# Best Practice Guidelines





Edition 1 February 2019



# **Table of Contents**

About Photomath	3
Best Practice Introduction	4
Learning a New Concept	5
Mastering a New Concept	6
Practicing a Mastered Concept	8
Appendix	10



# About Photomath

Photomath is an award-winning mobile app bringing math learning to everybody's fingertips. From basic arithmetic to advanced calculus, Photomath uses state-of-the-art technology to read math problems and provide step-by-step explanations on how to approach them.

## **Our Philosophy**

Our approach is to surface the right information at the right time to learners thereby helping them process and understand math problems using a device and format with which they are already familiar. At Photomath, we believe that math is an increasingly crucial skill, particularly as problem-solving and quantitative analysis become prerequisites for most occupations. Join us in our goal to give every student math superpowers.



# Best Practice Guidelines

These guidelines aim to provide educators and students with best practices to effectively use Photomath inside and outside the classroom. We've structured them in the spirit of a gradual release of responsibility methodology in both independent and collaborative learning settings. We want every student to assume responsibility for her/his own learning and become confident in her/his math learning capabilities. Thus, this document outlines best practices for using Photomath when learning a new concept, mastering a new concept, and then practicing a mastered concept. Similarly, these three areas can also be thought of as conceptual understanding, procedural fluency, and application of the concept. Examples of this gradual release of responsibility methodology for covering one specific learning objective and using Photomath can be found in the Appendix.

Our guidelines were created with contributions and guidance from educators on the frontlines of math classrooms. In working with leading educators, we hope these guidelines will help ensure students develop a deeper understanding of math concepts through the smart use of technology. We want students to appreciate the mathematics that underlies Photomath and to become skilled users with healthy habits with the ability to mentally check the feasibility of answers. In other words, these guidelines are meant to encourage students to avoid using Photomath for solely getting an answer without understanding the process (aka "cheating").

A huge thank you to <u>John Stevens</u>, Edtech Coach at Chaffey Joint Union High School District; Kevin Hoffman, Associate Manager of Innovative Learning at Aspire Public Schools; and Nada Mihovilic, 10th grade Math Teacher at AGM School, for their invaluable input in creating this document.

If you have any feedback or additional recommendations on how to use Photomath effectively, please email us <u>hello@photomath.app</u>.

## Learning a New Concept

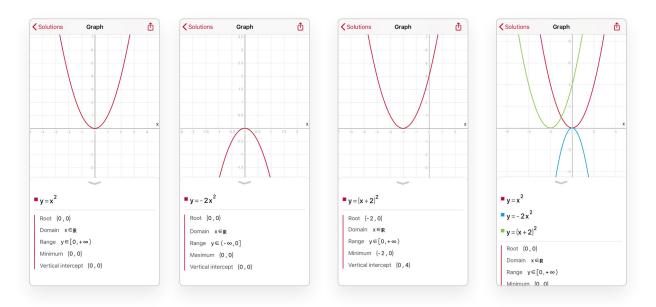
#### Best Practice 1 Explain New Concepts Using Examples through Photomath

Introduce and explain new concepts by walking through problems on Photomath. With step-by-step solutions, you can review problems ahead of time in preparation of teaching a lesson, identifying possible misconceptions, areas of difficulty, and points of clarification. In addition, you can walk through as many problems as necessary with your students, spending less time on writing steps on the chalkboard and more time discussing the why's and how's with your students.

Depending on your textbook, you can also scan problems directly from the book and walk through any number of examples. Feel free to choose whichever of the problems you think explains a concept best.

#### Best Practice 2 Developing Intuition through Visuals

Using Photomath's graphing function, teachers can demonstrate and/or ask what happens when a student adds different parts (like a constant or cosine) to a function. Photomath is one visual resource available to teachers to help develop students' intuition.



# **Mastering a New Concept**

#### Best Practice 3 Promote Mathematical Reasoning through Small Group Practice

Photomath should be used to explore math concepts so that students integrate this resource to enhance, not replace, their mathematical thinking. The emphasis on this type of exercise is not about finding an answer but developing the ability to see the structure and to appreciate the strategic approaches to problem-solving.

In small groups, have students scan problems, look at the step-by-step explanations provided, and then ask students how else they could solve the same equation.

