Positive Results for Mechanism Design without Money*

(Extended Abstract)

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ABSTRACT

Consider the problem of allocating multiple divisible goods to two agents in a strategy-proof fashion without the use of payments or priors. Previous work [1, 2] has aimed at implementing allocations that are competitive with respect to an appropriately defined measure of social welfare. These results have mostly been negative, proving that no dictatorial mechanism can achieve an approximation factor better than 0.5, and leaving open the question of whether there exists a non-dictatorial mechanism that outperforms this bound.

We provide a positive answer to this question by presenting an interesting non-dictatorial mechanism that achieves an approximation factor of 2/3 for this measure of social welfare. In proving this bound we also touch on the issue of fairness: we show that the proportionally fair solution, a well known fairness concept for money-free settings, is highly competitive with respect to social welfare. We then show how to use the proportionally fair solution to design our non-dictatorial strategy-proof mechanism.

Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*; I.2.11 [**Distributed Artificial Intelligence**]: Multiagent Systems

General Terms

Economics,Theory

Keywords

Prior-Free Mechanism Design; Proportional Fairness

1. INTRODUCTION

How does one allocate a collection of resources to a set of *strategic* agents without using money? This is a fundamental problem with many applications since in many scenarios payments cannot be solicited from agents; for instance, different teams compete for a set of shared resources in a firm, and the firm cannot solicit payments from the teams to make allocation decisions.

The lack of monetary rewards causes several problems to arise which make the design of useful allocation processes much more difficult. One of the arguably most significant obstacles is the difficulty of enforcing truthfulness on behalf of the strategic agents. The most useful tool in designing mechanisms that incentivize the agents to report their true preferences has been the use of payments that can help make undesirable allocations seem less appealing to them. The other practical issue is that the valuations of the agents need to be put on a common scale. When payments can be used, a standard approach is to measure the valuations in terms of *money*. In the absence of money, when maximizing social welfare (SW), one can define an appropriate scale-free solution by first normalizing the valuations of the agents so that these values add up to a common number (1 say) and then maximizing the welfare with these normalized values.

The problem of designing truthful mechanisms aiming to allocate divisible resources while maximizing this notion of social welfare was first studied by Guo and Conitzer [1]. They mainly focused on the special case of two items and two agents for which they presented a truthful mechanism that achieves a 0.829 approximation; they also showed that no truthful mechanism can achieve better than a 0.841 approximation, even for this very restricted setting. For the more general setting of many items and two agents they showed that no mechanism from a class of *increasing price* mechanisms (mechanisms using artificial currency for both linear and non-linear pricing) can guarantee an approximation factor better than 0.5. Subsequent work of Han et al. [2] extended these negative results, showing that even for the more general class of *swap-dictatorial* mechanisms, no mechanism can guarantee an approximation factor better than 0.5 when the number of items is unbounded. This class of swap-dictatorial mechanisms contains all mechanisms that first (randomly) choose one of the two agents and then allow her to choose her preferred bundle of items from a predefined set; the other agent receives the remaining items. Finally, another negative result from the work of Han et al. [2] showed that if both the number of agents and the number of items are unbounded, then no non-trivial approximation factor of the optimal SW can be achieved.

Therefore, the main open question that remains in this setting is whether useful truthful mechanisms for the two-agent case exist beyond the class of swap-dictatorial mechanisms and whether such mechanisms can achieve an approximation factor better than 0.5. We provide a positive

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answer to this question by presenting an interesting non-swap-dictatorial mechanism that breaks this bound of 0.5.

2. PRELIMINARIES

Let M denote the set of m items and N the set of n agents. Each agent $i \in N$ has a valuation v_{ij} for each item $j \in M$ and each item is divisible. The agent valuations are scaled so that $\sum_j v_{ij} = 1$ for each agent i. If agent i is allocated a fraction x_{ij} of each item j, then her valuation for that allocation x is $v_i(x) = \sum_j x_{ij} v_{ij}$.

Given a valuation bid vector from each agent (one bid for each item), we want to design a mechanism that outputs an allocation of items to agents. We restrict ourselves to truthful mechanisms, i.e. mechanisms which never return a more valuable allocation to an agent who reports a false bid. In designing such mechanisms we consider the objective which aims to output an allocation x (approximately) maximizing the social welfare, denoted $SW(x) = \sum_{i \in N} v_i(x)$. When referring to an approximation factor of a mechanism, this will be the minimum value of the ratio $SW(x)/SW(x^*)$ across all the relevant problem instances, where x is the output of the mechanism and x^* is the allocation that maximizes SW.

