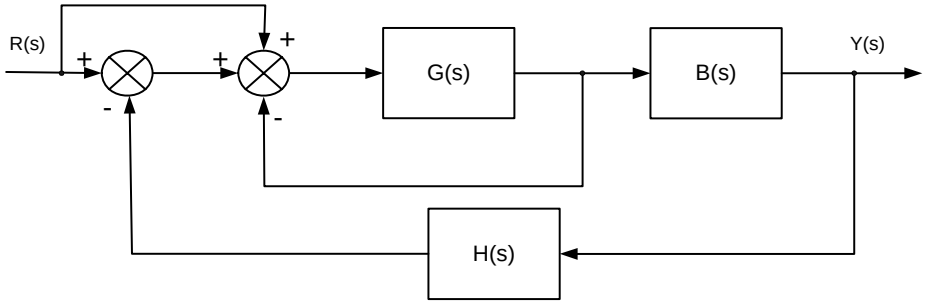
# MAKİNA MÜHENDİSLİĞİ BÖLÜMÜ
# OTOMATİK KONTROL DERSİ
# VİZE SINAVI SORULARI

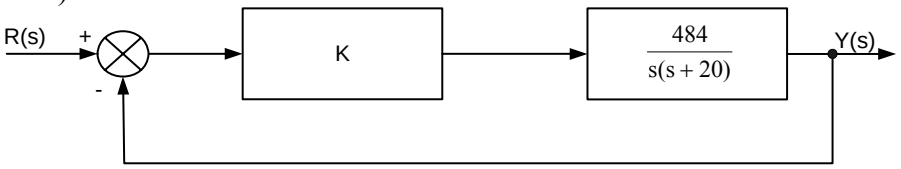
**Soru 1** (25 puan) Aşağıda blok diyagramı verilen sistemin genel transfer fonksiyonunu elde ediniz.




**Soru 2** (30 Puan) Aşağıda karakteristik denklemi verilen sistemin kararlı olmasını sağlayan K değerlerini belirleyiniz.

$$s^5 + 28s^4 + 302s^3 + 1550s^2 + 3525s + K = 0$$

**Soru 3** (30 Puan)




Şekilde blok diyagramı verilen kapalı çevrimli kontrol sisteminin

a) Genel transfer fonksiyonunu elde ediniz.
b) K=1 için birim anidarbe girişine verdiği cevabı elde ediniz.
c) Maksimum aşma yüzdesinin %10 değerinde olması istenirse K değeri ne seçilmelidir. Zaman sabiti ve yerleşme zamanının değerlerinide hesaplayınız.

**Soru 4** (25 puan)

$$\frac{Y(s)}{R(s)} = \frac{s+2}{(s+3)^2(s+0.5)}$$

Yukarıda genel transfer fonksiyonu verilen sistemin birim basamak girişine verdiği cevabı elde ediniz. Geçici ve sürekli rejim cevaplarını belirtiniz.

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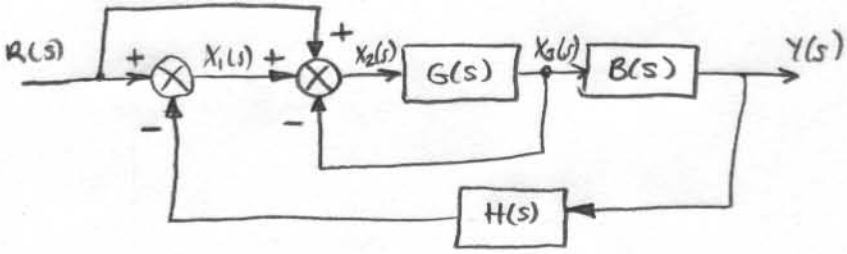
**ARALIK 2007** **ÖĞR. GÖR. DR. ERSUN YALÇIN**




$$X_1(s) = R(s) - H(s)\,Y(s) \qquad (1)$$

$$X_2(s) = R(s) + X_1(s) - X_3(s) \qquad (2)$$

$$X_3(s) = X_2(s)\,G(s) \qquad (3)$$

$$Y(s) = X_3(s)\,B(s) \qquad (4)$$

$(1) \rightarrow (2)$

$$X_2(s) = R(s) + R(s) - H(s)\,Y(s) - X_3(s)$$

$$X_2(s) = 2R(s) - H(s)\,Y(s) - X_3(s) \qquad (5)$$

$(5) \rightarrow (3)$

$$X_3(s) = \left(2R(s) - H(s)\,Y(s) - X_3(s)\right) G(s)$$

$$X_3(s)\left(1 + G(s)\right) = 2R(s)\,G(s) - H(s)\,Y(s)\,G(s)$$

$$X_3(s) = \frac{2G(s)}{1+G(s)}\,R(s) - \frac{H(s)\,G(s)}{1+G(s)}\,Y(s) \qquad (6)$$

