

Civil Engineering FE CBT Exam

Knowledge: Statics with 30 Example Problems

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1 Resultants of Force Systems

1.1 Core Principles & Theory

Forces have magnitude, direction, and a point or line of application. A general system reduces to a resultant force and a couple moment. A single resultant force is possible only when that moment can be represented by its line of action.

1.1.1 2D Vector Decomposition

A force $\mathbf{F}$ acting at an angle θ relative to the positive x -axis can be decomposed into orthogonal components:

$$F_x = F \cos \theta, \quad F_y = F \sin \theta \quad (1)$$

Conversely, if the horizontal and vertical components are known, the magnitude and angle are found by:

$$F = \sqrt{F_x^2 + F_y^2}, \quad \theta = \arctan\left(\frac{F_y}{F_x}\right) \quad (2)$$

In FE exam problems, direction is frequently represented using a slope triangle (often called a "rise and run" triangle) rather than an explicit angle. For a rise a and run b , the hypotenuse is $c = \sqrt{a^2 + b^2}$. The component resolution is:

$$F_x = F \left(\frac{b}{c}\right), \quad F_y = F \left(\frac{a}{c}\right) \quad (3)$$

This is particularly useful for the **5-12-13** and **3-4-5** right triangles, which are frequently reused by the NCEES.

1.1.2 3D Vector Operations

In three-dimensional spatial mechanics, a force vector $\mathbf{F}$ is written in Cartesian vector notation as:

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \quad (4)$$

The direction of a 3D force is described by coordinate directional angles $\theta_x, \theta_y, \theta_z$, which relate to the components via directional cosines:

$$\cos \theta_x = \frac{F_x}{F}, \quad \cos \theta_y = \frac{F_y}{F}, \quad \cos \theta_z = \frac{F_z}{F} \quad (5)$$

The unit vector $\mathbf{u}_F$ pointing in the direction of the force is:

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k} \quad (6)$$

The sum of the squares of the directional cosines is always equal to 1:

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \quad (7)$$

When the direction of a force is defined by a line of action passing through two points $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$, we first construct the position vector $\mathbf{r}_{AB}$:

$$\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k} \quad (8)$$

We find the distance $d_{AB} = |\mathbf{r}_{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$. Then, the unit vector is $\mathbf{u}_{AB} = \mathbf{r}_{AB}/d_{AB}$, and the force is expressed as:

$$\mathbf{F} = F\mathbf{u}_{AB} = F \left(\frac{x_B - x_A}{d_{AB}}\mathbf{i} + \frac{y_B - y_A}{d_{AB}}\mathbf{j} + \frac{z_B - z_A}{d_{AB}}\mathbf{k} \right) \quad (9)$$

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 94: Resultant (Two Dimensions)

The magnitude of the resultant F of n forces with components $F_{x,i}$ and $F_{y,i}$ is:

$$F = \left[\left(\sum F_{x,i} \right)^2 + \left(\sum F_{y,i} \right)^2 \right]^{1/2}$$

The direction angle is $\theta = \arctan \left(\frac{\sum F_{y,i}}{\sum F_{x,i}} \right)$.

Exam Pitfall Warning!

Sign Conventions in Components

Do not rely blindly on cos or sin formulas. Always draw or visualize the quadrants. If a force acts leftward, its x -component is negative; if it acts downward, its y -component is negative. When adding components to find resultants, algebraic sign tracking is the most common failure point.

2 Equivalent Force Systems

2.1 Core Principles & Theory

Any system of forces and moments acting on a rigid body can be simplified into a single statically equivalent system consisting of one **resultant force** acting at a chosen point and one **resultant moment** (couple). This concept is crucial for simplifying complex loading patterns prior to performing equilibrium or stress analysis.

2.1.1 Moments and Couples

The moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. Mathematically, the moment vector $\mathbf{M}_O$ about a point O is defined as the cross product of the position vector $\mathbf{r}$ (from O to the line of action of the force) and the force vector $\mathbf{F}$:

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad (10)$$

In Cartesian components, the cross product is evaluated using a determinant:

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \quad (11)$$

This yields the scalar moment components about the coordinate axes:

$$M_x = yF_z - zF_y, \quad M_y = zF_x - xF_z, \quad M_z = xF_y - yF_x \quad (12)$$

A **couple** consists of two parallel forces that are equal in magnitude, act in opposite directions, and are separated by a perpendicular distance d . Because the sum of forces of a couple is zero ($\sum \mathbf{F} = 0$), a couple creates a pure rotation without translation. The magnitude of the couple moment is:

$$M_c = Fd \quad (13)$$

Unlike force moments, the moment of a couple is a **free vector**, meaning its rotational effect is identical about any point on the body.

2.1.2 Distributed Loads

Civil engineering structures routinely carry distributed loads (e.g., wind, snow, water pressure, concrete self-weight). For 2D analysis, these loads are expressed as force per unit length, $w(x)$ (e.g., kN/m or lbs/ft). We simplify them into an equivalent concentrated load W_{eq} acting at the centroid $\bar{x}$ of the loading diagram:

$$W_{eq} = \int_0^L w(x) dx, \quad \bar{x} = \frac{\int_0^L xw(x) dx}{W_{eq}} \quad (14)$$

For common loading profiles, we bypass calculus and use standard geometric properties:

1. **Uniformly Distributed Load (Rectangle):** $w(x) = w_0$. The equivalent load is $W_{eq} = w_0L$, acting at the midpoint: $\bar{x} = L/2$.
2. **Triangularly Varying Load (Triangle):** $w(x)$ varies linearly from 0 to w_0 . The equivalent load is $W_{eq} = \frac{1}{2}w_0L$, acting at the centroid: $\bar{x} = \frac{2}{3}L$ from the 0 end, or $\frac{1}{3}L$ from the w_0 end.

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 94: Moments

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Where the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

Exam Pitfall Warning!

Trapezoidal Distributed Loads

Do not attempt to memorize or look up the centroid formula for a trapezoid. Instead, split the trapezoid into a rectangle of height w_1 and a right triangle of height $(w_2 - w_1)$. Calculate the equivalent forces for both shapes independently, then sum them to find the total force and locate the composite centroid using: $\bar{x} = \frac{\sum W_i \bar{x}_i}{\sum W_i}$.

3 Equilibrium of Rigid Bodies

3.1 Core Principles & Theory

A rigid body is in static equilibrium when both the net external force and the net external moment acting on it are equal to zero. This ensures that the body has neither translational acceleration nor rotational acceleration.

3.1.1 Equations of Equilibrium

In 2D coplanar analysis, the vector equilibrium equations reduce to three independent scalar equations:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_P = 0 \quad (15)$$

Where P is any arbitrary point in the plane of the structure. In 3D spatial analysis, we have six independent scalar equilibrium equations:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0 \quad (16)$$

3.1.2 Supports and Reactions

To solve for reactions, we replace structural connections with their equivalent reaction forces and moments:

- **Roller / Rocker / Smooth Surface:** Prevents translation perpendicular to the contact surface. Generates **1 reaction force** normal to the surface.
- **Pin / Hinge:** Prevents translation in any direction in the plane. Generates **2 orthogonal reaction forces** (R_x and R_y).
- **Fixed Support:** Prevents translation and rotation. Generates **3 reactions:** two orthogonal forces (R_x, R_y) and one reaction moment (M_R).
- **Cable / Link:** A cable carries tension only. A two-force pin-connected link carries force along its axis and can be in tension or compression.

