

Civil Engineering FE CBT Exam

Knowledge: Surveying with 9 Example Problems

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1 Angles, Distances, and Trigonometry

1.1 Angular Measurements & Conversions

Surveying measurements express angles in Degrees, Minutes, and Seconds (DMS), Decimal Degrees (DD), Radians, or Grads (Gons).

DMS to Decimal Degrees Conversion

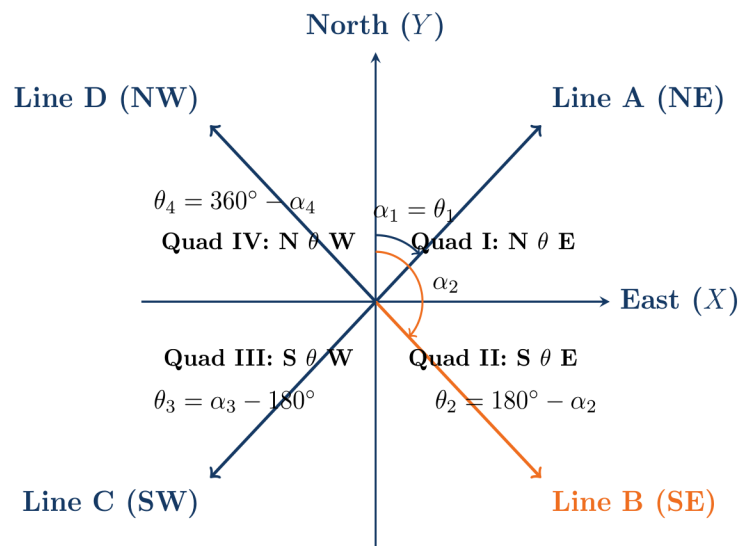
$$DD = \text{Degrees} + \frac{\text{Minutes}}{60} + \frac{\text{Seconds}}{3600}$$

$$\text{Degrees} = \lfloor DD \rfloor, \quad \text{Minutes} = \lfloor (DD - \text{Degrees}) \times 60 \rfloor, \quad \text{Seconds} = \lfloor ((DD - \text{Degrees}) \times 60) - \text{Minutes} \rfloor \times 60$$

1.2 Bearings and Azimuths

Direction in surveying is defined by Bearings or Azimuths relative to a reference meridian (True North, Magnetic North, or Assumed North).

- **Azimuth** (α): A continuous horizontal angle measured clockwise from North ($0^\circ \leq \alpha < 360^\circ$).
- **Bearing**: An acute horizontal angle measured from North or South toward East or West ($0^\circ \leq \theta \leq 90^\circ$). Formatted as **N/S θ E/W**.



Azimuth to Bearing Quadrant Formulas

Quadrant I (NE):	$0^\circ \leq \alpha < 90^\circ$	$\Rightarrow$ Bearing = N α E
Quadrant II (SE):	$90^\circ \leq \alpha < 180^\circ$	$\Rightarrow$ Bearing = S $(180^\circ - \alpha)$ E
Quadrant III (SW):	$180^\circ \leq \alpha < 270^\circ$	$\Rightarrow$ Bearing = S $(\alpha - 180^\circ)$ W
Quadrant IV (NW):	$270^\circ \leq \alpha < 360^\circ$	$\Rightarrow$ Bearing = N $(360^\circ - \alpha)$ W

1.3 Traverse Computations: Latitudes and Departures

A traverse consists of a series of connected lines (courses) with measured lengths (L) and directions (azimuth α or bearing θ).

Latitude and Departure Equations

$$\text{Latitude (Lat)} = \Delta Y = L \cos(\alpha) = \pm L \cos(\theta)$$

$$\text{Departure (Dep)} = \Delta X = L \sin(\alpha) = \pm L \sin(\theta)$$

where Latitude is North (+)/South (-) and Departure is East (+)/West (-).

1.3.1 Linear Closure Error and Compass (Bowditch) Rule

For a closed polygon traverse, the theoretical sum of Latitudes and Departures is zero ($\sum \text{Lat} = 0$, $\sum \text{Dep} = 0$). Field errors produce misclosures:

$$E_Y = \sum \text{Lat}, \quad E_X = \sum \text{Dep}$$

Traverse Error of Closure and Adjustment

$$\text{Linear Error of Closure (EC): } E_c = \sqrt{E_X^2 + E_Y^2}$$

$$\text{Relative Precision (Precision Ratio): } \text{RP} = \frac{E_c}{P} = \frac{1}{\frac{P}{E_c}} \quad (P = \text{Total Perimeter} = \sum L_i)$$

