

Civil Engineering FE CBT Exam

Knowledge: Mechanics of Materials with 10 Example Problems

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1 Shear and Bending Moment Diagrams

Calculating internal shear force (V) and bending moment (M) resultants across beam spans is the starting point for determining structural bending and shear stresses.

1.1 Sign Conventions

NCEES standards define positive internal shear and bending moments as follows:

- **Positive Shear Force (V):** An internal shear force causing clockwise rotation of a beam segment. On a free-body diagram of a left-hand cut section, positive shear acts **downward**; on a right-hand cut section, it acts **upward**.
- **Positive Bending Moment (M):** An internal moment causing concave upward bending curvature (the “smiling beam” convention). Positive bending moment causes **compression** in top fibers and **tension** in bottom fibers.

1.2 Calculus & Differential Relationships

The distributed load $w(x)$ (positive downward), internal shear force $V(x)$, and internal bending moment $M(x)$ obey first-order differential calculus relations:

$$w(x) = -\frac{dV(x)}{dx} \iff \Delta V = V_2 - V_1 = \int_{x_1}^{x_2} [-w(x)] dx \quad (1)$$

$$V(x) = \frac{dM(x)}{dx} \iff \Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V(x) dx \quad (2)$$

1.3 Graphical Construction Rules

Constructing shear and moment diagrams rapidly during the FE exam relies on polynomial degree relationships:

1. **Concentrated Downward Load (P):** Creates an instantaneous downward vertical jump in the shear diagram equal to magnitude P .
2. **Uniformly Distributed Load (w):** Creates a constant negative slope on the shear diagram (1st degree sloped line) and a parabolic curve (2nd degree polynomial) on the moment diagram.
3. **Triangular Load (Linear $w(x)$):** Creates a 2nd degree parabolic curve on the shear diagram and a 3rd degree cubic curve on the moment diagram.
4. **Zero Shear Point ($V = 0$):** Because $V(x) = dM/dx$, local maximum or minimum bending moments occur precisely where the shear force diagram crosses zero or changes sign!
5. **Net Area Integration (ΔM):** The change in bending moment between two points equals the net area under the shear force diagram over that interval.

Polynomial Degrees Across Diagrams

Applied Distributed Load $w(x)$	Shear Force Diagram $V(x)$	Bending Moment Diagram $M(x)$
No Load ($w = 0$)	Constant / Horizontal Line ($n = 0$)	Linear / Sloped Line ($n = 1$)
Uniform Load ($w = \text{const}$)	Linear / Sloped Line ($n = 1$)	Parabolic Curve ($n = 2$)
Linear Load ($w = c \cdot x$)	Parabolic Curve ($n = 2$)	Cubic Curve ($n = 3$)

2 Axial Stresses, Thermal Strains & Axial Deformations

Axial members carry loads directed along their central longitudinal axis, producing normal stresses (σ) and longitudinal strains (ϵ).

2.1 Uniaxial Stress-Strain Principles

Normal stress (σ) measures average internal force per unit cross-sectional area, while normal strain (ϵ) measures unit axial change in length:

$$\sigma = \frac{P}{A}, \quad \epsilon = \frac{\delta}{L_0} \quad (3)$$

Within the elastic range, Hooke's Law establishes proportionality via Young's Modulus of Elasticity (E):

$$\sigma = E\epsilon \quad \Rightarrow \quad \frac{P}{A} = E \left(\frac{\delta}{L} \right) \quad (4)$$

2.2 Elastic Axial Elongation Formula

Rearranging Hooke's law yields the core elongation formula for a uniform prismatic member under axial load P :

$$\delta = \frac{PL}{AE} \quad (5)$$

For stepped bars, multi-segment members, or variable axial forces, total elongation is evaluated by superposition or integration:

$$\delta_{\text{total}} = \sum_i \frac{P_i L_i}{A_i E_i} \quad \text{or} \quad \delta = \int_0^L \frac{P(x)}{A(x)E} dx \quad (6)$$

2.3 Thermal Expansion & Constrained Thermal Stress

Temperature variations ($\Delta T = T_{\text{final}} - T_{\text{initial}}$) produce thermal deformations independent of applied loads:

$$\delta_t = \alpha L \Delta T, \quad \epsilon_t = \alpha \Delta T \quad (7)$$

where α is the thermal expansion coefficient ($1/^\circ\text{C}$ or $1/^\circ\text{F}$).

