

Vehicle Dynamics for Racing Games

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Introduction

Vehicle dynamics is concerned with the movement of vehicles. In general, the movements of interest are braking, acceleration, and cornering. The forces imposed on the vehicle by the tires, gravity, and aerodynamics determine its behavior during these conditions. The vehicle and its components are studied to determine the vehicle's response to the forces produced during operation. This paper discusses methods that can be used to develop a realistic vehicle dynamics model for use in a racing simulation.

Equations of Motion

Vehicle dynamic behavior is described by three deceptively simple equations that govern lateral force, longitudinal force, and yaw moment:

$$\begin{aligned}\sum F_x &= Ma_x \\ \sum F_y &= Ma_y \\ \sum \tau_x &= I_{xx}\dot{\alpha}_x\end{aligned}$$

Equations through make it possible to develop a highly accurate simulation of a vehicle's motion. However, in order to provide useful information, you must take great care to accurately define the lateral and longitudinal forces. In order to discuss the forces, we must first define a standard coordinate system.

Coordinate Systems

Work in vehicle dynamics uses both world-fixed and vehicle-fixed coordinate systems. It is often necessary to use matrix transformation methods to convert back and forth between the two systems. In any vehicle dynamics simulation, there are some calculations that are better carried out in a particular coordinate system. See Figure 1, the SAE standard vehicle axis system. The vehicle fixed coordinate system is right-hand orthogonal, originates at the CG, and travels with the vehicle. We will use this standard coordinate system to describe the forces on the vehicle.

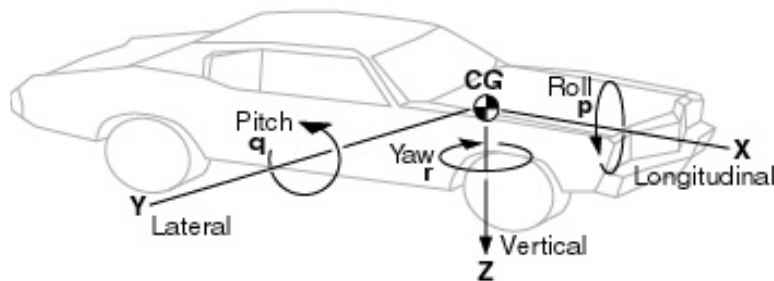


Figure 1 SAE Vehicle Axis System

Summary of Forces

Figure 2 depicts the significant forces acting on a vehicle. Subsequent sections will provide further details about these forces.

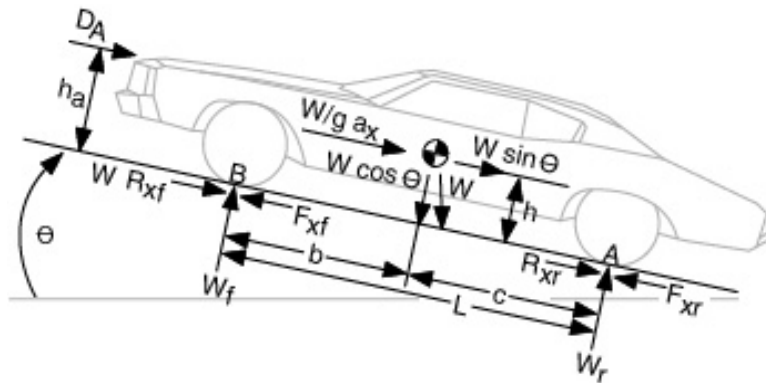


Figure 2 Significant Forces Acting on a Vehicle

W is the weight of the vehicle acting at its CG. On a grade, it may have a cosine component perpendicular to the road and a sine component parallel to the road.

If the vehicle is accelerating along the road, it is convenient to represent the effect by an equivalent inertial force known as a “d’Alembert force,” denoted as $W/g a_x$ acting at the CG opposite to the direction of the acceleration.

The tires will experience a normal force denoted by W_f and W_r .

Tractive forces F_{xf} and F_{xr} , and/or rolling resistance forces R_{xf} and R_{xr} may act in the ground plane at the tire contact patch.

Steering forces F_{yf} and F_{yr} act in the ground plane at the tire contact patch.

D_A is the aerodynamic force acting on the vehicle, usually represented as acting at a point above the ground indicated by the height h_a .

Most of these forces do not act at the center of rotation for the vehicle, and thus create moments.

Static Load Distribution

Static loads are the basis for determining the dynamic behavior of the vehicle, so this is an important first step in the analysis of vehicle forces.

