

Environmental Engineering FE CBT Exam

Knowledge: Groundwater, Soils, and Sediments with 15 Example Problems

Contents

1 Exam Map and Hydrogeologic Framework	4
1.1 What NCEES expects	4
1.2 The subsurface system	4
2 Soil Phase Relationships	5
2.1 Three phases	5
2.2 Unit weights	5
3 Grain Size, Plasticity, and Soil Classification	6
3.1 Grain-size parameters	6
3.2 Atterberg limits	6
3.3 Why classification matters in environmental work	6
4 Porosity, Effective Porosity, Specific Yield, and Retention	7
4.1 Total versus effective porosity	7
4.2 Specific yield and specific retention	7
4.3 Environmental significance	7
5 Hydraulic Head and Hydraulic Gradient	8
5.1 Hydraulic head	8
5.2 Hydraulic gradient	8
5.3 Potentiometric surface	8
6 Darcy's Law: Groundwater Discharge	9
6.1 Core relationship	9
6.2 Dimensional check	9
6.3 Assumptions behind the FE form	9
7 Hydraulic Conductivity and Intrinsic Permeability	10
7.1 Hydraulic conductivity	10
7.2 Intrinsic permeability	10
7.3 Qualitative trends	10
8 Laboratory Permeability Tests	11
8.1 Constant-head test	11
8.2 Falling-head test	11
8.3 Unit discipline	11
9 Layered Aquifers and Equivalent Conductivity	12
9.1 Why direction matters	12
9.2 Parallel to layers	12
9.3 Perpendicular to layers	12
9.4 Environmental application	12
10 Aquifer Types and Storage	13
10.1 Unconfined aquifer	13
10.2 Confined aquifer	13
10.3 Leaky and perched conditions	13
10.4 Conceptual test	13
11 Transmissivity, Storativity, and Specific Capacity	14
11.1 Transmissivity	14
11.2 Storativity	14
11.3 Specific capacity	14

Contents (continued)

12 Steady Unconfined Well Flow: Dupuit Formula	15
12.1 Geometry	15
12.2 Why squared heads appear	15
12.3 Assumptions	15
13 Steady Confined Well Flow: Thiem Equation	16
13.1 Thiem relationship	16
13.2 Equivalent drawdown form	16
14 Transient Confined Flow: Theis Solution	17
14.1 Theis drawdown equation	17
14.2 Physical trends	17
15 Cooper-Jacob Straight-Line Approximation	18
15.1 Late-time approximation	18
15.2 Transmissivity from one log cycle	18
15.3 Storativity from the time intercept	18
16 Superposition, Multiple Wells, and Boundary Effects	19
16.1 Superposition principle	19
16.2 Image wells	19
16.3 Capture and interference	19
17 Recharge, Water Balance, and Groundwater Gradients	20
17.1 Recharge concept	20
17.2 Groundwater budget	20
17.3 Recharge and gradients	20
18 Contaminant Transport Mechanisms	21
18.1 Advection	21
18.2 Mechanical dispersion and molecular diffusion	21
18.3 Sorption and reaction	21
19 Advection-Dispersion and Retardation	22
19.1 One-dimensional transport equation	22
19.2 Retardation factor	22
20 Partitioning: K_d, K_{oc}, K_{ow}, and Henry's Law	23
20.1 Distribution coefficient	23
20.2 Octanol-water partition coefficient	23
20.3 Henry's law	23
21 Decay, Transformation, and Plume Behavior	24
21.1 First-order decay	24
21.2 Mass versus concentration	24
21.3 Source persistence	24
22 Site Characterization and Monitoring	25
22.1 Conceptual site model	25
22.2 Monitoring wells	25
22.3 Sampling considerations	25
22.4 Data quality	25
23 Sediments, Settling, Erosion, and Stability	26
23.1 Sediment as an environmental medium	26
23.2 Settling tendency	26
23.3 Erosion and resuspension	26
24 Remediation of Soil and Groundwater	27
24.1 Pump-and-treat	27
24.2 Soil vapor extraction and air sparging	27

Contents (continued)

24.3 In-situ treatment	27
24.4 Ex-situ soil/sediment treatment	27
25 FE Formula Map and Rapid Review	28
25.1 High-yield equations	28
25.2 Decision sequence for CBT questions	28
26 15 Example Problems	29
27 Reference Basis and Exam Alignment	45

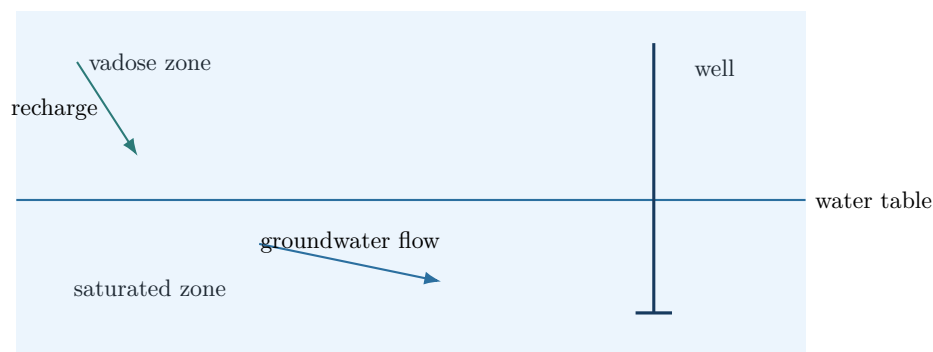
1 Exam Map and Hydrogeologic Framework

1.1 What NCEES expects

The Groundwater, Soils, and Sediments category is organized around four tasks: (1) recognize aquifer and soil properties, (2) calculate groundwater flow and velocity, (3) analyze drawdown around wells, and (4) select or evaluate remediation approaches. A productive study strategy is to connect every calculation to a physical picture of the subsurface.

1.2 The subsurface system

A typical profile contains an unsaturated (vadose) zone above the water table and a saturated zone below it. Water in the vadose zone occupies only part of the pore space; below the water table, interconnected pores are essentially water-filled. The **water table** is the surface where pore-water pressure is approximately atmospheric. A **confined aquifer** is bounded by lower-permeability material and can have a potentiometric surface above the top of the aquifer.



$$h = z + \frac{p}{\gamma_w} \quad \text{total hydraulic head} = \text{elevation head} + \text{pressure head}$$

FE exam note. Start by identifying whether the problem is about soil index properties, Darcy flow, a pumping well, or contaminant transport. This usually determines the equation family immediately.

