



Low rank perturbation of canonical forms

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Outline

- 1 Motivation
- 2 Preliminaries
- 3 Previous result: low rank of the coefficients
- 4 New result: low normal rank
- 5 Related work
- 6 Conclusions and Bibliography

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DAEs and canonical forms

D(ifferential)**A**(lgebraic)**E**(quation):

$$(1) \quad A_0 x(t) + A_1 x'(t) = f(t)$$

$A_0, A_1 \in \mathbb{C}^{m \times n}$, $x(t)$ unknown.

☞ Associated to the pencil: $A_0 + \lambda A_1$

☞ Canonical form of the pencil under (strict) equivalence:

$$E(A_0 + \lambda A_1)F = K_{A_0} + \lambda K_{A_1}$$

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Which canonical forms?

We are interested in canonical forms under **Strict equivalence** of **regular** matrix pencils: $E(A_0 + \lambda A_1)F$ (E, F nonsingular).

- $A_1 = -I$: **Jordan canonical form** (JCF) of A_0
- **Weierstrass canonical form** (WCF): regular pencils.

Low rank perturbations

$$A_0 + \lambda A_1 \rightsquigarrow (A_0 + \lambda A_1) + (B_0 + \lambda B_1) = (A_0 + B_0) + \lambda(A_1 + B_1)$$

with $B_0 + \lambda B_1$ of low rank (?????)

GOAL: Describe the generic change of the canonical form.

Low-rank perturbations arise in applied problems like...

- Structural modification of dynamical/vibrating systems (*pole-zero assignment*).
- Frequency compensation in electrical circuits.

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Low rank and genericity

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👉 **Genericity**: “Most likely behavior”

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The “classical” result for matrices

Theorem [Hörmander & Mellin, 1994], [Moro & Dopico, 2003]

$A \in \mathbb{C}^{n \times n}$, $\lambda_0 \in \sigma(A)$, $g = \dim \text{Nul}(A - \lambda_0 I)$.

For **generic** perturbations $B \in \mathbb{C}^{n \times n}$ with $r := \text{rank } B < g$:

The Jordan blocks of $A + B$ at λ_0 are the $g - \text{rank } B$ **smallest** Jordan blocks of A at λ_0 .

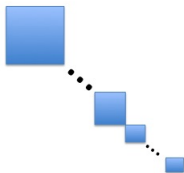
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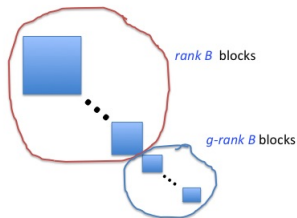
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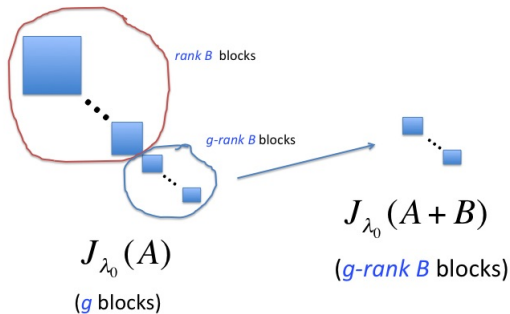
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Generic: $B \in \mathcal{M}_r \cap (\mathbb{C}^{n \times n} \setminus \mathcal{C})$

$\mathcal{M}_r = \{\text{matrices with rank} \leq r\}$

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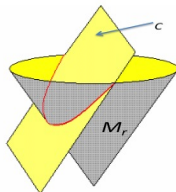
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Definition of genericity

Generic subset of \mathbb{C}^m : $\mathbb{C}^m \setminus C$, with C an algebraic set.

Problem: \mathcal{M}_r is an algebraic set...but **not irreducible !!!** ($r + 1$ irreducible components [D. & Dopico, 2008])

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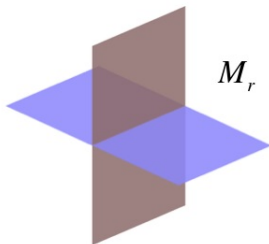
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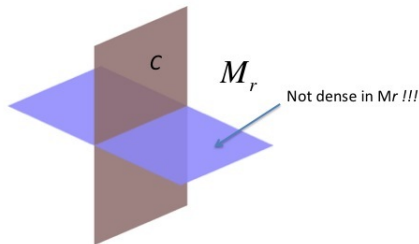
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Definition of low rank

