

Talking Generally or

Generally Talking in an Algebra Classroom

Listening to students while they problem solve can produce valuable results.

Karen Koellner, Mary Pittman, and Jeffrey Frykholm

"I am so tired of counting the cubes with different numbers of sides painted."

"Well, there is an algebraic way to do it, so we just need to count first and then we can find the pattern."

—*Eighth-grade students discuss working on the Painted Cube problem*

Have you ever had the opportunity to listen to a group of students as they venture into algebraic thinking for the first time? In this article, we share and analyze excerpts from conversations among four students. They think out loud, debate, and share ideas as they look for patterns and algebraic generalizations while solving a problem. Conversations will allow teachers to analyze common conceptions and misconceptions about students' algebraic thinking to provide insight into instructional decisions. This article will help us as teachers think about the following questions:

- How do small groups of students collectively discuss, debate, and apply strategies as they attempt to formulate generalizations and solutions for a given algebra problem?
- What is the nature of the classroom culture?
- How do problem tasks promote rich types of discussions?

Exploring these questions will allow teachers to assess students' algebraic reasoning and will help them determine the next steps in terms of instructional practice.

Among other things, algebra provides a structure and a language with which to talk about patterns. Although simply stated, the students whose quotes opened this article seem to understand the essence of what it means to generalize—to use the tools and language of algebra to articulate mathematical relationships in ways that are clear, concise, and powerful.

Karen Koellner, Karen.koellner@cudenver.edu, is associate professor of mathematics education at the University of Colorado at Denver. **Mary Pittman**, mary.pittman@bvsd.org, is the math director at Boulder Valley School District in Boulder, Colorado. **Jeffrey Frykholm** is associate professor of mathematics education at the University of Colorado at Boulder. The authors collaborated on a research project focusing on professional learning for middle school mathematics teachers, with a focus on students' algebraic reasoning.



This process of generalization may be contrasted with the typically less organized and less coherent language that students use when initially exploring a problem that involves patterns.

Generalization has become particularly important in mathematics education. Effective communication is essential as both a learning process and an outcome in mathematics. *Principles and Standards for School Mathematics* (NCTM 2000), for example, lists Communication as one of its five Process Standards, which students will need to function effectively in the twenty-first century. In the domain of algebra, communication goes beyond “sharing ideas and clarifying understanding” (NCTM 2000, p. 60) that are essential attributes in various strands of mathematics education. To reason algebraically requires some level of facility with language structure, vocabulary, and connections between ideas.

BACKGROUND: THE PROBLEM CONTEXT AND TASK

The conversation we analyze in this article occurs among four eighth-grade girls enrolled in an algebra class. These four students attend a school in which most students score “partially proficient” on state mathematics tests. The school’s population is primarily lower middle class and consists of 30 percent minority students. The teacher, Kristen Brennan, has taught for twenty-four years. At the time of this project, she was participating in a one-year professional development program about algebra that was conducted in part by the authors of this article. During the year, participants would work toward implementing, and subsequently critiquing, open-ended algebraic tasks that required students to generalize mathematical patterns. As part of her experience in the professional development program, Brennan was working toward the personal goal of improving student communication

in her classroom by focusing on group work and questioning strategies. The authors used two cameras to observe her classroom; one camera was focused on the teacher and the other on a small group of students. The conversations shared here were captured on the student camera. Other groups in this class were equally productive and used similar strategies to arrive at generalizations in this problem.

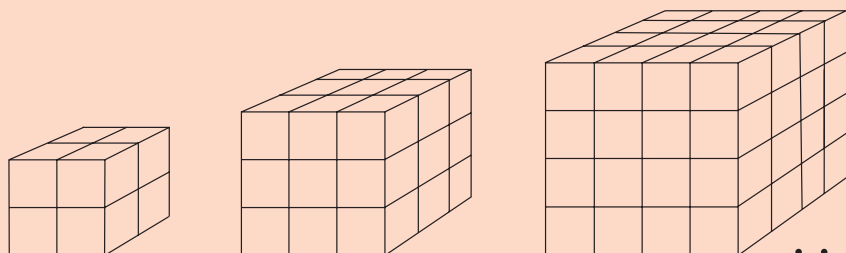
The four girls worked on the Painted Cube problem (see **fig. 1**). Throughout this article, we provide an analysis of the student conversations and pedagogical strategies that Brennan used to support student engagement and persistence in solving the task.