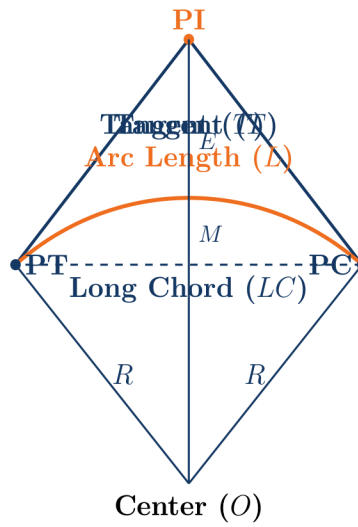
Compass Rule Corrections:

$$\text{Correction to Lat}_i = -\left(\frac{L_i}{P}\right) E_Y, \quad \text{Correction to Dep}_i = -\left(\frac{L_i}{P}\right) E_X$$

$$\text{Adjusted Lat}_i = \text{Lat}_i + \text{Correction to Lat}_i, \quad \text{Adjusted Dep}_i = \text{Dep}_i + \text{Correction to Dep}_i$$

1.4 Horizontal Circular Curves

Horizontal circular curves connect intersecting straight highway/railway tangents.



Horizontal Circular Curve Formulas (NCEES Handbook)

$$\begin{aligned}
 R &= \frac{5729.58}{D_a} \quad (\text{Arc definition}) & T &= R \tan \left(\frac{I}{2} \right) \\
 L &= RI \left(\frac{\pi}{180} \right) = 100 \frac{I}{D_a} & LC &= 2R \sin \left(\frac{I}{2} \right) \\
 E &= R \left[\sec \left(\frac{I}{2} \right) - 1 \right] = T \tan \left(\frac{I}{4} \right) & M &= R \left[1 - \cos \left(\frac{I}{2} \right) \right]
 \end{aligned}$$

Stationing Rule: Station **PT** = Station **PC** + **L** (Note: Station **PT** $\neq$ Station **PI** + **T**)

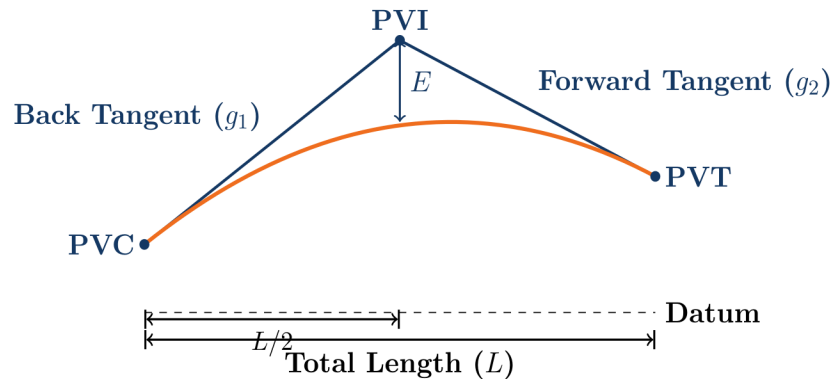
$$\text{Sub-chord Deflection Angle: } \delta = \frac{l}{L} \left(\frac{I}{2} \right) = \frac{l \cdot D_a}{200}$$

Where:

- R = Radius of curve, $I = \Delta$ = Intersection / Central Angle
- PC = Point of Curvature, PI = Point of Intersection, PT = Point of Tangency
 - D_a = degree of curve (100-ft arc definition); l = subarc length (chord-length use is approximate).
- T = Tangent distance, LC = Long chord length, E = External distance, M = Middle ordinate

1.5 Vertical Parabolic Curves

Vertical highway curves utilize equal-tangent parabolic curves to provide smooth transitions between grade lines g_1 and g_2 (expressed as decimals or percentages).



Vertical Parabolic Curve Formulas

Elevation of Point on Curve: $Y(x) = Y_{PVC} + g_1x + ax^2 = Y_{PVC} + g_1x + \left(\frac{g_2 - g_1}{2L}\right)x^2$

Parabola Constant: $a = \frac{g_2 - g_1}{2L}$, **Rate of Change of Grade:** $r = \frac{g_2 - g_1}{L}$

Tangent Offset at Any Distance x : $y(x) = ax^2 = \left(\frac{g_2 - g_1}{2L}\right)x^2$

Tangent Offset at PVI: $E_{signed} = (g_2 - g_1)L/8$ (decimal grades).
Magnitude: $|E| = AL/800$, where $A = |g_2 - g_1|$ in percent.

Location of Turning Point (High/Low Point): $x_m = -\frac{g_1}{2a} = \frac{g_1L}{g_1 - g_2}$

Rate of Vertical Curvature: $K = \frac{L}{A}$

2 Area Computations

Determining boundary areas is a fundamental surveying task performed using coordinate geometry or numerical approximation rules.

2.1 Method of Coordinates (Shoelace Formula)

When the coordinates (X_i, Y_i) or Latitudes/Departures of traverse vertices are known, the exact enclosed area is computed using Gauss's area formula (Shoelace formula).

