

Introduction to the Aggregate Marketing System Simulator

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Abstract

Advertising is becoming more and more complex, and there is a strong demand for measurement tools that are capable of keeping up. In tandem with new measurement problems and solutions, new capabilities for evaluating measurement methodologies are needed. Given the complex marketing environment and the multitude of analytical methods available, simulation can be an essential tool for evaluating and comparing analysis options.

This paper describes the Aggregate Marketing System Simulator (AMSS), a simulation tool capable of generating aggregate-level time series data related to marketing measurement (e.g., channel-level marketing spend, website visits, competitor spend, pricing, sales volume). It is flexible enough to model a wide variety of marketing situations that include different mixes of advertising spend, levels of ad effectiveness, types of ad targeting, sales seasonality, competitor activity, and much more. A key feature of AMSS is that it generates ground truth for marketing performance metrics, including return on advertising spend (ROAS) and marginal ROAS (mROAS). The capabilities provided by AMSS create a foundation for evaluating and improving measurement methods, including media mix models (MMMs), campaign optimization (Scott, 2015), and geo experiments (Vaver & Koehler, 2011), across complex modeling scenarios.

1 Introduction

The desire to create more effective marketing strategies drives strong demand for marketing performance measurement. Marketers need to understand the long and short term impact of media advertising, trade promotion, and other marketing tools in order to create effective tactical and strategic marketing plans. Quantitative analysis is expected to provide accurate, reliable measurement of key performance metrics and the effects of various marketing strategies.

When available, randomized experiments provide the most reliable way to measure the causal effects of advertising (Kohavi, Longbotham, Sommerfield, & Henne, 2009). Some marketing platforms provide an integrated experimental capability; Facebook and Google report incremental advertising effects measured through randomized experiments in their Conversion Lift and Brand Lift products, respectively (Google, 2011; Facebook, 2015). On the other hand, experimental capabilities in other media, such as national television, are rare. Also, while randomized experiments are well understood and offer reliable results if implemented properly, they can be expensive, impractical, or infeasible (Lewis & Rao, 2015; Kohavi et al., 2009). Measuring media effects across many channels, for example, requires complex experimental designs, significant advanced planning, and extended test periods. Accurately measuring small effects requires large sample size. Experimental designs also impose constraints on marketing flexibility. Finally, experimental studies generally provide only snapshot views of the marketing environment.

When randomized experiments are not viable, marketers must turn to other types of quantitative analysis. Many commonly used techniques, such as media mix modeling (MMM) and digital attribution, rely on either aggregate or user-level observational data (Dekimpe & Hanssens, 2000, 2010; Manchanda, Rossi, & Chintagunta, 2004). These methods generate conclusions by analyzing historical data, rather than data generated from an experiment. While historical data may be readily accessible, drawing causal conclusions from observational data is difficult, as summed up by the oft-repeated warning, “Correlation does not imply causation” (Holland, 1986; Ehrenberg & Barnard, 2000; Imbens & Rubin, 2015; Pearl, 2009; Chan & Perry, 2017). Drawing causal conclusions requires modeling assumptions concerning the nature of the marketing environment (e.g., how advertising changes user behavior, how ad channels interact, how pricing impacts sales, etc.). These assumptions will be inaccurate to varying degrees, with potentially harmful effects on the reliability of model-generated estimates. In many situations these assumptions are unstated and/or unidentified. They are also often unverified, and indeed in some cases, unverifiable. On top of this, results from observational models may be biased due to hidden (unobserved) confounding variables and issues related to data scope and data granularity (Lewis & Rao, 2015).

Both observational and experimental methods require evaluation and validation. Given the statistical issues that can affect the reliability of results derived from observational studies, it is important to have a source of truth against which the accuracy of estimates can be verified. Simulation can also be used to analyze experimental designs. A realistic ad system, which includes the complex interplay between consumers, marketing tools, and environmental phenomena, is also too complex for analytical validation. However, it is possible to represent these complexities in a simulated ad system. With simulation, it is feasible and inexpensive to consider a wide variety of scenarios for methodology evaluation and to run virtual experiments to measure marketing impact.¹ Working with simulated datasets generated by a simulated ad system, which has a marketing environment that is specified and known, allows modelers to explore statistical issues, verify model performance, and compare competing models. By simulating across a variety of reasonable market conditions, we can understand the situations under which methods do and do not work. These insights help modelers develop measurement methods that are robust to different market conditions.

The Aggregate Marketing System Simulator (AMSS) was developed to provide a mechanism for evaluating models based on observational and experimental aggregate time series data. AMSS is capable of capturing key aspects of consumer behavior, including complex purchase behavior and response to marketing techniques. It is also capable of replicating data features such as different levels of aggregation and hidden confounders. Data can be generated with varying degrees of ad system complexity to better understand the capabilities and limitations of analytical models. Furthermore, because the simulator makes it possible to run virtual experiments, it also provides ground truth for the evaluation of any measurement approach grounded in the analysis of aggregate data.

This paper begins with a detailed description of AMSS’s design and data generation methodology. We follow up by showing how simulated data can be used in model evaluation with an example application to media mix modeling.

¹A virtual experiment is implemented in the virtual world of the simulation rather than in the real world.

2 Data generating model

AMSS is designed to generate the aggregate time-series data resulting from natural consumer behavior and changes in this behavior due to marketing interventions. It does this by segmenting the consumer population into distinct groups based on several key features that characterize the consumer’s relationship with the category and the brand. Consumers in different segments have different media consumption patterns, responses to advertising, purchase behavior, etc. Over the course of time, a consumer’s relationship with the category and/or the brand may change in response to uncontrolled forces (e.g., seasonality and competitive activity) as well as advertiser-controlled marketing interventions. These changes are reflected in the simulation by the migration of consumers to segments that reflect their new mindsets. The changes in population segmentation then lead to changes in the aggregate behavior of the population. For example, marketing interventions increase advertiser sales by moving consumers to more favorable segments, which correspond to higher probabilities of making purchases in the category and/or purchasing the advertiser brand.

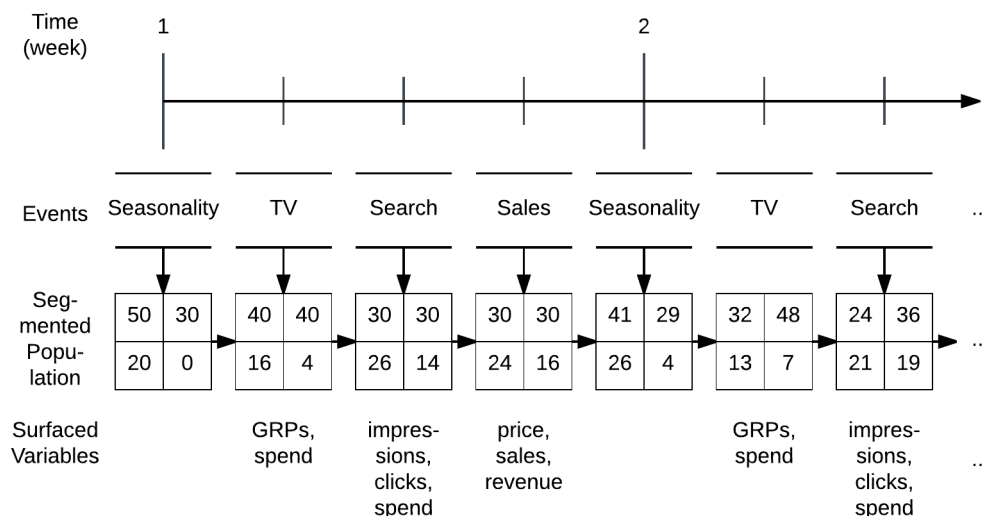


Figure 1: Overview of simulator structure

The process of modeling population migrations and generating observable variables is illustrated in Figure 1. This figure shows a simplified scenario in which the simulator tracks a population of one hundred individuals divided into four segments.² The various market forces driving consumer behavior are conceptualized as an ordered sequence of events that repeat across time intervals. Each event corresponds to a specific non-actionable or actionable force with its own impact on consumer disposition and behavior. Changes in consumer disposition are reflected by the migration of consumers from pre-event to post-event population segmentations. Also, each event generates observable data. In particular, Figure 1 depicts a scenario with four repeating events: seasonality, television, paid search, and sales. The ‘seasonality’ event reflects non-actionable forces that change the size of the market over time, and drives consumer migrations in and out of the market for the

²In Section 2.1, we discuss the segments implemented in AMSS. For now, we forgo this specification and think of these as generic indicators of behavioral tendencies (e.g., uninterested in the brand/category, investigating the category, ready to purchase).

category. Television and paid search are marketing interventions controlled by the advertiser. They drive migrations that are generally favorable to the advertiser, and they also generate related data such as the media volume and spend. The last event is the ‘sales’ event, which generates advertiser and competitor sales. It may also drive migrations; for example, a consumer’s loyalty to, or opinion of, the advertiser brand may change as part of a post-purchase evaluation process.

Due to the sequential nature of AMSS, each event impacts all subsequent events through changes to the population segmentation. This naturally supports the modeling of interactions between different marketing forces (e.g., television advertisements can encourage brand queries, thereby increasing the volume of search inventory) and is critical to accurately representing the complex relationships that can exist within a marketing system.

AMSS provides flexibility in the set of events, their design, and their sequencing. Modelers have the freedom to add or remove specific market forces from the simulation and to change event specifications and sequencing to reflect different models of consumer behavior. Examining the performance of analytical methods over a wide range of scenarios allows modelers to find and design methodologies that are robust to a wide range of statistical issues.

The sections below describe the simulation process in more detail. We first discuss how the population is segmented in Section 2.1. Section 2.2 focuses on how the migration of consumers between different segments reflects the impact of different market forces on the consumer mindset. Section 2.3 describes the models used for certain key marketing interventions in more detail; this includes a model for simulating the behavior of a traditional media channel and a model for paid search. The sales event is described in Section 2.4.

2.1 Population segmentation

AMSS conceptualizes the consumer mindset, with regard to both the category and the advertiser brand, as a discrete hidden variable; it then uses consumer mindset to segment the population. Between segments, differences in consumer mindset lead to differences in all aspects of consumer behavior, from media consumption and response to purchase behavior. The aggregate behavior of the population is determined by the size of each segment. For example, if a high proportion of consumers belong to segments corresponding to high brand loyalty, the advertiser brand will have high market share.

2.1.1 Features of the consumer mindset

The population is segmented in several dimensions, based on common concepts in modeling consumer behavior (Sharp, 2010; Millward Brown, 2009). In total, AMSS segments the population along six dimensions, each tracking a particular aspect of the consumer’s relationship with the category or the brand. The first three dimensions track the consumer’s relationship with the category and are referred to as ‘category states.’ The last three track the consumer’s relationship with the advertiser brand and are referred to as ‘brand states.’ Let $\mathbf{s} = (s_1, s_2, \dots, s_6)^\top$ be a population segment, with each s_l describing the consumer mindset in the l -th dimension. Let \mathcal{S}_l be the set of values the consumer mindset may take in the l -th dimension, so that $s_l \in \mathcal{S}_l$ for all $l \in \{1, \dots, 6\}$. The six category and brand states and the corresponding \mathcal{S}_l are listed in Table 1. More details on the meaning and usage of each category and brand state follow below.

State type l	Potential values \mathcal{S}_l
Category	
Market ($l = 1$)	in-market, out-of-market
Satiation ($l = 2$)	satiated, unsatiated
Activity ($l = 3$)	inactive, exploratory, purchase
Brand	
Favorability ($l = 4$)	unaware, unfavorable, neutral, somewhat favorable, favorable
Loyalty ($l = 5$)	switcher, loyal, competitor-loyal
Availability ($l = 6$)	low, average, high

Table 1: Individual category and brand states, and their potential values

Market state. The market state specifies whether members of the population should be considered part of the pool of potential customers for the category. This allows us to label part of the population as being entirely uninterested in the category, even under the most purchase-friendly conditions (e.g., high marketing pressure, low pricing). As an example, the market for a medication treating hypertension consists of only those individuals who have been diagnosed with the condition. Those not suffering from hypertension would never consider purchasing, regardless of any marketing interventions.

AMSS splits the population into ‘in-market’ and ‘out-of-market’ individuals. The number of individuals in the market for a certain category of goods can change over time. Some changes are seasonal, i.e., they repeat in a regular pattern, say once a year. For example, the travel category has an annual seasonality that responds to the school year, national holidays, weather patterns, etc. There can also be more general trends that affect ‘in-market’ population. Examples include the rising adoption of smartphones leads to a growing market for apps, the effect of economic factors on luxury goods, and the effect of gasoline prices on SUV sales. AMSS allows the modeler to specify both seasonal patterns and more general trends in the rate of market participation. See Section 2.2.1 for more details.

Satiation state. The satiation state specifies whether a person’s demand for the category has been satisfied by a past purchase. In AMSS, consumers are either ‘satiated’ or ‘un-satiated.’ Un-satiated individuals may become satiated after a purchase in the category; satiated individuals will eventually become un-satiated over time. Tracking satiation allows AMSS to model effects such as dips in sales following price-promotions. Both the advertiser’s own promotions and competitor promotions can create depressed demand in succeeding weeks, since this is a category-wide effect. In the real world, the time it takes satiation effects to wear off depends on the category. In categories where demand cycles quickly, such as perishable foods, satiation fades quickly. It fades more slowly in categories with longer intervals between purchases, such as travel or durable goods categories. In the simulation, the rate of decay is part of the model specification (see Section 2.2.2).

Activity state. The activity state tracks the consumer’s location along the path to purchase. AMSS segments the population into three activity states: ‘inactive,’ ‘exploratory,’ and ‘purchase.’ Consumers in different activity states have different media consumption behaviors, different responses to marketing, and different purchase behavior. Figure 2 shows an example of the actions a consumer might perform, and the marketing activity that might occur, in each activity state.

‘Inactive’ individuals are not currently engaged in any activities related to the category. Certainly,

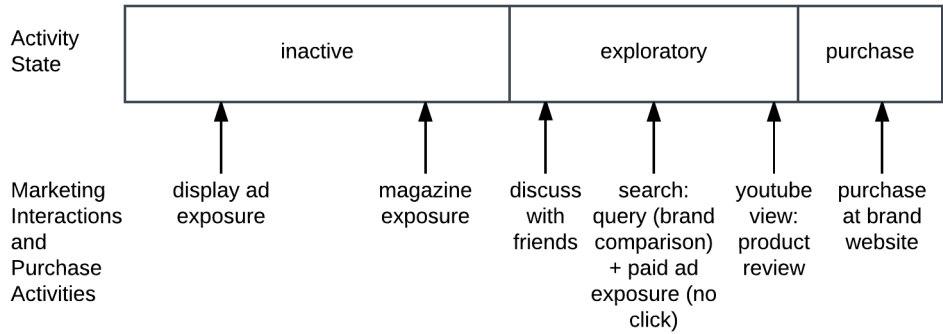


Figure 2: Example user actions at different activity states

they do not make purchases in the category. They also do not show any observable interest in the category, say by making category-related search queries or online site visits. ‘Exploratory’ individuals are considering making a purchase, and conducting related activities to help them make their decision. For example, the ‘exploratory’ population may make generic and branded search queries as part of the decision-making process. They still have not decided to make a purchase in the category. Individuals who reach the ‘purchase’ state are in the process of making a purchase. This may be at a brand-specific location such as the brand website, or a non-specific location such as a department store, grocery store, fare aggregator. The brand chosen by individuals who are in the process of making a purchase depends on factors such as brand favorability, brand loyalty, brand availability, and price at the point of purchase.

Tracking the consumer’s activity state allows the simulator to differentiate marketing tools that target different audiences, both in terms of reach and precision. Mass marketing media formats such as television and radio will reach consumers in a wide range of activity states, while other media like paid search will tend to target a smaller number of consumers further along the path to purchase. Tracking activity state also allows the simulator to follow natural and marketing-influenced progress along the path to purchase. It is necessary for the consumer to reach the ‘purchase’ state in order for the advertiser to make a sale.

