

UNITS AND MEASUREMENTS

▶ The **SI system** : It is the international system of units. At present internationally accepted for measurement. In this system there are seven fundamental and two supplementary quantities and their corresponding units are:

Quantity	Unit	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electric current	ampere	A
5. Temperature	kelvin	K
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol
Supplementary		
1. Plane angle	radian	rad
2. Solid angle	steradian	sr

- ▶ **Dimensions** : These are the powers to which the fundamental units are raised to get the unit of a physical quantity.
- ▶ **Uses of dimensions**
 - To check the correctness of a physical relation.
 - To derive relationship between different physical quantities.
 - To convert one system of unit into another.

$$n_1 u_1 = n_2 u_2$$

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

- ▶ **Significant figures** : In any measurement, the reliable digits plus the first uncertain digit are known as significant figures.
- ▶ **Error** : It is the difference between the measured value and true value of a physical quantity or the uncertainty in the measurements.
- ▶ **Absolute error** : The magnitude of the difference between the true value and the measured value is called absolute error.

$$\Delta a_1 = \bar{a} - a_1, \Delta a_2 = \bar{a} - a_2, \Delta a_n = \bar{a} - a_n$$

Mean absolute error

$$\bar{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n} = \frac{1}{n} \sum_{i=1}^n |\Delta a_i|$$

- ▶ **Relative error** : It is the ratio of the mean absolute error to its true value

or relative error = $\frac{\bar{\Delta a}}{a}$

- ▶ **Percentage error** : It is the relative error in per cent.

$$\text{Percentage error} = \left(\frac{\bar{\Delta a}}{a_{\text{mean}}} \right) \times 100\%$$

MOTION IN A STRAIGHT LINE

- ▶ Average speed, $V_{\text{av}} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$
- ▶ Average acceleration, $a_{\text{av}} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$
- ▶ The area under the velocity-time curve is equal to the displacement and slope gives acceleration.
- ▶ If a body falls freely, the distance covered by it in each subsequent second starting from first second will be in the ratio 1 : 3 : 5 : 7 etc.
- ▶ If a body is thrown vertically up with an initial velocity u , it takes u/g second to reach maximum height and u/g second to return, if air resistance is negligible.
- ▶ If air resistance acting on a body is considered, the time taken by the body to reach maximum height is less than the time to fall back the same height.
- ▶ For a particle having zero initial velocity if $s \propto t^\alpha$, where $\alpha > 2$, then particle's acceleration increases with time.
- ▶ For a particle having zero initial velocity if $s \propto t^\alpha$, where $\alpha < 0$, then particle's acceleration decreases with time.
- ▶ **Kinematic equations** :
 $v = u + a_t(t)$; $v^2 = u^2 + 2a_t(s)$
 $S = ut + \frac{1}{2} a_t(t)^2$; $S_n = u + \frac{a}{2}(2n-1)$
 applicable only when $|\bar{a}_t| = a_t$ is constant.
 a_t = magnitude of tangential acceleration, S = distance
- ▶ If acceleration is variable use calculus approach.
- ▶ **Relative velocity** : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

MOTION IN A PLANE

- ▶ If T is the time of flight, h maximum height, R horizontal range of a projectile, α its angle of projection, then the relations among these quantities.

$$h = \frac{gT^2}{8} \quad \dots\dots (1);$$

$$gT^2 = 2R \tan \alpha \quad \dots\dots (2);$$

$$R \tan \alpha = 4h \quad \dots\dots (3)$$

- ▶ $T = \frac{2u \sin \theta}{g}; h = \frac{u^2 \sin^2 \theta}{2g}$
- ▶ $R = \frac{u^2 \sin 2\theta}{g}; R_{\max} = \frac{u^2}{g}$ when $\theta = 45^\circ$
- ▶ For a given initial velocity, to get the same horizontal range, there are two angles of projection α and $90^\circ - \alpha$.
- ▶ The equation to the parabola traced by a body projected horizontally from the top of a tower of height y , with a velocity u is $y = gx^2/2u^2$, where x is the horizontal distance covered by it from the foot of the tower.
- ▶ Equation of trajectory is $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$, which is parabola.
- ▶ Equation of trajectory of an oblique projectile in terms of range (R) is $y = x \tan \theta \left(1 - \frac{x}{R}\right)$
- ▶ Maximum height is equal to n times the range when the projectile is launched at an angle $\theta = \tan^{-1}(4n)$.
- ▶ In a uniform circular motion, velocity and acceleration are constants only in magnitude. Their directions change.
- ▶ In a uniform circular motion, the kinetic energy of the body is a constant. $W = 0, \vec{a} \neq 0, \vec{P} \neq \text{constant}, \vec{L} = \text{constant}$
- ▶ Centripetal acceleration, $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$ (always applicable)
 $\vec{a}_r = \vec{\omega} \times \vec{v}$

