 4$
- B. Center $(-3, 4, -2)$, Radius $r = 16$
- C. Center $(3, -4, 2)$, Radius $r = 12$
- D. Center $(6, -8, 4)$, Radius $r = 2$

Solution

Correct Answer: A

Step 1: Group Variables and Complete the Square

$$(x^2 - 6x) + (y^2 + 8y) + (z^2 - 4z) = -13 \quad (108)$$

Add $(\frac{-6}{2})^2 = 9$, $(\frac{8}{2})^2 = 16$, and $(\frac{-4}{2})^2 = 4$ to both sides:

$$(x^2 - 6x + 9) + (y^2 + 8y + 16) + (z^2 - 4z + 4) = -13 + 9 + 16 + 4 \quad (109)$$

$$(x - 3)^2 + (y + 4)^2 + (z - 2)^2 = 16 \quad (110)$$

Step 2: Extract Center and Radius Comparing to $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$:

$$\text{Center} = (3, -4, 2), \quad r = \sqrt{16} = 4 \quad (111)$$

Problem 5: L'Hôpital's Rule for Indeterminate Limits

Evaluate the following limit:

$$L = \lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{x^3} \quad (112)$$

- A. 0
- B. $-9/2$
- C. $-3/2$
- D. $9/2$

Solution**Correct Answer: B****Step 1: Check Form at $x = 0$** $\frac{\sin(0)-0}{0} = \frac{0}{0}$ (Indeterminate). Apply L'Hôpital's Rule (Pass 1):

$$L = \lim_{x \rightarrow 0} \frac{3 \cos(3x) - 3}{3x^2} = \lim_{x \rightarrow 0} \frac{\cos(3x) - 1}{x^2} \quad \left(\text{Form } \frac{0}{0} \right) \quad (113)$$

Step 2: Apply L'Hôpital's Rule (Pass 2)

$$L = \lim_{x \rightarrow 0} \frac{-3 \sin(3x)}{2x} \quad \left(\text{Form } \frac{0}{0} \right) \quad (114)$$

Step 3: Apply L'Hôpital's Rule (Pass 3)

$$L = \lim_{x \rightarrow 0} \frac{-9 \cos(3x)}{2} = \frac{-9(1)}{2} = -4.5 = -\frac{9}{2} \quad (115)$$

Problem 6: Tangent Line Slope and DerivativeFind the equation of the tangent line to the function $f(x) = 2x^3 - 5x^2 + 4x - 1$ at $x = 2$.

- A. $y = 12x - 21$
- B. $y = 12x - 19$
- C. $y = 8x - 13$
- D. $y = 8x - 11$

Solution**Correct Answer: C****Step 1: Find Point of Tangency (x_0, y_0)** At $x_0 = 2$:

$$y_0 = f(2) = 2(2)^3 - 5(2)^2 + 4(2) - 1 = 16 - 20 + 8 - 1 = 3 \quad (116)$$

Point is $(2, 3)$.**Step 2: Find First Derivative $f'(x)$**

$$f'(x) = 6x^2 - 10x + 4 \quad (117)$$

Step 3: Evaluate Slope $m_t = f'(2)$

$$m_t = 6(2)^2 - 10(2) + 4 = 24 - 20 + 4 = 8 \quad (118)$$

Step 4: Form Tangent Line Equation

$$y - 3 = 8(x - 2) \implies y = 8x - 16 + 3 \implies y = 8x - 13 \quad (119)$$

Problem 7: Optimization of Beam Dimensions

A rectangular timber beam is to be cut from a circular log of diameter $D = 20$ inches. The bending strength of the beam is proportional to $b \cdot h^2$, where b is the width and h is the height. What width b maximizes the beam's bending strength?

- A. $b = 10.0$ in
- B. $b = 11.55$ in
- C. $b = 14.14$ in
- D. $b = 16.33$ in

Solution

Correct Answer: B

Step 1: Set Up Constraint and Objective Function By the Pythagorean Theorem: $b^2 + h^2 = D^2 = 20^2 = 400 \implies h^2 = 400 - b^2$.

Objective function to maximize:

$$S(b) = b \cdot h^2 = b(400 - b^2) = 400b - b^3 \quad (120)$$

Step 2: Take Derivative and Set to Zero

$$\frac{dS}{db} = 400 - 3b^2 = 0 \implies 3b^2 = 400 \implies b = \sqrt{\frac{400}{3}} = \frac{20}{\sqrt{3}} \approx 11.547 \text{ in} \quad (121)$$

Step 3: Check Second Derivative $\frac{d^2S}{db^2} = -6b < 0$ for $b > 0$, so $b \approx 11.55$ in yields a maximum.

Problem 8: Bounded Area Between Parabola and Line

Find the total area bounded between the parabola $y = x^2$ and the line $y = 2x + 3$.