When a member is restrained between unyielding rigid supports, thermal movement is prevented ($\delta_{\text{total}} = \delta_t + \delta_{\text{force}} = 0$). This constraint generates compressive reaction forces and thermal stress:

$$\alpha L \Delta T + \frac{PL}{AE} = 0 \quad \Rightarrow \quad \sigma = \frac{P}{A} = -E\alpha \Delta T \quad (8)$$

2.4 Poisson's Ratio & Lateral Strains

Under axial tension, structural materials stretch longitudinally and contract laterally. Poisson's ratio (ν) measures the negative ratio of lateral strain to longitudinal strain:

$$\nu = -\frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{axial}}} = -\frac{\Delta d/d_0}{\Delta L/L_0} \quad (9)$$

3 Torsional Stresses & Angular Deformations

Torsion occurs when moments (torque T) act about a structural member's longitudinal axis, producing shear stresses (τ) and angular twist (ϕ).

3.1 Torsional Shear Stress Formula

In elastic circular shafts, planar cross-sections rotate without distorting. Shear stress at radial distance r from the neutral center is:

$$\tau = \frac{Tr}{J} \quad \Rightarrow \quad \tau_{\max} = \frac{Tc}{J} = \frac{Td/2}{J} \quad (10)$$

where J is the polar moment of inertia and $c = d/2$ is outer radius.

3.2 Polar Moment of Inertia (J)

- Solid Circular Shaft (diameter d):

$$J = \frac{\pi d^4}{32} \quad \Rightarrow \quad \tau_{\max} = \frac{16T}{\pi d^3} \quad (11)$$

- Hollow Circular Shaft (outer diameter d_o , inner diameter d_i):

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} \quad (12)$$

3.3 Angle of Twist Formula

The total angle of twist (ϕ , in **radians**) for a shaft of length L and shear modulus G is:

$$\phi = \frac{TL}{GJ} \quad (13)$$

Convert radians to degrees using: $\phi_{\text{degrees}} = \phi_{\text{radians}} \times \left(\frac{180^\circ}{\pi}\right)$.

3.4 Relationships Between Elastic Constants

The Shear Modulus (Modulus of Rigidity, G) and Bulk Modulus (K) relate to E and Poisson's ratio (ν) via:

$$G = \frac{E}{2(1 + \nu)}, \quad K = \frac{E}{3(1 - 2\nu)} \quad (14)$$

3.5 Power Transmission in Rotating Shafts

Power (P) transmitted by a shaft rotating at angular frequency ω (rad/s) or f (Hz) is:

$$P = T\omega = 2\pi fT \quad (15)$$

4 Flexural Bending & Transverse Shear Stresses

Transverse loading produces simultaneous flexural bending moments (M) and shear forces (V), giving rise to normal bending stresses (σ) and transverse shear stresses (τ).

4.1 Flexural Stress Formula (The Elastic Bending Equation)

Normal stress (σ) at vertical distance y from the neutral centroidal axis (NA) is given by:

$$\sigma = -\frac{My}{I} \quad (16)$$

where I is the centroidal area moment of inertia.

Maximum bending stress occurs at extreme fibers furthest from the neutral axis ($y = c$):

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M}{S} \quad (17)$$

where $S = I/c$ is the **Elastic Section Modulus**. For a rectangular cross-section ($b \times h$):

$$I = \frac{bh^3}{12}, \quad c = \frac{h}{2}, \quad S = \frac{bh^2}{6} \quad (18)$$

Extreme Compression vs. Tension Fibers

For asymmetric cross-sections (e.g. T-beams or triangles), $c_{\text{top}} \neq c_{\text{bottom}}$. Always measure y directly from the centroid to the specific fiber requested in the exam question!

