

Artificial Intelligence

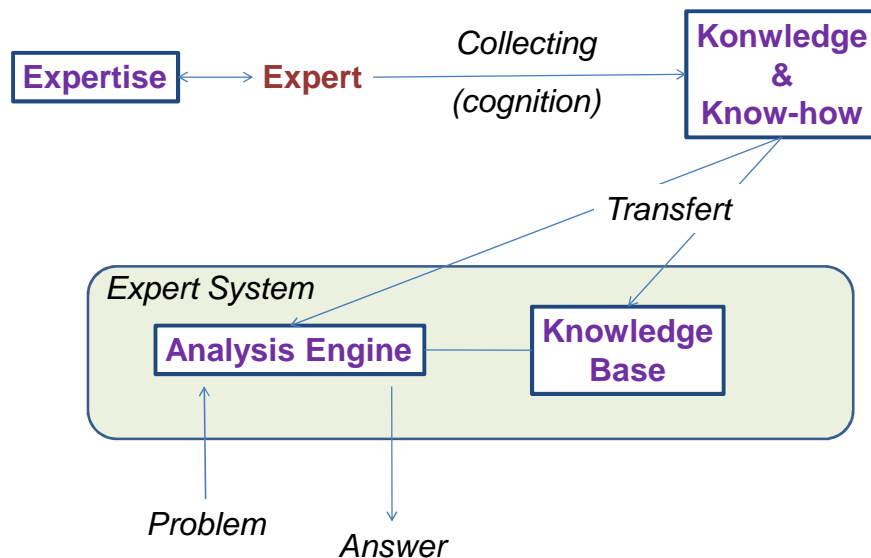
Logic and Expert Systems

© Hervé BARBOT, 2010 – www.proactitude.com

Introduction

(C) Hervé Barbot, 2009

3



(C) Hervé Barbot, 2009

4

System with Knowledge Base ?

- SBC (Système à Base de Connaissance)
 - Knowledge Base is related to a specific domain, a type of problem, ...
 - Explicit representation of the knowledge
- Transfert of the knowledge
 - Implies a change of :
 - Media / support
 - Form
 - Linguistic modeling
 - Power of expression / statements
 - Is it possible to transfert the complete knowledge that the system shall take into account?
 - Is it possible to transfert all the knowledge that the expert owns?
 - Enables reasoning
 - Is the form well adapted to the processing that are required for solving a specific problem?
 - Efficient!

(C) Hervé Barbot, 2009

5

Examples of knowledge base representation

- Triplets < object , attribute , value >
- Semantic networks
 - Nodes = concepts
 - Arcs = relations between concepts
- Logic – Predicates
 - Predicates represent facts, situations
 - Predicates can be defined based on conditions satisfied by other predicates
 - Equivalences are defined between logical expressions
 - A goal is defined by a predicate to satisfy
- Rules
 - Relations between:
 - Known information
 - Information to determine / verify / infer
 - Can include the execution of procedures

(C) Hervé Barbot, 2009

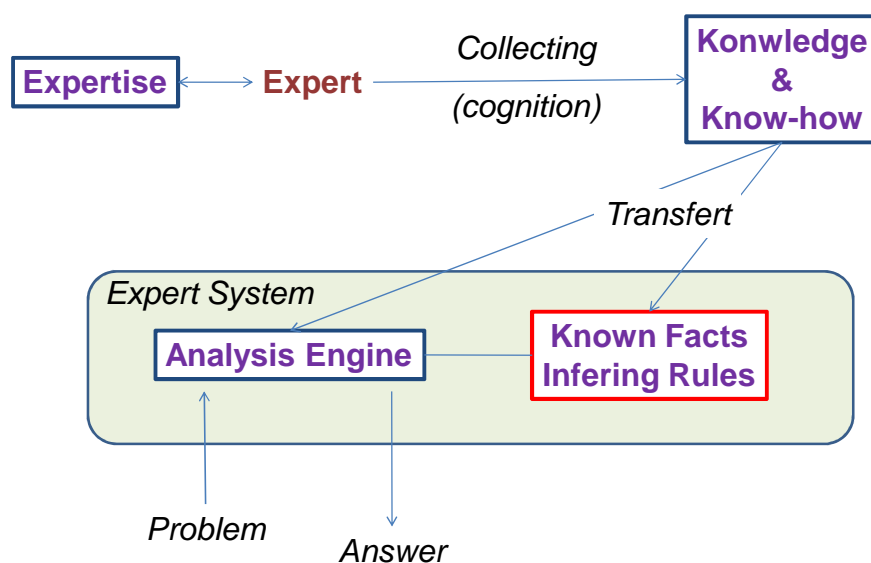
6

Rules-based reasoning

- Components :
 - Set of rules
 - Set of facts
 - Infering engine
- Principle
 - Find among the set of rules the ones that can be « fired », or « applied »:
 - If conditions are verified, then conclusions are applicable
 - Select one of these rules
 - This may need conflict resolution
 - « Fire » / execute the rule