Area by Coordinates Formula (NCEES Handbook)

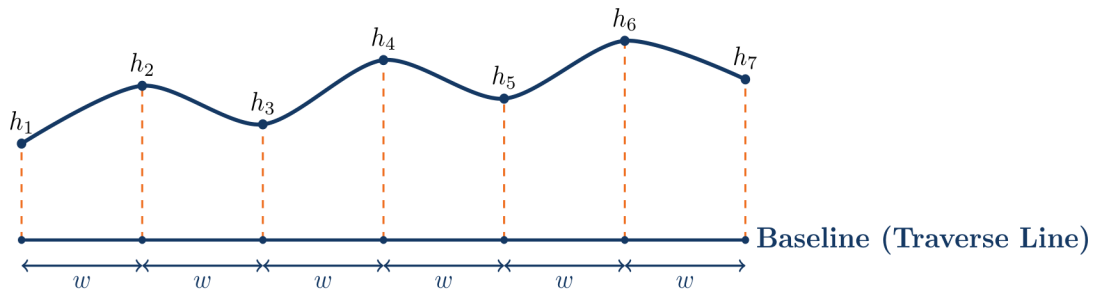
$$\text{Area} = \frac{1}{2} |X_A(Y_B - Y_N) + X_B(Y_C - Y_A) + X_C(Y_D - Y_B) + \cdots + X_N(Y_A - Y_{N-1})|$$

$$\text{Area} = \frac{1}{2} \sum_{i=1}^N (X_i Y_{i+1} - X_{i+1} Y_i) \quad (\text{with } X_{N+1} = X_1, Y_{N+1} = Y_1)$$

Note: X = Easting / Departure, Y = Northing / Latitude

2.2 Irregular Boundary Approximations

When property lines border natural features such as rivers, creeks, or curved roads, offsets ($h_1, h_2, \dots, h_n$) are measured at uniform intervals (w) along a straight baseline.



Trapezoidal Rule Formula

$$\text{Area} = w \left[\frac{h_1 + h_n}{2} + h_2 + h_3 + \dots + h_{n-1} \right] = w \left[\frac{h_1 + h_n}{2} + \sum_{i=2}^{n-1} h_i \right]$$

Assumes boundary segments between adjacent offsets are straight lines.

Simpson's 1/3 Rule Formula

$$\text{Area} = \frac{w}{3} \left[(h_1 + h_n) + 2 \sum h_{\text{odd}} + 4 \sum h_{\text{even}} \right]$$

$$\text{Area} = \frac{w}{3} [(h_1 + h_n) + 2(h_3 + h_5 + \dots + h_{n-2}) + 4(h_2 + h_4 + \dots + h_{n-1})]$$

MANDATORY CONSTRAINT: Requires an **odd** number of offset measurements (n), which corresponds to an **even** number of intervals ($n - 1$).

3 Earthwork and Volume Computations

Earthwork computations determine cut and fill volumes for highway corridors, site grading, trenches, and borrow pits.

3.1 Average End Area Formula

The Average End Area (AEA) method is the standard engineering approximation for route earthwork volumes between parallel cross sections spaced at distance L .

Average End Area Volume Formula

$$V = L \left(\frac{A_1 + A_2}{2} \right) \quad [\text{ft}^3 \text{ or } \text{m}^3]$$

$$\text{Volume in Cubic Yards (CY): } V_{\text{CY}} = \frac{L}{27} \left(\frac{A_1 + A_2}{2} \right) = \frac{L(A_1 + A_2)}{54}$$

where A_1 and A_2 are cross-sectional areas (ft^2) and L is section spacing (ft).

3.2 Prismoidal Formula

For a true prismoid, the prismoidal formula is exact; for general earthwork sections it is typically more accurate than the average-end-area method when cross-sectional area varies nonlinearly.

Prismoidal Volume Formula

$$V = L \left(\frac{A_1 + 4A_m + A_2}{6} \right) \quad [\text{ft}^3 \text{ or } \text{m}^3]$$

$$\text{Volume in Cubic Yards (CY): } V_{\text{CY}} = \frac{L(A_1 + 4A_m + A_2)}{162}$$

CRITICAL RULE: A_m is the area of the **mid-section** calculated using **mid-dimensions**,

$$\text{NOT the average of end areas } \left(A_m \neq \frac{A_1 + A_2}{2} \right)!$$

3.3 Borrow-Pit / Grid Method

Site excavation volumes for land leveling or borrow pits are calculated by dividing the layout into a grid of equal squares/rectangles of area $A_{\text{grid}} = b \times w$.

