

Civil Engineering FE CBT Exam

Knowledge: Structural Engineering with 27 Example Problems

Contents

1 Analysis of Statically Determinate Systems	3
1.1 Fundamental Equilibrium & Statics Principles.....	3
1.1.1 Support Conditions & Reaction Components.....	3
1.2 Internal Beam Force Sign Conventions & Differential Relations.....	3
1.2.1 Differential Equilibrium Equations for Beams	4
1.3 Truss Analysis: Method of Joints & Method of Sections	5
1.3.1 Method of Joints	5
1.3.2 Zero-Force Members	6
1.3.3 Method of Sections	6
1.4 Pin-Connected Frames and Three-Hinged Arches	6
1.4.1 Analysis Procedure for Frames	6
2 Deflection of Determinate Beams, Trusses & Frames	6
2.1 Elastic Beam Deflections: Governing Differential Equations.....	7
2.2 Standard Beam Deflection Formulas (Principle of Superposition).....	8
2.3 The Principle of Virtual Work (Unit Load Method)	8
2.3.1 Truss Deflections via Virtual Work	8
2.3.2 Flexural Beam & Frame Deflections via Virtual Work	9
3 Structural Determinacy & Stability Analysis.....	9
3.1 Static Determinacy vs. Indeterminacy	9
3.1.1 Determinacy Criteria for Two-Dimensional Pin-Jointed Trusses	10
3.1.2 Determinacy Criteria for Two-Dimensional Rigid Frames & Beams.....	10
3.2 Geometric Instability.....	10
3.2.1 Forms of Instability	11
4 Column Buckling & Boundary Conditions	11
4.1 Euler Elastic Buckling Theory	11
4.1.1 Derivation for Pin-Ended Column	11
4.2 Effective Length Factor (K)	12
4.3 Alignment Charts for Frame Columns (GA, GB)	13
5 Elementary Statically Indeterminate Structures	13
5.1 The Force Method of Analysis (Flexibility Method)	13
5.1.1 Step-by-Step Analysis Procedure	13
5.2 Fixed-End Moments (FEM)	14
5.3 Moment Distribution Method Overview	14
6 Design Loads, Load Combinations & Tributary Areas	14
6.1 ASCE 7 Structural Load Classifications	15
6.2 ASCE 7 LRFD Load Combinations.....	15
6.3 Tributary Areas & Load Distribution.....	15
6.3.1 One-Way vs. Two-Way Floor Framing.....	16
6.3.2 ASCE 7 Live Load Reduction.....	16
7 Design of Steel Components (ANSI/AISC 360-16)	16
7.1 AISC LRFD Specification Fundamentals	17
7.2 Tension Member Design.....	17
7.2.1 1. Tensile Yielding on Gross Section.....	17
7.2.2 2. Tensile Rupture on Net Effective Section	17
7.2.3 3. Block Shear Rupture.....	18
7.3 Flexural Member Design (Beams)	18
7.3.1 Flexural Limit States.....	18
7.3.2 Beam Shear Capacity	19
7.4 Compression Member Design (Columns)	19
8 Design of Reinforced Concrete Components (ACI 318-14)	19
8.1 ACI 318 Strength Design Principles.....	19

8.1.1 Material Properties & Assumptions.....	20
8.2 Flexural Design of Singly Reinforced Rectangular Beams.....	20
8.2.1 Equilibrium & Flexural Moment Capacity Equations.....	21
8.2.2 Strain Verification & Strength Reduction Factor (ϕ).....	21
8.2.3 Reinforcement Limits	21
8.3 Shear Design of Reinforced Concrete Beams.....	22
8.3.1 ACI Stirrup Spacing Limits	22
8.4 Concentric Short Concrete Column Design	22
9 Example Problems and Solutions (27 Problems)	24

1 Analysis of Statically Determinate Systems

1.1 Fundamental Equilibrium & Statics Principles

Structural analysis begins with the evaluation of equilibrium. For any two-dimensional (plane) structural system subject to coplanar force systems, static equilibrium requires that the vector sum of all external forces and moments acting on the system equals zero:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_z = 0 \quad (1.1)$$

In three-dimensional spatial structures, six independent scalar equations of equilibrium must be satisfied:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0 \quad (1.2)$$

Rigid body equilibrium forms the cornerstone of structural analysis. Before internal force distributions (such as bending moments, shear forces, and axial tension/compression) can be computed, all external boundary reaction forces at supports must be evaluated.

1.1.1 Support Conditions & Reaction Components

Support conditions dictate the kinematic boundary constraints imposed on a structural element. Table 1.1 summarizes standard two-dimensional support types, their corresponding reaction force components, and constrained degrees of freedom (DOF).

1.2 Internal Beam Force Sign Conventions & Differential Relations

Internal forces at any transverse cut along a beam member consist of axial force (N), shear force (V), and bending moment (M). Standard positive sign conventions dictate:

Axial Force (N): Positive in tension (pulling away from the cut face).

Table 1.1: Standard Two-Dimensional Support Types and Reactions

Support Type	Restrained Movement / Degrees of Freedom	Reaction Components
Roller / Rocker	Translation perpendicular to support surface	1 force (R_y or R_x)
Pin / Hinge	Translation in both horizontal and vertical directions	2 forces (R_x, R_y)
Fixed Support	Translation in x, y and rotation in z -plane	2 forces (R_x, R_y), 1 moment (M)
Internal Hinge	Relative rotation permitted; translation constrained	V, N continuous; $M = 0$ at hinge
Guided Roller	Rotation and vertical translation restrained	1 force (R_y), 1 moment (M)

Shear Force (V): Positive when creating a clockwise rotational moment on the member element.

Bending Moment (M): Positive when causing compression in the top fibers and tension in the bottom fibers ("concave upwards" or "smiling" curvature).

1.2.1 Differential Equilibrium Equations for Beams

Consider a beam segment subjected to a distributed load $w(x)$ acting downwards (positive downwards). Applying vertical force equilibrium and moment equilibrium to an infinitesimal element dx yields:

$$\frac{dV(x)}{dx} = -w(x) \implies V(x_2) - V(x_1) = - \int_{x_1}^{x_2} w(x) dx \quad (1.3)$$

$$\frac{dM(x)}{dx} = V(x) \implies M(x_2) - M(x_1) = \int_{x_1}^{x_2} V(x) dx \quad (1.4)$$

Furthermore, the curvature relationship ties bending moment directly to beam flexural rigidity EI :

$$\frac{d^2 M(x)}{dx^2} = -w(x), \quad EI \frac{d^2 y}{dx^2} = M(x) \quad (1.5)$$

Key Mathematical Relationships for Beam Diagrams

The slope of the shear diagram equals the negative magnitude of the distributed load:

$$\frac{dV}{dx} = -w(x).$$

The slope of the bending moment diagram equals the shear force: $\frac{dM}{dx} = V(x)$.

Peak bending moments ($M_{\max}$) occur at locations where shear crosses zero ($V(x) = 0$) or at concentrated load points.

Concentrated point loads produce abrupt vertical jumps in the shear diagram equal to the load magnitude.

Concentrated applied moments produce abrupt vertical jumps in the moment diagram equal to the moment magnitude.

1.3 Truss Analysis: Method of Joints & Method of Sections

A structural truss is an assemblage of slender members connected at their end points by friction-free pin connections. The fundamental assumptions governing ideal truss behavior are:

1. Members are connected at joints with frictionless pins.
2. Loads and support reactions are applied exclusively at the joints.
3. The centroidal axis of each member coincides with the line connecting joint centers.

Under these assumptions, all truss members experience only uniaxial axial forces (tension or compression).

1.3.1 Method of Joints

The Method of Joints isolates individual truss joints as free-body diagrams under concurrent force systems. For a plane truss joint, two scalar equilibrium equations apply:

$$\sum F_x = 0, \quad \sum F_y = 0 \quad (1.6)$$

Analysis proceeds sequentially through joints having no more than two unknown member forces.