2 Soil Phase Relationships

2.1 Three phases

Soil is represented as solids, water, and air. Let total volume be $V = V_s + V_v$, where $V_v = V_w + V_a$. The FE Reference Handbook provides the standard phase diagram and the most-used relationships.

$$\begin{aligned} e &= \frac{V_v}{V_s}, & n &= \frac{V_v}{V} = \frac{e}{1+e}, \\ S_r &= \frac{V_w}{V_v}, & w &= \frac{W_w}{W_s}, \\ G_s &= \frac{\rho_s}{\rho_w}, & \gamma_d &= \frac{W_s}{V} \end{aligned}$$

A useful identity follows directly from the phase diagram:

$$S_r e = w G_s \quad \Rightarrow \quad w = \frac{S_r e}{G_s}.$$

For saturated soil, $S_r = 1$. This makes the equation especially convenient for finding water content from void ratio and specific gravity.

2.2 Unit weights

For a saturated soil,

$$\gamma_{sat} = \frac{G_s + e}{1 + e} \gamma_w, \quad \gamma' = \gamma_{sat} - \gamma_w.$$

The submerged or effective unit weight γ' appears in seepage and effective-stress reasoning.

Interpretation: void ratio e compares void volume to solids volume; porosity n compares void volume to total volume. Porosity must be between 0 and 1, but void ratio can be greater than 1 for very loose or organic soils.

FE exam note. Keep n and e distinct. A common distractor uses $n = e$ or $n = e/(1 - e)$; the correct conversion is $n = e/(1 + e)$.

3 Grain Size, Plasticity, and Soil Classification

3.1 Grain-size parameters

The FE Handbook lists the standard gradation coefficients

$$C_u = \frac{D_{60}}{D_{10}}, \quad C_c = \frac{D_{30}^2}{D_{10}D_{60}},$$

where D_x is the particle diameter for which $x\%$ of the sample is finer by weight. Well-graded soils span a broad range of particle sizes; poorly graded soils are more uniform or gap-graded.

3.2 Atterberg limits

For fine-grained soils,

$$PI = LL - PL,$$

where LL is the liquid limit, PL is the plastic limit, and PI is the plasticity index. The Unified Soil Classification System (USCS) uses percent passing the No. 200 sieve, gradation, and Atterberg limits to assign symbols such as GW, SP, CL, or MH.

Property	Higher value generally indicates	FE interpretation
C_u	broader size distribution	potential for well-graded coarse soil
PI	greater plasticity	stronger clay-like behavior
Percent fines	more silt/clay fraction	lower permeability is often expected

3.3 Why classification matters in environmental work

Soil texture influences hydraulic conductivity, capillary retention, contaminant sorption, and the feasibility of remediation. Coarse sand and gravel usually transmit water readily and are favorable for pumping or flushing. Fine-grained soils may store substantial water but transmit it slowly, producing diffusion-limited remediation and long cleanup times.

FE exam note. If a question gives D_{10} , D_{30} , and D_{60} , expect C_u or C_c . If it gives LL and PL , expect PI or USCS interpretation.

4 Porosity, Effective Porosity, Specific Yield, and Retention

4.1 Total versus effective porosity

Total porosity n counts all void space. **Effective porosity** n_e counts the interconnected pore volume that participates in advective flow. Groundwater seepage velocity uses n_e , not necessarily total porosity.

$$v_s = \frac{q}{n_e} = \frac{Ki}{n_e}$$

4.2 Specific yield and specific retention

For an unconfined aquifer, drainage after the water table falls does not release all pore water. Some remains held by capillary forces. The storage partition is commonly written

$$n \approx S_y + S_r,$$

where S_y is specific yield and S_r is specific retention. Coarse materials tend to have larger S_y because gravity drainage is efficient. Fine materials can have high porosity but also high retention.

Important distinction: a clay can have high porosity yet very low hydraulic conductivity. Porosity measures storage space; hydraulic conductivity measures how easily water moves through connected pores.

4.3 Environmental significance

Effective porosity controls advective travel time, while sorption and diffusion can further slow contaminants relative to groundwater. In coarse aquifers, plume migration can be fast even if concentration attenuation is modest. In finer media, groundwater velocity may be slow but persistent dissolved-phase contamination can remain for long periods.

FE exam note. When the question asks for travel time, check whether it asks for Darcy velocity q or average linear/seepage velocity v_s . Use $v_s = q/n_e$ for particle travel.

5 Hydraulic Head and Hydraulic Gradient

5.1 Hydraulic head

Groundwater flows in response to differences in total hydraulic head. For low groundwater velocities, velocity head is negligible, so

$$h = z + \frac{p}{\gamma_w}.$$

The elevation z must be measured from a consistent datum. A piezometer reports hydraulic head directly as the water-level elevation relative to that datum.

5.2 Hydraulic gradient

Between two points separated by flow-path distance L ,

$$i = \frac{\Delta h}{L}.$$

The gradient is dimensionless. Flow is from high head to low head, which is why Darcy's law is often written with a negative sign.

Sign convention:

$$Q = -KA \frac{dh}{dx}.$$

If you are asked only for the magnitude of discharge, use $Q = KAi$ with $i = |\Delta h|/L$.

5.3 Potentiometric surface

In a confined aquifer, the potentiometric surface is the elevation to which water rises in a tightly cased well. It can lie above the top of the confined aquifer and can even rise above ground surface in an artesian condition.

FE exam note. Do not use vertical elevation difference alone unless the pressure-head component is already represented by measured water levels. In most piezometer problems, use water-level elevations directly to compute Δh .

6 Darcy's Law: Groundwater Discharge

6.1 Core relationship

The FE Reference Handbook provides Darcy's law as

$$Q = -KA \frac{dh}{dx}.$$

For a uniform one-dimensional flow path, the magnitude becomes

$$Q = KAi \quad \text{and} \quad q = \frac{Q}{A} = Ki$$

where K is hydraulic conductivity, A is area normal to flow, i is hydraulic gradient, and q is specific discharge (Darcy velocity).

