Backward error and conditioning of Fiedler linearizations

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Joint work with **F. Tisseur** (UoM)

Outline

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Goal

$P(\lambda) = \sum_{i=0}^{\ell} \lambda^i A_i$ a **regular** matrix polynomial ($A_i \in \mathbb{C}^{n\times n}$)

PEP ($P(\lambda)x = 0 = y^* P(\lambda)$) solved by **linearization.**

Classical linearizations: First and second Frobenius companion forms:

$$C_1(\lambda) = \lambda \begin{bmatrix} A_\ell & & & \\ & I_n & & \\ & & \ddots & \\ & & & I_n \end{bmatrix} + \begin{bmatrix} A_{\ell-1} & A_{\ell-2} & \cdots & A_0 \\ -I_n & 0 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ 0 & & -I_n & 0 \end{bmatrix}, \quad C_2(\lambda) = C_1(\lambda)^B$$

($(\cdot)^B$ means block-transposition).

New family of (companion) linearizations: Fiedler companion pencils ([Antoniou&Vologiannidis'04], [D., Dopico&Mackey'10]&['12], based on [Fiedler'03] for scalar polynomials) $\rightsquigarrow$ **many interesting features.**

Goal Analyze the numerical behavior of Fiedler linearizations (**b'err** and **conditioning**).

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Backward error of eigenpairs

Backward error of an approximated eigenpair $(\Delta P = \sum_{i=0}^{\ell} \Delta A_i)$

Right: $\eta_P(\lambda, x) := \min\{\varepsilon : \ (P(\lambda_0) + \Delta P(\lambda_0))x = 0, \ \|\Delta A_i\|_2 \le \varepsilon \|A_i\|_2, \ i = 0 : \ell\},$

Left: $\eta_P(\lambda, y) := \min\{\varepsilon : \ y^*(P(\lambda_0) + \Delta P(\lambda_0)) = 0, \ \|\Delta A_i\|_2 \le \varepsilon \|A_i\|_2, \ i = 0 : \ell\}.$

Practical formulas [Tisseur'00]:

$$\eta_P(\lambda, x) = \frac{\|P(\lambda)x\|_2}{\left(\sum_{j=0}^{\ell} |\lambda|^j \|A_j\|_2\right) \|x\|_2}, \quad \eta_P(\lambda, y) = \frac{\|y^* P(\lambda)\|_2}{\left(\sum_{j=0}^{\ell} |\lambda|^j \|A_j\|_2\right) \|y\|_2}.$$

$(\lambda, x), (\lambda, y)$ right/left eigenpairs of P
$(\lambda, v), (\lambda, w)$ corresponding right/left eigenpairs of a Fiedler linearization F_σ

We want to **compare** $\eta_P(\lambda, x), \eta_P(\lambda, y)$ with $\eta_{F_\sigma}(\lambda, v), \eta_{F_\sigma}(\lambda, w)$ (resp.)

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Condition number of eigenvalues

Condition number of a (simple) eigenvalue: $(\Delta P = \sum_{i=0}^{\ell} \Delta A_i)$

$$\kappa_P(\lambda) = \lim_{\varepsilon \to 0} sup \left\{ \frac{|\Delta\lambda|}{\varepsilon|\lambda|} : (P(\lambda+\Delta\lambda) + \Delta P(\lambda+\Delta\lambda))(v+\Delta v) = 0,\ \|\Delta A_i\|_2 \leq \varepsilon \|A_i\|_2,\ i = 0 : \ell \right\}.$$

Practical formulas [Tisseur'00]:

$$\kappa_P(\lambda) = \frac{\sum_{i=1}^{k} |\lambda|^i \|A_i\|_2}{|y^* P'(\lambda) x|} \|x\|_2 \|y\|_2.$$

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Fiedler pencils: Definition

$$M_0 = \operatorname{diag}(I_{n(\ell-1)}, -A_0), \quad M_\ell = \operatorname{diag}(A_\ell, I_{(\ell-1)n})$$

$$M_j = \begin{bmatrix} I_{n(\ell-j-1)} & & & \\ & A_j & -I_n & \\ & -I_n & 0 & \\ & & & I_{n(j-1)} \end{bmatrix}, \quad \text{for } j = 1, \ldots, \ell-1.$$