1.3.2 Zero-Force Members

Rapid identification of zero-force members simplifies truss analysis significantly under timed FE exam conditions:

Rule 1 (Two non-collinear members): If a joint connects only two non-collinear members and has no external load or support reaction applied to it, **both members are zero-force members**.

Rule 2 (Three members, two collinear): If a joint connects three members, two of which are collinear, and no external force or support reaction acts at that joint, the **third (non-collinear) member is a zero-force member**.

1.3.3 Method of Sections

When internal forces in specific non-adjacent members are required, the Method of Sections cuts an imaginary section line through up to three non-concurrent, non-parallel members. Applying all three equations of equilibrium ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M_O = 0$) to either isolated half permits direct calculation of the unknown forces without solving the entire truss.

1.4 Pin-Connected Frames and Three-Hinged Arches

Pin-connected frames contain at least one multi-force member—a structural element subjected to three or more forces, or distributed/moment loadings. Unlike trusses, frame members experience internal shear forces and bending moments in addition to axial forces.

1.4.1 Analysis Procedure for Frames

1. Draw an overall free-body diagram (FBD) of the entire frame structure to compute external support reactions.
2. Disassemble the frame into individual multi-force members and pin joints.
3. Apply equal and opposite reaction pairs (Newton's Third Law) at all internal pin connections.
4. Apply the three equations of equilibrium ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$) to each member component.

2 Deflection of Determinate Beams, Trusses & Frames

2.1 Elastic Beam Deflections: Governing Differential Equations

Structural deflection control is a core serviceability requirement. The relationship governing small-deflection elastic beam flexure is derived from Euler-Bernoulli beam theory:

$$EI \frac{d^2 y}{dx^2} = M(x) \quad (2.1)$$

where E is the modulus of elasticity, I is the moment of inertia about the neutral axis, $M(x)$ is the bending moment expression, and $y(x)$ is the vertical deflection. Integrating once yields the rotation (slope) $\theta(x) = dy/dx$:

$$EI\theta(x) = EI \frac{dy}{dx} = \int M(x) dx + C_1 \quad (2.2)$$

Integrating a second time yields the transverse displacement $y(x)$:

$$EIy(x) = \iint M(x) dx dx + C_1 x + C_2 \quad (2.3)$$

Integration constants C_1 and C_2 are evaluated using displacement and slope boundary conditions at beam supports.

Table 2.1: Standard Beam Boundary Conditions for Deflection Analysis

Support Condition	Location (x)	Displacement Constraint	Slope Constraint
Pinned Support	$x = 0$ or $x = L$	$y = 0$ at support	Unconstrained
Roller Support	$x = 0$ or $x = L$	$y = 0$ at support	Unconstrained
Fixed Support	$x = 0$	$y(0) = 0$	$\theta(0) = 0$
Free End	$x = L$	Unconstrained	θ : unconstrained $M = V = 0$ if end unloaded
Internal Hinge	$x = a$	$y(a^-) = y(a^+)$	Rotations may differ; $M(a) = 0$

2.2 Standard Beam Deflection Formulas (Principle of Superposition)

Under linear elastic conditions, total deflection caused by complex load combinations equals the algebraic sum of deflections produced by each individual load component acting alone.

Table 2.2: Standard Deflection and Slope Equations for Common Beam Cases

Beam Configuration	Max Deflection ($\Delta_{\max}$)	End Slope (θ_{end})	Location of $\Delta_{\max}$
Simply Supported, Center Load P	$\frac{PL^3}{48EI}$	$\frac{PL^2}{16EI}$	$x = L/2$
Simply Supported, Uniform Load w	$\frac{5wL^4}{384EI}$	$\frac{wL^3}{24EI}$	$x = L/2$
Cantilever, Tip Load P	$\frac{PL^3}{3EI}$	$\frac{PL^2}{2EI}$	Free End ($x = L$)
Cantilever, Uniform Load w	$\frac{wL^4}{8EI}$	$\frac{wL^3}{6EI}$	Free End ($x = L$)
Cantilever, Tip Moment M_0	$\frac{M_0L^2}{2EI}$	$\frac{M_0L}{EI}$	Free End ($x = L$)

2.3 The Principle of Virtual Work (Unit Load Method)

The Unit Load Method is an energy-based method derived from virtual work principles, valid for computing displacements and rotations in any linear elastic determinate or indeterminate structure.

2.3.1 Truss Deflections via Virtual Work

To calculate the deflection Δ at a specific joint in a specific direction: 1. Apply the actual external loads to the truss and compute real internal axial member forces N_i . 2. Remove all external loads, apply a single dimensionless unit virtual load (1.0) at the joint of interest along the desired direction, and calculate virtual internal member axial forces n_i . 3. Compute deflection using the Virtual Work equation:

Virtual Work Formula for Truss Joint Deflection

$$1.0 \cdot \Delta = \sum_{i=1}^m n_i \left(\frac{N_i L_i}{A_i E_i} + \alpha L_i \Delta T_i + \delta_{\text{fab},i} \right) \quad (2.4)$$

where:

n_i = internal virtual axial force in member i due to virtual unit load.

N_i = internal axial force in member i due to actual applied loads.

L_i, A_i, E_i = member length, cross-sectional area, and modulus of elasticity.

$\alpha, \Delta T_i$ = coefficient of thermal expansion and temperature change in member i .

$\delta_{\text{fab},i}$ = fabrication error (initial over/under-length) of member i .

2.3.2 Flexural Beam & Frame Deflections via Virtual Work

For beams and rigid frames where flexural deformation dominates over shear and axial strain, the virtual work equation simplifies to integration over member spans:

$$1.0 \cdot \Delta = \int_0^L \frac{m(x)M(x)}{EI} dx \quad (2.5)$$

For rotations at a specific point, apply a virtual unit moment (1.0 lb·ft or 1.0 kN·m) yielding rotation θ :

$$1.0 \cdot \theta = \int_0^L \frac{m_\theta(x)M(x)}{EI} dx \quad (2.6)$$

3 Structural Determinacy & Stability Analysis**3.1 Static Determinacy vs. Indeterminacy**

A structure is statically determinate if all unknown support reaction forces and internal forces can be uniquely evaluated using only static equilibrium equations. If unknown forces exceed available equilibrium equations, the system is statically indeterminate.

3.1.1 Determinacy Criteria for Two-Dimensional Pin-Jointed Trusses

For a plane truss containing m structural members, r independent reaction components, and j pin connections:

Plane Truss Determinacy Equations

$$m + r < 2j \implies \text{Statically Unstable (Mechanism)} \quad (3.1)$$

$$m + r = 2j \implies \text{Statically Determinate (if geometrically stable)} \quad (3.2)$$

$$m + r > 2j \implies \text{Statically Indeterminate (if stable)} \quad (3.3)$$

The degree of indeterminacy (i) is given by: $i = (m + r) - 2j$.

3.1.2 Determinacy Criteria for Two-Dimensional Rigid Frames & Beams

For planar rigid frames and beam systems containing m structural members, r support reaction components, j joints, and c internal release conditions (e.g., internal hinges where $c = 1$, internal rollers where $c = 2$):

Plane Frame/Beam Determinacy Equations

$$3m + r < 3j + c \implies \text{Statically Unstable} \quad (3.4)$$

$$3m + r = 3j + c \implies \text{Statically Determinate (if geometrically stable)} \quad (3.5)$$

$$3m + r > 3j + c \implies \text{Statically Indeterminate (if stable)} \quad (3.6)$$

The degree of indeterminacy (i) is given by: $i = (3m + r) - (3j + c)$.

3.2 Geometric Instability

Counting equations alone cannot establish stability. Equality of the counts is required for a *stable determinate* system; stable indeterminate systems have redundant constraints. Check support restraints and internal mechanisms separately:

3.2.1 Forms of Instability

1. **Parallel Reactions:** If all external support reactions are parallel, no horizontal equilibrium can be maintained under inclined loading ($\sum F_x \neq 0$).
2. **Concurrent Reactions:** If the lines of action of all support reactions intersect at a single common point O , the structure possesses zero rotational restraint about point O ($\sum M_O \neq 0$).
3. **Internal Collapse Mechanisms:** Un-triangulated member arrangements in trusses (e.g., four-bar pin linkage panels) or three collinear internal hinges in a beam system create kinematic mechanisms that move under load.