Brand favorability state. Brand favorability measures a consumer’s opinion of the brand. Generally, high levels of brand favorability correspond to higher probabilities of purchase of the advertiser’s brand. In AMSS, consumers are segmented into five groups based on brand favorability: ‘unaware,’ ‘unfavorable,’ ‘neutral,’ ‘somewhat favorable,’ and ‘favorable’ consumers. Marketing tools may increase brand sales by increasing brand favorability. Note that brand favorability can be high for multiple brands simultaneously, so high brand favorability does not automatically imply brand purchase or brand loyalty. For further details on how brand states, including brand favorability affect consumer purchase probabilities, refer to Section 2.4

Brand loyalty state. Brand loyalty is another key aspect of consumer behavior (Sharp, 2010, ch. 7). Consumers can be loyal to the advertiser’s brand, loyal to a competitor, or have divided loyalties. AMSS tracks brand loyalty through the brand loyalty state; consumers have state ‘loyal,’ ‘competitor-loyal,’ or ‘switcher.’ It is important to differentiate between brand favorability and brand loyalty. Brand loyalty is exclusive, as opposed to brand favorability; consumers can have highly favorable opinions of multiple brands. Consumers loyal to a particular brand have low

probability of buying from a competitor. Consumers that are loyal neither to the advertiser nor to its competitors are labeled as ‘switchers’; at time of purchase, these consumers choose from multiple brands based on price, convenience, and other factors.

Brand availability state. Brand availability refers to the physical and mental availability of the advertiser brand to the consumer (Sharp, 2010, ch. 12). In other words, brand availability is how physically or mentally easy the brand is to buy. Brand availability is physically affected by brand distribution, i.e., the presence of the brand at retail locations. Certain marketing interventions grab consumer attention for the brand or increase its convenience; these increase the mental availability of the brand. For example, point-of-purchase displays bring the advertised brand to the attention of the consumer through prominent placement at the point of sale. In the online space, search ads bring brands greater prominence on the search results page, increasing the brand’s mental availability. This can be key in drawing sales from ‘switchers’ with no strong brand preference.

In AMSS, brand availability can be ‘low,’ ‘average,’ or ‘high’ for each consumer. Consider, for example, modeling the impact of physical distribution on sales of breakfast cereal. If a brand of cereal is available at only 70% of grocery and convenience stores, brand availability should be ‘average’ for the 70% of consumers purchasing breakfast cereal from stores carrying the brand, and ‘low’ for the other 30% of the population. Efforts to increase distribution increase brand availability for some consumers from ‘low’ to ‘average.’ In-store displays increase brand availability for some consumers from ‘average’ to ‘high.’

2.1.2 Full segmentation

The set of all segments is \mathcal{S} , a subset of the Cartesian product $\mathcal{S}_1 \times \dots \times \mathcal{S}_6$ representing all combinations of the individual features of consumer mindset. Two rules are used to restrict \mathcal{S} to a total of 198 segments. First, only consumers that are both ‘in-market’ and ‘un-satiated’ may move out of the ‘inactive’ activity state. This enforces the concept that ‘out-of-market’ or ‘satiated’ individuals have no interest in making a purchase in the category and will not engage in purchase-related activities. As a result, there are 6 valid category segments $(s_1, s_2, s_3)^\top$, and these are listed in Table 2. Second, only consumers with ‘favorable’ brand favorability can be loyal to the advertiser. The 33 resulting brand segments $(s_4, s_5, s_6)^\top$ are listed in Table 3.

Market	Satiation	Activity
out-of-market	satiated	inactive
out-of-market	unsatiated	inactive
in-market	satiated	inactive
in-market	unsatiated	inactive
in-market	unsatiated	exploratory
in-market	unsatiated	purchase

Table 2: The six category segments.

2.2 Population Migration

Consumer mindsets change over time, due to the impact of various market forces. AMSS tracks the population associated with each of the segments $\mathbf{s} \in \mathcal{S}$ described in Section 2.1.

Favorability	Loyalty		Availability
unaware	switcher		low
unfavorable	switcher		low
neutral	switcher		low
somewhat favorable	switcher		low
favorable	switcher		low
favorable	loyal	×	average
unaware	competitor-loyal		high
unfavorable	competitor-loyal		high
neutral	competitor-loyal		high
somewhat favorable	competitor-loyal		high
favorable	competitor-loyal		high

Table 3: The 33 brand segments, from a cross of 11 brand favorability/loyalty states and 3 brand availability segments.

Recall from Figure 1, which graphically depicts the simulator structure, that a sequence of ordered events drives migration between segments. The modeler specifies an ordered sequence of events $k = 1, 2, \dots, K$ that is applied once within each time interval $t \in \{1, 2, \dots, T\}$. Let the size of the population assigned to a segment $\mathbf{s} \in \mathcal{S}$ before the k -th event of time interval t be $n_{t,k,\mathbf{s}}$. Also, we can consider groupings of segments. Thus for $\mathcal{A} \subseteq \mathcal{S}$, let $n_{t,k,\mathcal{A}} = \sum_{\mathbf{s} \in \mathcal{A}} n_{t,k,\mathbf{s}}$. The overall segmentation of the population at time t is denoted by the vector $\mathbf{n}_{t,k,\cdot} = (n_{t,k,\mathbf{s}})_{\mathbf{s} \in \mathcal{S}}$. The k -th event drives an update in the segmentation from $\mathbf{n}_{t,k,\cdot}$ to $\mathbf{n}_{t,k+1,\cdot}$, reflecting a change in consumer mindset. The changing consumer mindset affects observable consumer behavior, such as media consumption, product investigation, and purchase behavior. Flexibility in the design and ordering of the events allows AMSS to accommodate a variety of marketing environments.

Consumer migration is probabilistic and controlled by a sequence of transition matrices. The k -th event of the t -th time interval affects a population of size $\mathbf{a}_{t,k,\cdot}$, where $\mathbf{a}_{t,k,\mathbf{s}} \leq n_{t,k,\mathbf{s}}$ for all $\mathbf{s} \in \mathcal{S}$. Individuals in the affected population migrate between population segments according to a transition matrix $Q^{(t,k)} = (q_{\mathbf{s},\mathbf{s}'}^{(t,k)})_{\mathcal{S} \times \mathcal{S}}$, where $q_{\mathbf{s},\mathbf{s}'}^{(t,k)}$ is the probability of migration from segment \mathbf{s} to segment \mathbf{s}' . The size of the affected population in each segment, $\mathbf{a}_{t,k,\cdot}$, and the migration probabilities, $q_{\mathbf{s},\mathbf{s}'}^{(t,k)}$, are determined by the pre-event population segmentation, $\mathbf{n}_{t,k,\cdot}$, and the event specifications. The migration of individuals during the k -th event of the t -th time interval can be expressed as

$$\mathbf{m}_{t,k,\mathbf{s},\cdot} = (m_{t,k,\mathbf{s},\mathbf{s}'})_{\mathbf{s}' \in \mathcal{S}} \sim \text{Multinomial}(\mathbf{a}_{t,k,\mathbf{s}}, \mathbf{q}_{\mathbf{s},\cdot}^{(t,k)} = (q_{\mathbf{s},\mathbf{s}'}^{(t,k)})_{\mathbf{s}' \in \mathcal{S}}), \quad (1)$$

where $m_{t,k,\mathbf{s},\mathbf{s}'}$ is the number of people migrating from segment \mathbf{s} to segment \mathbf{s}' . The post-event population segmentation is thus

$$n_{t,k+1,\mathbf{s}'} = n_{t,k,\mathbf{s}'} - a_{t,k,\mathbf{s}'} + \sum_{\mathbf{s} \in \mathcal{S}} m_{t,k,\mathbf{s},\mathbf{s}'}, \quad \mathbf{s}' \in \mathcal{S}. \quad (2)$$

There are many different kinds of events, each driving consumer migration in its own particular way. For instance, even in the absence of marketing intervention, the disposition of consumers toward a category and brand will vary and evolve over time. Some events reflect these natural forces that drive changes in the consumer mindset. Other events reflect the effects of marketing interventions, such as media advertising. The action of making a purchase in the category can

also change the consumer mindset—for example, the consumer can become satiated or loyal. We discuss how consumers migrate between segments according to specific forces below.

2.2.1 Changing market size

The size of the market for a particular category of goods changes naturally over time. For example, the market size for a travel category such as vacation cruises is impacted by consumers’ leisure time and discretionary income. Variation in the market due to annual holiday and vacation schedules is seasonal. The travel market also responds to changes in the economy, which impact consumers’ discretionary income.

The natural migration of population between ‘in-market’ and ‘out-of-market’ segments accounts for both seasonal changes and more general trends in the population’s interest in the category. It is the first event within each time interval, i.e., $k = 1$. Thus the natural forces determine a starting point, with marketing interventions building on the resulting baseline. When natural migration brings a large number of people into ‘in-market’ segments, marketing forces have more opportunities to encourage brand purchases; periods with low market participation limit the effectiveness of marketing.

We define the market rate to be the proportion of the population with state ‘in-market.’ Let $\mathcal{A} = \{\mathbf{s} \in \mathcal{S} : s_1 = \text{‘in-market’}\}$ be the set of segments where the population is ‘in-market.’ The market rate before the k -th event of the t -th time interval is

$$r_{t,k} = n_{t,k,\mathcal{A}}/n_{t,k,\mathcal{S}}.$$

The simulator allows the modeler to specify target market rates $\boldsymbol{\rho} = (\rho_t)_{1:T}$ for each time interval. The target market rate is meant to reflect the modeler’s understanding of seasonal and nonseasonal trends in the size of the category. For convenience, the specification is split into a seasonal market rate $\boldsymbol{\rho}^{(seas)}$ and a trend $\boldsymbol{\rho}^{(trend)}$, such that $\rho_t = \rho_t^{(seas)} \cdot \rho_t^{(trend)}$. AMSS attempts to attain the desired market rate with minimal migration. If $r_{t,1} > \rho_t$, individuals in ‘in-market’ segments \mathcal{A} migrate to ‘out-of-market’ segments $\mathcal{S} \setminus \mathcal{A}$ with probability $(r_{t,1} - \rho_t)/r_{t,1}$, so that the expected post-migration market rate is

$$E[r_{t,2}] = r_{t,1} * \left[1 - \frac{r_{t,1} - \rho_t}{r_{t,1}} \right] = \rho_t.$$

Similarly, if $r_{t,1} < \rho_t$, individuals migrate from ‘out-of-market’ segments $\mathcal{S} \setminus \mathcal{A}$ to ‘in-market’ segments \mathcal{A} with probability $(\rho_t - r_{t,1})/(1 - r_{t,1})$. Then,

$$E[r_{t,2}] = r_{t,1} + (1 - r_{t,1}) * \frac{\rho_t - r_{t,1}}{1 - r_{t,1}} = \rho_t.$$

If newly ‘in-market’ consumers are ‘un-satiated,’ they may also change activity state. For more details on this migration, refer to Appendix B.1.

2.2.2 De-satiation

Consumers become satiated and no longer open to further purchases after making a purchase in a category; this occurs during the sales event (see Section 2.2.5). The concept of satiation makes it

possible, for example, to reproduce dips in sales following big price promotions. Following satiation, consumers will eventually de-satiate and once more become open to purchasing in the category. The migration of consumers from satiated to un-satiated segments is the second event of each time interval, i.e., $k = 2$. De-satiation is controlled by a parameter specifying the de-satiation rate λ , which is the probability that satiated individuals become un-satiated. If newly ‘un-satiated’ consumers are ‘in-market,’ they may also change activity state. For details, see Appendix B.2.

2.2.3 Marketing interventions

Marketing interventions attempt to influence consumer mindsets and drive brand sales; in AMSS, this is equivalent to driving population migration between segments. Generally, marketing is expected to move individuals from less favorable to more favorable states, and thus increase the total number of purchasers in the category and the proportion purchasing the advertiser brand rather than a competitor’s. Marketing can drive changes in activity state, brand favorability, brand loyalty, and brand availability. The other two states, marketing state and satiation state, cannot be affected by marketing interventions. Individuals that are ‘out-of-market’ and/or ‘satiated’ are limited to the activity state of ‘inactive.’ Thus, high levels of satiation or a small market size will reduce the number of activity state migrations driven by a media channel and prevent increasing brand interest from generating purchases.

The efficacy of a particular marketing intervention in driving population migration depends on the pre-intervention population segmentation $\mathbf{n}_{t,k}$, and the nature of the intervention itself. For example, the amount of migration driven by a media channel like television in a particular week depends on the size of the television audience, that audience’s frequency of ad exposure, and the efficacy of the ad campaign. More details on how AMSS models particular types of media channels can be found in Section 2.3.

2.2.4 Lagged Effects

Marketing activity can shift consumers into more desirable population segments (e.g., segments with higher brand favorability). Although these shifts are not permanent, they may persist beyond the immediate marketing experience and lead to a lagged increase in sales. In the simulation, lagged impact is controlled by the pressure of natural migrations that, in the absence of persistent marketing activity, push the allocation of population across segments toward a natural equilibrium. The stronger the pressure toward the natural equilibrium, the more short-lived the impact of marketing interventions.

Natural migration in each state that may be affected by marketing activity—activity state ($l = 3$), brand favorability state ($l = 4$), brand availability state ($l = 5$), and brand loyalty state ($l = 6$)—is controlled by a separate event $k = 3, 4, 5, 6$. Natural migration for the l -th state is specified by a transition matrix $\mathcal{Q}^{(l)}$, which determines both the natural equilibrium for that state and the strength of the pressure toward that equilibrium. Appendix B.3 provides more detail on the migration process for activity state. The related processes for other dimensions are completely analogous. Subsection B.3.1 focuses on the relationship between the pressure of natural migration and lagged effects.

2.2.5 Post-purchase migration

Each time interval finishes with the K -th event, the sales event.³ In addition to generating brand and competitor sales, the sales event drives population migration, since consumer mindsets sometimes change as a result of a purchase. In particular, the sales event has impact on brand favorability, brand loyalty, and satiation.

Following a successful purchase, consumers may become habituated to or develop a strong preference for a particular brand. During the sales event, some consumers will make a purchase in the category, from either the advertiser or competitor brands. Depending on the consumer’s satisfaction with the product, his or her brand favorability and/or brand loyalty may change. More detail on these migrations can be found in Section 2.4.

In addition to changes in consumer loyalty and brand favorability, all purchasers in the category become temporarily satiated. This satiation effect will decay in future time intervals according to the rate of de-satiation (see Section 2.2.2).

2.3 Modeling specific marketing interventions

One of the key features of AMSS is its ability to simulate the combined effects of multiple marketing interventions on a single consumer population, with each marketing intervention having its own characteristic behavior. For example, when simulating the effect of media advertising, it is possible to specify how advertising is implemented (e.g., target audience), how advertising works (e.g., increase category and product awareness), and how ad channels interact with one another (ad channel synergy).

AMSS uses a separate event to simulate the behavior (e.g., the channel’s audience targeting and ad effectiveness) of each ad channel, and then sequences these events into a custom simulation scenario. Each event is a function that takes the current population segmentation as input and returns an updated population segmentation, along with related output variables. Example output variables include media spend and media volume. Specification of events is standardized by ‘media modules,’ which provide a flexible framework for specifying interrelated media variables such as spend and volume, and their effect on the population segmentation. A particular media channel’s behavior is specified by the parameterization of an appropriate media module. For example, the paid search module specifies the manner in which query volume, paid impressions, paid clicks, and search spend are generated. Different parameterizations specifying the query rates and click-through rates for each population segment can be used to create separate branded and generic search events from the same module. We describe two example ‘media modules’ below.

2.3.1 Traditional media module

The traditional media module is sufficient for simulating the key features of a relatively simple media channel such as television. Modelers using the module can specify the values of parameters controlling the media channel’s audience size and composition, the media volume and spend, and the media effectiveness, according to their simulation needs.

³This is the sales event from the perspective of the advertiser, or the purchase event from the perspective of the consumer.

Figure 3 depicts the structure of the traditional media module. This module can be used to simulate the behavior of a traditional media format such as television or radio. This module has

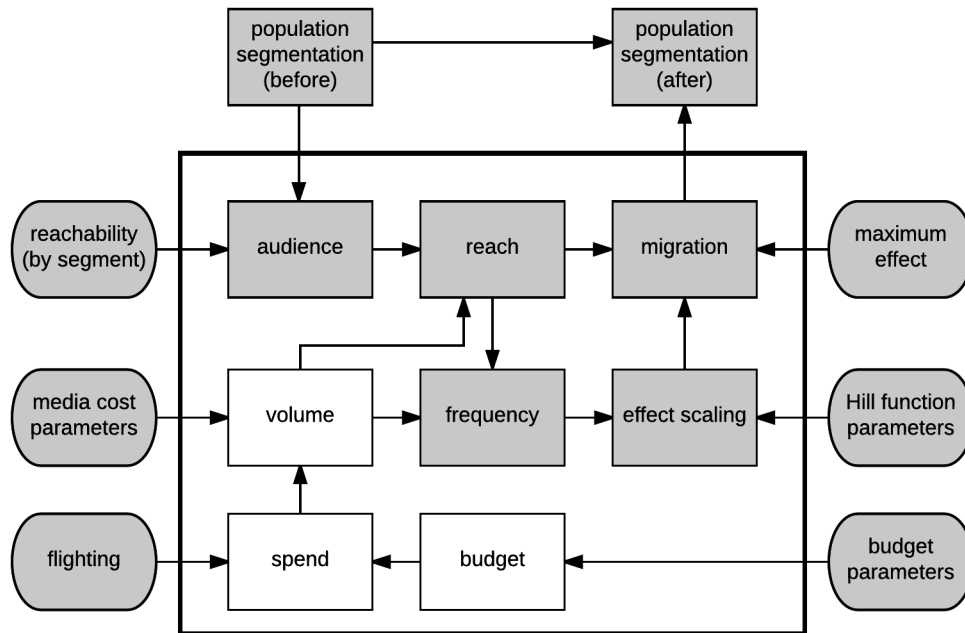


Figure 3: Design for the traditional media module. This figure depicts the dependencies between the modeler-specified input parameters (ovals) and the hidden (shaded) and observed (unshaded) variables (rectangles) generated by the module.

four main components: audience, spend, volume, and effect. Some of the variables generated by this module, such as the weekly spend, are observed, while others are hidden. In particular, the population migration caused by the media is calculated as the ‘effect’ of the media module, and this information is not typically available in a real-world analysis, such as an MMM study. Below, we describe how each of these components is calculated in the traditional media module. Further technical details can be found in Appendix C.1.

