

# Environmental Engineering FE CBT Exam

## Knowledge: Thermodynamics with 8 Example Problems

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# 1 Thermodynamics for FE Environmental

**Exam scope.** The FE Environmental CBT specification assigns approximately 3–5 questions to thermodynamics. The listed areas are: thermodynamic laws; energy, heat, and work; efficiencies and coefficients of performance; energy cycles and conversion; conduction, convection, and radiation; and behavior of ideal gases.

The goal is not to memorize a large thermodynamics course. Focus on recognizing the correct control volume or closed system, using the energy balance consistently, working in absolute temperature where required, and selecting the right heat-transfer mode.

## 1.1 Core vocabulary

- **System:** quantity of matter or region chosen for analysis.
- **Closed system:** no mass crosses the boundary; energy may cross as heat or work.
- **Open system/control volume:** mass and energy may cross the control surface.
- **State:** condition specified by properties such as  $P$ ,  $T$ ,  $V$ ,  $u$ ,  $h$ , and  $s$ .
- **Intensive property:** independent of system size (e.g.,  $T$ ,  $P$ , density).
- **Extensive property:** scales with mass (e.g.,  $U$ ,  $V$ , total enthalpy).
- **Specific property:** extensive property per unit mass, e.g.  $u = U/m$ ,  $v = V/m$ .

## 1.2 Frequently used energy quantities

$$E = U + KE + PE = U + \frac{mV^2}{2} + mgz$$

where  $U$  is internal energy,  $V$  is speed, and  $z$  is elevation. In many FE problems, changes in kinetic and potential energy are negligible unless velocities or elevations are emphasized.

## 1.3 Temperature scales

$$T(K) = T(^{\circ}C) + 273.15, \quad T(^{\circ}R) = T(^{\circ}F) + 459.67$$

Use absolute temperature for ideal-gas laws, Carnot efficiency, and radiation.

| Quantity      | SI        | Common USCS form |
|---------------|-----------|------------------|
| Pressure      | Pa, kPa   | psi, psia        |
| Energy        | J, kJ     | Btu, ft-lbf      |
| Power         | W, kW     | hp, Btu/hr       |
| Specific heat | kJ/(kg·K) | Btu/(lbm·°R)     |

**Exam tip:** Pressure in ideal-gas equations must be *absolute*. A gauge pressure must be converted before using  $PV = mRT$ .

## 2 First Law: Closed Systems

The first law expresses conservation of energy. Using the common sign convention that heat *into* a system is positive and work *done by* the system is positive,

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

For many stationary closed-system problems,

$$Q - W = \Delta U = m(u_2 - u_1)$$

For an ideal gas with constant specific heat,

$$\Delta U = mc_v(T_2 - T_1), \quad \Delta H = mc_p(T_2 - T_1)$$

**Boundary work.** For quasi-equilibrium compression or expansion,

$$W_b = \int_{V_1}^{V_2} P dV$$

Useful special cases include:

- Constant volume:  $W_b = 0$ .
- Constant pressure:  $W_b = P(V_2 - V_1)$ .
- Ideal-gas isothermal process:  $W_b = mRT \ln(V_2/V_1)$ .
- Polytropic process  $PV^n = C$ ,  $n \neq 1$ :

$$W_b = \frac{P_2 V_2 - P_1 V_1}{1 - n}.$$

### 2.2 Sign interpretation

- $Q > 0$ : heat enters the system;  $Q < 0$ : heat leaves.
- $W > 0$ : system does work on surroundings;  $W < 0$ : work is done on the system.
- Expansion at positive pressure usually gives  $W_b > 0$ ; compression gives  $W_b < 0$ .

For a closed, rigid tank, boundary work is zero because  $dV = 0$ . For an insulated rigid tank, both  $Q = 0$  and  $W_b = 0$ , so internal energy does not change unless another form of work crosses the boundary.

### 2.3 Problem-solving template

1. Define system and identify whether it is closed.
2. Write the first law before substituting numbers.
3. Cross out negligible terms only after checking the problem statement.
4. Use consistent energy units.
5. Interpret the sign of the final heat or work value physically.