BEGINNING CONVERSATIONS: STARTING WITH THE SIMPLEST CASE

When the algebra class began, three of the four girls were working on the Painted Cube problem. First, they

Fig. 1 The Painted Cube problem

A cube with edges of length 2 centimeters is built from 1-centimeter cubes. If you paint the faces of this cube and then break it into 1-centimeter cubes, how many cubes will be painted on three faces? How many will be painted on two faces? On one face? How many will be unpainted? What if the edge has a length different from 2? What if the length of the cube is 3 cm? 50 cm? n cm? (Driscoll 1999, p. 20).



attempted to answer the questions about a $3 \times 3 \times 3$ cube: How many cubes would be painted on only one face? On two faces? On three faces? Brennan encouraged the students to grapple with the problem in ways that helped them make sense of what the question was asking. One potential strategy was to organize their information in a table. The conversation below shows how Jenny was able to talk through the initial participation at more than one mathematical level of sophistication and how these “levels” acted as steps in the process of formalization. Note in particular how Jenny reduces the problem to a simpler case, which ultimately unlocks the solution for the $3 \times 3 \times 3$ example.

Abby: This is the part getting painted. [*Pointing to the faces of the $3 \times 3 \times 3$ cube*]

Abby: These blocks only have two faces painted. [*Pointing to the edge cubes*] And this block has three faces painted. [*Pointing to a corner*]

Jenny: And there is like one inside in the very middle with no sides painted.

Kaitlyn: So you mean all of the corner

blocks are painted on three sides?

Jenny: [*Ignores Kaitlyn’s question and continues explaining*] These center blocks are only painted once.

Abby: And there is a block inside that doesn’t get painted at all.

Jenny constructs a $2 \times 2 \times 2$ cube and begins to point to the corners of her new model. In the $2 \times 2 \times 2$ model, all of the cubes are, in fact, corners. She begins counting the painted faces of the corners.

Jenny: It would be one, two, three . . . one, two, three . . . [*Referring to the corner cubes*]

Kaitlyn: So there would be 8 [*corners, each with three painted faces*].

Kaitlyn then moves to the $3 \times 3 \times 3$ cube. She begins counting the corner cubes, each with three painted faces.

Kaitlyn: One, 2, 3, 4, . . . 8. So that would be 8. I think it is 8 for all of them. [*All cubes, regardless of their dimensions*]

Abby: Wait. [*Looking at the models and thinking*] Oh, she’s right!

Jenny: [*Counting all 8 corners again to confirm Abby’s assertion.*] Yeah, she’s right!

Abby: This is because there are 8 corners.

This dialogue is in contrast to what we might commonly see in a collaboration between middle school students in a mathematics classroom where one person finds the answers while others follow along and copy the work. Notice the degree to which all three girls are taking part in this conversation, for example, by clarifying one another’s ideas. All girls have models, and each is counting to find and affirm the solution. We contend that the accessibility of the problem and the ability to use models are factors in the girls’ engagement.

After using the $2 \times 2 \times 2$ model to recognize that there will always be eight corner cubes that have three faces painted (the discovery of a constant function), the students began studying other blocks of the cube that have fewer than three painted faces.

In the next excerpt, note the ways in which the problem context itself invites conversation, as students share their ideas about the physical structure of the cube. This is an important consideration, given that good problem contexts allow students to use models to make sense of the numbers or concepts inherent in the problem. Note the way the girls’ conversation moves toward the idea that there must be some kind of algebraic formula to represent (and simplify) the patterns inherent in the problem.

Abby: There are 6 [*blocks*] with only one side painted.

Jenny: No, I think there are 4 [*blocks*] with one side painted.

Abby: No, let me show you: 1, 2, 3, 4, 5, and 6.

Jenny: Oh yeah, I see. [*Pointing to the top and bottom, which were the faces she forgot*]

Abby: And I think there are 12 with two sides painted.