Borrow-Pit Grid Volume Formula

$$V = \frac{A_{\text{grid}}}{4} \left[1(\sum h_1) + 2(\sum h_2) + 3(\sum h_3) + 4(\sum h_4) \right]$$

where h_1, h_2, h_3, h_4 are cut/fill depths at grid corners participating in 1, 2, 3, or 4 grid squares.

$$\text{In Cubic Yards: } V_{\text{CY}} = \frac{A_{\text{grid}}}{4 \times 27} \left[\sum h_1 + 2 \sum h_2 + 3 \sum h_3 + 4 \sum h_4 \right]$$

3.4 Mass Haul Diagrams & Soil Volumetrics

A Mass Haul Diagram plots cumulative earthwork volume against stationing along a project corridor.

- **Rising Curve:** Indicates excess excavation (Cut).
- **Falling Curve:** Indicates excess embankment (Fill).
- **Peak / Valley:** Indicates points transitioning between cut and fill.
- **Balance Line:** Horizontal line intersecting the diagram; net volume between balance points is zero.

Soil Swell and Shrinkage Factors

$$\text{Swell Factor } (S_w) : S_w = \left(\frac{V_{\text{Loose}}}{V_{\text{Bank}}} - 1 \right) \times 100\%$$

$$\text{Shrinkage Factor } (S_h) : S_h = \left(1 - \frac{V_{\text{Compacted}}}{V_{\text{Bank}}} \right) \times 100\% = \left(\frac{\gamma_{d,\text{compacted}} - \gamma_{d,\text{bank}}}{\gamma_{d,\text{compacted}}} \right) \times 100\%$$

V_{Bank} = Undisturbed volume, V_{Loose} = Excavated volume, $V_{\text{Compacted}}$ = Final fill volume

4 Coordinate Systems

4.1 State Plane Coordinate (SPC) Systems

State Plane Coordinate Systems project geodetic latitude and longitude coordinates onto a flat planar surface (X = Easting, Y = Northing).

- **Lambert Conformal Conic Projection:** Utilizes a secant cone intersecting the ellipsoid at two standard parallels. Used primarily for states or zones with long **East-West** dimensions (e.g., Tennessee, Pennsylvania, Washington).
- **Transverse Mercator Projection:** Utilizes a secant cylinder intersecting the ellipsoid along two lines parallel to a central meridian. Used primarily for states or zones with long **North-South** dimensions (e.g., Illinois, Indiana, Florida).
- **Key Advantages:** Minimizes and controls projection distortion within each State Plane zone; widely used by state and local agencies and available in feet or meters.

4.2 Combined Scale Factor (Grid vs. Ground Distance)

Surveyed ground distances (D_{ground}) must be converted to map grid distances (D_{grid}) using scale factors.

Combined Scale Factor Equations

Combined Scale Factor (CSF) = Grid Scale Factor (k) $\times$ Elevation Scale Factor (S_e)

$$S_e = \frac{R}{R + h} = \frac{R}{R + H + N}$$

$$D_{\text{grid}} = D_{\text{ground}} \times \text{CSF} \quad D_{\text{ground}} = \frac{D_{\text{grid}}}{\text{CSF}}$$

where H is orthometric height (elevation), N is geoid height, and $h = H + N$ is ellipsoidal height.

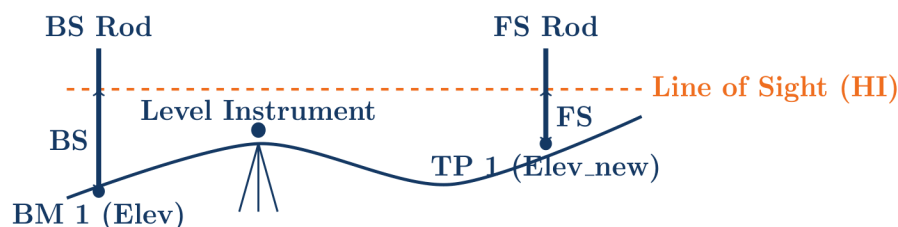
Use $R \approx 20,906,000 \text{ ft} \approx 6,371,000 \text{ m}$ for the mean Earth radius approximation.

5 Leveling & Field Methods

Leveling determines elevations of points relative to a vertical datum (e.g., NAVD88 or Mean Sea Level).

5.1 Differential Leveling Principles & Field Notes

Differential leveling utilizes a leveling instrument and a graduated rod to transfer elevation from a Benchmark (BM) to unknown points via Turning Points (TP).



Differential Leveling Equations

Height of Instrument (HI): $HI = \text{Elevation}_{\text{known}} + BS$

New Elevation: $\text{Elevation}_{\text{new}} = HI - FS$

Page Check (Arithmetic Verification): $\text{Initial Elev} + \sum BS - \sum FS = \text{Final Elev}$

$$\Rightarrow \sum BS - \sum FS = \text{Final Elev} - \text{Initial Elev}$$

5.2 Trigonometric Leveling with Total Station

Trigonometric leveling determines elevation differences using measured slope distance (SD) and zenith angle (ZA) or vertical angle (α).