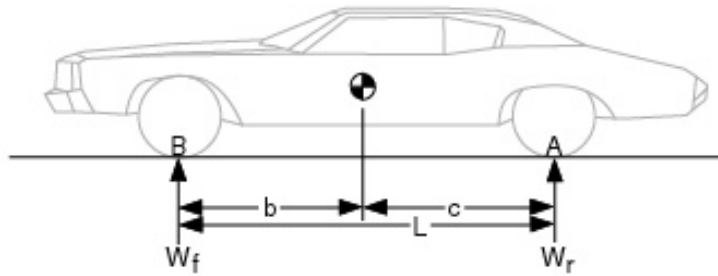


Figure 3 Static Load Distribution on Level Ground

A stationary vehicle on level ground has the load distribution shown in Figure 3. In this case, load distribution is strictly a function of vehicle geometry, and can be determined by summing the moments around the contact point A, leading to equations and :

$$W_{fs} = W \frac{c}{L}$$

$$W_{rs} = W \frac{b}{L}$$

where:

W_{fs} = front axle normal force

W_{rs} = rear axle normal force

b = longitudinal distance from CG to front axle

c = longitudinal distance from CG to rear axle

L = wheelbase

Example: a 1970 Chevelle ($m = 1765$ kg, $L = 2.84$ m, $b = 1.22$ m, $c = 1.62$ m) is sitting on level ground. The load on the front axle is $W_{fs} = 1765(1.62/2.84) = 1007$ kg. The load on the rear axle is $W_{rs} = 1765(1.22/2.84) = 758$ kg.

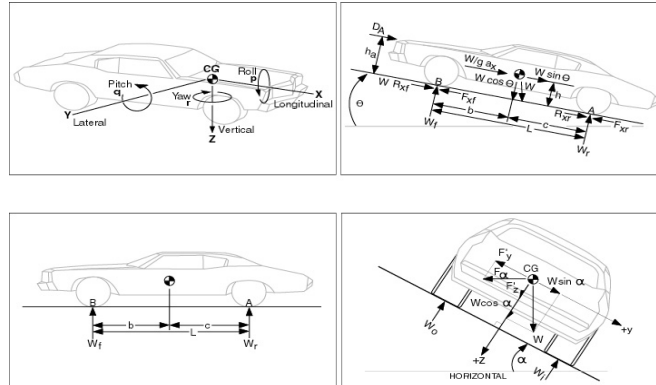


Figure 4 Load Distribution of a Vehicle on a Banked Surface

Figure 4 depicts the effect of a bank angle on the load distribution. A bank causes the load on the interior (lower) tires to increase, while the load on the exterior (upper) tires decreases. The formulas for the change in load (using the small angle approximation) on the tires are:

$$W_i = \frac{W_a}{2} + W_a \frac{h}{t} \alpha$$

$$W_o = \frac{W_a}{2} - W_a \frac{h}{t} \alpha$$

where:

h = height of CG above ground

α = bank angle, in radians

t = track width

W_a = axle load

Example: a 1970 Chevelle ($h = 0.6$ m, $t = 1.52$ m) sits on a banked road ($\alpha = 3^\circ$). In this case, $W_{of} = (1007/2) - 1007(0.6/1.52)(3\pi/180) = 483$ kg.

The effect of gradients is very similar to the effect to the effect of a bank angle. In this case, the lower tires become more heavily loaded. Refer to equations and :

$$W_{fs} = W \frac{c}{L} - W \frac{h}{L} \theta$$

$$W_{rs} = W \frac{b}{L} + W \frac{h}{L} \theta$$

where:

θ = gradient angle, in radians

Example: a 1970 Chevelle sits on an incline($\theta = 5^\circ$), pointing uphill. In this case, $W_{fs} = 1007 - 1765(0.6/2.84)(5\pi/180) = 974$ kg and $W_{rs} = 758 + 1765(0.6/2.84)(5\pi/180) = 791$ kg.

The small angle approximation is useful for bank (α) or gradient (θ) angles of up to 10 or 20 degrees. After that, it becomes necessary to derive equations including all sine and cosine terms.

Dynamic Load Transfer

In dynamic conditions, load can transfer to the front wheels (during braking), the rear wheels (during acceleration), and side to side (during cornering). Determining the axle loads under arbitrary conditions is an important step in the analysis of acceleration, braking, and cornering because the axle loads determine the tractive and steering forces available at each wheel, affecting acceleration, braking performance, and maximum speed.

The governing equations for acceleration and deceleration are:

$$W_f = \frac{c}{L}W - \frac{h}{L} \frac{W}{g} A_x$$

$$W_r = \frac{b}{L}W + \frac{h}{L} \frac{W}{g} A_x$$

where:

W_f = front axle load

W_r = rear axle load

Equations and work for both acceleration and deceleration provided you keep the sign of the acceleration (A_x) straight.