For example, there are several ways to solve the problem below:

$$\frac{x}{5} - 7 = \frac{x}{3} - 5$$

Photomath's approach starts by multiplying both sides of the equation by 15 before collecting the variable to one side of the equation. However, students can also:

OR

Collect the variable x onto one side of the equation: x/5-x/3=2 and then multiply by 15.

Students can also rewrite the problem as (x-35)/5=(x-15)/3 and then solve by cross-multiplying.

In some cases, Photomath will provide different approaches for problem-solving directly in the app. When this functionality exists for a problem, You may select a different solving method by scrolling up the solution card.

For example, students can solve a quadratic equation using the quadratic formula, factoring, or the least squares method. Have students work in small groups and discuss which approach makes the most sense for a given problem. Students can explain their reasoning to their small group as a think-pair-share or simply present their reasoning to the class at large.





#### Best Practice 4 Check Answers in Small Group Practice

Students should be encouraged to use Photomath to check their answers to practice problems, compare their solutions, and identify any missteps in their calculations. As students work through representative problems in small groups, students should first compare and understand the strategies used by their peers, making sure to give students the responsibility to make their strategies accessible to others. Students can also compare the group's solutions to Photomath's as another or similar means of approaching the problem.

It is very important to communicate expectations for how to use Photomath during the small groups and the reasons for those expectations, particularly at the beginning of the activity. We encourage teachers to reinforce the point that Photomath is not the arbiter of the "right answer". Students should discuss if one of their solutions is more efficient, more intuitive, etc. This is an opportunity for students to assess their own understanding, develop communication and justification strategies, and make mathematical connections. Photomath can also be used as a tool for reference if the entire group gets stuck.

# **Practicing a Mastered Concept**

#### Best Practice 5 Check Answers at Home, i.e. Ensuring Students Know the Fundamentals

Students should be encouraged to use Photomath to check their answers to homework problems, compare their solutions, and identify any missteps in their calculations. For students who need extra help or to review previous concepts, Photomath can be a resource to re-work through problems from class or through additional problems at their own pace, utilizing as many examples as necessary to understand how to approach certain types of problems.

We believe in the common adage in education that there is no such thing as a true 3rd grader. Photomath can help students work at their own pace, at their own level, and at their optimal time with as many examples and practice as needed.



#### Best Practice 6 Using Photomath for Calculations

Math can require a lot of practice until one masters a concept, at which point aspects of problem-solving become more executing calculations and application of key skills and concepts. Students can use Photomath to manually practice concepts that they are still mastering. Once students have demonstrated a solid understanding of a given fundamental concept, Photomath can be a helpful resource for running calculations. In other words, students should be the ones determining "What approach is needed?" and "Why is this the needed approach?" while technology like Photomath can help with the calculations.

The College Board and its AP exams have excellent examples of questions that drive students to think about how to apply math concepts to solve real problems. However, this approach can also be used in simpler problems.

For example:

Oil is being drained out of a vat through 2 pipelines at the rate of 680 gallons/ min. One pipeline releases 50 gallons/min more than the other. How much do each of the 2 pipelines drain?

- 1. Start by asking students to draw a visual of the problem and then discuss how they would label that diagram to represent the information in the problem.
- 2. Use student discussion (in groups or with the whole class) around the visuals to recognize that this question contains 2 simultaneous equations:

$$\begin{aligned} x - y &= 50\\ x + y &= 680 \end{aligned}$$

3. From here, students can scan and solve the simultaneous equations with Photomath

# Appendix

Example Exercises for Learning, Mastering, and Practicing a Concept using Photomath

# **Quadratic Function**

 $f(x) = ax^2 + bx + c$ 

Objective: Students will be able to determine the vertex of a quadratic function and whether it is a maximum or minimum. This exercise also highlights how to use the Photomath app to help achieve this stated objective. Students can work in pairs or in groups.

#### Learning a New Concept:

Using the examples of quadratic functions given in the table, students will analyze the functions by finding intervals at which these functions are increasing or decreasing, as well as the minima or maxima of the functions. They should record their findings by completing the accompanying table.

#### Mastering a New Concept:

In the second part of the exercise, students will use the Photomath app to determine the formula of a given function and analyze the function.

#### Practicing a New Concept:

In the third part, students will model a problem mathematically and apply their conclusions concerning the behavior of the quadratic function to solve the tasks. They should check their findings using the Photomath app.

