# Large Scale Distributed Acoustic Modeling With Back-off N-grams

Ciprian Chelba\* and Peng Xu and Fernando Pereira and Thomas Richardson

#### **Abstract**

The paper revives an older approach to acoustic modeling that borrows from n-gram language modeling in an attempt to scale up both the amount of training data and model size (as measured by the number of parameters in the model), to approximately 100 times larger than current sizes used in automatic speech recognition. In such a data-rich setting, we can expand the phonetic context significantly beyond triphones, as well as increase the number of Gaussian mixture components for the context-dependent states that allow it. We have experimented with contexts that span seven or more context-independent phones, and up to 620 mixture components per state. Dealing with unseen phonetic contexts is accomplished using the familiar back-off technique used in language modeling due to implementation simplicity. The back-off acoustic model is estimated, stored and served using MapReduce distributed computing infrastructure.

Speech recognition experiments are carried out in an N-best list rescoring framework for Google Voice Search. Training big models on large amounts of data proves to be an effective way to increase the accuracy of a state-of-the-art automatic speech recognition system. We use 87 000 hours of training data (speech along with transcription) obtained by filtering utterances in Voice Search logs on automatic speech recognition confidence. Models ranging in size between 20–40 million Gaussians are estimated using maximum likelihood training. They achieve relative reductions in word-error-rate of 11% and 6% when combined with first-pass models trained using maximum likelihood, and boosted maximum mutual information, respectively. Increasing the context size beyond five phones (quinphones) does not help.

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#### I. Introduction

As web-centric computing has grown over the last decade, there has been an explosion in the amount of data available for training acoustic and language models for speech recognition. Machine translation [1] and language modeling for Google Voice Search [2] have shown that using more training data is quite beneficial for improving the performance of statistical language models. The same holds true in many other applications as highlighted in [3]. Of equal importance is the observation that the increase in training data amount should be paired with an increase in the model size. This is the situation in language modeling, where word n-grams are the core features of the model and more training data leads to more parameters in the model. We propose a similar approach for automatic speech recognition (ASR) acoustic modeling that is conceptually simpler than established techniques, but more aggressive in this respect.

As a first step, it is worth asking how many training samples are needed to estimate a Gaussian well? Appendix A provides an answer for unidimensional data under the assumption that the n i.i.d. samples are drawn from a normal distribution of unknown mean and variance,  $\mathcal{N}(\mu, \sigma^2)$ . We can place an upperbound on the probability that the sample mean  $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$  is more than  $q \cdot \sigma$  away from the actual mean  $\mu$ , for q small. For example,  $P(|\overline{X} - \mu| > 0.06 \cdot \sigma) < 0.06$  when n = 983; similar values are obtained for the sample variance estimate.

Typical amounts of training data used for the acoustic model (AM) in ASR vary from 100 to 1000 hours. The frame rate in most systems is 100 Hz, (corresponding to advancing the analysis window in 10-millisecond steps), which means that about 360 million samples are used to train the 0.5 million or-so Gaussians in a common state-of-the-art ASR system. Assuming that n = 1000 frames are sufficient for robustly estimating a single Gaussian, then 1000 hours of speech would allow for training about 0.36 million Gaussians. This figure is quite close to values encountered in ASR practice, see Section IV-B, or Table VI in [4]. We can thus say that current AMs achieve *estimation efficiency*: the training data is fully utilized for robust estimation of model parameters.

Recent applications have led to availability of data far beyond that commonly used in ASR systems. Filtering utterances logged by the Google Voice Search service at an adequate ASR confidence threshold, (see [5] for an overview on various confidence measures for ASR), guarantees transcriptions that are close

to human annotator performance, e.g. we can obtain 87 000 hours of automatically transcribed speech at a confidence level of 0.8 or higher in the accuracy of the transcription. If we are to strive for estimation efficiency, then this much speech data would allow training of AMs whose size is about 40 million Gaussians. From a modeling point of view the question becomes: what is the best way to "invest" these parameters to meaningfully model the speech signal?

The most common technique for dealing with data sparsity when estimating context-dependent output distributions for HMM states is the well-known decision-tree (DT) clustering approach [6]. To make sure the clustered states have enough data for reliable estimation, the algorithm guarantees a minimum number of frames at each context-dependent state (leaf of the DT). The data at each leaf is modeled by a Gaussian mixture model (GMM). At the other end of the spectrum, states for which there is a lot more training data should have more mixture components. There is a vast amount of literature on such model selection techniques, see [7] for a recent approach, as well as an overview. [8] shows that an effective way of sizing the GMM output distribution in HMMs as a function of the amount of training data (number of frames n) is the log-linear rule:

$$\log(\text{num. components}) = \log(\beta) + \alpha \cdot \log(n)$$
 (1)

We take the view that we should estimate as many Gaussian components as the data allows for a given state, according to the robustness considerations in Appendix A. In practice one enforces both lower and upper thresholds on the number of frames for a given GMM (see Section IV-C for actual values used in our experiments), and thus the parameters  $\alpha$  and  $\beta$  in (1) are set such that the output distributions for states are estimated reliably across the full range of the data availability spectrum.

As a first direction towards increasing the model size when using larger amounts of training data, we choose to use longer phonetic context than the traditional triphones or quinphones: the phonetic context for an HMM state is determined by M context-independent (CI) phones to the left and right of the current phone and state. We experiment with values for  $M=1,\ldots,3$ , thus reaching the equivalent of 7-phones. For such large values of M not all M-phones (context dependent HMM states in our model), are encountered in the training data. At test time we deal with such unseen M-phones by backing-off, similar to what is done in n-gram language modeling: the context for an unseen M-phone encountered on test data is decreased gradually until we reach an M-phone that we have already observed in training.

The next section describes our approach to increasing the state space using back-off acoustic modeling, and contrasts it with prior work. Section III describes the back-off acoustic model (BAM) implementation using Google's distributed infrastructure, primarily MapReduce [9] and SSTable (immutable persistent

B-tree, see [10]), along similar lines to their use in large scale language modeling for statistical machine translation [1]. Section IV presents our experiments in an N-best list rescoring framework, followed by conclusions. The current paper is a more comprehensive description of the experiments reported in [11].

#### II. BACK-OFF N-GRAMS FOR ACOUSTIC MODELING

Consider a short utterance whose transcription is:  $W = \langle S \rangle$  action  $\langle S \rangle$ , and assume the pronunciation lexicon provides the following mapping to CI phones sil ae k sh ih n sil. We use  $\langle S \rangle$ ,  $\langle S \rangle$  to denote sentence boundaries, both pronounced as long silence sil.