- A. $32/3$
- B. 10.00
- C. 11.33
- D. 12.00

Solution**Correct Answer: A**

Step 1: Find Points of Intersection Set $x^2 = 2x + 3 \implies x^2 - 2x - 3 = 0 \implies (x - 3)(x + 1) = 0$. Limits of integration are $x_1 = -1$ and $x_2 = 3$.

Step 2: Set Up Definite Integral Upper curve is $y_T = 2x + 3$, lower curve is $y_B = x^2$:

$$A = \int_{-1}^3 [(2x + 3) - x^2] dx = \left[x^2 + 3x - \frac{x^3}{3} \right]_{-1}^3 \quad (122)$$

Step 3: Evaluate At $x = 3$: $(3)^2 + 3(3) - \frac{27}{3} = 9 + 9 - 9 = 9$. At $x = -1$: $(-1)^2 + 3(-1) - \frac{-1}{3} = 1 - 3 + \frac{1}{3} = -\frac{5}{3}$.

$$A = 9 - \left(-\frac{5}{3} \right) = 9 + \frac{5}{3} = \frac{32}{3} \approx 10.667 \quad (123)$$

Problem 9: Centroid of a Parabolic Region

Determine the y -coordinate centroid ($\bar{y}$) of the area bounded by $y = 4 - x^2$ and the x -axis from $x = -2$ to $x = 2$.

- A. $\bar{y} = 1.20$
- B. $\bar{y} = 1.60$
- C. $\bar{y} = 2.00$
- D. $\bar{y} = 2.40$

Solution**Correct Answer: B**

Step 1: Calculate Area A

$$A = \int_{-2}^2 (4 - x^2) dx = 2 \int_0^2 (4 - x^2) dx = 2 \left[4x - \frac{x^3}{3} \right]_0^2 = 2 \left(8 - \frac{8}{3} \right) = \frac{32}{3} \quad (124)$$

Step 2: Calculate First Moment about x -axis (M_x)

$$M_x = \frac{1}{2} \int_{-2}^2 [f(x)]^2 dx = \frac{1}{2} \int_{-2}^2 (4 - x^2)^2 dx = \int_0^2 (16 - 8x^2 + x^4) dx \quad (125)$$

$$M_x = \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = 32 - \frac{64}{3} + \frac{32}{5} = \frac{480 - 320 + 96}{15} = \frac{256}{15} \quad (126)$$

Step 3: Calculate Centroid $\bar{y}$

$$\bar{y} = \frac{M_x}{A} = \frac{256/15}{32/3} = \frac{256}{15} \cdot \frac{3}{32} = \frac{8}{5} = 1.60 \quad (127)$$

Problem 10: Volume of Revolution via Disk Method

Find the volume of the solid formed by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x -axis.

- A. 8π
- B. 16π
- C. 32π
- D. 64π

Solution

Correct Answer: A

Step 1: Apply Disk Method Formula

$$V = \pi \int_a^b [f(x)]^2 dx = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x dx \quad (128)$$

Step 2: Integrate

$$V = \pi \left[\frac{x^2}{2} \right]_0^4 = \pi \left(\frac{16}{2} - 0 \right) = 8\pi \approx 25.13 \text{ cubic units} \quad (129)$$

Problem 11: Matrix Determinant and Singularity

Find the values of k for which the matrix A is singular (non-invertible, $\det(A) = 0$):

$$A = \begin{bmatrix} 2 & k \\ 4 & 6 \end{bmatrix} \quad (130)$$

- A. $k = 1$
- B. $k = 3$
- C. $k = 6$
- D. $k = 12$

Solution

Correct Answer: B

Step 1: Compute Determinant

$$\det(A) = (2)(6) - (k)(4) = 12 - 4k \quad (131)$$

Step 2: Set Determinant to Zero

$$12 - 4k = 0 \implies 4k = 12 \implies k = 3 \quad (132)$$

Problem 12: Vector Dot Product and Angle

Given vectors $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{B} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, calculate the dot product $\mathbf{A} \cdot \mathbf{B}$ and the angle θ between them.

- A. $\mathbf{A} \cdot \mathbf{B} = -2$; $\theta = 97.7^\circ$
- B. $\mathbf{A} \cdot \mathbf{B} = 2$; $\theta = 82.3^\circ$
- C. $\mathbf{A} \cdot \mathbf{B} = 14$; $\theta = 21.2^\circ$
- D. $\mathbf{A} \cdot \mathbf{B} = -2$; $\theta = 105.0^\circ$

Solution

Correct Answer: A

Step 1: Compute Dot Product

$$\mathbf{A} \cdot \mathbf{B} = (3)(2) + (4)(-2) + (0)(1) = 6 - 8 + 0 = -2 \quad (133)$$

Step 2: Calculate Magnitudes

$$\|\mathbf{A}\| = \sqrt{3^2 + 4^2} = 5, \quad \|\mathbf{B}\| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3 \quad (134)$$

Step 3: Calculate Angle θ

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} = \frac{-2}{(5)(3)} = -\frac{2}{15} \approx -0.1333 \quad (135)$$

$$\theta = \arccos(-0.1333) = 97.66^\circ \approx 97.7^\circ \quad (136)$$

Problem 13: Geometric Progression in Infrastructure Maintenance

The maintenance cost of a bridge is \$10,000 in the first year and increases by 6% each year. What is the total maintenance cost over a 10-year period?

