

Civil Engineering FE CBT Exam

Knowledge: Water Resources & Environmental Engineering with 20 Example Problems

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1 Basic Hydrology & Watershed Analysis

1.1 The Hydrologic Budget Equation

Use a whole-watershed control volume including soil and groundwater. With no other external inflow, the water balance over the chosen interval is:

$$P - R - ET - G = \Delta S \quad (1.1)$$

P : precipitation; R : surface runoff; ET : evapotranspiration; G : net groundwater outflow; ΔS : total storage change. Infiltration is internal; annual ΔS need not be zero.

1.2 The Rational Method

The Rational Method estimates the peak surface runoff rate (Q) from small urban or agricultural watersheds ($A < 200$ acres or < 80 ha) assuming uniform rainfall intensity over the entire watershed duration equal to the time of concentration (t_c).

Rational Formula (USCS & SI)

US Customary System (USCS):

$$Q = C \cdot I \cdot A \quad (1.2)$$

where:

- Q = peak runoff rate in cubic feet per second (cfs)
- C = dimensionless runoff coefficient (0.05 to 0.95)
- I = rainfall intensity in inches per hour (in/hr)
- A = watershed drainage area in acres

Note: 1 acre · in/hr $\approx$ 1.0083 cfs $\approx$ 1.0 cfs (no empirical conversion needed).

SI Metric System:

$$Q = 0.00278 C I A \quad \text{or} \quad Q = \frac{C I A}{360} \quad (1.3)$$

where Q is in m^3/s , I is in mm/hr , and A is in hectares (ha).

1.2.1 Composite Runoff Coefficient ($C_{\text{composite}}$)

When a watershed contains sub-areas with different land uses, a weighted average coefficient is used:

$$C_{\text{composite}} = \frac{\sum (C_i A_i)}{\sum A_i} = \frac{C_1 A_1 + C_2 A_2 + \cdots + C_n A_n}{A_{\text{total}}} \quad (1.4)$$

Table 1.1: Typical Runoff Coefficients (C) for the FE Exam

Land Use / Surface Type	Runoff Coefficient (C)
Asphalt / Concrete Pavement	0.85–0.95
Roofs / Impervious Areas	0.75–0.95
Commercial Business Areas	0.70–0.95
Residential (Single Family)	0.30–0.50
Lawns / Heavy Soil (Flat $< 2\%$)	0.13–0.17
Lawns / Heavy Soil (Steep $> 7\%$)	0.25–0.35
Forests / Woodlands	0.10–0.20

1.3 Time of Concentration (t_c)

The time of concentration (t_c) is the time required for water to travel from the most hydrologically distant point in a watershed to the outlet. When a watershed drains through multiple flow paths or catchments in series/parallel:

- **Sheet Flow:** $t_{sheet}[h] = 0.007(nL)^{0.8}/(P_2^{0.5}S^{0.4})$. Use L in ft, P_2 in inches (2-year, 24-hour rain), S in ft/ft; limit sheet flow to 100 ft and multiply hours by 60 for minutes.
- **Shallow Concentrated Flow:** $v = 16.1345\sqrt{S}$ (unpaved) or $v = 20.3282\sqrt{S}$ (paved).
- **Pipe Flow / Channel Flow:** $t_{pipe} = \frac{L}{v}$, where L is sewer length (ft) and v is flow velocity (ft/s). Convert seconds to minutes ($\div 60$).
- **Controlling t_c :** For catchments discharging to a common point, the longest cumulative travel time governs the overall site t_c , which is then matched to the IDF curve to select design intensity I .

1.4 NRCS / SCS Rainfall-Runoff Method

The Natural Resources Conservation Service (NRCS), formerly Soil Conservation Service (SCS), method calculates total runoff depth Q (in inches) from total storm rainfall P (in inches) based on a land-use Curve Number (CN).

NRCS (SCS) Curve Number Equations

Max Basin Retention (S , inches):

$$S = \frac{1000}{CN} - 10 \quad (1.5)$$

Direct Runoff Depth (Q , inches):

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} = \frac{(P - 0.2S)^2}{P + 0.8S} \quad (\text{valid for } P > 0.2S) \quad (1.6)$$

where:

- P = total precipitation (inches)
- S = potential maximum retention after runoff begins (inches)
- CN = SCS Curve Number ($0 < CN \leq 100$)
- I_a = initial abstraction $\approx 0.2S$

1.4.1 Composite Curve Number ($CN_{\text{composite}}$)

Like runoff coefficients, heterogeneous soils and land uses require area-weighting:

$$CN_{\text{composite}} = \frac{\sum(CN_i A_i)}{\sum A_i} \quad (1.7)$$

1.4.2 Initial Abstraction (I_a)

Initial abstraction (I_a) includes all losses prior to runoff:

- Surface depression storage / retention
- Interception by vegetation
- Infiltration before surface saturation

FE Exam Trap: Initial abstraction does *not* include post-saturation direct runoff or channel transmission losses.

2 Basic Hydraulics & Open-Channel Flow

2.1 Manning's Equation

Manning's equation governs uniform, steady open-channel flow in gravity-driven channels, pipes flowing under gravity (sanitary/storm sewers), and natural streams.

Manning's Equation

Volumetric Flow Rate (Q):

$$Q = \frac{K}{n} A R_H^{2/3} S^{1/2} \quad (2.1)$$

Flow Velocity (v):

$$v = \frac{K}{n} R_H^{2/3} S^{1/2} \quad (2.2)$$

where:

- $K = 1.486$ for US Customary Units (ft, ft/s, cfs)
- $K = 1.000$ for SI Metric Units (m, m/s, m³/s)
- n = Manning's roughness coefficient (dimensionless)
- A = cross-sectional flow area (ft² or m²)
- P = wetted perimeter (ft or m)
- $R_H = A/P$ = hydraulic radius (ft or m)
- S = channel longitudinal bed slope (ft/ft or m/m)

2.2 Geometric Properties of Open Channels

Key Insight: For both a full circular pipe and a half-full circular pipe, the hydraulic radius is identical: $R_H = D/4$.