A triphone approach would model the three states of ih as sh-ih+n\_{1,2,3} using the DT clustering algorithm for tying parameters across various instances \*-ih+\*\_{1,2,3}, respectively. This yields the so-called context-dependent states in the HMM.

In contrast, BAM with M=3 extracts the following training data instances (including back-off) for the first HMM state of the ih instance in the example utterance above:

There are other possible back-off strategies, but we currently implement only the one above:

- if the M-phone is symmetric (same left and right context length), then back-off at both ends
- if not, then back-off from the longer end until the M-phone becomes symmetric, and proceed with symmetric back-offs from there on.

To achieve this we first compute the context-dependent state-level Viterbi alignment between transcription W and speech feature frames using the transducer composition  $H \circ C \circ L \circ W$ , where L, C, H denote respectively the pronunciation lexicon, context dependency tree, and HMM-to-state FST transducers [12]. From the alignment we then extract M-phones along with the corresponding sequence of speech feature frames. Each M-phone is uniquely identified by its key, e.g.  $ih_1 / ae k sh ___ n sil$ . The key is a string representation obtained by joining on / the central CI-state, i.e.  $ih_1 above$ , and the surrounding phonetic context, in this case  $ae k sh __ n sil$ ; main sil; main sil; main sil and order M-phones, we also collect back-off M-phones as outlined above. With each back-off we clone the frames from the maximal order M-phone to the back-off one. We found it useful to augment the phonetic context with word boundary information. The word boundary has its own symbol, and occupies its own context position.

All M-phone instances encountered in the training data are aggregated using MapReduce. For each M-phone that meets a threshold on the minimum number of frames aligned against itself, we estimate a GMM using the standard splitting algorithm [13], following the rule in (1) to size the GMM. The M-phones that have more frames than an upper threshold on the maximum number frames (256k in our experiments),<sup>1</sup> are estimated using reservoir sampling [14]. Variances that become too low are floored to a small value (0.00001 in our experiments).

# A. Comparison with Existing Approaches and Scalability Considerations

BAM can be viewed as a simplified version of DT state clustering that uses question sets consisting of atomic CI phones, queried in a pre-defined order. This very likely makes BAM sub-optimal relative to standard DT modeling, yet we prefer it due to ease of implementation in MapReduce.

The approach is not novel: [15] proposes a very similar strategy where the probability assigned to a frame by a triphone GMM is interpolated with probabilities assigned by left, right diphone GMMs, and CI phone GMMs, respectively. However, the modeling approach in BAM is not identical to [15] either: the former does indeed back-off in that it uses only the maximum order M-phone found in the model, whereas the latter interpolates up the back-off tree, and allows asymmetric back-offs. It is of course perfectly feasible to conceive BAM variants that come closer to the approach in [15] by using interpolation between M-phones at various orders.

Scalability reasons make the current BAM implementation an easier first attempt when using very large amounts of training data: a BAM with M=5 estimated on 87 000 hours of training data leads to roughly 2.5 billion (2 489 054 034) 11-phone types. DT building requires as sufficient statistics the single-mixture Gaussians for M-phones sharing the same central CI phone and state. Assuming uniformity across central phone and state identity, we divide the total number of M-phones by the number of phones (40) times the number of states/phone (3) to arrive at about 25 million different M-phones that share a given central state and phone. Storing a single-mixture Gaussian for each M-phone requires approximately 320 bytes  $(39 \cdot 4 \cdot 2)$ . Under the uniformity assumption above, the training data for each DT amounts to about  $25 \cdot 320 = 8$  GB of storage. It is more realistic to assume that some central CI phones will have ten times more M-phones than the average, leading to a memory footprint of 80 GB, which starts to become problematic although still feasible (perhaps by employing sampling techniques, or reducing the context size M).

<sup>&</sup>lt;sup>1</sup>For convenience, we use the "k" shorthand to denote thousands, e.g. we write 256k instead of 256 000; a value of 41 898 799 is rounded to 41 899k.

To avoid such scalability issues, we resort to MapReduce and streaming the data for M-phones sharing a given central triphone to the same Reducer, one maximal order M-phone at a time, as described in Section III. M-phones at lower orders  $1 \dots M-1$  are estimated by accumulating the data arriving at the Reducer into buffers of fixed capacity, using reservoir sampling [14] to guarantee a random sample of fixed size of the input data stream. With careful sorting of the M-phone stream (see Section III), M-1 reservoirs are sufficient for buffering data on the Reducer until all the data for a given M-phone arrives, the final GMM for a given M-phone is estimated and output, and the respective buffer is flushed. The reservoir size thus controls the memory usage very effectively. For example, when using 256k as the maximum number of frames for estimating a given GMM (equal to the maximum reservoir size), only 160 MB of RAM are sufficient for building a BAM with M=5.

Our approach to obtaining large amounts of training data is very similar to that adopted in [4]. Table VI there highlights the gains from using increasing amounts of training data from 375 hours to 2210 hours, and shows that past 1350 hours a system with 9k states and about 300k Gaussians gets diminishing returns in accuracy. Our modeling approach and its implementation using MapReduce allows both the use of significantly more training data and estimation of much larger models: in our experiments we used 87 000 hours of training data and built models of up to 1.1 million states and 40 million Gaussians.

# III. DISTRIBUTED ACOUSTIC MODELING

BAM estimation and run-time are implemented using MapReduce and SSTable, and draw heavily from the large language modeling approach for statistical machine translation described in [1].

# A. BAM Estimation Using MapReduce

Our implementation is guided by the large scale n-gram language model estimation work of [16]. MapReduce is a framework for parallel processing across huge datasets using a large number of machines. The computation is split in two phases: a *Map* phase, and a *Reduce* one. The input data is assumed to be a large collection of key-value pairs residing on disk, and stored in a distributed file system. MapReduce divides it up into *chunks*. Each such chunk is processed by a Map worker called a *Mapper*, running on a single machine, and whose entire lifetime is dedicated to processing one such data chunk.

Mappers are stateless, and for each input key-value pair in a given chunk they output one or more new key-value pairs; the computation of the new value, and the assignment of the new output key are left to the user code implementing a Mapper instance. The entire key space of the Map output is disjointly

partitioned according to a *sharding function*: for any key value output by Map, we can identify exactly one Reduce *shard*.