- A. \$131,808
- B. \$139,716
- C. \$159,084
- D. \$168,914

Solution**Correct Answer: A****Step 1: Identify Parameters of Geometric Series** First term $a_1 = 10,000$, common ratio $r = 1 + 0.06 = 1.06$, number of terms $n = 10$.**Step 2: Apply Sum Formula for Geometric Progression**

$$S_n = \frac{a_1(1 - r^n)}{1 - r} = \frac{10,000(1 - (1.06)^{10})}{1 - 1.06} \quad (137)$$

$$(1.06)^{10} \approx 1.790848 \quad (138)$$

$$S_{10} = \frac{10,000(1 - 1.790848)}{-0.06} = \frac{10,000(-0.790848)}{-0.06} = 131,808 \quad (139)$$

Total cost over 10 years is **\$131,808**.

Problem 14: First-Order Differential Equation with Integrating Factor

Solve the initial value problem:

$$\frac{dy}{dx} + 2y = 4, \quad y(0) = 5 \quad (140)$$

A. $y(x) = 2 + 3e^{-2x}$

B. $y(x) = 4 + e^{-2x}$

C. $y(x) = 2 + 5e^{-2x}$

D. $y(x) = 3 + 2e^{-2x}$

Solution**Correct Answer: A****Step 1: Identify $P(x)$ and $Q(x)$** In $\frac{dy}{dx} + P(x)y = Q(x)$: $P(x) = 2$ and $Q(x) = 4$.**Step 2: Compute Integrating Factor**

$$IF = e^{\int 2 dx} = e^{2x} \quad (141)$$

Step 3: Solve General Equation

$$y \cdot e^{2x} = \int 4e^{2x} dx = 2e^{2x} + C \implies y(x) = 2 + Ce^{-2x} \quad (142)$$

Step 4: Apply Initial Condition $y(0) = 5$

$$5 = 2 + Ce^0 \implies C = 3 \implies y(x) = 2 + 3e^{-2x} \quad (143)$$

Problem 15: Second-Order Differential Equation Classification

Find the general solution to the second-order homogeneous differential equation modeling structural damping:

$$y'' + 6y' + 9y = 0 \quad (144)$$

- A. $y(x) = C_1 e^{-3x} + C_2 e^{3x}$
- B. $y(x) = (C_1 + C_2 x) e^{-3x}$
- C. $y(x) = e^{-3x} (C_1 \cos(3x) + C_2 \sin(3x))$
- D. $y(x) = C_1 e^{-6x} + C_2 e^{-9x}$

Solution

Correct Answer: B

Step 1: Form Characteristic Equation

$$r^2 + 6r + 9 = 0 \implies (r + 3)^2 = 0 \implies r_1 = r_2 = -3 \quad (145)$$

Step 2: Classify Damping Regime Since the roots are real and repeated ($b^2 - 4ac = 36 - 36 = 0$), the system is ****Critically Damped****.

Step 3: Write General Solution

$$y(x) = (C_1 + C_2 x) e^{-3x} \quad (146)$$

Problem 16: Laplace Transform of Differential System

Find the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$ for the differential equation:

$$y'' + 4y = 8, \quad y(0) = 0, \quad y'(0) = 0 \quad (147)$$

- A. $Y(s) = \frac{8}{s(s^2+4)}$
- B. $Y(s) = \frac{8}{s^2+4}$
- C. $Y(s) = \frac{4}{s(s^2+4)}$
- D. $Y(s) = \frac{8}{s^2(s+4)}$

Solution**Correct Answer: A****Step 1: Apply Laplace Transform to Both Sides**

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = \mathcal{L}\{8\} \quad (148)$$

$$[s^2Y(s) - sy(0) - y'(0)] + 4Y(s) = \frac{8}{s} \quad (149)$$

Step 2: Substitute Initial Conditions $y(0) = 0, y'(0) = 0$

$$s^2Y(s) + 4Y(s) = \frac{8}{s} \implies Y(s)(s^2 + 4) = \frac{8}{s} \quad (150)$$

$$Y(s) = \frac{8}{s(s^2 + 4)} \quad (151)$$

Problem 17: Sample Standard Deviation of Concrete Test Cylinders

Five concrete test cylinders exhibit the following 28-day compressive strengths (in psi):

$$\{3800, 4000, 4100, 3900, 4200\} \quad (152)$$

What is the sample standard deviation s ?