(C) Hervé Barbot, 2009

7



(C) Hervé Barbot, 2009

9

Logic

(C) Hervé Barbot, 2009

10

Introductory example

■ Facts

- $\text{on_top}(C,A)$
- $\text{on_table}(A) \text{ on_table}(B)$
- $\text{free}(B) \text{ free}(C)$



■ Operations / Moving blocks:

« remove » & « pile » actions

- $R1: \text{free}(x) \Leftrightarrow \neg (\exists y, \text{on_top}(y,x))$
- $R2: \text{on_top}(y,x) \wedge \text{remove}(y,x) \Rightarrow \text{free}(x) \wedge \neg \text{on_top}(y,x)$
- $R3: \text{free}(x) \wedge \text{free}(y) \wedge \text{pile}(x,y) \Rightarrow \text{on_top}(x,y)$

■ How to move 'A' on top of 'B'?

i.e. how to obtain « $\text{on_top}(A,B)$ »?

(C) Hervé Barbot, 2009

11

■ Facts & Rules

$\text{on_top}(C,A) \text{ on_table}(A) \text{ on_table}(B) \text{ free}(B) \text{ free}(C)$

$R1: \text{free}(x) \Leftrightarrow \neg (\exists y, \text{on_top}(y,x))$

$R2: \text{on_top}(y,x) \wedge \text{remove}(y,x) \Rightarrow \text{free}(x) \wedge \neg \text{on_top}(y,x)$

$R3: \text{free}(x) \wedge \text{free}(y) \wedge \text{pile}(x,y) \Rightarrow \text{on_top}(x,y)$

■ « $\text{on_top}(A,B)$ »?

- R3 says: possible to obtain if we have

« $\text{free}(A) \wedge \text{free}(B) \wedge \text{pile}(A,B)$ »

- $\text{free}(B)=\text{true}$; $\text{pile}(A,B)$ is an action to execute

- « $\text{free}(A)$ »=?

R1 says: yes if $\neg (\exists y, \text{on_top}(y,A))$

- We have $\text{on_top}(C,A)$! ... How to obtain $\neg \text{on_top}(C,A)$?

Need to apply R2:

$\text{on_top}(C,A) \wedge \text{remove}(C,A) \Rightarrow \text{free}(A) \wedge \neg \text{on_top}(C,A)$

- Thus: execute « $\text{remove}(C,A)$ »:

« $\text{free}(A) \wedge \neg \text{on_top}(C,A)$ » becomes true

- Then execute « $\text{pile}(A,B)$ »:

the goal « $\text{on_top}(A,B)$ » becomes true!

(C) Hervé Barbot, 2009

12

Logic

- « Language » / Formal way to represent the knowledge
 - Logics = formal languages used in order to represent/translate:
 - Information (known facts)
 - Reasoning mechanisms
 - i.e. the method used to make conclusions/ take decisions based on known facts

(C) Hervé Barbot, 2009

26

Syntax + semantic

- Syntax
 - Made of words and symbols
 - Words + symbols = sentences
- Semantic
 - Meaning of a particular sentence in a specific environment
 - Meaning = truth value

(C) Hervé Barbot, 2009

27

Types of logic

■ Predicates - « ordre 0 »

une suite de symboles séparés par des conjonctions (et), des disjonctions (ou) ou des négations (non)

exemples:

"Socrate est un homme"	→	HommeSocrate
"Platon est un homme"	→	HommePlaton
"Platon et Socrate sont des hommes"	→	HommeSocrate \wedge HommePlaton

■ Propositions - « ordre 1 »

une suite de symboles, de variables et de relations avec des quantificateurs universels et existentiels

exemples:

– Homme(Socrate)	$\forall x$ Homme(x) \Rightarrow Mortel(x)
------------------	--

(C) Hervé Barbot, 2009

31

Propositions

(C) Hervé Barbot, 2009

33

- Enable the representaton of:

- Facts about the « world »: « Jean loves Marie »
- Negations: « Marie does not love Jean »
- Conjonctions & disjunctions
- Sentences with logical consequence: « Marie does not love Maris, she does not love Jean, neither »

La logique des propositions permet

- des faits sur le monde: "Jean aime Marie"
- des négations: "Marie n'aime pas Jean"
- des conjonctions et des disjonction
- des phrases avec "conséquence" "Jean aime Marie, elle aime Jean, elle aime Marie"

Une proposition est une expression vraie soit fausse.