Example: a 1970 Chevelle accelerates at 6 m/s^2 (a mild acceleration given the vehicle's capabilities). The front axle load is then $W_f = 1007 - (0.6)(1765)(5)/(2.84)(9.81) = 817$ kg. Likewise the rear axle load is then $W_r = 758 + (0.6)(1765)(5)/(2.84)(9.81) = 948$ kg.

The governing equation for lateral load transfer is:

$$F_{z_o} - F_{z_i} = 2\Delta F_z = 2F_y h_r / t + 2K_\phi \phi / t$$

where:

$$F_y = \text{lateral force} = F_{yi} + F_{yo}$$

$$h_r = \text{roll center height}$$

$$t = \text{tread (track width)}$$

$$K_\phi = \text{roll stiffness of the suspension} = K_{\phi f} + K_{\phi r}$$

$$\phi = \text{roll angle of the body}$$

Equation has two components. The $2F_y h_r / t$ component acts because of acceleration and instantaneously affects the load distribution. The $2K_\phi \phi / t$ term depends on the roll angle of the vehicle; naturally the body takes some time Δt to complete its rolling motion. To be completely accurate, this rolling motion must be integrated over time, taking into account the I_{xx} (roll axis) inertial properties of the vehicle.

The roll angle (ϕ) of the body is given by:

$$\phi = \frac{Wh_l V^2 / (Rg)}{K_{\phi f} + K_{\phi r} - Wh_l}$$

where:

$$h_l = \text{Roll axis to vertical CG distance}$$

$$R = \text{Radius of turn}$$

The roll rate is usually in the range of 3 to 7 degrees/g on passenger cars. Sports cars are often in the 1 to 2 degrees/g range.

Acceleration Performance

As power delivered to the wheels increases, acceleration eventually becomes limited either power or traction. In power limited acceleration, the vehicle reaches its peak acceleration because the engine cannot deliver any more power. In traction limited acceleration, the engine can and does deliver more power, but vehicle acceleration is limited because the tires cannot transmit any more driving force to the ground. Equation gives the maximum transmittable force:

$$F_x = \mu F_z$$

where μ is the coefficient of friction between the tire and the road. Note that μ depends on many factors, including load and velocity. If F_x exceeds this limit, the tire slips excessively and enters dynamic friction, where the coefficient of friction dramatically decreases, i.e., it breaks traction.

The acceleration of the vehicle (and therefore the longitudinal forces on the tires) at a given point can be determined via equation :

$$(M + M_r) a_x = \frac{(W + W_r)}{g} a_x = \frac{\tau_e N_{tf} \eta_{tf}}{r} - R_x - D_A - W \sin \theta$$

where:

M = Mass of the vehicle = W / g

M_r = Equivalent mass of the rotating components

a_x = Longitudinal acceleration

R_x = Rolling resistance forces

D_A = Aerodynamic drag force

T_e = engine torque at a given rpm (from dynamometer data)

N_{tf} = combined ratio of the transmission and final drive

η_{tf} = combined efficiency of the transmission and final drive

r = radius of the tire

Engine torque is measured at a steady speed on a dynamometer; thus, the actual torque delivered to the drivetrain is reduced by the amount required to accelerate the inertia of the rotating components (as well as any accessory loads). The combination of the two masses in the above equation is an “effective mass,” which accounts for the rotational inertia of the drivetrain components. The ratio of $(M+M_r)/M$ is the “mass factor.” The mass factor depends on the operating gear. Representative numbers are often estimated using equation [1], although they can also be calculated from the basic inertial properties of the components.

$$\text{Mass Factor} = 1 + 0.04N_{tf} + N_{tf}^2$$

In this complete form of the acceleration equation, there is no explicit solution. All terms except the grade term must be evaluated at speed. Fortunately, this is relatively easy to do with a spreadsheet, and we can easily evaluate the tractive force term on the right.

Example: consider the following plot of tractive force and total drag (rolling resistance plus aerodynamic drag) for a 1970 Chevelle. Note that the tractive force and total drag intersect at about 54 m/s (120 mph), which represents a theoretical top speed for the vehicle.