1. *Calculate the media audience.*

Each media channel has its own audience, i.e., the population that interacts with the media. This audience is the maximum population reachable through advertising in the media channel. Let a population segment’s *reachability* be the probability a consumer from the segment is part of the media channel’s audience during the current time interval. Heterogeneity in reachability across population segments reflects the media channel’s ability to target consumers with different levels of interest in the category and/or the brand. Mass marketing tools such as television will reach broad segments of the population, with very coarse targeting. More targeted media reach individuals with a prior interest in the category and/or the brand at higher rates.

In AMSS, the modeler specifies the reachability, $\pi_s^{(a)}$, of each population segment. Then the audience size, $a_{t,k,s}^*$ is calculated based on the current size of the segment, $n_{t,k,s}$, and the

reachability, $\pi_s^{(a)}$:

$$a_{t,k,s}^* \sim \text{Binomial}(n_{t,k,s}, \pi_s^{(a)}).$$

2. *Calculate the weekly spend.*

The weekly spend is calculated from the media channel’s budget and flying pattern. In the simulation settings, the time intervals are divided into groups called *budget periods*, and each budget period is given a target spend called the *budget*. The budget assigned to a budget period, say a year, is one of the main levers an advertiser can use to control their media plan. The budget is split into a weekly spend based on the flying pattern, which specifies the proportion of the budget to assign to each week in the budget period. To illustrate, suppose the modeler specifies a budget of \$100 to be spent over a 4-week budget period, with flying pattern (0.20, 0.00, 0.65, 0.15). Then the advertiser would spend \$20 the first week, \$0 the second, \$65 the third, and \$15 the fourth.

3. *Calculate media volume.*

The volume $v_{t,k,s}$, i.e., the total number of exposures to media k for consumers in segment s during time interval t , is calculated from the weekly spend based on a cost function. The simplest cost function is a unit cost per exposure c_t ; we allow for variability in the weekly unit cost through mean and variance parameters.

4. *Reach and frequency.*

The *reach* $a_{t,k,s}$ over a population segment s is defined as the number of consumers in the segment who are exposed to the advertiser’s ads in the k -th media channel at least once during the t -th time interval. It is calculated based on the audience size $a_{t,k,s}^*$ and the total volume of exposures for the segment $v_{t,k,s}$. This is accomplished via a normal approximation based on the assumption that ad exposures for each individual occur as independent Poisson processes. The *average frequency* is the average number of ad exposures among consumers with at least one ad exposure. It is calculated as

$$f_{t,k,s} = v_{t,k,s} / a_{t,k,s}.$$

5. *Update the population segmentation.*

The amount of migration in population segmentation driven by the k -th media channel during the t -th time interval depends on both the reach and frequency; the former determines the affected population (those with the potential to migrate) and the latter determines their migration probabilities.

Only consumers exposed to advertising have the potential to migrate to new population segments; thus the affected population in each segment is the reach $a_{t,k,s}$.

Migration probabilities depend on the frequency of ad exposure and the effectiveness of those ads. At high frequencies, consumers in the media audience will migrate between segments according to a transition matrix $\mathcal{Q}^{(k)}$ specifying maximal migration probabilities $\varphi_{s,s'}^{(k)}$ between segments $s \neq s'$. To reduce the parameter space, $\mathcal{Q}^{(k)}$ is defined as the product of successive transitions in each dimension affected by marketing ($l = 3, 4, 5, 6$). Transitions in the l -th dimension are specified by a matrix

$$\mathcal{Q}^{(k,l)} = \left(\varphi_{i,j}^{(k,l)} \right)_{S_l \times S_l}.$$

For example, transitions in activity state are determined by $\mathbf{Q}^{(k,3)}$. To specify a 20% chance of converting ‘inactive’ individuals to the activity state of ‘exploratory,’ and a 10% chance of converting them to an activity state of ‘purchase,’ a modeler would set the first row of $\mathbf{Q}^{(k,3)}$ to

$$\varphi_{\text{‘inactive’,}\cdot}^{(k,3)} = (0.7, 0.2, 0.1).$$

At lower frequencies, the probability $q_{\mathbf{s},\mathbf{s}'}^{(t,k)}$ of migrating from segment \mathbf{s} to some other segment $\mathbf{s}' \neq \mathbf{s}$ is less than $\varphi_{\mathbf{s},\mathbf{s}'}^{(k)}$. The maximal probability $\varphi_{\mathbf{s},\mathbf{s}'}^{(k)}$ is scaled against the frequency $f_{t,k,\mathbf{s}}$ of exposure according to a Hill equation. Hill equations are sigmoidal, making them convenient for parameterizing increasing (at small frequencies) and diminishing (at large frequencies) returns for media channels (Hill, 1910; Gesztelyi et al., 2012; Jin, Wang, Sun, Chan, & Koehler, 2017).

$$H(f; \kappa, \zeta) = \frac{1}{1 + (f/\kappa)^{-\zeta}}. \quad (3)$$

The parameter κ is the half maximal effective concentration (EC50), i.e., the frequency at which $H(f) = 1/2$. The parameter ζ is the maximal slope of the curve $H(f)$. We let

$$q_{\mathbf{s},\mathbf{s}'}^{(t,k)} = H(f_{t,k,\mathbf{s}}; \kappa, \zeta) \varphi_{\mathbf{s},\mathbf{s}'}^{(k)}, \quad \mathbf{s} \neq \mathbf{s}'.$$

The Hill transformation has several desirable properties; at the right limit, $H(\infty) = 1$, enforcing the definition of $\mathbf{Q}^{(k)}$ as defining the maximal migration probabilities, and the Hill transformation also creates diminishing returns as average frequency increases.

The traditional media module described here is sufficient for modeling the behavior of many traditional media channels, such as television, radio, and print. Using different parameterizations of this module, a modeler can include various media channels, each with its own reach, targeting, frequency of exposure, spend, and effectiveness in driving sales.

2.3.2 Paid search media module.

Some media channels have a different structure than the traditional media channel described above and require their own customized specifications. Paid search, for example, has limited inventory and an auction-based pricing system.

The search media module is depicted in Figure 4. Its design is in some ways analogous to that of the traditional media module, but added complexities allow search to interact with other media channels and seasonal changes in the population in complex ways. In particular, previous events in the simulation path that change the state of the population can affect the volume of paid search and its effectiveness. Activity state, for example, affects a consumer’s probability of conducting a relevant generic search, and both activity state and brand state can affect the consumer’s probability of conducting a relevant branded search. Brand favorability and brand loyalty also affect a consumer’s click-through rate (CTR), i.e., their probability of clicking on the brand’s paid search ads. The paid search module also accounts for other features of search, such as organic search activity in the absence of paid ads; this allows modelers to identify the incremental effect of paid search over organic search.

The process for generating search-related variables and calculating its effect on population segmentation is described below. More details can be found in Appendix C.2.

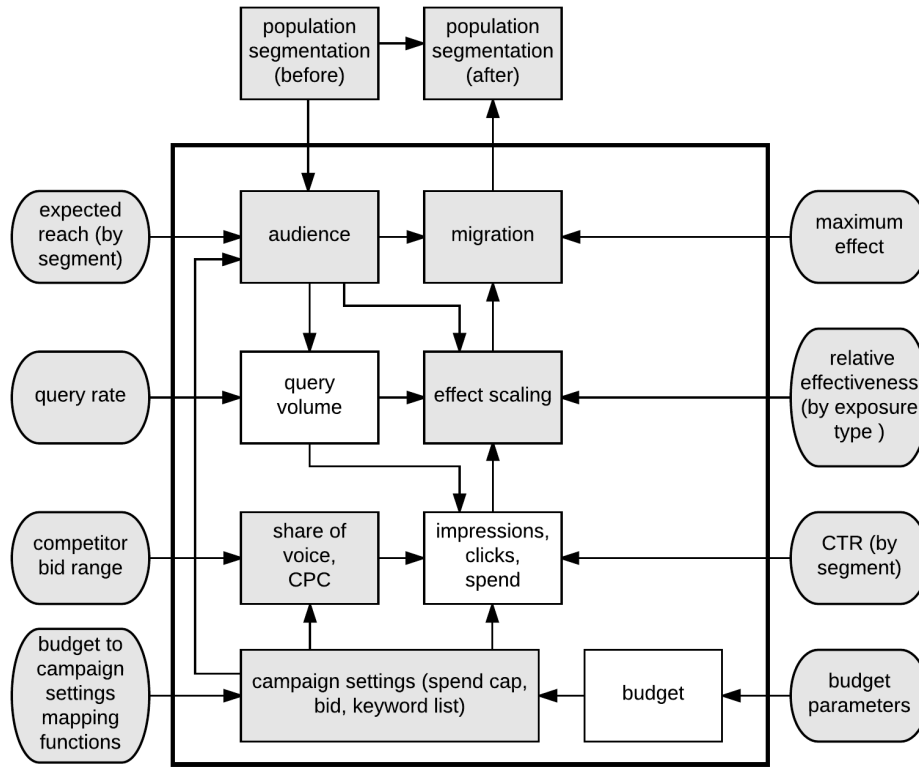


Figure 4: Design for the paid search module. This figure depicts the dependencies between the modeler-specified input parameters (ovals) and the hidden (shaded) and observed (unshaded) variables (rectangles) generated by the module.

1. *Determine the campaign settings from the budget.*

The modeler specifies how the campaign settings⁴ for search will change in response to the budget. As in the traditional media module, the budget represents the target spend in the channel over a fixed budget period, say a year. An advertiser may control search spend through adjustments to the weekly spend cap, the bid, or the keyword list.

(a) *Compute the weekly spend cap.*

The modeler specifies a function that maps the budget to a weekly spend cap. For example, the budget may be divided evenly among the weeks in the budget period. Or, an advertiser may have uncapped spend (cap = ∞), and affect the volume of paid search spend through other controls (b) and/or (c).

(b) *Compute the bid.*

The modeler specifies a function that maps the budget to a weekly bid.⁵ An advertiser may, for example increase its bid linearly with the yearly budget. Or, it could have a constant bid and adjust spend through the weekly spend cap (a) and/or the keyword list (c).

⁴Campaign settings include the weekly spend cap, the bid, and the keyword list.

⁵This is simplified to a single number for the campaign, rather than a per-keyword bid.

(c) *Compute the keyword list length.*

The modeler specifies a function that maps the budget to the length of the keyword list. As the keyword list grows, so does the volume of matching queries made by the population.

2. *Calculate query volume.*

Members of each population segment have different levels of interest in the category and/or the brand, and thus different probabilities of making relevant queries. This is an important consideration for studying media with different levels of targeting.

(a) *Determine the audience.*

The audience is the number of people making queries that match the keyword list. It is calculated based on the population of each segment, the probability members of each segment have of making a relevant query, and the proportion of those queries covered by the keyword list.

Consumers in different segments have different probabilities of making generic and branded queries, depending on their level of interest in the category and/or the brand. Paid search generally is targeted to higher levels of category and/or brand interest. Generic search ads, for example, target individuals in the ‘exploratory’ or ‘purchase’ activity states. Branded search ads targets segments with higher levels of brand favorability and brand loyalty. Targeting gives advertisers confidence that the population exposed to search ads has a higher and more immediate level of interest in the category and/or the brand. Modeling multiple channels with different levels of targeting allows modelers to explore challenges related to selection bias.

(b) *Determine the query volume.*

Each member of the audience makes queries matching the keyword list according to a Poisson process with rate $\lambda^{(k)}$, specified by the modeler.

3. *Calculate impressions, clicks, and spend.*

Based on the campaign settings and the query activity coming from each segment, we calculate the volume of paid search and the associated spend.

(a) *Calculate the average cost-per-click (CPC) and maximum share of voice (SOV).⁶*

The CPC c_t and the maximum SOV $p_t^{(u)}$ both increase with the advertiser’s bid c_t^* . The pricing of paid search is parameterized by the maximum bid $\eta_t^{(m)}$ at which the advertiser loses all auctions (and thus places no paid search ads) and the minimum bid $\eta_t^{(M)}$ necessary to win all auctions. These values bound the CPC, which is calculated as

$$c_t = \eta_t^{(m)} \vee c_t^* \wedge \eta_t^{(M)},$$

where $a \vee b$ is the maximum of a and b and $a \wedge b$ is the minimum. The maximum SOV increases linearly from 0 at $\eta_t^{(m)}$ to 1 at $\eta_t^{(M)}$:

$$p_t^{(u)} = 0 \vee \frac{c_t^* - \eta_t^{(m)}}{\eta_t^{(M)} - \eta_t^{(m)}} \wedge 1.$$

⁶The *maximum SOV* is the fraction of search queries matching the keyword list whose auctions the advertiser can win at the current bid. The final SOV attained by the advertiser may be less than the maximum when impressions are limited by the spend cap.

The parameters $\eta_t^{(m)}$ and $\eta_t^{(M)}$ can be adjusted to fit the modeler’s needs. For example, Google’s paid search system rewards ads for relevancy through its quality score system (Google, 2016). In branded search campaigns, the advertiser benefits from a competitive advantage and can win auctions at lower bids; the modeler can simulate this with low $\eta_t^{(m)}$ and $\eta_t^{(M)}$.

- (b) *Available impressions, available clicks.*

Based on the query volume $w_{t,S}$ and the SOV $p_t^{(u)}$, calculate the number of impressions available to the advertiser for purchase,

$$u_{t,S}^* | w_{t,S} \sim \text{Binomial}(w_{t,S}, p_t^{(u)}).$$

Then use the click-through rate (CTR) to calculate the number of clicks generated by those impressions. Individuals from different segments have different click-through rates (CTR), depending on their levels of category and brand interest.

- (c) *Calculate paid impressions, paid clicks, and spend.*

The spend is calculated by multiplying the number of paid clicks by the CPC. If the cost of the available clicks is less than the week’s spend cap, then the advertiser purchases all impressions available at the current bid. Otherwise, the number of paid impressions and paid clicks is limited to honor the spend cap.

4. *Update the population segmentation.*

Both paid and organic search can drive migration between segments. Each segment has individuals with different exposure to search. Individuals making queries may (a) see no paid ads from the advertiser, resulting in an organic-only experience (b) see the advertiser’s paid search ad but not click on it (though they may click on the organic result), or (c) see the paid ad and click on it. The probability of migrating to a new population segment depends on both the segment of origin and which of these groups the consumer belongs to. In particular, the migration probabilities assigned to exposure type (a) account for migrations driven by organic search results.

Migration probabilities are parameterized by

- A transition matrix $\mathcal{Q}^{(k)}$ specifying maximal transition probabilities between segments. As in the traditional media module, to reduce the parameter space, $\mathcal{Q}^{(k)}$ is specified via a series of transition matrices $\mathcal{Q}^{(k,l)}$ specifying transition probabilities in each dimension $l = 3, 4, 5, 6$ of the population segmentation that can be affected by marketing.
- The relative effectiveness $\psi^{(k)} = (\psi_w^{(k)}, \psi_u^{(k)}, \psi_v^{(k)})$ of each exposure type (organic search only, paid exposure without paid click, paid click) is used to scale the migration probabilities.

The relative effectiveness of each exposure type allows the modeler to control the incremental effect of paid search over organic activity. This incremental effect can be measured by setting the search budget to zero, which will force everyone making queries into group (a). See Section 3.2 for more discussion on measuring the incremental impact of a marketing channel.

