

Reg. No. :

SY-227

Name :

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2021

Part – III

Time : 2 Hours

MATHEMATICS

Cool-off time : 20 Minutes

Maximum : 60 Scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 20 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 20 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer the following questions from 1 to 29 upto a maximum score of 60.

Part - A

Questions from 1 to 10 carry 3 scores each.

(10 × 3 = 30)

1. Find the values of x for which

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} \quad (3)$$

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(i) Find $\text{adj } A$ (2)

(ii) Find $A \cdot \text{adj } A$. (1)

3. Find the value of k so that the function

$$f(x) = \begin{cases} kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$. (3)

4. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$. (3)

5. Find the rate of change of the area of a circle with respect to its radius r when $r = 5$ cm. (3)

6. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. (3)

7. Find the equation of a plane passing through the point $(1, 4, 6)$ and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$. (3)

8. (i) Which of the following can be the domain of the function $\cos^{-1}x$?

- (a) $(0, \pi)$ (b) $[0, \pi]$
(c) $(-\pi, \pi)$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (1)

- (ii) Find the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$. (2)

9. Find the area of a triangle with vertices $(-2, -3)$, $(3, 2)$ and $(-1, -8)$. (3)

10. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. (3)

Part - B

Questions from 11 to 22 carry 4 scores each.

(12 × 4 = 48)

11. Consider the matrices $A = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix}$ and $3A + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$

(i) Find the matrix B. (2)

(ii) Find AB. (2)

12. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1, 3, -6]$

(i) What is the order of AB? (1)

(ii) Verify $(AB)' = B'A'$. (3)

13. (i) If $xy < 1$, $\tan^{-1} x + \tan^{-1} y = \underline{\hspace{2cm}}$.

(a) $\tan^{-1} \frac{x-y}{1+xy}$ (b) $\tan^{-1} \frac{1-xy}{x+y}$

(c) $\tan^{-1} \frac{x+y}{1-xy}$ (d) $\tan^{-1} \frac{x+y}{1+xy}$ (1)

(ii) Prove that $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$. (3)

14. Find $\frac{dy}{dx}$

(i) $x^2 + xy + y^2 = 100$. (2)

(ii) $y = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$, $-1 \leq x \leq 1$. (2)

15. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is
- increasing
 - decreasing (4)
16. (i) Find the order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + \frac{3}{dt^2}s = 0$. (1)
- (ii) Find the general solution of the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$. (3)
17. Find a unit vector both perpendicular to the vectors if
- $$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k} \text{ and } \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}.$$
- (4)
18. Find the shortest distance between the skew lines
- $$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$
- (4)
19. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$. Find
- $P(A \cap B)$ (2)
 - $P(A|B)$ (1)
 - $P(A \cup B)$ (1)
20. (i) Let R be a relation on a set $A = \{1, 2, 3\}$, defined by $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Then the ordered pair to be added to R to make it a smallest equivalence relation is _____.
- $(2, 1)$
 - $(3, 1)$
 - $(1, 2)$
 - $(1, 3)$ (1)
- (ii) Determine whether the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y \text{ is divisible by } x\}$ is reflexive, symmetric and transitive. (3)

21. Find $\frac{dy}{dx}$

(i) x^x (2)

(ii) $x = 2at^2$; $y = at^4$ (2)

22. Integrate :

$$\int \frac{x}{(x+1)(x+2)} dx. \quad (4)$$

Part - C

Questions from 23 to 29 carry 6 scores each.