6.2 Dimensional check

$$[K] = \frac{L}{T}, \quad [A] = L^2, \quad [i] = 1 \quad \Rightarrow \quad [Q] = \frac{L^3}{T}.$$

This check is powerful when units are mixed between seconds and days.

6.3 Assumptions behind the FE form

Darcy's law is most reliable for laminar flow through porous media. It is used extensively for groundwater because pore-scale Reynolds numbers are usually low. The law describes bulk flow; it does not say that every water particle travels at q .

Common error: using q as the groundwater particle velocity. Because flow occurs only through connected pore space, average seepage velocity is larger: $v_s = q/n_e$.

FE exam note. A Darcy-law question should usually be solvable in three steps: compute gradient, compute cross-sectional area, then apply $Q = KAi$.

7 Hydraulic Conductivity and Intrinsic Permeability

7.1 Hydraulic conductivity

Hydraulic conductivity K combines properties of both the porous medium and the fluid. The PE Environmental reference material expresses the connection to intrinsic permeability k as

$$K = \frac{\rho g k}{\mu},$$

where ρ is fluid density and μ is dynamic viscosity.

7.2 Intrinsic permeability

Intrinsic permeability is a property of the porous medium with units of area. Hydraulic conductivity has units of velocity. Consequently, the same soil can have different K values for fluids with different viscosity or density.

Quantity	Depends on	Typical role
k	pore geometry only	material property
K	pore geometry + fluid	groundwater-flow calculation
$T = Kb$	K and aquifer thickness	well/aquifer flow capacity

7.3 Qualitative trends

Coarse, well-connected pores generally give higher K . Fine-grained soils generally give lower K , even when their total porosity is substantial. Fractured rock can have low matrix permeability but high effective flow capacity through fractures.

FE exam note. If the problem provides K directly, use it. Convert units before multiplying. A value in m/s cannot be inserted into an equation whose other quantities use days unless converted.

8 Laboratory Permeability Tests

8.1 Constant-head test

The FE Handbook gives

$$K = \frac{Q}{iAt_e},$$

where Q is the total volume collected during elapsed time t_e . With specimen length L and constant head difference Δh , $i = \Delta h/L$.

8.2 Falling-head test

For lower-permeability materials, the head decreases with time:

$$K = 2.303 \frac{aL}{At_e} \log_{10} \left(\frac{h_1}{h_2} \right),$$

where a is standpipe area, A is specimen area, and h_1, h_2 are initial and final heads.

Why the equations differ: constant-head testing keeps the driving gradient essentially constant. Falling-head testing integrates a gradually changing driving head, producing the logarithmic term.

8.3 Unit discipline

Use one length unit throughout the falling-head equation. If a , A , L , h_1 , and h_2 are in centimeters and t is in seconds, K is obtained in cm/s.

FE exam note. In a constant-head test, Q means collected *volume*, not volumetric flow rate, in the handbook form $K = Q/(iAt_e)$. This distinction is a frequent source of factor-of-time errors.

9 Layered Aquifers and Equivalent Conductivity

9.1 Why direction matters

Layering produces anisotropy even if each layer is individually isotropic. Flow parallel to bedding can preferentially use the most conductive layers. Flow perpendicular to bedding must cross every layer and is restricted by the least conductive material.

9.2 Parallel to layers

For horizontal flow through layers of thickness b_m ,

$$K_{h,avg} = \frac{\sum K_m b_m}{\sum b_m}.$$

This is a thickness-weighted arithmetic mean.

9.3 Perpendicular to layers

For vertical flow,

$$K_{v,avg} = \frac{\sum b_m}{\sum (b_m / K_m)}.$$

This is a thickness-weighted harmonic mean.

Interpretation: the harmonic mean is strongly controlled by the smallest K . A thin clay layer can dramatically reduce vertical leakage even if sand layers above and below it are highly permeable.

9.4 Environmental application

Layering influences plume shape, well capture, landfill-liner leakage, and the effectiveness of in-situ treatment. Preferential horizontal flow can spread contaminants laterally while vertical migration remains limited by low- K units.

FE exam note. Use arithmetic-type averaging for flow parallel to layers and harmonic-type averaging for flow perpendicular to layers.

10 Aquifer Types and Storage

10.1 Unconfined aquifer

The upper boundary is the water table. Pumping lowers the water table and dewateres pore space. Storage is governed mainly by specific yield, so unconfined storage coefficients are relatively large compared with confined aquifers.

10.2 Confined aquifer

A confined aquifer is bounded by lower-permeability units. Pumping lowers the potentiometric surface without immediately dewatering the aquifer thickness. Water is released primarily through compressibility of water and the aquifer skeleton.

10.3 Leaky and perched conditions

A leaky aquifer receives flow through a semiconfining unit. A perched water table can occur above the regional water table where local low-permeability material impedes downward movement.

Storage coefficient (storativity) S : volume of water released from or taken into storage per unit aquifer area per unit change in hydraulic head. It is dimensionless.

10.4 Conceptual test

When a confined well is pumped, the drawdown cone develops in the potentiometric surface. In an unconfined aquifer, the water table itself declines and the saturated thickness changes.

FE exam note. Before using a well equation, identify whether the aquifer is confined or unconfined and whether the question assumes steady or transient flow.

11 Transmissivity, Storativity, and Specific Capacity

11.1 Transmissivity

The FE Handbook defines transmissivity as

$$T = Kb$$

where b is saturated aquifer thickness. If K is in m/day and b is in m, T is in m²/day. Transmissivity summarizes the ability of the entire saturated thickness to transmit water.

11.2 Storativity

Storativity S controls how rapidly a transient pumping disturbance propagates. It appears in Theis and Cooper-Jacob relationships through the dimensionless parameter

$$u = \frac{r^2 S}{4Tt}.$$

Small S or long pumping time makes u smaller, generally increasing the extent of the drawdown influence.

11.3 Specific capacity

The FE Handbook identifies specific capacity as

$$\frac{Q}{D_w},$$

where D_w is well drawdown. It is an empirical performance measure for a particular well and pumping condition, often reported in gpm/ft or m³/day per m.