Table 2.1: Geometric Elements for Open Channels

Section Type	Area (A)	Wetted Perimeter (P)	Hydraulic Radius (R_H)
Rectangular (width b , depth y)	by	$b + 2y$	$\frac{by}{b + 2y}$
Trapezoidal (bottom b , depth y , side slope $z : 1$)	$(b + zy)y$	$b + 2y\sqrt{1 + z^2}$	$\frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}}$
Triangular (side slope $z : 1$, depth y)	zy^2	$2y\sqrt{1 + z^2}$	$\frac{zy^2}{2y\sqrt{1 + z^2}}$
Circular (Full) (diameter D)	$\frac{\pi D^2}{4}$	πD	$\frac{D}{4}$
Circular (Half-Full) (diameter D)	$\frac{\pi D^2}{8}$	$\frac{\pi D}{2}$	$\frac{D}{4}$

2.3 Specific Energy and Critical Flow

Specific energy (E) is the total energy head referenced to the channel bed:

$$E = y + \alpha \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2} \quad (2.3)$$

where kinetic energy correction factor $\alpha \approx 1.0$.

2.3.1 Critical Depth (y_c)

Critical flow occurs at minimum specific energy for a given discharge Q . The general condition for critical depth in any channel shape is:

$$\frac{Q^2}{g} = \frac{A^3}{T} \quad (2.4)$$

where T is the top width of the water surface.

Critical Depth in Rectangular Channels

For a rectangular channel of width b , defining unit discharge $q = Q/b$ (cfs/ft or $\text{m}^3/\text{s}/\text{m}$):

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{Q^2}{gb^2} \right)^{1/3} \quad (2.5)$$

Minimum Specific Energy at Critical Depth:

$$E_{\min} = \frac{3}{2}y_c = 1.5y_c \quad (2.6)$$

2.4 Froude Number and Flow Regimes

The Froude number (Fr) represents the ratio of inertial forces to gravitational forces:

$$Fr = \frac{v}{\sqrt{gy_h}} = \sqrt{\frac{Q^2 T}{gA^3}} \quad (2.7)$$

where hydraulic depth $y_h = A/T$. For a rectangular channel, $y_h = y$, so $Fr = v/\sqrt{gy}$.

- **Subcritical Flow** ($Fr < 1$): Flow depth $y > y_c$, low velocity, tranquil flow. Downstream conditions control flow depth.
- **Critical Flow** ($Fr = 1$): Flow depth $y = y_c$, minimum energy head.
- **Supercritical Flow** ($Fr > 1$): Flow depth $y < y_c$, high velocity, rapid flow. Upstream conditions control flow depth.

2.5 Hydraulic Jump

A hydraulic jump is an abrupt transition from supercritical flow ($Fr_1 > 1$, depth y_1) to subcritical flow ($Fr_2 < 1$, depth y_2) accompanied by significant energy dissipation.

Sequent (Conjugate) Depths in Rectangular Channels

$$y_2 = \frac{y_1}{2} \left(-1 + \sqrt{1 + 8Fr_1^2} \right) \quad (2.8)$$

Head Loss Across the Hydraulic Jump (ΔE):

$$\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^3}{4y_1 y_2} \quad (2.9)$$

3 Pumps & Hydraulic Machinery

3.1 Energy Equation with Pumps

Applying the steady, incompressible energy equation between upstream point 1 and downstream point 2:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_p = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_m \quad (3.1)$$

where h_p is total pump dynamic head added (ft or m), h_f is pipe friction head loss, and h_m represents minor losses.

3.2 Pump Horsepower Equations

Pump Power Equations (USCS & SI)

Water Horsepower (WHP, Hydraulic Power Output):

$$WHP = \frac{\gamma Q H}{550} \quad (3.2)$$

where $\gamma = 62.4 \text{ lb/ft}^3$, Q is in cfs, H is total dynamic head in feet. If Q is in gallons per minute (gpm):

$$WHP = \frac{Q(\text{gpm}) H(\text{ft}) SG}{3960} \quad (3.3)$$

Brake Horsepower (BHP, Shaft Power Input):

$$BHP = \frac{WHP}{\eta_{\text{pump}}} = \frac{\gamma Q H}{550 \eta_{\text{pump}}} = \frac{Q(\text{gpm}) H(\text{ft}) SG}{3960 \eta_{\text{pump}}} \quad (3.4)$$

Purchased / Electrical Motor Power ($W_{\text{purchased}}$):

$$W_{\text{purchased}} = \frac{BHP}{\eta_{\text{motor}}} = \frac{\gamma Q H}{550 \eta_{\text{pump}} \eta_{\text{motor}}} \quad (3.5)$$

SI Metric Units (Power in kW):

$$P_{\text{hydraulic}}(\text{kW}) = \frac{\rho g Q H}{1000} = \gamma Q H \quad (3.6)$$

$$P_{\text{brake}}(\text{kW}) = \frac{\gamma Q H}{\eta_{\text{pump}}} \quad (3.7)$$

3.3 Net Positive Suction Head (NPSH) & Cavitation

Cavitation occurs when local pressure inside the pump drops below liquid vapor pressure (P_{vp}), forming vapor bubbles that collapse destructively against the impeller.