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 94: Equilibrium Requirements

$$\sum \mathbf{F}_n = 0, \quad \sum \mathbf{M}_n = 0$$

FE Exam Strategy

Strategic Selection of Moment Centers

When solving for beam reactions, always choose your moment center at the pin/hinge support. Since a pin support has two unknown reaction forces, summing moments about the pin eliminates both of these unknowns from the equation, allowing you to solve for the roller reaction in a single, one-equation-one-unknown algebraic step.

4 Frames and Trusses

4.1 Core Principles & Theory

Trusses and frames are structural assemblages of members. Trusses consist entirely of pinned, straight, two-force members that carry strictly axial tension or compression. Frames contain at least one multi-force member subjected to bending, shear, and axial forces.

4.1.1 Truss Analysis Methods

There are two primary methods to analyze statically determinate planar trusses:

1. **Method of Joints:** We isolate individual pin joints and apply force equilibrium ($\sum F_x = 0$, $\sum F_y = 0$). Because all member forces pass through the pin, no moments are generated. This method is ideal when we need to find forces in multiple adjacent members near a support.
2. **Method of Sections:** We pass an imaginary cut through the entire truss, dividing it into two independent portions. We apply all three 2D equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M_P = 0$) to one of the isolated portions. This method is highly efficient for the FE exam because it allows us to solve for a single member in the middle of a truss in a single step by summing moments about the intersection point of the other two cut members.

4.1.2 Zero-Force Members

Identifying zero-force members by inspection saves valuable exam time. The two key rules from the FE Handbook are:

- **Rule 1:** If only two non-collinear members meet at a joint and there is no external load or support reaction at that joint, both members are zero-force members.
- **Rule 2:** If three members meet at a joint where two of the members are collinear, and there is no external load or support reaction at the joint, the third (non-collinear) member is a zero-force member.

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 95: Plane Truss

For a plane truss in equilibrium, the truss reactions and member forces can be determined using:

$$\sum \mathbf{F} = 0, \quad \sum \mathbf{M} = 0$$

Exam Pitfall Warning!

Tension vs. Compression Sign Conventions

When drawing a Free Body Diagram for Method of Joints or Sections, always assume unknown member forces are in **Tension** (pointing away from the joint or cut face). When you solve the equilibrium equations:

- A **positive** result confirms the member is in **Tension** (T).
- A **negative** result indicates the member is in **Compression** (C).

5 Centroid of Area

5.1 Core Principles & Theory

The centroid is the geometric center of a plane area. If the area is made of a homogeneous material, the centroid coincides with the center of gravity. Centroids are critical in structural mechanics because they locate the **neutral axis** of cross-sections subjected to bending, which is where bending stress is zero.

5.1.1 Composite Centroid Formula

For complex asymmetrical cross-sections (like T-beams, I-beams, or channels), we split the shape into a set of simple geometric components (rectangles, triangles, circles) whose centroids are easily defined. The composite centroid coordinates are:

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}, \quad \bar{y} = \frac{\sum y_i A_i}{\sum A_i} \quad (17)$$

Where A_i is the area of component i , and x_i, y_i are the coordinates of the individual component's centroid relative to a chosen reference axis.

5.1.2 Symmetry and Reference Axes

If an area possesses an axis of symmetry, the centroid must lie on that axis. If there are two axes of symmetry, the centroid is located precisely at their intersection. When setting up calculations:

1. Establish a clear, convenient reference coordinate system (e.g., place $x = 0$ at the leftmost boundary and $y = 0$ at the bottommost boundary).
2. Set up a tabular calculation to systematically track the area A_i , centroidal coordinates x_i, y_i , and the first moments of area $x_i A_i, y_i A_i$.

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 94: Centroids of Areas

The centroid coordinates $\bar{x}$ and $\bar{y}$ of a composite area are:

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}, \quad \bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

FE Exam Strategy

Negative Areas for Cutouts

If a cross-section has holes or cutouts, do not attempt to divide the surrounding area into multiple complex shapes. Instead, treat the cutout as a **negative area** in your composite centroid formula. This simplifies the geometry and significantly reduces calculation errors.

6 Area Moments of Inertia

6.1 Core Principles & Theory

The area moment of inertia (I , or second moment of area) is a geometric property of a cross-section that measures its resistance to bending (flexure) and deflection. In beam mechanics, bending stress is given by the flexure formula $\sigma = My/I$, showing that a larger moment of inertia directly leads to lower bending stresses.

6.1.1 Parallel Axis Theorem

If the moment of inertia about the centroidal axis $\bar{I}$ of a shape is known, the moment of inertia about any parallel axis I' separated by a perpendicular distance d is determined using the **Parallel Axis Theorem**:

$$I' = \bar{I} + Ad^2 \quad (18)$$

Where:

- I' = moment of inertia about the new, non-centroidal axis.
- $\bar{I}$ = centroidal moment of inertia of the shape (tabulated in the FE Handbook).
- A = cross-sectional area of the shape.
- d = perpendicular distance between the centroidal axis of the shape and the new axis.

For a composite section made of multiple simple shapes, we sum the shifted moments of inertia of each individual shape relative to the **common centroidal axis** of the composite section:

$$I_{composite} = \sum (\bar{I}_i + A_i d_i^2) \quad (19)$$

Where $d_i = |\bar{y}_{composite} - y_i|$ represents the vertical distance from the composite centroid to the individual shape's centroid.

6.1.2 Radius of Gyration

The radius of gyration (r , or sometimes represented as k) is a geometric property used to analyze columns and buckling. It represents the distance from an axis at which the entire area could be concentrated to yield the same moment of inertia:

$$r = \sqrt{\frac{I}{A}} \quad (20)$$

NCEES Reference Handbook Key Formula

FE Reference Handbook Page 95: Moment of Inertia Transfer Theorem

$$I_{x'} = I_{xc} + d_y^2 A, \quad r_x = \sqrt{\frac{I_x}{A}}$$

Exam Pitfall Warning!

Forgetting to Square the Distance Term

The most common algebraic mistake on the FE exam is failing to square the d term in the Parallel Axis Theorem ($I = \bar{I} + Ad$). Under exam stress, students often write $I = \bar{I} + Ad$ or double-count the centroid offset. Write out the formula Ad^2 on scratch paper first.

7 Static Friction

7.1 Core Principles & Theory

Friction is a tangential contact force that resists the relative sliding motion of two touching bodies. In engineering design, friction can be beneficial (traction, brakes, fasteners) or detrimental (energy loss, wear).

7.1.1 Coulomb Friction Model

The maximum static friction force $F_{f,max}$ that can develop before sliding begins (impending motion) is proportional to the normal force N :

$$F_f \leq \mu_s N \quad (21)$$

Where μ_s is the coefficient of static friction. Once the applied force exceeds this threshold, motion begins, and the friction force drops to the kinetic friction level:

$$F_f = \mu_k N \quad (22)$$

Where μ_k is the coefficient of kinetic friction (typically $\mu_k < \mu_s$).