- A. 141.4 psi
- B. 158.1 psi
- C. 126.5 psi
- D. 200.0 psi

Solution**Correct Answer: B****Step 1: Calculate Sample Mean $\bar{x}$**

$$\bar{x} = \frac{3800 + 4000 + 4100 + 3900 + 4200}{5} = \frac{20000}{5} = 4000 \text{ psi} \quad (153)$$

Step 2: Calculate Sum of Squared Deviations $\sum(x_i - \bar{x})^2$

$$(3800 - 4000)^2 = (-200)^2 = 40,000 \quad (154)$$

$$(4000 - 4000)^2 = 0^2 = 0 \quad (155)$$

$$(4100 - 4000)^2 = (100)^2 = 10,000 \quad (156)$$

$$(3900 - 4000)^2 = (-100)^2 = 10,000 \quad (157)$$

$$(4200 - 4000)^2 = (200)^2 = 40,000 \quad (158)$$

$$\sum(x_i - \bar{x})^2 = 40000 + 0 + 10000 + 10000 + 40000 = 100,000 \quad (159)$$

Step 3: Calculate Sample Variance and Standard Deviation

$$s^2 = \frac{100,000}{n - 1} = \frac{100,000}{4} = 25,000 \quad (160)$$

$$s = \sqrt{25,000} \approx 158.11 \text{ psi} \quad (161)$$

Problem 18: Binomial Probability in Structural Inspection

In a large structural welding project, the probability that a weld defect occurs is $p = 0.05$. If an inspector randomly examines 10 independent welds, what is the probability that exactly 1 weld has a defect?

- A. 0.3151
- B. 0.5987
- C. 0.0746
- D. 0.3774

Solution**Correct Answer: A****Step 1: Identify Binomial Formula** $n = 10$, $k = 1$, $p = 0.05$, $1 - p = 0.95$.

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (162)$$

Step 2: Calculate Combination and Probability

$$\binom{10}{1} = 10 \quad (163)$$

$$P(X = 1) = 10 \cdot (0.05)^1 \cdot (0.95)^9 = 10 \cdot 0.05 \cdot 0.630249 = 0.3151 \quad (164)$$

Problem 19: Standard Normal Distribution and Quality Target

The compressive strength of a soil-cement mix is normally distributed with mean $\mu = 500$ psi and standard deviation $\sigma = 40$ psi. What percentage of samples will have a compressive strength greater than 560 psi?

- A. 6.68%
- B. 93.32%
- C. 15.87%
- D. 2.28%

Solution**Correct Answer: A****Step 1: Calculate Z-score**

$$Z = \frac{X - \mu}{\sigma} = \frac{560 - 500}{40} = \frac{60}{40} = 1.50 \quad (165)$$

Step 2: Find Tail Probability From standard normal distribution tables: $\Phi(1.50) = P(Z \leq 1.50) = 0.9332$. The probability of exceeding 560 psi is:

$$P(X > 560) = 1 - \Phi(1.50) = 1 - 0.9332 = 0.0668 = 6.68\% \quad (166)$$

Problem 20: Least-Squares Linear Regression Line

An environmental engineer collects soil compaction data pairing moisture content $x(\%)$ and dry density $y(\text{pcf})$:

Sample	$x_i(\%)$	$y_i(\text{pcf})$	$x_i y_i$	x_i^2
1	10	110	1100	100
2	12	114	1368	144
3	14	117	1638	196
4	16	121	1936	256

Find the slope b of the least-squares linear regression equation $y = a + bx$.

- A. $b = 1.80 \text{ pcf}/\%$
- B. $b = 1.83 \text{ pcf}/\%$
- C. $b = 2.10 \text{ pcf}/\%$
- D. $b = 1.50 \text{ pcf}/\%$

Solution

Correct Answer: A

Step 1: Compute Summations for $n = 4$

$$\sum x_i = 10 + 12 + 14 + 16 = 52 \quad (167)$$

$$\sum y_i = 110 + 114 + 117 + 121 = 462 \quad (168)$$

$$\sum x_i y_i = 1100 + 1368 + 1638 + 1936 = 6042 \quad (169)$$

$$\sum x_i^2 = 100 + 144 + 196 + 256 = 696 \quad (170)$$

Step 2: Calculate Slope b

$$b = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2} = \frac{4(6042) - (52)(462)}{4(696) - (52)^2} \quad (171)$$

$$b = \frac{24168 - 24024}{2784 - 2704} = \frac{144}{80} = 1.80 \text{ pcf}/\% \quad (172)$$