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GOLDEN PROPORTION

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ABSTRACT
Why is the world so obsessed with the beautiful? What makes them beautiful? What do they have that catches the eye? Well it certainly isn't perfection -- there is no such thing. Although it is difficult to think otherwise sometimes, isn't it? Is it the balance of their facial features?

KEYWORDS: Golden divider, Fibonacci series, pentagon analysis.

Facial proportions and symmetry

The concept that 'ideal' proportions are the secret of beauty is perhaps the oldest idea regarding the nature of beauty. The ancient Egyptians had a great interest in art and beauty. The famous painted limestone figure of Queen Nefertiti (*c* 1350 BC) (Figure 1), with her harmonious facial proportions and symmetry, is an example of how the Egyptians immortalized the beauty of their kings and queens by depicting them, perhaps unrealistically, with 'ideal' facial proportions. In fact, the name Nefertiti literally means the 'Perfect One'.[1]

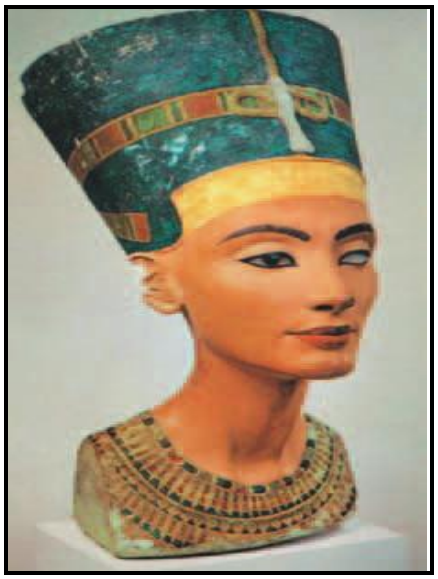


Fig. 1: Queen Nefertiti. The famous face is well proportioned and symmetrical (Berlin Museum).

Lesser dignitaries were not so honoured and had more realistic depictions in art and sculpture. The Egyptian proportional canons, however, used grids with meshes of equal-sized squares. This was to change with the age of Greek sculpture which, rather than featuring fixed units, described proportion between the parts of the whole human figure. In the course of his travels, the Greek mathematician Pythagoras (6thcentury BC) is extremely likely to have come into contact with the mathematical treatise of the Egyptians. He postulated that beauty could be explained through mathematical laws and laws of proportion. He proposed an explanation of beauty through a significant finding that plucking taut strings of proportionately different lengths produces harmonious notes. The difference in the proportionate lengths of the strings followed mathematical laws, and hence his explanation of laws of proportion. The term Pythagoras used to describe beauty was 'cosmos' as he felt that beauty was part of the mathematical order of the universe, hence the origin of the word 'cosmetic'.

Throughout the ages, painters and sculptors have attempted to establish ideal proportions for the human form, however, possibly the most famous of all axioms about ideal proportions is that of the Golden Proportion.[1]

Roman copies of one of his most famous statues, the 'Doryphorus' ('Spear Bearer'), still exist. This statue is itself often referred to as the 'Canon' because it embodies Polycleitus' views on the correct proportions of the ideal male form (Figure 2). In the 2nd century AD, the prominent Greek physician and philosopher Galen said, 'Beauty does not lie in the individual parts, but in the harmonius proportion of all the parts to all the others, as is stated in the Canon of Polycleitus'.

Fig. 2: Doryphorus ('Spear Bearer'). In the 5th century BC, Polycleitos wrote the *Canon* in which he laid down the guidelines for the ideal proportions of the human body, possibly founding them on precise numerical relationships described by Pythagoras. In this statue, also often referred to as the 'Canon', Polycleitos created the archetype of the Greek ideal of male beauty. (Naples Museum)

Phidias (*c* 490−430 BC), a contemporary of Polycleitus, was an Athenian famous as one of the most outstanding of all sculptors. He directed the construction and design of the Parthenon, the chief temple of the Greek goddess Athena on the hill of the Acropolis at Athens. The Parthenon itself, and the statues contained within it, were said to conform to 'ideal' proportions, with Phidias incorporating the Golden Proportion into the architectural design. It is said of Phidias that he alone had seen the exact image of the gods and that he revealed it to man. In ancient Greece, sculpture of the human form was used to represent the many gods. As these sculptures were constructed with ideal proportions, the belief arose that the better 'mortals' looked, the more god-like they were.[1]

Fig. 3: Aphrodite of Melos (known in French as 'Venus de Milo') is a representation of the classic Greek facial profile. The facial profile is orthognathic (orthos = correct; gnathos = jaw). The sweep from the forehead to the nasal tip is also almost straight. The vermilion border of the upper lip has a classic curve, which later served as the model for the Roman bow of love, termed 'Cupid's bow'. (Louvre, Paris).

Golden Proportion

This is a geometrical proportion in which a line AB is divided at a point C in such a way that AB/AC = AC/CB. That is, the ratio of the shorter section to the longer section of the line is equal to the ratio of the longer section to the whole line. This gives AC/AB the value 0.618, termed the Golden Number.

The point at which the line is divided is known as the Golden Section and is represented by the symbol Φ (Phi), derived from the name of the Greek sculptor Phidias. This proportion has classically been described as pleasing to the eye, the emphasis being upon the proportion of the parts to the whole. The prominent mathematician Euclid (*c*. 325−265 BC) described this in his treatise The Elements. In his edition of Euclid's Elements, the mathematician Luca Pacioli (1509) re-named the Golden Proportion the 'Divine Proportion' as he felt the concept could not be fully explained, and published a treatise entitled De Divina Proportione (On Divine Proportion) for which Leonardo da Vinci drew figures of symmetrical and proportionate faces and bodies.