7.1.2 Belt Friction and Band Brakes

When a belt, rope, or band wraps around a curved surface like a drum or pulley, friction allows for a large tension difference between the two sides of the belt. The relationship between the tension on the tight side F_1 (direction of impending motion) and the slack side F_2 is:

$$F_1 = F_2 e^{\mu_s \theta} \quad (23)$$

Where:

- F_1 = tension force on the tight side (always larger).
- F_2 = tension force on the slack side.
- μ_s = coefficient of static friction between the belt and drum.
- θ = angle of wrap/contact in **radians** (not degrees!).

The net torque T transmitted or resisted by the drum is:

$$T = (F_1 - F_2)r \quad (24)$$

Where r is the radius of the drum.

NCEES Reference Handbook Key Formula**FE Reference Handbook Page 95-96: Friction & Belt Friction**

$$F \leq \mu_s N, \quad F_1 = F_2 e^{\mu \theta}$$

Where θ is the total angle of contact in radians.

Exam Pitfall Warning!**The Belt Friction Radian Rule**

Never plug degrees directly into the exponent of the belt friction formula ($F_1 = F_2 e^{\mu \theta}$). You must convert degrees to radians: $\theta_{\text{radians}} = \theta_{\text{degrees}} \times \frac{\pi}{180^\circ}$. For a half-wrap (180°), use $\theta = \pi \approx 3.1416$.

8 Summary of Key TakeawaysTable 1: **High-Yield Statics Equations Summary**

Topic	Key Equation	Exam Strategy
Resultants	$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$	Use 5-12-13 and 3-4-5 ratios directly to avoid resolving angles.
Moments	$\mathbf{M} = \mathbf{r} \times \mathbf{F}$	$M_z = xF_y - yF_x$ for 2D moments about the z -axis.
Equilibrium	$\sum F_x = 0, \sum F_y = 0, \sum M = 0$	Sum moments at the pin support to eliminate multiple reactions.
Trusses	$F_{\text{chord}} = M_{\text{cut}}/h$ (suitable cut)	Use the Method of Sections to solve for a single member instantly.
Centroids	$\bar{y} = \sum y_i A_i / \sum A_i$	Treat cutouts as negative areas to simplify composite shapes.
Inertia	$I' = \bar{I} + Ad^2$	Ensure d is squared, and calculate the centroid first.
Friction	$F_1 = F_2 e^{\mu \theta}$	Always convert wrap angle θ to radians ($\pi = 180^\circ$).

9 Example Problems and Solutions

Problem 1: 2D Force Component Resolution

A bracket is subjected to a tension force of $F = 850$ N directed at an angle of 25° below the positive x -axis. What is the vertical (y) component of this force, using standard Cartesian coordinate conventions?

- A) -359.2 N
- B) -770.4 N
- C) 359.2 N
- D) 770.4 N

Solution

The force is in the fourth quadrant, meaning the horizontal component F_x is positive, and the vertical component F_y is negative. Using basic trigonometry:

$$F_y = -F \sin \theta$$

$$F_y = -850 \sin(25^\circ) = -850 \times 0.4226 = -359.23 \text{ N}$$

Thus, the vertical component is -359.2 N. **Correct Answer: A**

Problem 2: Guy Wire Geometry

A guy wire anchoring a transmission tower exerts a $3,250$ N tension force directed along a line that runs 5 m horizontally and 12 m vertically. What is the horizontal component of this tension force?

- A) $1,250$ N
- B) $1,500$ N
- C) $2,600$ N
- D) $3,000$ N

Solution

The slope of the line is defined by a rise of 12 m and a run of 5 m. We first determine the hypotenuse of the slope triangle:

$$c = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13 \text{ m}$$

This is a standard 5-12-13 right triangle. The horizontal component of the force is proportional to the horizontal run over the hypotenuse:

$$F_x = F \left(\frac{\text{run}}{\text{hypotenuse}} \right) = 3250 \times \left(\frac{5}{13} \right)$$

$$F_x = 250 \times 5 = 1250 \text{ N}$$

The horizontal component is $1,250$ N. **Correct Answer: A**

Problem 3: 2D Coplanar Resultant

Three coplanar forces act concurrently on a gusset plate:

- $F_1 = 300$ N acting horizontally to the right (0°).
- $F_2 = 450$ N acting vertically upwards (90°).
- $F_3 = 200$ N acting at 225° (pointing left and down).

What is the magnitude of the resultant force acting on the plate?

- A) 214 N
- B) 346.9 N
- C) 478 N
- D) 531 N

Solution

We resolve each force into its x and y components and sum them:

$$\sum F_x = F_{1,x} + F_{2,x} + F_{3,x} = 300 \cos(0^\circ) + 450 \cos(90^\circ) + 200 \cos(225^\circ)$$

$$\sum F_x = 300(1) + 450(0) + 200(-0.7071) = 300 - 141.42 = 158.58 \text{ N}$$

$$\sum F_y = F_{1,y} + F_{2,y} + F_{3,y} = 300 \sin(0^\circ) + 450 \sin(90^\circ) + 200 \sin(225^\circ)$$

$$\sum F_y = 300(0) + 450(1) + 200(-0.7071) = 450 - 141.42 = 308.58 \text{ N}$$

Now calculate the magnitude of the resultant force:

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{(158.58)^2 + (308.58)^2}$$

$$R = \sqrt{158.58^2 + 308.58^2} = 346.9 \text{ N}$$

Therefore, the resultant magnitude is 346.9 N. Correct Answer: B

Problem 4: 3D Vector Resultant

Three spatial forces act concurrently on a structural node:

- $\mathbf{F}_1 = 15\mathbf{i} + 25\mathbf{j} - 10\mathbf{k}$
- $\mathbf{F}_2 = 20\mathbf{i} - 5\mathbf{j} + 15\mathbf{k}$
- $\mathbf{F}_3 = -10\mathbf{i} + 40\mathbf{j} + 50\mathbf{k}$

What is the magnitude of the single equivalent resultant force vector $\mathbf{R}$?

- A) 25.0 N
- B) 55.0 N
- C) 60.0 N
- D) 85.1 N

Solution

To find the resultant force vector $\mathbf{R}$, we sum the components algebraically along the x , y , and z axes:

$$\mathbf{R} = \sum \mathbf{F} = (15 + 20 - 10)\mathbf{i} + (25 - 5 + 40)\mathbf{j} + (-10 + 15 + 50)\mathbf{k}$$

$$\mathbf{R} = 25\mathbf{i} + 60\mathbf{j} + 55\mathbf{k}$$

Now, we calculate the magnitude of $\mathbf{R}$:

$$R = |\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$R = \sqrt{25^2 + 60^2 + 55^2} = \sqrt{625 + 3600 + 3025} = \sqrt{7250} \approx 85.147 \text{ N}$$

The magnitude of the resultant force vector is 85.1 N. **Correct Answer: D**

Problem 5: 3D Moment Component

A force $\mathbf{F} = 40\mathbf{i} - 120\mathbf{j} - 30\mathbf{k}$ lbs acts at a point $P(0, 3, 4)$ ft relative to a coordinate origin. What is the magnitude of the moment component of this force about the z -axis (M_z)?