4.2 Transverse Shear Stress Formula

Shear forces generate horizontal and vertical shear stresses across beam cross-sections according to:

$$\tau = \frac{VQ}{Ib} \quad (19)$$

where:

- V = total transverse shear force at the cross-section.
- Q = first moment of area above (or below) the plane where shear stress is calculated: $Q = A'\bar{y}'$.
- I = total centroidal area moment of inertia of the entire cross-section.
- b = width of the cross-section at the plane evaluated.

4.3 Shear Distribution in Rectangular Sections

For a rectangular cross-section ($b \times h$), shear stress varies parabolically with depth, reaching maximum intensity at the neutral centroidal axis ($y = 0$):

$$\tau_{\max} = 1.5 \frac{V}{A} = \frac{3V}{2bh} \quad (20)$$

4.4 Composite Transformed Sections

For beams constructed from two dissimilar materials ($E_1 > E_2$), transform the section width using modular ratio n :

$$n = \frac{E_1}{E_2} \implies b_{\text{transformed}} = n \cdot b_1 \quad (21)$$

After finding transformed centroidal moment of inertia I_T , flexural stresses are calculated as:

$$\sigma_1 = -n \frac{My}{I_T}, \quad \sigma_2 = -\frac{My}{I_T} \quad (22)$$

5 Combined Stresses & Mohr's Circle

Structural components often experience combined axial load (P), flexural moment (M), torsion (T), and transverse shear force (V).

5.1 Superposition of Combined Normal Stresses

Normal stresses caused by axial load and bending moments superimpose linearly:

$$\sigma_{\text{combined}} = \pm \frac{P}{A} \pm \frac{M_{xy}}{I_x} \pm \frac{M_{yx}}{I_y} \quad (23)$$

5.2 2D Stress Transformation Equations

For a 2D stress element subjected to σ_x , σ_y , and τ_{xy} , the transformed normal stress ($\sigma_{x'}$) and shear stress ($\tau_{x'y'}$) at angle θ counterclockwise are:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta \quad (24)$$

$$\tau_{x'y'} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta \quad (25)$$

5.3 Mohr's Circle Construction Rules

Mohr's circle graphically represents 2D stress transformations on a coordinate system with σ horizontal and τ vertical:

1. **Center Coordinates (C):**

$$C = \frac{\sigma_x + \sigma_y}{2}, \quad 0 \quad (26)$$

2. **Circle Radius (R):**

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad (27)$$

3. **Principal Stresses (σ_a, σ_b):** Maximum and minimum normal stresses occur where shear stress is zero ($\tau = 0$):

$$\sigma_{a,b} = C \pm R = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad (28)$$

4. **Maximum In-Plane Shear Stress (τ_{max}):**

$$\tau_{\text{in-plane, max}} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad (29)$$

The normal stress associated with maximum shear stress equals average stress $\sigma_{\text{avg}} = C$.

5. **Principal Plane Orientation (θ_p):**

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (30)$$

5.4 Thin-Walled Pressure Vessels

Thin-walled shells ($r/t \geq 10$); the cylindrical axial stress below assumes closed ends.

- **Cylindrical Vessel:**

$$\text{Hoop Stress: } \sigma_1 = \frac{pr}{t}, \quad \text{Axial Stress: } \sigma_2 = \frac{pr}{2t} \quad (31)$$

- **Spherical Vessel:**

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \quad (32)$$

6 Beam Deflections & Column Buckling

6.1 Beam Deflection Differential Equations

The deflection curve $v(x)$ of an elastic beam is governed by differential equations:

$$EI \frac{d^4 v}{dx^4} = -w(x) \quad \Rightarrow \quad EI \frac{d^3 v}{dx^3} = V(x) \quad \Rightarrow \quad EI \frac{d^2 v}{dx^2} = M(x) \quad (33)$$

6.2 Euler Column Buckling

Euler buckling applies to elastic columns with idealized restraints. Check the elastic stress range; actual end conditions govern each effective length.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad (34)$$

Check both axes: $P_{cr,x} = \pi^2 EI_x / (K_x L_x)^2$ and $P_{cr,y} = \pi^2 EI_y / (K_y L_y)^2$. Use the smaller load. The smaller principal inertia governs only for equal effective lengths.

