Revisiting Stellar Orbits and the Sgr A* Quadrupole Moment

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Abstract

The "no-hair" theorem can, in principle, be tested at the center of the Milky Way by measuring the spin and the quadrupole moment of Sgr A* with the orbital precession of S-stars, measured over their full periods. Contrary to the original method, we show why it is possible to test the no-hair theorem using observations from only a single star, by measuring precession angles over a half-orbit. There are observational and theoretical reasons to expect S-stars to spin rapidly, and we have quantified the effect of stellar spin, via spin-curvature coupling (the leading-order manifestation of the Mathisson-Papapetrou-Dixon equations), on future quadrupole measurements. We find that they will typically suffer from errors of order a few percentage points, but for some orbital parameters, the error can be much higher. We re-examine the more general problem of astrophysical noise sources that may impede future quadrupole measurements, and find that a judicious choice of measurable precession angles can often eliminate individual noise sources. We have derived optimal combinations of observables to eliminate the large noise source of mass precession, the novel noise of spin-curvature coupling due to stellar spin, and the more complicated noise source arising from transient quadrupole moments in the stellar potential.

Shifts and Half-Shifts

To test the no-hair theorem, we need to determine five parameters: the mass of the black hole $M_\bullet$, the magnitude and two angles of its spin $\mathbf{J}_\bullet$, and the value of its quadrupole moment $Q_{2\bullet}$, and then verify or refute the relation $Q_{2\bullet} = -\frac{1}{c}\frac{J_\bullet^2}{M_\bullet}$ [3, 4]. Using measurements of the S2 orbital period, observers have already constrained the mass of Sgr A*. Using orbital perturbation theory, we can calculate the precessions per orbit of a star's Euler angles ($\delta\varpi$, $\delta\Omega$ and δi). We call these per-orbit precessions "full-shifts". Measuring the full-shifts of the ascending node and the inclination of two S-stars orbits would allow us to calculate the four remaining parameters $\mathbf{J}_\bullet$ and $Q_{2\bullet}$ needed to test the no-hair theorem [1]. In this approach, two stars are needed because there is not enough information in the full-shifts of a single star.

However, there is more information contained in the relativistic orbital motion that is hidden by a full orbit average. Specifically, we can use the precession completed after a half-orbit (the "half-shifts"), which in some cases are non-degenerate with the full-shifts. Combining the half-shifts with the full-shifts we will have enough independent equations to calculate the SMBH spin and quadrupole moment, to test the no-hair theorem without the need for a second star.

Sources of Noise

Stellar Perturbations: Previous studies [2] showed that the presence of other stars in the cluster around the SMBH can induce orbital precession at the same order of magnitude as relativistic effects. To a first-order approximation, the stellar distribution can be approximated as a smooth spherical cluster, which causes apsidal precession. Non-spherically symmetric perturbations, such as vector resonant relaxation (VRR), can also create a source of error by changing the orientation of the orbital planes. The stellar potential's quadrupole, the lowest order aspherical contribution in the multipole expansion, dominates over higher multipole moments. Therefore, we only considered the leading-order multipole moment.

Spin-Curvature Coupling (MPD Effect): There is substantial direct evidence that S-stars spin rapidly [7]. The spin of a test particle in a gravitational field will cause deviations from geodesic motion. Those deviations would add a new source of noise to the orbital precession measurements. In Fig. 1 we notice that MPD effects due to the star's spin are almost always subdominant to precession from the SMBH quadrupole moment, although spin-curvature coupling may set a noise floor of $\sim 1-10\%$ in future no-hair tests.

Tidal Force At very small pericenters, tidal interactions between the SMBH and the star can cause a level of precession that overwhelms the higher order GR shifts we are interested in. We show in Fig. 1 that tides are highly subdominant in the radii of current interest.

References

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Sources of Noise (continued)