Examples of solutions, using Photomath app, are shown at the end.

### Learning How to Find the Vertex of Quadratic Functions

#### Part 1a.

Analyze the functions given in the form  $f(x) = ax^2 + bx + c$ :

	vertex $T(x_0, y_0)$	minimum/maximum $f(\underline{\qquad}) = \underline{\qquad}$	interval at which it decreases	interval at which it increases
$f(x) = 3x^2 + 18x - 1$				
$f(x) = 0.3x^2 - 6x + 2$				
$f(x) = -4x^2 - 12x + 8$				
$f(x) = -5x^2 - 20x - 8$				
$f(x) = 2x^2 - 6x + 4.5$				
$f(x) = -x^2 + x - \frac{1}{4}$				

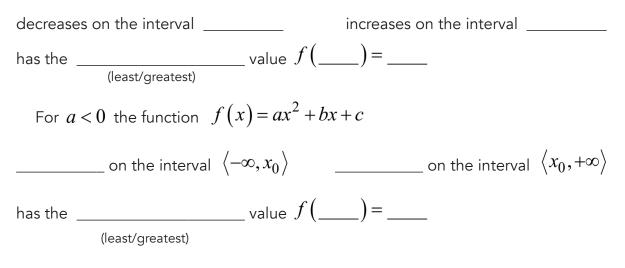
*Hint:* You can graph each function in Photomath to see a visual representation of each quadratic formula.

#### Part 1b.

#### Moving from specific examples to general concepts

The point  $T(x_0, y_0)$  is the vertex of the parabola determined by the equation  $y = ax^2 + bx + c$ . The following is true:

For 
$$a > 0$$
 the function  $f(x) = ax^2 + bx + c$ 

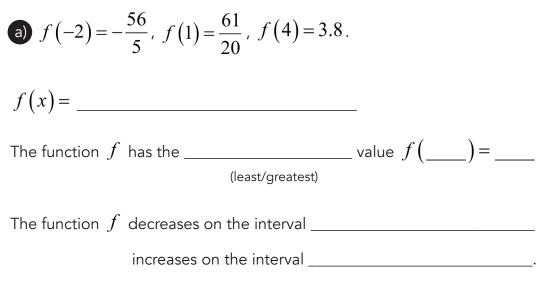




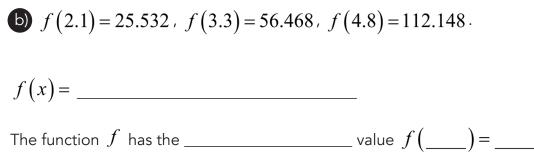
### Mastering the Quadratic Function

#### Part 2.

Determine the quadratic function in the form  $f(x) = ax^2 + bx + c$  using the expressions below:



 ${\it Hint: Solve for the values of a, b, and c using a system of equations.}$ 



(least/greatest)

## Applying the quadratic function

#### Part 3a.

#### Basic practice with quadratic function:

A manufacturer noticed that the profits for a product can be determined using the formula,  $p(x) = -7x^2 + 560x - 4680$ , where *p* represents profits in \$, and *x* is the number of manufactured products. What is the maximum profit?

#### Part 3b.

#### A challenging task to show off mastery and application of understanding $\stackrel{\smile}{\smile}$

In a very tight ending of a handball game, the two teams are tied. A second before the final whistle, the goalkeeper blocks the ball, catches the ball, and has no time to pass it to a player of her team. All the other players, including the opponents' goalkeeper are on her side of the court, so she decides to shoot at the opponents' goal. In the moment she takes her shot, the goalkeeper is 6 meters away from her goal line (aka one end of the court), and she shoots the ball from a height of 2.26 meters. The ball's trajectory is a parabola. At the height of 5.1 meters, the ball flies over a player who is 16 meters away from the goalkeeper. Finally, at the height of 4.2 meters, it flies over a player who is 9 meters from the goal line of the goal the ball is flying towards. If the handball court is 40 meters long, and the goal is 2 meters tall, will the goalkeeper score the goal and bring victory to her team?