NPSH Available (NPSHA)

$$NPSH_A = H_{pa} + H_s - \sum h_{L,s} - H_{vp} \quad (3.8)$$

or using absolute pressure/velocity at pump inlet:

$$NPSH_A = \frac{P_{\text{inlet}}}{\gamma} + \frac{v_{\text{inlet}}^2}{2g} - \frac{P_{\text{vapor}}}{\gamma} \quad (3.9)$$

where:

- $H_{pa} = P_{atm}/\gamma$ = atmospheric pressure head (ft or m, 34 ft for standard air)
- H_s = static suction head (positive if pump is below liquid level; negative if suction lift)
- $\sum h_{L,s}$ = total head losses in the suction piping (feet or meters)
- $H_{vp} = P_{vp}/\gamma$ = vapor pressure head at liquid operating temperature (feet or meters)

Cavitation Prevention Criterion: $NPSH_A \geq NPSH_R$ (Required NPSH from manufacturer curve).

4 Water Distribution Systems & Fire Flow

4.1 Pipe Friction Head Loss Equations

4.1.1 Hazen-Williams Equation

The Hazen-Williams equation is empirically calibrated for water flow in pressurized pipes at ambient temperatures:

Hazen-Williams Formulations

Flow Velocity (v , USCS):

$$v = 1.318 C R_H^{0.63} S^{0.54} \quad (\text{ft/s}) \quad (4.1)$$

Circular Pipe Friction Head Loss (h_f , USCS):

$$h_f = \frac{4.73 L}{C^{1.852} D^{4.87}} Q^{1.852} \quad (h_f, L, D \text{ in ft, } Q \text{ in cfs}) \quad (4.2)$$

Pressure Loss per foot (P_{loss} , psi/ft):

$$P_{loss} = \frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}} \quad (Q \text{ in gpm, } D \text{ in inches}) \quad (4.3)$$

where C is Hazen-Williams roughness factor (e.g., $C = 150$ for PVC/Plastic, $C = 140$ for Ductile Iron, $C = 130$ for Concrete, $C = 100$ for 20-yr old Cast Iron).

4.1.2 Darcy-Weisbach Equation

$$h_f = f \frac{L}{D} \frac{v^2}{2g} = \frac{8fLQ^2}{\pi^2 g D^5} \quad (4.4)$$

where f is Darcy friction factor obtained from the Moody diagram or Swamee-Jain equation based on Reynolds number ($Re = vD/\nu$) and relative roughness (ϵ/D).

4.2 Fire Flow Hydrant Testing

Fire flow testing measures available supply rate Q_R at a specified residual pressure (typically 20 psi minimum):

Fire Hydrant Flow Equations

Step 1: Hydrant Discharge during Test (Q_F , gpm):

$$Q_F = 29.83 C_d d^2 \sqrt{P_{\text{pitot}}} \quad (4.5)$$

where d is orifice diameter (inches), P_{pitot} is pitot pressure gauge reading (psi), C_d is outlet coefficient ($C_d = 0.90$ for smooth rounded, 0.80 for square sharp edge, 0.70 for square projecting into barrel).

Step 2: Rated Capacity at 20 psi (Q_R , gpm):

$$Q_R = Q_F \left(\frac{H_R}{H_F} \right)^{0.54} = Q_F \left(\frac{P_s - 20}{P_s - P_r} \right)^{0.54} \quad (4.6)$$

where P_s is static pressure (psi), P_r is residual pressure observed during test (psi), $H_R = P_s - 20$, $H_F = P_s - P_r$.

4.3 Fire Sprinkler Discharge

$$Q = K\sqrt{P} \quad (4.7)$$

where Q is discharge in gpm, P is operating pressure in psi, K is sprinkler orifice coefficient ($K = 5.6$ for standard 1/2 in, $K = 8.0$ for 17/32 in, $K = 11.2$ for 5/8 in).

5 Flood Control & Emergency Spillways

5.1 Spillway Functions

Reservoirs incorporate two distinct spillway structures:

- **Principal Spillway:** Handles routine, high-frequency inflow events to maintain target pool elevation.
- **Emergency Spillway:** Protects dam embankment integrity against catastrophic overtopping during extreme flood events (e.g., 100-year storm or Probable Maximum Flood PMF).

5.2 Weir Hydraulics

Weir Formulas

1. Rectangular Suppressed Weir (Full Channel Width $L = b$):

$$Q = C L H^{3/2} \quad (C = 3.33 \text{ USCS; } C = 1.84 \text{ SI}) \quad (5.1)$$

2. Rectangular Contracted Weir (End Contractions Present):

$$Q = C (L - 0.2H) H^{3/2} \quad (C = 3.33 \text{ USCS}) \quad (5.2)$$

3. Triangular 90° V-Notch Weir:

$$Q = C H^{5/2} \quad (C = 2.54 \text{ USCS; } C = 1.40 \text{ SI}) \quad (5.3)$$

where L is weir crest length (ft or m), H is head above crest measured upstream (ft or m).

6 Stormwater Quality & Management

6.1 Water Quality Volume (WQ_v)

Water Quality Volume (WQ_v) captures first-flush runoff containing concentrated land surface pollutants:

$$WQ_v = \frac{P R_v A}{12} \quad (\text{acre-feet}) \quad (6.1)$$

where P is rainfall depth for 90% storm event (typically 0.9–1.2 in), A is site area in acres, and R_v is volumetric runoff coefficient:

$$R_v = 0.05 + 0.009I \quad (6.2)$$

where I is percent imperviousness of the site ($0\% \leq I \leq 100\%$).