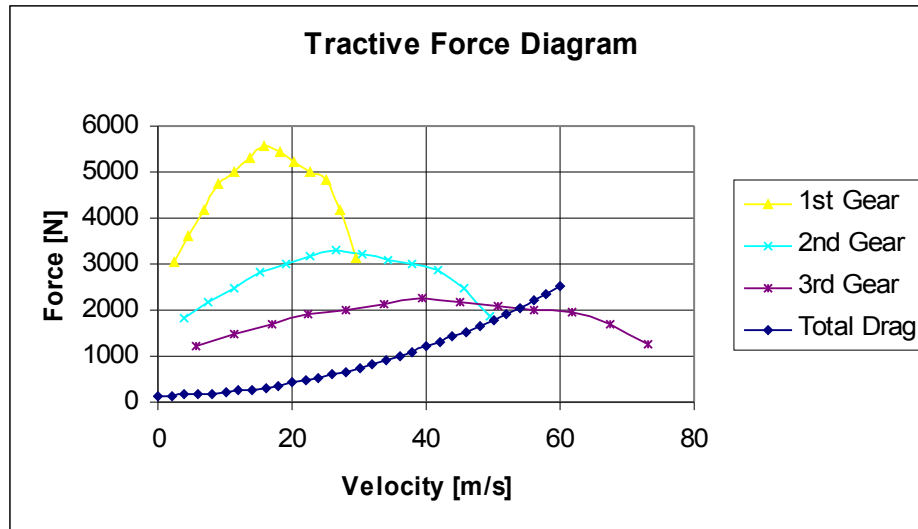


Figure 5 Tractive Force Diagram for 1970 Chevelle

Acceleration performance is also limited by other factors, such as suspension geometry, the drivetrain, and lateral load transfer caused by engine torque.

Braking Performance

The general equation for braking performance is:

$$(M) a_x = - \frac{(W)}{g} D_x = - F_{xf} - F_{xr} - R_x - D_A - W \sin \theta$$

where:

M = Mass of the vehicle = W / g

D_x = Linear deceleration

F_{xf} = Front axle braking force

F_{xr} = Rear axle braking force

R_x = Rolling resistance forces

D_A = Aerodynamic drag force

θ = Uphill grade

Braking causes the vehicle to decelerate, which causes load to transfer to the front of the vehicle. This is why brake pads are usually larger and heavier on the front of a vehicle. If they were not, the brakes would wear out too fast. Braking is usually limited by friction, just like acceleration performance.

The torque produced by the brake generates a braking force at the ground to decelerate the wheels and driveline components. Then:

$$F_b = \frac{(T_b - I_w \alpha_w)}{r}$$

where:

r = Rolling radius of the tires

I_w = Rotational inertia of the wheels and driveline components

α_w = Rotational deceleration of the wheels

As long as the wheels are rolling, the braking forces can be predicted using . However, the brake force can only increase to the limit of the friction coupling between the tire and road. The friction coupling depends on some small amount of slip occurring between the tire and the road. Various deformation processes cause braking force and slip to be coexistent. Slip of the tire is defined as:

$$\text{Slip} = \frac{V - \omega r}{V}$$

where:

V = Vehicle forward velocity

ω = Tire rotational speed (rad/sec)

Brake force (expressed as a coefficient F_x/F_z) for a dry surface is shown as a function of slip in Figure 6.

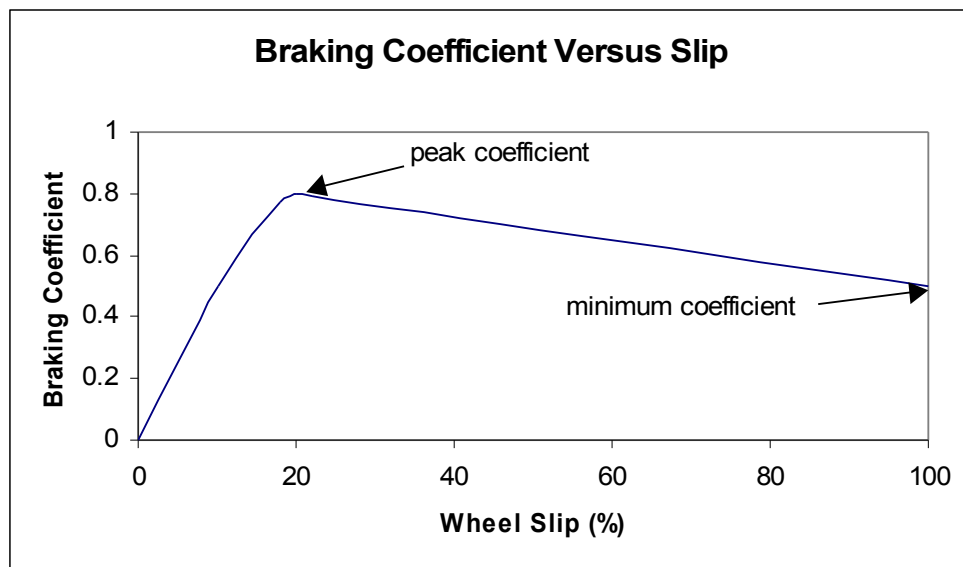


Figure 6 Braking Coefficient versus Slip

The brake coefficient increases up to about 10 or 20%, establishing the maximum braking force that can be obtained from the particular tire-road combination. This peak

coefficient is denoted by μ_p . At higher slip the coefficient diminishes, reaching its minimum value at 100% slip (wheel fully locked) and denoted by μ_s .