4 Column Buckling & Boundary Conditions

4.1 Euler Elastic Buckling Theory

Long slender compression members subject to axial compressive forces can collapse suddenly due to lateral instability (buckling) well before the material reaches its yield strength F_y . Euler's critical buckling load P_{cr} represents the maximum axial force a perfectly straight column can support without buckling flexurally.

4.1.1 Derivation for Pin-Ended Column

Consider a pin-ended column of length L subjected to axial compression P . Let $y(x)$ denote the lateral displacement profile. The differential equation of equilibrium at section x is:

$$EI \frac{d^2 y}{dx^2} + Py = 0 \implies \frac{d^2 y}{dx^2} + k^2 y = 0, \quad \text{where } k^2 = \frac{P}{EI} \quad (4.1)$$

The general solution is $y(x) = C_1 \sin(kx) + C_2 \cos(kx)$. Applying boundary conditions $y(0) = 0 \implies C_2 = 0$, and $y(L) = 0 \implies C_1 \sin(kL) = 0$. For a non-trivial solution ($C_1 \neq 0$), $\sin(kL) = 0 \implies kL = n\pi$. The fundamental mode ($n = 1$) yields Euler's critical buckling load:

Euler Critical Buckling Load & Critical Buckling Stress

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad (4.2)$$

Dividing by gross cross-sectional area A_g and substituting $I = A_g r^2$ gives Euler's critical stress σ_{cr} :

$$\sigma_{cr} = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad (4.3)$$

where:

E = Modulus of Elasticity (29,000 ksi for structural steel).

I : principal inertia for the axis being checked; use the governing KL/r .

KL = Effective column length (K = effective length factor).

r = Radius of gyration ($r = \sqrt{I/A}$).

KL/r = Column slenderness ratio.

4.2 Effective Length Factor (K)

The effective length factor K converts a column with arbitrary end restraints into an equivalent pin-ended column of length KL possessing the same critical buckling load.

Table 4.1: Theoretical vs. Recommended Design Values for Effective Length Factor K

Boundary Conditions (Top / Bottom)	Theoretical K	AISC Design K	Sway Tendency
Pin - Pin (Hinged - Hinged)	1.0	1.0	Sidesway Inhibited
Fixed - Fixed	0.5	0.65	Sidesway Inhibited
Fixed - Pin (Fixed - Hinged)	0.7	0.80	Sidesway Inhibited
Fixed - Free (Cantilever Column)	2.0	2.10	Sidesway Uninhibited
Fixed - Guided (Translating Fixed)	1.0	1.20	Sidesway Uninhibited
Pin - Guided	2.0	2.00	Sidesway Uninhibited

4.3 Alignment Charts for Frame Columns (GA, GB)

For columns embedded in rigid frames, K depends on relative flexural stiffnesses of rigidly connected columns and girders at top (A) and bottom (B) joints:

$$G = \frac{\sum \left(\frac{EI_c}{L_c} \right)}{\sum \left(\frac{EI_g}{L_g} \right)} \quad (4.4)$$

Braced Frames (Sidesway Inhibited): K ranges between 0.5 and 1.0.

Unbraced Frames (Sidesway Uninhibited): K ranges between 1.0 and ∞ .

Pinned Base Connection: Theoretical $G = \infty$ (use $G = 10.0$ in design).

Rigidly Fixed Base Connection: Theoretical $G = 0$ (use $G = 1.0$ in design).

5 Elementary Statically Indeterminate Structures

5.1 The Force Method of Analysis (Flexibility Method)

The Force Method resolves static indeterminacy by removing redundant constraints until a stable, statically determinate "primary structure" remains. The unknown redundant forces are then determined by enforcing geometric compatibility.

5.1.1 Step-by-Step Analysis Procedure

1. Determine the degree of indeterminacy $i = (3m + r) - (3j + c)$.
2. Select i redundant forces (B_y, C_y , etc.) and remove corresponding constraints to form the primary determinate structure.
3. Apply actual external loads to the primary structure and calculate displacement Δ_{B0} at redundant locations.
4. Remove external loads, apply a unit virtual redundant load ($B_y = 1.0$), and calculate flexibility coefficient f_{BB} (displacement per unit load).
5. Formulate compatibility equation and solve for the redundant force:

$$\Delta_{B0} + B_y \cdot f_{BB} = 0 \implies B_y = -\frac{\Delta_{B0}}{f_{BB}} \quad (5.1)$$

5.2 Fixed-End Moments (FEM)

Statically indeterminate structures subjected to span loadings develop Fixed-End Moments (FEM) at fully restrained ends. These values serve as foundational input for Moment Distribution and Stiffness Matrix methods.

Table 5.1: Fixed-End Moment Expressions for Standard Beam Members

Span Loading Condition	Left Moment (FEM_{AB})	Right Moment (FEM_{BA})
Uniform Load w over full span L	$-\frac{wL^2}{12}$	$+\frac{wL^2}{12}$
Concentrated Load P at midspan ($L/2$)	$-\frac{PL}{8}$	$+\frac{PL}{8}$
Concentrated Load P at distance a from A , b from B	$-\frac{Pab^2}{L^2}$	$+\frac{Pa^2b}{L^2}$
Linear Increasing Load w_0 (0 at A to w_0 at B)	$-\frac{w_0L^2}{30}$	$+\frac{w_0L^2}{20}$

5.3 Moment Distribution Method Overview

The Moment Distribution Method is an iterative displacement analysis technique:

Member Flexural Stiffness (k): For member fixed at far end: $k = \frac{4EI}{L}$. For member pinned at far end: $k = \frac{3EI}{L}$.

Distribution Factor (DF_i): At a rigid joint j : $DF_i = \frac{k_i}{\sum k_j}$.

Carry-Over Factor (COF): For prismatic beam elements, $COF = +0.5$ to a fixed far end.

6 Design Loads, Load Combinations & Tributary Areas

6.1 ASCE 7 Structural Load Classifications

Building design requires accounting for various environmental and operational load effects:

Dead Load (D): Permanent weight of structural elements, walls, floors, roofs, and fixed service equipment.

Live Load (L): Transient occupancy loads produced by human use, furniture, and movable equipment.

Roof Live Load (L_r), Snow (S), Rain (R): Gravity environmental loads acting on roofs.

Wind Load (W): Lateral and uplift pressure forces induced by wind velocity fields.

Seismic Load (E): Inertial forces resulting from earthquake ground motion accelerations.

6.2 ASCE 7 LRFD Load Combinations

Under Strength Design / Load and Resistance Factor Design (LRFD), structures must satisfy:

$$R_u \leq \phi R_n \quad (6.1)$$

where R_u is the required ultimate design load effect calculated from governing basic LRFD combinations:

ASCE 7-16 LRFD Load Combinations

$$\text{Combo 1: } 1.4D \quad (6.2)$$

$$\text{Combo 2: } 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad (6.3)$$

$$\text{Combo 3: } 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \quad (6.4)$$

$$\text{Combo 4: } 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \quad (6.5)$$

$$\text{Combo 5: } 1.2D + 1.0E + 1.0L + 0.2S \quad (6.6)$$

$$\text{Combo 6: } 0.9D + 1.0W \quad (\text{Governs for structural uplift / overturning}) \quad (6.7)$$

$$\text{Combo 7: } 0.9D + 1.0E \quad (\text{Governs for seismic overturning}) \quad (6.8)$$

6.3 Tributary Areas & Load Distribution

Structural floor systems distribute surface area pressures (q in lb/ft^2 or kPa) to supporting beams as linear line loads (w in lb/ft or kN/m).

6.3.1 One-Way vs. Two-Way Floor Framing

One-Way System: A slab supported on two opposite edges is one-way. With four-edge support, $L_2/L_1 > 2$ commonly indicates one-way behavior. Deck loads go to supporting joists or beams using their tributary widths.

$$w_{\text{beam}} = q \cdot b_{\text{trib}} = q \cdot \left(\frac{S_1}{2} + \frac{S_2}{2} \right) \quad (6.9)$$

Two-Way System: With suitable four-edge support and $L_2/L_1 \leq 2$, load acts in both directions. The 45° tributary construction is an approximate distribution model.

