

Environmental Engineering FE CBT Exam

Knowledge: Fluid Mechanics and Hydraulics with 20 Example Problems

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1 Fluid Properties, Units, and Dimensional Discipline

Fluid-mechanics errors on the FE exam often come from units rather than physics. Keep density ρ , specific weight γ , dynamic viscosity μ , and kinematic viscosity ν distinct:

$$\gamma = \rho g, \quad \nu = \frac{\mu}{\rho}.$$

For water near room temperature, $\rho \approx 998 \text{ kg/m}^3$ and $\gamma \approx 9.79 \text{ kN/m}^3$. In USCS, water weighs about 62.4 lbf/ft^3 . Standard gravity is 9.807 m/s^2 or 32.174 ft/s^2 .

Pressure units

$$1 \text{ Pa} = 1 \text{ N/m}^2, \quad 1 \text{ kPa} = 10^3 \text{ Pa}, \quad 1 \text{ psi} = 144 \text{ lbf/ft}^2.$$

Pressure head is pressure divided by specific weight:

$$h_p = \frac{p}{\gamma}.$$

Thus a pressure can be interpreted as an equivalent height of fluid.

Absolute versus gauge pressure

$$p_{abs} = p_{atm} + p_{gauge}.$$

Vacuum pressure indicates that the absolute pressure is below atmospheric. Use absolute pressure in gas-law and cavitation calculations; gauge pressure is commonly used in liquid piping when both ends are exposed to the same atmosphere.

Quick example

Water has $\mu = 1.00 \times 10^{-3} \text{ Pa s}$ and $\rho = 998 \text{ kg/m}^3$. Then

$$\nu = \frac{1.00 \times 10^{-3}}{998} = 1.00 \times 10^{-6} \text{ m}^2/\text{s}.$$

This value is the one used directly in $\text{Re} = VD/\nu$.

Sanity checks

A head is a length. Reynolds and Froude numbers are dimensionless. Pump head is not pressure until multiplied by γ . Power must end in watts, horsepower, or equivalent energy/time units.

1.1 Fluid Statics: Pressure Variation with Elevation

A static fluid has no shear stress caused by motion. Its pressure changes only because of the weight of fluid above or below the point of interest. For a fluid of constant density,

$$\frac{dp}{dz} = -\rho g$$

which integrates to

$$p_2 - p_1 = -\rho g(z_2 - z_1).$$

If point 2 is lower than point 1 by depth h , then

$$p_2 = p_1 + \rho gh.$$

This is the basis of hydrostatic head, manometers, and forces on submerged surfaces.

Pressure at a depth below a free surface

For a tank open to the atmosphere,

$$p_{gauge} = \rho gh = \gamma h.$$

The pressure distribution on a vertical wall is triangular because pressure increases linearly with depth.

Reservoir pressure

At a depth of 6.0 m in water,

$$p_g = (998)(9.807)(6.0) = 58.7 \text{ kPa}.$$

Absolute pressure would be approximately $58.7 + 101.3 = 160.0 \text{ kPa}$ at standard atmospheric pressure.

Force-analysis clues

If the question asks only for pressure at a point, use $p = \rho gh$. If it asks for the total force on a submerged gate or wall, integrate pressure over area or use the centroid relationship discussed later. If it asks for moment or center of pressure, geometry matters because deeper portions experience larger pressure.

Common trap

Elevation z in Bernoulli is measured upward from a datum, but depth h in hydrostatics is typically measured downward from the free surface. Keep the sign convention explicit.

1.2 Manometers and Multi-Fluid Pressure Balances

Manometers convert pressure differences into liquid-column height differences. The safest method is to walk from one point to another through the connected fluid columns:

- moving downward in a fluid increases pressure by ρgh ;
- moving upward decreases pressure by ρgh ;
- pressure is continuous across an interface between immiscible fluids.

For a simple differential manometer with equal height changes,

$$p_1 - p_2 = (\rho_m - \rho_f)gh$$

when the manometer liquid has density ρ_m and the pipe fluid has density ρ_f .

Differential pressure

A mercury manometer connects two points in a water line. The mercury-water interfaces differ by 0.20 m. Using $\rho_m = 13,600$ and $\rho_w = 998 \text{ kg/m}^3$,

$$\Delta p = (13,600 - 998)(9.807)(0.20) = 24.7 \text{ kPa.}$$

Barometers

A barometer is a special manometer for atmospheric pressure. If vapor pressure above the liquid is small,

$$p_{atm} \approx \rho gh.$$

For high-accuracy work, include the vapor pressure in the force balance.

FE method

Draw a horizontal reference line through each interface and annotate pressure changes. Never memorize a sign-heavy multi-fluid formula; pressure walking is faster and more reliable.

Check

The denser liquid must generally produce a larger pressure change per unit height. If your answer predicts the opposite, revisit the sign path.

1.3 Hydrostatic Force on Plane Surfaces

For a plane area submerged in a liquid, the resultant hydrostatic force equals pressure at the area centroid times area:

$$F_R = \rho g h_c A = \gamma h_c A$$

for gauge pressure with a free surface at atmospheric pressure. Here h_c is the vertical depth of the centroid below the free surface.

The line of action passes below the centroid for a vertical or inclined plane because pressure increases with depth. For a plane surface inclined at angle θ from the free surface,

$$h_{cp} = h_c + \frac{I_G \sin^2 \theta}{h_c A},$$

where I_G is the second moment of area about the centroidal axis parallel to the free surface. For a vertical plane, $\sin \theta = 1$.

Vertical rectangular gate

A 2 m wide by 3 m high vertical gate has its top at the water surface. Then $A = 6 \text{ m}^2$ and $h_c = 1.5 \text{ m}$.

$$F_R = (998)(9.807)(1.5)(6) = 88.1 \text{ kN}.$$

For the rectangle, $I_G = bh^3/12 = 2(3^3)/12 = 4.5 \text{ m}^4$. Thus

$$h_{cp} = 1.5 + \frac{4.5}{1.5(6)} = 2.00 \text{ m}.$$

Why this works

The resultant is the area under the pressure distribution. The center of pressure is a moment balance of that distributed pressure. A deeper centroid increases force; a taller surface also shifts the resultant farther below the centroid.