Other marketing interventions. The previously described modules suffice for describing the behavior of many common media channels. In addition, new modules can be written to fit the essential characteristics of other marketing interventions. By creating a specific sequence of these modules, each with its own parameterization, modelers can describe complex marketing environments.

2.4 Sales event

The final, K -th, event modeled over each time interval is the sales event. During this event, AMSS calculates the advertiser sales per segment, $\mathbf{y}_{t,\cdot} = (y_{t,\mathbf{s}})_{\mathbf{s} \in \mathcal{S}}$, and the competitor sales per segment, $\mathbf{z}_{t,\cdot} = (z_{t,\mathbf{s}})_{\mathbf{s} \in \mathcal{S}}$, for the current time interval t . In addition, post-purchase changes in consumer mindset cause changes in the population segmentation.

2.4.1 Calculating sales

AMSS takes into account the effects of pricing and competition on advertiser and competitor sales. It also takes into account differences in purchase behavior between consumers belonging to different population segments.

Let us first consider consumer purchase behavior in the absence of competition. In each segment \mathbf{s} whose consumers have purchase intent, i.e., $\mathbf{s} \in \{\mathbf{s}' \in \mathcal{S} : s'_3 = \text{'purchase'}\}$, the relationship between the advertiser's product's price and the probability of purchase by any consumer in the segment is specified by a linear demand curve. The demand curves in each segment are parameterize their y-intercept and negative slope,

- $\boldsymbol{\alpha} = (\alpha_{\mathbf{s}})_{\mathbf{s} \in \mathcal{S}}$. Each $\alpha_{\mathbf{s}} \in [0, 1]$ specifies the probability a consumer in segment \mathbf{s} will purchase the advertiser's brand at unit price 0, in the absence of competition.
- $\boldsymbol{\beta} = (\beta_{\mathbf{s}})_{\mathbf{s} \in \mathcal{S}}$. Each $\beta_{\mathbf{s}} \in (-\infty, \infty)$ specifies the decrease in purchase probability per unit increase in price, in the absence of competition. It controls the sensitivity of consumers in the segment to price. Generally, $\beta_{\mathbf{s}} > 0$ so that sales decrease as price increases.

Thus, in a competition free environment, each consumer in segment \mathbf{s} will purchase the advertiser's brand with probability

$$r_{t,\mathbf{s}}^{(y^*)} = (\alpha_{\mathbf{s}} - \beta_{\mathbf{s}} p_t) \vee 0 \wedge 1,$$

given product unit price $p_t > 0$ during time interval t .

The demand curves reflect differences in purchase behavior between segments. Generally, consumers in segments with high brand favorability, brand loyalty, and/or brand availability are more likely to purchase the advertiser's brand over its competitors' brands; thus these segments should have higher values of $\alpha_{\mathbf{s}}$ and smaller values of $\beta_{\mathbf{s}} > 0$. An example set of demand curves respecting these guidelines is plotted in Figure 5. Previously examined marketing interventions such as media advertising encourage sales by changing consumer mindset, causing migration to population segments with more favorable demand curves. Alternatively, pricing strategies, such as promotional discounts, encourage sales by moving consumers along the demand curve to lower price points with higher purchase probabilities.

To avoid the complexity of simulating competitor activity at the same level of detail as advertiser activity, competitor strength is summarized with two time-varying parameters in the sales module. They specify the current strength of the advertiser's competitors and the degree to which the competitor and advertiser sales replace each other:

- $\boldsymbol{\gamma} = (\gamma_{t,\mathbf{s}})_{(1:T) \times \mathcal{S}}$. Each $\gamma_{t,\mathbf{s}} \in [0, 1]$ specifies the probability consumers in segment \mathbf{s} will purchase the competitor's brand at time t when the advertiser's product's unit price is very high, and thus the advertiser is not making any sales.

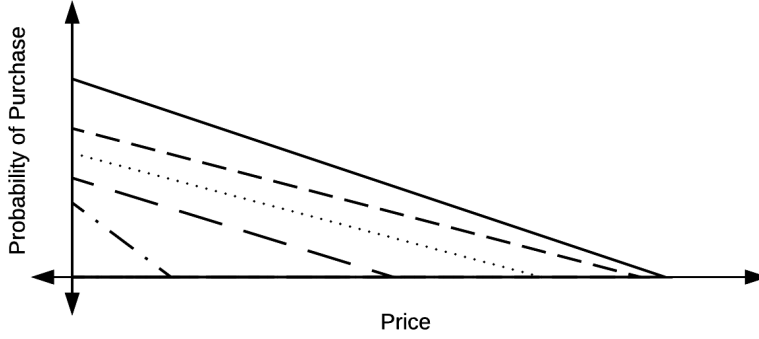


Figure 5: Demand curves for multiple segments in a competition-free environment. The plotted population segments have activity state ‘purchase,’ brand loyalty ‘switcher,’ and brand availability ‘average.’ The brand favorability is favorable (solid), somewhat favorable (dash), neutral (dot), unaware (long dash), or negative (dot-dash).

- $\omega = (\omega_{t,s})_{(1:T) \times \mathcal{S}}$. Each $\omega_{t,s} \in [0, 1]$ specifies the degree to which brand and competitor sales are replacements for each other. When $\omega_{t,s} = 1$, competitor sales are unaffected by advertiser pricing, and competitor sales replace advertiser sales to the greatest degree possible. This is the default setting for segments where consumers are ‘competitor-loyal,’ and is illustrated in the top plot of Figure 6. When $\omega_{t,s} = 0$, advertiser sales are unaffected by the presence of the competitor, and advertiser sales replace competitor sales to the greatest degree possible. This is the default setting for segments where consumers are ‘loyal.’ See the bottom plot in Figure 6 for illustration. By default, ‘switchers’ balance the tradeoff between the advertiser’s and its competitors’ brands with $\omega_{t,s} = 0.5$, as illustrated in the middle plot of Figure 6.

Given the price p_t , consumers purchase a competitor brand with probability

$$r_{t,s}^{(z)} = \left\{ \gamma_{t,s} - (1 - \omega_{t,s})r_{t,s}^{(y^*)} \right\} \vee 0.$$

They purchase the advertiser’s brand with probability

$$r_{t,s}^{(y)} = \max(r_{t,s}^{(y^*)}, \gamma_{t,s}) - r_{t,s}^{(z)}$$

The population of each segment is divided into those who did not make a purchase in the category, those who purchased the advertiser’s brand, and those who purchased a competitor’s brand. The size of each of these groups is, respectively, $a_{t,K,s}^{(w)}$, $a_{t,K,s}^{(y)}$, and $a_{t,K,s}^{(z)}$. These quantities are generated following a multinomial distribution:

$$\left(a_{t,K,s}^{(w)}, a_{t,K,s}^{(y)}, a_{t,K,s}^{(z)} \right) \sim \text{Multinomial} \left(n_{t,K,s}, \left(1 - r_{t,s}^{(y)} - r_{t,s}^{(z)}, r_{t,s}^{(y)}, r_{t,s}^{(z)} \right) \right).$$

Let $\lambda^{(y)}$ be the average number of units purchased by any purchaser in the category. The total brand sales $y_{t,s}$ from population segment s during time interval t is

$$y_{t,s} \sim a_{t,K,s}^{(y)} + \text{Poisson}(a_{t,K,s}^{(y)}(\lambda^{(y)} - 1)).$$

Revenue is calculated as the product of sales $y_{t,s}$ and price p_t .

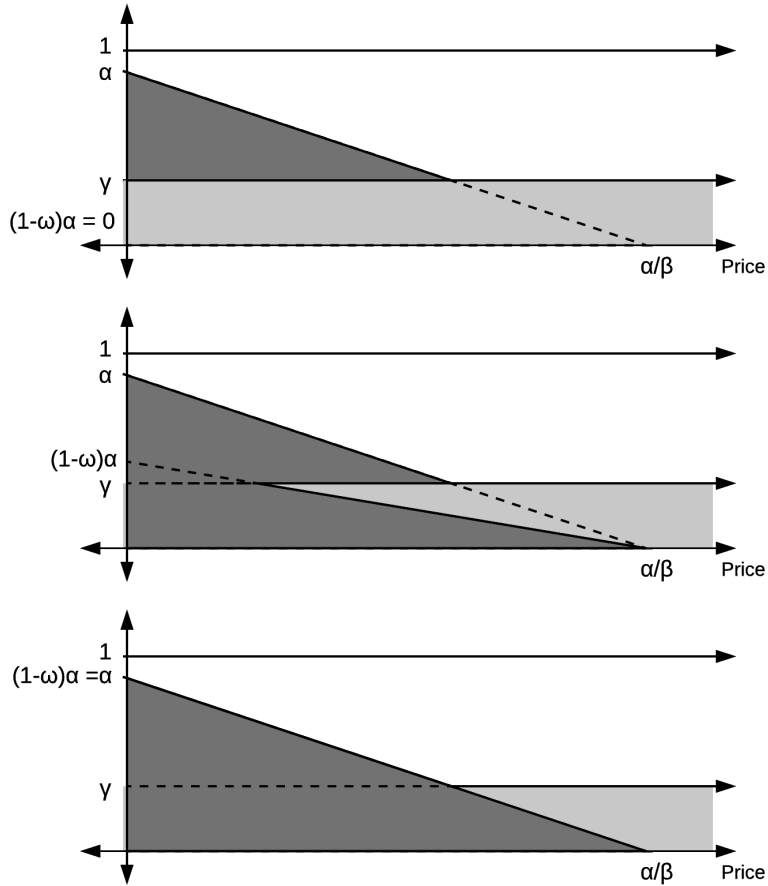


Figure 6: Plots of purchase probabilities vs price in a competitive environment. At any price p , the height of the lightly shaded region corresponds to the probability of purchase for the competitor’s brand, $r^{(z)}$. The combined height of the darker shaded region corresponds to the probability of purchase for the advertiser’s brand, $r^{(y)}$. The plots show changes in competitor vs advertiser sales at varying levels of ω : $\omega = 1$ (top), $\omega = 1/2$ (middle), $\omega = 0$ (bottom).

2.4.2 Post-purchase migration

In addition to calculating sales, revenue, and profit, the sales module also models post-purchase changes in consumer mindset. In the simulation model, this means updating the pre-event segmentation $\mathbf{n}_{t,K,\cdot}$ to the post-event segmentation, which is also the segmentation at the beginning of the next time interval, $\mathbf{n}_{t+1,1,\cdot}$. This is done as a combination of two migrations.

First, all purchasers become satiated. Let \mathcal{B} be the set of ‘un-satiated’ population segments. For any $\mathbf{s} \in \mathcal{B}$, let the size of the affected population undergoing migration be

$$a_{t,K,\mathbf{s}} = a_{t,K,\mathbf{s}}^{(y)} + a_{t,K,\mathbf{s}}^{(z)},$$

i.e., the number of people who made a purchase in the category. Let

$$g(\mathbf{s}) := (s_1, \text{‘satiated,’} s_3, \dots, s_6)^\top$$

be the ‘satiated’ population segment corresponding to any ‘un-satiated’ population segment $\mathbf{s} \in \mathcal{B}$. All consumers who made a purchase in the category during the current time interval become satiated, migrating from \mathbf{s} to $g(\mathbf{s})$. Thus, the transition probabilities between segments are

$$q_{\mathbf{s},\mathbf{s}'}^{(t,K)} = \mathbb{1}_{\{g(\mathbf{s})\}}(\mathbf{s}'),$$

where $\mathbb{1}_{\mathcal{A}}(x)$ indicates whether $x \in \mathcal{A}$. Migration is simulated based on $a_{t,K,\mathbf{s}}$ and $Q^{(t,K)}$ according to (1), (2).

In addition, the purchase experience can affect brand state; following a successful purchase, consumers may become habituated to or develop a strong preference for a particular brand. This migration is controlled by matrices $\mathcal{Q}^{(K,l)}$ for $l = 4, 5, 6$, and the migration process is analogous to the migrations driven by marketing interventions, except the frequency is ignored in the sales-driven migrations. See Appendix C.1 for a description of marketing intervention-driven migration.

3 Ground truth

For any simulation scenario, the simulator can be used to calculate ground truth for comparison with modeling results. Several types of results may be generated by a marketing analysis. Generally, they will be some variant of the following:

- Prediction: Estimate resulting KPI given specified changes in marketing strategy.
- Attribution: Attribute KPI across the existing set of media/marketing channels.
- Optimization: Propose marketing strategies that will increase KPI.

Counterfactual data generation is the key capability required to evaluate these analyses. The simulator is capable of running virtual experiments that allow the causal consequences of changes in the simulation settings, such as a change in the budget assigned to each media channel, to be identified.

3.1 Counterfactual scenarios

We consider a complete specification of the simulation parameters to be a single scenario. The simulator is capable of generating data from modified parameterizations which represent counterfactual scenarios. Comparing data from the original and counterfactual scenarios allows the modeler to observe the causal consequences of changes in the marketing environment. For example, a common comparison may be between a scenario with a positive media budget and a scenario with a media budget of zero. Comparing results from these two scenarios allows the modeler to observe the incremental impact of this media spend on sales.

The causal impact of a change in the simulation scenario can be very complex. Multiple variables can be affected by a single change, and perturbations in one part of the system can affect others through multiple pathways. Consider, for example the following scenario for two interconnected media. Suppose we are in a two media channel system, with the two media channels being television and paid search. Also suppose search has uncapped spend, with paid search volume proportional to query volume. Spend on television has multiple effects. It can

- Directly generate sales without any interaction with search
- Increase the number of related category and/or brand queries, which may increase the *volume* of paid search.
- Change the disposition of the population making queries, perhaps making them view the advertiser brand more favorably, which may change the *effectiveness* of paid search.

Running a virtual experiment that removes television advertising will show the combined impact of eliminating all of these pathways.

Counterfactual scenarios come in three types:

- Changes to actionable components of the simulation settings, such as the yearly budget assigned to each media channel.
- Changes to non-actionable components of the simulation settings, such as the seasonality, or competitor activity.
- Changes that require modification beyond the simulation settings. For example, fixing spend in paid search is not directly possible within the paid search media module. Doing so would require changes in the structure, dependencies, and campaign specification described in Section 2.3.

AMSS can run virtual experiments simulating the effects of either of the first two scenario types. However, certain parts of the simulation are not designed to be specifiable. For example, a modeler cannot simulate the effect of fixing sales to a newly specified level—sales cannot be specified, only generated based on dependencies embedded in the simulation. Similarly, in certain situations it is not possible to run a virtual experiment fixing search spend to a pre-specified level. When the weekly spend cap is not a limiting factor, search spend varies based on query volume, SOV, and CPC. Query volume fluctuates outside of the modeler’s control, and changes in other media channels will generally impact consumer mindset in ways that affect query volume. Furthermore, there can be multiple combinations of campaign settings that achieve a particular spend in search, but with differing impact on the consumer. This makes the problem of fixing search spend not well-defined. Thus, rather than specifying search spend when measuring media effectiveness, modelers should instead specify values for upstream variables such as the budget or the campaign settings.

3.2 Calculating the ground truth

Given a fully specified scenario, the simulator is capable of producing the observable data resulting from a random instance of that scenario. Ground truth is obtained empirically by generating multiple random instances of data. The ground truth θ may be a quantity such as the expected weekly sales resulting from the scenario. The simulator reports the ground truth with greater accuracy as the sample size, i.e., the number of datasets generated, grows. Larger samples are needed for accurate estimation in scenarios with more variability.

This process is used to report ground truth for quantities such as the return on advertising spend (ROAS) in a media channel. Consider a scenario with a particular marketing strategy $\mathbf{b} = (b_m)_{1:M}$, where b_m is the budget for m -th media channel. Let $m = 1$ represent television. To calculate the ROAS for television, generate N_1 datasets $D_{n_1}(\mathbf{b})$, $n_1 = 1, \dots, N_1$. Let $x_{n_1}(\mathbf{b})$ be the total ad spend in dataset $D_{n_1}(\mathbf{b})$, and let $y_{n_1}(\mathbf{b})$ be the revenue. Let \mathbf{b}' represent the counterfactual scenario that

is identical to \mathbf{b} in all respects, except that no budget is assigned to television. Thus, $b'_1 = 0$ and $b'_m = b_m \forall m \neq 1$. Generate N_2 datasets $D_{n_2}(\mathbf{b}')$, $n_2 = 1, \dots, N_2$ from scenario \mathbf{b}' , each with ad spend $x_{n_2}(\mathbf{b}')$ and revenue $y_{n_2}(\mathbf{b}')$. The ROAS θ is empirically estimated by

$$\hat{\theta} = \frac{1}{N_1 N_2} \sum_{n_1, n_2} \frac{y_{n_1}(\mathbf{b}) - y_{n_2}(\mathbf{b}')}{x_{n_1}(\mathbf{b}) - x_{n_2}(\mathbf{b}')}$$

By the law of large numbers, the ground truth reported by the simulator, $\hat{\theta}$, approaches the true ROAS, θ . The accuracy of the approximation can be estimated by calculating a margin of error from the variability in the empirical sample.