6.3.2 ASCE 7 Live Load Reduction

Recognizing low probability of full design live load occurring simultaneously over large floor areas, ASCE 7 permits live load reduction for members having influence area $K_{LL}A_T \geq 400 \text{ ft}^2$:

ASCE 7 Live Load Reduction Formula

$$L = L_0 \left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}} \right) \quad (\text{US Customary Units: ft}^2) \quad (6.10)$$

Design Limitations:

L shall not be less than $0.50L_0$ for members supporting one floor.

L shall not be less than $0.40L_0$ for members supporting two or more floors.

Heavy loads over 100 psf have restricted reductions; code exceptions govern.

Public-assembly loads have separate restrictions, not a blanket heavy-load exception.

Table 6.1: Live Load Element Factor (K_{LL}) Values

Structural Member Type	K_{LL}
Interior Columns & Exterior Columns without cantilever slabs	4
Edge Columns with cantilever slabs	3
Corner Columns with cantilever slabs	2
Interior Beams & Edge Beams without cantilever slabs	2
Cantilever Beams	1
One-way Slabs	1

7 Design of Steel Components (ANSI/AISC 360-16)

7.1 AISC LRFD Specification Fundamentals

Steel structural design governed by ANSI/AISC 360-16 uses LRFD criteria:

$$\phi R_n \geq R_u \quad (7.1)$$

where ϕ is the resistance factor ($\phi \leq 1.0$), R_n is nominal limit state strength, and R_u is required factored strength.

7.2 Tension Member Design

Structural steel tension members (angles, plates, channels, W-shapes) must be checked against three governing limit states:

7.2.1 1. Tensile Yielding on Gross Section

Yielding throughout member length results in excessive elongation:

$$P_n = F_y A_g, \quad \phi_t = 0.90 \implies \phi_t P_n = 0.90 F_y A_g \quad (7.2)$$

7.2.2 2. Tensile Rupture on Net Effective Section

Fracture across connection holes at member ends:

$$P_n = F_u A_e, \quad \phi_t = 0.75 \implies \phi_t P_n = 0.75 F_u A_e \quad (7.3)$$

where $A_e = U A_n$ is effective net area, F_u is specified minimum tensile strength, and U is shear lag factor.

Net Area (A_n) & Shear Lag Factor (U)

Net area accounts for bolt hole deductions (bolt diameter $d_b + 1/8$ in. standard allowance):

$$A_n = A_g - \sum (d_b + 1/8'')t + \sum \frac{s^2}{4g}t \quad (7.4)$$

where s is stagger pitch and g is gauge line spacing. For applicable cases:

$$U = 1 - \frac{\bar{x}}{L} \quad (7.5)$$

where $\bar{x}$ is connection eccentricity and L is connection length. Equation (7.5) is not universal; use the AISC-prescribed U for the specific member and connection.

7.2.3 3. Block Shear Rupture

Block shear involves tear-out failure along a perimeter containing shear planes and a perpendicular tension plane:

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs}F_u A_{nt}, \quad \phi_v = 0.75 \quad (7.6)$$

where $U_{bs} = 1.0$ for uniform tension stress ($U_{bs} = 0.5$ for non-uniform tension).

7.3 Flexural Member Design (Beams)

Flexural capacity of doubly symmetric compact I-shapes (W-shapes) depends on unbraced length L_b of the compression flange.

7.3.1 Flexural Limit States

1. Plastic Moment / Yielding ($L_b \leq L_p$):

$$M_n = M_p = F_y Z_x, \quad \phi_b = 0.90 \implies \phi_b M_n = 0.90 F_y Z_x \quad (7.7)$$

where Z_x is plastic section modulus.

2. Inelastic Lateral-Torsional Buckling ($L_p < L_b \leq L_r$):

$$M_n = C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (7.8)$$

3. Elastic Lateral-Torsional Buckling ($L_b > L_r$):

$$M_n = F_{cr} S_x \leq M_p, \quad \text{where } F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{J_c}{S_x h_0} \left(\frac{L_b}{r_{ts}} \right)^2} \quad (7.9)$$

Moment Modification Factor (C_b)

C_b accounts for non-uniform bending moment gradients along unbraced segment L_b :

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C} \quad (7.10)$$

where $M_{\max}$, M_A , M_B , M_C are absolute moments at maximum, 1/4-point, midspan, and 3/4-point of L_b .

7.3.2 Beam Shear Capacity

For unstiffened W-shape webs with $h/t_w \leq 2.24\sqrt{E/F_y}$:

$$V_n = 0.60F_y A_w C_{v1}, \quad \phi_v = 1.00 \implies \phi_v V_n = 1.00(0.60F_y dt_w) \quad (7.11)$$

7.4 Compression Member Design (Columns)

Column capacity under axial compression depends on flexural buckling about the weak axis:

AISC Column Compressive Strength ($\phi_c = 0.90$)

Calculate governing slenderness ratio $(KL/r)_{\max} = \max(K_x L_x / r_x, K_y L_y / r_y)$:

If $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$ (Inelastic Buckling):

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \quad (7.12)$$

If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ (Elastic Buckling):

$$F_{cr} = 0.877F_e \quad (7.13)$$

where Elastic Euler Buckling Stress $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$. Nominal strength $P_n = F_{cr} A_g$.

8 Design of Reinforced Concrete Components (ACI 318-14)

8.1 ACI 318 Strength Design Principles

Reinforced concrete structural design enforces ultimate strength compatibility:

$$\phi S_n \geq U \quad (8.1)$$

where U is factored load combination demand, S_n is nominal member strength, and ϕ is strength reduction factor based on cross-sectional strain conditions.

8.1.1 Material Properties & Assumptions

Concrete compressive strength f'_c (typically 3,000 – 6,000 psi).

Concrete elastic modulus: $E_c = 57,000\sqrt{f'_c}$ psi (normal-weight concrete).

Reinforcing steel yield strength $f_y = 60,000$ psi (Grade 60), $E_s = 29,000$ ksi.

Maximum usable concrete compressive strain at extreme fiber: $\epsilon_{cu} = 0.003$.

Concrete tensile strength ignored in flexural strength calculations.

8.2 Flexural Design of Singly Reinforced Rectangular Beams

At ultimate strength, actual complex parabolic concrete stress distribution is replaced by Whitney's Equivalent Rectangular Stress Block.

Whitney Rectangular Stress Block Parameters

Uniform concrete stress magnitude: $0.85f'_c$.

Stress block depth: $a = \beta_1 c$, where c is distance from extreme compression fiber to neutral axis.

Stress block parameter β_1 :

$$\beta_1 = \begin{cases} 0.85 & \text{for } f'_c \leq 4000 \text{ psi} \\ 0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right) \geq 0.65 & \text{for } f'_c > 4000 \text{ psi} \end{cases} \quad (8.2)$$

8.2.1 Equilibrium & Flexural Moment Capacity Equations

Equating total compressive force C in concrete to total tensile force T in steel ($T = A_s f_y$ assuming steel yields):

$$C = T \implies 0.85 f'_c b a = A_s f_y \implies a = \frac{A_s f_y}{0.85 f'_c b} \quad (8.3)$$

Nominal flexural moment capacity M_n evaluated about steel centroid:

$$M_n = C \left(d - \frac{a}{2} \right) = A_s f_y \left(d - \frac{a}{2} \right) = 0.85 f'_c b a \left(d - \frac{a}{2} \right) \quad (8.4)$$

8.2.2 Strain Verification & Strength Reduction Factor (ϕ)

Net tensile strain ϵ_t in extreme layer of tension steel dictates ϕ :

$$c = \frac{a}{\beta_1}, \quad \epsilon_t = \frac{d - c}{c} (0.003) \quad (8.5)$$

Tension-Controlled Section ($\epsilon_t \geq 0.005$): Ductile failure. $\phi = 0.90$.