Éléments de base:

- symboles de propositions: P, Q, ...
- phrases spéciales: Vrai, Faux

(C) Hervé Barbot, 2009

Syntaxe

- Propositions:

« P » « Q » « R »

- Operators (symbols):

\neg	\wedge	\vee	not	or	and
\Rightarrow	IF ... THEN ...				
\Leftrightarrow	... IF AND ONLY IF ...				

- Examples:

$\neg(P \wedge Q) \vee (P \Rightarrow (Q \Rightarrow R))$
 $P \wedge Q \wedge (Q \Rightarrow R)$

(C) Hervé Barbot, 2009

35

Truth table

P	$\neg P$	Q	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
V	F	V	V	V	V	V
V	F	F	F	V	F	F
F	V	V	F	V	V	F
F	V	F	F	F	V	V

(C) Hervé Barbot, 2009

36

Inference

- To infer = to establish the truth value of a sentence by applying syntactical modifications to the sentence
 - The sentence is the fact to prove
 - The modifications are made based on the « inferring rules »
 - Example

Knowledge
 $A = \text{true}$ $B = \text{true}$ $(A \wedge B) \Rightarrow C$

Inferring 'C'
 yes because A and B are both true, and $(A \wedge B) \Rightarrow C$

(C) Hervé Barbot, 2009

37

Predicates

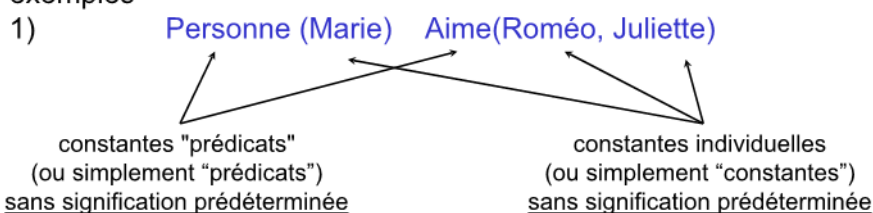
(C) Hervé Barbot, 2009

39

Syntax

- Au lieu des symboles propositionnels $P_1, P_2, Q \dots$, utiliser des **prédicats** et des **termes**:

- exemples



2) "Jean donne Fifi à Marie"
 Donne(Jean, Marie, Fifi)

3) **Égal(Plus(2,2), 4)**

(C) Hervé Barbot, 2009

40

Constantes	RoiJean, 2, UniGE, ...
Prédicats	Frère, >, ...
Fonctions	Sqrt(.), JambeGaucheDe(.), ...
Variables	x, y, a, b, ...
Connecteurs	$\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$
Égalité	=
Quantificateurs	$\forall \exists$

(C) Hervé Barbot, 2009

41

Formula

Phrases atomiques = $\text{prédicat}(\text{terme}_1, \dots, \text{terme}_n)$
ou $\text{terme}_1 = \text{terme}_2$

Terme = $\text{fonction}(\text{terme}_1, \dots, \text{terme}_n)$
ou *constante* ou *variable*

Exemples

- 1) Frère(RoiJean, RichardCoeurDeLion)
- 2) > (Longueur(JambeGaucheDe(Richard)), 15)

Phrases complexes = phrases atomiques réunies par des connecteurs

$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$

Exemples

- 1) Frère(RoiJean, Richard) \Rightarrow Frère(Richard, RoiJean)
- 2) > (1, 2) $\vee \leq$ (1, 2)
- 3) > (1, 2) $\wedge \neg >$ (1, 2)

(C) Hervé Barbot, 2009

42

Truth table

`my_predicate(A,B,C)`

is true if and only if

A, B and C belong alltogether to the relationship defined by `my_predicate`

(C) Hervé Barbot, 2009

44

Quantification

- Used to define properties on sets of objects

$\forall x \dots$ whatever the value of x , \dots is true

$\exists x \dots$ there exist a value of x such that \dots is true

(C) Hervé Barbot, 2009

45

▪ \exists <variables> <sentence>


« There exist someone who is intelligent outside EPITECH »

$\exists x \text{ human_being}(x) \wedge \text{intelligent}(x) \wedge \neg \text{epitech}(x)$

(C) Hervé Barbot, 2009

46

$\exists x \text{ Chien}(x)$ “il y a un chien”
 variable
 liée par \exists



les variables désignent des emplacements possibles pour des constantes:

- x est une variable libre dans $\text{Chien}(x)$
- x est une variable liée dans $\exists x \text{ Chien}(x)$

(C) Hervé Barbot, 2009

47

- \forall <variables> <sentence>

Everyone at Epitech is intelligent

$\forall x \text{ epitech}(x) \Rightarrow \text{intelligent}(x)$

(C) Hervé Barbot, 2009

48

Morgan law

...	<i>is the same as</i>	...
$\neg \exists x P$		$\forall x \neg P$
$\neg \forall x P$		$\exists x \neg P$
$\neg (P \vee Q)$		$\neg P \wedge \neg Q$
$\neg (P \wedge Q)$		$\neg P \vee \neg Q$

(C) Hervé Barbot, 2009

49

Example: genealogical tree▪ **Predicates:****Father(x,y)**

father(yves,bob), father(claire,jean), ...

Mother(x,y)

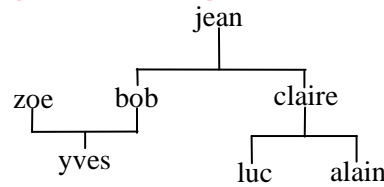
Allows a complete representation of the known facts

- Other relationships are derived from « father » and « mother » predicates by means of inferring rules:

$$\text{father}(x,y) \vee \text{mother}(x,y) \Rightarrow \text{parent}(x,y)$$

Or

$$\text{father}(x,y) \Rightarrow \text{parent}(x,y)$$

$$\text{mother}(x,y) \Rightarrow \text{parent}(x,y)$$


(C) Hervé Barbot, 2009

50

Summary

(C) Hervé Barbot, 2009

53

Logic

- « Language » / Formal way to represent the knowledge
 - Logics = formal languages used in order to represent/translate:
 - Information: known facts
 - Reasoning mechanisms: know-how
 - i.e. the method used to make conclusions/ take decisions based on known facts

(C) Hervé Barbot, 2009

54

Types of logic

- Predicates - « ordre 0 »
 - Sentences combined from symbols and operators 'and', 'or', 'not'
 - ManSocrate *Socrate is a man*
 - $\text{ManSocrate} \wedge \text{ManPlaton}$ *Socrate and Planton are men*
 - with logical operators $\Rightarrow, \Leftrightarrow$
 - $\text{ManSocrate} \Rightarrow \text{MortalSocrate}$ *If Socrate is a man, then Socrate is mortal*
- Propositions - « ordre 1 »
 - Symboles, variables, quantifiers (\exists, \forall)
 - $\text{Man}(\text{Socrate})$
 - $\forall x \text{ Man}(x) \Rightarrow \text{Mortal}(x)$ *All men are mortal*

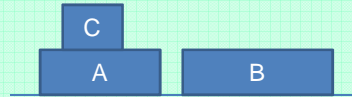
(C) Hervé Barbot, 2009

55

Example with predicates

■ Facts

- $\text{on_top}(C,A)$
- $\text{on_table}(A) \text{ on_table}(B)$
- $\text{free}(B) \text{ free}(C)$



■ Operations / Moving blocks:

« remove » & « pile » actions

- $R1: \text{free}(x) \Leftrightarrow \neg (\exists y, \text{on_top}(y,x))$
- $R2: \text{on_top}(y,x) \wedge \text{remove}(y,x) \Rightarrow \text{free}(x) \wedge \neg \text{on_top}(y,x)$
- $R3: \text{free}(x) \wedge \text{free}(y) \wedge \text{pile}(x,y) \Rightarrow \text{on_top}(x,y)$

■ How to move 'A' on top of 'B'?

i.e. how to obtain « $\text{on_top}(A,B)$ »?

(C) Hervé Barbot, 2009

56

■ Facts & Rules

$\text{on_top}(C,A) \text{ on_table}(A) \text{ on_table}(B) \text{ free}(B) \text{ free}(C)$

$R1: \text{free}(x) \Leftrightarrow \neg (\exists y, \text{on_top}(y,x))$

$R2: \text{on_top}(y,x) \wedge \text{remove}(y,x) \Rightarrow \text{free}(x) \wedge \neg \text{on_top}(y,x)$

$R3: \text{free}(x) \wedge \text{free}(y) \wedge \text{pile}(x,y) \Rightarrow \text{on_top}(x,y)$

■ « $\text{on_top}(A,B)$ »?