The key-value pairs output by all Mapper instances are routed to their corresponding Reduce shards by a *Shuffler*, using the sharding function mentioned above. The Shuffler also collates all values for a given key, and presents the tuple of values along with the key to the *Reducer* (a Reduce worker), as one of the many inputs for a given Reduce shard. It also sorts the keys for a given shard in lexicographic order, which is the order in which they are presented to the Reducer. Each Reduce worker processes all the key-value pairs for a given Reduce shard: the Reducer receives a key along with all the values associated with it as output by all the Mappers, and collected by the Shuffler; for a given input key, the Reducer processes all values associated with it, and outputs one single key-value pair. It is worth noting that the Reducer cannot change the identity of the key it receives as input. The output from the Reduce phase is stored in an *SSTable*: an immutable persistent B-tree<sup>2</sup> associative array of key-value pairs (both key and value are strings), that is partitioned according to the same sharding function as the one used by the MapReduce that produced it. Another SSTable feature is that it can be used as a distributed inmemory key-value serving system (*SSTable service*) with *S* servers (machines), each holding a partition containing 1/S of the total amount of data. This allows us to serve models larger than what would fit into the memory of a single machine.

Fig. 1 describes the training MapReduce, explained in more detail in the following two subsections.

- 1) Mapper: Each Mapper instance processes a chunk of the input data, one record at a time. Each record consists of a key-value pair; the value stores the waveform, the word level transcript for the utterance, and other elements. For each record arriving at the Mapper we:
  - generate the context-dependent state-level Viterbi alignment by finding the least cost path through the state space of the FST  $H \circ C \circ L \circ W$  using the the first-pass AM
  - extract maximal order M-phones along with speech frames, and output (M-phone key, frames) pairs
  - compute back-off M-phones and output (M-phone key, empty) pairs.

We note that in order to avoid sending too much data to the Reducer, we do not copy the frames to the back-off M-phones, which would lead to replicating the input data M times. To make sure that the data needed for estimating back-off M-phones is present at a given Reducer we resort to a few tricks:

• the sharding function takes as argument the central triphone. This guarantees that all M-phones sharing a given central triphone (sh-ih+n in our example), are handled by the same Reducer

<sup>&</sup>lt;sup>2</sup>A format similar to SSTable has been open-sourced as part of the LevelDB project http://code.google.com/p/leveldb/

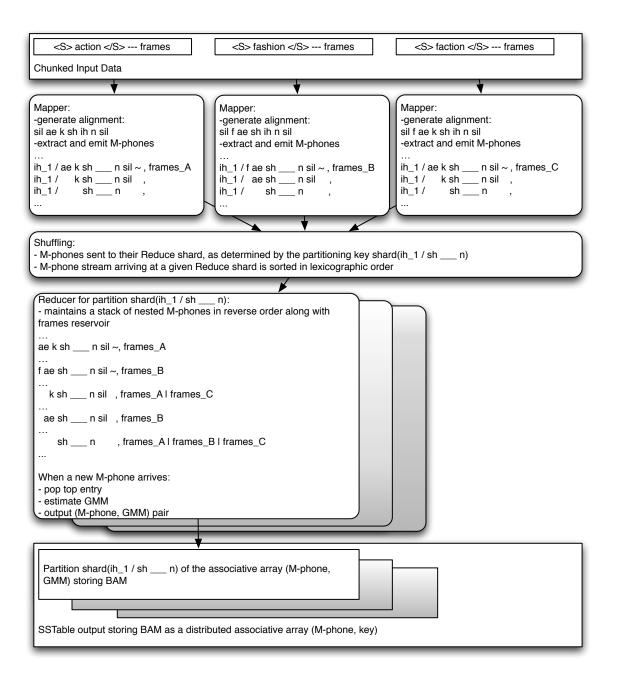


Fig. 1. MapReduce Estimation for Back-Off Acoustic Model

• the M-phones need to arrive at the Reducer in a certain order, since only the maximal order M-phones carry speech frame data. The sorting of the M-phone stream needs to be such that any given maximal order M-phone arrives at the Reducer before all of its back-off M-phones; this allows us to buffer the frames for all the back-off M-phones down to the central triphone state. We accomplish this by relying

on the implicit lexicographic sorting of the keys, and re-keying each M-phone before outputting it such that the context CI-phones are listed in proximity order to the central one; missing context CI-phones (due to utterance boundaries), are represented using ~ to ensure correct sorting. For example ih\_1 / ae k sh \_\_\_\_ n sil is actually keyed as: ih\_1 / sh n k sil ae ~, to guarantee that M-phones sharing the central triphone ih\_1 / sh \_\_\_\_ n are processed in order of longest to shortest context at the Reducer processing the partition partition (ih\_1 / sh \_\_\_\_ n).

2) Reducer: After shuffling, each M-phone has its frame data (if carrying any), collated and presented to the Reducer along with the M-phone key. Since the Reducer cannot change the key of the input, it needs to output the GMM for an M-phone when it arrives at the Reducer. The sorting described in Section III-A1 guarantees that the M-phones sharing the same central triphone arrive in the correct order (high to low M-phone order). Every time a maximal order M-phone arrives at the Reducer we estimate a GMM from its data (assuming the number of frames is above the lower threshold), and also accumulate its data in the *reservoirs* for all of its back-off M-phones which are buffered in a "first-in first-out" stack.

Reservoir sampling is a family of randomized algorithms for randomly choosing K samples from a list L containing n items, where n is either a very large or unknown number. Our implementation populates the reservoir with the first K samples to arrive at the Reducer. If more samples arrive after that, we draw a random index r in the range [0, current sample index - 1); if r < K we replace the sample at index r in the reservoir with the newly arrived one, and otherwise we ignore it. Every time a back-off M-phone arrives at the Reducer, it is guaranteed to be the same one as the M-phone at the top of the stack due to the sorting of the M-phone stream done by the Shuffler. We then:

- add to the reservoir at the top of the stack any frames that arrived at the Reducer with the current M-phone;
- pop the M-phone and the corresponding reservoir from the top of the stack;
- estimate the GMM for this back-off M-phone if the accumulated frames exceed the lower threshold on the minimum number of frames, or discard the M-phone and its data otherwise;
- output the pair (M-phone key, GMM).

Due to the particular sorting of the M-phone stream, the Reducer is guaranteed to have seen all the frame data for an M-phone when the GMM estimation takes place. The resulting SSTable stores the BAM as a distributed (partitioned) associative array (M-phone key, GMM).