**Exam tip:** Do not substitute enthalpy for internal energy in a closed-system balance unless the derivation specifically requires it. Enthalpy is especially convenient for flowing streams.

### 3 Open Systems and Steady-Flow Devices

For a steady one-inlet, one-outlet control volume, the steady-flow energy equation can be written per unit mass as

$$q - w_s = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

where  $w_s$  is shaft work done by the control volume. In rate form,

$$\dot{Q} - \dot{W}_s = \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right].$$

#### 3.1 Common FE simplifications

| Device              | Typical assumptions                         | Useful result                   |
|---------------------|---------------------------------------------|---------------------------------|
| Turbine             | adiabatic; $\Delta KE, \Delta PE \approx 0$ | $w_s \approx h_1 - h_2$         |
| Compressor          | adiabatic; $\Delta KE, \Delta PE \approx 0$ | work input $\approx h_2 - h_1$  |
| Pump                | incompressible liquid                       | $w_{in} \approx v(P_2 - P_1)$   |
| Nozzle              | adiabatic; no shaft work                    | $h_1 + V_1^2/2 = h_2 + V_2^2/2$ |
| Throttle valve      | adiabatic; no work; negligible $\Delta KE$  | $h_1 \approx h_2$               |
| Heat exchanger side | negligible shaft work                       | $q \approx h_2 - h_1$           |

#### 3.2 Mass balance at steady state

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

For a single inlet and outlet,  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .

#### 3.3 Isentropic efficiencies

$$\eta_t = \frac{h_1 - h_{2,actual}}{h_1 - h_{2s}}, \quad \eta_c = \frac{h_{2s} - h_1}{h_{2,actual} - h_1}$$

For turbines, the actual work output is less than the isentropic work. For compressors and pumps, the actual work input is greater than the isentropic requirement.

When a gas can be treated as ideal with nearly constant  $c_p$ , use  $h_2 - h_1 \approx c_p(T_2 - T_1)$  to convert the energy balance into a temperature calculation.

**Exam tip:** A compressor problem asking for power usually requires  $\dot{W} \approx \dot{m}c_p(T_2 - T_1)$  after applying the efficiency relation if an isentropic outlet state is involved.

## 4 Second Law, Entropy, and Reversibility

The second law determines the direction and quality of energy processes. It explains why heat flows spontaneously from a higher temperature to a lower temperature and why no heat engine can convert all heat input into work.

**Entropy balance.** For a closed system,

$$\Delta S = \int \frac{\delta Q}{T_b} + S_{gen}, \quad S_{gen} \geq 0$$

For a reversible process,  $S_{gen} = 0$ . Irreversibilities such as friction, mixing, heat transfer across a finite temperature difference, and throttling produce entropy.

For an ideal gas with constant specific heats,

$$\Delta s = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right)$$

or

$$\Delta s = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right).$$

### 4.2 Isentropic ideal-gas relations

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k}, \quad \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

where  $k = c_p/c_v$ .

### 4.3 Reversible vs. irreversible

- Reversible processes are ideal limits and maximize work output or minimize required work input.
- Adiabatic does *not* automatically mean isentropic. It is isentropic only when adiabatic and internally reversible.
- A throttling valve is approximately isenthalpic ( $h_1 = h_2$ ), but it is generally irreversible and entropy increases.

**Kelvin-Planck statement:** a cyclic heat engine cannot convert all heat received from a single reservoir into work.

**Clausius statement:** heat cannot, by itself, flow from a cold body to a hot body; external work is required for refrigeration.

On FE problems, the second law is often tested through Carnot limits, isentropic relations, efficiency bounds, or qualitative identification of impossible devices.

**Exam tip:** If a calculated heat-engine efficiency exceeds the corresponding Carnot efficiency, the proposed performance is impossible.

## 5 Ideal Gases and Gas Properties

The ideal-gas model is central to FE Environmental thermodynamics because air and dilute gas streams are often adequately modeled this way at moderate pressure.

### 5.1 Ideal-gas equation

$$PV = mRT, \quad Pv = RT$$

For a fixed mass,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

Pressure and temperature must be absolute.

