

This Question Paper contains 20 printed pages.

(Part - A & Part - B)

Sl.No.

050 (E)

(MARCH, 2024)
(SCIENCE STREAM)
(CLASS - XII)

પ્રશ્ન પેપરનો સેટ નંબર જોની
સામેનું વર્તુળ OMR શીટમાં
ઘટ્ટ કરવાનું રહે છે.

Set No. of Question Paper,
circle against which is to be
darken in OMR sheet.

16

Part - A : Time : 1 Hour / Marks : 50

Part - B : Time : 2 Hours / Marks : 50

(Part - A)

Time : 1 Hour]

[Maximum Marks : 50

Instructions :

- 1) There are 50 objective type (M.C.Q.) questions in Part - A and all questions are compulsory.
- 2) The questions are serially numbered from 1 to 50 and each carries 1 mark.
- 3) Read each question carefully, select proper alternative and answer in the OMR sheet.
- 4) The OMR sheet is given for answering the questions. The answer of each question is represented by (A) O, (B) O, (C) O and (D) O. Darken the circle ● of the correct answer with ball-pen.
- 5) Rough work is to be done in the space provided for this purpose in the Test Booklet only.
- 6) Set No. of Question Paper printed on the upper- most right side of the Question Paper is to be written in the column provided in the OMR sheet.
- 7) Use of simple calculator and log table is allowed, if required.
- 8) Notations used in this question paper have proper meaning.

- 1) If the lines $\frac{1-x}{3} = \frac{y-2}{1} = \frac{z-1}{2}$ and $\frac{x-2}{p} = \frac{y-1}{2} = \frac{z-2}{1}$ are perpendicular to each other, then $p =$ _____.

(A) $-\frac{2}{3}$

(B) 0

(C) $\frac{4}{3}$

(D) $-\frac{4}{3}$

Rough Work

- 2) For linear programming problem, the objective function is $Z = 3x + 2y$. If the corner points of the bounded feasible region are $(12, 0)$, $(4, 2)$, $(1, 5)$ and $(0, 10)$, then the maximum value of Z is _____.
- (A) 36 (B) 46
(C) 13 (D) 56
- 3) Corner points of the feasible region determined by the system of linear constraints are $(0, 3)$, $(1, 1)$ and $(3, 0)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum value of Z occurs at $(3, 0)$ and $(1, 1)$ is _____.
- (A) $p = 2q$ (B) $p = \frac{q}{2}$
(C) $p = 3q$ (D) $p = q$
- 4) The probability of obtaining an even number on each die, when a pair of dice is rolled is _____.
- (A) $\frac{1}{9}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{1}{36}$
- 5) Given that events A and B are such that $P(A) = 0.5$, $P(A \cup B) = 0.6$, $P(B) = K$. If A and B are mutually exclusive events then $K =$ _____.
- (A) 0.1 (B) 0.2
(C) 0.11 (D) 0

6) The relation $R = \{(a, b), (b, a)\}$ is defined on the set $\{a, b, c\}$, then R is _____.

- (A) Reflexive, but not symmetric and transitive
- (B) Symmetric, but not reflexive and transitive
- (C) Transitive, but not reflexive and symmetric
- (D) An equivalence relation

7) Function defined as $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x^6$, then _____.

- (A) f is one-one and onto.
- (B) f is many-one and onto.
- (C) f is one-one, but not onto.
- (D) f is neither one-one nor onto.

8) Let $A = \{1, 2, 3\}$, then number of equivalence relations containing $(1, 2)$ is _____.

- | | |
|-------|-------|
| (A) 4 | (B) 3 |
| (C) 2 | (D) 1 |

9) $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \text{_____}$.

- | | |
|----------------------|-----------|
| (A) $\frac{\pi}{2}$ | (B) π |
| (C) $\frac{5\pi}{6}$ | (D) 0 |

10) $\tan^{-1}\left(\tan\frac{31\pi}{6}\right) = \underline{\hspace{2cm}}.$

(A) $\frac{\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{31\pi}{6}$

(D) $-\frac{\pi}{6}$

11) If $\cos^{-1} x = y$, then $\underline{\hspace{2cm}}.$

(A) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

(B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $0 \leq y \leq \pi$

12) $\cos(\tan^{-1} x) = \underline{\hspace{2cm}}. (|x| < 1).$

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{x}{\sqrt{1+x^2}}$

13) The number of all possible matrices of order 3×2 with each entry 1 or 2 is $\underline{\hspace{2cm}}.$

(A) 512

(B) 64

(C) 32

(D) 128

14) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & -2 \\ 0 & 3 \end{bmatrix}$, then $AB =$ _____.