#### B. BAM Test Run-time Using SSTable Service

At test time we rescore N-best lists for each utterance using BAM. We load the model into an SSTable service with S servers, each holding 1/S of the data. For each hypothesis in the N-best list, we:

- generate the context-dependent state-level Viterbi alignment after composing  $H \circ C \circ L$  with the transcript W from the first-pass; the alignment is generated using the first-pass AM and saved with the hypothesis
- extract maximal order M-phones
- compute back-off M-phones
- add all M-phones to a pool initialized once per input record (utterance).

Once the pool is finalized, it is sent as a batch request to the SSTable service. The M-phones that are actually stored in the model are returned to the Mapper, and are used to rescore the alignment for each of the hypotheses in the N-best list. For each segment in the alignment we use the highest order M-phone that was retrieved from the BAM SSTable. If no back-off M-phones are retrieved for a given segment, we back-off to the first-pass AM score for that segment which is computed during the Viterbi alignment.

To penalize the use of lower order M-phones, the score for a segment with an M-phone of lower order o ( $o \ge 0$ ) than the maximum one M incurs a per-frame back-off cost. The order of an asymmetric M-phone is computed as the maximum of the left and right context lengths. The per-frame back-off cost reaches its maximum value when the model backs-off all the way to using the first-pass AM (DT clustered state), o = 0. To formalize, assume that we are using a GMM with Q components for modeling M-phone s, and that the order of s is o(s), computed as described above. The log-likelihood assigned to a frame g aligned against state s will be:

$$\log P_{s}(\boldsymbol{y}) = \log \sum_{q=1}^{Q} m_{q} \cdot P(\boldsymbol{y} | \boldsymbol{\mu}_{s,q}, \boldsymbol{\Sigma}_{s,q}) - f_{bo} \cdot (M - o(s))$$
(2)

where  $f_{\text{bo}} \geq 0$  is the per-frame back-off cost, and  $m_q$  are the mixture weights for each component of the GMM for state s:  $P(y|\mu_{s,q}, \Sigma_{s,q})$ .

The final score for each hypothesis W,  $\log P(W, A, V)$ , is computed by log-linear interpolation between the first-pass AM and that obtained from the second pass one (BAM, or first-pass AM if running sanity checks, see Table II), followed by the usual log-linear combination between AM and language

model (LM) scores:

$$\log P_{\text{AM}}(\boldsymbol{A}|\boldsymbol{W},\boldsymbol{V}) = \lambda \cdot \log P_{\text{1st pass}}(\boldsymbol{A}|\boldsymbol{W},\boldsymbol{V}) +$$

$$(1.0 - \lambda) \cdot \log P_{\text{2nd pass}}(\boldsymbol{A}|\boldsymbol{W},\boldsymbol{V}) -$$

$$\log(Z_1)$$

$$\log P(\boldsymbol{W},\boldsymbol{A},\boldsymbol{V}) = 1/w_{LM} \cdot \log P_{\text{AM}}(\boldsymbol{A}|\boldsymbol{W},\boldsymbol{V}) +$$

$$\log P_{\text{LM}}(\boldsymbol{W}) - \log(Z_2),$$

$$(4)$$

where A denotes the acoustic features, W denotes the word sequence in an N-best hypothesis,  $w_{LM}$  is the language model weight, P(W, A, V) is the probability assigned to the word sequence W and the corresponding acoustic features A by using V as a state-level alignment, and  $\log(Z_1)$ ,  $\log(Z_2)$  are normalization terms ignored in rescoring; both first-pass AM and BAM pair states with frames using the same state-level Viterbi alignment V computed using the first-pass AM.

#### IV. EXPERIMENTS

We ran our experiments on Google Voice Search training and test data. The subsections below detail the training and test setup, as well as the baseline acoustic models and their performance.

### A. Task Description

There are two training sets that we used in our experiments:

- maximum likelihood (ML) baseline: 1 million manually transcribed Voice Search spoken queries, consisting of 1300 hours of speech (468 887 097 frames);
- *filtered logs*: 110 million Voice Search spoken queries along with 1-best ASR transcript, filtered by confidence at 0.8 threshold, consisting of 87 000 hours of speech (31 530 373 291 frames). The query-level confidence score used for filtering training data transcriptions is derived using standard lattice-based word posteriors. The best baseline AM available, namely the boosted maximum mutual information (bMMI) baseline AM trained as we describe in Section IV-B is used for generating both transcriptions and confidence scores.

As development and test data we used two sets of manually transcribed data that do not overlap with the training data (the utterances originate from non-overlapping time periods in our logs). Let's denote them as data sets DEV, and TEST, consisting of 27 273 and 26 722 spoken queries (87 360 and 84 918 words), respectively. All query data used in the experiments (training, development and test), is anonymized.

As a first attempt at evaluating BAM, we carry out N-best list rescoring experiments with N=10. While 10-best may seem small, such N-best lists have approximately 7% oracle word-error-rate (WER)<sup>3</sup> on our development data set, starting from 15% WER baseline. Also, as shown in Section IV-G, about 80% of the test set achieves 0% oracle WER at 10-best, so there is plenty of room for improvement when doing 10-best list rescoring. In addition to this, very large LM rescoring experiments for the same task, e.g. [2], have shown that 10-best list rescoring was very close to full lattice rescoring.

#### B. First Pass Acoustic Models

The feature extraction front-end is common across all experiments:

- the speech signal is sampled at 8 kHz, and quantized linearly on 16 bits
- 13-dimensional perceptual linear predictive (PLP) coefficients [17] are extracted every 10 ms using a raised cosine analysis window of size 25 ms, consisting of 200 samples zero-padded to 512 samples
- 9 consecutive PLP frames around the current one are then stacked to form a 117-dimensional vector
- a joint transformation estimated using linear discriminant analysis (LDA) followed by semi-tied covariance (STC) modeling [18] reduces the feature vector down to 39 dimensions in a way that minimizes the loss from modeling the data with a diagonal covariance Gaussian distribution.

Since BAM uses ML estimation, we decided to use two baseline AMs in our experiments: an ML baseline AM that matches BAM training, and a discriminative (bMMI) baseline AM which produces the best available results on our development and test data. All models use diagonal covariance Gaussians.