6.3 Effective Length Factor (K) Standard Values

End Support Boundary Conditions	Theoretical K	Design K	Buckled Shape Feature
Pinned–Pinned	1.0	1.0	Single sine half-wave
Fixed–Fixed	0.5	0.65	Inflection points at $0.25L$
Fixed–Pinned	0.7	0.80	Inflection point at $0.30L$
Fixed–Free (Cantilever Column)	2.0	2.10	Double effective length

Design K values are approximate; assess the actual end restraint.

6.4 Critical Buckling Stress & Slenderness Ratio

Dividing P_{cr} by area A yields critical buckling stress σ_{cr} expressed using radius of gyration $r = \sqrt{I/A}$:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(KL/r)^2} \quad (35)$$

The parameter (KL/r) is defined as the ****Slenderness Ratio****. Higher slenderness dramatically reduces compressive column stability!

7 Example Problems and Solutions

The following 10 practice problems reflect the exact scope, difficulty, and format of Mechanics of Materials questions on the NCEES FE Civil CBT exam.

Problem 1: Shear & Moment Diagrams - Support Reaction & Zero-Shear Location

A simply supported beam of span $L = 9.0$ m carries a uniformly distributed load $w_1 = 200$ kN/m over segment $AB = 3.5$ m and $w_2 = 160$ kN/m over segment $BC = 5.5$ m. Ignoring beam self-weight, the vertical reaction force at support A (R_A) is to be calculated.

Question: What is the magnitude of reaction force R_A ?

- (A) 713.3 kN
- (B) 832.8 kN
- (C) 1,390.0 kN
- (D) 1,580.0 kN

Solution

Step 1: Calculate total applied force resultants and take moments about support C ($\sum M_C = 0$):

$$W_1 = 200 \text{ kN/m} \times 3.5 \text{ m} = 700 \text{ kN} \quad \text{acting at } (5.5 + 3.5/2) = 7.25 \text{ m from } C$$

$$W_2 = 160 \text{ kN/m} \times 5.5 \text{ m} = 880 \text{ kN} \quad \text{acting at } 5.5/2 = 2.75 \text{ m from } C$$

Step 2: Apply static equilibrium equation $\sum M_C = 0$:

$$R_A \cdot (9.0 \text{ m}) - (700 \text{ kN} \cdot 7.25 \text{ m}) - (880 \text{ kN} \cdot 2.75 \text{ m}) = 0$$

$$R_A \cdot 9.0 = 5075 + 2420 = 7495 \text{ kN} \cdot \text{m} \quad \implies \quad R_A = \frac{7495}{9.0} = 832.8 \text{ kN}$$

Correct Answer: (B)

Problem 2: Shear & Bending Moment Diagram Profile Identification

A beam supported at both ends carries a downward point load P at midspan. Which of the following describes the shape of the shear force diagram $V(x)$ and bending moment diagram $M(x)$ between the left support and point load?

- (A) $V(x)$ is linear sloped; $M(x)$ is parabolic curved.
- (B) $V(x)$ is constant horizontal; $M(x)$ is linear sloped.
- (C) $V(x)$ is constant horizontal; $M(x)$ is parabolic curved.
- (D) $V(x)$ is zero; $M(x)$ is constant horizontal.

Solution

Analysis: Between the support and point load, no distributed load acts ($w(x) = 0$). Because $dV/dx = -w = 0$, shear force $V(x)$ is ****constant**** (a horizontal line). Because $dM/dx = V = \text{constant}$, bending moment $M(x)$ is ****linear sloped****.

Correct Answer: (B)

Problem 3: Cable Strain & Equilibrium Analysis

A rigid horizontal beam is supported by two vertical steel cables (A and B). Each cable has Young's Modulus $E = 29,000$ ksi and cross-sectional area $A = 0.1 \text{ ft}^2$. In addition to a point load of 800 lb placed 4 ft from cable A along a 16 ft span, the beam itself weighs 200 lb (uniformly distributed).