(A) $\begin{bmatrix} 4 & 0 \\ 1 & -2 \\ 0 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 0 \\ 1 & -2 \end{bmatrix}$

(D) • not defined

15) If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A =$ _____.

(A) A

(B) $I - A$

(C) • I

(D) $3A$

16) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{10} =$ _____.

(A) $1024A$

(B) • $512A$

(C) $10A$

(D) A

17) If $f(\theta) = \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{vmatrix}$, then $f\left(\frac{\pi}{6}\right) =$ _____.

(A) • $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $-\frac{\sqrt{3}}{2}$

18) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$, then $|adj A| = \underline{\hspace{2cm}}$.

(A) 2

(B) • 4

(C) 8

(D) 6

19) If $A = \begin{bmatrix} 5 & -2 \\ 4 & 3 \end{bmatrix}$, then $A(adj A) = \underline{\hspace{2cm}}$.

(A) I

(B) A

(C) • 23 I

(D) 23 A

20) If area of a triangle is 3 sq. units with the vertices $(3, 5)$, $(2, 2)$ and $(k, 2)$, then $k = \underline{\hspace{2cm}}$.

(A) • 0, 4

(B) 0, -4

(C) 3, 1

(D) -3, 1

21) If $f(x) = \begin{cases} kx+1, & x \leq \frac{\pi}{2} \\ \sin x, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $k = \underline{\hspace{2cm}}$.

(A) $-\frac{2}{\pi}$ (B) $\frac{2}{\pi}$

(C) 1

(D) • 0

22) If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

- (A) y (B) $y - 1$
(C) 0 (D) does not exist

23) Differentiation of $\sin^2 x$ w.r.t. $\cos^2 x$ is $\underline{\hspace{2cm}}$.

- (A) $\tan^2 x$ (B) $-\tan^2 x$
(C) -1 (D) 1

24) The rate of change of the volume of a sphere with respect to radius r , at $r = 3$ cm is $\underline{\hspace{2cm}}$ cm^3/s .

- (A) 12π (B) 36π
(C) 24π (D) 81π

25) The total revenue in Rupees received from the sale of x units of a product is given by, $R(x) = x^2 + 6x + 5$. The marginal revenue, when $x = 20$ is ₹ $\underline{\hspace{2cm}}$.

- (A) 525 (B) 126
(C) 46 (D) 96

26) The function given by $f(x) = x^2 - 6x + 10$ is an increasing in $\underline{\hspace{2cm}}$ interval.

- (A) $(3, \infty)$ (B) $(-\infty, 3)$
(C) $(-3, 3)$ (D) $(0, 6)$

27) The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is _____.