The ML AM used in the first-pass is estimated on the ML baseline data in the usual staged approach:

- 1) three-state, CI phone HMMs with output distributions consisting of single Gaussian, diagonal covariance
- 2) standard DT clustering for triphones, producing 8k context-dependent states
- 3) GMM splitting, resulting in a model with 330k Gaussians:
  - the minimum number of frames  $N_{\min}$  for a given context-dependent state is 18k, enforced during DT building;
  - the maximum number of frames  $N_{\rm max}$  for a given context-dependent state is 256k; GMMs for states with more than the maximum number of frames are estimated by random sampling down to 256k frames

<sup>&</sup>lt;sup>3</sup>The oracle WER measures the WER along the hypothesis in the N-best list that is closest in string-edit distance to the transcription for that utterance.

• *varmix* estimation is used to determine the number of mixtures according to the amount of training data, as in (1) with  $\alpha = 0.3$ ,  $\beta = 2.2$ ; this amounts to 42 components when the number of frames n is at its minimum value of 18k, and 92 mixture components when it is at its maximum value of 256k.

The bMMI baseline AM is obtained by running an additional discriminative training stage on significantly more training data than the ML baseline:

4) bMMI training [19] on the ML baseline data augmented with 10 million Voice Search spoken queries (approximately 8000 hours) and 1-best ASR transcript, filtered by confidence.

Training and test are matched with respect to the first-pass AM used: experiments reporting development and test data results using the ML baseline AM use a BAM trained on alignments generated using the same ML baseline AM; likewise, when switching to the bMMI baseline AM we use it to generate training, development and test data alignments.

# C. N-best List Rescoring Experiments using ML Baseline AM

The development data is used to optimize the following parameters for BAMs trained on the ML baseline data, as well as 1%, 10% and 100% of the filtered logs data, respectively:

- model order  $M = 1, 2, \dots, 3$  (triphones to 7-phones),
- acoustic model weight in log-linear mixing of first-pass AM scores with the rescoring AM, (3):  $\lambda = 0.0, 0.2, 0.4, \dots, 1.0,$
- language model weight, (4):  $w_{LM} = 7, 12, ..., 22$ ,
- per-frame back-off weight, (2):  $f_{\rm bo}=0.0,0.2,\ldots,1.0.$

Across all experiments reported in this section we kept the following constant:

- the ML baseline AM is trained on the ML baseline data,
- minimum number of frames for an M-phone state  $N_{\min}$  is 4k except for one experimental condition setting it to 18k to compare against the ML baseline AM, see Table II,
- maximum number of frames (reservoir size),  $N_{\text{max}}$  for an M-phone state is 256k:
  - for the  $\alpha=0.3$  and  $\beta=2.2$  varmix setting this means a maximum number of 92 mixture components per state
  - for the  $\alpha=0.7$  and  $\beta=0.1$  varmix setting this means 620 mixture components per state; the GMM splitting becomes very slow for such large numbers of mixture components, so we only trained M=1 models for this setting.

TABLE I

MAXIMUM LIKELIHOOD BACK-OFF ACOUSTIC MODEL (BAM) RESULTS ON THE DEVELOPMENT SET, 10-BEST RESCORING, IN VARIOUS TRAINING AND TEST REGIMES.

| Model  | WER (S/D/I),               | No.       |
|--|----------------------------|-----------|
|  | (%)                        | Gaussians |
| TRAINING DATA = ML baseline data (1.3k hours)  |                            |           |
| ML baseline AM, $\lambda = 0.0, w_{LM} = 17$   | 19.1 (2.3/4.3/12.5)        | 327k      |
| ML baseline AM, $\lambda = 0.6$ , $w_{LM} = 17$  | 18.5 (2.2/4.2/12.0)        | 327k      |
| ML baseline AM, $\lambda = 1.0$ (first-pass), $w_{LM} = 17$  | <u>18.8</u> (2.3/4.3/12.2) | 327k      |
| TRAINING DATA = 100% filtered logs data (87k hours)  |                            |           |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 1$ , $\lambda = 0.0$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 18.0 (2.4/3.9/11.7)        | 3213k     |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 1$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 17.1 (2.2/3.8/11.1)        | 3213k     |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 2$ , $\lambda = 0.0$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 17.7 (2.0/4.2/11.6)        | 22 210k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 2$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 16.8 (2.0/3.9/10.9)        | 22 210k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.0$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 18.0 (2.0/4.2/11.8)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 16.9 (2.0/3.9/11.0)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.2$ | 16.8 (2.0/3.8/11.0)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.4$ | 16.8 (2.0/3.8/11.0)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.6$ | 16.8 (2.0/3.8/11.0)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.8$ | 16.9 (2.0/3.8/11.1)        | 41 899k   |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 1.0$ | 16.9 (2.0/3.8/11.1)        | 41 899k   |

1) Development Set Results: Table I shows the most relevant results when rescoring 10-best lists with BAM in the log-linear interpolation (4); S/D/I denotes Substitutions, Deletions and Insertions, respectively. We built and evaluated models for  $M=1,2,\ldots,5$  but as the results in Table I show, there is no gain in performance for values of M>2; since training such models is expensive, we stopped early on

experimenting with models at M=4,5, and as such Table I reports results for M=1,2,3 only.

The first three rows show the performance, and size (in number of Gaussians), of the ML AM baseline (stage 3 in Section IV-B) on the development set DEV. Somewhat surprisingly, there is a small gain (0.3% absolute) obtained by interpolating the first and second pass scores produced by the ML baseline AM for the same utterance, as well as a loss of 0.3% absolute when the N-best list is rescored with the same AM. We point out this oddity because the same second pass alignments are rescored with BAM, and hence this small improvement should not be credited to better modeling using BAM, but rather to

re-computation of alignments in the second pass for each N-best hypothesis individually. This discrepancy could be due to one or more possible sources of mismatch between the first-pass system and the N-best list rescoring one<sup>4</sup>:

- different frame level alignments for the same word hypothesis. This could happen due to the fact that the rescoring system uses extremely wide beams when computing the alignment for each hypothesis in the N-best list, as well as the fact that some optimizations in the generation of the first-pass static CLG FST network may not be matched when aligning a given hypothesis W using  $H \circ C \circ L \circ W$ ;
- slightly different acoustic model settings in computing the log-likelihood of a frame;
- slightly different front-end configurations between first-pass and rescoring.

The per-frame back-off (2) does not make any difference at all for M=1,2 models (we do not include the results in Table I since they are identical to those obtained under the  $f_{\rm bo}=0.0$  condition), and has a minimal impact on the M=3 model.