6.2 Lake Trophic Classification

Lakes are categorized based on primary biological productivity:

Table 6.1: Lake Trophic Status Thresholds

Trophic State	Chlorophyll a ($\mu\text{g/L}$)	Secchi Depth (m)	Total Phosphorus ($\mu\text{g/L}$)
Oligotrophic (Low productivity, clear)	< 2.6	> 4.0	< 12
Mesotrophic (Moderate productivity)	2.6–7.3	2.0–4.0	12–24
Eutrophic (High productivity, algae)	> 7.3	< 2.0	> 24

7 Collection Systems & Sanitary Sewers

7.1 Gravity Sewer Hydraulics

Sanitary sewers are designed to operate as open channels under gravity flow. Hydraulic calculations utilize Manning's equation in conjunction with circular conduit partial-flow curves.

Hydraulic Elements for Partial Circular Flow

For a circular pipe of diameter D filled to depth d :

- Ratio of depth to diameter: d/D
- Ratio of partial flow to full flow rate: q/Q_{full}
- Ratio of partial velocity to full velocity: v/V_{full}

Design Velocity Limits:

- **Minimum Velocity:** $v \geq 2.0$ ft/s (0.6 m/s) when half-full to maintain self-cleansing and prevent solids deposition.
- **Maximum Velocity:** $v \leq 10.0$ ft/s (3.0 m/s) to prevent internal pipe invert abrasion.

8 Groundwater Hydrogeology & Well Drawdown

8.1 Darcy's Law & Seepage Velocity

Darcy's Law governs fluid flow through porous media:

$$Q = K A i = K A \left(\frac{\Delta h}{L} \right) \quad (8.1)$$

where K is hydraulic conductivity (ft/day or m/s), A is total cross-sectional area perpendicular to flow, $i = -dh/dx$ is hydraulic gradient.

Darcy Velocity vs. Seepage Velocity

Darcy (Specific Discharge / Superficial Velocity, q):

$$q = \frac{Q}{A} = K i \quad (8.2)$$

Actual Seepage Velocity (v_s):

$$v_s = \frac{q}{n_e} = \frac{K i}{n_e} \quad (8.3)$$

where n_e is effective porosity ($0 < n_e < 1$). Since $n_e < 1$, $v_s > q$ always.

8.2 Steady-State Well Drawdown Equations

Dupuit/Thiem Unconfined Aquifer Drawdown

For steady pumping rate Q in an unconfined aquifer above a horizontal impermeable layer:

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln(r_2/r_1)} \quad (8.4)$$

where r_1, r_2 are radial distances from pumping well centerline, and h_1, h_2 are water table heights above baseline at observation wells ($h_i = H_{\text{initial}} - \text{drawdown}_i$).

Thiem Confined Aquifer Drawdown

For a confined aquifer of uniform thickness b and transmissivity $T = K \cdot b$:

$$Q = \frac{2\pi T (h_2 - h_1)}{\ln(r_2/r_1)} = \frac{2\pi K b (h_2 - h_1)}{\ln(r_2/r_1)} \quad (8.5)$$

9 Water Quality Chemistry & Environmental Standards

9.1 Environmental Regulations

General secondary-treatment standards include 30-day BOD_5 /TSS averages ≤ 30 mg/L and 85% removal; 7-day averages ≤ 45 mg/L. Permit terms and exceptions govern.

- 30-day average $BOD_5 \leq 30$ mg/L (minimum 85% removal).
- 30-day average Total Suspended Solids (TSS) ≤ 30 mg/L (minimum 85% removal).
- pH within 6.0–9.0.

9.2 Hardness & Water Softening Stoichiometry

Hardness is caused by multivalent metallic cations, primarily Ca^{2+} and Mg^{2+} , expressed as equivalent $CaCO_3$ (mg/L):

$$\text{Hardness (mg/L as } CaCO_3) = [Ca^{2+}] \left(\frac{50}{20.04} \right) + [Mg^{2+}] \left(\frac{50}{12.16} \right) \quad (9.1)$$

where equivalent weight of $CaCO_3 = 50.0$ g/eq, $Ca^{2+} = 20.04$ g/eq, $Mg^{2+} = 12.16$ g/eq. **9.2.1 Lime-Soda Softening Reactions**

1. $CO_2 + Ca(OH)_2 \rightarrow CaCO_3(s) + H_2O$
2. $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3(s) + 2H_2O$ (Carbonate Hardness Removal)
3. $CaSO_4 + Na_2CO_3 \rightarrow CaCO_3(s) + Na_2SO_4$ (Non-Carbonate Hardness Removal)

10 Testing Standards & Biochemical Oxygen Demand

10.1 Biochemical Oxygen Demand (BOD)

BOD measures dissolved oxygen consumed by microorganisms decomposing organic waste.

BOD Dilution Test Equations

5-Day BOD (BOD_5 , Unseeded Sample):

$$BOD_5 = \frac{DO_0 - DO_5}{P} = \frac{DO_0 - DO_5}{V_{\text{sample}}/V_{\text{bottle}}} \quad (10.1)$$

where standard test bottle volume $V_{\text{bottle}} = 300$ mL.

First-Order BOD Kinetics:

$$BOD_t = L_0 (1 - e^{-kt}) \quad (10.2)$$

where L_0 is ultimate BOD (mg/L), k is kinetic reaction constant (day^{-1}), t is incubation time (days).