4 Sample application to media mix modeling

AMSS is capable of modeling both simple and complex marketing environments. We demonstrate how the simulator may be used to test the performance of linear regression in estimating ROAS and marginal ROAS (mROAS). We start from an extremely simple ad system where basic linear regression performs well, and then consider model performance as we add a single complicating factor to the scenario; lagged ad impact of increasing strength.

4.1 Simple simulation

4.1.1 Specification

We begin with a simple ad system with two media variables, television and search. We avoid several complications that would make modeling more difficult. In particular,

- There are no lagged effects.
- There is no satiation effect; consumers remain in-market and active in the category after making a purchase.
- There is no interaction between the two media channels.
- The seasonality is made known to the model.
- Competitive pressure is constant over time.

The simulation tracks media spend, media volume, and sales across four years of weekly data, for a total of 208 weeks. The population size is 240 million. There is a strong seasonality in the rate of market participation over time (see Figure 7), which is generated by multiplying a sinusoidal time series centered vertically at 0.42, with amplitude 0.175 and a 52-week period by an AR1 time series with mean 1, standard deviation 0.1, and autocorrelation 0.3.

To avoid lagged effects, the activity state and brand favorability state naturally migrate back to their equilibrium proportions at the beginning of each time interval. For activity state, we use

$$\mathcal{Q}^{(3)} = Q^{(3)} = \begin{pmatrix} 0.60 & 0.30 & 0.10 \\ 0.60 & 0.30 & 0.10 \\ 0.60 & 0.30 & 0.10 \end{pmatrix} \quad (4)$$

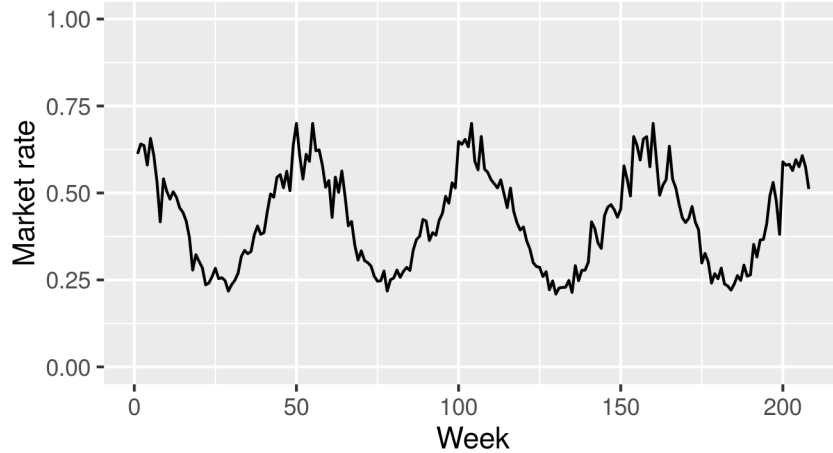


Figure 7: Market rate in the simple simulation.

for an equilibrium distribution of 60% inactive, 30% exploratory, and 10% purchasing, among ‘in-market’ and ‘un-satiated’ individuals. Brand favorability transitions according to

$$Q^{(4)} = Q^{(4)} = \begin{pmatrix} 0.03 & 0.07 & 0.65 & 0.20 & 0.05 \\ 0.03 & 0.07 & 0.65 & 0.20 & 0.05 \\ 0.03 & 0.07 & 0.65 & 0.20 & 0.05 \\ 0.03 & 0.07 & 0.65 & 0.20 & 0.05 \\ 0.03 & 0.07 & 0.65 & 0.20 & 0.05 \end{pmatrix}, \quad (5)$$

for an equilibrium distribution of 3% unaware, 7% negative, 65% neutral, 20% somewhat favorable, and 5% favorable.

The media channels are sequenced with television followed by paid search. Television is treated as a mass marketing channel with an audience comprised of 40% of the population of each segment. It has a relatively stable yearly budget of (109, 95, 84, 91) million dollars per year for years 1-4. There is strong signal for the effect of television due to large variability in the weekly spend (visible in the fighting pattern plotted in Figure 8) and a strong effect size (Chan & Perry, 2017).

Television’s impact on the population is limited to changes in brand favorability, with maximum effect

$$Q^{(tv,4)} = Q^{(tv,4)} \begin{pmatrix} 0.4 & 0 & 0.4 & 0.2 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The above matrix specifies transition probabilities between, in order, ‘unaware,’ ‘negative,’ ‘neutral,’ ‘somewhat favorable,’ and ‘favorable’ brand favorability states. Note that television is relatively effective at increasing awareness (40% of the population becomes neutral and 20% becomes somewhat favorable) and in changing neutral favorability to somewhat favorable (40% of neutral individuals will migrate to ‘somewhat favorable.’). The effect size scales with the frequency of exposure according to a Hill transformation (3) with a EC50 $\kappa = 1.56$ and maximal slope $\zeta = 1$ (see Figure 9).

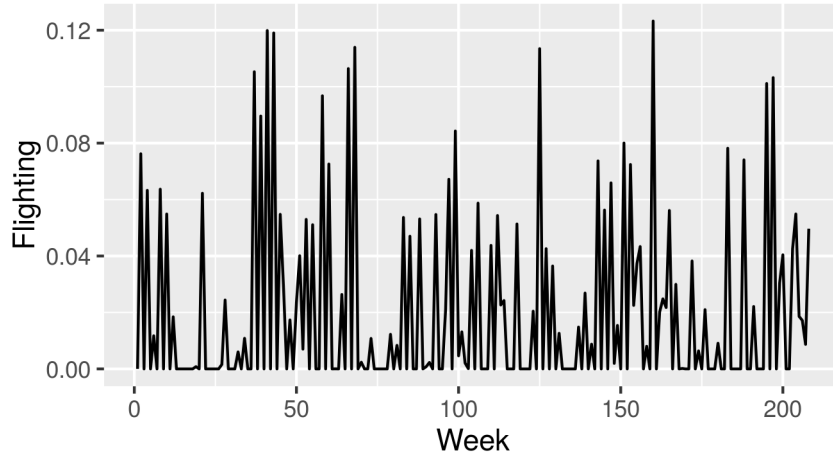


Figure 8: Flying for television. For each week t within a year, the total yearly budget is allocated in proportion to the flying coefficient f_t .

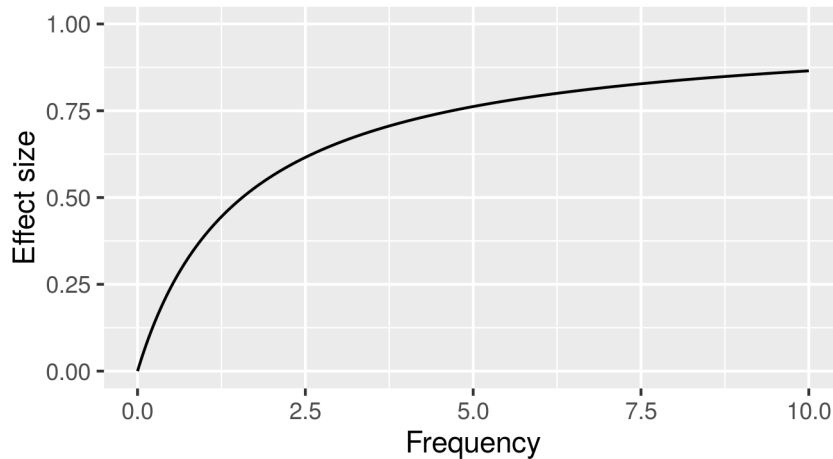


Figure 9: Effect size vs frequency of exposure to television shows diminishing returns.

Search has a more targeted audience, showing ads on queries from 1% of inactive, 30% of exploratory, and 40% of purchasing individuals. The CTR is higher in more favorable activity states. It is 0.005 for ‘inactive’ individuals to 0.08 for ‘exploratory’ individuals and 0.10 for ‘purchase’ individuals. Neither query rates nor click-through-rates depend on brand favorability, in order to avoid interaction effects between television and paid search in this simple scenario. The volume of paid impressions and clicks is controlled by the interaction of query volume and CTR with the search budget and its associated settings. In this simulation, budget affects neither the weekly spend cap nor the keyword. Both remain fixed; in particular, the weekly spend is uncapped. Instead, the budget is translated into changes in search spend through the advertiser’s bid. The bid increases linearly with budget, from a minimum of 0.8 at a budget of 0. The simulation sets a larger budget for search in each year of the simulation to create some variability in the paid search channel.

The effect of search on consumer behavior is entirely focused on activity state. Including effects on the consumer’s brand state would create interactions with the effect of television, which we are avoiding in this simulation. The maximal transition probabilities between activity states (see

Appendix C.2) are

$$\mathcal{Q}^{(\text{search},3)} = Q^{(\text{search},3)} = \begin{pmatrix} 0.05 & 0.95 & 0 \\ 0 & 0.85 & 0.15 \\ 0 & 0 & 1 \end{pmatrix}$$

In addition, relative probabilities of migration for individuals exposed to organic search only, paid impressions without paid clicks, and paid clicks, are set to $\psi^{\text{search}} = (0.0, 0.1, 1.0)$, respectively. Thus,

$$\psi_v^{\text{search}} \cdot \mathcal{Q}_{\text{inactive}, \text{exploratory}}^{(\text{search},3)} = 95\%$$

of ‘inactive’ users who click on a paid ad will become ‘exploratory’ and

$$\psi_v^{\text{search}} \cdot \mathcal{Q}_{\text{exploratory}, \text{purchase}}^{(\text{search},3)} = 15\%$$

of ‘exploratory’ users who click on the paid ad will become purchasers.⁷ Those who view the paid ad without clicking it will have a 9.5% chance or 1.5% chance of those same migrations, since $\psi_u^{\text{search}} = 0.1$. Those who make queries and only view organic and/or competitor paid results have a 0% chance of either migration.

In the sales module, the price is specified as a constant \$40 per unit. All members of the population are ‘switchers’ with average brand availability. Thus, the demand curve varies solely with brand favorability, and at \$40, the proportion of individuals who will make a brand purchase is set at (0.014, 0, 0.2, 0.3, 0.9) for the ‘unaware,’ ‘negative,’ ‘neutral,’ ‘somewhat favorable,’ and ‘favorable’ brand favorability states. We fix the probability of competitor strength across time with $\gamma \equiv 0.8$ and avoid competitor effects on advertiser sales by setting $\omega \equiv 0$.

As a final note, the first year of data is used as a burn-in period for the simulator, and only data from years 2-4 are observed and used in the analysis. From here on we refer to time within the dataset as years and weeks starting from the beginning of Year 2.

4.1.2 Data

We generated 1000 datasets corresponding to the scenario described above. Time series plots from one of these datasets provide an overview of the data generated using this particular parameter specification.

Figure 10 shows the media spend for both channels. Note that television has high variability, while search more closely follows the underlying seasonality of the market rate.

Figure 11 shows volume for paid search. The three types of search volume all closely follow the underlying seasonality of the market rate, and are highly correlated with each other within each budget period (year). Note that, as the budget increases from year to year, the increasing bid increases the advertiser’s SOV; as a result, the height of the dotted line (paid impressions) approaches that of the solid line (query volume) over time.

The *base sales* is the sales that the advertiser would generate in the absence of any advertising. When modeling, it is used to control for seasonality; we assume it is observed in order to avoid trouble with unobserved seasonality while focusing on other modeling issues, such as lagged media

⁷Note that despite the high transition probability of ‘inactive’ users becoming ‘exploratory’ users, this only applies to those ‘inactive’ users who are exposed to and click on the paid search ad. This is a relatively small group, since this example describes a targeted campaign with low exposure rates for ‘inactive’ users.

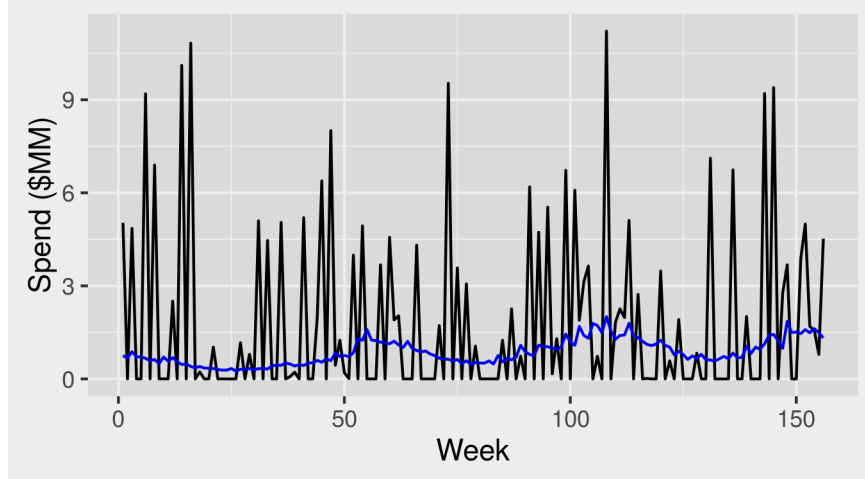


Figure 10: Media spend over time for television (black) and paid search (blue).

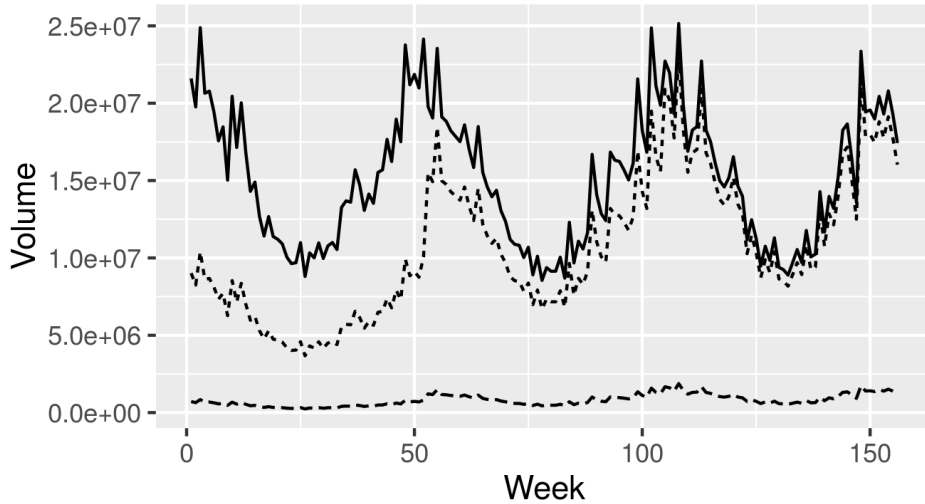


Figure 11: Search volume: query volume (solid), paid impressions (dotted), and paid clicks (dashed).

effects. We generate the base sales by running the counterfactual scenario with all media budgets set to 0. The average weekly sales over 1000 simulations from this no media scenario are reported as the base sales. Base sales are approximately 95% of the total sales. Both sales and base sales over time are plotted in Figure 12.

4.1.3 Ground truth

We calculate the ROAS and mROAS as described in Section 3.2. The ROAS for television was calculated using 1000 datasets generated with the original budget, and another 1000 datasets with a television budget of 0. The ROAS for paid search was calculated analogously. The calculation for the mROAS is similar to that of the ROAS; instead of generating a counterfactual with zero budget, we drop the budget of the specified media by 5% to calculate the incremental effect of the last 5% spent in the channel. The results, by year, are shown in Table 4.

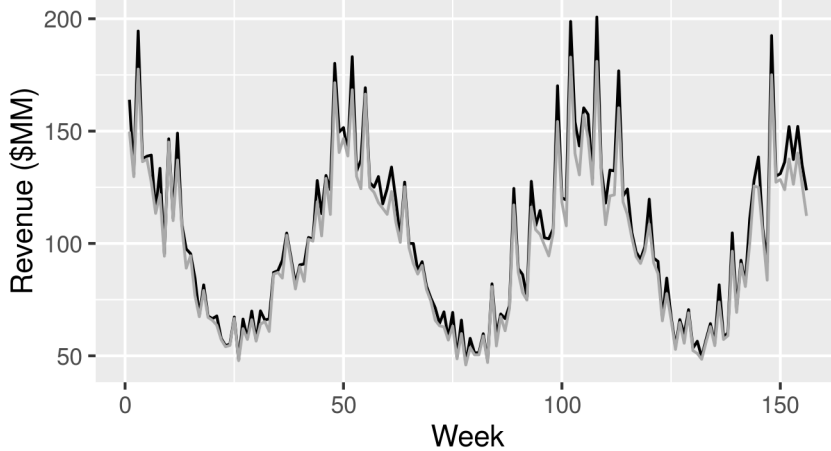


Figure 12: Sales (solid) and base sales (gray) vs time.

year	TV		Search	
	ROAS	mROAS	ROAS	mROAS
2	1.79 (0.00)	0.41 (0.02)	1.90 (0.00)	1.94 (0.03)
3	2.11 (0.00)	0.40 (0.02)	1.85 (0.00)	1.52 (0.03)
4	2.23 (0.00)	0.47 (0.02)	1.86 (0.00)	1.76 (0.08)

Table 4: ROAS and mROAS for the two media channels in the simple simulation. Margins of error reported in parentheses.