Another point worth making is that BAM stands on its own, at least in the N-best list rescoring framework investigated: comparing the rows for  $\lambda = 0.0$ , we observe that BAM improves over the ML baseline AM for all values M = 1, 2, 3, with the optimal value being M = 2.

Finally, the fact that the larger M=3 value does not improve performance over the M=2 model despite the availability of data to estimate GMMs reliably is an interesting result in its own right, suggesting that simply increasing the phonetic context beyond quinphones may in fact weaken the acoustic model.

2) Test Set Results: Table II shows the results of rescoring 10-best lists with the BAM in the log-linear interpolation setup of (4), along with the best settings as estimated on the development data.

The first training regime for BAM used the same training data as that used for the ML part of the baseline AM training sequence. When matching the threshold on the minimum number of frames to the threshold used for the baseline AM (18k), BAM ends up with fewer Gaussians than the baseline AM: 223k vs. 327k. This is not surprising, since no DT clustering is done, and the data is not used as effectively: many triphones (i.e., M=1) are discarded, along with their data. However, its performance matches that of the baseline AM in a 10-best list rescoring setup; no claims are made about the performance of such a model in the first pass. Lowering the threshold on the minimum number of frames to 4k (26 mixture

<sup>&</sup>lt;sup>4</sup>We tried our best to minimize this discrepancy but given the many parameter settings in an ASR system this task has proven to be very difficult. The small difference reported was the best we could achieve after spending a significant amount of time on this issue.

TABLE II

MAXIMUM LIKELIHOOD BACK-OFF ACOUSTIC MODEL (BAM) RESULTS ON THE TEST SET TEST, 10-BEST LIST

RESCORING, IN VARIOUS TRAINING REGIMES.

| Model   | WER (S/D/I),              | No.       |  |  |
|---|---------------------------|-----------|--|--|
|   | (%)                       | Gaussians |  |  |
| TRAINING DATA = ML baseline data (1.3k hours)   |                           |           |  |  |
| ML baseline AM, $\lambda = 0.0$ , $w_{LM} = 17$   | 12.4 (1.3/2.5/8.6)        | 327k      |  |  |
| ML baseline AM, $\lambda = 0.6$ , $w_{LM} = 17$   | 11.6 (1.2/2.3/8.1)        | 327k      |  |  |
| ML baseline AM, $\lambda = 1.0$ (first-pass), $w_{LM} = 17$   | <u>11.9</u> (1.2/2.4/8.3) | 327k      |  |  |
| TRAINING DATA = ML baseline data (1.3k hours)   |                           |           |  |  |
| BAM $N_{\min} = 18k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 1$ , $\lambda = 0.8$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ | 11.6 (1.2/2.2/8.2)        | 223k      |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 1$ , $\lambda = 0.8$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$  | 11.5 (1.2/2.2/8.1)        | 490k      |  |  |
| TRAINING DATA = 1% filtered logs data (870 hours)   |                           |           |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 2$ , $\lambda = 0.8$ , $w_{LM} = 17$ , $f_{\text{bo}} = 1.0$  | 11.3 (1.2/2.2/7.9)        | 600k      |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.7$ , $\beta = 0.1$ , $M = 1$ , $\lambda = 0.8$ , $w_{LM} = 12$ , $f_{\text{bo}} = 0.0$  | 11.4 (1.1/2.3/8.0)        | 720k      |  |  |
| TRAINING DATA = 10% filtered logs data (8.7k hours)   |                           |           |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 2$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.4$  | 10.9 (1.1/2.2/7.7)        | 3975k     |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.7$ , $\beta = 0.1$ , $M = 1$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$  | 10.9 (1.1/2.2/7.6)        | 4465k     |  |  |
| TRAINING DATA = 100% filtered logs data (87k hours)   |                           |           |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 2$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$  | 10.6 (1.0/2.2/7.4)        | 22 210k   |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.7$ , $\beta = 0.1$ , $M = 1$ , $\lambda = 0.6$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$  | 10.6 (1.2/2.0/7.3)        | 14 435k   |  |  |

components at  $\alpha = 0.3$ ,  $\beta = 2.2$ ), does increase the number of Gaussians in the model to 490k.

The second training regime for BAM uses the filtered logs data, in varying amounts: 1%, 10%, 100%, respectively. A surprising result is that switching from manually annotated data to the same amount of confidence filtered data provides a small absolute WER gain of 0.1–0.2%. This suggests that the confidence filtered data is just as good as the manually annotated data for training acoustic models that are used in an N-best list rescoring pass.

From then on, BAM steadily improves as we add more filtered logs training data in both the  $\alpha=0.3$ ,  $\beta=2.2$  and  $\alpha=0.7$ ,  $\beta=0.1$  setups, respectively: the first ten-fold increase in training data brings a 0.4–0.5% absolute WER reduction, and the second one brings another 0.3% absolute WER reduction. As shown in Fig. 2, the WER decreases almost linearly with the logarithm of the training data size.

The BAM WER gain amounts to 1.3% absolute reduction (11% relative) on the one-pass baseline

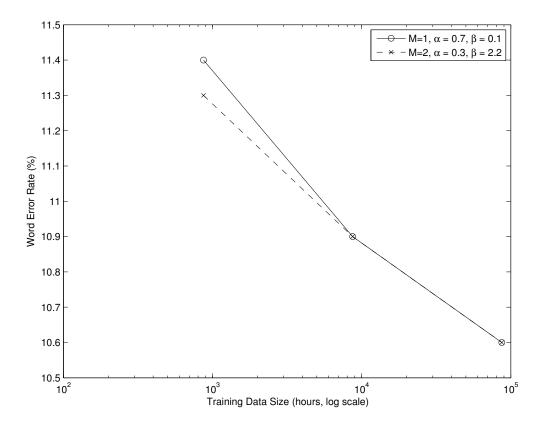


Fig. 2. ASR Word Error Rate as a Function of Training Data Size, for two different BAM configurations. The WER decreases almost linearly with the logarithm of the training data size, and it is marginally influenced by the BAM order (context size).

of 11.9% WER. Comparing the baseline results when using ML and bMMI models, respectively (see Tables II and III), we note that BAM does not fully close the 18% relative difference between the ML and the bMMI first-pass AMs performance, which leaves open the possibility that a discriminatively trained BAM would yield additional accuracy gains.

