CMU 17-651 Models of Software Systems Fall 2008

Homework #2: Logic

Garlan Due: Wednesday September 3, 2008

I. Propositional Logic

1. Construct truth tables for each of the following:
   a. $p \wedge (p \vee q)$
   b. $\neg p \wedge (p \vee (q \Rightarrow p))$
   c. $(p \Rightarrow q) \Rightarrow (\neg p \vee q)$
2. Which of the above sentences are:
   i. valid?
   ii. satisfiable?
   iii. contingent?
   iv. inconsistent?

   Briefly explain why.
3. Explain, using truth tables, why the following sentences have the same meaning:
   - $p \Rightarrow q$
   - $\neg(p \wedge \neg q)$

II. Predicate Logic

4. Which occurrences of the variables x and y are free and which are bound in each of the following? Briefly explain why.
   Note: Recall that a variable may be both bound and free in the same sentence.
   a. $(\exists y : N \bullet y > 2) \wedge (\forall x : N \bullet x + 1 > x)$
   b. $x = 2 * y$
   c. $(\exists y : N \bullet y > 2) \wedge (\forall x : N \bullet x > y)$
   d. $\forall x : N \bullet ((\exists y : N \bullet y > x) \wedge x = 2 * y)$
5. Translate the following sentences into predicate logic, using the translation key provided.
   Note: You may only use the standard universal and existential quantifiers ($\forall$ and $\exists$). Do *not* use the unique existential quantifier ($\exists!$).

E:	the set of elephants
A:	the set of animals
$G(x)$:	x is green
$E(x)$:	x is an elephant
$N(x, y)$:	the name of x is y

   a. Some elephants are green.
   b. All elephants are green.
   c. If an animal is green, it is an elephant.
   d. No green animal is an elephant.

e. There is *exactly one* green elephant, and his name is James.

6. Translate the following sentences into predicate logic, using the translation key provided.

   NOTE 1: You may assume the existence of = over elements of all sets.

   NOTE 2: You may only use the standard universal and existential quantifiers (∀ and ∃). Do *not* use the unique existential quantifier (∃!).

S:	the set of students
T:	the set of topics, which has *logic* and *models* as elements
$MSE(s)$:	s is an MSE student
$Likes(s, t)$:	student s likes topic t

   a. Some MSE students like logic.
   b. MSE students like logic.
   c. MSE students like only logic.
   d. No MSE student likes logic.
   e. If an MSE student likes logic then he/she likes Models.
   f. Exactly one MSE student likes Models

7. In this class we will be creating various models of an infusion pump. An infusion pump is a device used in hospitals to feed fluids intravenously to patients through one of several "infusion lines." Each line is a physical tube connected to a patient.

   Consider the following excerpt from a description of a typical pump provided to us by the Food and Drug Administration:

   A. An infusion line may become pinched causing the flow to be blocked. This will be recognized by the pump as an occlusion and will cause the pump to alarm.
      i. The mitigation is to straighten the line and re-start the pump.
      ii. Caregiver may silence the alarm during the procedure.

   B. The infusion line may become plugged. The pump will recognize an occlusion and alarm.
      i. The mitigation is to clear the infusion lines and re-start the pump.
      ii. Caregiver may silence the alarm during the procedure.

   C. Electrical failure may occur causing the pump to switch to battery operation.
      i. Pump will switch over to battery power and notify the caregiver visually.
      ii. Switch may not occur if the battery is not properly charged.

   Questions:

   a. Define some sets and predicates appropriate to this domain (similar to the elephant problem above).
   b. Using the sets and predicates you defined express the following statements in predicate logic:
      i.. An alarm will sound whenever the line is "pinched" or "plugged."
      ii.. If there is an electrical failure the battery power will be on unless the battery is not properly charged.
   c. Does your collection of predicates allow you to say "The alarm will continue to sound until the care giver turns it off." Why or why not?