(7 × 6 = 42)

23. (i) Construct a 3×2 matrix $A = [a_{ij}]$ whose elements are given by

$$a_{ij} = 3\hat{i} - \hat{j} \quad (2)$$

(ii) Express $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (4)

24. Solve the following system of equations by matrix method

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4 \quad (6)$$

25. (i) Let $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by

$$f = \{(1, 2), (3, 5), (4, 1)\} \text{ and } g = \{(1, 3), (2, 3), (5, 1)\}. \text{ Write down } \text{gof}. \quad (3)$$

(ii) Consider $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x + 1$. Show that f is invertible. Find the inverse of f . (3)

26. (i) Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$. (2)
- (ii) Find the equation of tangent to the above curve. (2)
- (iii) What is the maximum value of the function $\sin x + \cos x$? (2)

27. Integrate :

(i) $\int \sin x \sin (\cos x) dx$. (3)

(ii) $\int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$. (3)

28. Solve the following problem graphically

Maximise : $z = 3x + 2y$

Subject to : $x + 2y \leq 10$

$3x + y \leq 15,$

$x, y \geq 0$ (6)

29. (i) Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x -axis. (3)
- (ii) Find the area of the region bounded by two parabolas $y = x^2$ and $y^2 = x$. (3)

SECOND YEAR HIGHER SECONDARY EXAMINATION

MARCH 2021

Questions carry 3 Scores each **ANSWER KEY**

① $3 - x^2 = 3 - 8$

$$x^2 - 8 = 0$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

② i) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

ii) $A \cdot \text{Adj } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

③ LHL = $\lim_{x \rightarrow 5^-} f(x)$

$$= \lim_{x \rightarrow 5^-} kx + 1$$

$$= 5k + 1$$

$$\text{RHL} = \lim_{x \rightarrow 5^+} f(x)$$

$$= \lim_{x \rightarrow 5^+} 3x - 5$$

$$= 15 - 5$$

$$= 10$$

But $f(x)$ is continuous at 5.

$$\therefore \text{LHL} = \text{RHL} = f(5)$$

$$\therefore 5k + 1 = 10$$

$$5k = 9$$

$$k = \frac{9}{5}$$

④ $f(x) = x^2 + 2x - 8$

$f(x)$ is a polynomial function which is continuous in $[-4, 2]$ and differentiable in $(-4, 2)$

$$f(a) = f(-4)$$

$$= (-4)^2 + 2(-4) - 8$$

$$= 16 - 8 - 8$$

$$= 0$$

$$f(b) = f(2)$$

$$= 2^2 + 2(2) - 8$$

$$= 4 + 4 - 8$$

$$\therefore f(a) = f(b).$$

Then there exist c such that $f'(c) = 0$.

$$\text{But } f'(x) = 2x + 2$$

$$f'(c) = 0 \Rightarrow 2c + 2 = 0$$

$$2c = -2$$

$$c = -1 \in [-4, 2]$$

Hence Roll's Theorem is verified.

⑤ Area of a circle, $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$= 2\pi \times 5$$

$$= 10\pi \text{ cm}^2/\text{s}$$

$$\therefore r = 5$$

⑥ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$\vec{a} \cdot \vec{b} = 1 \times 7 + (3 \times -1) + (7 \times 8)$$

$$= 7 - 3 + 56$$

$$= 60$$

$$|\vec{b}| = \sqrt{7^2 + 1^2 + 8^2} = \sqrt{49 + 1 + 64} = \sqrt{114}$$

$$\therefore \text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{60}{\sqrt{114}}$$

⑦ Position vector of the point is $\hat{i} + 4\hat{j} + 6\hat{k}$

Normal vector is $\hat{i} - 2\hat{j} + \hat{k}$.

Vector equation

$$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$$

$$[\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

Cartesian equation

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$1(x - 1) - 2(y - 4) + 1(z - 6) = 0$$

$$x - 1 - 2y + 8 + z - 6 = 0$$

$$x - 2y + z + 1 = 0$$

⑧ 1) incorrect question

ii) $y = \cos^{-1}(-\frac{1}{2})$

$$\cos y = -\frac{1}{2}$$

$$= \cos(\pi - \frac{\pi}{3})$$

$$= \cos \frac{2\pi}{3} \therefore \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$$