Example: $C_1(\lambda) = \lambda M_\ell - M_{\ell-1} \cdots M_1 M_0, \quad C_2(\lambda) = \lambda M_\ell - M_0 M_1 \cdots M_{\ell-1},$

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Let $\sigma : \{0, 1, \ldots, \ell-1\} \to \{1, \ldots, \ell\}$ be a bijection. Then:

$$F_\sigma(\lambda) := \lambda M_\ell - M_{\sigma^{-1}(1)} \cdots M_{\sigma^{-1}(\ell)}.$$

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Fiedler pencils: Basic properties

- M_σ consists of $\ell \times \ell$ block partitioned matrices with $n \times n$ blocks which are: $(A_0, \dots, A_{\ell-1})$ + ($\ell - 1$ identities $-I_n$) + (0's).
- F_σ is **always** a strong linearization (even for singular matrix polynomials).
- There are $2^{\ell-1}$ (generically) different Fiedler pencils.

Definition:

σ has a consecution at $i \in \{0, 1, \dots, \ell-2\}$ if $\sigma^{-1}(i) < \sigma^{-1}(i+1)$, and σ has an inversion otherwise.

PCIS(σ):="PCIS(F_σ)"=$(k_0, \dots, k_{\ell-2})$, with $k_i = \begin{cases} 1, & \text{if } \sigma \text{ has a consecution at } i, \\ 0, & \text{if } \sigma \text{ has an inversion at } i. \end{cases}$

Example: PCIS(C_1)=$(0, \dots, 0)$, PCIS(C_2)=$(1, \dots, 1)$, PCIS(F_1)=$(1, 0, \dots, 0)$.

Then: $F_{\sigma_1} = F_{\sigma_2} \Leftrightarrow$ PCIS(σ_1)=PCIS(σ_2).

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Left and right eigencolumns

Right eigencolumn: $\mathscr{R}_\sigma(\lambda) \in \mathbb{C}^{n\ell \times n}$ such that

$$F_\sigma(\lambda)\mathscr{R}_\sigma(\lambda) = e_i \otimes P(\lambda) = \begin{bmatrix} 0 \\ \vdots \\ P(\lambda) \\ \vdots \\ 0 \end{bmatrix} \qquad \text{(for some } i\text{).}$$

Left eigencolumn: $\mathscr{L}_\sigma(\lambda) \in \mathbb{C}^{n\ell \times n}$ such that

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$i_0 = \min\{i : \sigma$ has a consecution at $i\}$, $j_0 = \min\{j : \sigma$ has an inversion at $j\}$.

This gives the correspondences:

(a) (λ, x) right eigenpair of $P \longrightarrow (\lambda, \mathscr{R}_\sigma(\lambda)x)$ right eigenpair of F_σ

(λ, y) left eigenpair of $P \longrightarrow (\lambda, \mathscr{L}_\sigma(\lambda)y)$ left eigenpair of F_σ

(b) (λ, v) right eigenpair of $F_\sigma \longrightarrow (\lambda, (e_{\ell-j_0}^T \otimes I)v)$ right eigenpair of P

(λ, w) left eigenpair of $F_\sigma \longrightarrow (\lambda, (e_{\ell-i_0}^T \otimes I)w)$ left eigenpair of P

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(a) (λ, x) right eigenpair of $P \longrightarrow (\lambda, \mathscr{R}_\sigma(\lambda)x)$ right eigenpair of F_σ

(λ, y) left eigenpair of $P \longrightarrow (\lambda, \mathscr{L}_\sigma(\lambda)y)$ left eigenpair of F_σ

(b) (λ, v) right eigenpair of $F_\sigma \longrightarrow (\lambda, (e_{\ell-j_0}^T \otimes I)v)$ right eigenpair of P

(λ, w) left eigenpair of $F_\sigma \longrightarrow (\lambda, (e_{\ell-i_0}^T \otimes I)w)$ left eigenpair of P