LAWS OF MOTION

- ▶ Newton's first law of motion or law of inertia : It is resistance to change.
- ▶ Newton's second law :
 $\vec{F} = m\vec{a}, \vec{F} = d\vec{p} / dt$
- ▶ Impulse : $\Delta\vec{p} = \vec{F}\Delta t, p_2 - p_1 = \int_1^2 \vec{F} dt$
- ▶ Newton's third law : $\vec{F}_{12} = -\vec{F}_{21}$
- ▶ Frictional force $f_s \leq (f_s)_{\max} = \mu_s R; f_k = \mu_k R$
- ▶ Circular motion with variable speed. For complete circles, the string must be taut in the highest position, $u^2 \geq 5ga$. Circular motion ceases at the instant when the string becomes slack, i.e., when $T = 0$, range of values of u for which the string does go slack is $\sqrt{2ga} < u < \sqrt{5ga}$.
- ▶ Conical pendulum : $\omega = \sqrt{g/h}$ where h is height of a point of suspension from the centre of circular motion.

- ▶ The acceleration of a lift
 $a = \frac{\text{actual weight} - \text{apparent weight}}{\text{mass}}$
If 'a' is positive lift is moving down, and if it is negative the lift is moving up.
- ▶ On a banked road, the maximum permissible speed V_{\max}
 $= \left(R_g \frac{u_s + \tan \theta}{1 - u_s \tan \theta} \right)^{1/2}$

WORK, ENERGY AND POWER

- ▶ Work done $W = FS \cos \theta$
- ▶ Relation between kinetic energy E and momentum, $P = \sqrt{2mE}$
K.E. = $1/2 mV^2$; P.E. = mgh
- ▶ If a body moves with constant power, its velocity (v) is related to distance travelled (x) by the formula $v \propto x^{3/2}$.
- ▶ Power $P = \frac{W}{t} = F.V$
- ▶ Work due to kinetic force of friction between two contact surfaces is always negative. It depends on relative displacement between contact surfaces. $W_{FK} = -F_K(S_{\text{rel}})$.
- ▶ $\Sigma W = \Sigma \Delta K, \Sigma W \Rightarrow$ total work due to all kinds of forces, $\Sigma \Delta K \Rightarrow$ total change in kinetic energy.
- ▶ $\Sigma W_{\text{conservative}} = -\Sigma \Delta U; \Sigma W_{\text{conservative}} \Rightarrow$ Total work due to all kinds of conservative forces.
 $\Sigma \Delta u \Rightarrow$ Total change in all kinds of potential energy.
- ▶ Coefficient of restitution $e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$
- ▶ The total momentum of a system of particles is a constant in the absence of external forces.

SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

- ▶ The centre of mass of a system of particles is defined as the point whose position vector is $R = \frac{\Sigma m_i r_i}{M}$
- ▶ The angular momentum of a system of n particles about the origin is $L = \Sigma_{i=1}^n r_i \times p_i;$
 $L = mvr = I\omega$
- ▶ The torque or moment of force on a system of n particles about the origin is $\tau = \Sigma r_i \times F_i$
- ▶ The moment of inertia of a rigid body about an axis is defined by the formula $I = \Sigma m_i r_i^2$
- ▶ The kinetic energy of rotation is $K = \frac{1}{2} I\omega^2$
- ▶ The theorem of parallel axes : $I_z' = I_z + Ma^2$
Theorem of perpendicular axes : $I_z = I_x + I_y$

- ▶ For rolling motion without slipping $v_{cm} = R\omega$. The kinetic energy of such a rolling body is the sum of kinetic energies

of translation and rotation : $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$

- ▶ A rigid body is in mechanical equilibrium if
 - (a) It is translational equilibrium i.e., the total external force on it is zero : $\Sigma F_i = 0$.
 - (b) It is rotational equilibrium i.e., the total external torque on it is zero : $\Sigma \tau_i = \Sigma r_i \times F_i = 0$.
- ▶ If a body is released from rest on rough inclined plane, then

for pure rolling $\mu_r \geq \frac{n}{n+1} \tan \theta$ ($I_c = nmr^2$)