Maestlin gave the first known calculation of the Golden Proportion as a decimal in a letter to his former pupil, the famous astronomer Johannes Kepler, in 1597.

Another often quoted concept, which gives some credence to the Golden Proportion, is the Fibonacci sequence.[1]

The Fibonacci Series

No consideration of the Golden Proportion can be complete without mention of the Fibonacci Series which is the complementary view of the Golden Proportion as illustrated below by the 3 Golden Mean Gauges. These numbers are also abundant in the beauty of nature and teeth.

Definition

In this series of numbers each term is the sum of the previous two terms as follows:

It is called the Fibonacci series after Leonardo of Pisa or (Filius Bonacci), alias Leonardo Fibonacci, born in 1175, whose great book The Liber Abaci (1202), on arithmetic, was a standard work for 200 years and is still considered the best book written on arithmetic. devised a number sequence in which each number is the sum of the two preceding numbers, ie 1, 1, 2, 3, 5, 8,13, 21, 34, 55 etc. In the 19th century, the mathematician Edouard Lucas coined the term Fibonacci sequence and scientists began to discover the numbers in nature, such as in the spirals of sunflower heads, the logarithmic spiral in snail shells and in animal horns. As the numbers increase in magnitude, the ratio between succeeding numbers approaches the Golden Proportion.[1]

FIBONACCI SERIES

Add:

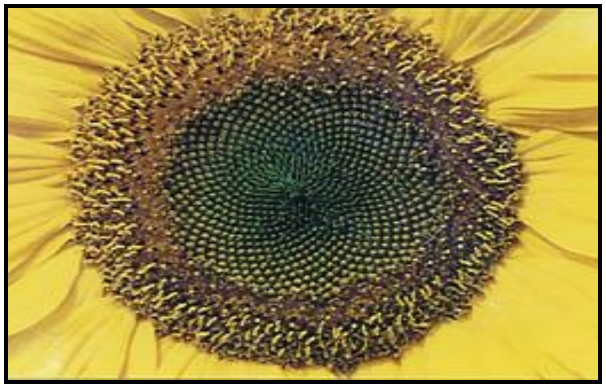
0	1	1	2	3	5	8	13	21	34	55	89	144	233
1	1	2	3	5	8	13	21	34	55	89	144	233	377
1	2	3	5	8	13	21	34	55	89	144	233	377	610

Divide:

0 ÷ 1 = 0
1 ÷ 1 = 1
2 ÷ 1 = 2
3 ÷ 2 = 1.5
5 ÷ 3 = 1.666
8 ÷ 5 = 1.6
13 ÷ 8 = 1.625
21 ÷ 13 = 1.615
34 ÷ 21 = 1.61904
55 ÷ 34 = 1.6176
89 ÷ 55 = 1.61818
144 ÷ 89 = 1.61797
233 ÷ 144 = 1.61805
377 ÷ 233 = 1.61802
610 ÷ 377 = 1.61803
987 ÷ 610 = 1.61803
1597 ÷ 987 = 1.61803

Fig. 4: A summation series. Note that the last number is a total of the previous two terms; the fifth addition divided by the fourth number is already a figure of 1.66. After this has been repeated fifteen times, the number 1.618 does not change thereafter.

One of the most spectacular examples of the Fibonacci Series in nature is in the head of the sunflower.



Sunflower

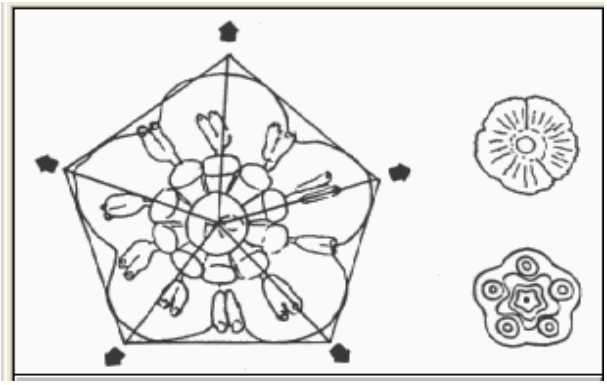


Fig. 5: On the left will be seen a schematic drawing of a flower; notice the pentagonal shape and the expression from a polar center. On the right is the same kind of geometric expression, but seen in seashells.

Scientists have measured the number of spirals in the sunflower head. They found, not only one set of short spirals going clockwise from the centre, but also another set of longer spirals going anti clockwise, These two beautiful sinuous spirals of the sun flower head reveal the astonishing double connection with the Fibonacci series.

The Golden Forms

The Golden Rectangle

If the altitude of a rectangle is constructed in the proportion as 1.0 and the base is made 1.618 times the altitude, this is called the golden rectangle.

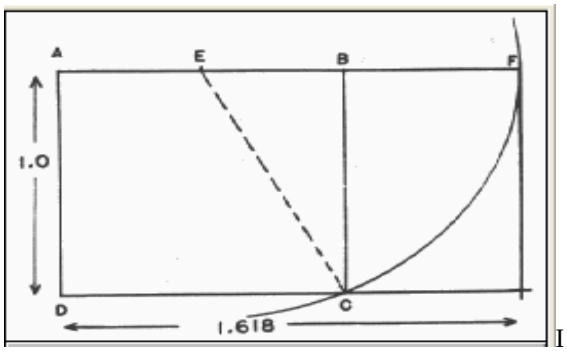


Fig. 6: The golden rectangle and its method of construction. By bisecting the line AB of a square and swinging an arc from that line to the parallel of AB at F, the golden rectangle can be constructed. The altitude of this rectangle being taken as 1, the base is 1.618. This particular rectangle is pleasing to the eye and to the senses.