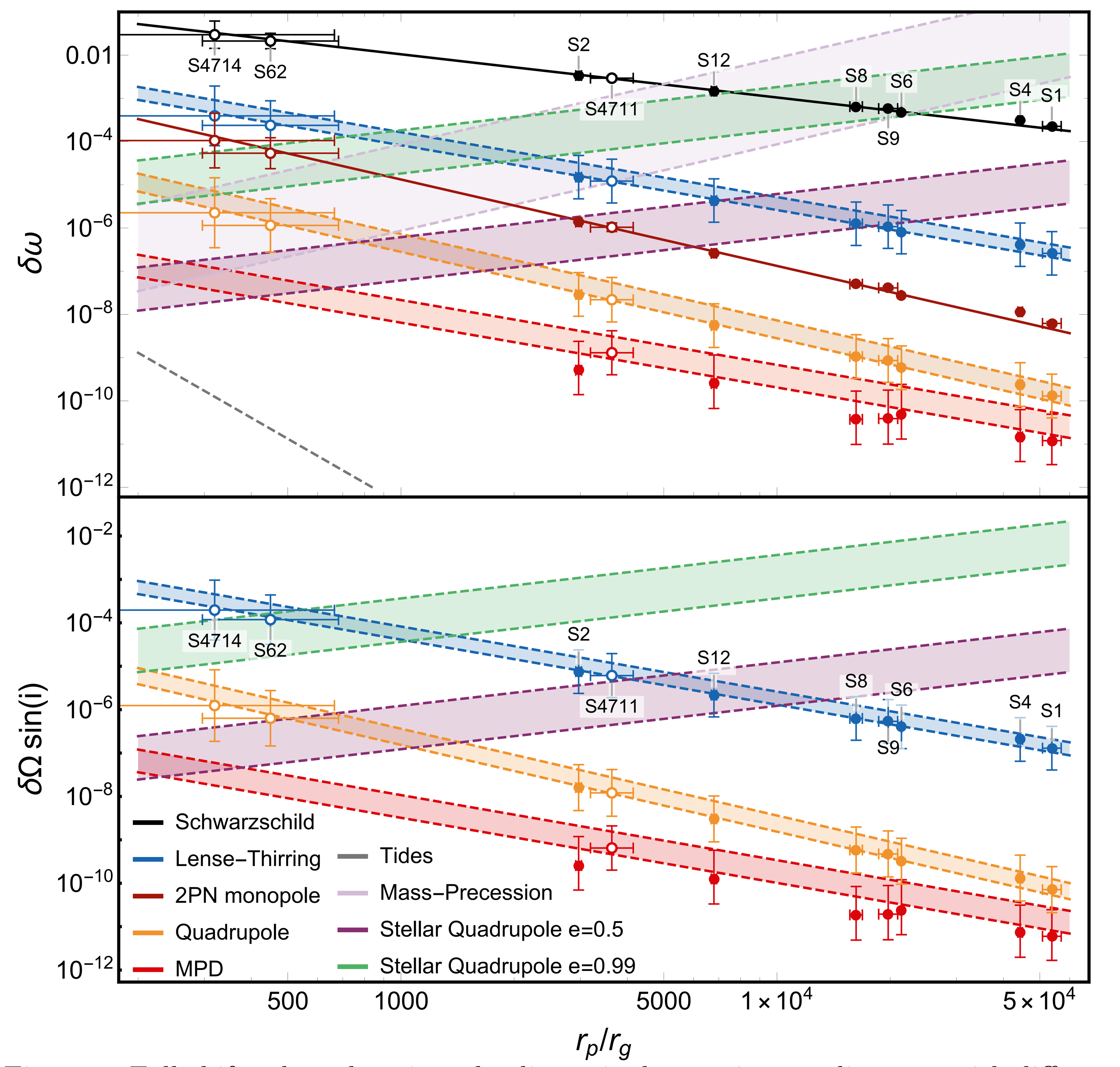

Figure 1: Full-shifts plotted against the dimensionless pericenter distance, with different effects color-coded. *Top*: The full-shift of the argument of the pericenter. *Bottom*: The full-shift of the longitude of the ascending node.

Minimizing the Errors

By careful algebraic calculations, we can tailor combinations of full- and half-shifts that by design will completely eliminate individual astrophysical noise sources. In Fig. 2 we show the percentage errors in the calculation of $Q_{2\bullet}$ estimation for three combinations of astrophysical noise, and five different combinations of observables:

1. The original method [1] using the full-shifts of two stars ($\delta\Omega$ and δi).
2. Using three full shifts ($\delta\varpi$, $\delta\Omega$ and δi) and the nodal half-shift ($\delta\Omega_{\frac{1}{2}}$) for a single star.
3. Using three full shifts ($\delta\varpi$, $\delta\Omega$ and δi) and the half-shift of the pericenter ($\delta\varpi_{\frac{1}{2}}$) for a single star.
4. Using two full shifts ($\delta\Omega$ and δi), the nodal half-shift ($\delta\Omega_{\frac{1}{2}}$), and the subtraction $\delta\varpi_{\rm sub} \equiv \delta\varpi - 2\delta\varpi_{\frac{1}{2}}$, for a single star.
5. Using shifts and half-shifts of two stars, in such a way as to remove the stellar quadrupole and mass precession noise.