Outline

Bounds for ratio between b'errs and condition numbers

Theorem

(λ, v) a (computed) right eigenpair of F_σ. Set $j_0 = \min\{j : \sigma \text{ has an inversion at } j\}$ and $x := (e^T_{\ell - j_0} \otimes I_n)v$. Then (λ, x) is an approximated right eigenpair of P and

$$\frac{\eta_P(\lambda, x)}{\eta_{F_\sigma}(\lambda, v)} \leq \ell^{5/2} \frac{\max(1, \max_i \|A_i\|_2)^2}{\min(\|A_0\|_2, \|A_\ell\|_2)} \frac{\|v\|_2}{\|x\|_2}.$$

There are similar bounds for left eigenpairs (λ, w) and $y := (e^T_{\ell - i_0} \otimes I_n)w$, with $i_0 = \min\{i : \sigma \text{ has a consecution at } i\}$.

Theorem

Let λ be a simple e-val of P. Then:

$$\frac{1}{\sqrt{2}\ell} \leq \frac{\kappa_{F_\sigma}(\lambda)}{\kappa_P(\lambda)} \leq \sqrt{2}\ell^4 \frac{\max(1, \max_i \|A_i\|_2)^3}{\min(\|A_0\|_2, \|A_\ell\|_2)}.$$

Small ratio if $\|A_i\|_2 \approx \|A_j\|_2$ for all $0 \leq i, j \leq \ell$ (well scaled).

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Motivation

Set $\rho = \frac{\max_i \|A_i\|_2}{\min(\|A_0\|_2, \|A_\ell\|_2)}$, and

$$F_\sigma^s := (\max_i \|A_i\|_2)^{-1} \cdot F_\sigma = \sum_{i=0}^{\ell} \lambda^i A_i^s \qquad (A_i^s = (\max_i \|A_i\|_2)^{-1} A_i).$$

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$$\frac{\max(1, max_i \|A_i^s\|_2)^3}{\min(\|A_0^s\|_2, \|A_\ell^s\|_2)} = \frac{\max(1, max_i \|A_i^s\|_2)^2}{\min(\|A_0^s\|_2, \|A_\ell^s\|_2)} = \rho$$

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Ratio between b'err and conditioning (scaled)

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Condition number using two different Fiedlers (scaled)

Set $F^s_{\sigma_i} = (\max_i \|A_i\|_2)^{-1} F_\sigma, \qquad (i = 1, 2).$

Then:

Theorem

If λ is a simple eigenvalue of $P(\lambda)$:

$$\frac{1}{\ell^3\sqrt{n}} \leq \frac{\kappa_{F^s_{\sigma_1}}(\lambda)}{\kappa_{F^s_{\sigma_2}}(\lambda)} \leq \ell^3\sqrt{n}.$$

Parameter scaling

Idea: Change of variables $\lambda \mapsto \alpha\lambda$, so that $P(\alpha\lambda) = \sum_{i=0}^{\ell} \lambda^i(\alpha^i A_i)$, and α minimizes:

$$\rho(\alpha) := \frac{\max_i \alpha^i \|A_i\|_2}{\min(\|A_0\|_2, \alpha^\ell \|A_\ell\|_2)}. \quad (1)$$

Theorem [Betcke, 2008]

The only minimizer of (1) is $\alpha_{opt} = (\|A_0\|_2 / \|A_\ell\|_2)^{1/\ell}$.

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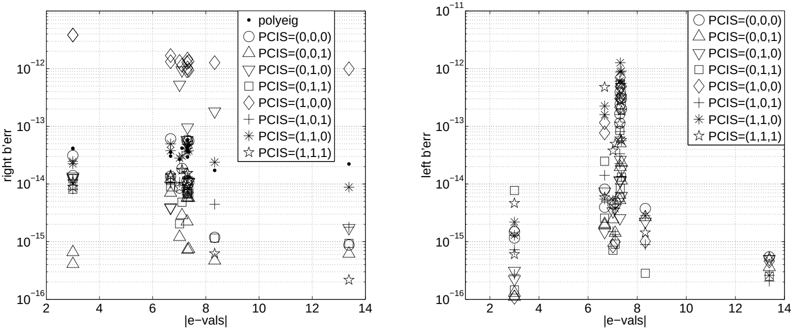
Outline