It was on such a scale that the Parthenon was built and it has endured for two millennia as a world attraction.

It is rare to see a square picture frame. Golden rectangular stationery, writing pads and 3 by 5 inch cards are known to be most popular. For example, examine the typical credit card. The golden section even entered commercial design and became a part of art, if not directly, certainly intuitively, probably since man first started to scratch forms on the walls of caves. It has harmony and balance yielding comfort and pleasure to the senses.[2]

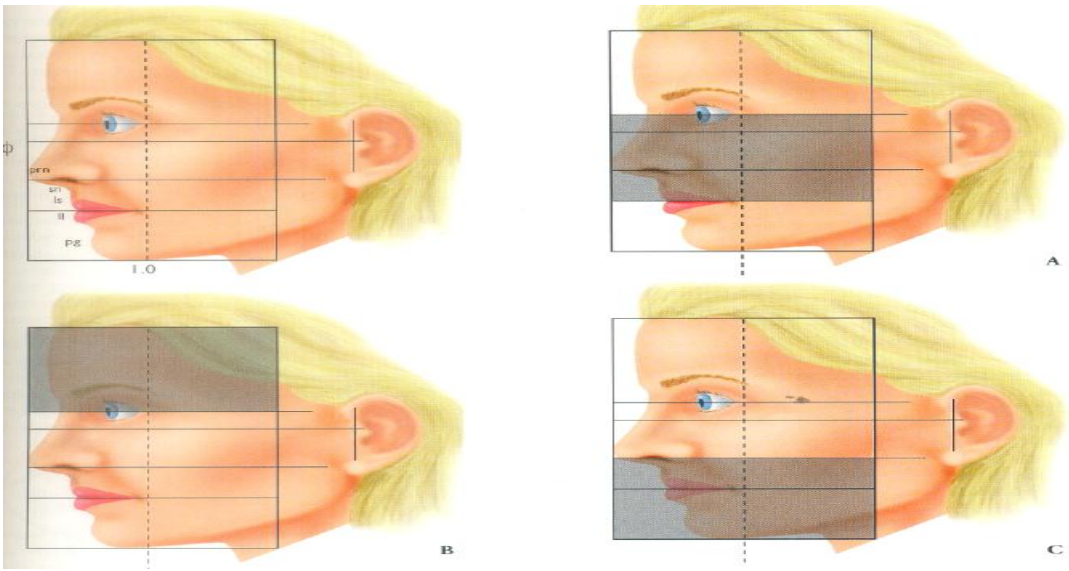


Fig. 7: Golden rectangle is formed from facial height and from the end of the nose, a bisection will be noted at the lateral canthus of the eye. Note the middle face rectangle (A, the upper face rectangle (B) and lower face rectangle(C) are equal in height.

The Pentagon Analysis

The five-pointed star has also been a striking symbol. The base of this star is the pentagon (Fig. 8).

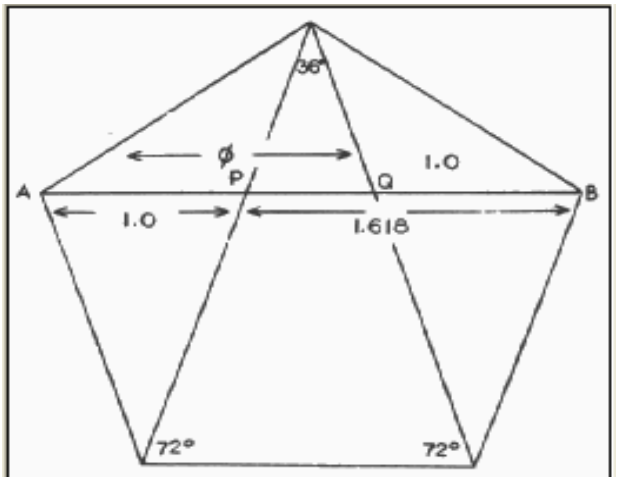


Fig. 8: An analysis of the pentagon. When two corners of the pentagon are connected and crossed with two other corners, the lines are sectioned in the golden section. It will further be noted that the repeated procedure will cut the original AB line again into a golden section on the opposite side; this leaves the area PQ being the congruent area and can serve reciprocally for the golden section to both ends. These two connections will also form a 36°, 72°, 72° triangle which is called the golden triangle.

The Golden Divider

To examine the validity of the fascinating Golden Proportion quickly and easily, an instrument, the GOLDEN MEAN GAUGE was developed by Dr. Levin to demonstrate this proportion. It has separate short arms with the divine measurements, 1.0 to 1.618 constructed between two legs of a simple divider, a third point is established between the two endpoints. The intersection would also remain at a point of the golden section as such dividers are spread. By use of golden divider, relationships can easily be identified for the teeth, the mouth, the jaws, the face and indeed the entire body.[2,3]

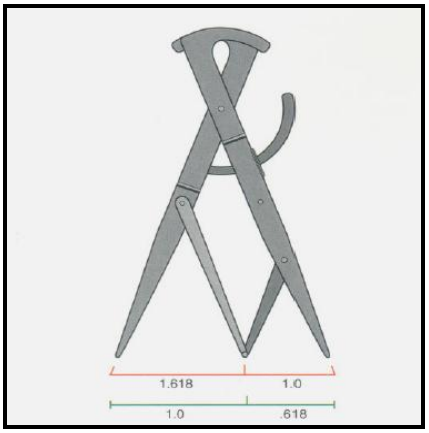


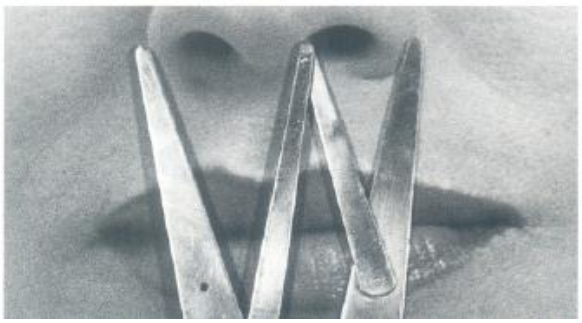
Fig. 9: The plan for construction of the golden divider, which maintains its proportions during expansion. This was designed for analysis of the face and to be used at the operating table by surgeons.