Let $y = \sin^{-1}(\frac{1}{2})$

$$\sin y = \frac{1}{2}$$

$$= \sin \frac{\pi}{6}$$

$$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}(-\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$$

$$= \frac{2\pi}{3} + 2 \times \frac{\pi}{6}$$

$$= \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= \frac{3\pi}{3}$$

$$= \pi$$

⑨
$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [-2(2+8) + 3(3+1) + 1(-24+2)]$$

$$= \frac{1}{2} [-20 + 12 + 22]$$

$$= -\frac{30}{2}$$

$$= \underline{\underline{15 \text{ sq. units}}}$$

10. $\frac{dy}{dx} - y = \cos x$

$$P = -1, Q = \cos x$$

$$I.F. = e^{\int -1 dx} = e^{-x}$$

Solution is,

$$y \cdot (I.F.) = \int (Q \times I.F.) dx + C$$

$$y \cdot e^{-x} = \int \cos x \cdot e^{-x} dx + C$$

$$I = \cos x \left(\frac{e^{-x}}{-1} \right) - \int (-\sin x) \cdot e^{-x} dx + C$$

$$= -\cos x e^{-x} - \int \sin x e^{-x} dx + C$$

$$= -\cos x e^{-x} \left[\sin x \left(\frac{e^{-x}}{-1} \right) \right.$$

$$\left. - \int \cos x (-e^{-x}) dx \right] + C$$

$$= -\cos x e^{-x} + \sin x e^{-x}$$

$$- \int \cos x e^{-x} dx$$

$$= -e^{-x} \cos x + e^{-x} \sin x - I$$

$$2I = (\sin x + \cos x) e^{-x}$$

$$I = \frac{(\sin n - \cos n)e^{-n}}{2}$$

∴ Solution is,

$$ye^{-n} = \frac{(\sin n - \cos n)e^{-n}}{2} + C$$

$$y = \frac{(\sin n - \cos n)}{2} + Ce^n$$

Questions Carry 4 Scores each.

11. i) $3A+B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$

$$3 \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix} + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix} + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$$

ii) $AB = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix} \begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 51 & -16 \\ 17 & 21 \end{bmatrix}$$

12. i) 3×3

ii) $AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} [1, 3, -6]$

$$= \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -20 \\ 5 & 15 & -30 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -20 & -30 \end{bmatrix} \quad \text{--- ①}$$

$$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

$$A' = [-2 \ 4 \ 5]$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} [-2 \ 4 \ 5]$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -20 & -30 \end{bmatrix} \quad \text{--- ②}$$

from ① & ②

$$(AB)' = B'A'$$

13. i) C, $\tan^{-1} \frac{n+y}{1-ny}$

ii) $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24}$

$$= \tan^{-1} \left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{48+77}{264}}{\frac{264-14}{264}} \right)$$

$$= \tan^{-1} \left(\frac{125}{250} \right)$$

$$= \tan^{-1} \left(\frac{1}{2} \right)$$

(14) i) $x^2 + xy + y^2 = 100$

differentiating w.r.t x .

$$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0.$$

$$(x + 2y) \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$= \frac{-(2x + y)}{(x + 2y)}$$

ii) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

put $x = \tan \theta$, $\theta = \tan^{-1} x$.

$$y = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2 \tan^{-1} x$$

differentiating w.r.t x

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2}$$

$$= \frac{2}{1+x^2}$$

(15) $f(x) = 2x^2 - 3x$

$$f'(x) = 4x - 3$$

$$f'(x) = 0 \Rightarrow 4x - 3 = 0$$

$$4x = 3$$

$$x = \frac{3}{4}$$

Intervals are $(-\infty, \frac{3}{4})$, $(\frac{3}{4}, \infty)$

i) $f'(x) > 0$ only if $4x - 3 > 0$

$$4x > 3$$

$$x > \frac{3}{4}$$

$\therefore f(x)$ strictly increasing on

$$\left(\frac{3}{4}, \infty\right)$$

ii) $f'(x) < 0$ only if $4x - 3 < 0$

$$4x < 3$$

$$x < \frac{3}{4}$$

$\therefore f(x)$ strictly decreasing on

$$(-\infty, \frac{3}{4}).$$

(16)

i) Order = 2

degree = 1

ii) $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$$\frac{dy}{1+y^2} = (1+x^2) dx$$

$$\therefore \int \frac{dy}{1+y^2} = \int (1+x^2) dx$$

$$\tan^{-1} y = x + \frac{x^3}{3} + C$$

(17) Unit vector perpendicular

to both $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{289 + 169 + 49} = \sqrt{507}.$$