Tire-road friction also varies with velocity, inflation pressure, road surface, and load.

Tracking wheel slip adds a great deal of realism to the simulation of braking. However, it requires careful tracking and integration of the wheel angular velocity and braking forces.

Environmental Forces

Braking and acceleration forces are applied by the vehicle; there are two primary environmental forces operating on a vehicle: aerodynamic loads and tire friction (rolling resistance).

The aerodynamic effect familiar to most people is aerodynamic drag, which is given by:

$$F_d = 0.5\rho C_d AV^2$$

where:

ρ = air density

C_d = drag coefficient

A = frontal area of the vehicle

V = velocity

C_d is usually determined by wind tunnel tests, although there are methods of estimating it from coast-down tests. Drag is strongly influenced by vehicle yaw (β). If the vehicle is yawing relative to the direction of the airflow around the vehicle, airflow separation will occur on the downward side. This causes an increase in the drag coefficient. Normally this increase is limited to 5 to 10% on passenger cars, but can be much larger on trucks.

Crosswinds can also produce large lateral aerodynamic forces. Crosswind components in a game are generally unsatisfactory for the user without some sort of force feedback indicating the direction and strength of the crosswind. Without the force feedback, the user gets frustrated by seemingly arbitrary changes of direction in the vehicle.

The pressure differential from the top to the bottom of the vehicle causes a lift force, just as airflow over a wing provides lift for an airplane. In the case of a vehicle, however, this is bad because it reduces the load forces on the tires, leading to a loss of control.

$$L_A = 0.5\rho V^2 C_L A$$

where:

C_L = aerodynamic lift coefficient

Example: consider a 1970 Chevelle operating at 100 mph (44.7 m/s). $A = 2.2 \text{ m}^2$, $\rho = 1.3 \text{ kg/m}^3$, $C_L = 0.5$, The lift force is 1428 N. The total load on the vehicle is around 17 kN, so even with this conservative estimate for C_L , the car just lost about 8% of its loading.

At zero wind angle, lift coefficients normally fall in the range of 0.3 to 0.5 for modern passenger cars, but under crosswind conditions this can increase dramatically. All sorts of devices are used to reduce the lift forces, or even provide negative lift (i.e., a push downward).

Rolling resistance is caused by the tire's resistance to deformation, scrubbing losses in the contact patch, tire slip, and air drag on the tire, among other things. Unlike aerodynamic forces, rolling resistance becomes effective as soon as the tire starts rotating. Aerodynamic loads only become equal to rolling resistance at 50-60 mph for modern passenger cars. The basic equation for rolling resistance is:

$$R_x = f_r W$$

where:

f_r = rolling resistance coefficient

It generally suffices to use the vehicle's static weight in equation . Taking account of the vehicle's dynamic load changes vastly increases the complication of the calculation without a significant increase in accuracy, which you can verify with a spreadsheet.

Important factors affecting the coefficient of rolling resistance include tire type (radial versus bias ply), temperature, inflation pressure, material, design, slip, velocity, and load. Because of these many inter-related factors, it is virtually impossible to develop a method of considering all these variables. However, there are several good, empirical estimates. One estimate such treats velocity as the significant variable, and provides linear speed dependence:

$$f_r = 0.01(1 + V / 100)$$

where:

V = speed in mph

There are several other ways to estimate the coefficient of rolling resistance, as discussed in references [1] and [4]. These books also present methods of taking some of the other effects into account, including tables of coefficients and relationships for different types of tires. It is possible to develop models that take most of the factors into account [1].

Tire Forces and Moments

Determining the cornering forces is probably the most difficult task in defining the forces acting on the vehicle. When the wheels are steered at some angle, δ , they develop lateral forces that turn (yaw) the vehicle. Getting an accurate model of these forces is a complex task.

In high speed cornering, the tires must develop lateral forces, and will experience lateral slip as they roll. The lateral force (denoted F_y) is called the cornering force. Slip angle (α) is the angle between the tire's direction of heading and the direction of travel. At a given tire load, the cornering force grows with slip angle as shown in Figure 7.