Divine Proportion Expressed In Face

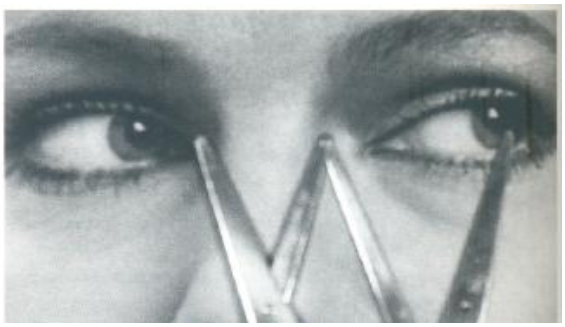
The golden proportion seem to be connected from the nose and mouth and to the teeth through the smile.

The relationship between the width of the nose at the level of the ala to the maxillary intercanine width at the cusp tips in the smiles of fashion models in magazine advertisements was studied. The relative proportions established a clinical working hypothesis for the nasal width and upper intercanine width in the smile of adult face. Smiling widens the nostrils slightly, thus esthetic connection occurs between the arch form and the structure.

Facial soft-tissue relations were studied from patients directly and from photographs. The points employed are depicted in the fig.10. These studies are conducted allegedly with the jaws and lips at physiologic rest.[2]



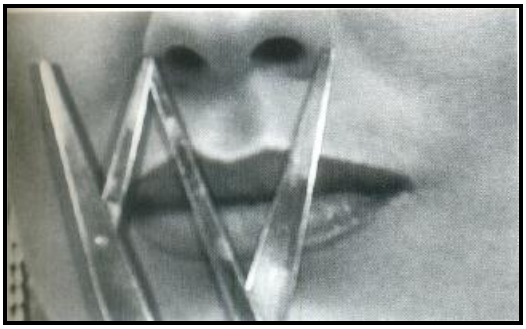
A



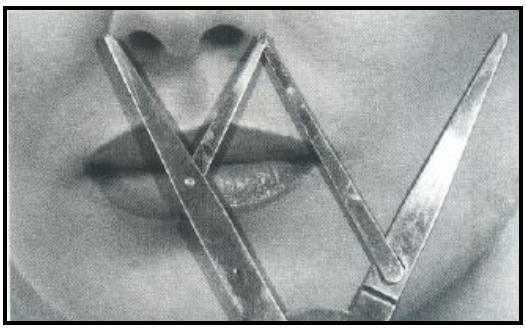
B

Fig. 10 A: Shows that one nostril compared to central columella and the nostril on the other side follows golden proportion.

Fig. 10 B: The width of the nasal bridge is found golden to the width of the lateral nares.



C



D

Fig. 10 C: The divider is not changed and shows that the small portion of the interdacryon is golden to the nose width.

Fig. 10 D: The small portion is set at the nose width at the alar width.

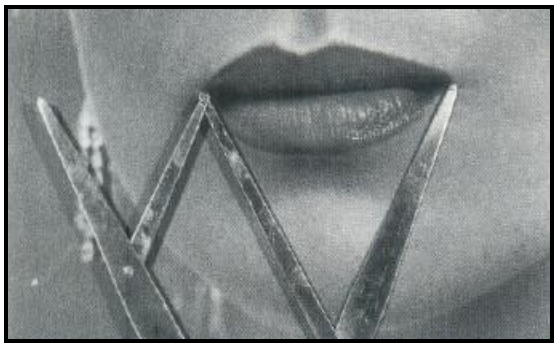


Fig. 10 E: Without alteration of the divider, the large portion equals the width of the mouth at cheilion (corner of the mouth).

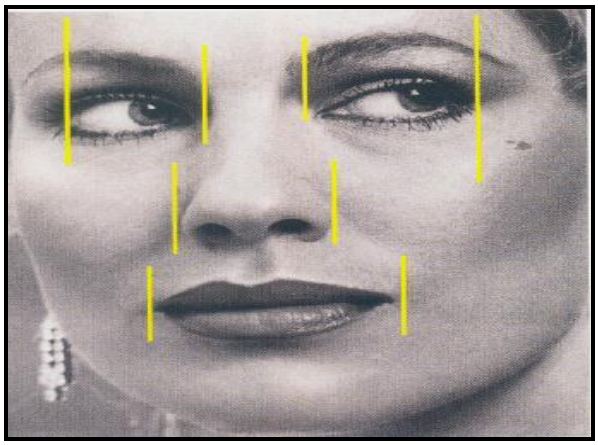


Fig. 11: The lines summarize the proportions seen in fig 10 as a four-step progression.

Vertical Divine Relations

Of greater significance to facial balance are the vertical proportions. These relationships start with the proportion of the upper lip to the lower lip (fig.12A).

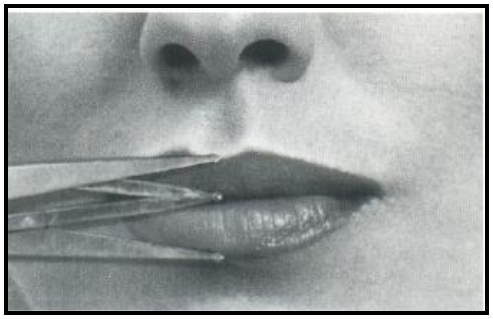


Fig. 12 A: The vertical golden proportion of the upper to the lower lip.