The specific gas constant is

$$R = \frac{\bar{R}}{M}$$

where  $\bar{R}$  is the universal gas constant and  $M$  is molecular weight. A commonly used SI value is

$$\bar{R} = 8.314 \text{ kJ}/(\text{kmol} \cdot \text{K}).$$

### 5.2 Specific heats

$$c_p - c_v = R, \quad k = \frac{c_p}{c_v}$$

With constant specific heats,

$$\Delta u = c_v \Delta T, \quad \Delta h = c_p \Delta T.$$

### 5.3 Density form

$$\rho = \frac{P}{RT}$$

This is useful for air-flow and emission calculations when  $P$  and  $T$  are known.

**Ideal-gas mixtures.** For a nonreacting ideal-gas mixture, Dalton's law gives

$$P = \sum_i P_i, \quad y_i = \frac{P_i}{P}$$

where  $y_i$  is the mole fraction. Mixture molecular weight can be found from

$$M_{mix} = \sum_i y_i M_i.$$

Then  $R_{mix} = \bar{R}/M_{mix}$ .

| If the problem gives...      | Think...                                       |
|------------------------------|------------------------------------------------|
| $P, V, T, m$                 | ideal-gas law                                  |
| Two states of fixed gas mass | combined ideal-gas relation                    |
| Air heating/cooling          | $\Delta h \approx c_p \Delta T$                |
| Rigid tank                   | $V = \text{constant}$ , boundary work = 0      |
| Gas composition              | mole fractions, partial pressures, mixture $M$ |

**Exam tip:** Never use Celsius or Fahrenheit directly in ratios such as  $T_2/T_1$  or in  $PV = mRT$ . Convert to Kelvin or Rankine first.

## 6 Ideal-Gas Processes

Recognizing the process often turns a long problem into one equation.

| Process              | Defining relation     | Key consequence                                              |
|----------------------|-----------------------|--------------------------------------------------------------|
| Constant volume      | $V = \text{constant}$ | $W_b = 0$ , so $Q = \Delta U$ if $\Delta KE = \Delta PE = 0$ |
| Constant pressure    | $P = \text{constant}$ | $W_b = P(V_2 - V_1)$                                         |
| Isothermal ideal gas | $T = \text{constant}$ | $\Delta U = 0$ , so $Q = W$ for a closed system              |
| Adiabatic            | $Q = 0$               | energy change comes from work and/or flow energy             |
| Isentropic ideal gas | $s = \text{constant}$ | $PV^k = C$ , $TV^{k-1} = C$                                  |
| Polytropic           | $PV^n = C$            | $W_b = (P_2V_2 - P_1V_1)/(1 - n)$                            |

### 6.1 Isothermal ideal-gas boundary work

$$W_b = mRT \ln \left( \frac{V_2}{V_1} \right) = mRT \ln \left( \frac{P_1}{P_2} \right)$$

### 6.2 Isentropic temperature-pressure relation

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{(k-1)/k}$$

This is commonly used for an idealized compressor or turbine.

### 6.3 Polytropic work

$$W_b = \frac{P_2V_2 - P_1V_1}{1 - n}, \quad n \neq 1$$

For  $n = 1$ , the process is isothermal for an ideal gas and the logarithmic work expression applies.

### 6.4 Useful physical checks

- Adiabatic compression of an ideal gas raises temperature.
- Adiabatic expansion lowers temperature.
- Isothermal ideal-gas expansion requires heat input to offset work output.
- Rigid-tank heating raises pressure when volume and mass are fixed.

A process description is part of the equation. Words such as rigid, insulated, constant pressure, isothermal, reversible, and steady state are not background details; each deletes or constrains terms in the balance.

## 7 Heat Engines, Refrigerators, and Heat Pumps

A heat engine receives heat  $Q_H$  from a high-temperature reservoir, produces net work  $W_{net}$ , and rejects heat  $Q_L$  to a lower-temperature reservoir.

$$W_{net} = Q_H - Q_L$$

### 7.1 Thermal efficiency

$$\eta = \frac{W_{net}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

The reversible Carnot efficiency is the maximum possible between two reservoirs:

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

where  $T_H$  and  $T_L$  are absolute temperatures.