Exam distinction

Use centroid depth for the magnitude of force. Use center-of-pressure depth only for the location of the force.

1.4 Buoyancy, Stability, and Displaced Volume

A submerged or floating object experiences an upward buoyant force equal to the weight of displaced fluid:

$$F_B = \rho_f g V_{disp} = \gamma_f V_{disp}.$$

For a floating body in equilibrium, buoyancy equals total weight:

$$W = F_B.$$

The fraction of a uniform floating object's volume that is submerged is approximately

$$\frac{V_{sub}}{V} = \frac{\rho_{object}}{\rho_f}.$$

Floating block

A sealed block has average density 650 kg/m^3 and floats in fresh water with $\rho_f = 998 \text{ kg/m}^3$.

The submerged fraction is

$$\frac{650}{998} = 0.651.$$

About 65.1% of the block volume is below the water surface.

Fully submerged objects

If an object is fully submerged, the displaced volume is the object's external volume. The apparent weight is

$$W_{app} = W - F_B.$$

If $F_B > W$, the object accelerates upward unless restrained.

Environmental applications

Buoyancy appears in submerged pipes, floating covers, storage tanks, oil-water separation devices, and submerged monitoring equipment. Groundwater uplift can also create buoyant forces on buried tanks and slabs.

Force direction

Buoyancy always acts through the center of displaced volume and vertically upward. Weight acts through the center of gravity.

2 Continuity and Conservation of Mass

For steady one-dimensional flow, conservation of mass is

$$\dot{m} = \rho AV = \text{constant}.$$

For incompressible liquid flow with constant density,

$$Q = AV, \quad A_1 V_1 = A_2 V_2.$$

This is the fastest equation in many FE problems. A decrease in cross-sectional area requires an increase in average velocity when flow rate is unchanged.

Pipe contraction

Water flows at $Q = 0.030 \text{ m}^3/\text{s}$ from a 0.20 m diameter pipe into a 0.10 m pipe.

$$A_1 = \frac{\pi(0.20)^2}{4} = 0.0314 \text{ m}^2, \quad V_1 = 0.955 \text{ m/s}.$$

$$A_2 = 0.00785 \text{ m}^2, \quad V_2 = 3.82 \text{ m/s}.$$

The fourfold area reduction produces a fourfold velocity increase.

Branches and junctions

At a junction under steady conditions,

$$\sum Q_{in} = \sum Q_{out}.$$

This relationship supplies one equation for pipe-network problems. Additional equations come from head-loss relationships and equal headloss between common nodes.

Unsteady storage form

When a tank level changes,

$$\frac{dV}{dt} = Q_{in} - Q_{out}.$$

For constant tank cross-sectional area A_t ,

$$A_t \frac{dh}{dt} = Q_{in} - Q_{out}.$$

This is the starting point for filling, draining, and equalization problems.

2.1 Bernoulli Equation, Energy Grade Line, and Hydraulic Grade Line

The mechanical-energy equation for incompressible steady flow between points 1 and 2 is commonly written

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L.$$

For most FE problems in ordinary pipe flow, $\alpha \approx 1$ unless otherwise stated.

The three fluid-energy terms are pressure head, elevation head, and velocity head. Pumps add head; turbines extract head; friction and fittings consume head.

Grade lines

Hydraulic grade line (HGL):

$$\text{HGL} = z + \frac{p}{\gamma}.$$

Energy grade line (EGL):

$$\text{EGL} = z + \frac{p}{\gamma} + \frac{V^2}{2g}.$$

The vertical separation between EGL and HGL is the velocity head.

Reservoir to nozzle

For a large reservoir discharging to atmosphere through a nozzle with negligible loss, the free-surface velocity is negligible and both endpoints are atmospheric. If the nozzle is 5 m below the surface,

$$V = \sqrt{2g(5)} = 9.90 \text{ m/s}.$$

Problem strategy

Cross out terms that are zero before substituting values. Large reservoirs imply $V \approx 0$ at the free surface. Open surfaces and free jets have atmospheric pressure, so gauge pressure is zero.

2.2 Reynolds Number and Flow Regimes

The Reynolds number compares inertial and viscous effects:

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}.$$

For circular pipe flow, the FE handbook characterizes flow as generally laminar for $\text{Re} < 2100$, transitional for approximately $2100 < \text{Re} < 10,000$, and fully turbulent for larger Reynolds number.

Flow classification

Water at 20°C has $\nu \approx 1.0 \times 10^{-6} \text{ m}^2/\text{s}$. In a 0.05 m diameter pipe at 0.03 m/s,

$$\text{Re} = \frac{(0.03)(0.05)}{1.0 \times 10^{-6}} = 1500,$$

so the flow is laminar.

Why the regime matters

For laminar circular-pipe flow, the Darcy friction factor is exactly

$$f = \frac{64}{\text{Re}}.$$

For turbulent flow, f depends on both Reynolds number and relative roughness ε/D and is read from a Moody diagram or calculated from an accepted correlation.

Froude number preview

Open-channel flow uses Froude number instead of Reynolds number to classify gravity-wave behavior:

$$\text{Fr} = \frac{V}{\sqrt{g D_h}}, \quad D_h = \frac{A}{T}.$$

Do not confuse hydraulic depth $D_h = A/T$ used in Froude number with hydraulic diameter $D_H = 4A/P$ used in noncircular internal flow.

2.3 Laminar Pipe Flow and Hagen-Poiseuille

For fully developed laminar flow in a circular tube, the velocity profile is parabolic. The centerline velocity is twice the average velocity:

$$V_{max} = 2V.$$

The Darcy friction factor is

$$f = \frac{64}{\text{Re}}.$$

The Hagen-Poiseuille relationship connects flow rate and pressure drop:

$$Q = \frac{\pi D^4 \Delta p}{128 \mu L}.$$

Equivalently,

$$\Delta p = \frac{128 \mu L Q}{\pi D^4}.$$

The D^4 dependence is extremely strong: a small increase in pipe diameter produces a large reduction in pressure drop at fixed Q .

Pressure drop

Oil with $\mu = 0.20$ Pa·s flows at $Q = 1.0 \times 10^{-4}$ m³/s through a $D = 0.025$ m tube of length $L = 10$ m.

$$\Delta p = \frac{128(0.20)(10)(1.0 \times 10^{-4})}{\pi(0.025)^4} = 20.9 \text{ kPa}.$$

You should verify that the Reynolds number is in the laminar range before using this equation.