In the most beautiful lips, when a unit of one is taken from a point midway on the cupid's bow to stomian (the point of contact of the lips), the lower lip height is greater and in divine proportion to the upper.[15]

Another vertical golden relationship, is observed when the length of the philtrum (from cupids bow to the base of the columella) is defined as 1.0, the vertical height of the upper and lower lips combined is 1.618 (fig. 12 B).

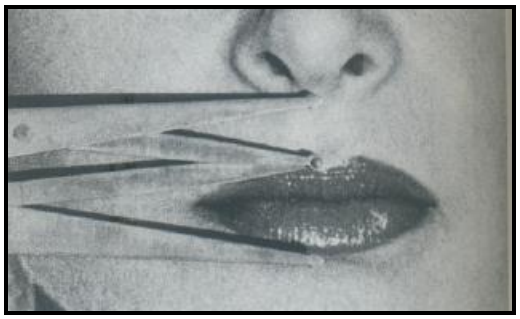


Fig. 12 B: The total lip height is golden to the philtrum as shown.

The distance from stomion to ala, taken as a unit of one, yields the distance from stomion to menton to be 1.618 (fig.12 C).

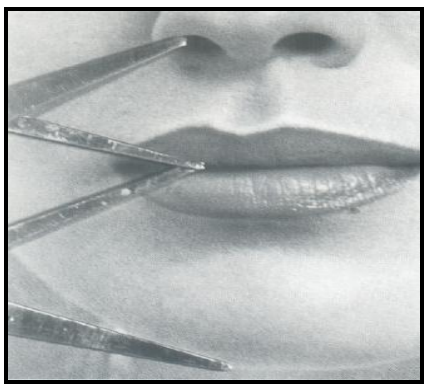


Fig. 12 C: The golden divider reveals a golden proportion from the chin (menton) to the lip embrasure (stomion) to the alar rim (al) of the nose.

Similarly, from stomion to ala was found to be golden to the distance from the level of the center of the eyes to the ala. Thus, in ideal face, the distance from the mouth to the chin (a harmonic equation). Other relationships in the analysis of the eye, nose, mouth and chin positions can be seen in fig. 12 D, fig.12 E.

Fig. 12 D: The relationship of the eye (medial canthus) to the nose to the chin is found to be golden in beautiful faces. This has been called "dynamic symmetry: or "dynamic balance".

Divine Proportions in the Human Dentition

With the foregoing findings, it was natural to seek divine proportions in the dentition. Data were available for consideration from several sources. Personal measurements made on thirty ideal normal subjects had been measured in arch dimensions.

Gardner's 12 findings in 104 stable, long-term, treated patients were available. Data were taken from G. V. Black's tables from standard dental anatomy texts.

Finally, the values found by Hamilton for tooth mass correlation charts, distributed by Dental Corporation of America, were used as reference. Thus, not just tooth size but "arch form in perspective" data were analyzed.

The lower incisor became a basic unit. The upper incisor was golden to the width of the lower incisor.

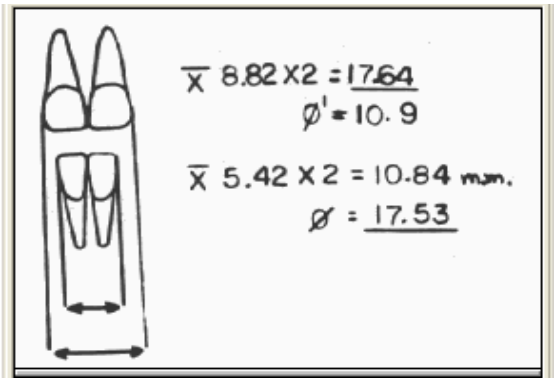


Fig. 13: Tables from research on the tooth size indicate that the value for the width of the lower central incisor is golden to the width of the upper incisors.

Taking both the upper and lower central incisors as a unit, the f relationship is a mark of dynamic symmetry. A progression is seen as the two upper incisors are compared to the four upper incisors— not in a straight line, but as they appear to the eye in the chord of the arc of the arch form.

Fig. 14

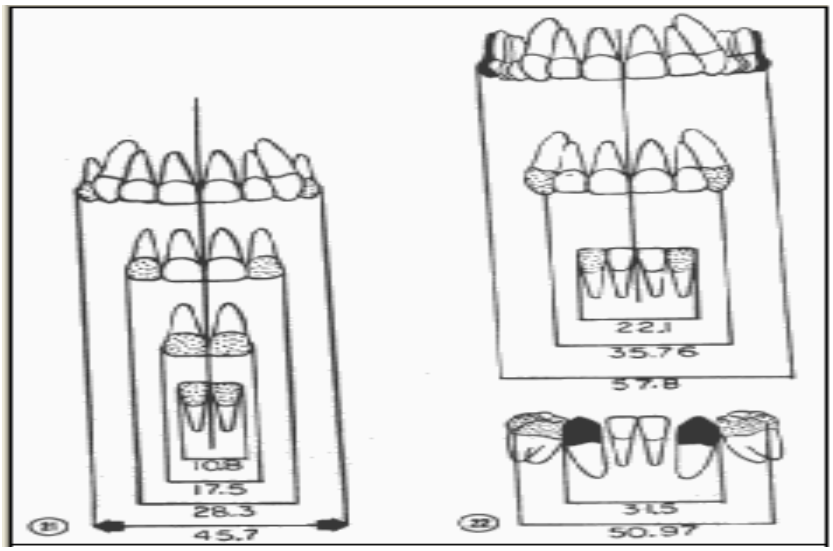


Fig. 15

Fig. 16: From Fig. 14, it will be seen that a progressive series obtains. A line from the lower and central incisors to the upper and central incisors to the distal apsects of the upper lateral incisors across the arch and then on to the width of the upper first premolars forms a progressive golden series.