The other group members try and find the 12 blocks. The group counts 11 blocks, with two painted faces. While sharing ideas, some confusion occurs among the group. Abby concentrates intently on the model and says that there are 12 blocks with two painted faces. The group counts the blocks and incorrectly answers 11. This counting mistake exasperates Abby. Jenny, again, expresses her belief that once they add more data to their chart, they will be able to uncover a pattern—a direct algebraic formula—that could be used to avoid having to tediously count all faces. Brennan encourages the students to look for patterns in their data. The students understand that if they can find patterns, they will not have to use physical models to count. This counting task will become exceedingly tedious as the model grows in size.

Despite their lingering confusion, mostly because of counting errors, the students have made significant progress in this initial conversation about the problem. The insights articulated at this stage are revisited as they continue to actively problem solve.

INCREMENTAL LEVELS: WORKING TOWARD GENERALIZATION

The structure of the Painted Cube problem allows for incremental levels of learning. For instance, many students can build the $3 \times 3 \times 3$ cube and use counting methods to isolate common categories of cubes (e.g., one painted face). Given that the problem requires that students find four different categories of cubes (no painted faces, one painted face, two painted faces, three painted faces), students can build confidence in their understanding of the patterns as they find small successes. Open-

ended tasks that are multifaceted and incorporate incremental levels of solutions may help students persist in problem solving. In the excerpts that follow, we also see how the girls create increasingly sophisticated solution strategies that move from simple exploration and counting to making generalizations from patterns they see. In the conversations highlighted below, the students build on their initial finding of linear and constant functions.

As background for this next excerpt, the students have just completed working with the $3 \times 3 \times 3$ and $4 \times 4 \times 4$ cubes by simply counting the number of cubes that fall into the various categories (zero painted faces, one painted face, and so on). They have recorded their findings in a table. They have just constructed a $5 \times 5 \times 5$ cube and are beginning to realize that their counting methods are proving less efficient as the size of the cube increases. Again, the nature of the problem is pushing them to start thinking about the patterns that might lead to generalized relationships.

At this point, Kaitlyn has led the group in counting the cubes. Her strategy has been accurate and relatively efficient. Abby is becoming frustrated with the counting method, and both she and Jenny have begun to turn their attention toward an algebraic formula to simplify their work. In addition, a fourth student, Nelly, has joined the group and is trying to familiarize herself with the problem.

Kaitlyn: Let's do ones first. That is the easiest. One, two, three, . . . nine. So it is 9 times 6, which is 54.

Nelly: We still count every time.

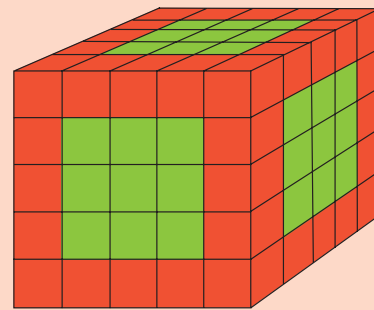
Abby: Sorry, but we can figure out something better. [*Implying that they look for a formula*]

Jenny: So, that's for one face.

Abby: Ahh. . .

Fig. 2 A $5 \times 5 \times 5$ face

This face of a $5 \times 5 \times 5$ cube illustrates the way that Kaitlyn describes the number of cubes in the center by removing the cubes on the outer layer.



Jenny: What is the “ahhh . . .” about?

Abby: There are 27 unpainted blocks [*for the $5 \times 5 \times 5$ cube*].

Nelly: How did you get that?

Abby: If you look at the side, you see there are like 3 blocks, the rest are corners. [*See fig. 2.*]

Kaitlyn: Oh yeah, I see, there are 9 and another 9 and another 9. [*Using her hands to show three layers of 9 blocks, each on the inside of the cube that has no paint*]