Unit Vector $\perp$ to both $\vec{a}$ and $\vec{b}$

$$= \frac{-17}{\sqrt{507}} \hat{i} + \frac{13}{\sqrt{507}} \hat{j} + \frac{7}{\sqrt{507}} \hat{k}$$

$$(18) d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= -3\hat{i} + 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = -3 + 0 - 6 = -9$$

$$\therefore d = \left| \frac{-9}{3\sqrt{2}} \right|$$

$$= \underline{\underline{\frac{3}{\sqrt{2}}}}$$

$$(19) i) P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$0.4 = \frac{P(A \cap B)}{0.8}$$

$$P(A \cap B) = 0.4 \times 0.8$$

$$= \underline{\underline{0.32}}$$

$$ii) P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \underline{\underline{0.64}}$$

$$iii) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.8 + 0.5 - 0.32$$

$$= \underline{\underline{0.98}}$$

$$(20) i) b, (3, 1)$$

$$ii) R = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (1,3), (1,4), (1,5), (1,6), (2,4), (2,6), (3,6) \}.$$

$(a,a) \in R$ for all $a \in A$.

$\therefore R$ is reflexive

$(1,2) \in R$ but $(2,1) \notin R$

$\therefore R$ is not symmetric.

If $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for $a \in A$

$\therefore R$ is transitive.

$\therefore R$ is reflexive, transitive but not symmetric

(21) i) Let $y = n^n$

taking logarithm

$$\log y = n \log n$$

differentiating w.r.t n .

$$\frac{1}{y} \frac{dy}{dn} = n \cdot \frac{1}{n} + \log n - 1$$

$$= 1 + \log n$$

$$\therefore \frac{dy}{dn} = y (1 + \log n)$$

$$= \underline{\underline{n^n (1 + \log n)}}$$

ii) $n = 2a + 2, y = a + 4$

$$\frac{dy}{dn} = \frac{\frac{dy}{da}}{\frac{dn}{da}}$$

$$y = a + 4$$

$$\frac{dy}{da} = 1$$

$$n = 2a + 2$$

$$\frac{dn}{da} = 2$$

$$\therefore \frac{dy}{dn} = \frac{1}{2}$$

$$= \underline{\underline{\frac{1}{2}}}$$

(22) $\int \frac{n}{(n+1)(n+2)} dn$

$$\text{Let } \frac{n}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

$$\therefore n = A(n+2) + B(n+1)$$

$$\text{Putting } n = -1, A = -1$$

$$\text{and } n = -2, B = 2$$

$$\therefore \frac{n}{(n+1)(n+2)} = \frac{-1}{n+1} + \frac{2}{n+2}$$

$$\therefore \int \frac{n}{(n+1)(n+2)} dn$$

$$= \int \frac{-1}{n+1} dn + \int \frac{2}{n+2} dn$$

$$= -\log |n+1| + 2 \log |n+2| + C$$

$$= -\log |n+1| + \log |n+2|^2 + C$$

$$= \log \left| \frac{n+2|^2}{n+1} \right| + C$$

$$= \underline{\underline{\log \left| \frac{(n+2)^2}{n+1} \right| + C}}$$

Questions carry 6 marks each.