Rolling with sliding $0 < \mu_s < \left(\frac{n}{n+1}\right) \tan \theta$;

$\frac{g \sin \theta}{n+1} < a < g \sin \theta$

GRAVITATION

- ▶ Newton's universal law of gravitation

Gravitational force $F = \frac{Gm_1m_2}{r^2}$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

- ▶ The acceleration due to gravity. (a) at a height h above the Earth's surface

$g(h) = \frac{GM_E}{(R_E + h)^2} = g \left(1 - \frac{2h}{R_E}\right)$ for $h \ll R_E$

$g(h) = g(0) \left(1 - \frac{2h}{R_E}\right)$ where $g(0) = \frac{GM_E}{R_E^2}$

- (b) at depth d below the Earth's surface is

$g(d) = \frac{GM_E}{R_E^2} \left(1 - \frac{d}{R_E}\right) = g(0) \left(1 - \frac{d}{R_E}\right)$

- (c) with latitude λ $g^1 = g - R\omega^2 \cos^2 \lambda$

- ▶ Gravitational potential $V_g = -\frac{GM}{r}$
- ▶ Intensity of gravitational field $I = \frac{GM}{r^2}$
- ▶ The gravitational potential energy $V = -\frac{Gm_1m_2}{r} + \text{constant}$
- ▶ The escape speed from the surface of the Earth is $v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$ and has a value of 11.2 km s^{-1} .
- ▶ Orbital velocity, $v_{orbi} = \sqrt{\frac{GM_E}{R_E}} = \sqrt{gR_E}$
- ▶ A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at an approximate distance of $4.22 \times 10^4 \text{ km}$ from the Earth's centre.

- ▶ Kepler's 3rd law of planetary motion.

$T^2 \propto a^3$; $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$

MECHANICAL PROPERTIES OF SOLIDS

- ▶ Hooke's law : stress \propto strain
- ▶ Young's modulus of elasticity $Y = \frac{F\Delta\ell}{A\ell}$
- ▶ Compressibility = $\frac{1}{\text{Bulk modulus}}$
- ▶ $Y = 3k(1 - 2\sigma)$
- ▶ $Y = 2n(1 + \sigma)$
- ▶ If S is the stress and Y is Young's modulus, the energy density of the wire E is equal to $S^2/2Y$.
- ▶ If α is the longitudinal strain and E is the energy density of a stretched wire, Y Young's modulus of wire, then E is equal to $\frac{1}{2}Y\alpha^2$

- ▶ Thermal stress = $\frac{F}{A} = Y \alpha \Delta\theta$

MECHANICAL PROPERTIES OF FLUIDS

- ▶ Pascal's law : A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel. Pressure exerted by a liquid column $P = h\rho g$
- ▶ Bernoulli's principle $P + \rho v^2/2 + \rho gh = \text{constant}$
- ▶ Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface.

- ▶ Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of viscosity η $F = -6\pi\eta av$.
- ▶ Terminal velocity $V_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$
- ▶ The surface tension of a liquid is zero at boiling point. The surface tension is zero at critical temperature.
- ▶ If a drop of water of radius R is broken into n identical drops, the work done in the process is $4\pi R^2 S(n^{1/3} - 1)$ and fall in temperature $\Delta q = \frac{3T}{J} \sqrt{\frac{1}{r} - \frac{1}{R}}$
- ▶ Two capillary tubes each of radius r are joined in parallel. The rate of flow is Q . If they are replaced by single capillary tube of radius R for the same rate of flow, then $R = 2^{1/4} r$.
- ▶ Ascent of a liquid column in a capillary tube $h = \frac{2s \cos \phi}{r\rho g}$
- ▶ Coefficient of viscosity, $n = -\frac{F}{A\left(\frac{dv}{dx}\right)}$
- ▶ Velocity of efflux $V = \sqrt{2gh}$