11 Water & Wastewater Treatment Operations

11.1 Sedimentation and Clarifiers

Clarifier Overflow & Loading Rates

Hydraulic Surface Overflow Rate (v_0):

$$v_0 = \frac{Q}{A_{\text{surface}}} \quad (\text{gpd/ft}^2 \text{ or } \text{m}^3/\text{m}^2 \cdot \text{day}) \quad (11.1)$$

Weir Overflow Rate (WOR):

$$WOR = \frac{Q}{\text{Weir Length}} \quad (\text{gpd/ft or } \text{m}^3/\text{m} \cdot \text{day}) \quad (11.2)$$

Hydraulic Detention Time (θ):

$$\theta = \frac{V_{\text{tank}}}{Q} \quad (11.3)$$

11.2 Trickling Filters (NRC Equation)

For a single-stage rock trickling filter:

National Research Council (NRC) Equation

$$E_1 = \frac{100}{1 + 0.0561 \sqrt{\frac{W}{VF}}} \quad (11.4)$$

where E_1 is percent BOD removal efficiency, W is influent BOD loading (lb/day), V is filter volume (10^3 ft^3), and recirculation factor F :

$$F = \frac{1 + R}{(1 + 0.1R)^2} \quad (R = \text{recirculation ratio } Q_R/Q_0) \quad (11.5)$$

11.3 Sludge Characteristics

$$V_{\text{sludge}} = \frac{W_s}{\left(\frac{s}{100}\right) \gamma S_g} \quad (11.6)$$

where W_s is dry solids weight (lb), s is percent solids content, $\gamma = 62.4 \text{ lb/ft}^3$, and S_g is specific gravity of wet sludge.

12 Example Problems and Solutions

Problem 1: Rational Method Peak Runoff

The 10-acre site has Basin 1 (4 acres, 6-min inlet time) draining through a 600-ft sewer at 2 ft/s to **Basin 2, the outlet** (6 acres, 10-min inlet time). The IDF gives $I = 5.0$ in./h at 11 min and $I = 4.0$ at 15 min. Use $C = 0.40$ to find peak discharge.

Solution

1. *Time of Concentration (t_c):*

- Path 1 (Sub-area A through sewer): $t_{c1} = 6 \text{ min} + \frac{600 \text{ ft}}{2 \text{ ft/s} \times 60 \text{ s/min}} = 6 + 5 = 11 \text{ min}.$
- Path 2 (Sub-area B direct): $t_{c2} = 10 \text{ min}.$

The controlling site time of concentration is $t_c = \max(11, 10) = 11 \text{ min}.$

2. *Rainfall Intensity:* From the IDF relationship at $t_c = 11 \text{ min}$, $I = 5.0 \text{ in/hr}.$

3. *Peak Discharge Calculation:*

$$Q = C_{\text{composite}}IA = 0.40 \times 5.0 \text{ in/hr} \times 10.0 \text{ acres} = 20.0 \text{ cfs}$$

Correct Answer: (B)

Problem 2: NRCS Curve Number Runoff Depth

A 12.5-acre development has land uses given below. For a 100-year storm event delivering 7.0 inches of rainfall, the direct runoff depth Q from the site is most nearly:

Land Use	Area (acres)	SCS Curve Number (CN)
Open Space (Good Lawns)	2.5	74
Residential (1/2-acre lots)	8.5	80
Impervious Pavement	1.5	98

- (A) 3.5 in (B) 4.8 in (C) 5.7 in (D) 7.0 in

Solution

1. *Weighted Composite Curve Number:*

$$CN_{\text{comp}} = \frac{(2.5 \times 74) + (8.5 \times 80) + (1.5 \times 98)}{12.5} = \frac{185 + 680 + 147}{12.5} = \frac{1012}{12.5} = 80.96 \approx 81$$

2. *Max Retention (S):*

$$S = \frac{1000}{CN} - 10 = \frac{1000}{80.96} - 10 = 12.35 - 10 = 2.35 \text{ inches}$$

3. *Direct Runoff Depth (Q):*

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} = \frac{[7.0 - 0.2(2.35)]^2}{7.0 + 0.8(2.35)} = \frac{(6.53)^2}{8.88} = \frac{42.64}{8.88} = 4.80 \text{ inches}$$

Correct Answer: (B)

Problem 3: Manning's Open Channel Flow in SI Units

A concrete-lined trapezoidal channel ($n = 0.012$) has a bottom width of 5.0 m, side slopes of 1 : 1 ($z = 1$), and conveys water at a flow depth of 3.0 m. The channel invert drops 2.0 m over a distance of 2.0 km. The flow rate conveyed by the channel is most nearly:

- (A) 70 m³/s (B) 93 m³/s (C) 105 m³/s (D) 150 m³/s

Solution

1. *Geometric Properties:*

- Flow area $A = (b + zy)y = (5 + 1 \times 3)(3) = 24 \text{ m}^2$.
- Wetted perimeter $P = b + 2y\sqrt{1 + z^2} = 5 + 6\sqrt{2} = 13.485 \text{ m}$.
- Hydraulic radius $R_H = A/P = 24/13.485 = 1.780 \text{ m}$.

2. *Channel Slope:* $S = \frac{\Delta z}{L} = \frac{2.0 \text{ m}}{2000 \text{ m}} = 0.001 \text{ m/m}$.

3. *Manning's Flow Rate ($K = 1.0$ for SI):*

$$Q = \frac{1}{0.012} (24)(1.780)^{2/3} (0.001)^{1/2} = 92.88 \text{ m}^3/\text{s} \approx 93 \text{ m}^3/\text{s}$$

Correct Answer: (B)

Problem 4: Hydraulic Jump Downstream Depth

An 8.0-ft wide rectangular channel carries a discharge of 200 cfs at an initial supercritical flow depth of $y_1 = 1.5$ ft. A hydraulic jump forms downstream. The depth downstream of the jump (y_2) is most nearly:
 (A) 2.8 ft (B) 3.6 ft (C) 4.4 ft (D) 5.2 ft

Solution

1. *Flow Velocity and Area Upstream:*

$$A_1 = by_1 = 8.0 \times 1.5 = 12.0 \text{ ft}^2, \quad v_1 = \frac{Q}{A_1} = \frac{200}{12.0} = 16.67 \text{ ft/s}$$