Compression-Controlled: $\epsilon_t \leq 0.002$ for Grade 60 steel; $\phi = 0.65$ (tied) or 0.75 (spiral). Other grades use the applicable ACI yield-strain boundary.

Transition Zone (Grade 60): $0.002 < \epsilon_t < 0.005$; interpolate ϕ .

$$\phi = 0.65 + (\epsilon_t - 0.002) \left(\frac{250}{3} \right) \quad (\text{for tied reinforcement}) \quad (8.6)$$

8.2.3 Reinforcement Limits

Minimum flexural steel area $A_{s,\min}$ prevents sudden brittle fracture upon concrete cracking:

$$A_{s,\min} = \max \left(\frac{3\sqrt{f'_c}}{f_y} b_w d, \frac{200}{f_y} b_w d \right) \quad (8.7)$$

8.3 Shear Design of Reinforced Concrete Beams

Ultimate shear design requires:

$$\phi_v V_n \geq V_u, \quad \phi_v = 0.75 \quad (8.8)$$

Nominal shear strength V_n combines concrete shear capacity V_c and steel stirrup capacity V_s :

$$V_n = V_c + V_s \quad (8.9)$$

ACI Shear Equations for Beams

1. Concrete Shear Strength Contribution V_c :

$$V_c = 2\lambda\sqrt{f'_c}b_wd \quad (\text{with } f'_c \text{ in psi, } \lambda = 1.0 \text{ for normal weight}) \quad (8.10)$$

2. Required Shear Reinforced Strength V_s :

$$V_s = \frac{V_u}{\phi_v} - V_c \quad (8.11)$$

3. Required Stirrup Spacing s (for vertical 2-leg stirrups with area $A_v = 2A_b$):

$$s = \frac{A_v f_{yt} d}{V_s} \quad (8.12)$$

8.3.1 ACI Stirrup Spacing Limits

If $V_u \leq 0.5\phi_v V_c$: No stirrups required.

If $0.5\phi_v V_c < V_u \leq \phi_v V_c$: Provide minimum stirrups:

$$s_{\max} = \min\left(\frac{d}{2}, 24 \text{ in.}\right), \quad \frac{A_{v,\min}}{s} = \max\left(\frac{0.75\sqrt{f'_c} b_w}{f_{yt}}, \frac{50b_w}{f_{yt}}\right) \quad (8.13)$$

If $V_s \leq 4\sqrt{f'_c}b_wd$: Maximum spacing $s_{\max} = \min(d/2, 24")$.

If $4\sqrt{f'_c}b_wd < V_s \leq 8\sqrt{f'_c}b_wd$: Maximum spacing $s_{\max} = \min(d/4, 12")$.

If $V_s > 8\sqrt{f'_c}b_wd$: Cross-section undersized (increase beam dimensions).

8.4 Concentric Short Concrete Column Design

Short concrete columns subjected to pure axial compression load are governed by ACI limits:

Axial Column Strength Formulas**Tied Columns ($\phi = 0.65$):**

$$\phi P_{n,\max} = 0.80\phi [0.85f'_c(A_g - A_{st}) + f_y A_{st}] \quad (8.14)$$

Spiral Columns ($\phi = 0.75$):

$$\phi P_{n,\max} = 0.85\phi [0.85f'_c(A_g - A_{st}) + f_y A_{st}] \quad (8.15)$$

Gross steel reinforcement ratio limitations: $0.01 \leq \rho_g = \frac{A_{st}}{A_g} \leq 0.08$.

9 Example Problems and Solutions

Problem 1: Comprehensive Shear & Moment Diagram Derivation

Problem: A simply supported beam of span $L = 24$ ft supports a uniform dead load $w = 1.5$ kips/ft across its entire length and a concentrated point load $P = 12$ kips at midspan ($x = 12$ ft). Determine:

- Vertical reaction forces at supports A ($x = 0$) and B ($x = 24$ ft).
- Shear force equations $V(x)$ for regions $0 < x < 12$ ft and $12 \text{ ft} < x < 24$ ft.
- Maximum bending moment $M_{\max}$ in the beam.

Solution

- Support Reactions:** Due to geometric and loading symmetry about midspan:

$$R_A = R_B = \frac{wL + P}{2} = \frac{(1.5 \text{ kips/ft})(24 \text{ ft}) + 12 \text{ kips}}{2} = \frac{36 + 12}{2} = 24.0 \text{ kips}$$

- Shear Force Equations:**

For $0 \leq x < 12$ ft: $V(x) = R_A - wx = 24.0 - 1.5x$. At $x = 0$, $V = +24$ kips.
Just left of midspan ($x = 12^-$), $V = 24 - 1.5(12) = +6.0$ kips.

At $x = 12$ ft, concentrated load $P = 12$ kips causes a downward jump: $V(12^+) = +6.0 - 12 = -6.0$ kips.

For $12 < x \leq 24$ ft: $V(x) = 24.0 - 12 - 1.5x = 12.0 - 1.5x$. At $x = 24$ ft, $V(24) = 12 - 1.5(24) = -24.0$ kips.

- Maximum Bending Moment:** Shear crosses zero at $x = 12$ ft. Integrating shear area from $x = 0$ to $x = 12$ ft:

$$M_{\max} = \text{Area under } V(x) = \frac{V(0) + V(12^-)}{2} \times 12 = \frac{24.0 + 6.0}{2} \times 12 = 15.0 \times 12 = 180.0 \text{ kip-ft}$$

Alternatively, using superposition of formulas:

$$M_{\max} = \frac{wL^2}{8} + \frac{PL}{4} = \frac{1.5(24)^2}{8} + \frac{12(24)}{4} = 108.0 + 72.0 = 180.0 \text{ kip-ft}$$

Problem 2: Zero-Force Member Identification in Pratt Truss

Problem: A planar Pratt truss features Joint K connected to Member AK (horizontal), Member KB (vertical), and Member KD (horizontal). No external load acts at Joint K , and AK and KD form a continuous straight horizontal line. Determine the force in vertical member KB .

Solution

Vertical member KB .

Applying $\sum F_y = 0$ perpendicular to the collinear line $AK - KD$ at Joint K :

$$\sum F_y = F_{KB} = 0 \implies F_{KB} = 0 \text{ (Zero-Force Member)}$$

Problem 3: Method of Sections Analysis

Problem: A Warren truss with panel span $s = 10$ ft and height $h = 12$ ft supports vertical downward loads of 20 kips at lower panel joints L_1, L_2, L_3 . Total span is 40 ft between pinned support L_0 and roller support L_4 . Determine the axial force in top chord member U_1U_2 .

Solution

- Support Reaction:** Total downward load $= 3 \times 20 = 60$ kips. By symmetry, $R_{L0} = R_{L4} = 30$ kips.
- Pass Imaginary Cut:** Pass a vertical section line through members U_1U_2 , U_1L_2 , and L_1L_2 . Isolate the left portion of the truss containing joints L_0, L_1, U_1 .
- Moment Equilibrium about Joint L_2 ($x = 20$ ft, $y = 0$):** Taking moments about L_2 eliminates diagonal member U_1L_2 and bottom chord L_1L_2 :

$$\sum M_{L2} = R_{L0}(20 \text{ ft}) - P_{L1}(10 \text{ ft}) + F_{U1U2}(h) = 0$$

$$(30 \text{ kips})(20 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) + F_{U1U2}(12 \text{ ft}) = 0$$

$$600 - 200 + 12F_{U1U2} = 0 \implies 400 + 12F_{U1U2} = 0 \implies F_{U1U2} = -\frac{400}{12} = -33.33 \text{ kips}$$

The negative sign indicates **Compression** of 33.33 kips.

Problem 4: Double Integration Method for Deflection

Problem: Derive the tip deflection equation for a cantilever beam of span L subjected to a uniform load w across its entire length using double integration.