Noncircular conduits

For many conduit shapes, use hydraulic diameter

$$D_H = \frac{4A}{P_w}$$

where P_w is wetted perimeter. This permits familiar Reynolds-number and friction relationships to be applied approximately to noncircular ducts.

2.4 Darcy-Weisbach Major Head Loss

The Darcy-Weisbach equation is the general energy-loss relation for fully developed pipe flow:

$$h_f = f \frac{L}{D} \frac{V^2}{2g}.$$

The corresponding pressure drop for a horizontal constant-diameter pipe is

$$\Delta p = \gamma h_f.$$

The Darcy friction factor f depends on Reynolds number and relative roughness ε/D .

Major loss

Water flows through a $D = 0.15$ m pipe, $L = 120$ m, at $V = 2.0$ m/s. If $f = 0.022$,

$$h_f = 0.022 \frac{120}{0.15} \frac{2.0^2}{2(9.807)} = 3.59 \text{ m}.$$

The corresponding pressure drop is approximately

$$\Delta p = (9.79 \text{ kN/m}^3)(3.59 \text{ m}) = 35.1 \text{ kPa}.$$

What changes the loss?

At fixed f , h_f scales linearly with L , inversely with D , and with V^2 . But at fixed flow rate, $V \propto 1/D^2$, so diameter changes are even more influential than the explicit L/D term suggests.

Darcy versus Fanning

The Moody chart in the FE handbook uses the Darcy friction factor. The Fanning factor is one-fourth of the Darcy factor. Mixing the two produces a factor-of-four error.

2.5 Moody Diagram, Relative Roughness, and Friction Factor

The Moody diagram provides the Darcy friction factor as a function of

$$\text{Re} = \frac{VD}{\nu}, \quad \frac{\varepsilon}{D}.$$

For laminar flow, the diagram collapses to $f = 64/\text{Re}$. In turbulent flow, roughness becomes important. At very high Reynolds number in fully rough flow, f depends primarily on relative roughness.

Moody-chart workflow

1. Compute $V = Q/A$.
2. Compute Re .
3. Compute relative roughness ε/D .
4. Enter the chart at Reynolds number and move to the roughness curve.
5. Read Darcy f and insert it into Darcy-Weisbach.

When an iterative equation is allowed, the Colebrook form is

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right).$$

A convenient explicit approximation for study is the Swamee-Jain form:

$$f \approx \frac{0.25}{\left[\log_{10} \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2}.$$

Interpretation

If two pipes carry the same fluid at the same Reynolds number, the rougher pipe has the larger ε/D , the larger f , and therefore the larger head loss.

CBT skill

Practice searching the electronic handbook for “Moody” so you can jump directly to the chart under time pressure.

2.6 Minor Losses: Entrances, Exits, Valves, and Fittings

Localized disturbances create additional energy losses:

$$h_m = K \frac{V^2}{2g}.$$

The coefficient K depends on fitting geometry, valve position, entrance shape, contraction, or expansion. If multiple fittings use the same local pipe velocity,

$$h_{m,tot} = \left(\sum K \right) \frac{V^2}{2g}.$$

Total pipe-system head loss is

$$h_L = f \frac{L}{D} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}.$$

Pipe with fittings

For $V = 2.5$ m/s, $fL/D = 16$, and $\sum K = 3.2$,

$$h_L = (16 + 3.2) \frac{2.5^2}{2(9.807)} = 6.12 \text{ m}.$$

Entrance and exit behavior

A sharp entrance causes separation and therefore a noticeable loss. A smoothly rounded entrance has a smaller K . A discharge from a pipe into a large reservoir typically loses essentially the entire velocity head, corresponding to $K \approx 1$.

Equivalent length

Some references convert fittings to an equivalent straight-pipe length:

$$K = f \frac{L_e}{D}.$$

Then major and minor losses can be combined using an adjusted total length.

Velocity selection

Use the velocity in the pipe section associated with that fitting. In systems with multiple diameters, do not use one velocity everywhere.

2.7 Hazen-Williams for Water Distribution

The Hazen-Williams equation is an empirical relation widely used for water-flow calculations. In velocity form,

$$V = k_1 C R_H^{0.63} S^{0.54},$$

where C is the Hazen-Williams roughness coefficient, $R_H = A/P_w$ is hydraulic radius, and $S = h_f/L$. The FE handbook gives $k_1 = 0.849$ in SI and 1.318 in USCS for the stated unit conventions.

For full circular pipes, the handbook also provides direct head-loss forms. In USCS with Q in cfs, D in ft, and L, h_f in ft,

$$h_f = 4.73 \frac{LQ^{1.852}}{C^{1.852} D^{4.87}}.$$

Interpretation of C

A larger C means a smoother pipe and therefore less head loss for the same flow. Plastic pipe commonly has a high C ; older cast iron has a lower value.

Sensitivity

At fixed L , D , and C , Hazen-Williams predicts approximately

$$h_f \propto Q^{1.852}.$$

Doubling flow increases head loss by about $2^{1.852} \approx 3.61$.

When to use

If the problem explicitly provides a Hazen-Williams C value or asks for a water-distribution result using the handbook equation, use Hazen-Williams. If the problem provides roughness ε , Reynolds number, or Moody data, use Darcy-Weisbach.

2.8 Pipes in Series, Parallel Pipes, and Network Logic

For pipes in series, the same flow passes through each segment:

$$Q_1 = Q_2 = \cdots = Q.$$

Total head loss is the sum:

$$h_{L,tot} = \sum_i h_{L,i}.$$

Different diameters therefore have different velocities even though the flow rate is the same.

For parallel branches joining the same two nodes, the head loss between the nodes is equal for every branch:

$$h_{L,1} = h_{L,2} = \cdots$$

and continuity gives

$$Q_{tot} = Q_1 + Q_2 + \cdots .$$

Identical parallel pipes

Two identical parallel pipes connect the same reservoirs. Because their resistance relations are identical and their head losses are equal, the total flow divides equally:

$$Q_1 = Q_2 = \frac{Q_{tot}}{2}.$$

If one branch has larger resistance, it carries less flow.