THERMAL PROPERTIES OF MATTER

- Relation between different temperature scales :

$$\frac{C}{100} = \frac{F-32}{100} = \frac{K-273}{100}$$

- The coefficient of linear expansion (α_ℓ), superficial (β) and volume expansion (α_v) are defined by the relations :

$$\frac{\Delta \ell}{\ell} = \alpha_\ell \Delta T ; \quad \frac{\Delta A}{A} = \beta \Delta T ; \quad \frac{\Delta V}{V} = \alpha_v \Delta T$$

$$\alpha_v = 3\alpha_\ell ; \quad \beta = 2\alpha_\ell$$

- In conduction, heat is transferred between neighbouring parts of a body through molecular collisions, without any flow of matter. The rate of flow of heat $H = KA \frac{T_C - T_D}{L}$, where K is the thermal conductivity of the material of the bar.
- Convection involves flow of matter within a fluid due to unequal temperatures of its parts.
- Radiation is the transmission of heat as electromagnetic waves.
- Stefan's law of radiation : $E = \sigma T^4$, where the constant σ is known as Stefan's constant = $5.67 \times 10^{-8} \text{ w m}^{-2} \text{ k}^{-4}$.
- Wein's displacement law : $\lambda_m T = \text{constant}$, where constant is known as Wein's constant = $2.898 \times 10^{-3} \text{ mk}$.
- Newton's law of cooling: $\frac{dQ}{dt} = -k (T_2 - T_1)$; where T_1 is the temperature of the surrounding medium and T_2 is the temperature of the body.
- Heat required to change the temperature of the substance, $Q = mc\Delta\theta$
c = specific heat of the substance
- Heat absorbed or released during state change $Q = mL$
L = latent heat of the substance
- Mayer's formula $c_p - c_v = R$

THERMODYNAMICS

- First law of thermodynamics: $\Delta Q = \Delta U + \Delta W$, where ΔQ is the heat supplied to the system, ΔW is the work done by the system and ΔU is the change in internal energy of the system.

- In an isothermal expansion of an ideal gas from volume V_1 to V_2 at temperature T the heat absorbed (Q) equals the work done (W) by the gas, each given by

$$Q = W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

- In an adiabatic process of an ideal gas $PV^\gamma = TV^{\gamma-1} = \frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$, where $\gamma = \frac{C_p}{C_v}$.
- Work done by an ideal gas in an adiabatic change of state from (P_1, V_1, T_1) to (P_2, V_2, T_2) is $W = \frac{nR (T_1 - T_2)}{\gamma - 1}$
- The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

- Second law of thermodynamics:** No engine operating between two temperatures can have efficiency greater than that of the Carnot engine.

- Entropy or disorder $S = \frac{\delta Q}{T}$

KINETIC THEORY

- Ideal gas equation $PV = nRT$
- Kinetic theory of an ideal gas gives the relation $P = \frac{1}{3} nm\bar{v}^2$, Combined with the ideal gas equation it yields a kinetic interpretation of temperature.

$$\frac{1}{2} nm\bar{v}^2 = \frac{3}{2} k_B T, \quad v_{\text{rms}} = (\bar{v}^2)^{1/2} = \sqrt{\frac{3k_B T}{m}}$$

- The law of equipartition of energy is stated thus: the energy for each degree of freedom in thermal equilibrium is $1/2 (k_B T)$
- The translational kinetic energy $E = \frac{3}{2} k_B NT$. This leads to a relation $PV = \frac{2}{3} E$.
- Degree of freedom : Number of directions in which it can move freely.
- Root mean square (rms) velocity of the gas

$$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

- Most probable speed $V_{\text{mp}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2KT}{m}}$

- Mean free path $\lambda = \frac{KT}{\sqrt{2}\pi d^2 P}$

OSCILLATIONS

- Displacement in SHM : $Y = a \sin \omega t$ or, $y = a \cos \omega t$
- The particle velocity and acceleration during SHM as functions of time are given by,

$$v(t) = -\omega A \sin(\omega t + \phi) \text{ (velocity),}$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t) \text{ (acceleration)}$$

Velocity amplitude $v_m = \omega A$ and acceleration amplitude $a_m = \omega^2 A$.

- A particle of mass m oscillating under the influence of a Hooke's law restoring force given by $F = -kx$ exhibits simple harmonic motion with $\omega = \sqrt{\frac{k}{m}}$ (angular frequency),

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ (period)}$$

Such a system is also called a linear oscillator.

- Time period for conical pendulum $T = 2\pi \sqrt{\left(\frac{\ell \cos \theta}{g} \right)}$ where θ angle between string & vertical.

- Energy of the particle $E = k + u = \frac{1}{2} m\omega^2 A^2$

WAVES

- ▶ The displacement in a sinusoidal wave $y(x, t) = a \sin(kx - \omega t + \phi)$ where ϕ is the phase constant or phase angle.

- ▶ Equation of plane progressive wave :

$$= a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

- ▶ Equation of stationary wave :

$$Y = 2a \sin \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}$$

- ▶ The speed of a transverse wave on a stretched string $v = \sqrt{T/\mu}$.