Solution

1. **Bending Moment Expression:** Setting origin $x = 0$ at fixed support (A) and $x = L$ at free tip (B): Reactions at fixed end $x = 0$: $R_A = wL$ (upwards), $M_A = \frac{wL^2}{2}$ (counterclockwise). Bending moment equation at cut section x :

$$M(x) = -M_A + R_A x - \frac{wx^2}{2} = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

2. **First Integration (Slope):**

$$EI \frac{dy}{dx} = \int M(x) dx = -\frac{wL^2}{2}x + \frac{wLx^2}{2} - \frac{wx^3}{6} + C_1$$

Boundary condition at fixed end $x = 0$: $\frac{dy}{dx}(0) = 0 \implies C_1 = 0$.

3. **Second Integration (Deflection):**

$$EI y(x) = \int EI \frac{dy}{dx} dx = -\frac{wL^2 x^2}{4} + \frac{wL x^3}{6} - \frac{wx^4}{24} + C_2$$

Boundary condition at fixed end $x = 0$: $y(0) = 0 \implies C_2 = 0$.

4. **Tip Deflection ($x = L$):**

$$EI y(L) = -\frac{wL^4}{4} + \frac{wL^4}{6} - \frac{wL^4}{24} = wL^4 \left(-\frac{6}{24} + \frac{4}{24} - \frac{1}{24} \right) = -\frac{3wL^4}{24} = -\frac{wL^4}{8}$$

Thus, tip downward displacement magnitude is $\Delta_{\max} = \frac{wL^4}{8EI}$.

Problem 5: Truss Joint Deflection via Virtual Work

Problem: A 3-member pin-connected truss consists of joints $A(0,0)$, $B(12\text{ ft},0)$, and $C(6\text{ ft},8\text{ ft})$. Support A is a pin and support B is a roller. Members are AC , BC , and AB . All members have $A = 2.0\text{ in.}^2$ and $E = 29,000\text{ ksi}$. A downward load $P = 30\text{ kips}$ acts at C . Calculate the vertical deflection Δ_{Cv} at C .

Solution

1. Geometry & Lengths: $L_{AC} = L_{BC} = \sqrt{6^2 + 8^2} = 10\text{ ft} = 120\text{ in.}$ and $L_{AB} = 12\text{ ft} = 144\text{ in.}$

2. Support Reactions & Real Member Forces (N_i): By symmetry, $A_y = B_y = 15\text{ kips}$ and $A_x = 0$. Taking member tension as positive, joint C gives

$$-2N_{AC} \left(\frac{8}{10} \right) - 30 = 0 \Rightarrow N_{AC} = N_{BC} = -18.75\text{ kips (compression)}.$$

At joint A , $N_{AB} + 0.6N_{AC} = 0$, so $N_{AB} = +11.25\text{ kips (tension)}$.

3. Virtual Member Forces (n_i): Apply a 1.0-kip downward unit load at C . Then $n_{AC} = n_{BC} = -0.625\text{ kip}$ and $n_{AB} = +0.375\text{ kip}$.

4. Virtual Work Summation:

$$\Delta_{Cv} = 2 \frac{(-0.625)(-18.75)(120)}{(2.0)(29,000)} + \frac{(0.375)(11.25)(144)}{(2.0)(29,000)} = 0.05897\text{ in.} \approx 0.0590\text{ in. downward.}$$

Problem 6: Frame Determinacy Evaluation

Problem: A planar frame consists of 3 rigid members, 4 rigid joints, 1 internal hinge ($c = 1$), a fixed support ($r = 3$) at Joint A, and a pin support ($r = 2$) at Joint D. Evaluate stability and determinacy.

Solution

Members $m = 3$, Reactions $r = 3 + 2 = 5$, Joints $j = 4$, Internal releases $c = 1$.

Left-hand side: $3m + r = 3(3) + 5 = 14$.

Right-hand side: $3j + c = 3(4) + 1 = 13$.

Since $14 > 13$, the frame is statically indeterminate to the 1st degree ($i = 14 - 13 = 1$), assuming the frame is geometrically stable.

Problem 7: Propped Cantilever Beam via Force Method

Problem: A uniform beam of span L and rigidity EI is fixed at A ($x = 0$) and propped with a roller support at B ($x = L$), supporting a uniform load w . Find prop reaction B_y .

Solution

Redundant selection: Roller reaction B_y (1° indeterminate).

Primary structure: Cantilever beam fixed at A , free at B .

Downward deflection at B due to w : $\Delta_{B0} = \frac{wL^4}{8EI}$ (downwards).

Upward deflection at B due to unit force $B_y = 1$: $f_{BB} = \frac{1 \cdot L^3}{3EI}$ (upwards).

Compatibility: $-\frac{wL^4}{8EI} + B_y \left(\frac{L^3}{3EI} \right) = 0 \implies B_y = \frac{3}{8}wL$.

Problem 8: Statics - Truss Reaction Forces**Topic 11.A: Statically Determinate Systems**

A planar truss spans $L = 36$ ft between a pin support at A ($x = 0$) and a roller support at B ($x = 36$ ft). Concentrated vertical loads of 12 kips and 18 kips act downwards at $x = 12$ ft and $x = 24$ ft, respectively. Determine vertical reaction force R_B at support B .

Options: (A) 13.0 kips (B) 15.0 kips (C) 16.0 kips (D) 17.0 kips

Solution

Taking moment equilibrium about support A ($\sum M_A = 0$ counterclockwise positive):

$$\sum M_A = (12 \text{ kips})(12 \text{ ft}) + (18 \text{ kips})(24 \text{ ft}) - R_B(36 \text{ ft}) = 0$$

$$144 + 432 - 36R_B = 0 \implies 576 = 36R_B \implies R_B = \frac{576}{36} = 16.0 \text{ kips}$$

Correct Answer: (C)

Problem 9: Zero-Force Member Identification**Topic 11.A: Statically Determinate Systems**

In a Howe roof truss, Joint J connects three members: horizontal bottom chord members IJ and JK , and vertical web member DJ . No external loads or support reactions are applied at Joint J . How many zero-force members are connected at Joint J ?

Options: (A) 0 (B) 1 (C) 2 (D) 3

Solution

Applying Rule 2 for zero-force members: Joint J connects three members (IJ, JK, DJ), two of which (IJ and JK) are collinear along the horizontal bottom chord. Since no external force acts at Joint J , equilibrium perpendicular to the collinear chord line requires $F_{DJ} = 0$. Thus, Member DJ is a zero-force member (1 member total).

Correct Answer: (B)

Problem 10: Beam Shears & Bending Moments**Topic 11.A: Statically Determinate Systems**

A simply supported beam of span $L = 20$ ft carries a uniform load $w = 2.4$ kips/ft over its entire length. Determine maximum bending moment $M_{\max}$ in the beam.

Options: (A) 96.0 kip-ft (B) 120.0 kip-ft (C) 144.0 kip-ft (D) 192.0 kip-ft

Solution

For a simply supported beam under uniform load w , maximum moment occurs at midspan ($x = L/2 = 10$ ft):

$$M_{\max} = \frac{wL^2}{8} = \frac{(2.4 \text{ kips/ft})(20 \text{ ft})^2}{8} = \frac{2.4 \times 400}{8} = 120.0 \text{ kip-ft}$$

Correct Answer: (B)

Problem 11: Beam Deflection via Integration**Topic 11.B: Deflections**

A cantilever beam of length $L = 12$ ft with flexural rigidity $EI = 45,000$ kip-ft² supports a concentrated load $P = 15$ kips at its free tip. Calculate maximum downward deflection at the tip.

Options: (A) 0.192 in. (B) 0.576 in. (C) 0.691 in. (D) 2.304 in.

Solution

Using standard cantilever tip deflection formula:

$$\Delta_{\max} = \frac{PL^3}{3EI} = \frac{(15 \text{ kips})(12 \text{ ft})^3}{3(45,000 \text{ kip-ft}^2)} = \frac{15 \times 1728}{135,000} = 0.192 \text{ ft}$$

Converting to inches: $\Delta_{\max} = 0.192 \text{ ft} \times 12 \text{ in./ft} = 2.304 \text{ in.}$

Correct Answer: (D)

Problem 12: Virtual Work Truss Deflection**Topic 11.B: Deflections**

A pin-connected truss member has axial load $N = +80$ kips (tension), length $L = 120$ in., cross-sectional area $A = 4.0$ in.², and elastic modulus $E = 29,000$ ksi. Under a virtual unit load applied at a joint, internal virtual force in this member is $n = +0.75$. Compute member contribution to joint deflection.