(A) $(2, 2)$

(B) $(0, 0)$

(C) $(2\sqrt{2}, 0)$

(D) $(2\sqrt{2}, 4)$

28) $\int \frac{\operatorname{cosec}^2 x}{\sec^2 x} dx = \text{_____} + C.$

(A) $\tan x - x$

(B) $-\cot x - x$

(C) $\cot x - x$

(D) $-\cot x + x$

29) $\int \frac{1}{x + x \log x} dx = \text{_____} + C.$

(A) $1 + \log x$

(B) $\log |\log x|$

(C) $\log |\log ex|$

(D) $\frac{(1 + \log x)^2}{2}$

30) $\int \frac{1}{e^x + 1} dx = \text{_____} + C.$

(A) $\log \left| \frac{e^x}{e^x + 1} \right|$

(B) $\log \left| \frac{e^x + 1}{e^x} \right|$

(C) $\log \left| \frac{1}{e^x + 1} \right|$

(D) $\log \left| \frac{e^x - 1}{e^x + 1} \right|$

31) $\int \frac{dx}{x^2 + 2x + 5} = \underline{\hspace{2cm}} + C.$

(A) $\tan^{-1}\left(\frac{x+1}{2}\right)$

(B) $\frac{1}{2} \tan^{-1}\left(\frac{x+1}{2}\right)$

(C) $\tan^{-1}(x+1)$

(D) $\frac{1}{2} \tan^{-1}(x+1)$

32) $\int_{-1}^1 \sin^7 x \cdot \cos^6 x \, dx = \underline{\hspace{2cm}}.$

(A) -1

(B) 2

(C) 0

(D) 1

33) $\int e^x \tan x (1 + \tan x) \, dx = \underline{\hspace{2cm}} + C.$

(A) $e^x (\tan x - 1)$

(B) $e^x \tan x$

(C) $e^x \sec x$

(D) $e^x (\tan x + 1)$

34) $\int_0^{2\pi} \sin^3 x \cos^2 x \, dx = \underline{\hspace{2cm}}.$

(A) 2π

(B) -1

(C) 1

(D) 0

35) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos^{\frac{1}{2}} x}{\cos^{\frac{1}{2}} x + \sin^{\frac{1}{2}} x} dx = \underline{\hspace{2cm}}.$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{12}$

(D) $\frac{\pi}{2}$

36) The area bounded by the curve $y = \cos x$ between $x = \frac{\pi}{2}$ and

$x = \frac{3\pi}{2}$ is $\underline{\hspace{2cm}}.$

(A) 1

(B) 2

(C) 3

(D) 4

37) Area of the region bounded by the curve $y^2 = 4x$, y -axis and the line $y = 3$ is $\underline{\hspace{2cm}}.$

(A) $\frac{9}{2}$

(B) 3

(C) $\frac{9}{4}$

(D) 2

38) The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = 0$ and $x = 1$ is given by $\underline{\hspace{2cm}}.$

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

39) The order of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is } \underline{\hspace{2cm}}.$$

(A) • 3

(B) 4

(C) 2

(D) undefined

40) The number of arbitrary constants in the particular solution of a differential equation of fourth order are .

(A) • 0

(B) 4

(C) 3

(D) 2

41) The integrating factor of the differential equation

$$x \frac{dy}{dx} + 2y = x^2 \ (x \neq 0) \text{ is } \underline{\hspace{2cm}}.$$

(A) $2 \log x$

(B) $\log x$

(C) $\frac{2}{x}$

(D) • x^2

42) The general solution of a differential equation $\frac{ydx - xdy}{y} = 0$ is .

(A) $y = Cx^2$

(B) • $y = Cx$

(C) $x = Cy^2$

(D) $xy = C$

43) The vector in the direction of a vector $\vec{a} = 4\hat{i} + 3\hat{j} - 2\hat{k}$, that has magnitude $2\sqrt{29}$ is _____.

(A) $\frac{4}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} - \frac{2}{\sqrt{29}}\hat{k}$

(B) $4\hat{i} + 3\hat{j} - 2\hat{k}$

(C) $8\hat{i} + 6\hat{j} - 4\hat{k}$

(D) $\frac{2}{\sqrt{29}}\hat{i} + \frac{3}{2\sqrt{29}}\hat{j} - \frac{1}{\sqrt{29}}\hat{k}$

44) The direction cosines of the vector joining the points A(1, 2, -3) and B(-1, -2, 1), directed from A to B is _____.

(A) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

(B) $\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

(C) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

(D) $\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

45) The angle between the vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ is _____.

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

46) The projection of the vector $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ on the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is _____.

(A) $\frac{10}{\sqrt{6}}$

(B) $\frac{\sqrt{10}}{6}$

(C) $\frac{\sqrt{10}}{17}$

(D) $\frac{10}{\sqrt{17}}$

47) The area of a parallelogram whose adjacent sides are $\vec{a} = \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j}$ is _____.

(A) $2\sqrt{21}$

(B) $\sqrt{42}$

(C) $\sqrt{21}$

(D) $\frac{1}{2}\sqrt{21}$

48) For the vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}| = \frac{2}{3}$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is _____.

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

49) Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$, then the vector equation of the line is _____.