Trigonometric Leveling Equations

Horizontal Distance (HD): $HD = SD \sin(ZA) = SD \cos(\alpha)$

Vertical Distance (VD): $VD = SD \cos(ZA) = SD \sin(\alpha)$

Target Elevation: $\text{Elev}_B = \text{Elev}_A + h_i + VD - h_r$

where h_i = Height of Instrument, h_r = Height of Prism Pole/Target.

5.3 Curvature and Refraction Corrections

Curvature and atmospheric refraction can affect long sight readings; significance depends on required precision.

Combined Curvature and Refraction Correction

$$C_{c+r} = 0.0206M^2 \quad [\text{ft, with } M \text{ in thousands of feet}]$$

$$C_{c+r} = 0.0675K^2 \quad [\text{m, with } K \text{ in kilometers}]$$

$$\text{Corrected Rod Reading} = \text{Measured Reading} - C_{c+r}$$

5.4 Systematic Errors in Taping and Total Station Distances

Surveying measurements must be corrected for systematic environmental and mechanical errors:

- **Tape Thermal Expansion:** $C_T = k(T - T_0)L$, where $k = 0.0000065/^\circ\text{F}$ for steel tapes, $T_0 = 68^\circ\text{F}$ standard temperature.
- **Tape Tension Correction:** $C_P = \frac{(P - P_0)L}{AE}$, where P_0 is standard tension, A is tape cross-sectional area, E is Young's Modulus.
- **Atmospheric PPM Correction (EDM/Total Station):**
Use the manufacturer's EDM atmospheric model, with its reference pressure, temperature, wavelength, and humidity assumptions.

5.5 Construction Staking and Slope Stakes

Slope stakes mark the intersection of a cut or fill slope with the natural ground surface (known as the *catch point*).

Slope Stake Catch Point Equation

$$\text{Distance from Centerline } (d) = \frac{W}{2} + s \times h$$

where W is roadbed width, s is side-slope ratio (e.g., $s = 2$ for a 2:1 slope), and h is cut/fill depth.
This simplified relation assumes level transverse ground and a uniform side slope.

6 Key Formula Quick Reference Summary

Table 1: NCEES FE Civil Surveying Formulas Summary

Topic / Concept	NCEES FE Handbook Formula
DMS to Decimal Degrees	$DD = \text{Deg} + \frac{\text{Min}}{60} + \frac{\text{Sec}}{3600}$
Latitude & Departure	$\text{Lat} = L \cos(\alpha), \quad \text{Dep} = L \sin(\alpha)$
Closure Error & Compass Rule	$E_c = \sqrt{E_X^2 + E_Y^2}, \quad \text{Corr}_{\text{Lat}} = -\left(\frac{L_i}{P}\right) E_Y, \quad \text{Corr}_{\text{Dep}} = -\left(\frac{L_i}{P}\right) E_X$
Horizontal Curve Radius	$R = \frac{5729.58}{D_a}$
Horizontal Curve Tangent	$T = R \tan(I/2)$
Horizontal Curve Length	$L = RI(\pi/180) = 100(I/D_a)$
Horizontal Curve Chord	$LC = 2R \sin(I/2)$
Vertical Curve Elevation	$Y(x) = Y_{\text{PVC}} + g_1 x + \left(\frac{g_2 - g_1}{2L}\right) x^2$
Vertical Curve High/Low Point	$x_m = \frac{-g_1 L}{g_2 - g_1}$
Shoelace Area Formula	$\text{Area} = \frac{1}{2} \left \sum X_i (Y_{i+1} - Y_{i-1}) \right $
Trapezoidal Area Rule	$\text{Area} = w \left[\frac{h_1 + h_n}{2} + h_2 + h_3 + \cdots + h_{n-1} \right]$
Simpson's 1/3 Area Rule	$\text{Area} = \frac{w}{3} \left[(h_1 + h_n) + 2 \sum h_{\text{odd}} + 4 \sum h_{\text{even}} \right]$
Average End Area Volume	$V_{\text{CY}} = \frac{L(A_1 + A_2)}{54}$
Prismoidal Volume	$V_{\text{CY}} = \frac{L(A_1 + 4A_m + A_2)}{162}$
Borrow-Pit Grid Volume	$V_{\text{CY}} = \frac{A_{\text{grid}}}{108} \left[\sum h_1 + 2 \sum h_2 + 3 \sum h_3 + 4 \sum h_4 \right]$
Differential Leveling	$\text{HI} = \text{Elev} + \text{BS}, \quad \text{Elev}_{\text{new}} = \text{HI} - \text{FS}$
Trigonometric Leveling	$\text{Elev}_B = \text{Elev}_A + h_i + SD \cos(ZA) - h_r$

7 Example Problems and Solutions

Problem 1: Angle Conversion to Decimal Degrees

Example Conversion: An angle of $114^{\circ}15'24''$ converts to decimal degrees as:

$$DD = 114 + \frac{15}{60} + \frac{24}{3600} = 114 + 0.25 + 0.006667 = 114.256667 \approx 114.2567^{\circ}$$

The following 8 exam-style practice problems represent the quantitative difficulty and topics encountered on the NCEES FE Civil CBT examination.