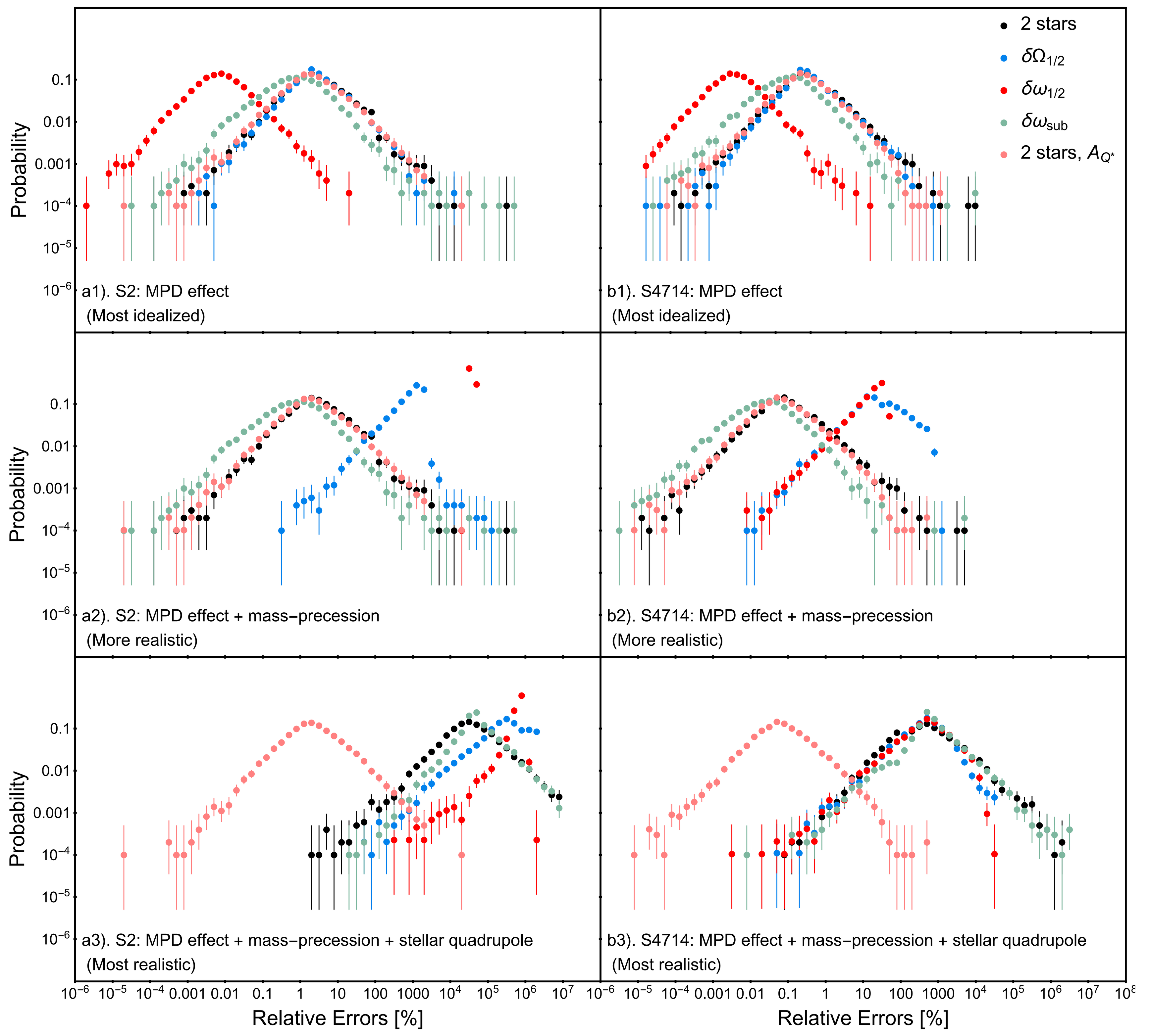

Figure 2: Histograms showing relative errors in the quadrupole measurements due to *top*: MPD effects, *middle*: MPD effects and mass precession, and *bottom*: MPD effects, mass precession and a stellar quadrupole moment, for *left*: S2-like orbit, and *right*: S4714-like orbit [8].