### 7.2 Refrigerator coefficient of performance

$$COP_R = \frac{Q_L}{W_{in}}$$

### 7.3 Heat-pump coefficient of performance

$$COP_{HP} = \frac{Q_H}{W_{in}} = COP_R + 1$$

Carnot limits are

$$COP_{R,C} = \frac{T_L}{T_H - T_L}, \quad COP_{HP,C} = \frac{T_H}{T_H - T_L}.$$

### 7.4 Interpretation

- Efficiency is always less than 1 for a heat engine.
- A COP may be greater than 1 because it measures heat moved per unit work input.
- Smaller temperature lift ( $T_H - T_L$ ) generally means a larger refrigeration/heat-pump COP.

**Energy-conversion chain.** Environmental systems may combine thermal and mechanical devices: fuel or waste heat  $\rightarrow$  thermal energy  $\rightarrow$  mechanical work  $\rightarrow$  electricity. Overall efficiency is commonly the product of stage efficiencies,

$$\eta_{overall} = \eta_1 \eta_2 \eta_3 \cdots$$

### 7.5 Refrigeration ton

$$1 \text{ ton of refrigeration} = 12,000 \text{ Btu/hr} \approx 3.516 \text{ kW}$$

**Exam tip:** For Carnot equations, convert reservoir temperatures to absolute units before substitution. A difference in temperatures may use K or °C numerically, but a temperature ratio cannot.

## 8 Conduction and Thermal Resistance

The FE Environmental specification explicitly includes conduction within the thermodynamics topic.

### 8.1 Fourier's law for one-dimensional steady conduction

$$\dot{Q} = -kA \frac{dT}{dx}$$

For a plane wall of thickness  $L$  with constant  $k$ ,

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L}$$

The thermal resistance form is

$$\dot{Q} = \frac{\Delta T}{R_{th}}, \quad R_{cond} = \frac{L}{kA}.$$

For a cylindrical wall,

$$R_{cond,cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}.$$

### 8.2 Convection resistance

$$R_{conv} = \frac{1}{hA}.$$

For resistances in series,

$$R_{total} = \sum_i R_i, \quad \dot{Q} = \frac{T_{hot} - T_{cold}}{R_{total}}.$$

A plane wall exposed to convection on both sides has

$$R_{total} = \frac{1}{h_1A} + \frac{L}{kA} + \frac{1}{h_2A}.$$

### 8.3 Composite walls

$$R_{total} = \frac{1}{h_iA} + \sum_j \frac{L_j}{k_jA} + \frac{1}{h_oA}$$

for layers in series and common area  $A$ .

### 8.4 Heat rate vs. heat flux

$$\dot{q}'' = \frac{\dot{Q}}{A}$$

Heat rate has units W; heat flux has units W/m<sup>2</sup>.

Thermal resistance problems are analogous to electrical resistance: temperature difference acts like voltage, heat rate like current, and resistances in series add.

**Exam tip:** Check whether the area changes. For cylindrical conduction, area varies with radius, so use the logarithmic cylindrical-wall expression rather than  $L/(kA)$ .

## 9 Convection and Radiation

**Convection.** Newton's law of cooling is

$$\dot{Q} = hA(T_s - T_\infty)$$

where  $h$  is the convective heat-transfer coefficient,  $T_s$  is surface temperature, and  $T_\infty$  is bulk-fluid temperature.

Convection may be **forced** (fan, blower, pump) or **natural/free** (buoyancy driven). Unless a correlation is requested, FE problems often provide  $h$  directly.

**Radiation.** For a small gray surface exchanging radiation with large surroundings,

$$\dot{Q}_{rad} = \varepsilon \sigma A (T_s^4 - T_{sur}^4)$$

where

$$\sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4).$$

Temperatures in the fourth-power term must be absolute.

### 9.3 Surface properties

$$\alpha + \rho + \tau = 1$$

where  $\alpha$  is absorptivity,  $\rho$  reflectivity, and  $\tau$  transmissivity. For an opaque surface,  $\tau = 0$ . A blackbody has  $\varepsilon = 1$ .