Resistance viewpoint

Many head-loss equations can be written approximately as

$$h_L = RQ^n.$$

For Darcy-Weisbach with an approximately constant turbulent friction factor, $n \approx 2$. For Hazen-Williams, $n = 1.852$. This form makes parallel-pipe comparisons easier.

Network questions

At each junction: apply continuity. Around a path between two fixed-head nodes: apply energy. For loops: the algebraic sum of head losses is zero after a consistent sign convention is chosen.

3 Centrifugal Pumps and the System Curve

A pump adds mechanical energy to a liquid. In the energy equation, pump head h_p is positive. The pump performance curve typically shows decreasing head as flow increases. The system curve shows the head required by the piping system:

$$H_{sys} = H_{static} + KQ^2$$

for a system dominated by turbulent friction and minor losses.

The operating point is the intersection of pump and system curves. At that point, pump head equals required system head and both have the same flow rate.

Simple operating point

Suppose a pump curve can be approximated by

$$H_p = 30 - 800Q^2$$

and a system curve by

$$H_s = 10 + 1200Q^2$$

with H in m and Q in m^3/s . Setting them equal,

$$30 - 800Q^2 = 10 + 1200Q^2 \Rightarrow Q = 0.100 \text{ m}^3/\text{s}.$$

The operating head is $H = 22 \text{ m}$.

Curve shifts

Closing a valve increases system resistance, steepening the system curve and reducing flow. Increasing pump speed shifts the pump curve to a higher head/flow operating range according to the affinity laws.

Exam vocabulary

“Operating point” means the pump-curve/system-curve intersection, not the best-efficiency point unless the problem states they coincide.

3.1 Pump Power and Efficiency

Hydraulic power transferred to the liquid is

$$P_{fluid} = \rho g Q H = \gamma Q H.$$

If pump efficiency is η_p ,

$$P_{shaft} = \frac{P_{fluid}}{\eta_p}.$$

If motor efficiency is also included,

$$P_{input} = \frac{\rho g Q H}{\eta_p \eta_m}.$$

Pump input power

A pump delivers $Q = 0.040 \text{ m}^3/\text{s}$ against $H = 28 \text{ m}$ with pump efficiency 78% and motor efficiency 92%.

$$P_{fluid} = (998)(9.807)(0.040)(28) = 10.96 \text{ kW}.$$

$$P_{input} = \frac{10.96}{0.78(0.92)} = 15.3 \text{ kW}.$$

Head from pressure rise

If elevation and velocity changes across the pump are negligible,

$$H \approx \frac{p_2 - p_1}{\gamma}.$$

If suction and discharge pipe diameters or elevations differ significantly, use the full energy equation around the pump.

Units

In SI, $\rho g Q H$ directly gives watts. In USCS, hydraulic power in ft-lbf/s can be converted using $1 \text{ hp} = 550 \text{ ft lbf/s}$.

Efficiency logic

Input power must exceed fluid power. If your calculated required input power is smaller than $\rho g Q H$, the efficiency was applied in the wrong direction.

3.2 Pumps in Series/Parallel, Affinity Laws, and NPSH

For pumps in series, heads add at approximately the same flow:

$$H_{series}(Q) = H_1(Q) + H_2(Q).$$

For identical pumps in parallel, flow rates add at approximately the same head:

$$Q_{parallel}(H) = Q_1(H) + Q_2(H).$$

Series operation is therefore useful for more head; parallel operation is useful for more flow.

Affinity laws

For geometrically similar pump operation at the same impeller diameter,

$$Q \propto N, \quad H \propto N^2, \quad P \propto N^3.$$

Thus

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}, \quad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2, \quad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3.$$

NPSH concept

Net positive suction head available measures how far the absolute suction head is above vapor-pressure head after suction-side losses. To avoid cavitation,

$$\text{NPSH}_A > \text{NPSH}_R$$

with a suitable margin. Higher fluid temperature raises vapor pressure and generally reduces NPSH available.

Speed change

If pump speed increases by 10%, then ideally

$$Q_2 = 1.10Q_1, \quad H_2 = 1.21H_1, \quad P_2 = 1.331P_1.$$

4 Open-Channel Geometry and Hydraulic Elements

Open-channel flow has a free surface exposed to the atmosphere and is driven primarily by gravity. Geometric quantities are essential:

$$R_H = \frac{A}{P_w}, \quad D_h = \frac{A}{T},$$

where A is flow area, P_w is wetted perimeter, and T is top width. Hydraulic radius R_H appears in Manning's equation; hydraulic depth D_h appears in the Froude number.

Rectangular channel

For bottom width b and depth y ,

$$A = by, \quad P_w = b + 2y, \quad T = b, \quad R_H = \frac{by}{b + 2y}.$$

Trapezoidal channel

With side slope z horizontal to 1 vertical,

$$A = y(b + zy), \quad P_w = b + 2y\sqrt{1 + z^2}, \quad T = b + 2zy.$$

Hydraulic radius

A rectangular channel has $b = 3$ m and $y = 1$ m. Then

$$A = 3.0 \text{ m}^2, \quad P_w = 5.0 \text{ m}, \quad R_H = 0.60 \text{ m}.$$

The hydraulic depth is $D_h = A/T = 1.0$ m.

Full circular pipe

For a full circular pipe, $R_H = A/P = D/4$ and the hydraulic diameter is $D_H = 4A/P = D$.

4.1 Manning Equation and Uniform Open-Channel Flow

For steady uniform open-channel flow,

$$V = \frac{K}{n} R_H^{2/3} S^{1/2}, \quad Q = AV.$$

The FE handbook uses $K = 1.0$ for SI and $K = 1.486$ for USCS. The Manning roughness coefficient n characterizes boundary resistance, while S is the energy-grade-line slope; under uniform flow it is commonly approximated by channel-bed slope.

Rectangular channel discharge

A concrete rectangular channel has $b = 2.5$ m, $y = 1.2$ m, $n = 0.015$, and $S = 0.0015$.

$$A = 3.00 \text{ m}^2, \quad P_w = 2.5 + 2(1.2) = 4.90 \text{ m}, \quad R_H = 0.612 \text{ m}.$$

$$V = \frac{1}{0.015} (0.612)^{2/3} (0.0015)^{1/2} = 1.86 \text{ m/s}.$$

$$Q = AV = 5.58 \text{ m}^3/\text{s}.$$

Sensitivity

Manning flow increases with hydraulic radius and slope, and decreases with roughness. Because $V \propto S^{1/2}$, quadrupling slope approximately doubles velocity if geometry and n remain fixed.