$(6) \rightarrow (4)$

$$Y(s) = \left(\frac{2G(s)}{1+G(s)}\,R(s) - \frac{H(s)\,G(s)}{1+G(s)}\,Y(s)\right) B(s)$$

$$Y(s)\left(1 + \frac{H(s)\,G(s)\,B(s)}{1+G(s)}\right) = \frac{2\,G(s)\,B(s)}{1+G(s)}\,R(s)$$

$$Y(s)\left(\frac{1+G(s)+H(s)\,G(s)\,B(s)}{1+G(s)}\right) = \frac{2\,G(s)\,B(s)}{1+G(s)}\,R(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{2\,G(s)\,B(s)}{1+G(s)+H(s)\,G(s)\,B(s)}}$$

Soru 2 Çözümü

$$s^5 + 28\,s^4 + 302\,s^3 + 1550\,s^2 + 3525\,s + K = 0$$

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 302 | 3525 |
| $s^4$ | 28 | 1550 | K |
| $s^3$ | $a_1$ | $a_2$ | |
| $s^2$ | $b_1$ | K | |
| $s^1$ | $c_1$ | | |
| $s^0$ | K | | |

$$a_1 = \frac{28 \times 302 - 1550}{28} = \frac{6906}{28}$$

$$a_2 = \frac{28 \times 3525 - K}{28} = 3525 - \frac{1}{28} K$$

$$b_1 = \frac{a_1\,1550 - 28\,a_2}{a_1} = 1550 - \frac{(28 \times 3525 - K)}{\frac{6906}{28}}$$

$$b_1 = 1550 - \frac{(28^2 \times 3525 - 28K)}{6906}$$

$$b_1 = \frac{1550 \times 6906 - 28^2\,3525 + 28\,K}{6906}$$

$$b_1 = \frac{7940700 + 28\,K}{6906} = 1149.826238 + \frac{K}{246.6428571}$$

$$b_1 = 1149.826238 + 0.0040544\,K$$

$$c_1 = \frac{b_1\,a_2 - a_1\,K}{b_1} = \frac{\frac{1}{6906}(7940700 + 28K)\left(3525 - \frac{1}{28}K\right) - \frac{6906}{28}K}{\frac{1}{6906}(7940700 + 28K)}$$

Koşullar

$b_1 > 0 \qquad \frac{7940700 + 28K}{6906} > 0$

$c_1 > 0 \qquad \frac{\frac{1}{6906}(7940700 + 28K)(3525 - \frac{1}{28}K) - \frac{6906}{28}K}{\frac{1}{6906}(7940700 + 28K)} > 0$

$K > 0$

$b_1 > 0$ koşulundan

$$K > -\frac{7940700}{6906} = -1149.82623$$

$c_1 > 0$ koşulundan

$$\frac{1}{6906}\left(7940700 \times 3525 + 28 \times 3525\, K - \frac{7940700}{28}K - K^2\right) - \frac{6906}{28}K > 0$$

$$\frac{1}{6906}\left(7940700 \times 3525 + \left(28 \times 3525 - \frac{7940700}{28} - \frac{6906^2}{28}\right)K - K^2\right) > 0$$

$$\frac{1}{6906}\left(7940700 \times 3525 - \frac{52869936}{28}K - K^2\right) > 0$$

$$\frac{1}{6906}\left(7940700 \times 3525 - 1888212\, K - K^2\right) > 0$$

$$\frac{7940700}{6906}\left(3525 - 0.237789\, K - 1.2593348\, 10^{-7} K\right) > 0$$

$$\Delta = (-0.237789)^2 - 4\,(-1.2593348\, 10^{-7}\, 3525)$$

$$\Delta = 0.125831927$$

$-1962290.6139$ $14707.4726$

| $-$ | $+$ | $-$ |
|---|---|---|

$$-1902920.61391827 < K < 14707.4726$$

Buradan K değerinin

$$0 < K < 14707.4726$$

olması durumunda sistem kararlıdır.

Soru 3

$$\frac{Y(s)}{R(s)} = \frac{K\,\frac{484}{s(s+20)}}{1+K\,\frac{484}{s(s+20)}} = \frac{484K}{s(s+20)+K\,484}$$

$$\frac{Y(s)}{R(s)} = \frac{484K}{s^2+20s+484K}$$

) $K=1 \qquad R(s)=1$

$$\frac{Y(s)}{R(s)} = \frac{484}{s^2+20s+484}$$

$$s^2+20s+484=0$$

$$\Delta = 20^2 - 4\ 484$$

$$\Delta = -1536$$

$$s_{1,2} = \frac{-20 \mp \sqrt{-1536}}{2} = -10 \mp i\sqrt{384}$$

$$s_{1,2} = -10 \mp i\ 19.595918$$

$$Y(s) = \frac{484}{s^2+20s+484} = \frac{a_1}{s+10-i\sqrt{384}} + \frac{a_2}{s+10+i\sqrt{384}}$$