**Combined convection and radiation.** A hot exposed surface may lose heat by both mechanisms:

$$\dot{Q}_{total} = hA(T_s - T_\infty) + \varepsilon \sigma A(T_s^4 - T_{sur}^4).$$

**Steady surface energy balance.** If heat conducted through a wall reaches an exterior surface and then leaves by convection and radiation,

$$\dot{Q}_{cond} = \dot{Q}_{conv} + \dot{Q}_{rad}.$$

| Mode              | Driving force                        | Key property          |
|-------------------|--------------------------------------|-----------------------|
| <b>Conduction</b> | temperature gradient                 | $k$                   |
| <b>Convection</b> | surface-fluid temperature difference | $h$                   |
| <b>Radiation</b>  | absolute-temperature fourth powers   | $\varepsilon, \sigma$ |

**Exam tip:** Radiation equations are especially sensitive to temperature. Converting 25°C to 298.15 K is essential before taking the fourth power.

## 10 FE Thermodynamics Strategy and Formula Map

### 10.1 A fast decision sequence

1. **Identify the system:** closed mass or open control volume?
2. **Classify the process:** steady, transient, rigid, insulated, isothermal, isentropic, etc.
3. **Write the governing balance:** mass, energy, entropy, or heat-transfer resistance.
4. **Use the property model:** ideal gas, incompressible liquid, constant specific heat, etc.
5. **Check units and absolute quantities:** absolute pressure and absolute temperature when required.
6. **Check physical plausibility:** signs, efficiency bounds, and temperature trends.

### 10.2 High-yield formula map

Ideal gas

$$PV = mRT, \quad \rho = P/(RT)$$

$$c_p - c_v = R, \quad k = c_p/c_v$$

$$COP_R = Q_L/W, \quad COP_{HP} = Q_H/W$$

Closed-system energy

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

Conduction

$$\dot{Q} = kA \frac{\Delta T}{L}$$

Steady-flow energy

$$\dot{Q} - \dot{W}_s = \dot{m} \Delta h + \Delta(V^2/2) + g\Delta z$$

#### 9.1 Convection

$$\dot{Q} = hA(T_s - T_\infty)$$

Isentropic ideal gas

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k}$$

#### 9.2 Radiation

Heat engine

$$\eta = 1 - \frac{Q_L}{Q_H}$$

$$\eta_C = 1 - \frac{T_L}{T_H}$$

$$\dot{Q} = \varepsilon \sigma A (T_s^4 - T_{sur}^4)$$

Thermal resistance

Refrigeration/heat pump

$$\dot{Q} = \Delta T/R_{total}$$

### 10.3 Common mistakes to avoid

- Using gauge pressure in  $PV = mRT$ .
- Using °C or °F in temperature ratios, Carnot equations, or radiation.
- Confusing  $c_p$  with  $c_v$ .
- Treating adiabatic as automatically isentropic.
- Reversing compressor/turbine efficiency definitions.
- Forgetting that heat-pump COP and refrigerator COP use different useful heat terms.
- Forgetting convection resistances at wall boundaries.

**Primary exam references used for scope and notation:** NCEES FE Environmental CBT Exam Specifications and NCEES FE Reference Handbook 10.6 (Thermodynamics and Heat Transfer sections).

## 11 8 Example Problems

### Example Problem 1: Ideal-Gas Heating

**Problem.** A rigid 2.00 m<sup>3</sup> tank contains air at 120 kPa absolute and 25°C. The air is heated to 95°C. Treat air as an ideal gas. What is the final pressure?

**Choices:** (A) 95 kPa (B) 120 kPa (C) 148 kPa (D) 456 kPa

**Solution.** The tank is rigid and the mass is fixed, so  $V$  and  $m$  are constant. For an ideal gas,

$$\frac{P_1 V}{T_1} = \frac{P_2 V}{T_2}$$

Thus,

$$P_2 = P_1 \frac{T_2}{T_1}.$$

Convert to absolute temperature:

$$T_1 = 25 + 273.15 = 298.15 \text{ K}$$

$$T_2 = 95 + 273.15 = 368.15 \text{ K}.$$

Then

$$P_2 = (120 \text{ kPa}) \left( \frac{368.15}{298.15} \right) = 148.2 \text{ kPa}.$$

$$P_2 \approx 148 \text{ kPa}$$

**Answer:** (C)

Why this works: at fixed mass and volume, the ideal-gas law gives  $P \propto T$  on an absolute temperature scale.