This brief conversation illustrates how the students use spatial reasoning to support the development of a generalization for the number of inner cubes (see **fig. 2**). Again, finding tasks that allow for a concrete visual representation is helpful as students move from concrete ideas to more generalized solutions. The visual approach (stemming from the actual physical model) helps them see and identify the cubic function. The students are able to verbalize the dimensions of the cube within the cube for the $5 \times 5 \times 5$ model, but it is not clear if they can move beyond this instance to generalize an algebraic formula. The next excerpt from the dialogue points

toward this development. We would like to emphasize how the iterative nature of the task pushes the students in this direction. In addition, Brennan has stopped by to ask this group to refer to their chart and see if they can find any patterns. After determining how many cubes had three painted faces (every cube will have 8 corners), and subsequently determining the size of the unpainted cube within the cube (no painted faces), the group loops back to the starting context to tackle the number of cubes with two painted faces. As we see below, they approach this task with a growing sense that pattern generalization may be helpful. Using their previous visual models and tables, Kaitlyn points out that increasing the dimension of the cube by 1 (e.g., going from a $3 \times 3 \times 3$ to a $4 \times 4 \times 4$) increases the number of cubes with two painted faces by 12.

Jenny: No, it's not because it goes from . . .

Abby: No, because it goes from 5 to 10. [*Referring to the $5 \times 5 \times 5$ to the $10 \times 10 \times 10$ size cubes*]

Jenny: Oh wait, if it is going [*up by*] 12, it would be 12×10 , and that is 120. So if it goes up by 12s all the way, 12×10 is 120.

Abby: But it doesn't start out at 12. It starts out at 3. So what about 2 and 1? [*Referring to the cube size. Abby thinks silently for a moment.*] It would be the number minus 2 times 12. Because 5 minus 2 is 3, and 3 times 12 is 36.

Nelly: If you figured out what 5 is, then all you do is multiply 2 to get to 10 and all you do is times by 5 because it is 50.

Abby: Let me think. If what Nelly says is true, then you do 36×2 , which is 72.

Kaitlyn: Perhaps we should try using a smaller cube like the $2 \times 2 \times 2$.

Tasks that allow visual representation help students as they move from concrete ideas to generalized solutions

At this point in the conversation, Abby has provided a generalized rule for finding the number of cubes with two painted faces. Nelly, however, is pursuing what she thinks is an easier formula based largely on the data recorded in her table. Nelly has an entry for a $5 \times 5 \times 5$ cube, followed by an entry for a $10 \times 10 \times 10$ cube; she skips the $6 \times 6 \times 6$, $7 \times 7 \times 7$, and so on, entries in between. She reasons that since 5×2 is 10, the girls can simply multiply the answers they got for the $5 \times 5 \times 5$ cube by 2 to obtain the correct numbers for the $10 \times 10 \times 10$ cube.

Jenny is not sure which conjecture is correct. Abby listens carefully to Nelly and tries out her method. Abby is unconvinced and attempts to explain why this method does not work. At the same time, Nelly tries to articulate why her method appears to work. Kaitlyn follows up on the idea she shared in the conversation. She constructs a $2 \times 2 \times 2$ cube in hopes that a smaller case will provide some insight, but because the $2 \times 2 \times 2$ cube consists entirely of corner cubes, this method proves ineffective. Without a consensus, the girls invited the teacher into the conversation about the number of cubes with two painted faces. Each member of the group was participating and justifying her ideas or expressing confusion about a particular aspect of the discussion.

In the following conversation, notice how the teacher redirects the groups of students toward more productive thinking.

Nelly: [*Asking the teacher*] We have a question on our thing. We stopped at the $10 \times 10 \times 10$ cube. Couldn't you just multiply by 2?

Brennan: Let's see if that works. How many cubes did we use in the very beginning?

Abby: Two times two was 8.

Brennan: It was a $2 \times 2 \times 2$, and we used 8 cubes. Then you did a $3 \times 3 \times 3$. How many blocks did you use then?

Abby: Twenty-seven.

Brennan: Did you have all of the 27 cubes accounted for? Let's see, there's . . . 26, 27. [*Adds the number of cubes in each column to determine the total number of cubes*] So you accounted for all the cubes. For the $4 \times 4 \times 4$, how many cubes do you have to account for?

Abby: Oh, that's how we can check it!

This excerpt shows the effective nature of Brennan's questioning. Since the class period is almost over, Brennan points the students toward a systematic method for assessing the accuracy of their work. The students confirm the accuracy of their results written in their table up to the $5 \times 5 \times 5$ entry. The conversation continues when Brennan looks at Nelly's data for the $10 \times 10 \times 10$ cube.

Brennan: Now for the $10 \times 10 \times 10$.