Specific capacity is not the same as aquifer transmissivity, although higher-transmissivity aquifers often support higher specific capacity when well losses are comparable.

FE exam note. If a problem asks only for specific capacity, do not overcomplicate it: divide the pumping rate by drawdown using consistent units.

12 Steady Unconfined Well Flow: Dupuit Formula

12.1 Geometry

For a fully penetrating well in an unconfined aquifer under idealized steady radial flow, the FE Handbook provides Dupuit's formula:

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln(r_2/r_1)}.$$

Here h_1 and h_2 are saturated thicknesses measured from the impermeable aquifer base at radii r_1 and r_2 .

12.2 Why squared heads appear

In unconfined flow, saturated thickness changes with radius. The radial flow area therefore depends on the local water-table elevation. Integrating this changing flow area produces the h^2 terms.

Check the sign: for a pumping well, $h_2 > h_1$ and $r_2 > r_1$, so both numerator and denominator are positive and $Q > 0$ as a pumping-rate magnitude.

12.3 Assumptions

Typical Dupuit assumptions include homogeneous and isotropic aquifer properties, predominantly horizontal flow, steady state, negligible well radius effects beyond the chosen r_1 , and a fully penetrating well.

FE exam note. If the problem gives an unconfined well and two observation radii with saturated thicknesses, look for the squared-head form.

13 Steady Confined Well Flow: Thiem Equation

13.1 Thiem relationship

For steady radial flow in a confined aquifer, the FE Handbook gives

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}.$$

Because aquifer thickness is approximately constant, transmissivity $T = Kb$ can be taken outside the radial integration.

13.2 Equivalent drawdown form

If $s = H - h$, then differences in head can be replaced by differences in drawdown. Between two radii,

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln(r_2/r_1)}.$$

The closer observation point generally has the greater drawdown.

Case	Main variable	FE steady equation cue
Unconfined	saturated thickness varies	$h_2^2 - h_1^2$
Confined	thickness nearly constant	$h_2 - h_1$ with T

FE exam note. Use natural logarithm in the handbook forms. If you use base-10 logarithms, the numerical coefficient must be changed accordingly.

14 Transient Confined Flow: Theis Solution

Reference basis. The FE Environmental specification explicitly lists Theis drawdown. The FE Reference Handbook identifies the topic but does not print the full Theis equation in the groundwater pages; therefore this page is supplemental exam knowledge.

14.1 Theis drawdown equation

For an ideal confined aquifer subjected to constant-rate pumping,

$$s(r, t) = \frac{Q}{4\pi T} W(u), \quad u = \frac{r^2 S}{4Tt},$$

where $W(u)$ is the Theis well function

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy.$$

14.2 Physical trends

Drawdown increases with pumping rate Q and time t , and decreases with transmissivity T and radial distance r . Storativity controls how quickly the pressure response spreads through the aquifer.

For many FE-style problems, $W(u)$ may be provided, tabulated, approximated, or the problem may give enough information to use the Cooper-Jacob approximation instead.

FE exam note. Compute u first. Then use the given or approximated $W(u)$ and substitute into $s = QW(u)/(4\pi T)$. Keep Q and T on the same time basis.

15 Cooper-Jacob Straight-Line Approximation

Reference basis. The FE Environmental specification lists Jacob drawdown. This approximation is supplemental theory used to interpret late-time pumping-test data.

15.1 Late-time approximation

For sufficiently small u , the Theis solution can be approximated by

$$s = \frac{2.3Q}{4\pi T} \log_{10} \left(\frac{2.25Tt}{r^2 S} \right).$$

At a fixed observation well, drawdown plotted against $\log_{10} t$ becomes approximately linear.

15.2 Transmissivity from one log cycle

If Δs is the drawdown increase over one log cycle of time,

$$T = \frac{2.3Q}{4\pi \Delta s}$$

15.3 Storativity from the time intercept

If the straight line extrapolates to $s = 0$ at time t_0 ,

$$S = \frac{2.25Tt_0}{r^2}.$$

The approximation is a late-time/small- u method. It is not universally valid at very early pumping times, near complex boundaries, or when aquifer assumptions are violated.

FE exam note. If a semilog pumping-test question gives “drawdown per log cycle,” the shortcut for T is usually the intended route.

16 Superposition, Multiple Wells, and Boundary Effects

16.1 Superposition principle

For linear confined-aquifer drawdown models, the effects of multiple pumping or injection wells can be added algebraically:

$$s_{total} = \sum_j s_j.$$

Pumping contributions are treated as positive drawdown; injection can be represented as negative pumping (recovery or buildup).

16.2 Image wells

Idealized boundaries can be represented with image wells. A no-flow boundary is modeled by an image pumping well of the same sign; a constant-head boundary is modeled by an image well of opposite sign. The method converts a boundary problem into a superposition problem.

16.3 Capture and interference

When cones of depression overlap, each well experiences drawdown caused by its own pumping plus neighboring wells. This can reduce available drawdown margin and well yield.

Concept check: superposition changes drawdown, not aquifer properties. T and S remain properties of the aquifer model unless pumping changes the physical system enough to violate the assumptions.

FE exam note. When several wells operate simultaneously, calculate each contribution at the point of interest using its own radius, then add the drawdowns.

17 Recharge, Water Balance, and Groundwater Gradients

17.1 Recharge concept

Recharge is water that crosses the water table and replenishes groundwater storage. Sources include infiltration from precipitation, irrigation return flow, losing streams, and engineered infiltration. Discharge occurs to wells, streams, springs, evapotranspiration from shallow groundwater, and other aquifers.

17.2 Groundwater budget

A control-volume statement is

$$\Delta S = I - O,$$

where I and O are inflow and outflow volumes over the selected time interval. For a long-term steady condition, $\Delta S \approx 0$ and inflow approximately balances outflow.

17.3 Recharge and gradients

Recharge can create groundwater mounds and alter local gradients. Pumping does the opposite by creating cones of depression. Flow lines tend to cross equipotential lines approximately at right angles in isotropic media.

A groundwater divide separates flow systems, much like a surface-water watershed divide. However, groundwater divides need not coincide with surface topography and can shift when pumping changes hydraulic heads.