**FE check.** Heating a rigid gas container should increase pressure. The result is about 23.5% higher because the absolute temperature increased by about 23.5%.

**Example Problem 2: Closed-System First Law**

**Problem.** A closed rigid tank contains 4.0 kg of air. Its temperature rises from 20°C to 70°C. Take  $c_v = 0.718 \text{ kJ}/(\text{kg} \cdot \text{K})$ . Neglect kinetic and potential energy changes. How much heat is transferred to the air?

**Choices:** (A) 71.8 kJ    (B) 143.6 kJ    (C) 200 kJ    (D) 287 kJ

**Solution.** For a closed system,

$$Q - W = \Delta U.$$

Because the tank is rigid, boundary work is zero:

$$W = 0.$$

For ideal-gas air with constant  $c_v$ ,

$$\Delta U = mc_v(T_2 - T_1).$$

The temperature difference is the same in K and °C:

$$\Delta T = 70 - 20 = 50 \text{ K}.$$

Therefore,

$$Q = (4.0)(0.718)(50) = 143.6 \text{ kJ}.$$

$Q \approx 144 \text{ kJ into the air}$

**Answer: (B)**

**Exam tip:** For temperature *differences*, 1 K has the same magnitude as 1°C. Absolute conversion is unnecessary when only  $\Delta T$  appears.

**Example Problem 3: Compressor Power**

**Problem.** Air flows through an adiabatic compressor at  $\dot{m} = 0.80$  kg/s. The inlet temperature is 300 K and the actual outlet temperature is 430 K. Take  $c_p = 1.005$  kJ/(kg · K) and neglect kinetic and potential energy changes. What compressor power input is required?

**Choices:** (A) 52 kW (B) 78 kW (C) 105 kW (D) 346 kW

**Solution.** For a steady adiabatic compressor with negligible kinetic and potential energy changes,

$$\dot{W}_{in} = \dot{m}(h_2 - h_1).$$

For ideal-gas air with constant  $c_p$ ,

$$h_2 - h_1 = c_p(T_2 - T_1).$$

Thus,

$$\dot{W}_{in} = (0.80)(1.005)(430 - 300) \text{ kJ/s.}$$

$$\dot{W}_{in} = 104.5 \text{ kW.}$$

$\dot{W}_{in} \approx 105 \text{ kW}$

**Answer: (C)**

A compressor requires work input. If the outlet temperature is higher than the inlet temperature in an adiabatic compressor, the positive power input is physically consistent.

**Example Problem 4: Isentropic Compression**

**Problem.** Air is compressed isentropically from 100 kPa and 300 K to 500 kPa. Assume  $k = 1.40$ . What is the ideal outlet temperature?

**Choices:** (A) 346 K (B) 475 K (C) 580 K (D) 1500 K

**Solution.** For an isentropic ideal-gas process,

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k}.$$

Substitute:

$$T_2 = 300 \left( \frac{500}{100} \right)^{(1.40-1)/1.40}.$$

The exponent is

$$\frac{0.40}{1.40} = 0.2857.$$

Therefore,

$$T_2 = 300(5)^{0.2857} \approx 300(1.584) = 475 \text{ K}.$$

$$T_2 \approx 475 \text{ K}$$

**Answer: (B)**

**Exam tip:** An isentropic compression raises temperature. If your result is below 300 K, recheck the pressure ratio or exponent.

**Example Problem 5: Heat Engine and Carnot Limit**

**Problem.** A heat engine receives 900 kW of heat from a reservoir at 800 K and rejects 540 kW to a reservoir at 300 K. Determine (1) the actual thermal efficiency and (2) whether it exceeds the Carnot limit.

**Solution.** The net work output is

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L = 900 - 540 = 360 \text{ kW}.$$

The actual efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{360}{900} = 0.400 = 40.0\%.$$

The Carnot limit between the same reservoirs is

$$\eta_C = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{800} = 0.625 = 62.5\%.$$

Since

$$40.0\% < 62.5\%,$$

the actual engine does not violate the Carnot limit.

$$\boxed{\eta = 40.0\%, \quad \eta_C = 62.5\%}$$

The Carnot efficiency is an upper bound, not an expected real-engine efficiency. Real systems have irreversibilities and therefore lower performance.