Problem 2: Angular Conversion & Bearing Analysis

Problem Statement:

A survey boundary line has an azimuth of $244^{\circ}18'36''$ measured clockwise from North. What is the equivalent quadrant bearing of this course?

Options:

- (A) N $64^{\circ}18'36''$ W
- (B) S $64^{\circ}18'36''$ W
- (C) S $24^{\circ}18'36''$ W
- (D) N $24^{\circ}18'36''$ E

Solution

1. Convert azimuth to decimal degrees:

$$\alpha = 244 + \frac{18}{60} + \frac{36}{3600} = 244 + 0.30 + 0.01 = 244.31^{\circ}$$

2. Determine quadrant for azimuth 244.31° : Since $180^{\circ} \leq 244.31^{\circ} < 270^{\circ}$, the line lies in **Quadrant III (South-West)**.
3. Apply Quadrant III formula from NCEES Reference Handbook:

$$\theta = \alpha - 180^{\circ} = 244^{\circ}18'36'' - 180^{\circ}00'00'' = 64^{\circ}18'36''$$

4. Therefore, the quadrant bearing is **S $64^{\circ}18'36''$ W**.

Correct Answer: (B)

Problem 3: Traverse Balancing via Compass Rule**Problem Statement:**

A closed 3-course field survey traverse ABC yields the following unadjusted course lengths, latitudes, and departures:

Course	Length (ft)	Latitude (ft)	Departure (ft)	Northing Y (ft)	Easting X (ft)
AB	95.49	-82.62	-47.88	1000.00	1000.00
BC	88.96	-32.17	+82.94	—	—
CA	120.26	+115.12	-34.79	—	—

Using the **Compass (Bowditch) Rule**, the adjusted coordinates of Point B are most nearly:

Options:

- (A) N 917.38 ft, E 952.12 ft
- (B) N 917.28 ft, E 952.04 ft
- (C) N 917.49 ft, E 952.21 ft
- (D) N 917.15 ft, E 951.95 ft

Solution

1. Calculate Total Perimeter (P):

$$P = 95.49 + 88.96 + 120.26 = 304.71 \text{ ft}$$

2. Calculate Misclosure Errors (E_Y and E_X):

$$E_Y = \sum \text{Lat} = -82.62 - 32.17 + 115.12 = +0.33 \text{ ft}$$

$$E_X = \sum \text{Dep} = -47.88 + 82.94 - 34.79 = +0.27 \text{ ft}$$

3. Compute Compass Rule Corrections for Course AB ($L_{AB} = 95.49 \text{ ft}$):

$$\text{Correction to Lat}_{AB} = -\left(\frac{L_{AB}}{P}\right) E_Y = -\left(\frac{95.49}{304.71}\right) (+0.33) = -0.1034 \text{ ft}$$

$$\text{Correction to Dep}_{AB} = -\left(\frac{L_{AB}}{P}\right) E_X = -\left(\frac{95.49}{304.71}\right) (+0.27) = -0.0846 \text{ ft}$$

4. Calculate Adjusted Latitude and Departure for Course AB :

$$\text{Adj Lat}_{AB} = -82.62 + (-0.1034) = -82.7234 \text{ ft}$$

$$\text{Adj Dep}_{AB} = -47.88 + (-0.0846) = -47.9646 \text{ ft}$$

5. Calculate Adjusted Coordinates for Point B :

$$\text{Northing}_B = Y_A + \text{Adj Lat}_{AB} = 1000.00 - 82.7234 = \mathbf{917.2766 \text{ ft} \approx 917.28 \text{ ft}}$$

$$\text{Easting}_B = X_A + \text{Adj Dep}_{AB} = 1000.00 - 47.9646 = \mathbf{952.0354 \text{ ft} \approx 952.04 \text{ ft}}$$

Correct Answer: (B)

Problem 4: Horizontal Curve Parameter Computation**Problem Statement:**

A horizontal highway circular curve is designed with a radius $R = 500$ ft and a deflection angle $I = 40^\circ$. The Point of Curvature (PC) is located at station $10 + 00.00$. Calculate the length of curve (L), long chord (LC), and the stationing of the Point of Tangency (PT).