$$a_1 = \left[\frac{484}{(s+10-i\sqrt{384})(s+10+i\sqrt{384})}\,(s+10-i\sqrt{384})\right]_{s=-10+i\sqrt{384}}$$

$$a_1 = \frac{484}{(-10+i\sqrt{384}+10+i\sqrt{384})}$$

$$Y(s) = -i\frac{484}{2\sqrt{384}}\frac{1}{s+10-i\sqrt{384}} + i\frac{484}{2\sqrt{384}}\frac{1}{s+10+i\sqrt{384}}$$

$$y(t) = -i\frac{242}{\sqrt{384}}e^{(-10+i\sqrt{384})t} + i\frac{242}{\sqrt{384}}e^{(-10-i\sqrt{384})t}$$

$$y(t) = \frac{242}{\sqrt{384}}e^{-10t}\left(-i\,e^{i\sqrt{384}\,t} + i\,e^{-i\sqrt{384}\,t}\right)$$

$$y(t) = \frac{242}{\sqrt{384}}e^{-10t}\left(-i\left(\cos\sqrt{384}\,t + i\sin\sqrt{384}\,t\right) + i\left(\cos\sqrt{384}\,t - i\sin\sqrt{384}\,t\right)\right)$$

$$y(t) = \frac{484}{\sqrt{384}}e^{-10t}\sin\sqrt{384}\,t$$

c) $M_p = \%\,10$

$$M_p = 100\,e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$10 = 100\,e^{-\xi\frac{\pi}{\sqrt{1-\xi^2}}}$$

$$\frac{10}{100} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\ln 0.1 = -\frac{\xi\pi}{\sqrt{1-\xi^2}}\underbrace{\ln e}_{1}$$

$$(\ln 0.1)^2 = \frac{\xi^2\pi^2}{1-\xi^2} \rightarrow (1-\xi^2)(\ln 0.1)^2 = \xi^2\pi^2$$

$$(\ln 0.1)^2 = \xi^2(\pi^2 + (\ln 0.1)^2)$$

Karakteristik denklem

$$s^2 + 20s + 484K \equiv s^2 + 2\xi\omega_n s + \omega_n^2 K$$

$$2\xi\omega_n = 20 \qquad \omega_n = \frac{20}{2\xi} \qquad \omega_n = \frac{20}{2\,0.5912}$$

$$\omega_n = 16.9147 \text{ rad/s}$$

$$\omega_n^2 = 484K \qquad K = \frac{484}{\omega_n^2}$$

$$K = \frac{484}{16.9147^2} = 1.6917$$

Soru 4

$$\frac{Y(s)}{R(s)} = \frac{s+2}{(s+3)^2 (s+0.5)}, \quad R(s) = \frac{1}{s}$$

$$Y(s) = \frac{s+2}{s(s+3)^2(s+0.5)} = \frac{a_1}{s} + \frac{a_2}{(s+0.5)} + \frac{b_2}{(s+3)^2} + \frac{b_1}{(s+3)}$$

$$a_1 = \left[ \frac{s+2}{s(s+3)^2(s+0.5)} \, s \right]_{s=0} = \frac{2}{3^2 \, 0.5} = \frac{4}{9}$$

$$a_2 = \left[ \frac{s+2}{s(s+3)^2(s+0.5)} \, (s+0.5) \right]_{s=-0.5} = \frac{-0.5+2}{-0.5(-0.5+3)^2} = \frac{1.5}{-3.125} = -0.48$$

$$b_2 = \left[ \frac{s+2}{s(s+3)^2(s+0.5)} \, (s+3)^2 \right]_{s=-3} = \frac{-3+2}{-3(-3+0.5)} = \frac{-1}{-3(-2.5)} = -\frac{10}{75}$$

$$= -0.1333$$

$$b_1 = \left[ \frac{d}{ds} \left[ \frac{s+2}{s(s+3)^2(s+0.5)} \, (s+3)^2 \right] \right]_{s=-3}$$

$$= \left[ \frac{s(s+0.5) - (s+2)(2s+0.5)}{s^2 (s+0.5)^2} \right]_{s=-3} = \frac{-3(-3+0.5) - (-3+2)(2(-3)+0.5)}{(-3)^2 (-3+0.5)^2}$$

$$= \frac{2}{56.25} = 0.0356$$

$$Y(s) = \frac{4}{9} \frac{1}{s} - 0.48 \frac{1}{s+0.5} - \frac{10}{75} \frac{1}{(s+3)^2} + 0.0356 \frac{1}{s+3}$$

$$y(t) = \frac{4}{9} 1(t) - 0.48 \, e^{-0.5t} - \frac{10}{75} t e^{-3t} + 0.0356 \, e^{-3t}$$

$0.44$ $\quad -0.1333$