Question: The strain (ϵ) in cable A is most nearly:

- (A) 0.002
- (B) 1.67×10^{-4}
- (C) 1.67×10^{-6}
- (D) 2.0×10^{-9}

Solution

Step 1: Determine reaction force T_A in Cable A via moment equilibrium about cable B ($\sum M_B = 0$): Beam span $L = 4 \text{ ft} + 12 \text{ ft} = 16 \text{ ft}$. Self-weight $W_{\text{beam}} = 200 \text{ lb}$ acts at midspan (8 ft from B). Applied load $P = 800 \text{ lb}$ acts 12 ft from B .

$$T_A \cdot (16 \text{ ft}) - (800 \text{ lb} \cdot 12 \text{ ft}) - (200 \text{ lb} \cdot 8 \text{ ft}) = 0$$

$$T_A \cdot 16 = 9600 + 1600 = 11,200 \text{ lb} \cdot \text{ft} \quad \Rightarrow \quad T_A = 700 \text{ lb}$$

Step 2: Convert cross-sectional area to square inches:

$$A_{\text{cable}} = 0.1 \text{ ft}^2 \times (144 \text{ in}^2/\text{ft}^2) = 14.4 \text{ in}^2$$

Step 3: Calculate axial stress σ_A and strain ϵ_A :

$$\sigma_A = \frac{T_A}{A_{\text{cable}}} = \frac{700 \text{ lb}}{14.4 \text{ in}^2} = 48.61 \text{ psi} = 0.04861 \text{ ksi}$$

$$\epsilon_A = \frac{\sigma_A}{E} = \frac{0.04861 \text{ ksi}}{29,000 \text{ ksi}} = 1.676 \times 10^{-6} \text{ in/in}$$

Correct Answer: (C)

Problem 4: Average Shear Stress in Double-Shear Clevis Bolt

A clevis connection links a steel bracket to a wooden block using a 20 mm diameter steel bolt. The applied tensile load is $P = 15 \text{ kN}$. Assuming double shear across the bolt cross-sections, the average shear stress in the bolt is most nearly:

- (A) 23.9 MPa
- (B) 47.7 MPa
- (C) 95.5 MPa
- (D) 191.0 MPa

Solution

Step 1: Identify shear planes: A clevis connection subjects the bolt to ****double shear****, meaning load P is divided across two shear planes ($V = P/2 = 7.5$ kN).

Step 2: Calculate bolt cross-sectional area (A_{bolt}):

$$A_{\text{bolt}} = \frac{\pi d^2}{4} = \frac{\pi (20 \text{ mm})^2}{4} = 314.16 \text{ mm}^2$$

Step 3: Calculate average shear stress (τ_{avg}):

$$\tau_{\text{avg}} = \frac{P}{2A_{\text{bolt}}} = \frac{15,000 \text{ N}}{2 \times 314.16 \text{ mm}^2} = \frac{15,000}{628.32} = 23.87 \text{ N/mm}^2 = 23.9 \text{ MPa}$$

Correct Answer: (A)

Problem 5: Composite Axial Member Extension

A stepped bar consists of a steel segment ($L_1 = 250$ cm, cross-section 3 cm $\times$ 3 cm, $E_{\text{steel}} = 200$ GPa) connected to an aluminum segment ($L_2 = 300$ cm, cross-section 2 cm $\times$ 2 cm, $E_{\text{alum}} = 69$ GPa). The axial tensile force P required to produce a total member elongation of $\delta = 5.0$ mm is most nearly:

- (A) 18.2 kN
- (B) 40.8 kN
- (C) 121.5 kN
- (D) 248.0 kN

Solution

Step 1: Convert all units to Newtons and millimeters (1 GPa = 10^3 N/mm²):

- Steel: $L_1 = 2500$ mm, $A_1 = 30 \times 30 = 900$ mm², $E_1 = 200,000$ N/mm².
- Aluminum: $L_2 = 3000$ mm, $A_2 = 20 \times 20 = 400$ mm², $E_2 = 69,000$ N/mm².