2. *Upstream Froude Number (Fr_1):*

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{16.67}{\sqrt{32.2 \times 1.5}} = \frac{16.67}{6.95} = 2.40 \quad (> 1.0 \text{ supercritical})$$

3. *Sequent Depth (y_2):*

$$y_2 = \frac{y_1}{2} \left(-1 + \sqrt{1 + 8Fr_1^2} \right) = \frac{1.5}{2} \left(-1 + \sqrt{1 + 8(2.40)^2} \right)$$

$$y_2 = 0.75(-1 + \sqrt{47.08}) = 0.75(-1 + 6.861) = 4.396 \approx 4.4 \text{ ft}$$

Correct Answer: (C)

Problem 5: Pump Brake Horsepower Calculation

A centrifugal pump delivers 100 gpm of water ($\gamma = 62.4 \text{ lb/ft}^3$) against a total dynamic head of 80 ft. If the pump operating efficiency is 75%, the required Brake Horsepower (BHP) is most nearly:
 (A) 1.8 HP (B) 2.7 HP (C) 3.6 HP (D) 4.5 HP

Solution

1. *Convert Flow Rate to cfs:*

$$Q = \frac{100 \text{ gpm}}{448.83 \text{ gpm/cfs}} = 0.2228 \text{ cfs}$$

2. *Calculate Water Horsepower (WHP):*

$$WHP = \frac{\gamma QH}{550} = \frac{62.4 \times 0.2228 \times 80}{550} = \frac{1112.2}{550} = 2.022 \text{ HP}$$

Or directly using gpm formula: $WHP = \frac{100 \times 80}{3960} = 2.020 \text{ HP}.$

3. *Calculate Brake Horsepower (BHP):*

$$BHP = \frac{WHP}{\eta_{\text{pump}}} = \frac{2.020}{0.75} = 2.693 \approx 2.7 \text{ HP}$$

Correct Answer: (B)

Problem 6: Net Positive Suction Head Available (NPSHA)

A pump is installed 5.0 ft above the surface of an open water sump (suction lift). Atmospheric pressure head is 34.0 ft, suction pipe friction losses are 2.0 ft, and the vapor pressure head of water at the operating temperature is 10.0 ft. The NPSHA is most nearly:

- (A) 12.0 ft (B) 17.0 ft (C) 27.0 ft (D) 31.0 ft

Solution

1. *Identify NPSHA Equation for Suction Lift:*

$$NPSHA = H_{pa} - H_s - \sum h_{L,s} - H_{vp}$$

2. *Substitute Values:*

- $H_{pa} = 34.0$ ft
- Static lift $H_s = 5.0$ ft (subtracted because pump is above sump)
- Suction friction $\sum h_{L,s} = 2.0$ ft
- Vapor pressure $H_{vp} = 10.0$ ft

$$NPSHA = 34.0 - 5.0 - 2.0 - 10.0 = 17.0 \text{ ft}$$

Correct Answer: (B)

Problem 7: Fire Hydrant Rated Capacity at 20 psi

A fire hydrant flow test is conducted. Static pressure is measured at 70 psi. When a 2.5-inch outlet ($C_d = 0.70$, square projecting into barrel) is opened, a pitot pressure of 45 psi and residual pressure of 45 psi are observed. The rated fire flow capacity at 20 psi residual pressure is most nearly:

- (A) 875 gpm (B) 1270 gpm (C) 1560 gpm (D) 1850 gpm

Solution

1. *Test Flow Rate (Q_F):*

$$Q_F = 29.83 C_d d^2 \sqrt{P_{\text{pitot}}} = 29.83 \times 0.70 \times (2.5)^2 \times \sqrt{45}$$

$$Q_F = 20.881 \times 6.25 \times 6.708 = 875.4 \text{ gpm}$$

2. *Rated Flow Capacity at 20 psi (Q_R):*

$$Q_R = Q_F \left(\frac{P_s - 20}{P_s - P_r} \right)^{0.54} = 875.4 \left(\frac{70 - 20}{70 - 45} \right)^{0.54} = 875.4 \left(\frac{50}{25} \right)^{0.54}$$

$$Q_R = 875.4(2.0)^{0.54} = 875.4 \times 1.4539 = 1272.8 \text{ gpm} \approx 1270 \text{ gpm}$$

Correct Answer: (B)

Problem 8: Contracted Rectangular Weir Discharge

A sharp-crested rectangular weir with two end contractions has a crest length of $L = 10.0$ ft. Water flows over the weir at a head of $H = 2.5$ ft. Using $C = 3.33$, the discharge over the weir is most nearly:
 (A) 98 cfs (B) 125 cfs (C) 131 cfs (D) 145 cfs

Solution

1. *Apply Contracted Weir Equation:*

$$Q = C(L - 0.2H)H^{3/2}$$

2. *Substitute Parameters:*

$$L - 0.2H = 10.0 - 0.2(2.5) = 10.0 - 0.5 = 9.5 \text{ ft}$$

$$H^{3/2} = (2.5)^{1.5} = 3.9528$$

$$Q = 3.33(9.5)(3.9528) = 125.05 \approx 125 \text{ cfs}$$

Correct Answer: (B)

Problem 9: Minimum Sanitary Sewer Slope

A 12-inch (1.0-ft) diameter concrete sanitary sewer pipe ($n = 0.013$) must maintain a minimum velocity of 2.0 ft/s when flowing half-full. The minimum required pipe drop per 100 ft of length is most nearly:
 (A) 0.05 ft (B) 0.15 ft (C) 0.20 ft (D) 0.44 ft

Solution

1. *Half-Full Hydraulic Radius:* $R_H = D/4 = 1.0/4 = 0.25$ ft.