Options: (A) 0.0248 in. (B) 0.0455 in. (C) 0.0621 in. (D) 0.0828 in.

Solution

Applying Virtual Work member displacement equation $\delta = n \frac{NL}{AE}$:

$$\delta = (0.75) \left[\frac{(80 \text{ kips})(120 \text{ in.})}{(4.0 \text{ in.}^2)(29,000 \text{ ksi})} \right] = 0.75 \left[\frac{9600}{116,000} \right] = 0.75 \times 0.082758 = 0.06207 \text{ in.} \approx 0.0621 \text{ in.}$$

Correct Answer: (C)

Problem 13: Structural Determinacy of Plane Truss

Topic 11.D: Determinacy & Stability

A stable plane truss has $m = 17$ members, $j = 10$ joints, and $r = 4$ reaction components. Classify the structure.

Options: (A) Statically Unstable (B) Statically Determinate (C) Statically Indeterminate 1° (D) Statically Indeterminate 2°

Solution

Applying plane truss determinacy criteria equation:

$$m + r = 17 + 4 = 21, \quad 2j = 2(10) = 20$$

Since $m + r > 2j$ ($21 > 20$), the truss is statically indeterminate to the degree $i = 21 - 20 = 1^\circ$.

Correct Answer: (C)

Problem 14: Structural Determinacy of Rigid Frame

Topic 11.D: Determinacy & Stability

A stable 2D rigid frame has $m = 6$ members, $j = 6$ rigid joints, $r = 6$ reaction components (two fixed bases), and $c = 2$ internal hinges. Determine degree of static indeterminacy.

Options: (A) 2° (B) 4° (C) 6° (D) 8°

Solution

Applying plane frame determinacy formula:

$$i = (3m + r) - (3j + c) = [3(6) + 6] - [3(6) + 2] = (18 + 6) - (18 + 2) = 24 - 20 = 4^\circ$$

Correct Answer: (B)

Problem 15: Euler Column Buckling Load

Topic 11.C: Column Buckling

A W10 × 45 steel column ($A_g = 13.3 \text{ in.}^2$, $I_x = 248 \text{ in.}^4$, $I_y = 53.4 \text{ in.}^4$) has length $L = 18 \text{ ft}$ and pinned ends ($K_x = K_y = 1.0$). Assuming $E = 29,000 \text{ ksi}$, calculate critical Euler buckling load P_{cr} .

Options: (A) 328 kips (B) 398 kips (C) 1230 kips (D) 1510 kips

Solution

Buckling occurs about the weak axis (minimum moment of inertia $I_y = 53.4 \text{ in.}^4$).

Converting column length to inches: $KL = 1.0 \times 18 \text{ ft} \times 12 \text{ in./ft} = 216 \text{ in.}$

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2} = \frac{\pi^2 (29,000 \text{ ksi})(53.4 \text{ in.}^4)}{(216 \text{ in.})^2} = \frac{15,283,326}{46,656} = 327.5 \text{ kips} \approx 328 \text{ kips}$$

Correct Answer: (A)

Problem 16: Column Slenderness Ratio & Critical Stress

Topic 11.C: Column Buckling

A steel column with radius of gyration $r_y = 2.0$ in. has an effective length $KL = 20$ ft. Calculate column slenderness ratio $(KL/r)_y$.

Options: (A) 10 (B) 120 (C) 140 (D) 240

Solution

Converting effective length to inches: $KL = 20 \text{ ft} \times 12 \text{ in./ft} = 240 \text{ in.}$

$$\frac{KL}{r} = \frac{240 \text{ in.}}{2.0 \text{ in.}} = 120$$

Correct Answer: (B)

Problem 17: Fixed-End Moments for Indeterminate Beam

Topic 11.E: Indeterminate Structures

A 24-ft beam fixed at both ends carries a concentrated force $P = 32$ kips at midspan (12 ft from each end). Compute magnitude of fixed-end moment FEM_{AB} .

Options: (A) 64 kip-ft (B) 96 kip-ft (C) 128 kip-ft (D) 192 kip-ft

Solution

Applying standard Fixed-End Moment formula for center point load on fixed-fixed beam:

$$|FEM_{AB}| = \frac{PL}{8} = \frac{(32 \text{ kips})(24 \text{ ft})}{8} = 96.0 \text{ kip-ft}$$

Correct Answer: (B)

Problem 18: Propped Cantilever Force Method

Topic 11.E: Indeterminate Structures

A propped cantilever beam of span $L = 16$ ft supports a concentrated load $P = 20$ kips at midspan ($a = b = 8$ ft). Determine vertical reaction force B_y at the simple roller prop support using force method compatibility.

Options: (A) 6.25 kips (B) 10.00 kips (C) 13.75 kips (D) 16.00 kips

Solution

Deflection at prop B due to load P on cantilever: $\Delta_{B0} = \frac{Pa^2}{6EI}(3L - a) = \frac{20(8)^2}{6EI}(3(16) - 8) = \frac{20(64)(40)}{6EI} = \frac{8533.33}{EI}$. Flexibility coefficient at B : $f_{BB} = \frac{L^3}{3EI} = \frac{16^3}{3EI} = \frac{1365.33}{EI}$. Compatibility equation: $\Delta_{B0} - B_y f_{BB} = 0 \implies B_y = \frac{8533.33}{1365.33} = 6.25 \text{ kips.}$

Correct Answer: (A)

Problem 19: ASCE 7 Factored Load Combination**Topic 11.F: Design Loads**

A roof girder is subjected to dead load $D = 40$ kips, roof live load $L_r = 25$ kips, and wind uplift $W = -35$ kips. Calculate the governing ultimate LRFD design load demand U for maximum downward force.

Options: (A) 25.5 kips (B) 56.0 kips (C) 60.5 kips (D) 88.0 kips

Solution

Evaluate ASCE 7 LRFD combinations with downward force positive and floor live load $L = 0$.

$$1.4D = 1.4(40) = 56.0 \text{ kips.}$$

$$1.2D + 0.5L_r = 48 + 12.5 = 60.5 \text{ kips.}$$

$$1.2D + 1.6L_r + \max(L, 0.5W) = 48 + 40 + \max(0, -17.5) = \mathbf{88.0 \text{ kips.}}$$

$$1.2D + W + 0.5L_r = 48 - 35 + 12.5 = 25.5 \text{ kips.}$$

$$0.9D + W = 36 - 35 = 1.0 \text{ kip.}$$

The governing downward design demand is **88.0 kips**; the favorable wind term is not retained when $L = 0$ is larger.

Correct Answer: D

Problem 20: Tributary Load on Interior Beam**Topic 11.F: Design Loads**

An interior floor beam spaced at $S = 8$ ft center-to-center has a span $L = 25$ ft. The floor deck carries uniform Dead Load $q_D = 60$ psf and Live Load $q_L = 80$ psf. Calculate factored ultimate linear load w_u (kip/ft) on the beam using LRFD Combo 2 ($1.2D + 1.6L$).

Options: (A) 1.12 kips/ft (B) 1.60 kips/ft (C) 2.00 kips/ft (D) 2.24 kips/ft

Solution

Calculating factored surface load q_u :

$$q_u = 1.2q_D + 1.6q_L = 1.2(60 \text{ psf}) + 1.6(80 \text{ psf}) = 72 + 128 = 200 \text{ psf} = 0.200 \text{ ksf}$$

Multiplying by tributary width $b_{\text{trib}} = 8$ ft:

$$w_u = q_u \times b_{\text{trib}} = (0.200 \text{ ksf})(8 \text{ ft}) = 1.60 \text{ kips/ft}$$

Correct Answer: (B)

Problem 21: Steel Tension Member Tensile Yielding**Topic 11.G: Steel Design**

A Grade A572 Gr. 50 steel plate ($F_y = 50$ ksi, $F_u = 65$ ksi) has gross dimensions $t = 0.5$ in. by $b = 8.0$ in.. Compute design tensile yielding strength $\phi_t P_n$.