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$$\text{rank } B < \dim \text{Nul}(A - \lambda_0 I) \implies \lambda_0 \in \sigma(A + B)$$

☞ For **regular pencils** $\rightsquigarrow A_0 + \lambda A_1$ (unperturbed), $B_0 + \lambda B_1$ (perturbation), $A_0 + B_0 + \lambda(A_1 + B_1)$ (perturbed)

▶ **1st approach:** $\text{rank}(B_0 + \lambda_0 B_1) < \dim \text{Nul}(A_0 + \lambda_0 A_1) \implies \lambda_0 \in \sigma(A_0 + B_0 + \lambda(A_1 + B_1))$

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Change of the WCF

$$g = \dim \text{Nul}(A_0 + \lambda_0 A_1)$$

$n_1 \geq \dots \geq n_g$: sizes of Jordan blocks at λ_0 in $A_0 + \lambda A_1$

Theorem [D., Dopico & Moro, 2008]

Set $\rho_0 = \text{rank}(B_0 + \lambda_0 B_1)$, $\rho_1 = \text{rank } B_1$. If $\rho_0 < g$ and

$$\text{nrnk}(B_0 + \lambda B_1) = \rho_0 + \rho_1 := r,$$

then **generically** the Jordan blocks at λ_0 in $A_0 + B_0 + \lambda(A_1 + B_1)$ are obtained by **removing the first r** terms in the list:

$$n_1, \dots, n_g, \overbrace{1, \dots, 1}^{\rho_1}$$

👉 **Different** from the generic behavior for matrices (some 1×1 additional blocks may appear) !!!!

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Generic pencils with $\text{nrank} - r$???

The way we had constructed the low-rank perturbations:

$$B_0 + \lambda B_1 = \underbrace{B_0 + \lambda_0 B_1}_{\text{rank } -\rho_0} + (\lambda - \lambda_0) \underbrace{B_1}_{\text{rank } -\rho_1}, \quad r := \rho_0 + \rho_1$$

Provided that $r < g \leq n$, then (generically):

$$\text{nrank}(B_0 + \lambda B_1) = \rho_0 + \rho_1 := r,$$

does not give **generic** pencils with $\text{nrank} - r$!!!!!!!

Hint: $B_0 + \lambda B_1$ above has λ_0 as **eigenvalue** (with geometric multiplicity ρ_1)
(generic nrank -deficient pencils **do not have eigenvalues at all**)

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Genericity and new approach

👉 **Generic set in \mathcal{M}_r :** **Dense open subset** of \mathcal{M}_r

Approach:

(1) $\mathcal{M}_r = C_0 \cup \dots \cup C_r$

(2) For each $s = 0, 1, \dots, r$:

$\Phi_s : \mathbb{C}^m \rightarrow C_s$, with $C_s = \Phi_s(\mathbb{C}^m)$ **KEY**

Analyze what happens in: $\Phi(\mathbb{C}^m \setminus G_s)$, with G_s an algebraic set.

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Construction of pencils with fixed nrank $- r$

For each $s = 0, 1, \dots, r$, set:

$$C_s := \left\{ \underbrace{v_1(\lambda)}_{\text{deg}=0} w_1(\lambda)^T + \dots + \underbrace{v_s(\lambda)}_{\text{deg}=0} w_s(\lambda)^T + v_{s+1}(\lambda) \underbrace{w_{s+1}(\lambda)}_{\text{deg}=0}^T + \dots + v_r(\lambda) \underbrace{w_r(\lambda)}_{\text{deg}=0}^T \right\}$$

($v_i(\lambda), w_j(\lambda)$ are polynomial vectors with $\text{deg } v_i(\lambda), w_j(\lambda) \leq 1$).

Then:

$$\mathcal{M}_r = C_0 \cup C_1 \cup \dots \cup C_r.$$

Define a “coefficient map” (**surjective**):

$$\phi_s : \mathbb{C}^{3m} \rightarrow C_s$$

that assigns the coefficients of $v_i(\lambda), w_j(\lambda)$, for $i, j = 1, \dots, r$.

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Guess: C_0, \dots, C_r are the **irreducible components** of \mathcal{M}_r (we have proved it for $r = 1$).