2. *Manning's Velocity Equation:*

$$v = \frac{1.486}{n} R_H^{2/3} S^{1/2} \Rightarrow 2.0 = \frac{1.486}{0.013} (0.25)^{2/3} S^{1/2}$$

$$2.0 = 114.308 \times 0.39685 \times S^{1/2} = 45.364 S^{1/2}$$

$$S^{1/2} = \frac{2.0}{45.364} = 0.04409 \Rightarrow S = (0.04409)^2 = 0.001944 \text{ ft/ft}$$

3. *Drop per 100 ft:*

$$\text{Drop} = S \times 100 = 0.001944 \times 100 = 0.1944 \text{ ft} \approx 0.20 \text{ ft}$$

Correct Answer: (C)

Problem 10: Unconfined Aquifer Hydraulic Conductivity

An unconfined aquifer has an initial saturated thickness of 180 ft above an impermeable layer. A pumping well discharges at $Q = 50$ gpm ($9625 \text{ ft}^3/\text{day}$). Two observation wells located at $r_1 = 400$ ft and $r_2 = 800$ ft record steady-state drawdowns of 15 ft and 11 ft respectively. The hydraulic conductivity K (ft/day) is most nearly:
 (A) 1.2 ft/day (B) 1.6 ft/day (C) 4.8 ft/day (D) 8.5 ft/day

Solution

1. Calculate Saturated Heights at Observation Wells:

$$h_1 = 180 - 15 = 165 \text{ ft}, \quad h_2 = 180 - 11 = 169 \text{ ft}$$

2. Dupuit/Thiem Unconfined Formula:

$$Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2/r_1)} \Rightarrow K = \frac{Q \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)}$$

3. Substitute Values:

$$h_2^2 - h_1^2 = (169)^2 - (165)^2 = 28561 - 27225 = 1336 \text{ ft}^2$$

$$\ln(800/400) = \ln(2) = 0.69315$$

$$K = \frac{9625 \times 0.6931}{\pi \times 1336} = \frac{6671.56}{4197.17} = 1.59 \text{ ft/day} \approx 1.6 \text{ ft/day}$$

Correct Answer: (B)

Problem 11: Ultimate BOD Calculation

An unseeded 5-day BOD test is performed on domestic wastewater using a standard 300-mL bottle with 6.0 mL of wastewater sample. Initial dissolved oxygen is $DO_0 = 8.0$ mg/L and $DO_5 = 4.0$ mg/L. If the BOD reaction rate constant is $k = 0.30 \text{ day}^{-1}$ (base e), the ultimate BOD (L_0) is most nearly:
 (A) 115 mg/L (B) 162 mg/L (C) 200 mg/L (D) 257.4 mg/L

Solution

1. Calculate BOD_5 :

$$P = \frac{V_{\text{sample}}}{V_{\text{bottle}}} = \frac{6.0 \text{ mL}}{300 \text{ mL}} = 0.020$$

$$BOD_5 = \frac{DO_0 - DO_5}{P} = \frac{8.0 - 4.0}{0.020} = \frac{4.0}{0.020} = 200 \text{ mg/L}$$

2. Calculate Ultimate BOD (L_0):

$$BOD_5 = L_0(1 - e^{-kt}) \Rightarrow 200 = L_0(1 - e^{-0.30 \times 5})$$

$$1 - e^{-1.50} = 1 - 0.2231 = 0.7769$$

$$L_0 = \frac{200}{1 - e^{-1.5}} = 257.44 \approx 257.4 \text{ mg/L}$$

Correct Answer: (D)

Problem 12: Primary Clarifier Overflow Rate

A rectangular primary clarifier processes a wastewater flow rate of 2.0 MGD (2,000,000 gpd). The tank dimensions are 60 ft long, 20 ft wide, and 10 ft deep. The hydraulic surface overflow rate (v_0) is most nearly:
 (A) 800 gpd/ft² (B) 1200 gpd/ft² (C) 1667 gpd/ft² (D) 2000 gpd/ft²

Solution

1. *Plan View Surface Area:* $A_{\text{surface}} = L \times W = 60 \text{ ft} \times 20 \text{ ft} = 1200 \text{ ft}^2$.
2. *Surface Overflow Rate:*

$$v_0 = \frac{Q}{A_{\text{surface}}} = \frac{2,000,000 \text{ gpd}}{1200 \text{ ft}^2} = 1666.7 \text{ gpd/ft}^2 \approx 1667 \text{ gpd/ft}^2$$

Correct Answer: (C)

Problem 13: Total Hardness as CaCO₃

A groundwater sample contains 60 mg/L of Ca²⁺ and 24 mg/L of Mg²⁺. The total hardness expressed as mg/L as CaCO₃ is most nearly:
 (A) 150 mg/L (B) 200 mg/L (C) 248.4 mg/L (D) 310 mg/L

Solution

1. *Equivalent Weight Ratios:* $\text{Ca}^{2+} \rightarrow 50/20.04 = 2.495$, $\text{Mg}^{2+} \rightarrow 50/12.16 = 4.112$.
2. *Total Hardness:*

$$TH = 60(50/20.04) + 24(50/12.16) \approx 248.4 \text{ mg/L as CaCO}_3$$

Correct Answer: (C)

Problem 14: Seepage Velocity through Soil

A soil sample has a hydraulic conductivity $K = 2.0 \times 10^{-3}$ cm/s and an effective porosity $n_e = 0.30$. Under a hydraulic gradient $i = 0.05$, the actual seepage velocity (v_s) in cm/s is most nearly:
 (A) 1.0×10^{-4} (B) 3.3×10^{-4} (C) 1.0×10^{-3} (D) 3.3×10^{-3}

Solution

1. *Darcy Velocity (q):*

$$q = K i = (2.0 \times 10^{-3}) \times 0.05 = 1.0 \times 10^{-4} \text{ cm/s}$$

2. *Seepage Velocity (v_s):*

$$v_s = \frac{q}{n_e} = \frac{1.0 \times 10^{-4}}{0.30} = 3.33 \times 10^{-4} \text{ cm/s}$$

Correct Answer: (B)

Problem 15: Critical Depth in a Rectangular Channel

A rectangular open channel 6.0 ft wide carries a total flow rate $Q = 144$ cfs. The critical flow depth (y_c) is most nearly:

- (A) 1.8 ft (B) 2.6 ft (C) 3.2 ft (D) 4.1 ft

Solution

1. *Unit Discharge (q):* $q = Q/b = 144/6.0 = 24.0$ cfs/ft.