Fig. 17: Another progressive series. The width of the four lower incisors is golden to the upper canines which, in turn, are golden to the width of the upper second molars. Another relationship is seen from the distal aspect of the lower canines to the buccal groove of the lower first molars.

Taking data from charts, the next progression is the width of the upper first premolars as also viewed from the anterior (Fig. 18). Thus, a rhythm is seen in the natural normal ideal occlusion with the lower incisor as a basic unit and Φ for the upper central incisors, Φ^2 for the lateral incisor widths and Φ^3 for premolars. It can be imagined what happens to esthetics and beauty with teeth missing and collapsed arches in the first premolar areas.

A second series of divine proportions was discovered in the teeth. Starting with the widths of all four lower incisors (across the arch) as 1.0 value, a Φ relation to the tips of the upper canine widths was found (Fig. 15). A Φ^2 relationship to the four lower incisors was found at the widths of the upper second molars. Therefore, the molar width for the upper molars is 2.618 times the width of the arc of the four lower incisors in great beauty. Thus, in the broad smile, there is harmony from the lower to the upper arch and harmony within the upper arch itself.

A third golden proportion was seen from the distal aspect of the lower canines. This measurement as a base revealed the lower first molars at the mesial cusps to be in the Φ relationship. Thus, the normal human dentition represents a concert of harmony— undoubtedly a factor in natural selection at the subliminal level.

Association of Divine Relations with Denture and Face

Any magazine will show smiling faces to portray happiness with the product advertised. Ten photographs were studied by Ricketts to test possible association. By dropping vertical lines from the lateral margin of the nose during the smile, the nasal width was found almost

consistently to be the same as the upper intercanine width at the canine tips (Fig. 16).

However, the ala are drawn slightly outward in a broad smile. If this is true (and ten patients do not prove a complete theory), then the four lower incisors form a basic unit of 1.0 with Φ relations going to the nose to the mouth Φ^2 to the eyes Φ^3 and the head Φ^4. Lower incisor width would be 0.2360 the width of the eyes. Who would ever have suspected that upper intercanine width could be related to the distance between the outer canthus of the eyes (0.3819)?

Who would have suspected that anyone could measure the width of the eyes to determine a possible indication for stripping and positioning of the four lower incisors?

Who would believe that the width of the eyes could be used to help determine the need for removal of a lower incisor? Or who would have believed that prediction of the orbit would be related in any way to an indication for arch expansion?

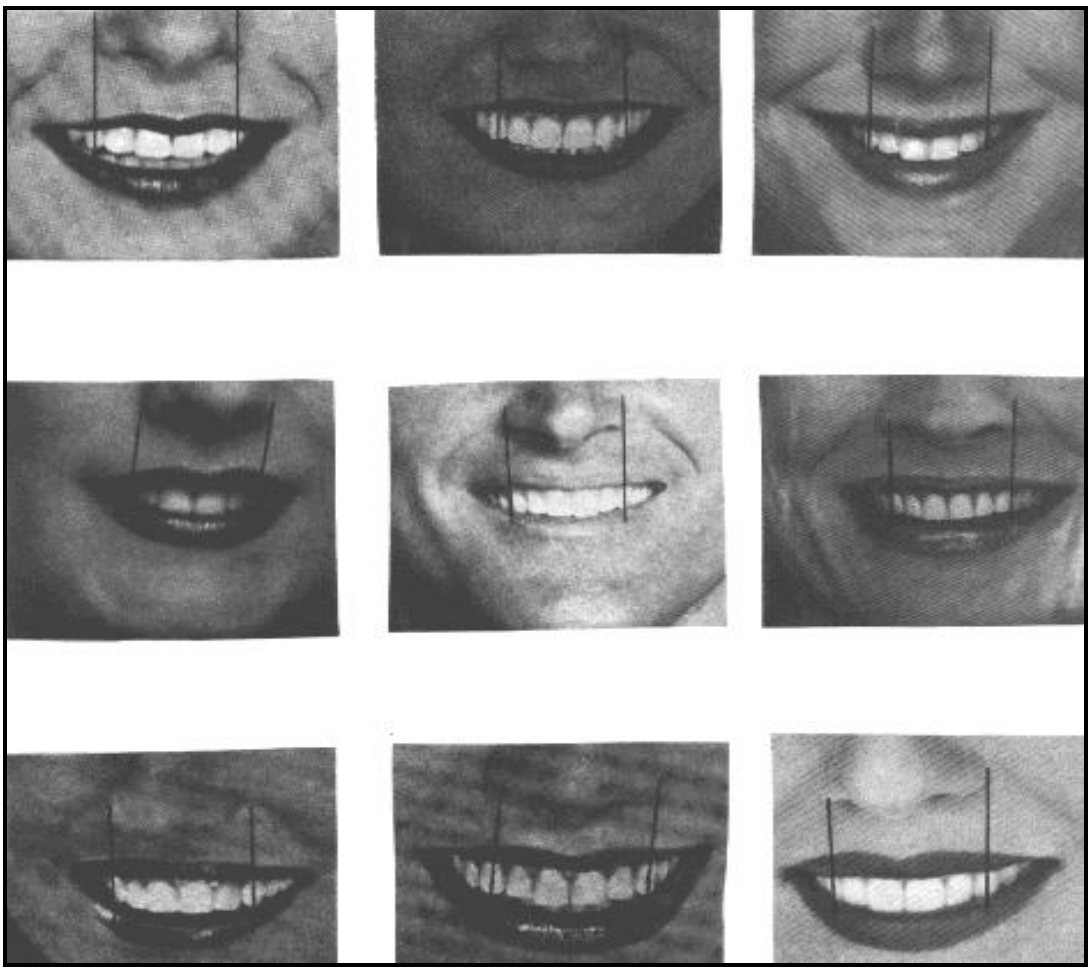


Fig. 18: Direct photocopies from current magazines. A line was drawn from the ala of the nose vertically in each of nine different subjects. Note the strong relationship from the ala of the nose to the tip of the canines. During the smile the nostrils are spread slightly.