- R3 says: possible to obtain if we have
« $\text{free}(A) \wedge \text{free}(B) \wedge \text{pile}(A,B)$ »
- $\text{free}(B)=\text{true}$; $\text{pile}(A,B)$ is an action to execute
- « $\text{free}(A)$ »=?
R1 says: yes if $\neg (\exists y, \text{on_top}(y,A))$
- We have $\text{on_top}(C,A)$! ... How to obtain $\neg \text{on_top}(C,A)$?
Need to apply R2:
 $\text{on_top}(C,A) \wedge \text{remove}(C,A) \Rightarrow \text{free}(A) \wedge \neg \text{on_top}(C,A)$
- Thus: execute « $\text{remove}(C,A)$ »:
« $\text{free}(A) \wedge \neg \text{on_top}(C,A)$ » becomes true
- Then execute « $\text{pile}(A,B)$ »:
the goal « $\text{on_top}(A,B)$ » becomes true!

(C) Hervé Barbot, 2009

57

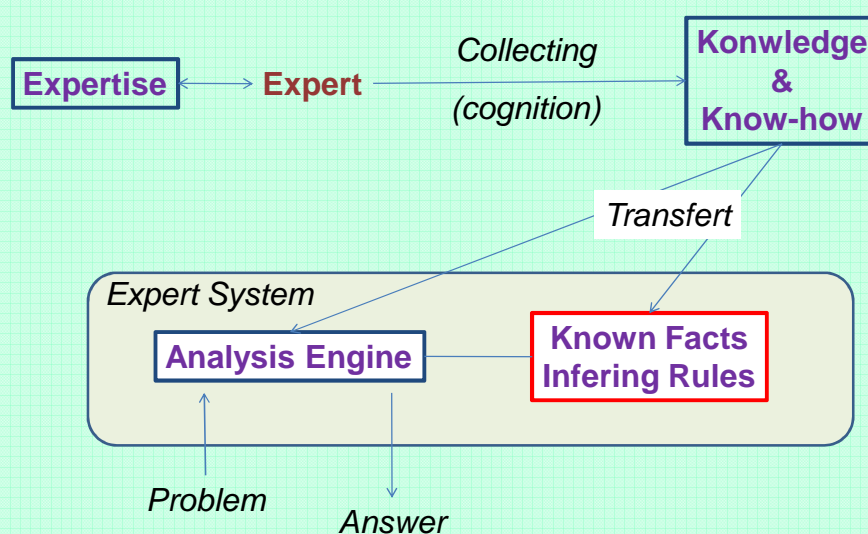
Rules-based reasoning

- Components :
 - Set of rules *if ...conditions... then ...conclusion/action...*
 - Set of facts *what is known*
 - Infering engine
- Principle
 - Find among the set of rules the ones that can be « fired », or « applied »:
If conditions are verified, then conclusions are applicable
 - Select one of these rules
This may need conflict resolution
 - « Fire » / execute the rule

(C) Hervé Barbot, 2009

58

Rules-based expert system



(C) Hervé Barbot, 2009

59

Hypothesis : rules such as

IF A and B and ... THEN X

(C) Hervé Barbot, 2009

60

AND/OR Tree or Graph

(C) Hervé Barbot, 2009

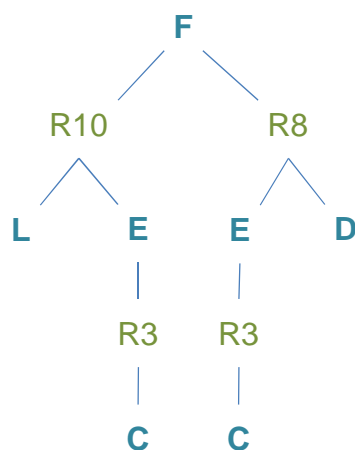
61

*Example*R1 : A \rightarrow B R6 : M+L \rightarrow AR2 : A \rightarrow C R7 : I+B \rightarrow DR3 : C \rightarrow E R8 : E+D \rightarrow FR4 : M \rightarrow C R9 : K+F \rightarrow HR5 : I+K \rightarrow A R10 : L+E \rightarrow F

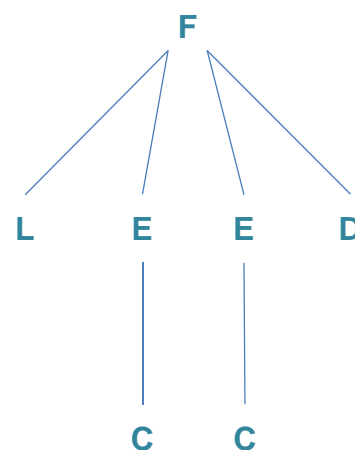
(C) Hervé Barbot, 2008

64

R3: $C \Rightarrow E$
 R8: $E \wedge D \Rightarrow F$
 R10: $L \wedge E \Rightarrow F$