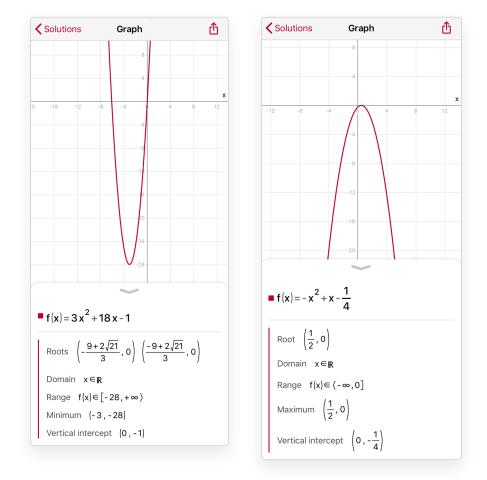
### 🔀 photomath

## SOLUTIONS

#### Part 1a Solution:

Analyze the functions given in the form  $f(x) = ax^2 + bx + c$ :

	$T\left(x_0, y_0\right)$	minimum/maximum $f(\underline{\qquad}) = \underline{\qquad}$	interval at which it decreases	interval at which it increases
$f(x) = 3x^2 + 18x - 1$	(-3,-28)	min. $f(-3) = -28$	$\langle -\infty, -3 \rangle$	$\langle -3, +\infty \rangle$
$f(x) = -x^2 + x - \frac{1}{4}$	$\left(\frac{1}{2},0\right)$	$f\left(\frac{1}{2}\right) = 0$	$\left\langle \frac{1}{2}, +\infty \right\rangle$	$\left\langle -\infty, \frac{1}{2} \right\rangle$



#### Part 1b Solution.

The point  $T(x_0, y_0)$  is the vertex of the parabola determined by the equation  $y = ax^2 + bx + c$ . The following is true:

For a > 0 the function  $f(x) = ax^2 + bx + c$ decreases on the interval  $\langle -\infty, x_0 \rangle$ increases on the interval  $\langle x_0, +\infty \rangle$ has the least value  $f(x_0) = y_0$ .

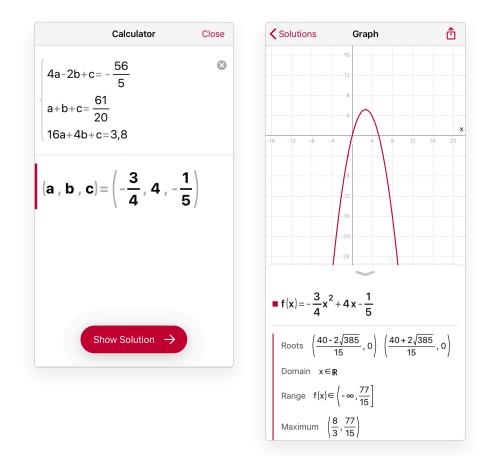
For a < 0 the function  $f(x) = ax^2 + bx + c$ increases on the interval  $\langle -\infty, x_0 \rangle$ decreases on the interval  $\langle x_0, +\infty \rangle$ has the greatest value  $f(x_0) = y_0$ 



#### Part 2 Solution:

Determine the quadratic function in the form  $f(x) = ax^2 + bx + c$  using the expressions below:

a) 
$$f(-2) = -\frac{56}{5}$$
,  $f(1) = \frac{61}{20}$ ,  $f(4) = 3.8$ .



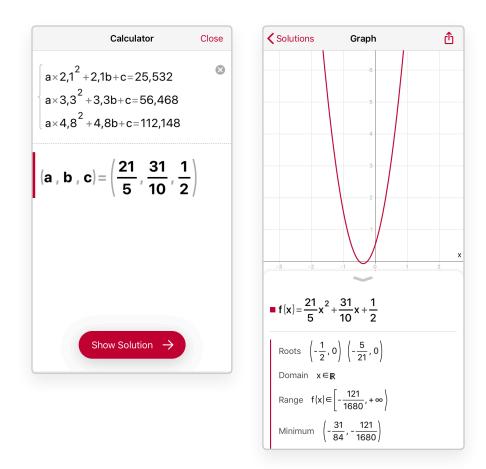
$$f(x) = -\frac{3}{4}x^2 + 4x - \frac{1}{5}$$

The function f has the greatest value  $f\left(\frac{8}{3}\right) = \frac{77}{15}$ .