- ▶ Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of sound wave in a fluid having bulk modulus B and density μ is $v = \sqrt{B/\rho}$.

- ▶ The speed of longitudinal waves in a metallic bar is $v = \sqrt{Y/\rho}$

For gases, since $B = \gamma P$, the speed of sound is $v = \sqrt{\gamma P/\rho}$

- ▶ The interference of two identical waves moving in opposite directions produces standing waves. For a string with fixed ends, standing wave $y(x, t) = [2a \sin kx] \cos \omega t$

- ▶ The separation between two consecutive nodes or antinodes is $\lambda/2$.

- ▶ A stretched string of length L fixed at both the ends vibrates with frequencies $f = \frac{1}{2L} v$.

The oscillation mode with lowest frequency is called the fundamental mode or the first harmonic.

- ▶ A pipe of length L with one end closed and other end open (such as air columns) vibrates with frequencies given by

$$f = \left(n + \frac{1}{2} \right) \frac{v}{2L}, n = 0, 1, 2, 3, \dots$$

The lowest frequency given by $v/4L$ is the fundamental mode or the first harmonic.

Open organ pipe $n_1 : n_2 : n_3 : \dots : 1, 2, 3, \dots$, $n = \frac{V}{2l}$

- ▶ Beats arise when two waves having slightly different frequencies, f_1 and f_2 and comparable amplitudes, are superposed. The beat frequency $f_{\text{beat}} = f_1 - f_2$

- ▶ The Doppler effect is a change in the observed frequency of a wave when the source S and the observer O moves relative

to the medium. $f = f_0 \left(\frac{v \pm v_0}{v \pm v_s} \right)$

ELECTRO-STATICS

- ▶ Coulomb's Law : \vec{F}_{21} = force on q_2

due to $q_1 = \frac{k(q_1 q_2)}{r_{21}^2} \hat{r}_{21}$ where $k = \frac{1}{4\pi\epsilon_0}$
 $= 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$

- ▶ Electric field due to a point charge q has a magnitude $|q|/4\pi\epsilon_0 r^2$

- ▶ Field of an electric dipole in its equatorial plane

$$E = \frac{-\vec{p}}{4\pi\epsilon_0 (a^2 + r^2)^{3/2}} \cong \frac{-\vec{p}}{4\pi\epsilon_0 r^3}, \text{ for } r \gg a$$

Dipole electric field on the axis at a distance r from the centre:

$$\vec{E} = \frac{2\vec{p}r}{4\pi\epsilon_0 (r^2 - a^2)^2} \cong \frac{2\vec{p}}{4\pi\epsilon_0 r^3} \text{ for } r \gg a$$

Dipole moment $\vec{p} = q2a$

- ▶ In a uniform electric field \vec{E} , a dipole experiences a torque $\vec{\tau}$ given by $\vec{\tau} = \vec{p} \times \vec{E}$ but experiences no net force.

The flux $\Delta\phi$ of electric field \vec{E} through a small area element $\Delta\vec{S}$ is given by $\Delta\phi = \vec{E} \cdot \Delta\vec{S}$

- ▶ Gauss's law: The flux of electric field through any closed surface S is $1/\epsilon_0$ times the total charge enclosed i.e., Q
- ▶ Thin infinitely long straight wire of uniform linear charge

density λ : $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$

- ▶ Infinite thin plane sheet of uniform surface charge density σ

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

- ▶ Thin spherical shell of uniform surface charge density σ :

$$\vec{E} = \frac{\sigma}{4\pi\epsilon_0 r^2} \hat{r} \quad (r \geq R); \vec{E} = 0 \quad (r < R)$$

- ▶ Electric Potential : $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$.

- ▶ An equipotential surface is a surface over which potential has a constant value.

- ▶ Potential energy of two charges q_1, q_2 at \vec{r}_1, \vec{r}_2 is given by

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}, \text{ where } r_{12} \text{ is distance between } q_1 \text{ and } q_2.$$

- ▶ Capacitance $C = Q/V$, where Q = charge and V = potential difference

- ▶ For a parallel plate capacitor (with vacuum between the plates), $C = \epsilon_0 \frac{A}{d}$.

- ▶ The energy U stored in a capacitor of capacitance C , with charge Q and voltage V is

$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

- ▶ For capacitors in the series combination,

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In the parallel combination, $C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$ where C_1, C_2, C_3, \dots are individual capacitances.