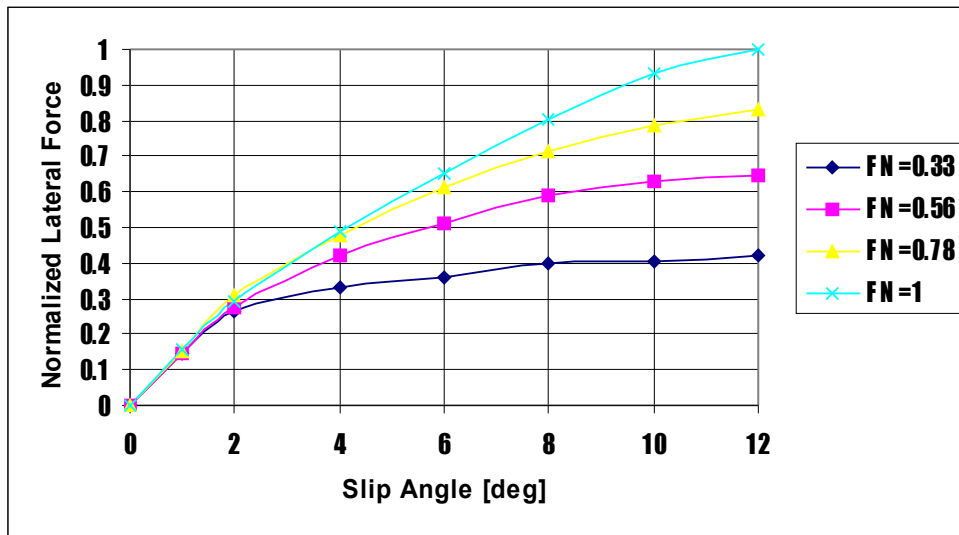


Figure 7 Normalized Lateral Force Versus Slip Angle for a Bias Ply Tire

At low slip angles, the relationship is linear; hence, the cornering force is given by:

$$F_y = C_\alpha \alpha$$

where C_α is the cornering stiffness, and is defined as the slope of the curve for F_y versus α at $\alpha = 0$.

This linear representation only holds true out to about 5 degrees, which is generally insufficient for use in a game, where the user will spend a great deal of time performing extreme (high slip angle) maneuvers. To simulate these conditions, it is necessary to develop accurate methods to calculate lateral force as a function of slip angle and load. Tire lateral forces developed from slip angle and camber angle are also influenced by inflation pressure, aspect ratio, and rim size.

In most cases, the following equations for slip angles suffice. Reference [3] discusses several nonlinear models.

$$\alpha_{fi} = \beta + \frac{\omega b}{L} - \delta_i$$

$$\alpha_r = \beta - \frac{\omega c}{L}$$

The angle β is known as the sideslip angle. It is the angle between the longitudinal axis and the actual direction of travel. Sideslip angle is determined by:

$$\beta = \tan^{-1}\left(\frac{V_x}{V_z}\right)$$

Determining the slip angle (and hence the lateral force exerted by the tire) is then a matter of having accurate information on the tire load, steering wheel angle, and angular velocity of the vehicle.

The inclination of a wheel outward from the body of the vehicle is known as the camber angle. Camber on a wheel produces an additional lateral force known as “camber thrust.” Camber angle produces much less lateral force than slip angle. About 4 to 6 degrees of camber are required to produce the same lateral force as 1 degree of slip angle on a bias-ply tire. Camber stiffness on radial tires is generally much lower than on bias-ply tires; hence as much as 10 or 15 degrees is required on a radial. Camber thrust is additive to the cornering force from slip angle.

Modeling the lateral force versus slip angle curves is a delicate task. Third order polynomials work well, but require a different set of coefficients for each load region in order to develop the necessary accuracy. Pacejka magic formulas [3] are a method of curve fitting lateral force versus load and slip angle, using either eleven or seventeen coefficients. Typically, these coefficients have to be derived from tire data (reference [2] is a good source) using constrained nonlinear optimization or genetic algorithm techniques. Pacejka magic formulas are very accurate, but are mathematically tricky and computationally expensive to implement.

In addition to lateral forces, tires also generate an aligning moment when operating at a slip angle. Aligning moment arises because the lateral force does not actually act directly on the normal axis of the tire; it acts at some small distance p behind the central point of the tire, which of course causes a moment. In most cases, the aligning moment always acts to oppose the moments developed by the lateral forces, thus acting to stabilize the yaw (spin) of the vehicle. In extreme slip angle conditions, however, the aligning moment can become negative, thus adding to the other moments and destabilizing the vehicle. Aligning moment is also a function of load. See Figure 8, which depicts aligning moment versus slip angle for a bias ply tire.