FE exam note. When a problem mixes volume and rate, write a time-based balance first. Convert recharge depth over area to volume before comparing it with pumping volume.

18 Contaminant Transport Mechanisms

18.1 Advection

Advection moves dissolved contaminants with the average groundwater motion. A first estimate of travel time is

$$t = \frac{L}{v_s}, \quad v_s = \frac{Ki}{n_e}.$$

18.2 Mechanical dispersion and molecular diffusion

Velocity variations within pores spread a solute plume even if all water moves in the same general direction. Molecular diffusion moves solute down concentration gradients and can be important in low-velocity or low-permeability zones.

18.3 Sorption and reaction

Sorption to soil or sediment slows some contaminants relative to groundwater. Chemical or biological transformation can reduce parent-compound mass, while daughter products may appear.

Process	Main effect	Typical control
Advection	translates plume	seepage velocity
Dispersion	spreads plume	heterogeneity / dispersivity
Diffusion	moves down concentration gradient	molecular diffusion coefficient
Sorption	retards dissolved migration	K_d , organic carbon, mineral surfaces
Decay	reduces parent concentration	reaction rate constant

FE exam note. Separate hydraulic travel from contaminant travel. Groundwater velocity comes from Darcy flow; contaminant velocity may then be divided by a retardation factor.

19 Advection-Dispersion and Retardation

19.1 One-dimensional transport equation

A common conceptual model for a reacting dissolved contaminant is

$$R \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - v \frac{\partial C}{\partial x} - \lambda RC,$$

where D is a dispersion coefficient, v is average linear groundwater velocity, R is retardation factor, and λ is a first-order decay constant.

19.2 Retardation factor

The environmental reference material gives

$$R = 1 + \frac{\rho_b}{n} K_d$$

where ρ_b is dry bulk density, n is porosity, and K_d is the soil-water distribution coefficient. Units of ρ_b and K_d must be chosen so their product is dimensionless after division by n .

For a linear equilibrium sorption model, the contaminant migration velocity is approximately

$$v_c = \frac{v}{R}.$$

Thus $R = 1$ indicates no retardation, while $R > 1$ indicates slower contaminant migration than groundwater.

FE exam note. Check density units carefully. A convenient pairing is ρ_b in kg/L and K_d in L/kg.

20 Partitioning: K_d , K_{oc} , K_{ow} , and Henry's Law

20.1 Distribution coefficient

For equilibrium linear sorption,

$$K_d = \frac{C_{soil}}{C_{water}}.$$

The organic-carbon normalized partition coefficient is related by

$$K_d = K_{oc}f_{oc}$$

where f_{oc} is the mass fraction of organic carbon in soil or sediment.

20.2 Octanol-water partition coefficient

$$K_{ow} = \frac{C_{octanol}}{C_{water}}.$$

Large K_{ow} values usually indicate greater hydrophobicity and a greater tendency for organic chemicals to partition away from water into organic phases.

20.3 Henry's law

Henry's law relates dissolved and gas-phase concentrations at equilibrium. A relatively large Henry's-law tendency supports volatilization-based approaches such as air stripping or soil vapor extraction when other site conditions are favorable.

Remediation interpretation: large K_d or K_{oc} can reduce dissolved mobility but may increase the amount of contaminant mass retained on solids, extending cleanup time. High volatility can make vapor-phase recovery effective in the vadose zone.

FE exam note. For a retardation question with K_{oc} and organic carbon fraction, first calculate K_d , then calculate R .

21 Decay, Transformation, and Plume Behavior

21.1 First-order decay

A frequently used screening model is

$$C(t) = C_0 e^{-\lambda t}, \quad \lambda = \frac{\ln 2}{t_{1/2}}.$$

If a contaminant moves while decaying, the travel time used in the exponential should reflect the relevant transport velocity, including retardation when appropriate.

21.2 Mass versus concentration

A decreasing concentration does not always mean contaminant mass is destroyed. Dilution and dispersion can lower concentration without reducing total mass. Biodegradation, hydrolysis, radioactive decay, or other reactions can reduce parent mass.

21.3 Source persistence

Residual NAPL or strongly sorbed mass can act as a continuing source. Pump-and-treat systems often remove dissolved mass quickly at first, then exhibit concentration rebound as slower domains release stored contaminant.

CBT reasoning: distinguish physical removal, phase transfer, immobilization, and destruction. For example, activated carbon transfers contaminant to a solid phase; incineration or complete biodegradation can destroy the parent compound.

FE exam note. If half-life is given, convert it to λ before applying a first-order model unless the problem offers a direct half-life relationship.

22 Site Characterization and Monitoring

22.1 Conceptual site model

A defensible subsurface investigation connects source, release mechanism, hydrostratigraphy, transport pathways, receptors, and time. Characterization should identify both physical controls (stratigraphy, heads, K) and chemical controls (phase, solubility, sorption, degradation).

22.2 Monitoring wells

A well network is designed to define background conditions, plume extent, migration direction, and vertical distribution. Water-level measurements should be referenced to surveyed elevations so hydraulic heads can be compared among wells.

22.3 Sampling considerations

Representative soil and groundwater data require appropriate locations, depths, quality-control samples, and methods that minimize cross-contamination. Field measurements can include pH, conductivity, temperature, dissolved oxygen, and oxidation-reduction potential when relevant to the contaminant and remedy.

Hydraulic interpretation: use measured heads to map equipotential contours. Groundwater flow is approximately perpendicular to equipotential contours in an isotropic aquifer and directed from high head to low head.

22.4 Data quality

The FE Handbook includes statistical and data-quality material elsewhere. In groundwater work, measurement uncertainty matters because small head differences can translate into uncertain gradients and flow directions.

FE exam note. When asked where to place a downgradient well, first determine the direction of decreasing hydraulic head, not simply the direction of decreasing ground-surface elevation.

23 Sediments, Settling, Erosion, and Stability

23.1 Sediment as an environmental medium

Sediment can store hydrophobic organic contaminants and metals, especially where fine particles and organic matter accumulate. Resuspension can return contaminants to the water column; deposition can bury contaminated layers.

23.2 Settling tendency

For small spherical particles in laminar settling, Stokes' law gives

$$v_t = \frac{g(\rho_p - \rho_f)d^2}{18\mu}.$$

Settling velocity increases strongly with particle diameter because of the d^2 dependence in the Stokes regime.