**Example Problem 6: Refrigerator COP**

**Problem.** A refrigeration unit removes heat from a cold space at a rate of 18 kW while consuming 6.0 kW of electric power. Find (1) the refrigerator COP and (2) the heat-rejection rate to the room.

**Solution.** The refrigerator COP is

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{18}{6.0} = 3.0.$$

Energy conservation for the cycle gives

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in}.$$

Therefore,

$$\dot{Q}_H = 18 + 6 = 24 \text{ kW}.$$

|                                                |
|------------------------------------------------|
| $COP_R = 3.0, \quad \dot{Q}_H = 24 \text{ kW}$ |
|------------------------------------------------|

**Check.** For the same device operating as a heat pump,

$$COP_{HP} = \frac{24}{6} = 4.0 = COP_R + 1.$$

|                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------|
| <b>Exam tip:</b> COP is not an efficiency and can exceed 1. The device moves heat in addition to converting work input into heat. |
|-----------------------------------------------------------------------------------------------------------------------------------|

**Example Problem 7: Composite Wall Heat Loss**

**Problem.** A  $10 \text{ m}^2$  wall consists of  $0.10 \text{ m}$  of material A with  $k_A = 0.50 \text{ W}/(\text{m} \cdot \text{K})$  and  $0.08 \text{ m}$  of material B with  $k_B = 0.20 \text{ W}/(\text{m} \cdot \text{K})$ . Indoor and outdoor convection coefficients are  $h_i = 10$  and  $h_o = 25 \text{ W}/(\text{m}^2 \cdot \text{K})$ . Indoor air is  $22^\circ\text{C}$  and outdoor air is  $2^\circ\text{C}$ . Find the steady heat-loss rate.

**Solution.** For series thermal resistances,

$$R_{tot} = \frac{1}{h_i A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_o A}.$$

Compute each term:

$$R_i = \frac{1}{(10)(10)} = 0.010 \text{ K/W}$$

$$R_A = \frac{0.10}{(0.50)(10)} = 0.020 \text{ K/W}$$

$$R_B = \frac{0.08}{(0.20)(10)} = 0.040 \text{ K/W}$$

$$R_o = \frac{1}{(25)(10)} = 0.004 \text{ K/W}.$$

Thus,

$$R_{tot} = 0.074 \text{ K/W}.$$

The temperature difference is  $20 \text{ K}$ , so

$$\dot{Q} = \frac{20}{0.074} = 270 \text{ W}.$$

$$\boxed{\dot{Q} \approx 270 \text{ W}}$$

**Exam tip:** In a plane composite wall with common area, conduction and convection resistances are added directly in series.

**Example Problem 8: Combined Convection and Radiation**

**Problem.** A  $1.5 \text{ m}^2$  hot metal surface is at  $120^\circ\text{C}$  in a room where the air and large surrounding surfaces are at  $25^\circ\text{C}$ . The convection coefficient is  $h = 8.0 \text{ W}/(\text{m}^2 \cdot \text{K})$  and emissivity is  $\varepsilon = 0.80$ . Estimate the total heat-loss rate by convection plus radiation. Use  $\sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$ .

**Solution.** Convection:

$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (8.0)(1.5)(120 - 25) = 1140 \text{ W}.$$

For radiation, convert temperatures to Kelvin:

$$T_s = 393.15 \text{ K}, \quad T_{sur} = 298.15 \text{ K}.$$

Then

$$\begin{aligned} \dot{Q}_{rad} &= \varepsilon \sigma A (T_s^4 - T_{sur}^4). \\ \dot{Q}_{rad} &= (0.80)(5.67 \times 10^{-8})(1.5) (393.15^4 - 298.15^4) \approx 1090 \text{ W}. \end{aligned}$$

Therefore,

$$\dot{Q}_{total} = 1140 + 1090 \approx 2230 \text{ W}.$$

$$\boxed{\dot{Q}_{total} \approx 2.23 \text{ kW}}$$

At elevated surface temperatures, radiation can be comparable to convection. Always use absolute temperature in the  $T^4$  radiation term.