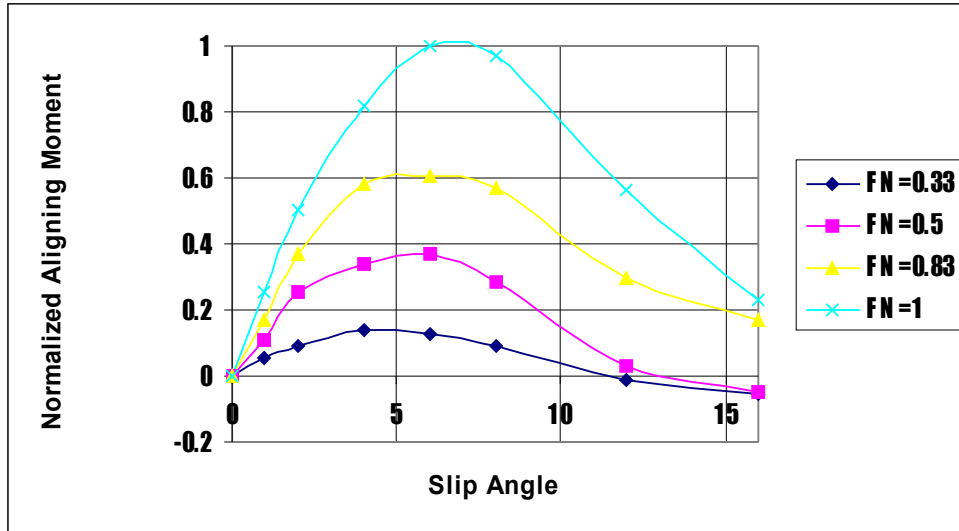


Figure 8 Normalized Aligning Moment Versus Slip Angle for a Bias Ply Tire

A “traction circle” is a graphical representation of the friction limits and forces on the tire. The traction circle is often actually a traction ellipse, because friction coefficients are often different in the lateral (μ_y) and longitudinal (μ_x) directions. The friction limit of the tire is determined by μF_z ; therefore, the tire can exert lateral force, brake force, or a combination of the two, in positive or negative directions. However, *in no case can the vector total of the forces exceed the friction limit*. This is why it’s critical to correctly calculate the dynamic load on the tire. Figure 9 depicts an example traction circle depicting force vectors for braking [A], cornering [B], and combined acceleration and cornering [C].

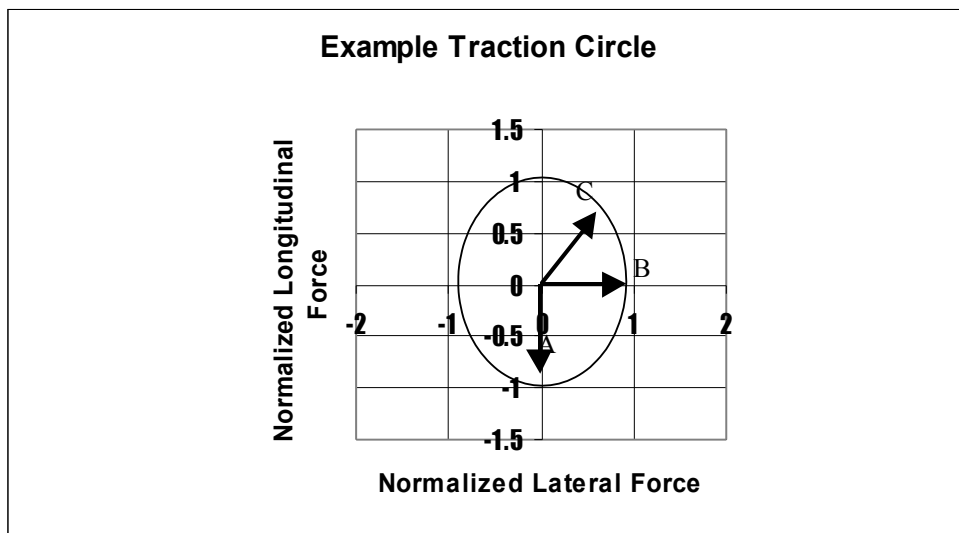


Figure 9 Example Traction Circle

Theoretically, the tire only needs to be just wide enough to provide contact at one point. The reason why wider tires provide better traction is that the coefficient of friction is a theoretical limit, dependent upon good contact and a variety of factors. The wider the tire, the more likely that the tire will have good contact and approach its theoretical maximum. With

very soft tires, the rubber actually deforms into the cracks and ridges of the driving surface, providing a further “friction” effect.