Options: (A) 150 kips (B) 180 kips (C) 195 kips (D) 260 kips

Solution

Calculating gross area $A_g = b \times t = 8.0 \times 0.5 = 4.0$ in.². Applying LRFD tensile yielding limit state equation ($\phi_t = 0.90$):

$$\phi_t P_n = \phi_t F_y A_g = 0.90(50 \text{ ksi})(4.0 \text{ in.}^2) = 180.0 \text{ kips}$$

Correct Answer: (B)

Problem 22: Steel Tension Member Net Area & Rupture**Topic 11.G: Steel Design**

The same 1/2-in. $\times$ 8-in. A572 Gr. 50 steel plate ($F_u = 65$ ksi) contains two standard holes for 3/4-in.-diameter bolts in a single transverse line. Assuming shear lag factor $U = 1.0$, determine the design tensile rupture strength $\phi_t P_n$.

Options: (A) 152.3 kips (B) 175.5 kips (C) 195.0 kips (D) 214.5 kips

Solution

Design bolt hole diameter $d_h = d_b + 1/8'' = 3/4'' + 1/8'' = 0.875$ in. Net area $A_n = A_g - 2d_h t = 4.0 \text{ in.}^2 - 2(0.875 \text{ in.})(0.5 \text{ in.}) = 4.0 - 0.875 = 3.125 \text{ in.}^2$. Effective net area $A_e = U A_n = 1.0(3.125) = 3.125 \text{ in.}^2$. Applying LRFD tensile rupture limit state equation ($\phi_t = 0.75$):

$$\phi_t P_n = \phi_t F_u A_e = 0.75(65 \text{ ksi})(3.125 \text{ in.}^2) = 152.34 \text{ kips} \approx 152.3 \text{ kips}$$

Correct Answer: (A)

Problem 23: Steel Beam Plastic Moment Capacity**Topic 11.G: Steel Design**

A **compact** W16 $\times$ 31 section has $F_y = 50$ ksi and $Z_x = 54.0 \text{ in.}^3$, with full lateral bracing ($L_b \leq L_p$). Find its LRFD flexural design strength $\phi_b M_n$.

Options: (A) 180 kip-ft (B) 202.5 kip-ft (C) 225 kip-ft (D) 270 kip-ft

Solution

Since unbraced length $L_b \leq L_p$, plastic moment yields governing limit state ($\phi_b = 0.90$):

$$M_n = M_p = F_y Z_x = (50 \text{ ksi})(54.0 \text{ in.}^3) = 2700 \text{ kip-in.}$$

Converting to kip-ft: $M_p = 2700/12 = 225.0$ kip-ft. Applying resistance factor $\phi_b = 0.90$:

$$\phi_b M_n = 0.90(225.0 \text{ kip-ft}) = 202.5 \text{ kip-ft}$$

Correct Answer: (B)

Problem 24: Concrete Beam Whitney Stress Block Depth

Topic 11.H: Concrete Design

A rectangular concrete beam ($f'_c = 4000$ psi, $\beta_1 = 0.85$) has width $b = 12$ in. and effective depth $d = 20$ in.. It is reinforced with 3 No. 8 bars ($A_s = 2.37 \text{ in.}^2$, $f_y = 60$ ksi). Calculate Whitney rectangular stress block depth a .

Options: (A) 2.85 in. (B) 3.49 in. (C) 4.11 in. (D) 5.20 in.

Solution

Equating concrete compression force C to steel tension force T :

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2.37 \text{ in.}^2)(60 \text{ ksi})}{0.85(4 \text{ ksi})(12 \text{ in.})} = \frac{142.2}{40.8} = 3.485 \text{ in.} \approx 3.49 \text{ in.}$$

Correct Answer: (B)

Problem 25: Concrete Beam Design Flexural Capacity

Topic 11.H: Concrete Design

For the beam in **Problem 24** ($a = 3.49$ in., $A_s = 2.37 \text{ in.}^2$, $f_y = 60$ ksi, $d = 20$ in.), verify $\epsilon_t \geq 0.005$ ($\phi = 0.90$) and calculate ϕM_n .

Options: (A) 194.7 kip-ft (B) 216.8 kip-ft (C) 240.9 kip-ft (D) 260.0 kip-ft

Solution

Neutral axis depth $c = a/\beta_1 = 3.485/0.85 = 4.10$ in. Tensile strain $\epsilon_t = \frac{d-c}{c}(0.003) = \frac{20-4.10}{4.10}(0.003) = \frac{15.90}{4.10}(0.003) = 0.0116 > 0.005 \implies \phi = 0.90$ (Tension-Controlled). Nominal flexural capacity M_n :

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = (2.37)(60) \left(20 - \frac{3.485}{2} \right) = 2596.2 \text{ kip}\cdot\text{in.}$$

Converting to kip-ft: $M_n = 2596.2/12 = 216.35$ kip-ft. Factored moment capacity: $\phi M_n = 0.90(216.35) = 194.7$ kip-ft.

Correct Answer: (A)

Problem 26: Concrete Beam Shear Reinforcement**Topic 11.H: Concrete Design**

A rectangular beam ($b_w = 14$ in., $d = 22$ in., $f'_c = 4000$ psi) carries factored shear load $V_u = 65$ kips. Using $\phi_v = 0.75$, calculate concrete shear capacity $\phi_v V_c$ and determine required nominal stirrup capacity V_s .

Options: (A) $\phi V_c = 29.2$ kips, $V_s = 47.7$ kips (B) $\phi V_c = 39.0$ kips, $V_s = 34.7$ kips
(C) $\phi V_c = 41.2$ kips, $V_s = 28.8$ kips (D) $\phi V_c = 52.0$ kips, $V_s = 17.3$ kips

Solution

Concrete shear contribution $V_c = 2\lambda \sqrt{f'_c} b_w d$:

$$V_c = 2(1.0)\sqrt{4000}(14 \text{ in.})(22 \text{ in.}) = 2(63.2455)(308) = 38,959 \text{ lbs} = 38.96 \text{ kips}$$

Factored concrete shear strength: $\phi_v V_c = 0.75(38.96 \text{ kips}) = 29.22 \text{ kips} \approx 29.2 \text{ kips}$. Since $V_u > \phi_v V_c$ ($65 > 29.22$), shear reinforcement is required:

$$V_s = \frac{V_u}{\phi_v} - V_c = \frac{65 \text{ kips}}{0.75} - 38.96 \text{ kips} = 86.67 - 38.96 = 47.71 \text{ kips} \approx 47.7 \text{ kips}$$

Correct Answer: (A)

Problem 27: Concentric Tied Concrete Column Capacity**Topic 11.H: Concrete Design**

A tied concrete column ($20 \text{ in.} \times 20 \text{ in.}$, $A_g = 400 \text{ in.}^2$) is reinforced with 8 No. 9 bars ($A_{st} = 8.0 \text{ in.}^2$). Given $f'_c = 5000$ psi and $f_y = 60$ ksi, calculate maximum design axial strength $\phi P_{n,\max}$ ($\phi = 0.65$).

Options: (A) 1000.5 kips (B) 1115.9 kips (C) 1220.8 kips (D) 1415.2 kips

Solution

Applying ACI maximum axial capacity formula for tied columns ($\phi = 0.65$):

$$P_o = 0.85f'_c(A_g - A_{st}) + f_y A_{st} = 0.85(5 \text{ ksi})(400 - 8) + (60 \text{ ksi})(8.0)$$

$$P_o = 4.25(392) + 480 = 1666.0 + 480 = 2146.0 \text{ kips}$$

Multiplying by ACI tied column reduction factors (0.80 cap and $\phi = 0.65$):

$$\phi P_{n,\max} = 0.80\phi P_o = 0.80(0.65)(2146.0 \text{ kips}) = 0.52 \times 2146.0 = 1115.92 \text{ kips} \approx 1115.9 \text{ kips}$$

Correct Answer: (B)