(23) i) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$$a_{ij} = 3i - j$$

$$a_{11} = 3 - 1 = 2$$

$$a_{12} = 3 - 2 = 1$$

$$a_{21} = 3 \times 2 - 1 = 5$$

$$a_{22} = 3 \times 2 - 2 = 4$$

$$a_{31} = 3 \times 3 - 1 = 8$$

$$a_{32} = 3 \times 3 - 2 = 7$$

$$\therefore A = \begin{bmatrix} 2 & 1 \\ 5 & 4 \\ 8 & 7 \end{bmatrix}$$

$$\text{ii) } A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$$

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$A' = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 4 & 4 \\ 4 & -8 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A + A') + \frac{1}{2}(A - A') = A$$

$$\text{ie } \begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$$

$$\textcircled{24} \quad Ax = B$$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$|A| = -17 \neq 0$$

$$\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$x' = A^{-1}B$$

$$= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, y = 2, z = 3$$

(25) i) $g \circ f(1) = g(f(1)) = g(2) = 3$
 $g \circ f(3) = g(f(3)) = g(5) = 1$
 $g \circ f(4) = g(f(4)) = g(1) = 3.$

ii) Let $y = 2x + 1$

$$2x = y - 1$$

$$x = \frac{y-1}{2}$$

g is the inverse of f . if
 $f \circ g = g \circ f$.

Let $g(x) = \frac{x-1}{2}$

$$f \circ g(x) = f(g(x))$$

$$= f\left(\frac{x-1}{2}\right)$$

$$= 2\left(\frac{x-1}{2}\right) + 1$$

$$= x - 1 + 1$$

$$= x$$

$$g \circ f(x) = g(f(x))$$

$$= g(2x+1)$$

$$= \frac{2x+1-1}{2}$$

$$= \frac{2x}{2}$$

$$= x.$$

$$g \circ f(x) = f \circ g(x) = x.$$

$\therefore f$ is invertible.

$$\therefore f^{-1}(x) = \frac{x-1}{2}$$

(26)

i) $\frac{dy}{dx} = 3x^2 - 1$

$$\begin{aligned} \frac{dy}{dx} \text{ at } x=2 &= 3 \times 2^2 - 1 \\ &= 3 \times 4 - 1 \\ &= 11 \end{aligned}$$

ii) $x = 2$

$$y = 2^3 - 2 = 8 - 2 = 6.$$

$\therefore$ Equation of the tangent with slope 11 and passing through (2,6) is given by

$$y - y_0 = m(x - x_0)$$

$$y - 6 = 11(x - 2)$$

$$y - 6 = 11x - 22$$

$$11x - y - 22 + 6 = 0$$

$$11x - y - 16 = 0$$

iii) $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x$
 $= 1 + \sin 2x$

Maximum value of $\sin 2x$ is 1.

$$\therefore (\sin x + \cos x)^2 = 1 + 1 = 2$$

$$\therefore \sin x + \cos x = \sqrt{2}$$

Maximum value of $\sin x + \cos x$ is $\sqrt{2}$

(27)

i) $\int \sin x \sin(\cos x) dx.$

put $t = \cos x$

$$dt = -\sin x dx.$$

$$\int \sin(\cos x) \sin x dx$$

$$= \int \sin t (-dt)$$

$$= -\int \sin t dt.$$

$$= -\cos t + C$$

$$= \cos t + C$$

$$= \cos(\cos x) + C$$

$$ii) \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\int \frac{\tan^{-1} x}{1+x^2} dx$$

Put $t = \tan^{-1} x$
 $dt = \frac{1}{1+x^2} dx$

$$= \int t dt$$

$$= \frac{t^2}{2} + C = \frac{(\tan^{-1} x)^2}{2} + C$$

$$\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx = \left[\frac{(\tan^{-1} x)^2}{2} \right]_0^1$$

$$= \frac{(\tan^{-1} 1)^2}{2} - \frac{(\tan^{-1} 0)^2}{2}$$

$$= \frac{(\frac{\pi}{4})^2}{2} - 0$$

$$= \frac{\pi^2}{32}$$

(28) $x + 2y = 10$

x	y
0	5
10	0

$3x + y = 15$

x	y
0	15
5	0

Corner Points

$z = 3x + 2y$

A(0,0)