An allocation x is Pareto Efficient if there exists no allocation x' such that $v_i(x') \geq v_i(x)$ for all $i \in N$ and $v_{i'}(x') > v_{i'}(x)$ for some $i' \in N$. An allocation x is *Proportionally Fair* (PF) if it is Pareto efficient and additionally, for any other allocation x' the aggregate proportional change to the valuations is not positive, i.e.:

$$\sum_{i \in N} \frac{v_i(x') - v_i(x)}{v_i(x)} \leq 0.$$

3. MAIN RESULTS

In the full paper we first present a very efficient $O(m \log m)$ time algorithm for computing the PF allocation for two-agent instances; we then prove that the social welfare of the PF allocation x_{PF} is a very good approximation of the optimal social welfare. The theorem that follows is an indication that social welfare and fairness are well aligned goals in this setting:

THEOREM 1. *For problem instances with two agents and multiple items the PF social welfare satisfies:*

$$\frac{SW(x_{PF})}{SW(x^*)} \geq \frac{2\sqrt{3}+3}{4\sqrt{3}} \approx 0.933.$$

We then define the following non-swap-dictatorial mechanism, which we call the *Partial Allocation* (PA) mechanism:

Mechanism 1: Partial Allocation

1 Compute the PF allocation: x_{PF}.
2 Let $v_{\text{A}}, v_{\text{B}} \in [0,1]$ be the agents' valuations for x_{PF}.
3 Agent A receives a fraction v_{B} of her PF allocation.
4 Agent B receives a fraction v_{A} of her PF allocation.

The types of problem instances for which this mechanism performs poorly are the ones where, for example, the two agents have the same valuations. Quite surprisingly, we show that this very interesting mechanism is truthful:

LEMMA 1. *The Partial Allocation mechanism is truthful.*

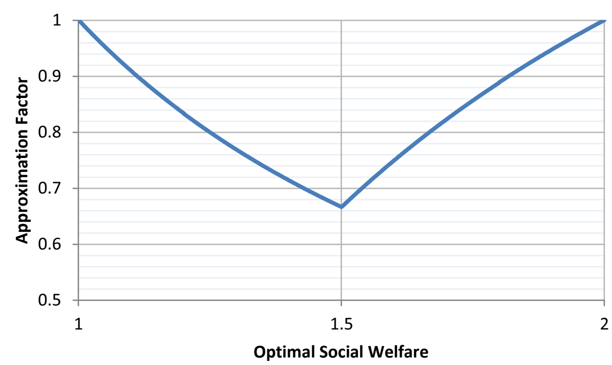

Figure 1: The approximation factor of Max as a function of the optimal social welfare value $SW(x^*)$.

We also consider the simple swap-dictatorial mechanism that cuts each item in half and, for each item, allocates one half to agent A and the other to agent B. This mechanism is clearly truthful since the final allocation is independent of the agents' reported values. Unlike the Partial Allocation mechanism, this mechanism performs poorly for problem instances for which the two agents are interested in disjoint sets of items. Using this intuition, we propose the following non-swap-dictatorial mechanism that combines the two mechanisms and outperforms all swap-dictatorial mechanisms in terms of the guaranteed approximation factor:

Mechanism 2: Max

1 Compute the allocation of the PA mechanism.
2 Compute the allocation of the dictatorial mechanism.
3 Output the allocation with the greater social welfare.

It is not common that combining two truthful mechanisms in this fashion will yield a truthful mechanism. Nevertheless, we prove that this is indeed the case for the Max mechanism:

LEMMA 2. *The Max mechanism is truthful.*

For the appproximation factor of this truthful mechanism we then prove the following bounds (depicted in Figure 1):

THEOREM 2. *For problem instances with two agents and multiple items the allocation x_m of Max satisfies:*

$$\frac{SW(x_m)}{SW(x^*)} \geq \begin{cases} \frac{1}{SW(x^*)} & \textit{when } SW(x^*) \leq 3/2 \\ 2 - \frac{2}{SW(x^*)} & \textit{when } SW(x^*) > 3/2. \end{cases}$$

These bounds imply that the Max mechanism guarantees an approximation factor of $2/3$, which substantially outperforms the best previously known factor of 0.5.

COROLLARY 1. *For problem instances with two agents and multiple items the Max mechanism satisfies $\frac{SW(x_m)}{SW(x^*)} \geq \frac{2}{3}$.*

4. REFERENCES

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