- A) 120 lb-ft
- B) -160 lb-ft
- C) -480 lb-ft
- D) -600 lb-ft

Solution

The moment of a force about the origin is given by $\mathbf{M} = \mathbf{r} \times \mathbf{F}$. The position vector of the point is $\mathbf{r} = 0\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ft, meaning $x = 0$, $y = 3$, $z = 4$. The force components are $F_x = 40$, $F_y = -120$, $F_z = -30$. Using the formula for the z -component of the moment (M_z):

$$M_z = xF_y - yF_x$$

$$M_z = (0 \times -120) - (3 \times 40) = 0 - 120 = -120 \text{ lb-ft}$$

The magnitude is 120 lb-ft. **Correct Answer: A**

Problem 6: Force-Couple System Simplification

A beam is subjected to a 15 kN downward force at $x = 1.5$ m and a 35 kN downward force at $x = 4.5$ m. If these forces are replaced by a single equivalent force F_{eq} acting at a distance d from the origin, what is the magnitude of the equivalent force and its coordinate d ?

- A) $F_{eq} = 50$ kN, $d = 3.0$ m
- B) $F_{eq} = 50$ kN, $d = 3.6$ m
- C) $F_{eq} = 154$ kN, $d = 3.9$ m
- D) $F_{eq} = 154$ kN, $d = 4.2$ m

Solution

1. Sum the forces vertically:

$$F_{eq} = \sum F = 15 + 35 = 50 \text{ kN (downward)}$$

2. Equate the moments about the origin ($x = 0$):

$$M_{eq} = \sum M = (15 \text{ kN} \times 1.5 \text{ m}) + (35 \text{ kN} \times 4.5 \text{ m})$$

$$M_{eq} = 22.5 + 157.5 = 180 \text{ kN} \cdot \text{m}$$

3. Find the coordinate d of the single equivalent force:

$$F_{eq} \times d = M_{eq} \implies 50 \times d = 180$$

$$d = \frac{180}{50} = 3.6 \text{ m}$$

Thus, the equivalent force system is a single force of 50 kN acting at $d = 3.6$ m. **Correct**

Answer: B

Problem 7: Triangular Distributed Load

A cantilever retaining wall must resist hydrostatic soil pressure. The lateral soil load acts on a vertical stem of height 6 m as a triangularly varying distributed load, which starts at 0 kN/m at the top ($y = 6$ m) and reaches a maximum of 18 kN/m at the base ($y = 0$ m). What is the equivalent concentrated force and its vertical distance from the base?

- A) 54 kN at 2.0 m from the base
- B) 54 kN at 4.0 m from the base
- C) 108 kN at 2.0 m from the base
- D) 108 kN at 3.0 m from the base

Solution

1. The equivalent concentrated force is the area of the triangular loading diagram:

$$W_{eq} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 18 \text{ kN/m} \times 6 \text{ m} = 54 \text{ kN}$$

2. The centroid of a triangle is located at one-third of the height from the larger base. In this case, the larger base (18 kN/m) is at the bottom ($y = 0$). Therefore, the centroid $\bar{y}$ is:

$$\bar{y} = \frac{1}{3} \times h = \frac{1}{3} \times 6 \text{ m} = 2 \text{ m from the base}$$

Thus, the equivalent force is 54 kN acting at 2.0 m from the base. **Correct Answer: A**

Problem 8: Composite Distributed Load

A beam of length 4 m is subjected to a trapezoidal distributed load. The loading intensity is $w_1 = 4$ kN/m at the left support ($x = 0$) and increases linearly to $w_2 = 10$ kN/m at the right support ($x = 4$ m). What is the magnitude of the equivalent concentrated force?

- A) 16 kN
- B) 24 kN
- C) 28 kN
- D) 40 kN

Solution

We divide the trapezoidal load into two simple shapes:

- A rectangle of uniform intensity $w_{rect} = 4$ kN/m over 4 m.
- A triangle of linearly varying intensity starting at 0 at the left and reaching $w_{tri} = w_2 - w_1 = 10 - 4 = 6$ kN/m at the right.

Calculate the equivalent force for each:

$$F_{rect} = w_{rect} \times L = 4 \text{ kN/m} \times 4 \text{ m} = 16 \text{ kN}$$

$$F_{tri} = \frac{1}{2} \times w_{tri} \times L = \frac{1}{2} \times 6 \text{ kN/m} \times 4 \text{ m} = 12 \text{ kN}$$

Sum the individual equivalent forces to find the total equivalent force:

$$F_{total} = F_{rect} + F_{tri} = 16 + 12 = 28 \text{ kN}$$

Correct Answer: C

Problem 9: Simply Supported Beam reactions

A simply supported beam of length 9 m is pin-supported at A and roller-supported at D . It carries a concentrated vertical load of 55 kN acting downwards at $x = 3$ m and a uniformly distributed load of 15 kN/m acting downwards from $x = 6$ m to the right end at $x = 9$ m. What is the vertical reaction force at roller support D ?

- A) 36.9 kN
- B) 44.2 kN
- C) 55.8 kN
- D) 63.8 kN

Solution

1. Draw the FBD of the beam. The reactions are A_x and A_y at pin A , and D_y at roller D .
2. Replace the distributed load with its equivalent concentrated force:

$$W_{eq} = 15 \text{ kN/m} \times 3 \text{ m} = 45 \text{ kN}$$

This equivalent force acts at the centroid of the distributed load. The load is placed from $x = 6$ to $x = 9$, so its midpoint is:

$$x_{centroid} = 6 + \frac{3}{2} = 7.5 \text{ m}$$

3. Take moments about pin A ($\sum M_A = 0$) to solve directly for D_y :

$$\sum M_A = 0 \implies (D_y \times 9 \text{ m}) - (55 \text{ kN} \times 3 \text{ m}) - (45 \text{ kN} \times 7.5 \text{ m}) = 0$$

$$9D_y - 165 - 337.5 = 0$$

$$9D_y = 502.5 \implies D_y = \frac{502.5}{9} = 55.83 \text{ kN}$$

Thus, the reaction at support D is 55.8 kN. **Correct Answer: C**

Problem 10: Cantilever Beam reactions

A 3 m cantilever beam is fixed at the left support A and completely free at the right end B . It carries a uniformly distributed load of 4 kN/m acting downwards over its entire length and a clockwise couple of 10 kN · m applied at B . What is the magnitude of the reaction moment at the fixed support A ?

- A) 10 kN · m
- B) 18 kN · m
- C) 28 kN · m
- D) 46 kN · m

Solution

1. Replace the distributed load with its equivalent concentrated force:

$$W_{eq} = 4 \text{ kN/m} \times 3 \text{ m} = 12 \text{ kN}$$

Acting at the midpoint: $\bar{x} = 1.5 \text{ m}$ from fixed support A . 2. Draw the FBD of the cantilever beam. At the fixed support A , we have a vertical reaction force A_y (upward) and a counterclockwise reaction moment M_A . 3. Sum the moments about A to achieve rotational equilibrium:

$$\sum M_A = 0 \implies M_A - (W_{eq} \times 1.5 \text{ m}) - M_{couple} = 0$$

$$M_A - (12 \text{ kN} \times 1.5 \text{ m}) - 10 \text{ kN} \cdot \text{m} = 0$$

$$M_A - 18 - 10 = 0 \implies M_A = 28 \text{ kN} \cdot \text{m}$$

The reaction moment at support A is 28 kN · m. **Correct Answer: C**

Problem 11: Cable Tension System

A cable system supports a hung load of 200 lbs at point B . Cable BA runs from B to support A with a horizontal run of 4 ft and vertical rise of 1 ft. Cable BC runs from B to support C with a horizontal run of 5 ft and vertical rise of 4 ft. What is the magnitude of the tension force T_{BC} in cable BC to maintain equilibrium?