Common trap

Use wetted perimeter, not total perimeter. The free-surface width is not included in P_w .

4.2 Specific Energy and Critical Depth

Specific energy in an open channel is the energy relative to the channel bottom:

$$E = y + \alpha \frac{V^2}{2g}.$$

For most FE applications, take $\alpha \approx 1$. For a given discharge, specific energy has a minimum at critical flow.

For a rectangular channel with unit discharge $q = Q/b$,

$$y_c = \left(\frac{q^2}{g} \right)^{1/3}.$$

At critical depth, the Froude number is one.

Critical depth

A 4 m wide rectangular channel carries $Q = 12 \text{ m}^3/\text{s}$. Then $q = 3 \text{ m}^2/\text{s}$ and

$$y_c = \left(\frac{3^2}{9.807} \right)^{1/3} = 0.972 \text{ m}.$$

Alternate depths

For a specific-energy value above the minimum, two depths can carry the same discharge: a shallow supercritical depth and a deeper subcritical depth. These are called alternate depths.

Control sections

Critical flow often occurs at hydraulic controls such as weirs, flumes, channel contractions, or slope transitions. Recognizing a critical section can simplify a discharge calculation because one additional relationship, $\text{Fr} = 1$, becomes available.

Do not confuse

Critical depth is an energy concept. Normal depth is the uniform-flow depth associated with Manning's equation for a given discharge, slope, geometry, and roughness.

4.3 Froude Number and Hydraulic Jumps

The Froude number compares inertial effects with gravity-wave effects:

$$\text{Fr} = \frac{V}{\sqrt{gD_h}}, \quad D_h = \frac{A}{T}.$$

Classification:

$$\text{Fr} < 1 \text{ subcritical}, \quad \text{Fr} = 1 \text{ critical}, \quad \text{Fr} > 1 \text{ supercritical}.$$

Subcritical flow is deep and slower; supercritical flow is shallow and faster.

A hydraulic jump is the rapid transition from supercritical to subcritical flow. It dissipates energy and increases depth. For a rectangular channel, the sequent-depth ratio is

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8\text{Fr}_1^2} - 1 \right).$$

Hydraulic jump

If $y_1 = 0.40$ m and $\text{Fr}_1 = 3.0$,

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{73} - 1) = 3.772,$$

so

$$y_2 = 1.51 \text{ m}.$$

Environmental relevance

Hydraulic jumps are intentionally used in energy-dissipation basins downstream of gates, spillways, and culverts. They can also improve mixing but may create scour if not controlled.

Information direction

In supercritical flow, disturbances cannot propagate upstream; in subcritical flow, downstream controls can influence upstream depth.

4.4 Culverts and Channel Controls

A culvert is a closed conduit that often operates partly full and therefore combines open-channel and pipe-flow concepts. FE problems may focus on identifying controlling conditions rather than performing a full design.

Inlet control versus outlet control

Under inlet control, capacity is governed mainly by entrance geometry and upstream headwater. Downstream conditions have limited influence. Under outlet control, capacity depends on the entire barrel system: length, roughness, slope, tailwater, entrance/exit losses, and sometimes full-pipe friction.

Energy approach

A generic culvert energy balance can be organized as

$$H_{up} - H_{down} = h_{entrance} + h_f + h_{exit} + \Delta \left(\frac{V^2}{2g} \right).$$

For a full barrel, Darcy-Weisbach or another specified pipe-loss relation may be used. For partially full uniform flow, Manning geometry becomes relevant.

Key hydraulic elements

Headwater is the upstream water-surface elevation relative to the culvert inlet. Tailwater is the downstream water depth or elevation. Increasing tailwater can submerge the outlet and shift the controlling condition.

Concept check

If a culvert is long and rough with a submerged outlet, barrel friction and tailwater are likely important; this points toward outlet-control reasoning. If a short culvert has a sharp entrance and free outfall, inlet geometry may dominate.

CBT advice

Do not over-design. Use the variables and control regime explicitly given in the problem and the handbook relationship supplied or referenced.

5 Flow Measurement: Pitot Tubes and Venturi Meters

A Pitot tube measures stagnation pressure. For incompressible flow at the same elevation,

$$V = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{2g \frac{p_0 - p_s}{\gamma}}.$$

The measured pressure difference is the velocity pressure.

A Venturi meter uses a smooth area contraction. Combining continuity and Bernoulli gives, for a horizontal incompressible meter,

$$Q = C_d A_2 \sqrt{\frac{2\Delta p / \rho}{1 - (A_2/A_1)^2}}$$

where C_d accounts for nonideal behavior.

Venturi calculation

A horizontal water line contracts from $D_1 = 0.20$ m to $D_2 = 0.10$ m with $\Delta p = 30$ kPa and $C_d = 0.98$. Since $A_2/A_1 = 0.25$,

$$Q = 0.98(0.007854) \sqrt{\frac{2(30,000)/998}{1 - 0.25^2}} = 0.0617 \text{ m}^3/\text{s}.$$

Meter selection

Venturi meters have relatively low permanent head loss because pressure recovers in the diffuser. Orifice plates are simpler and cheaper but create more irreversible loss.

Continuity is always present

The pressure difference in a differential-pressure meter is converted to a velocity difference through Bernoulli, and continuity links the two velocities.

5.1 Orifices and Tank Drainage

For a small orifice discharging from a large reservoir to atmosphere,

$$Q = C_d A_o \sqrt{2gh}.$$

The head h is measured from the upstream free surface to the centroid of the orifice. C_d combines contraction and velocity effects.

