



Total No. of Questions : 24

Total No. of Printed Pages : 3



Part - III
MATHEMATICS - PAPER - I(A)
(English Version)

Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of three Sections-A, B and C.

SECTION - A**I. Very Short Answer Type Questions :****10x2=20**

- (i) Answer *all* the questions.
(ii) Each question carries two marks.

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ for all $x \in \mathbb{R}$, then find $(g \circ f)(x)$.

2. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 25}$.

3. Define symmetric matrix and give an example.

4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and $\det A = 45$ then find x .

5. If $\overrightarrow{OA} = \vec{i} + \vec{j} + \vec{k}$, $\overrightarrow{AB} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\overrightarrow{BC} = \vec{i} + 2\vec{j} - 2\vec{k}$ and $\overrightarrow{CD} = 2\vec{i} + \vec{j} + 3\vec{k}$ then find the vector $\overrightarrow{OD}$.

6. Find the vector equation of the plane passing through the points $\vec{i} - 2\vec{j} + 5\vec{k}$, $-5\vec{j} - \vec{k}$ and $-3\vec{i} + 5\vec{j}$.

7. Let $\vec{a}$ and $\vec{b}$ be non-zero, non-collinear vectors. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

8. Find the value of $\sin 330^\circ \cdot \cos 120^\circ + \cos 210^\circ \cdot \sin 300^\circ$

9. If $A - B = \frac{3\pi}{4}$, then show that $(1 - \tan A)(1 + \tan B) = 2$.

10. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \log_e 3$.

SECTION - B

5x4=20

II. Short Answer Type Questions :

- (i) Answer any five questions.
(ii) Each question carries four marks.

11. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then prove that A is invertible and

$$A^{-1} = \frac{\text{Adj}A}{\det A}.$$

✓ 12. If the points whose position vectors are $3\vec{i} - 2\vec{j} - \vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-\vec{i} + \vec{j} + 2\vec{k}$ and $4\vec{i} + 5\vec{j} + \lambda\vec{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

✓ 13. If $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find the value of $(\vec{a} \times \vec{b}) \cdot \vec{c}$.

14. For $A \in \mathbb{R}$, prove that $\cos A \cos\left(\frac{\pi}{3} + A\right) \cos\left(\frac{\pi}{3} - A\right) = \frac{1}{4} \cos 3A$ and hence deduce that

$$\cos \frac{\pi}{9} \cdot \cos \frac{2\pi}{9} \cdot \cos \frac{3\pi}{9} \cdot \cos \frac{4\pi}{9} = \frac{1}{16}.$$

15. Solve $\cot^2 x - (\sqrt{3} + 1) \cot x + \sqrt{3} = 0$; $0 < x < \frac{\pi}{2}$.

16. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

17. In a ΔABC , show that $\frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin(B + C)}$.



SECTION C

5x7=35

III. Long Answer Type Questions :

- (i) Answer any five questions.
 (ii) Each question carries Seven marks.

✓18. Let $f: A \rightarrow B$ be a bijection. Then prove that $f \circ f^{-1} = I_B$ and $f^{-1} \circ f = I_A$.

✓19. By using Mathematical Induction, to prove the statement :

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}, \forall n \in \mathbb{N}.$$

✓20. Show that
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

✓21. Solve $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$ by using matrix inversion method.

22. Find the shortest distance between the Skew lines.

$$\vec{r} = (6\vec{i} + 2\vec{j} + 2\vec{k}) + t(\vec{i} - 2\vec{j} + 2\vec{k}) \text{ and } \vec{r} = (-4\vec{i} - \vec{k}) + s(3\vec{i} - 2\vec{j} - 2\vec{k}).$$

23. If $A + B + C = \frac{\pi}{2}$, then prove that $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$.

✓ 24. In ΔABC , prove that $r + r_1 + r_2 - r_3 = 4R \cos C$.