- A) 55 lbs
- B) 243.9 lbs
- C) 255 lbs
- D) 350 lbs

Solution

Let's resolve the cables into components using their geometric ratios: For Cable BA : run = 4, rise = 1. The hypotenuse is $\sqrt{4^2 + 1^2} = \sqrt{17} \approx 4.123$. The components of tension T_{BA} are:

$$T_{BA,x} = -T_{BA} \frac{4}{\sqrt{17}}, \quad T_{BA,y} = T_{BA} \frac{1}{\sqrt{17}}$$

For Cable BC : run = 5, rise = 4. The hypotenuse is $\sqrt{5^2 + 4^2} = \sqrt{41} \approx 6.403$. The components of tension T_{BC} are:

$$T_{BC,x} = T_{BC} \frac{5}{\sqrt{41}}, \quad T_{BC,y} = T_{BC} \frac{4}{\sqrt{41}}$$

Set up the horizontal and vertical force equilibrium at joint B :

$$\sum F_x = 0 \implies -T_{BA} \frac{4}{\sqrt{17}} + T_{BC} \frac{5}{\sqrt{41}} = 0 \implies T_{BA} = T_{BC} \frac{5\sqrt{17}}{4\sqrt{41}} = T_{BC} \times 0.8043$$

$$\sum F_y = 0 \implies T_{BA} \frac{1}{\sqrt{17}} + T_{BC} \frac{4}{\sqrt{41}} - 200 = 0$$

Substitute the expression for T_{BA} into the vertical equation:

$$\left(T_{BC} \frac{5\sqrt{17}}{4\sqrt{41}} \right) \frac{1}{\sqrt{17}} + T_{BC} \frac{4}{\sqrt{41}} = 200$$

$$T_{BC} \frac{5}{4\sqrt{41}} + T_{BC} \frac{4}{\sqrt{41}} = 200$$

$$\frac{T_{BC}}{\sqrt{41}} \left(\frac{5}{4} + 4 \right) = 200 \implies \frac{T_{BC}}{\sqrt{41}} (5.25) = 200$$

$$T_{BC} = \frac{200 \times \sqrt{41}}{5.25} = T_{BC} = 243.9 \text{ lbs}$$

Therefore, $T_{BC} = 243.9$ lbs. Correct Answer: B

Problem 12: Sheave Pulley System

An ideal block-and-tackle has three sheaves in the moving block, giving six cable segments supporting the moving block. It lifts a 1,500 N component. What tension P must be applied to the free end of the cable to maintain equilibrium, neglecting cable mass and friction?

- A) 250 N
- B) 500 N
- C) 750 N
- D) 1,500 N

Solution

For a mechanical sheave-pulley block (block and tackle) with n sheaves in the moving block, the load is shared equally among the $2n$ cable segments supporting the moving block. Thus, the mechanical advantage is $2n$ (or $2 \times 3 = 6$ supporting cables). The relationship between applied force P and weight W is:

$$P = \frac{W}{2n} = \frac{1500 \text{ N}}{2 \times 3} = \frac{1500}{6} = 250 \text{ N}$$

Therefore, the applied force P required is 250 N. **Correct Answer: A**

Problem 13: 3D Spatial Table Support reactions

A triangular steel plate of negligible weight is supported horizontally by three vertical legs located at its corners: $A(0,0)$, $B(3,0)$, and $C(0,4)$ (dimensions in meters). If a vertical concentrated load of 1,200 N acts downwards on the plate at coordinate $(1,1)$, what is the vertical reaction force at leg B ?

- A) 300 N
- B) 400 N
- C) 500 N
- D) 600 N

Solution

The system is in vertical and rotational equilibrium:

$$\sum F_z = R_A + R_B + R_C - 1200 = 0$$

Take moments about the y -axis (which lies along the line $x = 0$, containing points A and C):

$$\sum M_{y\text{-axis}} = 0 \implies (R_B \times x_B) - (F_{load} \times x_{load}) = 0$$

Since $x_B = 3$ m and the load is at $x_{load} = 1$ m:

$$(R_B \times 3) - (1200 \times 1) = 0$$

$$3R_B = 1200 \implies R_B = 400 \text{ N}$$

Similarly, taking moments about the x -axis ($y = 0$, containing A and B):

$$\sum M_{x\text{-axis}} = 0 \implies (R_C \times y_C) - (F_{load} \times y_{load}) = 0$$

$$(R_C \times 4) - (1200 \times 1) = 0 \implies R_C = 300 \text{ N}$$

By vertical equilibrium:

$$R_A = 1200 - 400 - 300 = 500 \text{ N}$$

Thus, the reaction force at leg B is 400 N. **Correct Answer: B**

Problem 14: Method of Sections

A simply supported 30-m Warren truss is 5 m high. A 30-kN load acts at lower joint G , 5 m from A . A section cuts horizontal upper chord BC at height 5 m and members CG and GF , whose lines of action pass through G . Find the force in BC .

- A) 15 kN (Compression)
- B) 20 kN (Tension)
- C) 25 kN (Compression)
- D) 25 kN (Tension)

Solution

- Find support reactions from overall equilibrium: $R_Z(30) = 30(5)$, so $R_Z = 5$ kN and $R_A = 25$ kN.
- Cut BC , CG , and GF and retain the left-hand portion. Take moments about G ; the cut forces in CG and GF , and the load at G , have zero moment about G .
- Assume the upper-chord force is tension, pointing to the right on the left portion. With counterclockwise moments positive:

$$\begin{aligned}\sum M_G &= -R_A(5) - F_{BC}(5) = 0 \\ -25(5) - F_{BC}(5) &= 0 \Rightarrow F_{BC} = -25 \text{ kN}\end{aligned}$$

The negative sign reverses the assumed tension arrow: the upper chord is in **compression**.

Correct Answer: C (25 kN compression).

Problem 15: Method of Joints

At joint C , a 20-kN force acts downward and an 8-kN force leftward. Member BC runs **down-left from** C , with rise 3 m and run 5 m. Member AC extends horizontally left from C . Find the axial force in AC .