NLEVP collection: `mirror` (quartic, 9×9)

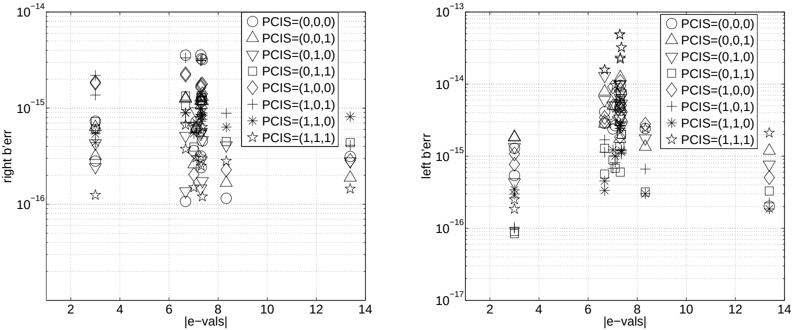


(a) Right eigenpairs (b) Left eigenpairs

Figure: B'err for right and left eigenpairs for e-vals λ with $|\lambda| > 1$, computed with `polyeig` and all Fiedler linearizations (**unscaled**).

rank $A_4 =$ rank $A_0 = 1 \Rightarrow$ At least 7 infinite and 7 zero e-vals (it has 9 of each)

NLEVP collection: `mirror` (quartic, 9×9) (ctd.)



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$\operatorname{rank} A_4 = \operatorname{rank} A_0 = 1 \Rightarrow$ At least 7 infinite and 7 zero e-vals (it has 9 of each)

NLEVP collection: `mirror` (quartic, 9×9) (ctd. II)

PCIS	`polyeig`	(000)	(001)	(010)	(011)	(100)	(101)	(110)	(111)
0 e-val	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
	8.1e-30	0	0	0	0	0	0	0	0
	-2.4e-23	0	0	0	0	0	0	0	0
	-7.2e-17	0	0	0	0	0	0	0	0
	-1.0e-16	0	0	0	0	0	0	0	-3.3e-21
	-1.6e-16	0	0	0	0	0	0	0	3.2e-18
	3.4e-15	0	0	0	0	0	0	-2.7ie-8	-1.9e-8
	-3.0e-14	0	0	0	-5.2e-21	0	0	2.7ie-8	1.9e-8
∞ e-val	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
	Inf	Inf	Inf	Inf	Inf	Inf	-Inf	-Inf	Inf
	Inf	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf	Inf

Table: Zero and infinite eigenvalues computed using `polyeig` and all Fiedler linearizations.

NLEVP collection: `relative_pose_5pt` (cubic, 10×10)

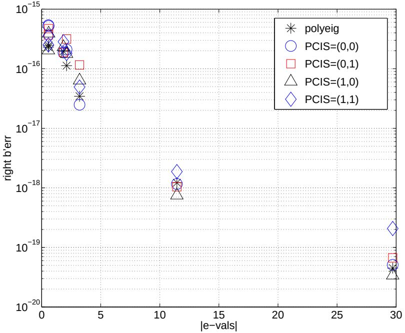
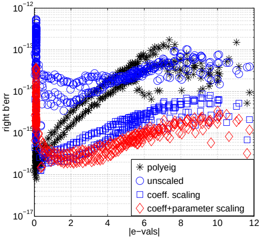


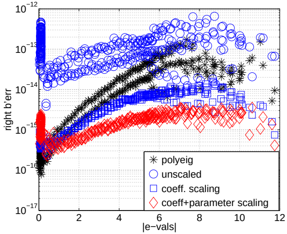
Figure: Comparison of b'errs for right eigenpairs of all 10 finite eigenvalues, computed with `polyeig` and all four Fiedler linearizations (with **scaled** coefficients).

$$\operatorname{rank} A_3 = 1 \Rightarrow \text{At least 9 infinite e-vals (it has 20)}$$