As Fig. 3 shows, the best predictor for model performance is the number of mixture components, which is consistent with the results on development data, and across the two different  $\alpha$ ,  $\beta$  settings we experimented with. The best model order M is between M=1 and M=3 (depending on the maximum number of mixtures/state allowed in the model). In fact, with enough mixtures per M-phone, triphones (i.e., M=1) perform just as well as quinphones (i.e., M=2) or 7-phones.

# D. N-best List Rescoring Experiments using bMMI Baseline AM

When switching to using the bMMI AM (stage 4 in Section IV-B) as the first-pass model in both training and test, the baseline results are significantly better, see Table III. Despite the fact that it is not

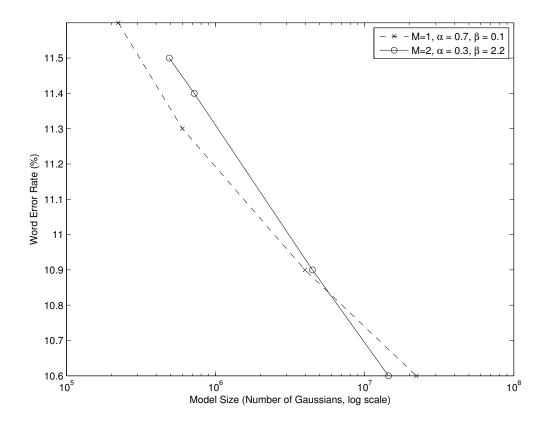


Fig. 3. ASR Word Error Rate as a Function of Model Size, for two different BAM configurations. The WER decreases almost linearly with the logarithm of the model size (measured in number of Gaussians), and it is marginally influenced by the BAM order (context size).

discriminatively trained, BAM still provides 0.6% absolute (6% relative) reduction in WER.

# E. M-phone Hit Ratios and Other Training Statistics

Similar to n-gram language modeling, we can compute M-phone *hit ratios* at various orders: the percentage of M-phones encountered in the test data (10-best hypotheses), with left, right context of length l, r, respectively, that are also present in the model (and thus there is no need to back-off further); Table IV shows the values for BAM trained on the filtered logs data (87 000 hours). M-phones at query boundaries do not have symmetric context, which explains the non-zero off-diagonal values. The maximal order M-phones (sum across last row and column), amount to 42.3% of the total number of M-phones encountered on 10-best list rescoring, with 23.6% at the highest order 3, 3.

We also note that only on 1.1% of test segments do we back-off out of the M-phones stored in BAM, and use the GMM stored with the clustered state in the first-pass AM. This shows convincingly that

| Model  | WER (S/D/I),   | No.       |  |  |  |
|--|--|-----------|--|--|--|
|  | (%)  | Gaussians |  |  |  |
| TRAINING DATA = ML baseline data (1.3k hours) + 10k hours filtered logs data   | TRAINING DATA = ML baseline data (1.3k hours) + 10k hours filtered logs data |           |  |  |  |
| bMMI baseline AM, $\lambda = 0.0, w_{LM} = 17$   | 10.2 (1.1/1.7/7.4)   | 327k      |  |  |  |
| bMMI baseline AM, $\lambda = 0.6, w_{LM} = 17$   | 9.7 (1.1/1.6/7.0)  | 327k      |  |  |  |
| bMMI baseline AM, $\lambda = 1.0$ (first-pass), $w_{LM} = 17$  | <u>9.8</u> (1.1/1.6/7.1)   | 327k      |  |  |  |
| TRAINING DATA = 100% filtered logs data (87k hours)  |  |           |  |  |  |
| BAM $N_{\min} = 4k$ , $\alpha = 0.3$ , $\beta = 2.2$ , $M = 3$ , $\lambda = 0.8$ , $w_{LM} = 17$ , $f_{\text{bo}} = 0.0$ ) | 9.2 (1.0/1.6/6.7)  | 40 360k   |  |  |  |

as both the amount of training data and the model size increase, the DT clustering of triphone states is no longer necessary as a means to cope with triphones that are unseen or have too little training data. Tables V and VI show the distribution of Gaussian mixtures, and M-phone types at various orders,

| left, right  |      |       |       |       |
|--------------|------|-------|-------|-------|
| context size | 0    | 1     | 2     | 3     |
| 0            | 1.1% | 0.1%  | 0.2%  | 4.3%  |
| 1            | 0.1% | 26.0% | 0.9%  | 3.4%  |
| 2            | 0.7% | 0.9%  | 27.7% | 2.2%  |
| 3            | 3.8% | 2.9%  | 2.0%  | 23.6% |

respectively. The total number of Gaussian mixtures in the model is 41 898 799, and the total number of M-phone types is 1 146 359, achieving our goal of scaling the AM 100 times larger than the size of the first-pass AM, which consists of 0.3 million Gaussians and about 8k context-dependent states.

# F. Data Flow in Training MapReduce

The filtered logs training data consists of approximately 110 million Voice Search spoken queries, or 87 000 hours of speech, or 31.5 billion frames; on disk it is stored as compressed SSTables at around

TABLE V  ${\it Number of Gaussian \, Mixtures \, at \, Various \, M-phone \, Orders \, for \, BAM \, Using \, M=3 \, (7\text{-phones}) \, Trained \, on }$  the Filtered Logs Data

| left, right |         |           |            |            |
|-------------|---------|-----------|------------|------------|
| context     |         |           |            |            |
| size        | 0       | 1         | 2          | 3          |
| 0           | 0       | 9802      | 138 694    | 776 488    |
| 1           | 9619    | 3 193 495 | 581 846    | 1 072 242  |
| 2           | 143 940 | 613 401   | 17 519 632 | 1 134 640  |
| 3           | 843 282 | 1 274 683 | 1 127 789  | 13 459 246 |

TABLE VI  ${\it Number of M-phone Types at Various Orders for BAM Using } M=3~(7\mbox{-phones}) {\it Trained on the Filtered Logs Data}$ 

| left, right |        |        |         |         |
|-------------|--------|--------|---------|---------|
| context     |        |        |         |         |
| size        | 0      | 1      | 2       | 3       |
| 0           | 0      | 114    | 1902    | 14 384  |
| 1           | 115    | 55 551 | 11 942  | 27 426  |
| 2           | 2124   | 12 673 | 491 528 | 32 248  |
| 3           | 15 858 | 33 035 | 32 717  | 414 742 |

5.7 TB. The output of the Map phase consists of about 8.54 TB uncompressed data, which is processed by the Reduce function.