- A) 8.0 kN
- B) 22.0 kN
- C) 25.3 kN
- D) 39.1 kN

Solution

1. Draw the FBD of joint C . Assume forces in members AC and BC are in tension (pulling away from joint C). Member BC has components $F_{BC,x}$ (leftward) and $F_{BC,y}$ (downward/upward).
2. Set up the vertical force equilibrium:

$$\sum F_y = 0 \implies -20 \text{ kN} - F_{BC,y} = 0 \implies F_{BC,y} = -20 \text{ kN}$$

This negative sign indicates the vertical component of the diagonal member points upward (Compression). 3. Relate horizontal and vertical components of member BC using the slope triangle (rise 3, run 5):

$$\frac{F_{BC,x}}{F_{BC,y}} = \frac{5}{3} \implies F_{BC,x} = \frac{5}{3} F_{BC,y} = \frac{5}{3} \times (-20) = -33.33 \text{ kN}$$

4. Set up the horizontal force equilibrium:

$$\sum F_x = 0 \implies -F_{AC} - 8 \text{ kN} - F_{BC,x} = 0$$

$$-F_{AC} - 8 - (-33.33) = 0$$

$$-F_{AC} - 8 + 33.33 = 0 \implies F_{AC} = 25.33 \text{ kN}$$

The force in horizontal member AC is 25.33 kN in tension. **Correct Answer: C**

Problem 16: Zero-Force Member Identification

A symmetric roof truss consists of collinear chords and vertical/diagonal bracing. Joint H connects two collinear top chord members GH and HI , and a vertical web member HC . There is no external load or support reaction applied at joint H . The vertical web member HC is a:

- A) Tension member
- B) Compression member
- C) Zero-force member
- D) Indeterminate member

Solution

Apply the zero-force member rules to joint H : - Joint H has three members meeting: GH , HI , and HC . - Members GH and HI are collinear top chord members. - There is no external force or reaction at joint H . According to **Rule 2**, because we have three members meeting at a joint where two are collinear and the joint is unloaded, the third non-collinear member (HC) must carry zero force. Thus, HC is a zero-force member. **Correct Answer: C**

Problem 17: Frame Pin Reaction Analysis

A symmetric 2D pin-connected frame consists of two legs AB and CB pinned at B and pinned to the ground at A and C (18 ft apart horizontally). A vertical concentrated load of 7 kips acts downwards at the midpoint of CB (horizontal distance 13.5 ft from A). Neglecting frame self-weight, what is the vertical reaction force at ground support C ?

- A) 1.75 kips
- B) 3.50 kips
- C) 5.25 kips
- D) 7.00 kips

Solution

Take moments about support A ($\sum M_A = 0$) for the entire frame:

$$\sum M_A = 0 \implies (C_y \times 18 \text{ ft}) - (7 \text{ kips} \times 13.5 \text{ ft}) = 0$$

$$18C_y = 94.5$$

$$C_y = \frac{94.5}{18} = 5.25 \text{ kips}$$

Thus, the vertical reaction at support C is 5.25 kips. **Correct Answer: C**

Problem 18: Machine Linkage Analysis

A symmetric A-frame has equal legs at 60° to the horizontal, a top pin C , and **smooth feet with vertical reactions only**. A 1,000-lb load hangs at C . A horizontal tie BD joins the leg midpoints. Find its axial tension.

- A) 288 lbs
- B) 500 lbs
- C) 577 lbs
- D) 1,000 lbs

Solution

1. Each smooth foot supplies a vertical reaction only. Symmetry gives $A_y = 500$ lb. Take moments about top pin C for left leg AC , with tie attached at midpoint B (lengths $CB = L$ and $CA = 2L$):

The tie bar connects at midpoint B (length L from A and C):

$$\sum M_C = 0 \implies (A_y \times 2L \cos 60^\circ) - (T_{BD} \times L \sin 60^\circ) = 0$$

$$500 \times 2L \times 0.5 = T_{BD} \times L \times 0.866$$

$$500L = 0.866LT_{BD}$$

$$T_{BD} = \frac{500}{0.866} = 577.35 \text{ lbs}$$

The axial tension force in the tie-bar is 577 lbs. **Correct Answer: C**

Problem 19: T-Section Centroid

A composite concrete T-beam cross-section consists of a horizontal flange and a vertical web.

- Flange: 300 mm wide $\times$ 50 mm thick.
- Web: 50 mm wide $\times$ 250 mm high.

What is the vertical distance $\bar{y}$ from the bottommost fiber of the web to the centroid of the section?

- A) 115 mm
- B) 150 mm
- C) 175 mm
- D) 206.8 mm

Solution

1. Divide the T-section into two rectangular elements:

- Element 1 (Web): $w_1 = 50$ mm, $h_1 = 250$ mm.
- Element 2 (Flange): $w_2 = 300$ mm, $h_2 = 50$ mm.

2. Define the reference axis $y = 0$ at the bottommost fiber of the web. 3. Calculate the area and individual centroids:

$$A_1 = 50 \text{ mm} \times 250 \text{ mm} = 12,500 \text{ mm}^2$$

$$y_1 = \frac{250}{2} = 125 \text{ mm}$$

$$A_2 = 300 \text{ mm} \times 50 \text{ mm} = 15,000 \text{ mm}^2$$

The flange sits on top of the web, so its bottom edge is at $y = 250$ mm:

$$y_2 = 250 + \frac{50}{2} = 275 \text{ mm}$$

4. Apply the composite centroid formula:

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{(125 \times 12,500) + (275 \times 15,000)}{12,500 + 15,000}$$

$$\bar{y} = 5,687,500 / 27,500 = 206.818 \text{ mm} = 206.8 \text{ mm}$$

Therefore, the centroid is 206.8 mm above the bottom of the web. Correct Answer: D

Problem 20: Composite Channel Centroid

An asymmetrical structural steel channel section is positioned like an inverted “U”.

- Top horizontal plate: 10 in. wide $\times$ 0.5 in. thick.
- Two vertical side flanges: each 4 in. high $\times$ 0.5 in. thick.

What is the vertical distance $\bar{y}$ from the bottom edges of the side flanges to the centroid of the composite channel section?

A) 1.52 in. B) 2.58 in. C) 2.96 in. D) 3.25 in.

Solution

1. Divide the section into the top plate and the two side flanges.

For the two side flanges combined:

$$A_f = 2(0.5)(4) = 4.0 \text{ in}^2, \quad y_f = \frac{4}{2} = 2.0 \text{ in.}$$

For the top plate:

$$A_t = (10)(0.5) = 5.0 \text{ in}^2, \quad y_t = 4 + \frac{0.5}{2} = 4.25 \text{ in.}$$

2. Apply the composite-centroid relation:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(4.0)(2.0) + (5.0)(4.25)}{4.0 + 5.0} = \frac{29.25}{9.0} = 3.25 \text{ in.}$$

Therefore, the centroid is **3.25 in. above the bottom edges of the side flanges.**

Correct Answer: D

Problem 21: Semicircle Centroid with Cutout

A homogeneous concrete architectural component consists of a semicircle of radius $R = 6$ m with a rectangular cutout of width 2 m and height 1 m at the center of its flat base. What is the distance $\bar{y}$ from the flat base to the centroid of the remaining area?