The function f decreases on the interval  $\left\langle -\infty, \frac{8}{3} \right\rangle$ , and increases on the interval  $\left\langle \frac{8}{3}, +\infty \right\rangle$ .



## **b** f(2.1) = 25.532, f(3.3) = 56.468, f(4.8) = 112.148.



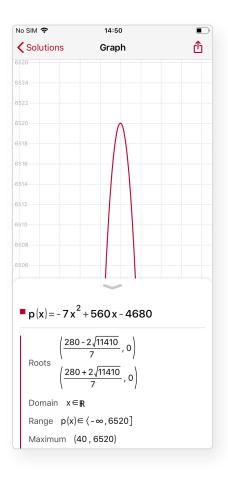
$$f(x) = \frac{21}{5}x^2 + \frac{31}{10}x + \frac{1}{2}$$
  
The function  $f$  has the least value  $f\left(-\frac{31}{84}\right) = -\frac{121}{1680}$   
The function  $f$  decreases on the interval  $\left\langle -\infty, -\frac{31}{84} \right\rangle$ , and increases on the interval  $\left\langle -\frac{31}{84}, +\infty \right\rangle$ .



#### Part 3a Solution:

#### Basic practice with quadratic formula:

A manufacturer noticed that the profits for a product can be determined using the formula,  $p(x) = -7x^2 + 560x - 4680$ , where *p* represents profits in \$, and *x* is the number of manufactured products. What is the maximum profit?



The maximum profit is \$6520.

#### Part 3b:

#### A challenging task to show off mastery and application of understanding 🙂

In a very tight ending to a handball game, the two teams are tied. A second before the final whistle, the goalkeeper blocks the ball, catches the ball, and has no time to pass it to a player of her team. All the other players, including the opponents' goalkeeper are on her side of the court, so she decides to shoot the opponents' goal. In the moment she shoots, the goalkeeper is 6 meters away from her goal line (aka one end of the court), and she shoots the ball from a height of 2.26 meters. The ball's trajectory is a parabola. At the height of 5.1 meters, the ball flies over a player who is 16 meters away from the goalkeeper. Finally, at the height of 4.2 meters, it flies over a player who is 9 meters from the goal line of the goal the ball is flying towards. If the handball court is 40 meters long, and the goal is 2 meters tall, will the goalkeeper score the goal and bring victory to her team?

First, the arc of the ball is a parabola and follows the format of the equation below, where x represents the distance to the goal line of the shooting goalkeeper and f(x) represents the height of the ball:

$$f(x) = ax^2 + bx + c$$

Next, we want to find the values of a, b, and c, using the information in the question.

- 1. At 6 meters from the first goal line, the height of the ball is 2.26 meters.
- 2. At (16 +6) meters from the first goal line, the height of the ball is 5.1 meters.
- 3. At (40-9) meters from the first goal line, the height of the ball is 4.2 meters.

$$f(6) = 2.26 \implies 36a + 6b + c = 2.26$$

 $f(22) = 5.1 \implies a \cdot 22^2 + 22b + c = 5.1$ 

$$f(31) = 4.2 \implies a \cdot 31^2 + 31b + c = 4.2$$

🔀 photomath

Input the simultaneous equations into Photomath, to calculate values of a, b, and c.

$$f(x) = -0.0111x^2 + 0.4883x - 0.2702$$

Finally, we calculate the height of the ball at the end goal line (aka the end of the court and the goal that is being shot at).

$$f(40) = 1.5018$$

No SIM 🗢	<sup>14:51</sup> Calculator	Close
$\begin{cases} y = -0,01 \\ x = 40 \end{cases}$	11x <sup>2</sup> +0,4883x-0	),2702 🙁
( <b>x</b> , <b>y</b> )=	(40 , 1,5018)	
	Show Solution $\rightarrow$	

Solutions
Solve the system of equations Using Elimination method
$\begin{cases} 36a+6b+c=2,26\\ 484a+22b+c=5,1\\ 961a+31b+c=4,2 \end{cases}$
Solution [a , b , c]=(-0,0111 , 0,4883 , -0,2702)
Show Solving Steps $\rightarrow$
Solve the system of equations
<ul> <li>Using Elimination method</li> </ul>
Using Cramer's rule

The ball crosses the goal line at a height of 1.5018 meters, so the goalkeeper will score and his team wins!