Of the total of 184 million M-phones encountered in the training data (including back-offs), only one million pass the lower threshold on the number of frames (4k); of those, approximately 36k (3.6%) have more frames than the upper threshold (256k), and are estimated using reservoir sampling.

Most time is spent during GMM splitting in the Reduce phase. The estimation takes about 48 hours on 1000 Reduce workers; at half-time, there are approximately 10% Reduce partitions still being worked on: since we need to use our own partitioning function, the Reduce partitions are fairly uneven, with the largest partition being about 70 GB (a lot of the data sent to the largest 3–5 reduce partitions is silence frames), and the smallest about 2 GB.

The size on disk for the largest models we built is about 30 GB. For N-best list rescoring we load the generated data into an in-memory key-value serving system with 100 servers, each holding 1/100 of the model stored uncompressed for faster look-up.

# G. Validation Setup

To verify the correctness of our implementation we set up a validation training and test bed:

- train on the development set by keeping all the data and M-phones by setting the minimum number of frames  $N_{\min} = 1$ ;
- test on the subset of the development data with 0% WER at 10-best (the manual transcription is among the 10-best hypotheses extracted in the first-pass). This is a significant part of the development set, approximately 80% (21751/27273) utterances. The first-pass AM achieves 7.6% WER on this subset;
- minimize the effect of the language model and first-pass AM scores by setting  $w_{LM} = 0.1, \lambda = 0.0$  in (3-4), and use only the AM score assigned by BAM.

The intent behind choosing this setup is that as the order M increases, BAM should "memorize" the alignments on the training set (even M-phones with a single training frame are retained in the model), and severely penalize mismatched alignments from N-best competitors to the correct transcript at test time. It is for this reason that we choose to test on the subset which contains the correct transcription (used in training) in the N-best list.

The results are presented in Table VII for various context types, and model orders M. A surprising result is the fact that a triphone equivalent BAM (M=1) that does not use word boundary information is significantly weaker than its counterpart that uses that information. Increasing the model order improves performance in both context settings. The residual WER is due to homophones.

 ${\it TABLE~VII}$  Word-Error-Rates in Validation Setup, Using Various Context Types as Well as Model Orders M

| Context type    | M | WER, (%) |
|-----------------|---|----------|
| CI phones       | 1 | 4.5      |
| CI phones       | 5 | 1.5      |
| + word boundary | 1 | 1.8      |
| + word boundary | 5 | 0.6      |

#### V. CONCLUSION AND FUTURE WORK

We find these results very encouraging, proving that large scale distributed acoustic modeling has the potential to greatly improve the quality of ASR. Expanding phonetic context is not really productive: "more model" by increasing M > 2 yields no gain in accuracy, so we still need to find alternative ways to fully exploit the large amounts of data that are now available. The best predictor for ASR performance is the model size, as measured by the number of Gaussians in the model.

State clustering using DTs as a means of coping with data sparsity may no longer be necessary: only 1.1% of the state-level segments on test data alignments back-off to the DT clustered GMM. It remains to be seen if DTs have other modeling advantages: since we used a rescoring framework, and the first-pass alignments are generated with an AM that uses DT state clustering, it is still very much part of the core modeling approach presented. In addition to that, our best results are obtained by interpolating BAM with the baseline AM.

Obvious future work items that are perfectly feasible at this scale include: DT state tying, re-computing alignments in BAM ML training, and discriminative GMM training.

Another possible direction exploits the BAM ability to deal with large phonetic contexts, and large amounts of data. In the early stages of this work we successfully built BAMs with M=5, but their performance on development data did not justify experimenting further with such large models. It would be interesting to build BAMs by starting from the surface form of words (using letters as context elements instead of phones) and inferring the HMM topology for each unit in a purely data-driven manner, along the lines described in Section 3.6 of [20].

It seems that we literally have more data than we know what to do with, and better modeling techniques at large scale are needed. Non-parametric modeling techniques may be well suited to taking advantage of such large amounts of data.

#### APPENDIX A

#### HOW MUCH DATA IS NEEDED TO ESTIMATE A GAUSSIAN WELL?

Consider n i.i.d. samples  $X_1,...,X_n$  drawn from a normal distribution  $\mathcal{N}(\mu,\sigma^2)$ . We would like an upper-bound on the probability that the sample mean estimate  $\overline{X} = \frac{1}{n} \sum_{i=1}^n X_i$  is more than  $q \cdot \sigma$  away from the actual mean, with  $q \in (0,1)$ .

If  $X_1,...,X_n \sim \mathcal{N}(\mu,\sigma^2)$  then  $(\overline{X}-\mu)/(\sigma/\sqrt{n}) \sim \mathcal{N}(0,1)$ . Thus  $P(|\overline{X}-\mu|>q\cdot\sigma)=P(|Z|>q\cdot\sqrt{n})=2\cdot\Phi(-\sqrt{n}\cdot q)$  where Z is the standard normal i.e.  $Z\sim\mathcal{N}(0,1)$ , and  $\Phi$  is the cumulative distribution function (CDF) for a standard normal random variable. Thus  $P(|\overline{X}-\mu|>q\cdot\sigma)< p$  is

equivalent to choosing n such that:  $2 \cdot \Phi(-\sqrt{n} \cdot q) < p$ . In Matlab this can be easily calculated as

 $n = (icdf('Normal', (1-p/2), 0, 1)/q)^2$  and in R as

 $n = (qnorm(1-p/2)/q)^2$ , see Table VIII for a few sample values.

TABLE VIII

Number of Samples n Needed to Estimate the Mean of a Normal Distribution within  $q \cdot \sigma$  of the Actual Mean, with Probability Lower than p

| p    | q    | n    |
|------|------|------|
| 0.05 | 0.05 | 1537 |
| 0.06 | 0.06 | 983  |
| 0.07 | 0.07 | 670  |
| 0.08 | 0.08 | 479  |
| 0.10 | 0.10 | 271  |
| 0.15 | 0.15 | 95   |

A "good" value for the sample size is  $n = 300, \dots, 1000$ . We also note that if the sample size is this large then the statement will still hold approximately true even if the population is not normal, since by the central limit theorem  $\overline{X}$  will be very close to normal even if the population is not.

A similar derivation can be carried out for the sample variance estimate: assuming normality,  $(n-1) \cdot S^2/\sigma^2$  follows a  $\chi^2$  distribution on n-1 d.f.

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