- A) 2.14 m
- B) 2.55 m
- C) 2.62 m
- D) 3.12 m

Solution

1. Calculate the properties of the semicircle (Element 1): The area of a semicircle is:

$$A_1 = \frac{\pi R^2}{2} = \frac{\pi \times 6^2}{2} = 18\pi \approx 56.55 \text{ m}^2$$

The centroid of a semicircle is located at:

$$y_1 = \frac{4R}{3\pi} = \frac{4 \times 6}{3\pi} = \frac{8}{\pi} \approx 2.546 \text{ m}$$

2. Calculate the properties of the rectangular cutout (Element 2): Since it is a cutout, treat its area as negative:

$$A_2 = -(\text{width} \times \text{height}) = -(2 \text{ m} \times 1 \text{ m}) = -2 \text{ m}^2$$

The centroid of the rectangle relative to the base is:

$$y_2 = \frac{h}{2} = \frac{1}{2} = 0.5 \text{ m}$$

3. Apply the composite centroid formula:

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{(2.546 \times 56.55) + (0.5 \times -2)}{56.55 - 2}$$

$$y_1 A_1 = \frac{8}{\pi} \times 18\pi = 144 \text{ m}^3$$

$$y_2 A_2 = 0.5 \times -2 = -1.0 \text{ m}^3$$

$$\bar{y} = \frac{144 - 1.0}{56.55 - 2} = \frac{143.0}{54.55} \approx 2.621 \text{ m}$$

The centroid distance is approximately **2.62 m**. **Correct Answer: C**

Problem 22: Composite Plate Centroid

A right triangle (base 3 m, height 3 m, vertical edge at the left) is joined to the right of a 4-m by 3-m rectangle. Measure x from the rectangle's left edge. Find the composite centroid $\bar{x}$.

- A) 2.00 m
- B) 2.82 m
- C) 3.15 m
- D) 3.50 m

Solution

1. Divide the plate into two elements:

- Element 1 (Rectangle): $w_1 = 4$ m, $h_1 = 3$ m.
- Element 2 (Triangle): $b_2 = 3$ m, $h_2 = 3$ m.

2. Calculate areas and horizontal centroids: For Rectangle:

$$A_1 = 4 \times 3 = 12 \text{ m}^2, \quad x_1 = \frac{4}{2} = 2.0 \text{ m}$$

For Triangle:

$$A_2 = \frac{1}{2} \times 3 \times 3 = 4.5 \text{ m}^2$$

The triangle starts at $x = 4$ m. Its centroid is at $1/3$ of the base from the left (straight) edge:

$$x_2 = 4 + \frac{1}{3} \times b_2 = 4 + \frac{1}{3} \times 3 = 5.0 \text{ m}$$

3. Apply the composite centroid formula:

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{(2.0 \times 12) + (5.0 \times 4.5)}{12 + 4.5}$$

$$\bar{x} = \frac{24 + 22.5}{16.5} = \frac{46.5}{16.5} = 2.81818 \approx 2.82 \text{ m}$$

The horizontal centroid is **2.82 m**. **Correct Answer: B**

Problem 23: Moment of Inertia of a T-Section

A structural flange-and-web T-section has:

- Flange: 10 cm wide $\times$ 2 cm thick.
- Web: 2 cm wide $\times$ 6 cm high.

Determine the moment of inertia (I_{xx}) about the horizontal centroidal axis of the composite section.

- A) 120 cm^4
- B) 162.7 cm^4
- C) 170.7 cm^4
- D) 300 cm^4

Solution

1. Compute component areas and centroid locations:

$$\text{Web: } A_1 = 2(6) = 12 \text{ cm}^2, \quad y_1 = 3.0 \text{ cm.}$$

$$\text{Flange: } A_2 = 10(2) = 20 \text{ cm}^2, \quad y_2 = 7.0 \text{ cm.}$$

2. Calculate the composite centroid:

$$\bar{y} = [12(3.0) + 20(7.0)] / (12 + 20) = 176/32 = 5.50 \text{ cm}$$

3. Calculate centroidal moments of inertia ($I_{bar} = bh^3/12$):

$$I_{bar1} = 2(6^3)/12 = 36.00 \text{ cm}^4$$

$$I_{bar2} = 10(2^3)/12 = 6.67 \text{ cm}^4$$

4. Distances from each component centroid to the composite centroid:

$$d_1 = |5.50 - 3.00| = 2.50 \text{ cm}; \quad d_2 = |7.00 - 5.50| = 1.50 \text{ cm}$$

5. Apply the Parallel Axis Theorem:

$$I_1 = 36.00 + 12(2.50)^2 = 111.00 \text{ cm}^4$$

$$I_2 = 6.67 + 20(1.50)^2 = 51.67 \text{ cm}^4$$

6. Sum the component inertias:

$$I_{xx} = 111.00 + 51.67 = 162.67 \text{ cm}^4 = 162.7 \text{ cm}^4$$

Correct Answer: B

Problem 24: Built-Up Steel beam Moment of Inertia

A built-up section consists of a W30x211 ($I_{xx} = 10,300 \text{ in}^4$, $A = 62 \text{ in}^2$) and a C15x50 ($A = 14.7 \text{ in}^2$) welded to the top flange. The channel is oriented so its tabulated $I_{yy} = 11 \text{ in}^4$ acts about the composite horizontal x-x axis. The channel centroid is 15.47 in. and the wide-flange centroid is 3.67 in. from the composite centroid. Find the total composite I_{xx} .

- B) $11,135 \text{ in}^4$
 C) $14,664 \text{ in}^4$
 D) $17,212 \text{ in}^4$

Solution

Apply the Parallel Axis Theorem to the composite steel shape: For the Wide Flange (W30x211):

$$I_{WF} = \bar{I}_{WF} + A_{WF}d_{WF}^2 = 10300 \text{ in}^4 + 62 \text{ in}^2 \times (3.67 \text{ in})^2$$

$$I_{WF} = 10300 + 62 \times 13.469 = 10300 + 835.07 = 11,135.07 \text{ in}^4$$

For the Channel (C15x50) welded on top:

$$I_{Ch} = \bar{I}_{Ch} + A_{Ch}d_{Ch}^2 = 11 \text{ in}^4 + 14.7 \text{ in}^2 \times (15.47 \text{ in})^2$$

$$I_{Ch} = 11 + 14.7 \times 239.32 = 11 + 3518.02 = 3,529.02 \text{ in}^4$$

Sum the shifted moments of inertia to get the composite second moment of area:

$$I_{\text{composite}} = 11,135.07 + 3,529.02 = 14,664.09 \text{ in}^4 = 14,664 \text{ in}^4$$

The composite moment of inertia is approximately $14,664 \text{ in}^4$. Correct Answer: C

Problem 25: Radius of Gyration Calculation

A column has area $A = 32 \text{ cm}^2$ and **principal centroidal** inertias $I_{xx} = 288 \text{ cm}^4$ and $I_{yy} = 72 \text{ cm}^4$ ($I_{xy} = 0$). Find its minimum radius of gyration.

- A) 1.50 cm
 B) 2.25 cm
 C) 3.00 cm
 D) 9.00 cm

Solution

The radius of gyration is defined by $r = \sqrt{I/A}$. The minimum radius of gyration correspond to the minimum moment of inertia (I_{min}):

$$I_{min} = I_{yy} = 72 \text{ cm}^4$$

Calculate r_{min} :

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{72 \text{ cm}^4}{32 \text{ cm}^2}} = \sqrt{2.25} = 1.50 \text{ cm}$$

The minimum radius of gyration is 1.50 cm. **Correct Answer: A**

Problem 26: Moment of Inertia about Base Axis

A solid rectangular steel bar has a width of $b = 4$ cm and a height of $h = 9$ cm. What is the moment of inertia of this bar about its base axis (the horizontal boundary line along the bottom fiber, $y = 0$)?