Step 2: Express total elongation as function of P ($\delta = \delta_1 + \delta_2$):

$$\delta = P \left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right]$$

$$\frac{L_1}{A_1 E_1} = \frac{2500}{900 \times 200,000} = 1.3889 \times 10^{-5} \text{ mm/N}$$

$$\frac{L_2}{A_2 E_2} = \frac{3000}{400 \times 69,000} = 1.0870 \times 10^{-4} \text{ mm/N}$$

Step 3: Solve for force P :

$$5.0 \text{ mm} = P [1.3889 \times 10^{-5} + 1.0870 \times 10^{-4}] = P [1.2259 \times 10^{-4}]$$

$$P = \frac{5.0}{1.2259 \times 10^{-4}} = 40,786 \text{ N} \approx 40.8 \text{ kN}$$

Correct Answer: (B)

Problem 6: Maximum Torsional Shear Stress in a Solid Shaft

A solid steel drive shaft of diameter $d = 10$ cm (0.10 m) transmits an applied torque $T = 50$ kN $\cdot$ m. The maximum torsional shear stress (τ_{max}) developed in the shaft is most nearly:

- (A) 25.5 MPa
- (B) 127.3 MPa
- (C) 254.6 MPa
- (D) 509.3 MPa

Solution

Step 1: Calculate polar moment of inertia (J):

$$J = \frac{\pi d^4}{32} = \frac{\pi (0.10 \text{ m})^4}{32} = 9.8175 \times 10^{-6} \text{ m}^4$$

Step 2: Calculate maximum torsional shear stress ($\tau_{\max} = Tc/J$): Outer radius $c = d/2 = 0.05 \text{ m}$. Torque $T = 50,000 \text{ N} \cdot \text{m}$.

$$\tau_{\max} = \frac{50,000 \text{ N} \cdot \text{m} \times 0.05 \text{ m}}{9.8175 \times 10^{-6} \text{ m}^4} = 254,647,908 \text{ Pa} \approx 254.6 \text{ MPa}$$

Correct Answer: (C)

Problem 7: Torsional Angle of Twist in Solid Shaft

A 4.0 m long solid aluminum shaft ($d = 12$ cm, $E = 69$ GPa, Poisson's ratio $\nu = 0.33$) is subjected to torque $T = 50$ kN · m. The maximum angle of twist (ϕ) at the free end is most nearly:

- (A) 0.43°
- (B) 5.42°
- (C) 21.7°
- (D) 310.5°

Solution

Step 1: Calculate Shear Modulus G from elastic constants:

$$G = \frac{E}{2(1 + \nu)} = \frac{69 \text{ GPa}}{2(1 + 0.33)} = \frac{69}{2.66} = 25.94 \text{ GPa} = 25.94 \times 10^9 \text{ N/m}^2$$

Step 2: Calculate polar moment of inertia J ($d = 0.12$ m):

$$J = \frac{\pi(0.12)^4}{32} = 2.0358 \times 10^{-5} \text{ m}^4$$

Step 3: Calculate angle of twist ϕ in radians ($\phi = TL/GJ$):

$$\phi = \frac{50,000 \text{ N} \cdot \text{m} \times 4.0 \text{ m}}{(25.94 \times 10^9 \text{ N/m}^2) \times (2.0358 \times 10^{-5} \text{ m}^4)} = 0.3787 \text{ radians}$$

Step 4: Convert radians to degrees:

$$\phi_{\text{deg}} = 0.3787 \text{ rad} \times \left(\frac{180^\circ}{\pi} \right) = 21.7^\circ$$

Correct Answer: (C)

Problem 8: Flexural Stress in Triangular Beam Cross-Section

A beam has a triangular section, $b = 40$ mm and $h = 60$ mm, with its **base at the bottom and apex at the top**. A sagging moment $M = 1.2$ kN·m compresses the top. Find the neutral-axis distance to the top and maximum compression.

- (A) $y = 2.0$ cm; $\sigma_{\text{comp}} = 100.0$ MPa
- (B) $y = 3.0$ cm; $\sigma_{\text{comp}} = 150.0$ MPa
- (C) $y = 4.0$ cm; $\sigma_{\text{comp}} = 200.0$ MPa
- (D) $y = 4.0$ cm; $\sigma_{\text{comp}} = 400.0$ MPa

Solution

Step 1: Centroid location for a triangular cross-section: The centroid is located $h/3 = 2.0$ cm from the base and $2h/3 = 4.0$ cm from the top apex. The extreme top compression fiber distance is $y_{\text{top}} = 4.0$ cm = 40 mm.