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Change of the WCF revisited

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$n_1 \geq \dots \geq n_g$: partial multiplicities of $A_0 + \lambda A_1$ at λ_0

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$$\phi_s : \mathbb{C}^{3m} \rightarrow C_s$$

Theorem [D., Dopico, submitted]

For each $s = 0, 1, \dots, r$, there is a **generic** set $G_s \subseteq \mathbb{C}^{3m}$ such that, for all $B_0 + \lambda B_1 \in \phi_s(G_s)$, the Jordan blocks of $A_0 + B_0 + \lambda(A_1 + B_1)$ at λ_0 have sizes $n_{r+1} \geq \dots \geq n_g$.

The largest r blocks disappear !!!!!

Change of the WCF revisited

$$g = \dim \text{Nul}(A_0 + \lambda_0 A_1)$$

$n_1 \geq \dots \geq n_g$: partial multiplicities of $A_0 + \lambda A_1$ at λ_0

$$\mathcal{M}_r = C_0 \cup C_1 \cup \dots \cup C_r.$$

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$$\mathcal{M}_r = \mathcal{C}_0 \cup \mathcal{C}_1 \cup \dots \cup \mathcal{C}_r.$$

$$\phi_s : \mathbb{C}^{3rn} \rightarrow \mathcal{C}_s$$

Theorem [D., Dopico, submitted]

There is a **generic** (dense open) set $G \subseteq \mathcal{M}_r$ such that, for all $B_0 + \lambda B_1 \in G$, the Jordan blocks of $A_0 + B_0 + \lambda(A_1 + B_1)$ at λ_0 have sizes $n_{r+1} \geq \dots \geq n_g$.

The largest r blocks disappear !!!!!

Outline

- 1 Motivation
- 2 Preliminaries
- 3 Previous result: low rank of the coefficients
- 4 New result: low normal rank
- 5 Related work**
- 6 Conclusions and Bibliography

Singular pencils and regular matrix polynomials

Generic behavior also know for:

- **Smith form** of **regular matrix polynomials** (1st approach).
- **Kronecker Canonical Form** of singular matrix pencils (under the assumption that the **perturbed** pencil is **still singular !!**).

Structured perturbations

👉 **Slightly different** behavior due to the restrictions imposed by the **structure**.

Known results for:

- J -Hamiltonian matrices (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2011]
- Selfadjoint matrices and sign characteristics (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2012]
- Symplectic, Orthogonal, and Unitary matrices matrices (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, 2014]
- J -Hamiltonian and H -symmetric (real) and sign characteristic (rank-1 perturbations) [Mehl, Mehrmann, Ran, & Rodman, submitted 2014]
- T -alternating, T -palindromic, and Symmetric pencils (rank-1 perturbations) [Batzke, 2014]
- H -selfadjoint, J -Hamiltonian matrices (rank- r perturbations) [Batzke, Mehl, Ran, & Rodman, submitted 2015]








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Conclusions

- We have presented a description for the **generic change** of the **WCF** of regular matrix pencils under **low rank** perturbations.
- The way how these perturbations are **constructed** is important: we use a **decomposition** of \mathcal{M}_r into **$r + 1$ subsets** (irreducible components ???) that **“parameterize”** \mathcal{M}_r .
- The meaning of **genericity** has been analyzed (related to the construction of the perturbations !!).
- Still much work to be done: generic rank- r perturbations of matrix polynomials / structured pencils; allow for singular perturbed pencils, (low rank) distance to singularity, ...

Some bibliography

-  [L. Batzke.](#)
LAA, 458 (2014) 638–670
-  [F. De Terán, F. M. Dopico.](#)
SIMAX, 29 (2007) 496–529
LAA, 430 (2009) 579–586
-  [F. De Terán, F. M. Dopico, J. Moro.](#)
SIMAX, 30 (2008) 538–547
-  [L. Hörmander, A. Mellin.](#)
Math. Scand., 75 (1994) 255–262
-  [C. Mehl, V. Mehrmann, A. Ran, L. Rodman.](#)
[A series of papers, including:](#)
LAA, 435 (2011) 687–716
LAA, 436 (2012) 4027–4042
Oper. Matrices, 7 (2013) 381–398
BIT, 54 (2014) 219–255
-  [J. Moro, F. M. Dopico.](#)
SIMAX, 25 (2003) 495–506
-  [S. V. Savchenko.](#)
Funct. Anal. Appl., 38 (2004) 69–71