- A) 729 cm⁴
- B) 972 cm⁴
- C) 1,215 cm⁴
- D) 1,458 cm⁴

Solution

Using the Parallel Axis Theorem: The centroidal moment of inertia of a rectangle is:

$$\bar{I} = \frac{bh^3}{12} = \frac{4 \times 9^3}{12} = \frac{4 \times 729}{12} = 243 \text{ cm}^4$$

The area of the rectangle is:

$$A = b \times h = 4 \times 9 = 36 \text{ cm}^2$$

The centroid of the rectangle is located at $d = h/2 = 4.5$ cm from the base. Apply the Parallel Axis Theorem:

$$I_{base} = \bar{I} + Ad^2 = 243 + 36 \times (4.5)^2 = 243 + 36 \times 20.25$$

$$I_{base} = 243 + 729 = 972 \text{ cm}^4$$

Shortcut: The moment of inertia of a rectangle about its base axis is $I_{base} = bh^3/3$:

$$I_{base} = \frac{bh^3}{3} = \frac{4 \times 9^3}{3} = \frac{2916}{3} = 972 \text{ cm}^4$$

Both methods yield 972 cm⁴. **Correct Answer: B**

Problem 27: Restrained Double-Block system

Two blocks rest stacked on a horizontal surface. The top block has a weight of 7 kips and is restrained horizontally by a cable pinned to a wall. The bottom block has a weight of 3 kips and a horizontal pulling force P is applied to it. The coefficient of static friction between the two blocks is 0.55, and between the bottom block and the floor is 0.40. What is the minimum force P required to cause the bottom block to move?

- A) 3.85 kips
- B) 4.00 kips
- C) 7.85 kips
- D) 9.40 kips

Solution

1. Draw the FBD of the bottom block (weight $W_B = 3$ kips). The vertical forces are: - Downward force from top block ($W_T = 7$ kips). - Downward self-weight ($W_B = 3$ kips). - Upward normal reaction from the floor (N_{floor}).

$$\sum F_y = 0 \implies N_{floor} - 7 \text{ kips} - 3 \text{ kips} = 0 \implies N_{floor} = 10 \text{ kips}$$

2. The contact surfaces generate friction resisting pulling force P : - Friction at the interface between top and bottom block ($F_{friction,top}$): The normal force at this top interface is $N_{top} = 7$ kips.

$$F_{friction,top} = \mu_{top} N_{top} = 0.55 \times 7 \text{ kips} = 3.85 \text{ kips}$$

- Friction at the floor interface ($F_{friction,floor}$):

$$F_{friction,floor} = \mu_{floor} N_{floor} = 0.40 \times 10 \text{ kips} = 4.00 \text{ kips}$$

3. Set up the horizontal force equilibrium for the bottom block:

$$\sum F_x = 0 \implies P - F_{friction,top} - F_{friction,floor} = 0$$

$$P = 3.85 + 4.00 = 7.85 \text{ kips}$$

The force P required to initiate movement is 7.85 kips. **Correct Answer: C**

Problem 28: Block on Incline with Horizontal Force

A block with mass $m = 200$ kg rests on an inclined plate with angle $\phi = 15^\circ$. The coefficient of static friction is 0.35. A force P acts horizontally through the center of mass. What is the magnitude of horizontal force P that will cause motion to begin up the plane?

- A) 1,100 N
- B) 1,338 N
- C) 1,400 N
- D) 1,710 N

Solution

1. Calculate weight force: $W = mg = 200 \text{ kg} \times 9.81 \text{ m/s}^2 = 1962 \text{ N}$. 2. Align coordinates with incline: x -axis parallel to incline (up is positive), y -axis normal to incline. Resolve W and P along these axes:

$$W_x = -W \sin 15^\circ = -1962 \sin 15^\circ = -507.8 \text{ N}$$

$$W_y = -W \cos 15^\circ = -1962 \cos 15^\circ = -1895.1 \text{ N}$$

$$P_x = P \cos 15^\circ = 0.9659P$$

$$P_y = -P \sin 15^\circ = -0.2588P$$

3. Set up normal force equilibrium:

$$N + P_y + W_y = 0 \implies N = 1895.1 + 0.2588P$$

4. Set up friction force at impending slip up the plane (friction force $F_f = \mu_s N$ acts down the plane):

$$\sum F_x = P_x + W_x - F_f = 0 \implies 0.9659P - 507.8 - 0.35(1895.1 + 0.2588P) = 0$$

$$0.9659P - 507.8 - 663.3 - 0.0906P = 0$$

$$0.8753P = 1171.1 \implies P = 1,338 \text{ N}$$

The horizontal force P is approximately 1,338 N. Correct Answer: B

Problem 29: Wedge Friction

A massless 10° wedge rests on a rough horizontal floor and lifts a 500-N block constrained by a **smooth vertical guide**. The block–wedge and wedge–floor coefficients are both $\mu = 0.25$. A horizontal force pushes the wedge to lift the block. Find the required force.

A) 250 N B) 292 N C) 348 N D) 392 N

Solution

For impending upward motion, friction opposes sliding at both rough interfaces. Let N_b be the block–wedge normal force and $\phi = \tan^{-1} \mu = 14.036^\circ$.

Block equilibrium: The smooth guide supplies horizontal reaction only. Thus $N_b(\cos \alpha - \mu \sin \alpha) = W$.

Wedge equilibrium: The floor normal is $N_f = W$, so floor friction is μW . Horizontal equilibrium gives

$$P = N_b(\sin \alpha + \mu \cos \alpha) + \mu W$$

$$P = W[\tan(\alpha + \phi) + \mu]$$

$$P = 500[\tan(10^\circ + 14.036^\circ) + 0.25] = 347.993 \text{ N.}$$

Correct Answer: C (approximately 348 N).

Problem 30: Band Brake Belt Friction

A simple band brake system uses a flexible belt wrapped around a rotating drum of radius $r = 0.35 \text{ m}$. The belt has an angle of contact of 180° (π radians). The coefficient of static friction is $\mu_s = 0.40$. If the tension on the slack side of the belt is $F_2 = 100 \text{ N}$, what is the maximum friction torque that can be developed on the drum before slipping begins?

- A) $35.0 \text{ N} \cdot \text{m}$
 B) $75.4 \text{ N} \cdot \text{m}$
 C) $88.0 \text{ N} \cdot \text{m}$
 D) $123.0 \text{ N} \cdot \text{m}$

Solution

1. Use the belt friction formula to calculate the tension on the tight side F_1 :

$$F_1 = F_2 e^{\mu_s \theta}$$

The angle of wrap is 180° , which converted to radians is $\theta = \pi \approx 3.1416$.

$$F_1 = 100 \times e^{0.40 \times \pi} = 100 \times e^{1.2566} = 100 \times 3.5136 = 351.36 \text{ N}$$

2. Calculate the friction torque T resisted by the drum:

$$T = (F_1 - F_2)r$$

$$T = (351.36 \text{ N} - 100 \text{ N}) \times 0.35 \text{ m}$$

$$T = 251.36 \text{ N} \times 0.35 \text{ m} = 87.98 \text{ N} \cdot \text{m} \approx 88.0 \text{ N} \cdot \text{m}$$

The maximum friction torque is approximately $88.0 \text{ N} \cdot \text{m}$. **Correct Answer: C**