CURRENT ELECTRICITY

- ▶ Electric current, $I = \frac{q}{t}$
- ▶ Current density j gives the amount of charge flowing per second per unit area normal to the flow, $\vec{J} = nqv_d$
- ▶ Mobility, $\mu = \frac{V_d}{E}$ and $V_d = \frac{I}{Ane}$
- ▶ Resistance $R = \rho \frac{\ell}{A}$, ρ = resistivity of the material
- ▶ Equation $\vec{E} = \rho \vec{J}$ another statement of Ohm's law, ρ = resistivity of the material.
- ▶ Ohm's law $I \propto V$ or $V = RI$
- ▶ (a) Total resistance R of n resistors connected in series $R = R_1 + R_2 + \dots + R_n$
- ▶ (b) Total resistance R of n resistors connected in parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
- ▶ Kirchhoff's Rules – (a) Junction rule: At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
- ▶ (b) Loop rule: The algebraic sum of changes in potential around any closed loop must be zero.
- ▶ The Wheatstone bridge is an arrangement of four resistances R_1, R_2, R_3, R_4 . The null-point condition is given by $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
- ▶ The potentiometer is a device to compare potential differences. The device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources. Internal resistance $r = R \left(\frac{\ell_1}{\ell_2} - 1 \right)$
- ▶ RC circuit : During charging : $q = CE(1 - e^{-t/RC})$
During discharging : $q = q_0 e^{-t/RC}$
- ▶ According to Joule's Heating law, $H = I^2 R t$

MAGNETISM

- ▶ The total force on a charge q moving with velocity v i.e., Lorentz force. $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$.
- ▶ A straight conductor of length ℓ and carrying a steady current I experiences a force \vec{F} in a uniform external magnetic field \vec{B} , $\vec{F} = I\vec{\ell} \times \vec{B}$, the direction of ℓ is given by the direction of the current.
- ▶ Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{r}}{r^3}$.
- ▶ The magnitude of the magnetic field due to a circular coil of radius R carrying a current I at an axial distance x from the centre is $B = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$.

- ▶ The magnitude of the field B inside a long solenoid carrying a current I is : $B = \mu_0 nI$. For a toroid one obtains, $B = \frac{\mu_0 NI}{2\pi r}$.
- ▶ Ampere's Circuital Law : $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I$, where I refers to the current passing through S .
- ▶ Force between two long parallel wires $F = \frac{\mu_0 I_1 I_2}{2\pi a} N m^{-1}$. The force is attractive if currents are in the same direction and repulsive currents are in the opposite direction.
- ▶ For current carrying coil $\vec{M} = NI\vec{A}$; torque = $\vec{\tau} = \vec{M} \times \vec{B}$
- ▶ Conversion of (i) galvanometer into ammeter, $S = \left(\frac{I_g}{I - I_g} \right) G$
- ▶ (ii) galvanometer into voltmeter, $S = \frac{V}{I_g} - G$
- ▶ The magnetic intensity, $\vec{H} = \frac{\vec{B}_0}{\mu_0}$.
- ▶ The magnetisation \vec{M} of the material is its dipole moment per unit volume. The magnetic field B in the material is, $\vec{B} = \mu_0(\vec{H} + \vec{M})$
- ▶ For a linear material $\vec{M} = \chi \vec{H}$. So that $\vec{B} = \mu \vec{H}$ and χ is called the magnetic susceptibility of the material. $\mu = \mu_0 \mu_r$; $\mu_r = 1 + \chi$.

ELECTRO-MAGNETIC INDUCTION

- ▶ The magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$, where θ is the angle between \vec{B} & \vec{A} .
- ▶ Faraday's laws of induction : $\epsilon = -N \frac{d\phi_B}{dt}$
- ▶ Lenz's law states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it.
- ▶ The induced emf (motional emf) across ends of a rod $\epsilon = B\ell v$
- ▶ The self-induced emf is given by, $\epsilon = -L \frac{dI}{dt}$
 L is the self-inductance of the coil. $L = \frac{\mu_0 N^2 A}{\ell}$
- ▶ A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1). $\epsilon_1 = -M_{12} \frac{dI_2}{dt}$, M_{12} = mutual inductance of coil 1 w.r.t coil 2. $M = \frac{\mu_0 N_1 N_2 A}{\ell}$
- ▶ Growth of current in an inductor, $i = i_0 [1 - e^{-Rt/L}]$
For decay of current, $i = i_0 e^{-Rt/L}$

ALTERNATING CURRENT

For an alternating current $i = i_m \sin \omega t$ passing through a resistor R , the average power loss P (averaged over a cycle) due to joule heating is $(1/2)i_m^2 R$.