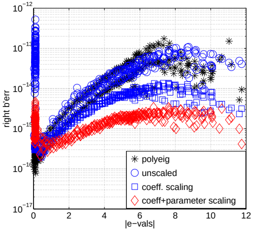
NLEVP collection: `plasma_drift` (cubic, 128×128)



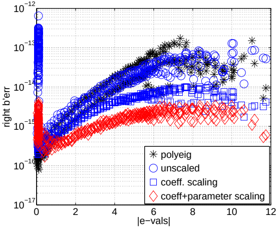
(a) PCIS=(0,0)



(b) PCIS=(0,1)



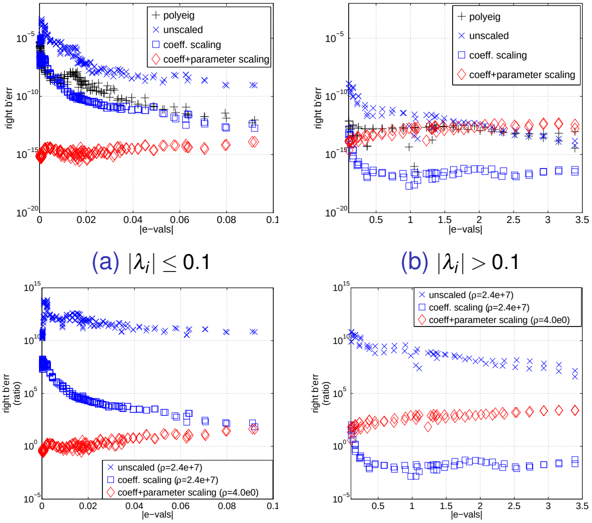
(c) PCIS=(1,0)



(d) PCIS=(1,1)

Figure: B'err for right eigenpairs computed with `polyeig` and all Fiedler linearizations.

NLEVP collection: `orr_sommerfeld` (quartic, 64×64)



(a) $|\lambda_i| \leq 0.1$

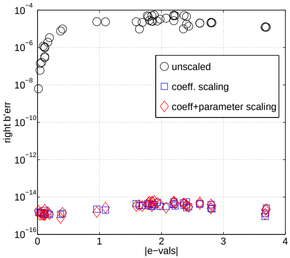
(b) $|\lambda_i| > 0.1$

(c) $|\lambda_i| \leq 0.1$ (ratio)

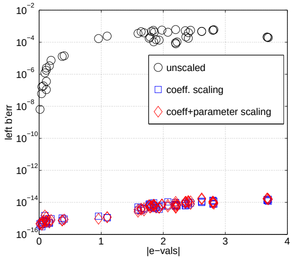
(d) $|\lambda_i| > 0.1$ (ratio)

Figure: B'err for right eigenpairs (F_σ with PCIS(σ)=(1, 0, 1))

$A_0 = 10^6\,\mathtt{rand}(20), A_1 = 10^7\,\mathtt{rand}(20), A_2 = 10^{-3}\,\mathtt{rand}(20), A_3 = 10\,\mathtt{rand}(20), A_4 = 10^6\,\mathtt{rand}(20)$

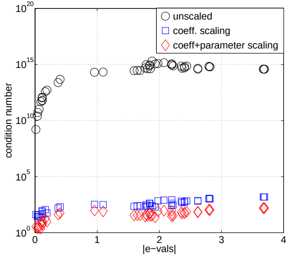


Right b'err

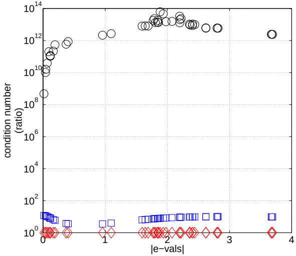


Left b'err

$(\rho \approx 10)$



Condition number



Condition number (ratio)

Conclusions

- Fiedler linearizations are as reliable as classical Frobenius companion linearizations.
- If the polynomial is badly scaled, there may be relevant differences (in b'err and conditioning) when solving the PEP using different Fiedler linearizations.
- Coefficient scaling seems to improve conditioning and b'err....though not always!
- Parameter scaling gives very promising results.
- Tropical scaling????
- Still much work to be done: analyze what happens with 0 and ∞ e-vals; is there any optimal Fiedler linearization?, ...

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