Graphical variations

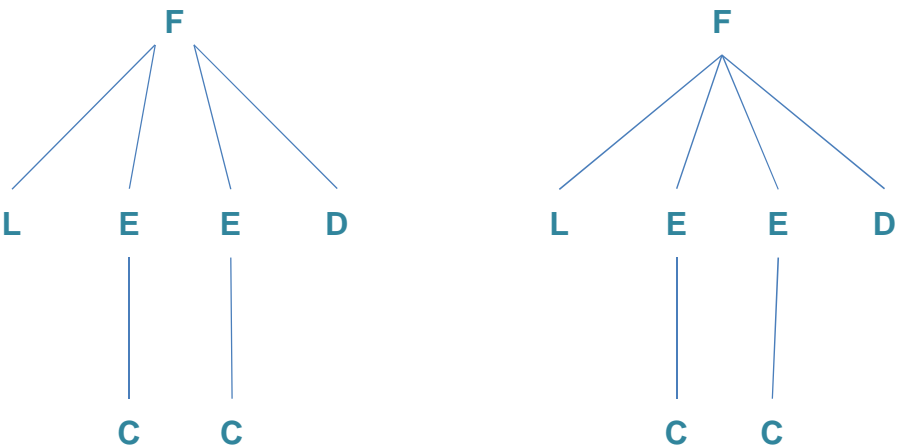
Rules are explicitly shown



Rules are not shown

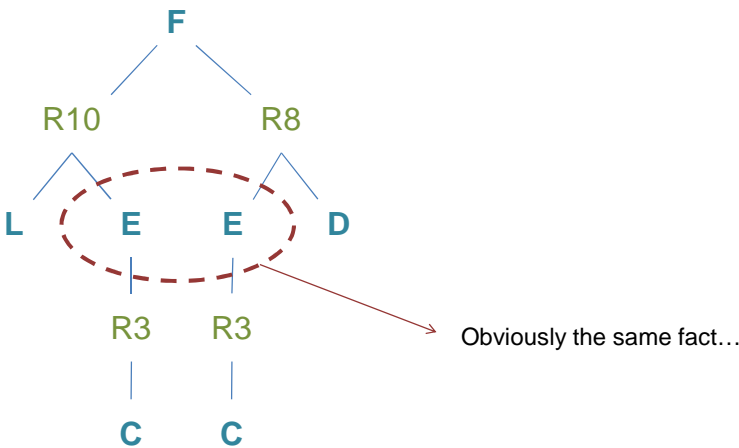
(C) Hervé Barbot, 2009

66



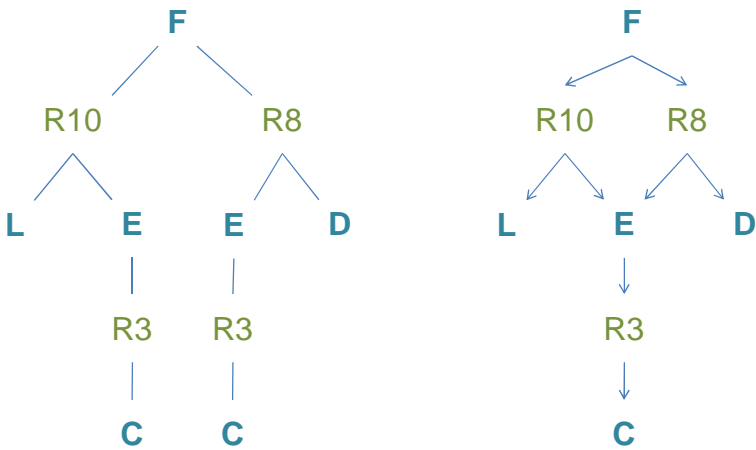
Is it the same?