2. *Critical Depth:*

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{(24.0)^2}{32.2} \right)^{1/3} = \left(\frac{576}{32.2} \right)^{1/3} = (17.888)^{1/3} = 2.615 \text{ ft} \approx 2.6 \text{ ft}$$

Correct Answer: (B)

Problem 16: Fire Sprinkler Discharge

A standard fire sprinkler head ($K = 5.6$) operates at a residual water pressure $P = 25.0$ psi. The discharge rate (Q) in gpm is most nearly:

- (A) 18.0 gpm (B) 28.0 gpm (C) 35.0 gpm (D) 42.0 gpm

Solution

1. *Formula:* $Q = K\sqrt{P} = 5.6 \times \sqrt{25.0} = 5.6 \times 5.0 = 28.0$ gpm. **Correct Answer: (B)**

Problem 17: Water Quality Volume Runoff Coefficient

A 20-acre commercial development is designed with 70% impervious surfaces ($I = 70\%$). For a design storm depth $P = 1.0$ inch, the required Water Quality Volume (WQ_v) in acre-feet is most nearly:

- (A) 0.85 ac-ft (B) 1.13 ac-ft (C) 1.50 ac-ft (D) 2.10 ac-ft

Solution

1. *Volumetric Runoff Coefficient (R_v):*

$$R_v = 0.05 + 0.009(70) = 0.05 + 0.63 = 0.68$$

2. *Water Quality Volume:*

$$WQ_v = \frac{P R_v A}{12} = \frac{1.0 \times 0.68 \times 20}{12} = \frac{13.6}{12} = 1.133 \text{ ac-ft} \approx 1.13 \text{ ac-ft}$$

Correct Answer: (B)

Problem 18: Trickling Filter BOD Removal Efficiency

A single-stage rock trickling filter receives an influent BOD loading $W = 2000$ lb/day. The filter volume is $V = 40 \times 10^3$ ft³ ($V = 40$) and operates with a recirculation ratio $R = 1.0$. The BOD removal efficiency (E_1) is most nearly:

- (A) 65% (B) 76.4% (C) 84% (D) 92%

Solution

1. *Recirculation Factor (F):*

$$F = \frac{1 + R}{(1 + 0.1R)^2} = \frac{1 + 1.0}{(1 + 0.1)^2} = \frac{2.0}{(1.1)^2} = \frac{2.0}{1.21} = 1.653$$

2. *NRC Efficiency Equation:*

$$E_1 = \frac{100}{1 + 0.0561 \sqrt{\frac{W}{VF}}} = \frac{100}{1 + 0.0561 \sqrt{\frac{2000}{40 \times 1.653}}}$$

$$\frac{2000}{66.116} = 30.25 \Rightarrow \sqrt{30.25} = 5.50$$

$$E_1 = 100 / [1 + 0.0561(5.50)] = 76.42\% = 76.4\%$$

Correct Answer: (B)

Problem 19: Wet Sludge Volume Calculation

A wastewater treatment plant produces 1000 lb/day of dry primary sludge solids ($W_s = 1000$ lb). The sludge has a moisture content $p = 95\%$ (solids content $s = 5\%$) and a wet sludge specific gravity $S_g = 1.02$. The volume of wet sludge produced daily in cubic feet is most nearly:

- (A) 150 ft³ (B) 314 ft³ (C) 450 ft³ (D) 620 ft³

Solution

1. *Sludge Volume Formula:*

$$V_{\text{sludge}} = \frac{W_s}{\left(\frac{s}{100}\right) \gamma S_g} = \frac{1000}{\left(\frac{5}{100}\right) \times 62.4 \times 1.02}$$

$$V_{\text{sludge}} = \frac{1000}{0.05 \times 63.648} = \frac{1000}{3.1824} = 314.22 \text{ ft}^3 \approx 314 \text{ ft}^3$$

Correct Answer: (B)

Problem 20: Hazen-Williams Head Loss in USCS Units

A 1000-ft long ductile iron pipe ($C = 140$) with a diameter $D = 12$ inches (1.0 ft) carries a flow rate $Q = 2.0$ cfs. The head loss n (h_f) in feet is most nearly:

- (A) 1.5 ft (B) 1.81 ft (C) 4.2 ft (D) 6.1 ft

Solution

1. *Hazen-Williams Head Loss Formula:*

$$h_f = \frac{4.73 L Q^{1.852}}{C^{1.852} D^{4.87}}$$

2. *Evaluate Components:*

- $Q^{1.852} = (2.0)^{1.852} = 3.610$
- $C^{1.852} = (140)^{1.852} = 9432.6$
- $D^{4.87} = (1.0)^{4.87} = 1.0$

3. *Calculate h_f :*

$$h_f = \frac{4.73(1000)(2.0)^{1.852}}{140^{1.852}(1.0)^{4.87}} = 1.81 \text{ ft}$$

Correct Answer: (B)