E.m.f, $E = E_0 \sin \omega t$

Root mean square (rms) current

$$I = \frac{i_m}{\sqrt{2}} = 0.707 i_m, E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

- The average power loss over a complete cycle $P = VI \cos \phi$. The term $\cos \phi$ is called the power factor.
- An ac voltage $v = v_m \sin \omega t$ applied to a pure inductor L , drives a current in the inductor $i = i_m \sin (\omega t - \pi/2)$, where $i_m = v_m/X_L$. $X_L = \omega L$ is called inductive reactance.
- An ac voltage $v = v_m \sin \omega t$ applied to a capacitor drives a current in the capacitor: $i = i_m \sin (\omega t + \pi/2)$. Here,

$$i_m = \frac{v_m}{X_C}, X_C = \frac{1}{\omega C} \text{ is called capacitive reactance.}$$

An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} (X_L = X_C)$.

- Impedance $z = \sqrt{R^2 + (x_L - x_C)^2}$
- Transformation ratio, $K = \frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{I_P}{I_S}$
- Step up transformer : $N_S > N_P; E_S > E_P; I_P > I_S$
- Step down transformer $N_P > N_S; E_P > E_S$ and $I_P < I_S$
- The quality factor Q defined by $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$ is an indicator of the sharpness of the resonance, the higher value of Q indicating sharper peak in the current.

RAY OPTICS

Reflection is governed by the equation $\angle i = \angle r$ and refraction by the Snell's law, $\sin i / \sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane.

Mirror equation: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Magnification $M = \frac{v}{u} = \frac{I}{O}$

- Prism Formula $n_{21} = \frac{n_2}{n_1} = \frac{\sin [(A + D_m)/2]}{\sin (A/2)}$, where D_m is the angle of minimum deviation.
- Dispersion is the splitting of light into its constituent colours. The deviation is maximum for violet and minimum for red.
- Scattering $\propto \frac{1}{\lambda^4}$

Dispersive power $\omega = \frac{\delta_v - \delta_r}{\delta}$, where δ_v, δ_r are deviation of violet and red and δ the deviation of mean ray (usually yellow).

For refraction through a spherical interface (from medium 1 to 2 of refractive index n_1 and n_2 , respectively)

$$\frac{n_2}{v} - \frac{n_1}{v} = \frac{n_2 - n_1}{R}$$

Refractive index of a medium $\mu = \frac{C}{V}$ ($C = 3 \times 10^8$ m/s)

$$r = \frac{1}{\sin C} \text{ (C = Critical angle)}$$

Condition for TIR : 1. Ray of light must travel from denser to rarer medium 2. Angle of incidence in denser medium > critical angle.

Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Lens maker's formula : $\frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

The power of a lens $P = 1/f$. The SI unit for power of a lens is dioptre (D): $1 D = 1 \text{ m}^{-1}$.

If several thin lenses of focal length f_1, f_2, f_3, \dots are in contact, the effective focal $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$

The total power of a combination of several lenses $P = P_1 + P_2 + P_3 + \dots$

Chromatic aberration if satisfying the equation

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \text{ or in terms of powers } \omega_1 P_1 + \omega_2 P_2 = 0.$$

For compound microscope $M = \frac{V_0}{u_0} \left(1 + \frac{D}{f_e} \right)$

when final image at D

$$M = \frac{V_0}{u_0} \cdot \frac{D}{f_e} \text{ when final image at infinity.}$$

WAVE OPTICS

Wavefront : It is the locus of all the particles vibrating in the same phase.

The resultant intensity of two waves of intensity $I_0/4$ of phase difference ϕ at any

points $I = I_0 \cos^2 \left[\frac{\phi}{2} \right]$,

where I_0 is the maximum density.

Intensity $I \propto (\text{amplitude})^2$

Condition for dark band : $\delta = (2n - 1) \frac{\lambda}{2}$, for bright band :

$$\delta = m\lambda$$

▶ Fringe width $\beta = \frac{D\lambda}{d}$

▶ A thin film of thickness t and refractive index μ appears dark by reflection when viewed at an angle of refraction r if

$$2\mu t \cos r = n\lambda \quad (n = 1, 2, 3, \text{etc.})$$

▶ A single slit of width a gives a diffraction pattern with a central maximum. The intensity falls to zero at angles of $\pm \frac{\lambda}{a}, \pm \frac{2\lambda}{a}$, etc.