23.3 Erosion and resuspension

Erosion begins when bed shear stress exceeds a critical value that depends on particle size, cohesion, consolidation, and biological effects. Fine cohesive sediments can behave very differently from noncohesive sand.

Environmental implication: remediation options for contaminated sediments include monitored natural recovery, capping, dredging/excavation, and treatment or disposal. Remedy selection depends on contaminant mobility, benthic exposure, resuspension risk, and long-term stability.

FE exam note. For a Stokes-law calculation, verify that the problem intends laminar settling and use consistent density and viscosity units.

24 Remediation of Soil and Groundwater

24.1 Pump-and-treat

Extraction wells hydraulically capture contaminated groundwater and send it to ex-situ treatment. It is useful for plume containment and dissolved-phase mass removal but can become slow when contaminants are strongly sorbed, trapped in low-permeability zones, or sustained by NAPL.

24.2 Soil vapor extraction and air sparging

Soil vapor extraction (SVE) applies vacuum to the vadose zone and is most effective for volatile contaminants in permeable unsaturated soils. Air sparging injects air below the water table, enhancing volatilization and sometimes aerobic biodegradation; SVE is often paired with it to capture vapors.

24.3 In-situ treatment

Common families include bioremediation, chemical oxidation or reduction, permeable reactive barriers, enhanced flushing, thermal treatment, and immobilization. Effectiveness depends on delivering reagents to contaminated zones.

24.4 Ex-situ soil/sediment treatment

Excavation or dredging allows direct handling and may be followed by soil washing, stabilization/solidification, thermal treatment, biological treatment, or off-site disposal.

Method	Best suited to	Key limitation
Pump-and-treat	dissolved groundwater plume	slow tailing / rebound
SVE	volatile vadose-zone contaminants	poor in tight/wet soils
Bioremediation	biodegradable contaminants	needs favorable geochemistry
Capping	contaminated sediment isolation	long-term integrity
Excavation/dredging	accessible source mass	handling/disposal impacts

FE exam note. Match the technology to contaminant phase, volatility, biodegradability, and subsurface permeability.

25 FE Formula Map and Rapid Review

25.1 High-yield equations

$$\begin{aligned}
 n &= \frac{e}{1+e}, & S_r e &= w G_s, \\
 i &= \frac{\Delta h}{L}, & Q &= K A i, \\
 q &= K i, & v_s &= \frac{q}{n_e}, \\
 T &= K b, & \text{specific capacity} &= \frac{Q}{D_w}, \\
 Q_{Dupuit} &= \frac{\pi K (h_2^2 - h_1^2)}{\ln(r_2/r_1)}, & Q_{Thiem} &= \frac{2\pi T (h_2 - h_1)}{\ln(r_2/r_1)}, \\
 u &= \frac{r^2 S}{4 T t}, & s_{Theis} &= \frac{Q W(u)}{4\pi T}, \\
 T_{Jacob} &= \frac{2.3Q}{4\pi \Delta s}, & R &= 1 + \frac{\rho_b}{n} K_d, \\
 K_d &= K_{oc} f_{oc}, & v_c &= \frac{v_s}{R}.
 \end{aligned}$$

25.2 Decision sequence for CBT questions

1. Identify the physical system: soil index problem, regional Darcy flow, steady well flow, transient pumping, transport, or remediation.
2. Sketch head locations and define the datum or radii.
3. Convert all units before substitution, especially seconds versus days.
4. Decide whether velocity means Darcy velocity or seepage velocity.
5. For wells, decide confined/unconfined and steady/transient.
6. For contaminants, calculate hydraulic travel first, then apply retardation or decay if needed.
7. Sanity-check the answer: flow follows decreasing head, seepage velocity exceeds Darcy velocity, and $R \geq 1$ for positive sorption.

Reference basis. Exam specifications: Groundwater, Soils, and Sediments - basic hydrogeology; groundwater flow; drawdown (Dupuit, Jacob, Theis, Thiem); and soil/sediment/groundwater remediation. FE Reference Handbook 10.6 supplies the core soil, permeability, Darcy, transmissivity, storativity, Dupuit, Thiem, partitioning, and remediation relationships used throughout this guide.

26 15 Example Problems

Groundwater, Soils, and Sediments

Each problem is designed to reinforce a high-yield calculation or interpretation from the 25 study pages. Solutions show the equation setup, unit handling, numerical work, and a short CBT-focused check.

Reference basis: NCEES FE Reference Handbook 10.6 and FE Environmental CBT Exam Specifications. The Theis and Cooper-Jacob equations are included as supplemental theory because the exam specification names these methods while the reference handbook does not print their full equations in the groundwater section.

Example Problem 1: Soil phase relationships

Problem. A partially saturated soil has void ratio $e = 0.65$, specific gravity of solids $G_s = 2.68$, and degree of saturation $S_r = 75\%$. Determine (a) porosity and (b) gravimetric water content.

Solution

Porosity follows from the void-ratio relationship:

$$n = \frac{e}{1 + e} = \frac{0.65}{1.65} = 0.3939.$$

Thus

$$n \approx 0.394 \text{ (39.4\%) }.$$

Use the phase identity $S_r e = w G_s$:

$$w = \frac{S_r e}{G_s} = \frac{(0.75)(0.65)}{2.68} = 0.1819.$$

Therefore

$$w \approx 18.2\%.$$

Check. Porosity is less than 1 and is smaller than the void ratio. Water content is plausible for a partially saturated mineral soil.

Example Problem 2: Darcy discharge

Problem. Hydraulic conductivity is $K = 3.0 \times 10^{-4}$ m/s. Two piezometers 80 m apart along the flow direction have heads of 18.6 m and 15.0 m. The aquifer cross-sectional area normal to flow is 12 m^2 . Find the groundwater discharge magnitude.

Solution

$$i = \frac{18.6 - 15.0}{80} = 0.045.$$

Darcy's law gives

$$Q = KAi = (3.0 \times 10^{-4})(12)(0.045) = 1.62 \times 10^{-4} \text{ m}^3/\text{s}.$$

Converting to m^3/day :

$$Q = (1.62 \times 10^{-4})(86400) = 14.0 \text{ m}^3/\text{day}.$$

$$Q = 1.62 \times 10^{-4} \text{ m}^3/\text{s} \approx 14.0 \text{ m}^3/\text{day}$$

FE exam note. The positive answer is a magnitude; actual flow is from the 18.6 m head toward the 15.0 m head.