For a submerged orifice between two reservoirs, the driving head is the difference in free-surface elevations or equivalent piezometric head:

$$Q = C_d A_o \sqrt{2g\Delta h}.$$

Free jet

A 50 mm diameter orifice has $C_d = 0.62$ and is 3.0 m below the free surface.

$$A_o = \frac{\pi(0.05)^2}{4} = 1.963 \times 10^{-3} \text{ m}^2$$

$$Q = 0.62(1.963 \times 10^{-3})\sqrt{2(9.807)(3)} = 9.33 \times 10^{-3} \text{ m}^3/\text{s}.$$

Unsteady draining

As a tank drains, h decreases, so outflow decreases. Combine

$$A_t \frac{dh}{dt} = -C_d A_o \sqrt{2gh}$$

with separation of variables to obtain the drain time between two water levels. For constant tank area,

$$t = \frac{2A_t}{C_d A_o \sqrt{2g}} \left(\sqrt{h_1} - \sqrt{h_2} \right).$$

Steady vs unsteady

The orifice equation can be used instantaneously in both cases, but tank drainage requires an accumulation equation because the driving head changes with time.

5.2 Weirs and Flumes

Weirs are open-channel flow-measurement devices that relate discharge to upstream head. The FE handbook provides commonly used forms.

For a suppressed rectangular weir,

$$Q = CLH^{3/2}.$$

For a contracted rectangular weir,

$$Q = C(L - 0.2H)H^{3/2}.$$

For a 90° V-notch weir,

$$Q = CH^{5/2}.$$

The handbook gives unit-specific C values; use the constant that matches SI or USCS.

Suppressed rectangular weir

In SI, $C = 1.84$. For $L = 2.0$ m and $H = 0.40$ m,

$$Q = 1.84(2.0)(0.40)^{3/2} = 0.931 \text{ m}^3/\text{s}.$$

V-notch sensitivity

Because $Q \propto H^{5/2}$, a V-notch is sensitive to low flows and is useful when discharge varies over a wide range.

Flumes

A flume accelerates flow through a controlled throat. Head measured upstream is related empirically to discharge. Compared with a sharp-crested weir, a flume often causes less head loss and can pass sediment more effectively.

Measurement principle

All these devices create a predictable hydraulic control so that one easily measured depth or pressure difference can be mapped to flow rate.

5.3 Blowers, Gas Pressure Rise, and Efficiency

Environmental systems use blowers for aeration, air stripping, pneumatic transport, and gas handling. Unlike ordinary liquid-pump calculations, gas density can change significantly with pressure and temperature, so compression relationships matter.

The FE handbook gives a blower-power relationship for air in the form

$$P_W = \frac{WRT_1}{Cne} \left[\left(\frac{P_2}{P_1} \right)^n - 1 \right],$$

where W is weight flow of air, R is the engineering gas constant, T_1 is absolute inlet temperature, P_1 and P_2 are absolute pressures, $n \approx 0.283$ for air, e is efficiency, and C is the unit-conversion constant stated in the handbook.

Important requirements

Use absolute temperature and absolute pressure in compression ratios. Gauge pressure cannot be placed directly in P_2/P_1 .

Series and parallel blowers

The same qualitative rule as pumps applies: series machines primarily increase achievable pressure rise; parallel machines primarily increase flow capacity. The actual operating point still comes from the machine curve and the system resistance curve.

Efficiency effect

If ideal gas-power demand is 12 kW and blower efficiency is 75%, the required shaft power is

$$P_{shaft} = \frac{12}{0.75} = 16 \text{ kW}.$$

Pressure ratio

Convert both inlet and outlet pressures to absolute pressure before taking a ratio.

5.4 Steady and Unsteady Flow

A steady flow has no time variation at a fixed point: $\partial()/\partial t = 0$ for the flow variables of interest. Steady does not mean zero velocity. An unsteady flow changes with time, as in tank filling, storm sewer surcharging, pump startup, or reservoir drawdown.

Control-volume mass balance

The general storage form is

$$\frac{dM}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}.$$

For incompressible liquid in a tank,

$$\rho \frac{dV}{dt} = \rho Q_{in} - \rho Q_{out} \Rightarrow \frac{dV}{dt} = Q_{in} - Q_{out}.$$

Constant-area tank

With $V = A_t h$,

$$A_t \frac{dh}{dt} = Q_{in} - Q_{out}(h).$$

If outflow is through an orifice, $Q_{out} = C_d A_o \sqrt{2gh}$, producing a nonlinear differential equation.

Constant net inflow

A 50 m² tank receives 0.20 m³/s and discharges 0.05 m³/s. Then

$$\frac{dh}{dt} = \frac{0.15}{50} = 0.0030 \text{ m/s}.$$

The level rises 1.08 m in 6 minutes if the rates remain constant.

Quasi-steady reasoning

In slowly varying systems, a steady-flow equation may be applied instantaneously while a storage balance updates the changing state. Tank drainage is the standard example.

6 Integrated Problem-Solving Workflow

Many FE questions intentionally combine two or three ideas. A disciplined sequence prevents unnecessary algebra.

Closed conduit

1. Use continuity to obtain velocities from Q and diameters.
2. Determine Reynolds number and friction factor if needed.
3. Compute major and minor losses.
4. Apply Bernoulli/energy equation for pressure, pump head, or flow.

Open channel

1. Compute area, wetted perimeter, hydraulic radius, and top width.
2. Use Manning for uniform-flow relations.
3. Use Fr to classify the regime.
4. Use specific energy or momentum relations when transitions or hydraulic jumps occur.

Pump system

Build a system head curve from static lift plus friction. Set the pump curve equal to the system curve. Then compute power using $P = \rho g Q H / \eta$.

Combined check

Suppose a pump must move water through a long pipe to a reservoir 18 m higher. If friction at the target flow is 7 m and minor losses are 2 m, the pump must supply approximately

$$H_p = 18 + 7 + 2 = 27 \text{ m}$$

when both reservoirs are open to atmosphere and free-surface velocities are negligible.

Always ask

What is conserved? Mass, mechanical energy, momentum, or storage? Which terms are negligible? Which coefficient is supplied by the problem or handbook?