0

B(5,0)

15

C(4,3)

18

Maximum

D(0,5)

10

$3x + 6y = 30$

$3x + y = 15$

$5y = 15$

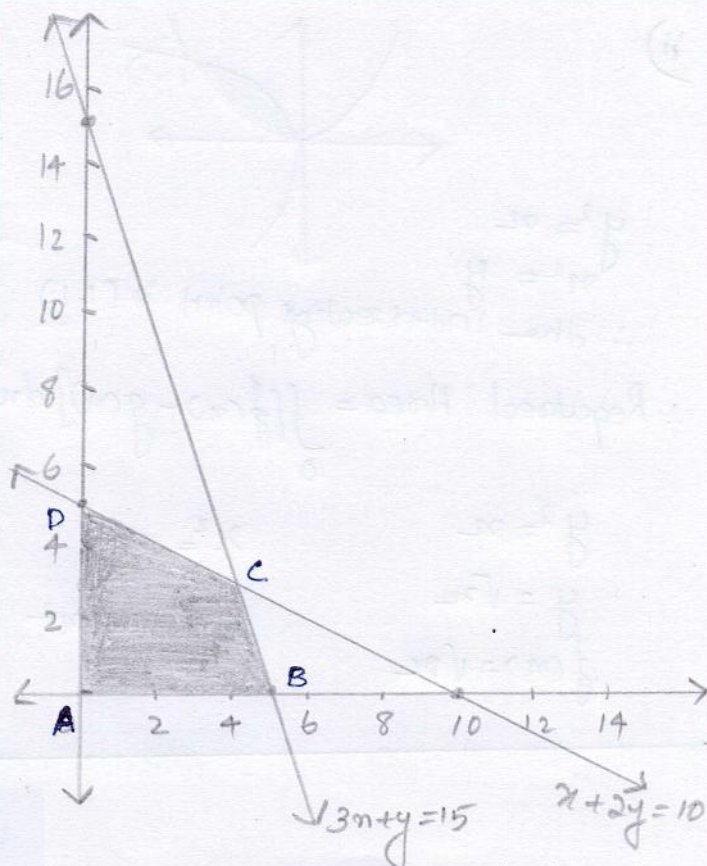
$y = 3$

$\therefore x = 10 - 2y$

$= 10 - 6$

$= 4$

$(4, 3)$



Maximum value is 18 at (4,3).

(29) i)

Required Area = $\int_1^4 y dx$

$= \int_1^4 \sqrt{x} dx$

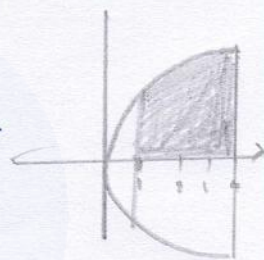
$= \frac{2}{3} \left[x^{3/2} \right]_1^4$

$= \frac{2}{3} \left[4^{3/2} - 1^{3/2} \right]$

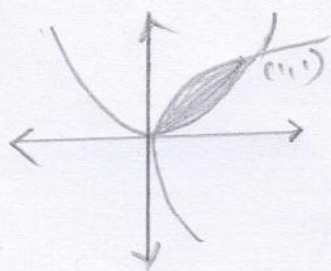
$= \frac{2}{3} [8 - 1]$

$= \frac{2}{3} \times 7$

$= \frac{14}{3}$



ii)



$$y^2 = x$$

$$x^2 = y$$

$\therefore$ the intersecting point is (1,1)

$$\therefore \text{Required Area} = \int_0^1 [f(x) - g(x)] dx$$

$$y^2 = x$$

$$y = \sqrt{x}$$

$$f(x) = \sqrt{x}$$

$$x^2 = y$$

$$\therefore g(x) = x^2$$

$$\therefore \text{Area} = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{3} \right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \underline{\underline{\frac{1}{3}}}$$