

Mathematics

(Common for Humanities, Sc & Agri Groups)

(Evening Session)

Time : Three Hours]

[Maximum Marks : 90

- I. i If $\cos^{-1} x = y$, then 1
- (a) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$ (b) $0 < y < \pi$
- (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 \leq y \leq \pi$
- ii If A and B are symmetric matrices of same order then $AB+BA$ is : 1
- (a) Skew-symmetric matrix (b) symmetric matrix
- (c) null matrix (d) identity matrix
- iii If $f(x) = \begin{cases} kx - 1 & , \quad x \leq 2 \\ 3x - 5 & , \quad x > 2 \end{cases}$ $x = 5$ is continuous at $x=5$ then value of k is : 1
- (a) $\frac{3}{5}$ (b) $\frac{11}{5}$
- (c) $\frac{8}{5}$ (d) $\frac{9}{5}$
- iv $\int_{-1}^1 x^{11} \cos^4 x \, dx$ is equal to : 1
- (a) $\frac{1}{9}$ (b) $\frac{-1}{9}$
- (c) 0 (d) $\frac{1}{8}$
- v $\int_0^{\sqrt{3}} \frac{dx}{1+x^2}$ is equal to : 1
- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{12}$ (d) $\frac{7\pi}{12}$
- vi Degree of differential equation $\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx}\right)^2 = 0$ is; 1
- (a) 0 (b) 1
- (c) 2 (d) 3
- vii Position vector of mid-point of vector joining the points P (8,3,-4) and Q (4,-7,-2) is 1
- (a) $3\hat{i} + 2\hat{j} + \hat{k}$ (b) $6\hat{i} - 2\hat{j} - 3\hat{k}$
- (c) $8\hat{i} + 3\hat{j} - 8\hat{k}$ (d) $\hat{i} + \hat{j} + 3\hat{k}$
- viii If line makes angles $135^\circ, 90^\circ, 45^\circ$ with X,Y and Z axes respectively then its direction ratios are : 1
- (a) $\langle 1, -1, 1 \rangle$ (b) $\langle 0, 1, 1 \rangle$
- (c) $\langle -1, 0, 1 \rangle$ (d) $\langle 0, 1, -1 \rangle$

- 2 Show that function $R \rightarrow R$ given by $f(x) = x^2$ is neither one – one nor onto. 3

OR

Show that relation $R = \{(L_1, L_2) : L_2 \text{ is parallel to } L_1\}$, defined on the set of all lines, is an equivalence relation.

3 Prove that $2 \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{39}{37}$ 3

4 Express $\begin{bmatrix} 2 & 1 \\ -1 & 8 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrices 3

OR

Using determinates find the equation on line passing from the points (1,4) and (-1,2).

5 If $y = (x \sin x)^x$ then find $\frac{dy}{dx}$. 3

6 Verify Rolle's theorem for $f(x) = x^2 - 8x + 15, x \in [3,5]$. 3

7 Using differentials find the approximate value of $(0.028)^{1/3}$. 3

8 Evaluate $\int \frac{3x+1}{\sqrt{x^2+4x+9}} dx$ 3

OR

Evaluate $\int \frac{(x^2-3)e^x}{(x+3)^2} dx$

9 Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}} dx$ 3

10 Using integration find the area of triangle with vertices (5,1), (5,3) and (7,0). 3

11 Find the particular solution of the differential equation $\cos x \frac{dy}{dx} = y \sin x; y = 1 \text{ when } x = 0$ 3

12 Solve $\frac{dy}{dx} + 3y = \cos x$ 3

13 If $|\vec{a}| = 2, |\vec{b}| = 3$ and $|\vec{c}| = 4$ and each of them is perpendicular to the sum of other two them find value is $|\vec{a} + \vec{b} + \vec{c}|$. 3

OR

Using vectors find the area of the triangle whose vertices are (1,1,-1), (2,3,1) and (1,3,5).

14 Probability of solving specific problem independently by A,B and C are $\frac{3}{7}, \frac{1}{3}$ and $\frac{2}{5}$ respectively, if all of them try to solve the problem independently then find the probability that problem will be solved. 3

15 A pair of dice thrown 5 times. If getting a doublet is considered success then find the probability of exactly 2 successes, 3

16 Solve the following system of linear equation by matrix method: 5

$$2x + 3y + 4z = 4, x - 2y - z = -2, 3x - y + z = 0$$

OR

Using elementary transformation find the inverse of $\begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$.

17 A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 20 m. find the dimensions of the window to admit maximum light through the whole opening. 5

18 Find the distance of the (2,3,-1) from the point of intersection of the line 5

$$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(3\hat{i} + 6\hat{j} + 2\hat{k}) \text{ and the plane } \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3.$$

OR

Find the shortest distance between the lines:

$$\vec{r} = \hat{i} + \hat{j} - \lambda(3\hat{i} + \hat{j} - 2\hat{k}) \text{ and } \vec{r} = 2\hat{i} + 3\hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + 4\hat{k})$$

19 Graphically minimize $Z = 7x + 4y$ subject to the constraints

$$5x + 2y \geq 100, 3x + 5y \geq 150, 9x + 8y \leq 360, x \geq 0, y \geq 0.$$

5