4.1.4 Modeling

For this illustration, we use linear regression to model the effect of media spend on sales. In order to simplify the task of fitting the model, we assume the seasonality $\mathbf{x}^{(seas)}$ is observed; in reality, advertiser knowledge of measures of seasonality, such as the base sales (used here), is imperfect. Our model formula is:

$$\begin{aligned}
 y_t = & \beta_0 + \beta_{tv}x_t^{(tv)} + \beta_{search}x_t^{(search)} + \beta_{seas}x_t^{(seas)} \\
 & + \beta_{tv,seas}x_t^{(tv)}x_t^{(seas)} + \beta_{search,seas}x_t^{(search)}x_t^{(seas)} \\
 & + \beta_{tv^{1/2}}\left(x_t^{(tv)}\right)^{1/2} + \beta_{tv^{1/3}}\left(x_t^{(tv)}\right)^{1/3} + \epsilon_t.
 \end{aligned} \tag{6}$$

where $\epsilon_t \sim N(0, \sigma^2)$ for some $\sigma^2 \geq 0$. The media variables are the weekly spend in each media channel. This is clearly a very simplified representation of the data-generating model, but it produces reasonable results (see Table 5).

We show the distribution of the estimates of ROAS and mROAS for each media in Figure 13. We compare the model estimates of ROAS and mROAS to the ground truth calculated by simulating from appropriate counterfactuals, as specified in Section 3.2. The bias and mean squared error (MSE) of the model estimates are reported in Table 5. There is a small amount of bias in the estimates of ROAS and mROAS for television, due to lack of model fit for the nonlinear effect of television.

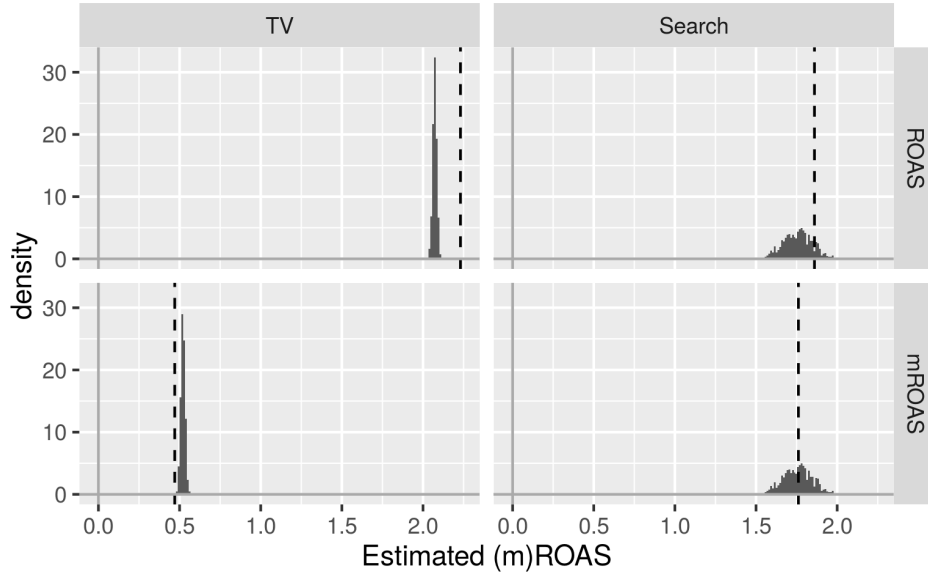


Figure 13: Histograms of the regression estimates for ROAS and mROAS for television and paid search, calculated over 1000 datasets using basic linear regression. The true values provided by the simulator are plotted as vertical lines.

metric	TV		Search	
	ROAS	mROAS	ROAS	mROAS
bias	-0.16	0.05	-0.10	-0.00
MSE	0.15	0.05	0.14	0.09

Table 5: Bias and MSE of estimates of ROAS and mROAS for television and search in the simple simulation.

4.2 Simulation with lagged effects

4.2.1 Specification

Next, we examine the effect of lagged effects for television on model performance. In the simple simulation, ad effects do not last beyond the week in which ad exposure occurs. This is because the segmentation of the population across hidden states is reset at the beginning of each time interval, thanks to row-identical transition matrices controlling the natural migration of activity state and brand favorability state. We add lag by increasing weight on the diagonal of the transition matrix $\mathcal{Q}^{(4)}$ to modify the natural migration of brand favorability, as described in Section B.3.1. The increasing tendency of the population to remain in the segment in which it finished the previous time interval means that changes in the population segmentation caused by marketing interventions persist beyond the initial time period of exposure. We gradually increase the amount of lag by running simulations with $\mathcal{Q}^{(4)} = \mathcal{Q}_\alpha^{(4)}$ where $\mathcal{Q}_\alpha^{(4)} = (1 - \alpha)\mathcal{Q}^{(4)} + \alpha I_5$, for $\alpha = 0.1, 0.2, \dots, 0.5$. In addition, we hold the total advertising effect (nearly) constant by scaling the transition matrix $\mathcal{Q}^{(\text{tv},4)}$, which controls the initial impact of television, by the same α . In the simulation with lag α , $\mathcal{Q}_\alpha^{(\text{tv},4)} = (1 - \alpha)\mathcal{Q}^{(\text{tv},4)} + \alpha I_5$. All other aspects of the scenario remain as in the simple simulation.

4.2.2 Ground truth

Table 6 contains the same-year impact of television and search, calculated in Year 4. The same-year impact of television decreases slightly with increasing α , since television’s initial impact was normalized to create a constant total impact, but some of the total impact of spend in a particular year occurs during the beginning of the next year.

lag α	TV		Search	
	ROAS	mROAS	ROAS	mROAS
0	2.23	0.47	1.86	1.76
0.1	2.22	0.46	1.86	1.83
0.2	2.20	0.47	1.86	1.94
0.3	2.18	0.49	1.86	1.78
0.4	2.15	0.47	1.86	1.85
0.5	2.11	0.41	1.86	1.69

Table 6: ROAS and mROAS for the two media channels in Year 4, at several values for lag.

4.2.3 Modeling

The original model specified in (6) does not account for lagged effects, and is expected to perform poorly as the lagged effect grows stronger. A simple way to account for lag is to include spend from earlier time intervals in the linear regression. We add six additional terms to our initial regression model (6) to account for past television spend:

$$\begin{aligned}
 y_t = & \beta_0 + \beta_{tv}x_t^{(tv)} + \beta_{tv,1}x_{t-1}^{(tv)} + \beta_{tv,2}x_{t-2}^{(tv)} + \beta_{tv,3}x_{t-3}^{(tv)} \\
 & + \beta_{tv^{1/2}}\left(x_t^{(tv)}\right)^{1/2} + \beta_{tv^{1/2},1}\left(x_{t-1}^{(tv)}\right)^{1/2} \\
 & + \beta_{tv^{1/2},2}\left(x_{t-2}^{(tv)}\right)^{1/2} + \beta_{tv^{1/2},3}\left(x_{t-3}^{(tv)}\right)^{1/2} \\
 & + \beta_{tv^{1/3}}\left(x_t^{(tv)}\right)^{1/3} + \beta_{search}x_t^{(search)} + \beta_{seas}x_t^{(seas)} \\
 & + \beta_{tv,seas}x_t^{(tv)}x_t^{(seas)} + \beta_{search,seas}x_t^{(search)}x_t^{(seas)}. \tag{7}
 \end{aligned}$$

Let Model 1 refer to the original model (6), and let Model 2 refers to the new model with lag terms (7).

We use the MSE to evaluate Model 1 and Model 2’s ability to estimate the ROAS and the mROAS of television in Year 4 over increasingly strong lag effects; the results are shown in Table 7. In addition, Figure 14 shows the distribution, over 1000 datasets, of the estimates of the ROAS and the mROAS from each model. It shows that the increasing MSE of Model 1 is driven by the bias. As the amount of lag grows, Model 1 increasingly underestimates the impact of television. Model 2 mostly corrects this bias in exchange for somewhat larger variance. Note that the correction is not perfect; the bias still becomes more negative as the lag increases. This same set of simulations could be used to evaluate alternative models that might be proposed to more accurately account for lag.

lag α	ROAS		mROAS	
	Model 1	Model 2	Model 1	Model 2
0	0.025	0.002	0.003	0.013
0.1	0.191	0.005	0.003	0.006
0.2	0.470	0.010	0.023	0.001
0.3	0.819	0.017	0.068	0.002
0.4	1.215	0.032	0.096	0.005
0.5	1.618	0.067	0.097	0.005

Table 7: MSE of estimates of television ROAS and mROAS in Year 4, generated by the two regression models, as the amount of lag in the true model increases.

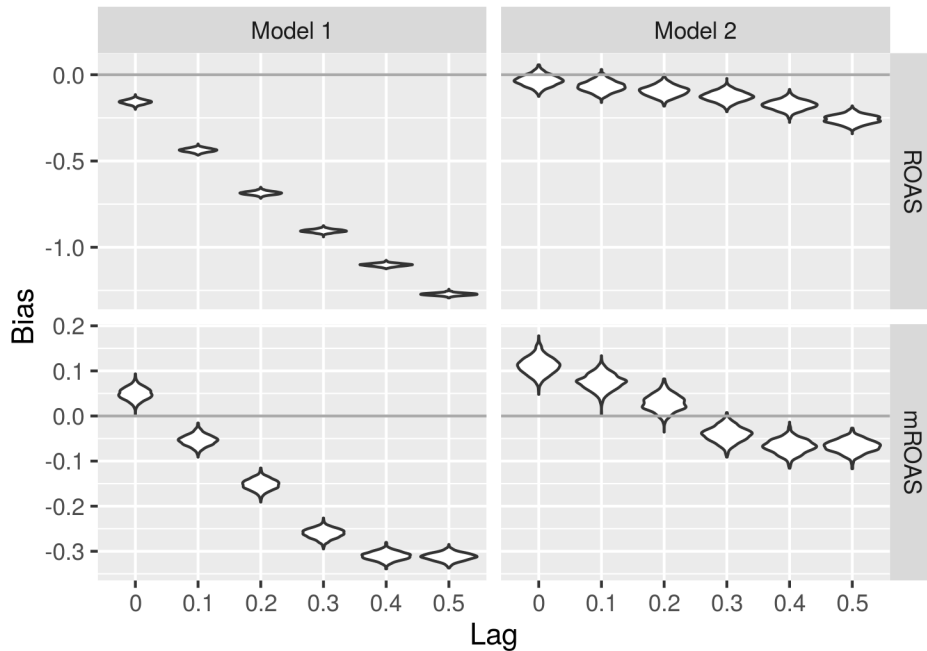


Figure 14: Bias in estimates of the ROAS (top) and mROAS (bottom) of television in Year 4, from Model 1 (left) and Model 2 (right). The distribution of estimates at each level of lag is plotted as a violin plot.

5 Concluding Remarks

The increasingly complex marketplace has led to a strong demand for measurement in marketing. Numerous methodologies have been developed in response to a wide variety of measurement needs. Given the complex marketing environment and the multitude of analytical methods that are available, simulation is an essential tool for evaluating and comparing analysis options. This paper introduced AMSS, a simulation system created to model marketing and its effect on consumer behavior. AMSS is a very flexible and general framework that can capture many important aspects of marketing and sales generation. It can be used to evaluate and guide the development of multiple measurement methods, both observational and experimental.

Here we used a very simple investigation of the performance of regression-based modeling to illustrate the use of AMSS in model evaluation and comparison. While it is not the goal of this paper to recommend a particular Media Mix Model, AMSS provides the foundation for creating

a systematic process for doing so. There are many challenging modeling issues related to MMM that can also be studied using AMSS (Chan & Perry, 2017). These will be described in future publications.

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A Notation

Throughout the paper, we utilize the following conventions for mathematical notation.

Notation	Represents	Examples
lowercase bold	vectors	$\mathbf{s}, \mathbf{n}_{t,k}$,
capital	matrices	$Q^{(t,k)}, I_3$
Script capitals	sets	$\mathcal{S}, \mathcal{A}, \mathcal{B}$
Greek	parameters	Q, λ, κ

Table 8: Mathematical notation

B Detailed mechanics of population migration

B.1 Changing market size

We define the set of ‘in-market’ population segments

$$\mathcal{A} = \{\mathbf{s} \in \mathcal{S} : s_1 = \text{‘in-market’}\} \quad (8)$$

and the set of ‘un-satiated’ population segments

$$\mathcal{B} = \{\mathbf{s} \in \mathcal{S} : s_2 = \text{‘un-satiated’}\}. \quad (9)$$

The market rate before the k -th event of the t -th time interval is $r_{t,k} = n_{t,k,\mathcal{A}}/n_{t,k,\mathcal{S}}$. Natural changes in the market rate occur during event $k = 1$. During this event, individuals migrate between segments in \mathcal{A} and $\mathcal{S} \setminus \mathcal{A}$, to attain (in expectation) the target market rate ρ_t . The cases where $r_{t,1} > \rho_t$ and $r_{t,1} < \rho_t$ are described separately, below. When $r_{t,1} = \rho_t$, no migration occurs.

Case 1, $r_{t,1} < \rho_t$. When the current market rate is less than the target market rate, i.e., $r_{t,1} < \rho_t$, individuals migrate from $\mathcal{S} \setminus \mathcal{A}$ to \mathcal{A} . Thus, the affected population is the set of consumers in $\mathcal{S} \setminus \mathcal{A}$ and the size of the affected population in any segment \mathbf{s} is

$$a_{t,1,\mathbf{s}} = n_{t,1,\mathbf{s}} \mathbb{1}_{\mathcal{S} \setminus \mathcal{A}}(\mathbf{s}),$$

where $\mathbb{1}_{\mathcal{A}}(\mathbf{s})$ indicates whether $\mathbf{s} \in \mathcal{A}$.

Changes in market state generally happen without affecting the other dimensions of consumer mindset. The sole exception is activity state; when un-satiated consumers enter the market, they have probability γ_j of migrating to activity state $j \in \mathcal{S}_3$, where γ_j is a modeler-specified distribution across activity states such that $\sum_{j \in \mathcal{S}_3} \gamma_j = 1$. Satiated consumers must remain inactive. For each $j \in \mathcal{S}_3$, let $h_j^{(1)}$ be the function that maps segments $\mathbf{s} \in \mathcal{S} \setminus \mathcal{A}$ to the corresponding segment in \mathcal{A} with activity state j . Then,

$$h_j^{(1)}(\mathbf{s}) := \begin{cases} (\text{'in-market,'} s_2, j, \dots, s_6)^\top, & \mathbf{s} \in \mathcal{B} \\ (\text{'in-market,'} s_2, \text{'inactive,'} s_4, s_5, s_6)^\top, & \text{o.w.} \end{cases}, \quad (10)$$

and consumers originating in segment \mathbf{s} migrate to segments $\mathbf{s}' \in \{h_j^{(1)}(\mathbf{s}) : j \in \mathcal{S}_3\}$. Then, for all $\mathbf{s} \in \mathcal{S} \setminus \mathcal{A}$ and $\mathbf{s}' \in \mathcal{S}$, the transition probabilities are

$$q_{\mathbf{s}, \mathbf{s}'}^{(t,1)} = \left(\frac{1 - \rho_t}{1 - r_{t,1}} \right) \mathbb{1}_{\{\mathbf{s}\}}(\mathbf{s}') + \sum_{j \in \mathcal{S}_3} \gamma_j \left(\frac{\rho_t - r_{t,1}}{1 - r_{t,1}} \right) \mathbb{1}_{\{h_j^{(1)}(\mathbf{s})\}}(\mathbf{s}').$$

The migrations $m_{\mathbf{s}, \mathbf{s}'}^{(t,1)}$ are randomly generated according to (1), and the post-migration population segmentation $\mathbf{n}_{t,2}$ is calculated according to (2).