Analysis of the Cephalometric Matrix

Extensive computer research and clinical analysis kept pointing to the use of certain points and planes for reference, not only for convenience but also for biologic significance. With all these golden associations in the soft-tissue face and hard-tissue teeth, the next issue was the possible skeletal and dental relations.

By now Ricketts was using "golden calipers" constructed in such a way as to expand or contract, maintaining the golden section relationship. It was, therefore, rather easy to start exploring computer composites made from samples of normal occlusions. Ricketts had already established the normal composites for the computer program. In addition, in 1979 a sample of thirty ideal normal, thirty-two-tooth, racially unmixed adult males from Peru in South America was sent for analysis by Dr. Maria Castro. These were perhaps the most bewitchingly beautiful examples of ideal occlusions. All were traced in detail by Ricketts and digitized and processed by the technicians of Rocky Mountain Data Systems. The result was the composite shown in Fig. 19.

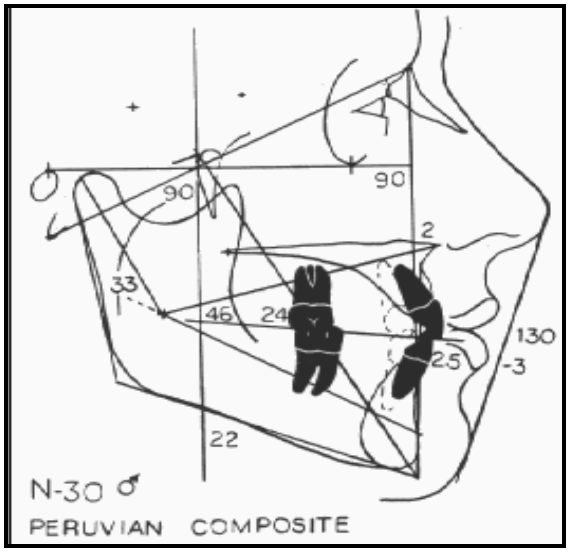


Fig. 19: A composite of thirty Peruvian nationals showing the relationships in the Ricketts summary analysis. The breakdown of these will be seen in Figs. 25, 26 and 27.

Eight divine proportions have been discovered so far and seem to match three other normal composites available for comparison.[2]

It will be remembered that the large unit used as a unit of 1.0 will show the smaller unit as 0.618 or 61.8 percent of the length of the original dimension. It is still a golden relation, but termed a negative value. It may be more convenient to describe the longer side first. The findings are as follows:

1. **Corpus axis length Φ to condyle axis length (to condyle tip) (Fig. 20).**
2. **Anterior cranial fossa length SN to posterior cranial fossa length S Ba (Fig. 21).**

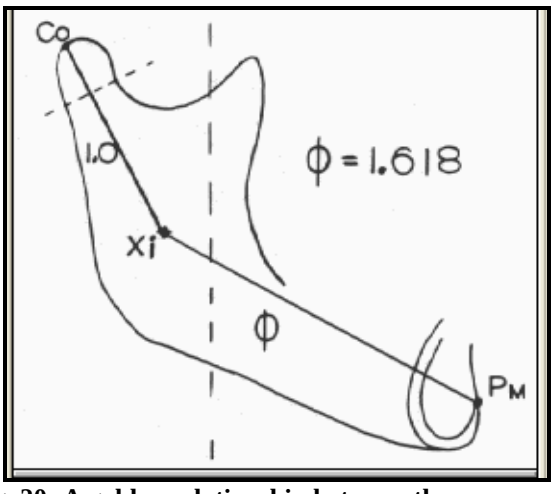


Fig. 20: A golden relationship between the corpus axis (Xi Pm) and the condylar axis (Xi Co) which makes the mandible a beautifully pleasing structure.

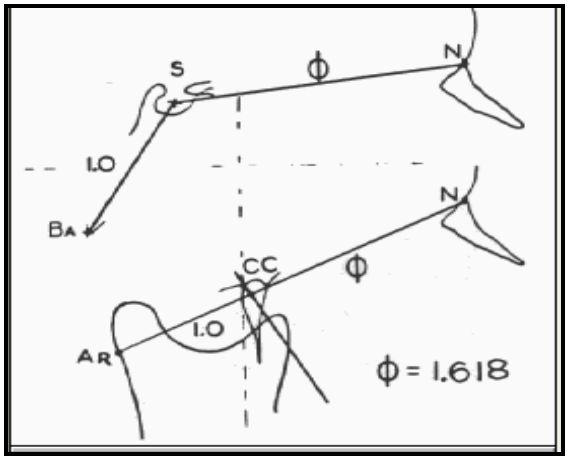


Fig. 21: Two golden relationships— SN to SBA and CCN to CC articulare.