Example Problem 3: Seepage velocity and travel time

Problem. A groundwater system has Darcy velocity $q = 2.5 \times 10^{-5}$ m/s and effective porosity $n_e = 0.28$. Estimate the average seepage velocity and the travel time for water to move 150 m.

Solution

$$v_s = \frac{q}{n_e} = \frac{2.5 \times 10^{-5}}{0.28} = 8.93 \times 10^{-5} \text{ m/s.}$$

Then

$$t = \frac{150}{8.93 \times 10^{-5}} = 1.68 \times 10^6 \text{ s.}$$

Convert to days:

$$t = \frac{1.68 \times 10^6}{86400} = 19.4 \text{ days.}$$

$v_s = 8.93 \times 10^{-5} \text{ m/s,} \quad t \approx 19.4 \text{ days}$
--

Why not use q directly? Darcy velocity is based on the full bulk cross-sectional area. Actual flow is concentrated in connected pore space.

Example Problem 4: Constant-head permeability test

Problem. In a constant-head test, 0.012 m^3 of water is collected in 180 s. The soil specimen is 0.50 m long with cross-sectional area 0.040 m^2 , and the constant head difference is 0.35 m. Determine K .

Solution

$$i = \frac{0.35}{0.50} = 0.70.$$

Using the FE Handbook form,

$$K = \frac{Q}{iAt_e} = \frac{0.012}{(0.70)(0.040)(180)} = 2.38 \times 10^{-3} \text{ m/s}.$$

$$K \approx 2.38 \times 10^{-3} \text{ m/s}$$

FE exam note. Here Q is collected volume, not discharge rate. The explicit t_e in the denominator confirms this interpretation.

Example Problem 5: Falling-head permeability test

Problem. A falling-head test uses a standpipe area $a = 4.0 \text{ cm}^2$ and specimen area $A = 80 \text{ cm}^2$. The soil specimen length is 20 cm. Over 300 s, the head drops from 90 cm to 30 cm. Determine K in cm/s.

Solution

$$K = 2.303 \frac{aL}{At} \log_{10} \left(\frac{h_1}{h_2} \right).$$

Substitute:

$$K = 2.303 \frac{(4)(20)}{(80)(300)} \log_{10}(3).$$

Since $\log_{10}(3) = 0.4771$,

$$K = 3.66 \times 10^{-3} \text{ cm/s}.$$

$$K \approx 3.66 \times 10^{-3} \text{ cm/s} = 3.66 \times 10^{-5} \text{ m/s}$$

Unit check. All areas and lengths were kept in centimeters, so the resulting conductivity is in cm/s.

Example Problem 6: Equivalent hydraulic conductivity

Problem. An aquifer consists of a 4 m layer with $K_1 = 1.0 \times 10^{-3}$ m/s over a 6 m layer with $K_2 = 2.0 \times 10^{-5}$ m/s. Find the equivalent hydraulic conductivity for (a) horizontal flow parallel to layers and (b) vertical flow perpendicular to layers.

Solution

Horizontal:

$$K_h = \frac{K_1 b_1 + K_2 b_2}{b_1 + b_2} = \frac{(10^{-3})(4) + (2 \times 10^{-5})(6)}{10} = 4.12 \times 10^{-4} \text{ m/s.}$$

Vertical:

$$K_v = \frac{10}{4/(10^{-3}) + 6/(2 \times 10^{-5})} = 3.29 \times 10^{-5} \text{ m/s.}$$

$$K_h = 4.12 \times 10^{-4} \text{ m/s,} \quad K_v = 3.29 \times 10^{-5} \text{ m/s}$$

FE exam note. The much smaller vertical value shows how the low-permeability layer controls cross-layer leakage.

Example Problem 7: Transmissivity

Problem. A confined aquifer has $K = 8.0 \times 10^{-5}$ m/s and thickness $b = 18$ m. Determine (a) transmissivity in m^2/day and (b) discharge per meter of aquifer width under hydraulic gradient $i = 0.006$.

Solution

$$T = Kb = (8.0 \times 10^{-5})(18) = 1.44 \times 10^{-3} \text{ m}^2/\text{s}.$$

Convert to daily units:

$$T = (1.44 \times 10^{-3})(86400) = 124.4 \text{ m}^2/\text{day}.$$

For unit width, discharge per width is $q' = Ti$:

$$q' = (124.4)(0.006) = 0.746 \text{ m}^2/\text{day}.$$

$$\boxed{T \approx 124 \text{ m}^2/\text{day}, \quad q' \approx 0.746 \text{ m}^2/\text{day}}$$

Example Problem 8: Dupuit unconfined-well flow

Problem. A well pumps steadily from an unconfined aquifer with $K = 2.0 \times 10^{-4}$ m/s. The saturated thickness is 10.0 m at $r_1 = 0.15$ m and 13.0 m at $r_2 = 60$ m. Estimate the pumping rate using Dupuit's formula.

Solution

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln(r_2/r_1)}.$$
$$Q = \frac{\pi (2.0 \times 10^{-4}) (13^2 - 10^2)}{\ln(60/0.15)}.$$

Now $13^2 - 10^2 = 69$ and $\ln(400) = 5.991$:

$$Q = 7.24 \times 10^{-3} \text{ m}^3/\text{s}.$$

$$Q \approx 7.24 \times 10^{-3} \text{ m}^3/\text{s} \approx 626 \text{ m}^3/\text{day}$$

FE exam note. Unconfined steady flow uses the squared saturated-thickness difference.

Example Problem 9: Thiem confined-well flow

Problem. A confined aquifer has transmissivity $T = 250 \text{ m}^2/\text{day}$. During steady pumping, the potentiometric head is **24.3 m at $r_1 = 0.25 \text{ m}$ and 27.5 m at $r_2 = 100 \text{ m}$. Find the pumping rate**

Solution

$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}.$$
$$Q = \frac{2\pi(250)(27.5 - 24.3)}{\ln(100/0.25)} = \frac{5026.5}{5.991} = 839 \text{ m}^3/\text{day}.$$

$$Q \approx 8.39 \times 10^2 \text{ m}^3/\text{day}$$

Check. Both head difference and log radius ratio are positive, giving a positive pumping-rate magnitude.