Case 2, $r_{t,1} > \rho_t$. When the current market rate is greater than the target market rate, i.e., $r_{t,1} > \rho_t$, individuals migrate from \mathcal{A} to $\mathcal{S} \setminus \mathcal{A}$. Thus, the affected population consists of out-of-market consumers in \mathcal{A} , and the size of the affected population in each segment $\mathbf{s} \in \mathcal{S}$ is

$$a_{t,1,\mathbf{s}} = n_{t,1,\mathbf{s}} \mathbb{1}_{\mathcal{A}}(\mathbf{s}).$$

When consumers leave the market, they also become 'inactive.' Thus, individuals migrate from $\mathbf{s} \in \mathcal{A}$ to

$$(h_{s_3}^{(1)})^{-1}(\mathbf{s}) = (\text{'out-of-market,'} s_2, \text{'inactive,'} s_4, s_5, s_6)^\top.$$

This migration occurs with probability $(r_{t,1} - \rho_t)/r_{t,1}$, so that the expected post-migration market rate is the target rate, $E[r_{t,2}] = \rho_t$. The migration probabilities can be written as the transition matrix $Q^{(t,k)}$, with entries

$$q_{\mathbf{s}, \mathbf{s}'}^{(t,1)} = \left(\frac{\rho_t}{r_{t,1}} \right) \mathbb{1}_{\{\mathbf{s}\}}(\mathbf{s}') + \left(\frac{r_{t,1} - \rho_t}{r_{t,1}} \right) \mathbb{1}_{\{(h_{s_3}^{(1)})^{-1}(\mathbf{s})\}}(\mathbf{s}').$$

The migrations $m_{\mathbf{s}, \mathbf{s}'}^{(t,1)}$ are randomly generated according to (1), and the post-migration population segmentation $\mathbf{n}_{t,2}$ is calculated according to (2).

B.2 De-satiation

Recall from (9) that \mathcal{B} is the set of 'un-satiated' segments. During the de-satiation process, the affected population is the satiated population, and the size of the affected population in each segment is

$$a_{t,2,\mathbf{s}} = n_{t,2,\mathbf{s}} \mathbb{1}_{\mathcal{S} \setminus \mathcal{B}}(\mathbf{s}).$$

Each member of the affected population de-satiates with probability λ . Some of the de-satiating consumers are ‘in-market’; these consumers migrate to activity states $j \in \mathcal{S}_3$ with probability γ_j .⁸ For each $j \in \mathcal{S}_3$, let $h_j^{(2)}$ map segments in $\mathcal{S} \setminus \mathcal{B}$ to corresponding segments in \mathcal{B} with activity state j . We define

$$h_j^{(2)}(\mathbf{s}) := \begin{cases} (s_1, \text{‘un-satiated,’ } j, \dots, s_6)^\top, & \mathbf{s} \in \mathcal{A} \\ (s_1, \text{‘un-satiated,’ ‘inactive,’ } s_4, s_5, s_6)^\top, & \text{o.w.} \end{cases}. \quad (11)$$

Then the affected population $\mathbf{a}_{t,2,\mathbf{s}}$ migrates from $\mathbf{s} \in \mathcal{S} \setminus \mathcal{B}$ to $\mathbf{s}' \in \mathcal{S}$ with probability

$$q_{\mathbf{s},\mathbf{s}'}^{(t,2)} = (1 - \lambda) \mathbb{1}_{\{\mathbf{s}\}}(\mathbf{s}') + \lambda \sum_{j \in \mathcal{S}_3} \gamma_j \mathbb{1}_{\{h_j^{(2)}(\mathbf{s})\}}(\mathbf{s}').$$

B.3 Natural migration in activity state

Natural migration in activity state is limited to consumers in segments $\mathbf{s} \in \mathcal{A} \cap \mathcal{B}$, i.e., consumers that are both ‘in-market’ and ‘un-satiated.’ Thus, the affected population is

$$\mathbf{a}_{t,3,\mathbf{s}} = n_{t,3,\mathbf{s}} \mathbb{1}_{\mathcal{A} \cap \mathcal{B}}(\mathbf{s}).$$

The tendency of the population to gravitate toward an equilibrium distribution across activity states is specified by a transition matrix $\mathbf{Q}^{(3)} = (\mathbf{Q}_{i,j}^{(3)})_{\mathcal{S}_3 \times \mathcal{S}_3}$. Entry $\mathbf{Q}_{i,j}$ represents the probability that an individual currently in activity state i will transition to activity state j . Note that, since $\mathbf{Q}^{(3)}$ is a transition matrix, $\sum_{j \in \mathcal{S}_3} \mathbf{Q}_{i,j} = 1$ for all $i \in \mathcal{S}_3$.

Migration occurs separately for each dimension. During this migration, consumers do not migrate with respect to any other dimensions $l \neq 3$. For every $j \in \mathcal{S}_3$, define the function mapping segments $\mathbf{s} \in \mathcal{A} \cap \mathcal{B}$ to the corresponding segment with activity state j ,

$$h_j(\mathbf{s}) := (s_1, s_2, j, s_4, s_5, s_6)^\top. \quad (12)$$

Then consumers migrate from $\mathbf{s} \in \mathcal{A} \cap \mathcal{B}$ to $\mathbf{s}' \in \mathcal{S}$ with transition probability

$$q_{\mathbf{s},\mathbf{s}'}^{(t,3)} = \left(1 - \mathbf{Q}_{s_3,s_3'}\right) \mathbb{1}_{\{\mathbf{s}\}}(\mathbf{s}') + \mathbf{Q}_{s_3,s_3'} \mathbb{1}_{\{h_{s_3'}(\mathbf{s})\}}(\mathbf{s}').$$

The migrations $m_{\mathbf{s},\mathbf{s}'}^{(t,3)}$ are randomly generated according to (1), and the post-migration population segmentation $\mathbf{n}_{t,3,\cdot}$ is calculated according to (2).

Migrations between brand favorability, brand availability, and brand loyalty states are completely analogous, except in the restrictions on the affected population. Recall that activity state migration is restricted to states $\mathbf{s} \in \mathcal{A} \cap \mathcal{B}$ that are both ‘in-market’ and ‘un-satiated.’ Favorability migration is restricted to segments with a loyalty state of ‘switcher’ or ‘competitor-loyal’; ad exposure won’t change the favorability state of an individual who is ‘loyal’. Similarly, only individuals with the favorability state ‘favorable’ can transition to the loyalty state ‘loyal’ during loyalty migration.

⁸This is the same γ_j used previously in Section B.1 to calculate transition probabilities for un-satiated consumers who enter the market.

B.3.1 Relationship to lagged effects

The transition matrix $\mathcal{Q}^{(3)}$ determines the equilibrium distribution and the strength of the lagged effect. Consider, for example, the following transition matrix:

$$Q_0 = (q_{i,j})_{\mathcal{S}_3 \times \mathcal{S}_3} = \begin{pmatrix} 0.50 & 0.30 & 0.20 \\ 0.50 & 0.30 & 0.20 \\ 0.50 & 0.30 & 0.20 \end{pmatrix}. \quad (13)$$

The matrix Q_0 is row-identical, i.e., $q_{i,j} = q_{i',j}$ for all $i, i' \in \mathcal{S}_3$. Thus, no matter the current activity state of a consumer in $\mathcal{A} \cap \mathcal{B}$, he or she has probability 0.50 of migrating to the ‘inactive’ state, probability 0.30 of migrating to the ‘exploratory’ state, and probability 0.20 of migrating to the ‘purchase’ state. We divide $\mathcal{C} = \mathcal{A} \cap \mathcal{B}$ into three disjoint sets, based on activity state:

$$\begin{aligned} \mathcal{C}_1 &= \{\mathbf{s} \in \mathcal{C} : s_3 = \text{‘inactive’}\} \\ \mathcal{C}_2 &= \{\mathbf{s} \in \mathcal{C} : s_3 = \text{‘exploratory’}\} \\ \mathcal{C}_3 &= \{\mathbf{s} \in \mathcal{C} : s_3 = \text{‘purchase’}\} \end{aligned}$$

Given any segmentation $\mathbf{n} = (n_{t,3,\mathcal{C}_1}, n_{t,3,\mathcal{C}_2}, n_{t,3,\mathcal{C}_3})$ of the population over activity states, $Q_0^\top \mathbf{n} = \|\mathbf{n}\| (0.50, 0.30, 0.20)^\top$. Thus, the pressure of natural migration immediately brings the population back to the equilibrium distribution across activity states, $\mathbf{x} = (0.50, 0.30, 0.20)^\top$, and changes in activity state brought on by marketing pressure have no lag. More generally, the matrix $\mathcal{Q}^{(3)}$ specifies a system with no lag if and only if it is row-identical:

$$\mathcal{Q}^{(3)} \mathbf{n} = \|\mathbf{n}\| \mathbf{x} \quad \forall \mathbf{n} \Leftrightarrow \mathcal{Q}^{(3)} = (\mathbf{x}, \mathbf{x}, \mathbf{x})^\top.$$

Now consider transition matrices Q_α of the form

$$Q_\alpha = (1 - \alpha)Q_0 + \alpha I_3, \quad (14)$$

where I_3 refers to the 3×3 identity matrix. The amount of lag in activity state varies with α . For example, when $\alpha = 0$, (14) is consistent with the definition of Q_0 in (13) and there is no lag. When $\alpha = 1$, $Q_1 = I_3$ and lagged effects are of infinite duration. In other words, the effect of marketing interventions on activity state is permanent. To illustrate this, suppose some media changes the distribution of the consumers in \mathcal{C} from the equilibrium \mathbf{x} to $\mathbf{n} \neq \mathbf{x}$. Then $Q_1^\top \mathbf{n} = \mathbf{n}$ and the initial impact propagates into all subsequent time intervals, until new marketing interventions change the distribution once again. For $\alpha \in (0, 1)$,

$$Q_\alpha^\top \mathbf{n} = (1 - \alpha)Q_0^\top \mathbf{n} + \alpha I_3 \mathbf{n} = (1 - \alpha)\|\mathbf{n}\| \mathbf{x} + \alpha \mathbf{n}.$$

As α increases, the pressure of migration towards the natural equilibrium \mathbf{x} weakens, resulting in stronger lagged impact.

In general, a valid system is specified by any $\mathcal{Q}^{(3)}$ such that $0 \leq \varphi_{i,j}^{(3)} \leq 1$ for all $(i, j) \in \mathcal{S}_3 \times \mathcal{S}_3$ and $\sum_{j \in \mathcal{S}_3} \varphi_{i,j}^{(3)} = 1$ for all $i \in \mathcal{S}_3$. Larger diagonal entries $\varphi_{i,i}^{(3)}$ result in stronger probabilities of staying in one’s current state and thus stronger lagged impact, while larger off-diagonal entries $\varphi_{i,j \neq i}^{(3)}$ result in faster mixing and weaker lag.

C Media module specification

C.1 Traditional media module

Here, we describe the process used to generate media variables and population migration in the traditional media module in more detail.

Audience. Every media has an audience, which is the number of people in each population segment who engage with the specified media. It is the number of viewers of a television program or the number of households reading a particular newspaper, and is randomly generated based on the population and reachability of each segment. Suppose the traditional media module is applied as the k -th event of the t -th time interval. The reachability for each segment \mathbf{s} , $\pi_{\mathbf{s}}^{(a)}$, is the probability a member of \mathbf{s} is exposed to the media, and also is the maximum reach of advertising in the media channel. For each segment $\mathbf{s} \in \mathcal{S}$, given the probability that members of the segment are in the audience, $\pi_{\mathbf{s}}^{(a)}$, and the pre-event population segmentation, $\mathbf{n}_{t,k,\cdot}$, the media audience is

$$a_{t,k,\mathbf{s}}^* \sim \text{Binomial}(n_{t,k,\mathbf{s}}, \pi_{\mathbf{s}}^{(a)}).$$

In the traditional media module, $\pi_{\mathbf{s}}^{(a)}$ depends only on the activity state, s_3 , associated with segment \mathbf{s} . The module can easily be modified to allow $\pi^{(a)}$ to depend on other aspects of the population segmentation.

Budget. The traditional media module uses the budget assigned to the media channel to determine the values of dependent variables, such as the weekly spend and media volume. This budget is an observed, advertiser-controlled variable. It is defined as the target spend of the media channel over a specific range of time intervals known as the budget period. For example, an advertiser could assign one million dollars to the newspaper channel in the fourth quarter of 2016.

Budget is specified by two sets of parameters: $\boldsymbol{\nu} = (\nu_t)_{1:T}$ specifies the budget period of each time interval $t \in \{1, \dots, T\}$, and $\boldsymbol{\beta} = (\beta_u)_{\{u\}}$ specifies the budget assigned to each budget period u . Then, each time interval $t \in \{1, \dots, T\}$ has budget period $u_t = \nu_t$ and budget $b_t = \beta_{\nu_t}$. Since the budget is the target spend, later, when we calculate the media spend, $\mathbf{x} = (x_t)_{1:T}$, the total spend over a particular budget period is expected to match the budget:

$$b_u \approx \sum_{\{t:u_t=u\}} x_t \quad \forall u.$$

Spend. The spend for media channel during time interval t in segment \mathbf{s} , $x_{t,\mathbf{s}}$, is generated based on the budget variables b_t, u_t , a parameter φ_t that specifies the fighting pattern for the spend, and the media audience $a_{t,\mathbf{s}}$. Fighting specifies the distribution of budget across the time intervals within a budget period, and sums to one over each budget period, i.e., $\sum_{\{t:u_t=u\}} \varphi_t = 1$ for all u . The spend in each segment is proportional to both the fighting φ_t and the audience $a_{t,\mathbf{s}}$, so that the spend per segment is

$$x_{t,\mathbf{s}} = b_t \varphi_t \frac{a_{t,k,\mathbf{s}}}{a_{t,k,\mathcal{S}}},$$

where $a_{t,k,\mathcal{S}} = \sum_{\mathbf{s} \in \mathcal{S}} a_{t,\mathbf{s}}$ is the total audience over all segments at time t .

Volume. This module calculates media volume from media spend based on unit cost. Unit cost may vary over time. For example, the size and composition of a television audience varies from

week to week and this changes the cost per exposure. Since unit cost can vary from week to week, AMSS allows specification of the mean $\mu^{(c)}$ and standard deviation $\sigma^{(c)}$ of the unit cost. The unit cost at time t is

$$c_t \sim \text{Gamma} \left(\left(\mu^{(c)} / \sigma^{(c)} \right)^2, \mu^{(c)} / \left(\sigma^{(c)} \right)^2 \right),$$

where $\text{Gamma}(\alpha, \beta)$ refers to the gamma distribution with shape α and rate β . The volume in each segment is

$$v_{t,k,s} = x_{t,s} / c_t.$$

Usually, the aggregate volume over all segments, $v_{t,S} = \sum_{s \in \mathcal{S}} v_{t,k,s}$, is observed. The traditional media module also generates some other common measures of media volume. For example, television volume data often comes in the form of gross rating points (GRP's), which are calculated as reach times frequency, i.e., $100v_{t,S}/n_{t,k,S}$.

Reach and frequency. The reach and frequency are determined by the size of the audience $a_{t,k,s}^*$ and the volume of exposures $v_{t,k,s}$ in each segment. Let $v_{t,k,s,i}$ be the number of ad exposures for the i -th individual in segment s during the k -th event of time interval t . Assuming that the number of exposures for each individual in the segment follows an independent Poisson process, the number of exposures per individual conditioned on the total volume of exposures $v_{t,k,s}$ follows a multinomial distribution with equal weight assigned to each individual:

$$(v_{t,k,s,i})_{i=1:a_{t,k,s}^*} | v_{t,k,s} \sim \text{Multinomial} \left(v_{t,k,s}, \left(\frac{1}{a_{t,k,s}^*}, \dots, \frac{1}{a_{t,k,s}^*} \right) \right).$$

Then the probability that a specific individual is not reached by the media channel is $p(a_{t,k,s}^*, v_{t,k,s})$, where

$$p(a, v) = \left(1 - \frac{1}{a} \right)^v \approx e^{-v/a}.$$

The number of individuals not reached by the media channel has first moment

$$\mu_1 = a_{t,k,s}^* \cdot p(a_{t,k,s}^*, v_{t,k,s})$$

and second moment

$$\mu_2 = \mu_1 \cdot (1 + (a_{t,k,s}^* - 1) \cdot p(a_{t,k,s}^* - 1, v_{t,k,s})).$$

These moments are used to simulate the reach $a_{t,k,s}$ of the k -th media during the t -th time interval in segment s , via a normal approximation. Here *reach* is defined in the absolute sense, as the total number of individuals exposed at to the media advertising at least once. The average frequency is then $v_{t,k,s}/a_{t,k,s}$.