Jenny: That would be 1000 cubes.

Brennan: Can you account for 1000? [*The group adds silently.*] You have nothing close to 1000. Were you thinking that this was your answer for 6 or 7? [*Noting the $6 \times 6 \times 6$ entry in the table*]

Jenny: What we were thinking was since 5 is half of 10, so we figured you just multiply by 2.

Brennan: Does that work?

Jenny: No.

Brennan helped the students use reasoning to consider their solutions for the $10 \times 10 \times 10$ cube. Before leaving the group, she helps guide them toward some generalized patterns by asking them how the data in their table might be helpful in finding patterns that could be used for cubes of even larger dimensions.

Abby: [Restating her idea to Brennan] It goes up by 12 but starts at 3 not 1.

Brennan: Can you guys relate the 3 to the 12? [Pointing to the columns in her table] What about the 4 to the 24 or the 5 to the 36?

Jenny: That doesn't work if you take 12 minus 2.

Abby: No, not like that. If you take the cubes size of 3 and subtract 2 and then times 12 it works.

Jenny: Oh I think I see it. You have to subtract 2 from the start because of the corners I think.

Abby: And then 4 minus 2 times 12 is 24, and 5 minus 2 times 12 is 36. So that works!

Jenny: So, okay, what do we get for 10?

Abby: Ten minus 2 is 8 and times 12 is 96.

Jenny: Yeah! I get it; 96 for 10.

Kaitlyn: Ninety-six for how many faces painted?

Jenny: Two, so then I can do 50. You take 50 minus 2 and that is 48 and then times 12 is 576.

Abby: I think if we add all of our $5 \times 5 \times 5$ numbers up we will get the total. You know they will be accounted for.

Brennan provides enough guidance for the group to look at the table from a new perspective. Then Abby

Generalization has become particularly important in mathematics education

is able to move the group toward a generalization. The students return to consider Abby's original idea about cubes with two painted faces. Abby helps the group think through her conjecture that the number of cubes with two painted faces could be found by subtracting 2 from the length of one side of the cube, then multiplying that number by 12. (For example, a $4 \times 4 \times 4$ cube will have $(4 - 2)(12) = 24$ cubes that have two painted faces.) Jenny recognizes that one must subtract 2 from the length of the cube because 2 of the edge cubes (the corners) have three painted faces. She has used the model to construct her understanding of the algebraic formula. Kaitlyn, on the other hand, is unsure of how the algebraic formula has been derived. The group agrees with Abby, recognizing the power of this generalized formula for identifying similarly painted cubes in larger iterations of the problem.

SUMMARY: CONVERSATIONS AND GENERALIZATION

Supporting students' algebraic thinking and communication while they are working toward a generalization of mathematical relationships can be a complex endeavor. The Painted Cube problem, in particular, encouraged students to build and manipulate a model through which they could see the relationship among the model, their generalizations, and ultimately the direct formulas they discovered. As we have tried to illustrate in this

article, this kind of algebraic generalization does not emerge haphazardly. We have highlighted several ways that the task and the teacher's pedagogical decisions and prompting helped students move from arithmetic reasoning to algebraic generalization. These essential features are summarized below.

1. Algebraic problem tasks must be open ended. Students need a concrete way to represent the problem as they grapple with it and formulate solutions.
2. The task should be multifaceted and provide incremental learning opportunities for students. For example, once the students found the constant function (i.e., there will always be 8 corner cubes with three painted faces, regardless of the size of the cube), they experienced initial success that motivated them to seek out use more complicated patterns and use generalizations.
3. Teachers must provide ample opportunities for students to work together as they share ideas, strategies, and problem solve. Brennan gave her students many opportunities to work in groups to solve challenging problems. She helped them focus on strategies that would allow the group to function efficiently and productively. In her classroom, each group member is responsible for engaging in the problem and being able to articulate solution strategies.
4. Teachers must give careful thought to the kinds of questions and prompts given to students. Brennan provides guidance for her students without actually giving them the specific strategies or solutions that will unlock the problem. We see the confluence of problem-specific questions with more general pedagogical goals and decisions that allow students to travel the path toward algebraic generalization.