Example Problem 10: Specific capacity

Problem. A pumping well produces 180 gpm after the water level stabilizes at a drawdown of 12 ft. Determine the well specific capacity.

Solution

The FE Handbook defines specific capacity as pumping rate divided by drawdown:

$$SC = \frac{Q}{D_w} = \frac{180 \text{ gpm}}{12 \text{ ft}}.$$

$$SC = 15 \text{ gpm/ft}$$

Interpretation

This is a performance metric for the well under the tested condition. It should not automatically be treated as a fundamental aquifer property because well efficiency and well losses can affect the observed drawdown.

FE exam note. This is intentionally a one-step question. Do not replace the requested specific capacity with transmissivity unless the problem explicitly asks for an aquifer-property estimate.

Example Problem 11: Theis transient drawdown

Problem. A confined aquifer is pumped at $Q = 900 \text{ m}^3/\text{day}$. Transmissivity is $T = 120 \text{ m}^2/\text{day}$ and storativity is $S = 1.5 \times 10^{-4}$. At $r = 100 \text{ m}$ after $t = 2.0$ days, calculate drawdown. For the resulting u , use $W(u) = 5.89$.

Solution

$$u = \frac{r^2 S}{4Tt} = \frac{(100)^2 (1.5 \times 10^{-4})}{4(120)(2)} = 1.56 \times 10^{-3}.$$

Then

$$s = \frac{QW(u)}{4\pi T} = \frac{(900)(5.89)}{4\pi(120)} = 3.52 \text{ m}.$$

$$\boxed{s \approx 3.52 \text{ m}}$$

FE exam note. The well function is dimensionless. Keep Q and T on the same time basis so drawdown comes out in units of length.

Example Problem 12: Cooper-Jacob pumping-test analysis

Problem. A confined aquifer is pumped at $800 \text{ m}^3/\text{day}$. At an observation well, drawdown increases by 1.40 m for each log cycle of time. The straight line extrapolates to zero drawdown at $t_0 = 0.015 \text{ day}$. The observation well is 60 m from the pumping well. Estimate T and S .

Solution

$$T = \frac{2.3Q}{4\pi\Delta s} = \frac{2.3(800)}{4\pi(1.40)} = 104.6 \text{ m}^2/\text{day}.$$

Then

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25(104.6)(0.015)}{60^2} = 9.81 \times 10^{-4}.$$

$$T \approx 105 \text{ m}^2/\text{day}, \quad S \approx 9.8 \times 10^{-4}$$

FE exam note. Storativity is dimensionless. A numerical answer with length or time units attached to S indicates a unit mistake.

Example Problem 13: Partitioning and retardation

Problem. For an organic contaminant, $K_{oc} = 120$ L/kg and soil organic-carbon fraction is $f_{oc} = 0.015$. Soil bulk density is 1.65 kg/L and porosity is $n = 0.32$. Calculate K_d and the retardation factor R .

Solution

$$K_d = K_{oc}f_{oc} = (120)(0.015) = 1.80 \text{ L/kg.}$$

Then

$$R = 1 + \frac{\rho_b}{n}K_d = 1 + \frac{1.65}{0.32}(1.80) = 10.28.$$

$$K_d = 1.80 \text{ L/kg,} \quad R \approx 10.3$$

Unit pairing. kg/L multiplied by L/kg is dimensionless, as required inside the retardation expression.

Example Problem 14: Retarded contaminant travel time

Problem. Average groundwater seepage velocity is 0.20 m/day. A contaminant has retardation factor $R = 5.0$. Neglecting dispersion and decay, estimate the time for the contaminant center of mass to travel 80 m.

Solution

The retarded contaminant velocity is

$$v_c = \frac{v}{R} = \frac{0.20}{5.0} = 0.040 \text{ m/day.}$$

Therefore

$$t = \frac{80}{0.040} = 2000 \text{ days} = 5.48 \text{ yr.}$$

$$t \approx 2000 \text{ days} \approx 5.5 \text{ years}$$

FE exam note. Retardation slows solute migration relative to groundwater. If your retarded travel time is shorter than the water travel time, revisit the calculation.

Example Problem 15: Pump-and-treat mass removal

Problem. A pump-and-treat system extracts 250 m³/day of groundwater containing 1.8 mg/L of a contaminant. The treatment process removes 85% of influent contaminant mass. Estimate the contaminant mass removed in 30 days.

Solution

Convert flow to liters per day:

$$250 \text{ m}^3/\text{day} = 250,000 \text{ L/day}.$$

Influent contaminant mass rate is

$$(250,000)(1.8) = 450,000 \text{ mg/day}.$$

Removed mass rate:

$$(450,000)(0.85) = 382,500 \text{ mg/day}.$$

Over 30 days:

$$M = (382,500)(30) = 11,475,000 \text{ mg} = 11.475 \text{ kg}.$$

$$M \approx 11.5 \text{ kg removed in 30 days}$$

Interpretation. Pump-and-treat performance depends on both hydraulic capture and mass transfer from sorbed, NAPL, or low-permeability zones; a simple mass-rate calculation does not guarantee plume cleanup.

27 Reference Basis and Exam Alignment

Source	Use in this guide
NCEES FE Environmental CBT Exam Specifications	Topic boundaries and question range: Groundwater, Soils, and Sediments; basic hydrogeology; groundwater flow; drawdown; remediation.
NCEES FE Reference Handbook, Version 10.6	Soil phase relationships and classification; hydraulic conductivity tests; Darcy's law; seepage velocity; transmissivity; storativity; Dupuit and Thiem equations; environmental partitioning and site-remediation relationships.
Supplemental groundwater theory	Theis transient drawdown and Cooper-Jacob late-time approximation, included because both are explicitly named in the FE Environmental exam specification.

Final review strategy: practice recognizing which equation family applies before reaching for the calculator. On CBT questions, equation selection and unit consistency are often more important than lengthy algebra.