Effect. The key feature of a media channel is its effect on consumer mindset, which is realized by the migration of consumers between population segments. This migration is controlled by the reach and frequency of the media.

Recall from the introduction to population migration, in Section 2.2, that the migration process is limited to an affected population $a_{t,k,s}$; in the traditional media module, this is equivalent to the reach. At maximal frequencies of media exposure, the affected population will migrate between segments following the maximal transition probabilities, which are specified by $\mathcal{Q}^{(k)} = (\varphi_{s,s'}^{(k)})_{\mathcal{S} \times \mathcal{S}}$. Specifying the matrix $\mathcal{Q}^{(k)}$ directly is tedious, due the large number of entries. Instead, the modeler specifies transition matrices

$$\mathcal{Q}^{(k,l)} = \left(\varphi_{i,j}^{(k,l)} \right)_{\mathcal{S}_l \times \mathcal{S}_l}, \quad l = 3, 4, 5, 6,$$

specifying a sequence of transitions in each dimension affected by marketing, i.e., for activity state, brand favorability, brand loyalty, and brand availability. Each $\mathcal{Q}^{(k,l)}$ is expanded to an $\mathcal{S} \times \mathcal{S}$ matrix $\mathcal{Q}^{*(k,l)}$ with entries

$$\mathcal{Q}^{*(k,l)}_{\mathbf{s},\mathbf{s}'} = \begin{cases} \mathcal{Q}_{s_l,s'_l}^{(k,l)} \prod_{l' \neq l} \mathbb{1}_{\{s_{l'}\}}(s'_{l'}) & \mathbf{s} \in \mathcal{R}_l \subseteq \mathcal{S} \\ \mathbb{1}_{\{\mathbf{s}\}}(\mathbf{s}') & \text{otherwise} \end{cases},$$

and $\mathcal{Q}^{(k)}$ is the matrix product of these transition matrices, equivalent to performing the transitions specified by each $\mathcal{Q}^{*(k,l)}$ in sequence:

$$\mathcal{Q}^{(k)} = \mathcal{Q}^{*(k,3)} \mathcal{Q}^{*(k,4)} \mathcal{Q}^{*(k,5)} \mathcal{Q}^{*(k,6)}.$$

We let \mathcal{R}_l be the set of population segments that are considered valid points of origin for migration in the l -th dimension. For example, activity state is only changeable for consumers that are ‘in-market’ and ‘un-satiated.’ Thus, $\mathcal{R}_3 = \mathcal{A} \cap \mathcal{B}$, where \mathcal{A} and \mathcal{B} are defined as in Equations (8), (9). Let

$$\mathcal{D} = \{\mathbf{s} \in \mathcal{S} : s_4 = \text{‘favorable’}\}$$

and let

$$\mathcal{E} = \{\mathbf{s} \in \mathcal{S} : s_5 = \text{‘loyal’}\}.$$

Then brand favorability migration is limited to $\mathcal{R}_4 = \mathcal{S} \setminus \mathcal{E}$, and brand loyalty migration is limited to $\mathcal{R}_5 = \mathcal{D}$. There are no limitations on brand availability migration, so $\mathcal{R}_6 = \mathcal{S}$.

When ad volume is very large, all members of the audience migrate according to the maximal effect $\mathcal{Q}^{(k)}$, i.e., $Q^{(t,k)} = \mathcal{Q}^{(k)}$. This is generally not the case. Instead, consumers migrate between segments with probabilities that depend on the average frequency of exposure $f_{t,k,\mathbf{s}}$. For each segment, we calculate an effect size, $e_{t,\mathbf{s}} \in [0, 1]$, and use this to scale the transition probabilities against the frequency of exposure $f_{t,k,\mathbf{s}} = v_{t,k,\mathbf{s}}/a_{t,k,\mathbf{s}}$. The effect size $e_{t,\mathbf{s}}$ is calculated from the frequency using the Hill equation (3):

$$e_{t,\mathbf{s}} = H(f_{t,k,\mathbf{s}}; \kappa, \zeta) = \frac{1}{1 + (f_{t,k,\mathbf{s}}/\kappa)^{-\zeta}}.$$

The sigmoid shape of the Hill function enforces diminishing returns of advertising against higher frequency of exposure, and the limited maximal effectiveness of the media. The audience migrates according to the transition matrix

$$Q^{(t,k)} = \text{diag}(\mathbf{e}_{t,\cdot}) \mathcal{Q}^{(k)} + \text{diag}(1 - \mathbf{e}_{t,\cdot}) I_{\#\mathcal{S}},$$

where for any vector \mathbf{v} , $\text{diag}(\mathbf{v})$ is the diagonal matrix with diagonal \mathbf{v} and for any dimensionality d , I_d is the identity matrix of size $d \times d$. This results in entries

$$q_{\mathbf{s},\mathbf{s}'}^{(t,k)} = \begin{cases} e_{t,\mathbf{s}} \mathcal{Q}_{\mathbf{s},\mathbf{s}'}^{(k)} & \mathbf{s} \neq \mathbf{s}' \\ (1 - e_{t,\mathbf{s}}) + e_{t,\mathbf{s}} \mathcal{Q}_{\mathbf{s},\mathbf{s}}^{(k)} & \mathbf{s} = \mathbf{s}' \end{cases}$$

Thus, the migration effect of the media on a segment \mathbf{s} is scaled so the probability of migration away from \mathbf{s} is proportional to $e_{t,\mathbf{s}}$. The updated population segmentation $\mathbf{n}_{t,k+1,\cdot}$ is calculated from $\mathbf{n}_{t,k,\cdot}$, $\mathbf{a}_{t,k,\cdot}$, and $Q^{(t,k)}$ as usual (see (1), (2)).

C.2 Paid search media module

Here, we describe the process used to generate media variables and effects in the paid search media module in more detail.

Campaign settings. As in the traditional media module, downstream variables depend on the budget assigned to the media channel. However, rather than choosing spend according to a flighting pattern, in search advertisers choose campaign settings such as a daily spend cap, a keyword list (the list of search terms on which the advertiser is bidding), and a maximum cost per click (which is equivalent to a bid). The simulator allows the modeler to specify the relationship between budget and each of these quantities. For example, a decrease in budget may be realized via a lowering of bids, and thus a lowering of the CPC and SOV. Alternatively, lowering the budget may be realized via the throttling of paid search volume through a lower spend cap, a contraction of the keyword list, or any combination of the above.

Audience. The calculation of audience size, $\mathbf{a}_{t,k,\cdot}$, in paid search is analogous to that in the traditional media module, except that the probability that individuals from a segment will make matching queries depends on more than just activity state. The probability individuals make matching queries also depends on brand state; this is essential in modeling branded search campaigns. Since the audience is limited to those making queries matching the keyword list, the keyword list length is an additional factor that controls the size of the audience.

Volume and spend. In paid search, there are three types of volume: query, impression, and click volume. The audience in segment \mathbf{s} generates queries following a Poisson distribution with rate $\lambda^{(k)}$. The total number of queries is

$$w_{t,\mathbf{S}}|\mathbf{a}_{t,k,\cdot} \sim \text{Poisson}(\lambda^{(k)}\mathbf{a}_{t,k,\mathbf{S}}).$$

Weekly spend is not directly specifiable through a flighting pattern. Instead, the total number of impressions available to the advertiser, $u_{t,\mathbf{S}}^*$, depends on the query volume, $w_{t,\mathbf{S}}$, and the maximum SOV, $p_t^{(u)}$, that the advertiser can achieve in the current bid landscape. Any query generates an available impression with probability $p_t^{(u)}$, so that the total number of impressions available to the advertiser for the query volume is

$$u_{t,\mathbf{S}}^*|w_{t,\mathbf{S}} \sim \text{Binomial}(w_{t,\mathbf{S}}, p_t^{(u)}).$$

The advertiser's ability to win auctions, and the price at which it wins them, depends on its bid c_t^* . Recall that paid search pricing is parameterized by the minimum $\eta_t^{(m)}$ and maximum $\eta_t^{(M)}$ CPC during each time interval t . In reality, paid search pricing depends on the actions of competitor advertisers, who are also making the full set of decisions we describe for the advertiser. In order to limit the burden on the modeler, we use a simplified competitor model in which the auction environment is summarized by the range $[\eta_t^{(m)}, \eta_t^{(M)}]$. This simplifies the competitive auction dynamics. For example, it doesn't allow competitors to change their bids in response to the advertiser's actions.

The CPC, c_t , increases with the bid, c_t^* , and is bounded by $[\eta_t^{(m)}, \eta_t^{(M)}]$:

$$c_t = \eta_t^{(m)} \vee c_t^* \wedge \eta_t^{(M)}.$$

When the bid c_t^* is less than the minimum CPC $\eta_t^{(m)}$, the maximum SOV the advertiser can attain is 0, and for $c_t^* > \eta_t^{(M)}$, the maximum SOV is 1. We calculate the maximum SOV for c_t^* within the range $[\eta_t^{(m)}, \eta_t^{(M)}]$ by linear interpolation, so that

$$p_t^{(u)} = 0 \vee \frac{c_t^* - \eta_t^{(m)}}{\eta_t^{(M)} - \eta_t^{(m)}} \wedge 1.$$

In addition to search volume being limited by the number of impressions available, it may also be limited by the advertiser's spend cap, $x_t^{(M)}$, which is translated into a cap on the number of clicks, $v_t^{(M)}$, through the CPC c_t :

$$v_t^{(M)} = x_t^{(M)} / c_t.$$

Each impression generates a click via a segment-specific CTR, $\pi_s^{(v)}$. The CTR is higher for more favorable and loyal states. The overall CTR is

$$p_t^{(v)} = \sum_{\mathbf{s} \in \mathcal{S}} \frac{a_{t,k,\mathbf{s}} \pi_{\mathbf{s}}^{(v)}}{a_{t,k,\mathcal{S}}}.$$

This calculation weights $\pi_s^{(v)}$ by the probability that an impression came from segment \mathbf{s} . From the CTR, we can derive the number of impressions $u_t^{(M)}$ needed to reach the click limit $v_t^{(M)}$ imposed by the spend cap:

$$u_t^{(M)} - v_t^{(M)} \sim \text{NegBin}(v_t^{(M)}, p_t^{(v)}),$$

where $\text{NegBin}(r, p)$ represents the number of failures until the r -th success given independent trials with probability of success p .

If the maximum number of clicks is realized within the available number of impressions, the simulator will generate $v_t^{(M)}$ clicks using $u_t^{(M)}$ impressions at time t . Else, the number of clicks is not limited by the spend cap and can be calculated from the hypergeometric distribution:

$$u_{t,\mathcal{S}} | u_{t,\mathcal{S}}^*, u_t^{(M)} = \min\{u_{t,\mathcal{S}}^*, u_t^{(M)}\},$$

$$v_{t,\mathcal{S}} | u_{t,\mathcal{S}}^*, u_t^{(M)} \leq u_{t,\mathcal{S}}^*, v_t^{(M)} = \begin{cases} v_t^{(M)} & u_t^{(M)} \leq u_{t,\mathcal{S}}^* \\ \text{Hyper}(v_t^{(M)}, u_t^{(M)} - v_t^{(M)}, u_{t,\mathcal{S}}^*) & u_t^{(M)} > u_{t,\mathcal{S}}^* \end{cases},$$

where $\text{Hyper}(m, n, k)$ represents the number of successes in k draws from a population with m successes and n failures. This reduces to the binomial distribution for $m = n = \infty$.

For the next step of calculating the effect of paid search, it is important to know the total number of queries, impression, and clicks coming from each segment. These volumes are given by the following multinomial distributions:

$$\mathbf{v}_{t,\cdot} | v_{t,\mathcal{S}} \sim \text{Multinomial} \left(v_{t,\mathcal{S}}, \frac{(a_{t,k,\mathbf{s}} p_{\mathbf{s}}^{(v)})_{\mathbf{s} \in \mathcal{S}}}{\sum_{\mathbf{s} \in \mathcal{S}} a_{t,k,\mathbf{s}} p_{\mathbf{s}}^{(v)}} \right),$$

$$\mathbf{u}_{t,\cdot} - \mathbf{v}_{t,\cdot} | u_{t,\mathcal{S}}, v_{t,\mathcal{S}} \sim \text{Multinomial} \left(u_{t,\mathcal{S}} - v_{t,\mathcal{S}}, \frac{(a_{t,k,\mathbf{s}} (1 - p_{\mathbf{s}}^{(v)}))_{\mathbf{s} \in \mathcal{S}}}{\sum_{\mathbf{s} \in \mathcal{S}} a_{t,k,\mathbf{s}} (1 - p_{\mathbf{s}}^{(v)})} \right),$$

$$\mathbf{w}_{t,\cdot} - \mathbf{u}_{t,\cdot} | w_{t,\mathcal{S}}, u_{t,\mathcal{S}} \sim \text{Multinomial}(w_{t,\mathcal{S}} - u_{t,\mathcal{S}}, \mathbf{a}_{t,k,\cdot} / a_{t,k,\mathcal{S}}).$$

The spend in each segment $\mathbf{s} \in \mathcal{S}$ is $x_{t,\mathbf{s}} = v_{t,k,\mathbf{s}} c_t$.

Effect. The calculation of effect is analogous to the calculations in the traditional media model. The key difference is in the formulation of the effect size, $e_{t,\mathbf{s}}$. We allow $e_{t,\mathbf{s}}$ to vary by query volume $w_{t,\mathbf{s}}$ ⁹, impressions $u_{t,\mathbf{s}}$, and clicks $v_{t,k,\mathbf{s}}$. Given the audience and the volume of search, we can calculate

- the number of people who made a query on the keyword list but were not served the advertiser’s paid search ad
- the number of people who were served a paid search ad but did not click on it (e.g., they may have clicked on a competitor’s result, or one of the advertiser’s organic results)
- the number of people who saw and clicked on the advertiser’s paid search ad.

Let $n_{t,\mathbf{s}}^{(w)}$ be the number of individuals from segment \mathbf{s} making queries, let $n_{t,\mathbf{s}}^{(u)}$ be the number viewing paid impressions, and let $n_{t,\mathbf{s}}^{(v)}$ be the number making paid clicks. Conditional on the aggregate number of queries $w_{t,\mathbf{s}}$ distributed over the query-making population $a_{t,\mathbf{s}}$, the number of queries made by each individual follows a $\text{Multinomial}(w_{t,\mathbf{s}}, (1/a_{t,\mathbf{s}}, \dots, 1/a_{t,\mathbf{s}})^\top)$ distribution. Based on this, we calculate the first and second moments for $n_{t,\mathbf{s}}^{(w)}$ and simulate from a normal approximation. The calculation of the number of people exposed to paid impressions given the total number of impressions $u_{t,\mathbf{s}}$ and the number of individuals making queries $n_{t,\mathbf{s}}^{(w)}$ is analogous, as is the calculation of the number of people making paid clicks given the total number of paid clicks $v_{t,k,\mathbf{s}}$ and the number of individuals exposed to paid impressions $n_{t,\mathbf{s}}^{(u)}$.

For query volume w , paid impressions u , and paid clicks v , the modeler specifies the ratios $\psi_w^{(k)}$, $\psi_u^{(k)}$, and $\psi_v^{(k)}$ of each search volume type’s effectiveness and the maximum effect of search. The relative effectiveness of each volume type is used to scale the effect of search on individuals who made queries without seeing a paid impression, on those who saw a paid impression without clicking, and on those who made a paid click. Generally, paid clicks will be the most effective volume type and attain relative effectiveness $\psi_v^{(k)} = 1$. The effectiveness of organic search is specified by $\psi_w^{(k)}$. It is possible to assign a high value to $\psi_w^{(k)}$ when simulating the behavior of branded search to account for paid search replacing organic search rather than adding incremental impact.

In addition, the simulator simplifies the effect-frequency relationship so that an individual will feel the full effect of any search volume at a frequency of 1. Then, the effect size depends only on the effectiveness and reach of each volume type, and can be calculated as the following weighted sum:

$$e_{t,\mathbf{s}} = \psi_w^{(k)} \frac{n_{t,\mathbf{s}}^{(w)} - n_{t,\mathbf{s}}^{(u)}}{a_{t,k,\mathbf{s}}} + \psi_u^{(k)} \frac{n_{t,\mathbf{s}}^{(u)} - n_{t,\mathbf{s}}^{(v)}}{a_{t,k,\mathbf{s}}} + \psi_v^{(k)} \frac{n_{t,\mathbf{s}}^{(v)}}{a_{t,k,\mathbf{s}}}.$$

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⁹The query volume’s effect on $e_{t,\mathbf{s}}$ can be thought of as the effect on a searcher of seeing the organic search results

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