Cornering Performance

When discussing cornering, there are two regimes of interest: low speed cornering and high speed cornering. In low speed cornering (parking lot speeds), a kinematic bicycle model generally suffices. This is not a region of extreme maneuvers, and we are not generally very interested in the (lateral) forces and moments on the vehicle. In a game, just about any way that you want to handle this region of steering is fine.

In the high speed cornering regime, equation describes how the steering angle (δ) must be changed with the radius of the turn, or the lateral acceleration, $V^2/(gR)$.

$$\delta = \frac{180}{\pi} \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) \frac{V^2}{gR}$$

This is often written in shorthand as follows:

$$\delta = \frac{180}{\pi} \frac{L}{R} + K a_y$$

where:

K = Understeer gradient (deg/g)

a_y = Lateral acceleration (g)

The term $[W_f/C_{\alpha f} - W_r/C_{\alpha r}]$ determines the magnitude and direction of the steering input required to maintain the turn. It has two terms, each of which is the ratio of the axle load to the cornering stiffness of the tires. This is called the “understeer gradient,” denoted by the symbol K . Three possibilities for the value of K exist:

1) Neutral Steer: $W_f / C_{\alpha f} = W_r / C_{\alpha r} \rightarrow K = 0 \rightarrow \alpha_f = \alpha_r$

In this case, the balance of the vehicle is such that the force of the lateral acceleration at the CG causes an identical increase in slip angle at both front and rear wheels. No change in steering angle is required as the speed varies (in a constant radius turn).

2) Understeer: $W_f / C_{\alpha f} > W_r / C_{\alpha r} \rightarrow K > 0 \rightarrow \alpha_f > \alpha_r$

In a constant radius turn, the front wheels slip sideways more than the rear wheels. Thus, to develop the required lateral force to maintain the turn, the front wheels must be steered at a greater angle.

3) Oversteer: $W_f / C_{\alpha_f} < W_r / C_{\alpha_r} \rightarrow K < 0 \rightarrow \alpha_f < \alpha_r$

The lateral acceleration causes the slip on the rear wheels to increase more than at the front. The outward drift on the rear wheels causes the front wheels to turn inwards, thus diminishing the radius of the turn, which increases the lateral acceleration. The process continues in a feedback loop unless the steering angle is reduced in order to maintain the radius of the turn.

These definitions provide a quantitative measure of the meaning of oversteer and understeer, and a way to calculate it. You can influence understeer and oversteer behavior by changing the weight balance of the vehicle, adjusting suspension stiffness, adjusting camber geometry, and modifying the tires. You can quantify the car's steering behavior in your game and tune it to operate the way you want to.

The suspension obviously affects the cornering behavior, since it influence the load on the tires. Possible factors include the roll moment distribution, camber change and geometry, roll steer, compliance steer, and various tractive force effects.

Summary of Forces and Moments

Once you have determined all of the forces exerted by (and acting upon) the vehicle, calculating the response is simply a matter of adding up all the forces. In the lateral direction, the tire lateral forces are the only contribution. In the longitudinal direction, care must be taken to sum all aerodynamic loads, rolling resistance loads, and acceleration and braking forces. These forces can be summed and equations used to determine the accelerations acting on the vehicle. The accelerations must be integrated over time to determine vehicle velocity and position.

The yaw rate (angular velocity) of the vehicle can be determined from equation , taking care to include all the relevant tire forces. Manipulating the equation produces the following equation for angular acceleration:

$$\alpha_z = \frac{1}{I_{zz}} \left(b \left(\sum F_{yf} \right) - c \left(\sum F_{yr} \right) + \frac{t_f}{2} \left(\sum F_{xf} \right) + \frac{t_r}{2} \left(\sum F_{xr} \right) + \sum \tau_a \right)$$

The vehicle also has angular acceleration and velocity in the x (roll) and y (pitch) directions, and these can be determined in a similar fashion.

The equation for angular acceleration can be integrated over time to determine the angular velocity.

Conclusions

To accurately simulate the operation of a vehicle, you must have knowledge of the various forces acting on the vehicle. These include static and dynamic loads, acceleration and braking forces, steering forces, aerodynamic loads, and rolling resistance. Knowledge of these

loads provides a means to model cornering performance through the integration of the equations of motion.

These are simple methods taken from the field of vehicle dynamics, which can be easily used in modern driving games. With today's personal computers, there is no longer a need to rely on abstract, simplified models of vehicle dynamics. The full simulation of the vehicle's behavior is possible.

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