(A) $\vec{r} = 3\hat{i} + 7\hat{j} - 2\hat{k} + \lambda(5\hat{i} - 4\hat{j} + 6\hat{k})$

(B) $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$

(C) $\vec{r} = 3\hat{i} + 7\hat{j} + 2\hat{k} + \lambda(5\hat{i} - 4\hat{j} + 6\hat{k})$

(D) $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

50) The angle between the pair of lines $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2}$ and

$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6}$ is _____

(A) $\sin^{-1}\left(\frac{17}{21}\right)$

(B) $\cos^{-1}\left(\frac{17}{21}\right)$

(C) $\sin^{-1}\left(\frac{19}{21}\right)$

(D) $\cos^{-1}\left(\frac{19}{21}\right)$

050 (E)

(MARCH, 2024)
(SCIENCE STREAM)
(CLASS - XII)

(Part - B)**Time : 2 Hours/****[Maximum Marks : 50]****Instructions :**

- 1) Write in a clear legible handwriting.
- 2) There are three sections in Part - B of the question paper and total 1 to 27 questions are there.
- 3) All questions are compulsory. Internal options are given.
- 4) The numbers at right side represent the marks of the question.
- 5) Start new section on new page.
- 6) Maintain sequence.
- 7) Use of simple calculator and log table is allowed, if required.
- 8) Use the graph paper to solve the problem of L.P.

SECTION - A

- From the following question no. 1 to 12 answer any 8 questions as directed. [16]
(Each carries 2 marks)

1) Prove that : $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$. [2]

2) Prove that : $\sin^{-1} \left(2x\sqrt{1-x^2} \right) = 2\sin^{-1} x$, where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$. [2]

3) If $x^x + y^x = 1$, then find $\frac{dy}{dx}$. [2]

4) Obtain $\int \frac{dx}{\sqrt{8+3x-x^2}}$. [2]

- 5) Find the area of the region bounded by the ellipse $9x^2 + 16y^2 = 144$. [2]
- 6) Find the area of the region bounded by the line $y = 3x + 2$, the x -axis and the ordinates $x = -1$ and $x = 1$. [2]
- 7) Find the general solution of the differential equation : [2]
 $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$.
- 8) If vertices of triangle are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$, determine the type of triangle they form. [2]
- 9) Find the Cartesian equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines : [2]
 $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
- 10) Find vector and the Cartesian equations of the line through the point $(5, 2, -4)$ and which is parallel to the vector $3\hat{i} - 2\hat{j} + 8\hat{k}$. [2]
- 11) Given that A and B are events such that $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.2$. Find $P(A|B)$ and $P(B|A)$. [2]
- 12) An unbiased die is thrown twice. Let the event E be 'odd number on the first throw' and F the event 'odd number on the second throw'. Check the independence of events E and F. [2]

SECTION - B

- From the following question no. : 13 to 21 answer any 6 questions as directed. [18]
(Each carries 3 marks)

13) Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive. [3]

14) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 = 6A^2 - 7A - 2I$. [3]

15) Using the cofactor of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$. [3]

16) If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} - 2 = 0$. [3]

17) Find the maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval $x \in [1, 3]$.
Find the maximum value of the same function in $x \in [-3, -1]$. [3]

18) The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ . [3]

19) Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$. [3]

20) Solve the following Linear Programming problem graphically : [3]
For $Z = -3x + 4y$
Subject to $x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$.
Find minimum and maximum values of Z .

21) In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B? [3]

SECTION - C

- From the following question no. : 22 to 27 answer any 4 questions as directed.
(Each carries 4 marks) [16]

22) Find x , if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$. [4]

23) Using matrix method, solve the system of equations: [4]

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 6, \frac{1}{y} + \frac{3}{z} = 11, \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 0$$

24) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\left(\frac{d^2 y}{dx^2} \right)_{t=\frac{\pi}{4}}$. [4]

25) Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$. [4]

26) Evaluate: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$. [4]

27) Find the particular solution of the differential equation [4]

$$2y e^{\frac{x}{y}} dx + \left(y - 2x e^{\frac{x}{y}} \right) dy = 0,$$

given that $x = 0$ when $y = 1$.