67



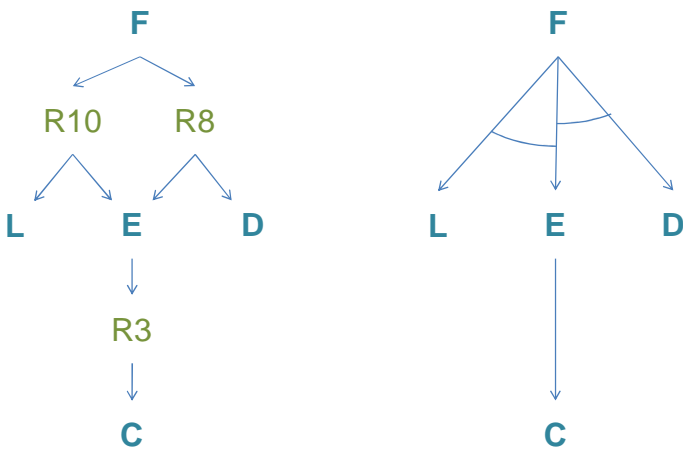
(C) Hervé Barbot, 2009

68



(C) Hervé Barbot, 2009

69



(C) Hervé Barbot, 2009

70

Chaînage avant

Forward chaining

(C) Hervé Barbot, 2009

71

Infering rules:
R1: $A \rightarrow B$
R2: $A \rightarrow C$
R3: $C \rightarrow E$
R4: $M \rightarrow C$
R5: $I \& K \rightarrow A$
R6: $M \& L \rightarrow A$
R7: $I \& B \rightarrow D$
R8: $E \& D \rightarrow F$
R9: $K \& F \rightarrow H$
R10: $L \& E \rightarrow F$

Known facts:
I
L
M

Problem statement :

According to:

Facts that we know are true
representing the initial state of the search,
Representing the actual description of the world

Infering rules (they are true as well!...)
representing the know-how of the problem

What other facts can we determine as true?

(C) Hervé Barbot, 2009

72

Lorsqu'un nouveau fait p est ajouté à la base de connaissances:

pour chaque règle telle que p s'unifie avec un prémisses
et si les autres prémisses sont connus
alors ajouter la conclusion à la base de connaissances et
continuer le chaînage

- le chaînage avant est piloté par les données ("*data-driven*")

càd, on infère des propriétés et des catégories à partir des
séquences perceptives

(C) Hervé Barbot, 2009

73

- Iterate on:
 - Select a rule that is applicable
 - That is for which the « conditions » part is true
 - Apply the rule
 - That is: add the conclusion in the « known facts » set
- Stop when there is no more applicable rule
- Applying a rule adds known facts
 - Thus, a rule not applicable at one time may be applicable later...

(C) Hervé Barbot, 2009

74

Inferring rules:

R1: $A \rightarrow B$
 R2: $A \rightarrow C$
 R3: $C \rightarrow E$
 R4: $M \rightarrow C$
 R5: $I \& K \rightarrow A$
 R6: $M \& L \rightarrow A$
 R7: $I \& B \rightarrow D$
 R8: $E \& D \rightarrow F$
 R9: $K \& F \rightarrow H$
 R10: $L \& E \rightarrow F$

Initial facts = {I,L,M}

Rule applied	Knowledge
	ILM
R4	ILMC
R6	ILMCA
R1	ILMCAB
R2	ILMCAB <i>no new fact added!</i>
R3	ILMCABE
R7	ILMCABED
R8	ILMCABEDF
R10	ILMCABEDF <i>no new fact added!</i>

Order may differ depending on the strategy to select a rule at each step!

But the result is the same...

(C) Hervé Barbot, 2009

75

```
facts_set forward_chaining(
    rules_set rules
    facts_set initial_facts)

rules_set to_fire ← rules
facts_set facts ← initial_facts

WHILE there_is_applicable_rule(to_fire,facts) DO

    rule r ← select_rule(to_fire,facts)
    to_fire ← to_fire - r
    facts ← facts + conclusion(r)

ENDWHILE

RETURN facts
```

(C) Hervé Barbot, 2009

76

■ Variants

- Each time a rule is selected, add its conclusions in the knowledge base and fire (new) rules accordingly

See previous exemple and pseudo-code

- Fire rules only based on the initial facts
 - Think about rules with actions as conclusions: allow to determine what could be done, but without taking the (final) decision)

(C) Hervé Barbot, 2009

77

Chaînage arrière

Backward chaining

(C) Hervé Barbot, 2009

78

Infering rules:

R1: $A \rightarrow B$
 R2: $A \rightarrow C$
 R3: $C \rightarrow E$
 R4: $M \rightarrow C$
 R5: $I \& K \rightarrow A$
 R6: $M \& L \rightarrow A$
 R7: $I \& B \rightarrow D$
 R8: $E \& D \rightarrow F$
 R9: $K \& F \rightarrow H$
 R10: $L \& E \rightarrow F$

Known

facts:

I
L
M

Problem statement :

According to:

Facts that we know are true
 representing the initial state of the search,
 Representing the actual description of the world

Infering rules (they are true as well!...)
 representing the know-how of the problem

Is a given fact (e.g. F) true or not ?

(C) Hervé Barbot, 2009

79

- Iterate on all rules concluding to the fact:
 - Check if conditions are true
 - If yes, decide that the fact is true

- Checking if a condition is true is recursion!