Options:

- (A) $PT = 13 + 49.07$, $LC = 342.02$ ft
- (B) $PT = 13 + 42.02$, $LC = 349.07$ ft
- (C) $PT = 13 + 49.07$, $LC = 349.07$ ft
- (D) $PT = 13 + 42.02$, $LC = 342.02$ ft

Solution**1. Calculate Curve Length (L):**

$$L = R \cdot I \cdot \left(\frac{\pi}{180} \right) = 500 \cdot 40 \cdot \left(\frac{\pi}{180} \right) = \frac{20000\pi}{180} = 349.0658 \text{ ft} \approx 349.07 \text{ ft}$$

2. Calculate Long Chord (LC):

$$LC = 2R \sin \left(\frac{I}{2} \right) = 2(500) \sin (20^\circ) = 1000(0.342020) = 342.0201 \text{ ft} \approx 342.02 \text{ ft}$$

3. Calculate PT Stationing:

$$\text{Station PT} = \text{Station PC} + L = (10 + 00.00) + 349.07 \text{ ft} = \mathbf{13 + 49.07}$$

Correct Answer: (A)

Problem 5: Vertical Parabolic Curve High Point**Problem Statement:**

A symmetrical vertical crest curve of length $L = 600$ ft connects an initial grade $g_1 = +3.0\%$ to a final grade $g_2 = -2.0\%$. The Point of Vertical Curvature (PVC) is located at Station $20 + 00.00$ with an elevation of 542.50 ft. The stationing and elevation of the high point on the curve are most nearly:

Options:

- (A) Station $23 + 60.00$, Elevation 547.90 ft
- (B) Station $23 + 60.00$, Elevation 542.50 ft
- (C) Station $22 + 40.00$, Elevation 547.90 ft
- (D) Station $23 + 00.00$, Elevation 546.10 ft

Solution

1. Calculate Distance x_m to High Point:

$$x_m = \frac{g_1 L}{g_1 - g_2} = \frac{(+0.03)(600)}{+0.03 - (-0.02)} = \frac{18}{0.05} = 360.00 \text{ ft from PVC}$$

2. Determine Stationing of High Point:

$$\text{High Point Station} = \text{Station PVC} + x_m = (20 + 00.00) + 360.00 \text{ ft} = \mathbf{23 + 60.00}$$

3. Calculate Elevation at High Point:

$$a = \frac{g_2 - g_1}{2L} = \frac{-0.02 - 0.03}{2(600)} = \frac{-0.05}{1200} = -0.000041667 \text{ ft}^{-1}$$

$$Y(x_m) = Y_{\text{PVC}} + g_1 x_m + a x_m^2$$

$$Y(360) = 542.50 + (0.03)(360) + (-0.000041667)(360)^2$$

$$Y(360) = 542.50 + 10.80 - 5.40 = \mathbf{547.90 \text{ ft}}$$

Correct Answer: (A)

Problem 6: Polygon Area by Shoelace Coordinates**Problem Statement:**

A land parcel $ABCD$ has surveyed coordinates (Easting X , Northing Y) in feet as follows: $A(700, 300)$, $B(700, 100)$, $C(1000, 200)$, $D(900, 450)$. The enclosed area in acres is most nearly (1 acre = 43,560 ft²):

Options:

- (A) 1.34 acres
- (B) 1.43 acres
- (C) 1.47 acres
- (D) 1.52 acres

Solution**1. Apply Shoelace Coordinate Formula:**

$$\text{Area} = \frac{1}{2} |X_A(Y_B - Y_D) + X_B(Y_C - Y_A) + X_C(Y_D - Y_B) + X_D(Y_A - Y_C)|$$

2. Substitute Coordinates:

$$\begin{aligned} X_A(Y_B - Y_D) &= 700(100 - 450) = 700(-350) = -245,000 \\ X_B(Y_C - Y_A) &= 700(200 - 300) = 700(-100) = -70,000 \\ X_C(Y_D - Y_B) &= 1000(450 - 100) = 1000(350) = +350,000 \\ X_D(Y_A - Y_C) &= 900(300 - 200) = 900(100) = +90,000 \end{aligned}$$

3. Sum Cross Products and Calculate Area:

$$\Sigma = -245,000 - 70,000 + 350,000 + 90,000 = +125,000$$

$$\text{Area} = \frac{1}{2} |125,000| = 62,500 \text{ ft}^2$$

4. Convert Square Feet to Acres:

$$\text{Area in Acres} = \frac{62,500 \text{ ft}^2}{43,560 \text{ ft}^2/\text{acre}} = \mathbf{1.4348} \text{ acres} \approx \mathbf{1.43} \text{ acres}$$

Correct Answer: (B)

Problem 7: Simpson's 1/3 Rule vs Trapezoidal Rule**Problem Statement:**

Boundary offsets measured from a straight baseline at a uniform spacing of $w = 25$ ft are recorded as: $h_1 = 134$ ft, $h_2 = 137$ ft, $h_3 = 145$ ft, $h_4 = 147$ ft, $h_5 = 150$ ft, $h_6 = 146$ ft, $h_7 = 131$ ft. The estimated area bounded by the irregular shoreline using **Simpson's 1/3 Rule** is most nearly:

Options:

- (A) 21,090 ft²
 (B) 21,110 ft²
 (C) 21,460 ft²
 (D) 21,220 ft²

Solution

1. **Verify Simpson's Rule Applicability:** Number of offsets $n = 7$ (odd number). Number of intervals $n - 1 = 6$ (even number). Simpson's 1/3 rule applies.
2. **Identify Offset Categories:**

$$\text{First and Last: } h_1 = 134, \quad h_7 = 131 \implies h_1 + h_7 = 265$$

$$\text{Even Offsets } (h_2, h_4, h_6) : 137 + 147 + 146 = 430$$

$$\text{Odd Offsets } (h_3, h_5) : 145 + 150 = 295$$

3. **Apply Simpson's 1/3 Rule Formula:**

$$\text{Area} = \frac{w}{3} \left[(h_1 + h_7) + 4 \sum h_{\text{even}} + 2 \sum h_{\text{odd}} \right]$$

$$\text{Area} = \frac{25}{3} [265 + 4(430) + 2(295)] = \frac{25}{3} [265 + 1720 + 590]$$

$$\text{Area} = \frac{25}{3} [2575] = 21,458.33 \text{ ft}^2$$

Note: Comparing with Trapezoidal Rule:

$$\text{Area}_{\text{Trap}} = 25 \left[\frac{134 + 131}{2} + 137 + 145 + 147 + 150 + 146 \right] = 25[132.5 + 725] = 21,437.5 \text{ ft}^2$$

$$\text{Area} \approx 21,460 \text{ ft}^2$$

Correct Answer: (C)

Problem 8: Earthwork Cut Volume Comparison**Problem Statement:**

A 200-ft long swale excavation ($L = 200$ ft) increases uniformly in size from Station 10 + 00 to Station 12 + 00. Station cross-sectional areas are:

- Station 10 + 00: $A_1 = 2.0 \text{ ft}^2$
- Station 11 + 00 (mid-section computed from mid-dimensions): $A_m = 4.375 \text{ ft}^2$
- Station 12 + 00: $A_2 = 7.5 \text{ ft}^2$

Using the **Prismoidal Formula**, the total required volume in cubic yards (CY) is most nearly:

Options:

- (A) 33 CY
- (B) 34 CY
- (C) 35 CY
- (D) 36 CY

Solution**1. Apply Prismoidal Volume Formula:**

$$V_{\text{ft}^3} = L \left(\frac{A_1 + 4A_m + A_2}{6} \right)$$

2. Substitute Given Values ($L = 200$ ft, $A_1 = 2.0$, $A_m = 4.375$, $A_2 = 7.5$):

$$V_{\text{ft}^3} = 200 \left(\frac{2.0 + 4(4.375) + 7.5}{6} \right) = 200 \left(\frac{2.0 + 17.5 + 7.5}{6} \right)$$

$$V_{\text{ft}^3} = 200 \left(\frac{27.0}{6} \right) = 200(4.5) = 900.0 \text{ ft}^3$$

3. Convert Volume to Cubic Yards (CY):

$$V_{\text{CY}} = \frac{900.0 \text{ ft}^3}{27 \text{ ft}^3/\text{CY}} = \mathbf{33.33 \text{ CY} \approx 33 \text{ CY}}$$

Comparison with Average End Area:

$$V_{\text{AEA}} = \frac{200}{27} \left(\frac{2.0 + 7.5}{2} \right) = \frac{200(4.75)}{27} = 35.18 \text{ CY}$$

The Prismoidal formula accounts for non-linear volume variation and yields 33 CY.

Correct Answer: (A)

Problem 9: Trigonometric Leveling with Total Station**Problem Statement:**

A total station is set up over Benchmark BM1 (Elevation = 518.30 ft) with an instrument height $h_i = 4.75$ ft. A rod sighting to Point A records a slope distance $SD = 61.25$ ft, a zenith angle $ZA = 88^\circ 17' 00''$, and a reflector target height $h_r = 4.75$ ft. The elevation at Point A is most nearly:

Options:

- (A) 513.64 ft
- (B) 517.31 ft
- (C) 520.13 ft
- (D) 522.96 ft

Solution**1. Convert Zenith Angle to Decimal Degrees:**

$$ZA = 88^\circ 17' = 88 + \frac{17}{60} = 88.28333^\circ$$

2. Calculate Vertical Distance (VD):

$$VD = SD \cos(ZA) = 61.25 \cos(88.28333^\circ) = 61.25(0.029957) = +1.8349 \text{ ft}$$

3. Calculate Elevation at Point A:

$$\text{Elev}_A = \text{Elev}_{\text{BM1}} + h_i + VD - h_r$$

Since $h_i = h_r = 4.75$ ft, the target height cancels out:

$$\text{Elev}_A = 518.30 + 4.75 + 1.8349 - 4.75 = 518.30 + 1.8349 = 520.13 \text{ ft}$$

Correct Answer: (C)