6.1 Formula Map, Search Terms, and Common Traps

Formula map

Problem type	Go-to relationship
Static pressure	$p_2 - p_1 = -\rho g(z_2 - z_1)$
Continuity	$Q = AV$
Energy	Bernoulli with h_p, h_t, h_L
Flow regime	$Re = VD/\nu$
Pipe friction	$h_f = f(L/D)V^2/(2g)$
Water distribution	Hazen-Williams
Uniform channel	Manning
Critical flow	$Fr = 1$ or $y_c = (q^2/g)^{1/3}$
Pump power	$P = \rho g Q H / \eta$
Orifice	$Q = C_d A \sqrt{2gh}$
Weir	$Q \propto H^{3/2}$ or $H^{5/2}$
Unsteady storage	$dV/dt = Q_{in} - Q_{out}$

Useful handbook search terms

“Moody”, “Manning”, “Hazen-Williams”, “weir”, “orifice”, “Venturi”, “blower”, “pump”, “Reynolds”, “hydraulic jump”, “specific energy”, and “NPSH”.

Common traps

- Using diameter in inches in an equation that requires feet or meters.
- Using gauge instead of absolute pressure in gas compression.
- Using top width instead of wetted perimeter in $R_H = A/P_w$.
- Using hydraulic radius where hydraulic depth is required in the Froude number.
- Treating a pump efficiency as a multiplier when input power is requested.
- Reading the Fanning friction factor instead of the Darcy factor.
- Forgetting that parallel branches have equal head loss between common nodes.
- Forgetting that a large reservoir surface has negligible velocity.

Final check

Before selecting an answer, verify sign, magnitude, units, and physical trend. A 20-second sanity check can prevent a correct method from becoming a wrong answer.

7 20 Example Problems

The following problems are original FE-style practice questions. Each includes four choices and a concise worked solution. Try each problem before reading the solution.

Example Problem 1: Hydrostatic pressure

A water tank is open to the atmosphere. What is the gauge pressure 7.5 m below the free surface? Use $\rho = 998 \text{ kg/m}^3$.

- A. 65.1 kPa
- B. 73.4 kPa
- C. 82.6 kPa
- D. 99.8 kPa

Answer: B

$$p = \rho gh = (998)(9.807)(7.5) = 73.4 \text{ kPa.}$$

Example Problem 2: Hydrostatic force on a gate

A vertical rectangular gate is 2.0 m wide and 2.4 m high, with its top at the water surface. What is the resultant hydrostatic force? Use $\rho = 998 \text{ kg/m}^3$.

- A. 28.2 kN
- B. 56.4 kN
- C. 67.7 kN
- D. 112.8 kN

Answer: B

$$A = (2.0)(2.4) = 4.8 \text{ m}^2 \text{ and centroid depth } h_c = 1.2 \text{ m. Thus } F = \rho gh_c A = (998)(9.807)(1.2)(4.8) = 56.4 \text{ kN.}$$

Example Problem 3: Differential manometer

A mercury manometer connects two points in a water line. The interface elevation difference is 0.15 m. Estimate the pressure difference. Take $\rho_{Hg} = 13,600$ and $\rho_w = 998 \text{ kg/m}^3$.

- A. 14.7 kPa
- B. 18.5 kPa
- C. 20.0 kPa
- D. 29.4 kPa

Answer: B

For equal pipe elevations, $\Delta p = (\rho_{Hg} - \rho_w)gh = (13600 - 998)(9.807)(0.15) = 18.5 \text{ kPa}$.

Example Problem 4: Continuity

Water flows at $0.050 \text{ m}^3/\text{s}$ through a 0.25 m diameter pipe that contracts to 0.125 m. What is the velocity in the smaller pipe?

- A. 1.02 m/s
- B. 2.04 m/s
- C. 4.07 m/s
- D. 8.15 m/s

Answer: C

$A_2 = \pi(0.125)^2/4 = 0.01227 \text{ m}^2$. Then $V_2 = Q/A_2 = 0.050/0.01227 = 4.07 \text{ m/s}$.

Example Problem 5: Bernoulli from reservoir to nozzle

A large reservoir discharges to atmosphere through a nozzle 8.0 m below the free surface. Neglect losses. What is the jet velocity?

- A. 8.86 m/s
- B. 12.5 m/s
- C. 15.7 m/s
- D. 19.6 m/s

Answer: B

At both free surface and jet, gauge pressure is zero; reservoir velocity is negligible. $V = \sqrt{2gh} = \sqrt{2(9.807)(8)} = 12.5 \text{ m/s}$.

Example Problem 6: Reynolds number

Water with $\nu = 1.0 \times 10^{-6} \text{ m}^2/\text{s}$ flows at 0.08 m/s in a 40 mm diameter pipe. What is the Reynolds number and regime?

- A. 320, laminar
- B. 3,200, transitional
- C. 32,000, turbulent
- D. 320,000, turbulent

Answer: B

$\text{Re} = VD/\nu = (0.08)(0.040)/(10^{-6}) = 3200$, which falls in the transitional range used by the FE handbook.

Example Problem 7: Darcy-Weisbach

A 100 m long, 0.20 m diameter pipe carries water at 2.5 m/s. If $f = 0.020$, what is the major head loss?

- A. 1.59 m
- B. 3.19 m
- C. 6.37 m
- D. 12.7 m

Answer: B

$$h_f = f(L/D)V^2/(2g) = 0.020(100/0.20)(2.5^2/[2(9.807)]) = 3.19 \text{ m.}$$

Example Problem 8: Hagen-Poiseuille

Oil with $\mu = 0.10 \text{ Pa}\cdot\text{s}$ flows at $2.0 \times 10^{-4} \text{ m}^3/\text{s}$ through a 0.030 m diameter, 8 m long tube. Assuming laminar flow, what pressure drop is predicted by Hagen-Poiseuille?

- A. 2.01 kPa
- B. 8.05 kPa
- C. 20.1 kPa
- D. 80.5 kPa

Answer: B

$$\Delta p = 128\mu LQ/(\pi D^4) = 128(0.10)(8)(2.0 \times 10^{-4})/[\pi(0.03)^4] = 8.05 \text{ kPa.}$$

Example Problem 9: Minor losses

A pipe has $V = 3.0$ m/s and fittings with total $\sum K = 4.0$. What is the total minor head loss?

- A. 0.92 m
- B. 1.84 m
- C. 3.67 m
- D. 7.34 m

Answer: B

$$h_m = \sum K V^2 / (2g) = 4(3^2) / [2(9.807)] = 1.84 \text{ m.}$$

Example Problem 10: Hazen-Williams

In a water-distribution line, all quantities except flow remain unchanged. If flow doubles, by approximately what factor does Hazen-Williams head loss increase?