▶ Amplitude of resultant wave $R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$

▶ Intensity of wave $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

▶ Brewster law : $\mu = \tan i_p$

MODERN PHYSICS

▶ Energy of a photon $E = hv = \frac{hc}{\lambda}$

▶ Momentum of a photon $P = \frac{h}{\lambda}$

▶ Einstein's photoelectric equation

$$\frac{1}{2}mv_{\max}^2 = V_0 e = hv - \phi_0 = h(\nu - \nu_0)$$

▶ Mass defect, $\Delta M = (Zm_p + (A - Z)m_n) - M$; $\Delta E_b = \Delta M c^2$
 1 amu = 931 MeV

▶ $E_n = -\frac{Z^2}{n^2} \times 13.6 \text{ eV}$ (For hydrogen like atom)

▶ According to Bohr's atomic model, angular momentum for the electron revolving in stationary orbit, $mvr = nh/2\pi$

▶ Radius of the orbit of electron $r = \frac{n^2 h^2}{4\pi^2 m k z e^2}$

▶ Bragg's law : $2d \sin \theta = n\lambda$.

▶ Radius of the nucleus $R = R_0 A^{1/3}$

▶ Law of radioactive decay : $N = N_0 e^{-\lambda t}$.

Activity = $\frac{dN}{dt} = -\lambda N$ (unit is Becquerel)

▶ Half life period, $T_{1/2} = \frac{0.693}{\lambda}$

▶ Characteristics X-rays : $\lambda_{K\alpha} < \lambda_{L\alpha}$

Moseley law : $\nu = a(Z - b)^2$

▶ Pure semiconductors are called 'intrinsic semiconductors'. The presence of charge carriers (electrons and holes) number of electrons (n_e) is equal to the number of holes (n_h).

▶ The number of charge carriers can be changed by 'doping' of a suitable impurity in pure semiconductors known as extrinsic semiconductors (n-type and p-type).

▶ In n-type semiconductors, $n_e \gg n_h$ while in p-type semiconductors $n_h \gg n_e$.

▶ n-type semiconducting Si or Ge is obtained by doping with pentavalent atoms (donors) like As, Sb, P, etc., while p-type Si or Ge can be obtained by doping with trivalent atom (acceptors) like B, Al, In etc.

▶ In forward bias (n-side is connected to negative terminal of the battery and p-side is connected to the positive), the barrier is decreased while the barrier increases in reverse bias.

▶ Diodes can be used for rectifying an ac voltage (restricting the ac voltage to one direction).

▶ Zener diode is one such special purpose diode. In reverse bias, after a certain voltage, the current suddenly increases (breakdown voltage) in a Zener diode. This property has been used to obtain voltage regulation.

▶ The important transistor parameters for CE-configuration are:

Input resistance

Output resistance

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

$$r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C} \right)_{I_B}$$

Current amplification factor, $\beta = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE}}$

The voltage gain of a transistor amplifier in common emitter configuration is:

$A_v = \left(\frac{v_o}{v_i} \right) = \beta \frac{R_C}{R_B}$, where R_C and R_B are respectively the resistances in collector and base sides of the circuit.

▶ The important digital circuits performing special logic operations are called logic gates. These are: OR, AND, NOT, NAND, and NOR gates. NAND gate is the combination of NOT and AND gate. NOR gate is the combination of NOT and OR gate.

▶ Transmitter, transmission channel and receiver are three basic units of a communication system.

▶ Two important forms of communication system are: Analog and Digital. The information to be transmitted is generally in continuous waveform for the former while for the latter it has only discrete or quantised levels.

▶ Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier signal by a process known as modulation.

In the process of modulation, new frequencies called sidebands are generated on either side.

▶ If an antenna radiates electromagnetic waves from a height h_T , then the range $d_T = \sqrt{2Rh_T}$ $R =$ radius of earth.

▶ Effective range, $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
 $h_T =$ height of transmitting antenna; $h_R =$ height of receiving antenna

▶ Critical frequency $V_c = 9(N_{\max})^{1/2}$
 where $N_{\max} =$ no. density of electrons/ m^3

▶ Skip distance, $D_{\text{skip}} = 2h \left(\frac{V_{\max}}{V_c} \right)^2 - 1$

$h =$ height of reflecting layer of atmosphere.

▶ Power radiated by an antenna $\propto \frac{1}{\lambda^2}$

COMMUNICATION SYSTEMS