This article provides a close look at natural and typical conversations that a group of students might have as they use reasoning to move toward a generalization of algebraic relationships. These types of conversations are important and empowering for both students and teachers. Students, of course, become comfortable exploring patterns, detecting trends, and making generalizations. Teachers, on the other hand, may use these conversations as important windows into the thinking of their students. We contend that conversations such as those highlighted in this article allow teachers to assess students' abilities to engage, persist, and solve for generalizations individually and as group members. Abby, for example, provided leadership for her group as she verbalized her mathematical thinking. Her thoughts moved along a continuum—first counting faces of the cube, then recording her

findings, then seeking patterns in her chart. In the end, she expressed generalizations inherent in the problem. Nelly, on the other hand, had a misconception related to arithmetic that became a limiting factor in her pursuit of the problem. Recall her notion that because 5×2 is 10, they could multiply all numbers of a $5 \times 5 \times 5$ cube times 2 to find the solutions for a $10 \times 10 \times 10$ cube. This misconception may not have been found without the group interaction. Subsequently, Brennan might not have been able to pose questions to help the group self-assess so that Nelly could realize her misunderstanding.

Jenny, on the other hand, was easily swayed by Abby and Nelly throughout the process. The way she reiterated the other girls' solutions was interesting, yet she was not able to discern and make sense of the correct answers, indicating that she may not have understood the algebraic ideas. We also

observed the actions and participation of Kaitlyn who, although involved in the conversation and affirming of her peers, did not aid the group *conceptually* in its attempt to move forward.

We contend that each student played a role in the group process. Although the students may be at different levels in terms of their algebraic understanding, their conversations helped each of them progress toward a more sophisticated understanding of the specific problem, and perhaps more broadly, toward a greater understanding of the power of algebraic generalization.

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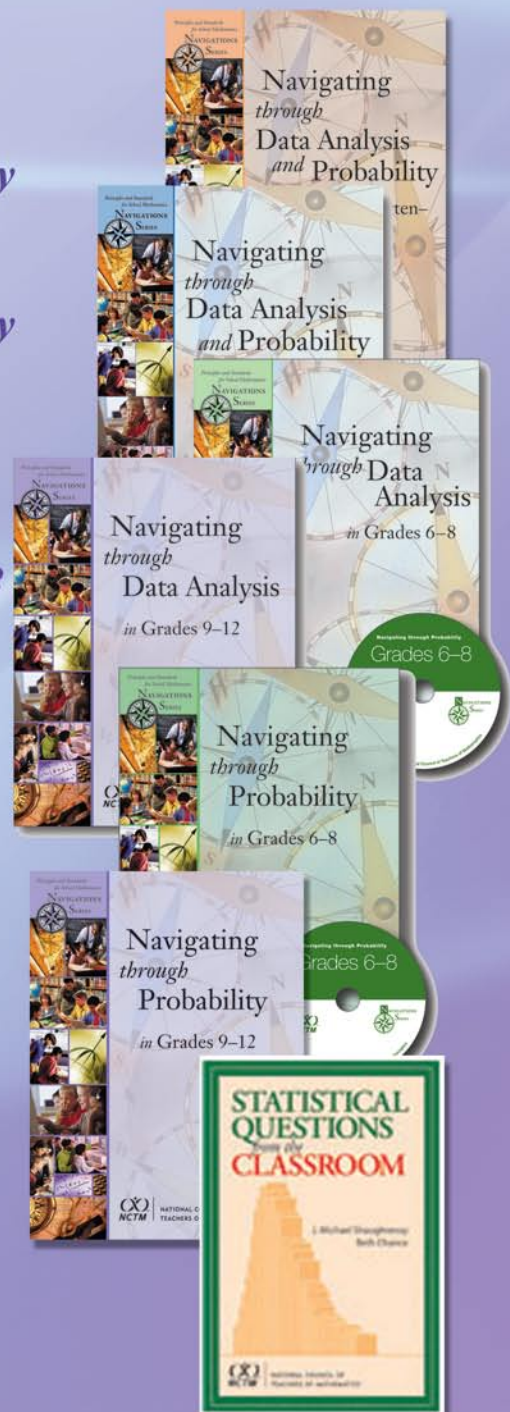
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