- A. 2.0
- B. 2.8
- C. 3.6
- D. 4.0

Answer: C

Hazen-Williams gives $h_f \propto Q^{1.852}$, so the factor is $2^{1.852} \approx 3.61$.

Example Problem 11: Parallel pipes

Two identical pipes are installed in parallel between the same two reservoirs and together carry $0.080 \text{ m}^3/\text{s}$. Neglect interaction at junctions. What flow passes through each pipe?

- A. $0.020 \text{ m}^3/\text{s}$
- B. $0.040 \text{ m}^3/\text{s}$
- C. $0.080 \text{ m}^3/\text{s}$
- D. $0.160 \text{ m}^3/\text{s}$

Answer: B

Identical branches have the same head loss relation and the same end nodes, so the flow splits equally: $Q_1 = Q_2 = 0.080/2 = 0.040 \text{ m}^3/\text{s}$.

Example Problem 12: Pump power

A pump delivers $0.030 \text{ m}^3/\text{s}$ of water through a 35 m head rise. If pump efficiency is 75%, what shaft power is required?

- A. 10.3 kW
- B. 13.7 kW
- C. 17.8 kW
- D. 23.5 kW

Answer: B

Fluid power is $\rho g Q H = (998)(9.807)(0.030)(35) = 10.28 \text{ kW}$. Shaft power = $10.28/0.75 = 13.7 \text{ kW}$.

Example Problem 13: Pump operating point

A pump has $H_p = 40 - 1000Q^2$ and the system has $H_s = 10 + 2000Q^2$, with H in m and Q in m^3/s . What is the operating flow?

- A. $0.050 \text{ m}^3/\text{s}$
- B. $0.075 \text{ m}^3/\text{s}$
- C. $0.100 \text{ m}^3/\text{s}$
- D. $0.173 \text{ m}^3/\text{s}$

Answer: C

Set curves equal: $40 - 1000Q^2 = 10 + 2000Q^2$. Thus $30 = 3000Q^2$, $Q^2 = 0.01$, and $Q = 0.100 \text{ m}^3/\text{s}$.

Example Problem 14: Manning rectangular channel

A rectangular channel is 3 m wide, 1 m deep, with $n = 0.015$ and slope $S = 0.001$. What is the discharge?

- A. $2.84 \text{ m}^3/\text{s}$
- B. $3.16 \text{ m}^3/\text{s}$
- C. $4.50 \text{ m}^3/\text{s}$
- D. $6.32 \text{ m}^3/\text{s}$

Answer: C

$A = 3$, $P = 3 + 2 = 5$, $R_H = 0.6$. $V = (1/0.015)(0.6)^{2/3}(0.001)^{1/2} = 1.50 \text{ m/s}$. Hence $Q = AV = 4.50 \text{ m}^3/\text{s}$.

Example Problem 15: Critical depth and Froude

A 2 m wide rectangular channel carries $8 \text{ m}^3/\text{s}$. What is the critical depth?

- A. 0.74 m
- B. 1.18 m
- C. 1.48 m
- D. 2.00 m

Answer: B

Unit discharge $q = Q/b = 4 \text{ m}^2/\text{s}$. $y_c = (q^2/g)^{1/3} = (16/9.807)^{1/3} = 1.18 \text{ m}$.

Example Problem 16: Hydraulic jump

A rectangular channel has upstream depth $y_1 = 0.50 \text{ m}$ and upstream Froude number $Fr_1 = 4.0$. What is the sequent depth y_2 ?

- A. 1.00 m
- B. 1.76 m
- C. 2.59 m
- D. 4.00 m

Answer: C

$y_2/y_1 = \frac{1}{2}[\sqrt{1 + 8(4)^2} - 1] = \frac{1}{2}(\sqrt{129} - 1) = 5.179$. Therefore $y_2 = 2.59 \text{ m}$.

Example Problem 17: Orifice flow

A 40 mm diameter orifice with $C_d = 0.62$ is 2.5 m below a reservoir free surface. What is the discharge?

- A. 3.0 L/s
- B. 5.46 L/s
- C. 8.80 L/s
- D. 12.6 L/s

Answer: B

$$A = \pi(0.04)^2/4 = 1.257 \times 10^{-3} \text{ m}^2. \quad Q = C_d A \sqrt{2gh} = 0.62(1.257 \times 10^{-3}) \sqrt{2(9.807)(2.5)} = 5.46 \times 10^{-3} \text{ m}^3/\text{s} = 5.46 \text{ L/s}.$$

Example Problem 18: Rectangular weir

A suppressed rectangular weir in SI units has length 1.5 m and head 0.30 m. Using $C = 1.84$, what is the discharge?

- A. 0.227 m³/s
- B. 0.454 m³/s
- C. 0.680 m³/s
- D. 0.907 m³/s

Answer: B

$$Q = CLH^{3/2} = 1.84(1.5)(0.30)^{3/2} = 0.454 \text{ m}^3/\text{s}.$$

Example Problem 19: Venturi meter

A horizontal water Venturi contracts from area $A_1 = 0.020 \text{ m}^2$ to $A_2 = 0.010 \text{ m}^2$. The pressure drop is 15 kPa and $C_d = 1.00$. What is the flow? Use $\rho = 1000 \text{ kg/m}^3$.

- A. $0.0100 \text{ m}^3/\text{s}$
- B. $0.0200 \text{ m}^3/\text{s}$
- C. $0.0632 \text{ m}^3/\text{s}$
- D. $0.0447 \text{ m}^3/\text{s}$

Answer: C

$$A_2/A_1 = 0.5. \quad Q = A_2 \sqrt{[2\Delta p/\rho]/[1 - (A_2/A_1)^2]} = 0.010 \sqrt{30/0.75} = 0.0632 \text{ m}^3/\text{s}.$$

Example Problem 20: Blower power concept

A blower requires 18 kW of ideal fluid power at the operating point. If overall blower efficiency is 72%, what input power is required?

- A. 13.0 kW
- B. 18.0 kW
- C. 25.0 kW
- D. 32.4 kW

Answer: C

Input power is ideal/fluid power divided by efficiency: $P_{in} = 18/0.72 = 25.0 \text{ kW}$.