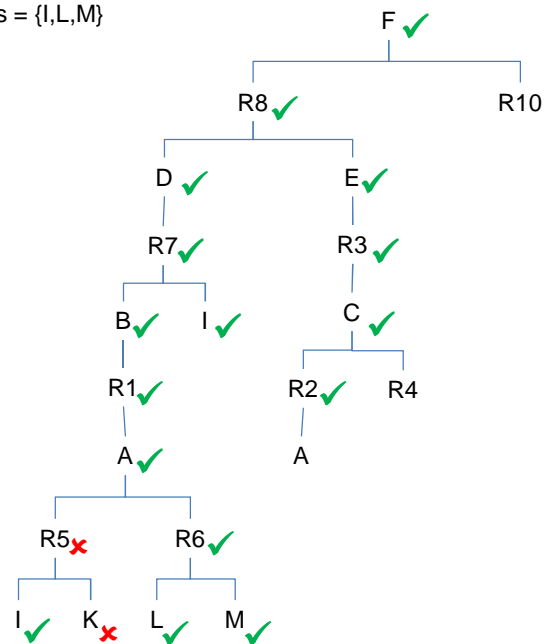
(C) Hervé Barbot, 2009

80

Inferring rules:

R1: $A \rightarrow B$
 R2: $A \rightarrow C$
 R3: $C \rightarrow E$
 R4: $M \rightarrow C$
 R5: $I \& K \rightarrow A$
 R6: $M \& L \rightarrow A$
 R7: $I \& B \rightarrow D$
 R8: $E \& D \rightarrow F$
 R9: $K \& F \rightarrow H$
 R10: $L \& E \rightarrow F$

Initial facts = {I, L, M}
 F ?



(C) Hervé Barbot, 2009

97

If all rules are of the form :

IF a set of facts that shall be true

THEN a set of facts that will be considered as true

- i.e. only the « and » operator is used

$$\text{truth_value}(F) = \text{OR}_{R, F \in C(R)} \text{fire_ability}(R)$$

$$\text{fire_ability}(R) = \text{AND}_{F, F \in P(R)} \text{truth_value}(F)$$

$C(R)$ =set of facts as conclusion of a rule R

$P(R)$ =set of facts in the condition part of rule R

(C) Hervé Barbot, 2009

98

```

bool truth_value: fact      F      IN
                  rules_set rules  IN/OUT
                  facts_set facts  IN/OUT

IF F ∈ facts THEN RETURN true

WHILE there_is_a_concluding_rule(rules,F) DO

    rule r ← select_a_concluding_rule(rules,F)
    rules ← rules - r
    IF fire_ability(r,rules,facts)
        THEN facts ← facts + F
        RETURN true

ENDWHILE

RETURN false

```

(C) Hervé Barbot, 2009

99

```

bool fire_ability: rule      R      IN
                  rules_set rules  IN/OUT
                  facts_set facts  IN/OUT

facts_set facts_to_check ← condition(R)

WHILE facts_to_check ≠ ∅ DO

    fact F ← select_fact(facts_to_check)
    IF truth_value(F,rules,facts) = false
        THEN RETURN false
    facts_to_check ← facts_to_check - F

ENDWHILE

RETURN true

```

(C) Hervé Barbot, 2009

100

After the « basic » rules and chaining algorithms...

Let's see more issues!

(C) Hervé Barbot, 2009

101

« OR » in conditions

- IF A OR B THEN C

- IF A THEN C
IF B THEN C

(C) Hervé Barbot, 2009

102

« NOT »

- Domain of value :
 { true , undetermined }
 or
 { true , false , undetermined }

(C) Hervé Barbot, 2009

103

General boolean expressions

- IF <bool_expression> THEN A

```

bool fire_ability: rule R
                        rules_set rules  IN/OUT
                        facts_set facts  IN/OUT

```

*Evaluate according to the semantic tree of the
boolean expression*

```

RETURN true

```

(C) Hervé Barbot, 2009

104

Multiple conclusions

- IF A THEN X AND Y AND Z
- IF A THEN X OR Y
 - At least one of 'X' and 'Y' shall be set to 'true'
 - How to do if we don't know that X or Y is false?
- IF A THEN X XOR Y
 - Exactly one shall be true
 - How to do if both are 'undetermined'?

(C) Hervé Barbot, 2009

105

Interactive backward chaining

- Idea:
ask the user to help the system
- Principle 1:
the system is an expert, and shall investigate as much as possible before requiring any help
- Principle 2:
ask as few help as possible

(C) Hervé Barbot, 2009

106