Step 2: Moment of inertia of a triangle about its centroid ($I = bh^3/36$):

$$I = \frac{40 \text{ mm} \times (60 \text{ mm})^3}{36} = \frac{40 \times 216,000}{36} = 240,000 \text{ mm}^4 = 2.4 \times 10^{-7} \text{ m}^4$$

Step 3: Calculate maximum flexural compression stress ($\sigma = My_{\text{top}}/I$): Bending moment $M = 1.2 \text{ kN} \cdot \text{m} = 1.2 \times 10^6 \text{ N} \cdot \text{mm}$.

$$\sigma_{\text{comp}} = \frac{1.2 \times 10^6 \text{ N} \cdot \text{mm} \times 40 \text{ mm}}{240,000 \text{ mm}^4} = 200.0 \text{ N/mm}^2 = 200.0 \text{ MPa}$$

Correct Answer: (C)

Problem 9: Maximum Transverse Shear Stress in Rectangular Beam

A rectangular aluminum beam of width $b = 4$ cm (40 mm) and height $h = 6$ cm (60 mm) carries a transverse vertical shear force $V = 7,000$ N. The maximum transverse shear stress ($\tau_{\max}$) developed at the neutral centroidal axis is most nearly:

- (A) 1.46 MPa
- (B) 2.92 MPa
- (C) 4.38 MPa
- (D) 5.83 MPa

Solution

Step 1: Calculate cross-sectional area ($A = b \cdot h$):

$$A = 40 \text{ mm} \times 60 \text{ mm} = 2400 \text{ mm}^2$$

Step 2: Apply rectangular section max shear formula ($\tau_{\max} = 1.5V/A$):

$$\tau_{\max} = 1.5 \times \frac{7,000 \text{ N}}{2400 \text{ mm}^2} = 1.5 \times 2.9167 \text{ N/mm}^2 = 4.375 \text{ N/mm}^2 = 4.38 \text{ MPa}$$

Correct Answer: (C)

Problem 10: Eccentric Column Combined Stress & Mohr's Circle

A column bracket carries an eccentric vertical compressive force $P = 15$ kN positioned at eccentricity $e = 50$ cm (0.50 m) from the column axis. The column cross-section is a square $10 \text{ cm} \times 10 \text{ cm}$ ($A = 0.01 \text{ m}^2$, section modulus $S = 1.667 \times 10^{-4} \text{ m}^3$).

Question: The maximum combined compressive stress ($\sigma_{\max}$) at the column base is most nearly:

- (A) 1.5 MPa
- (B) 43.5 MPa
- (C) 45.0 MPa
- (D) 46.5 MPa

Solution

Step 1: Calculate direct axial compressive stress ($\sigma_{\text{axial}} = P/A$):

$$\sigma_{\text{axial}} = \frac{15,000 \text{ N}}{0.01 \text{ m}^2} = 1,500,000 \text{ Pa} = 1.5 \text{ MPa} \quad (\text{Compression})$$

Step 2: Calculate bending moment ($M = P \cdot e$) and bending stress ($\sigma_{\text{bending}} = M/S$): Moment $M = 15 \text{ kN} \times 0.50 \text{ m} = 7.5 \text{ kN} \cdot \text{m} = 7,500 \text{ N} \cdot \text{m}$.

$$\sigma_{\text{bending}} = \frac{7,500 \text{ N} \cdot \text{m}}{1.667 \times 10^{-4} \text{ m}^3} = 45,000,000 \text{ Pa} = 45.0 \text{ MPa}$$

Step 3: Combine axial and bending compressive stresses on the compressive side:

$$\sigma_{\text{max, comp}} = \sigma_{\text{axial}} + \sigma_{\text{bending}} = 1.5 \text{ MPa} + 45.0 \text{ MPa} = 46.5 \text{ MPa}$$

Correct Answer: (D)