3. **Basal or cranial anterior base length (cc to NO Φ to cranial center to articulare [ar]) (Fig. 22).**
4. **Length of the hard palate ANS-PNS Φ to depth of nasopharynx (Fig. 23) and point A to PNS to posterior margin of the condyle neck.**

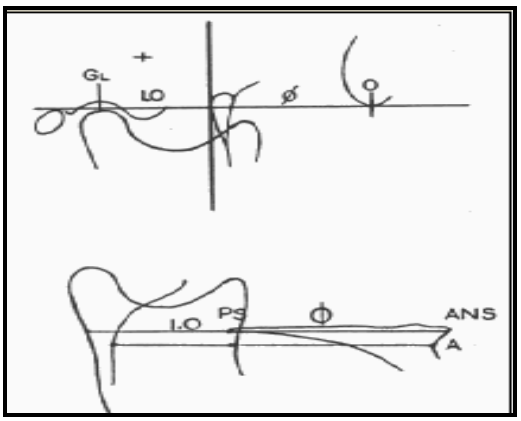


Fig. 22,23: Other golden relationships were seen in the distances from orbit to pterygoid vertical and the pterygoid vertical to the top of the glenoid fossae. Below will be seen ANS-PNS to the posterior border of the mandible; another relationship is A to the anterior border of the mandible to the outline of the pharynx.

5. **Anterior length of Frankfort plane (PtV to orbitale) Φ to PtV to glenoid fossa (GL) (Fig. 24).**
6. **Vertical height of point A to Pm Φ to A to the Frankfort plane (Fig. 24).**

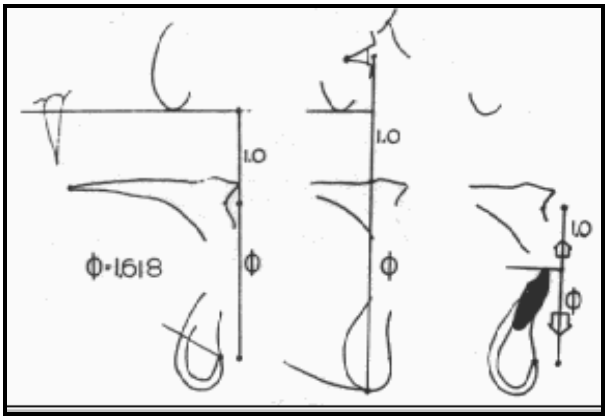


Fig. 24: Three vertical relationships were seen to be golden in cephalometrics. On the left will be seen the Frankfort plane to point A to PM as a golden structural relationship. In the center will be seen the eye to the floor of the nose to menton as another golden relationship. However, the most useful is seen on the right, where the lower incisor is golden to point A and to point PM.

7. Palate at incisive canal to menton Φ to canthus of eye (Fig. 26).
8. Height of the lower incisor tip from Pm Φ to distance of incisor tip to point A (Fig. 26).[2]

Divine Proportions in the Frontal Cephalometric View

The frontal proportions were determined from a computer composite of the adult sample of eighty-two normal occlusions collected by the Foundation for Orthodontic Research. The width of transverse dimensions would apply for the teeth except for greater enlargement in structures more distant from the film in the posteroanterior exposure (Fig. 27).

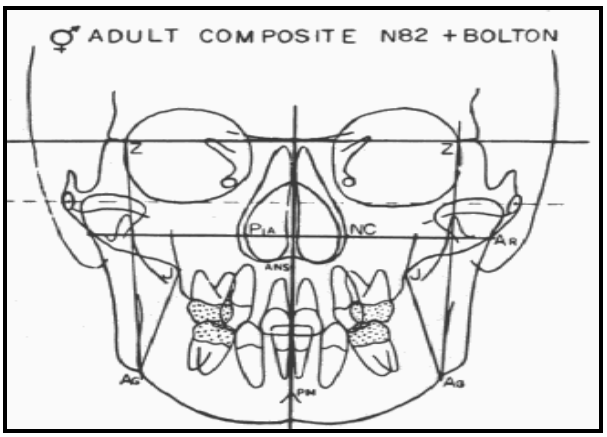


Fig. 27: An adult composite was made from a combination of the eighty-two normal patients of the Foundation for Orthodontic Research. Note the points used: Z = zygomatic frontal suture, medial border; AG = antegonial tubercle; J = jugal process; PiA = periform aperture; NC = widest portion of the nasal cavity; AR = lateral articulare; ANS = anterior nasal spine; PM = protuberance menti at the convergence of the trigonim mentali.

Certain golden relationships were found in the upper, middle, and lower face.

In the upper face, the distance from point Z at the lateral border of the orbit at the zygomatofrontal suture to the medial border of the orbit (or point dacryon) was golden to the interdacryon distance. This makes the bridge of the nose a congruent area between the two orbits (Fig. 25).

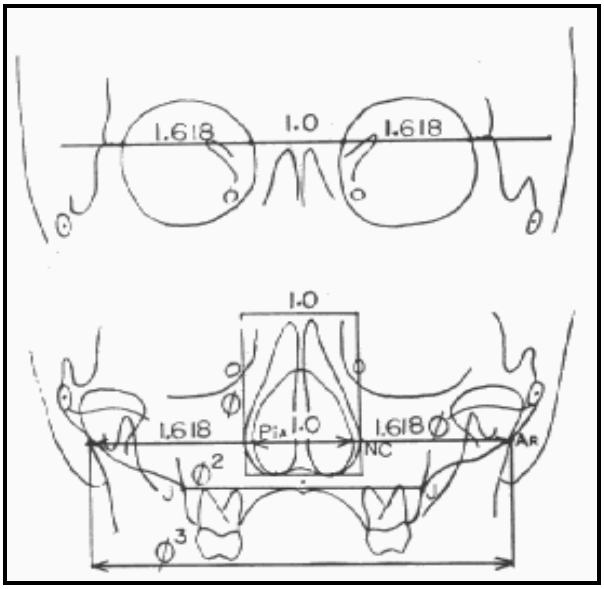


Fig. 25: The golden relationship of the orbit to the nasal width at dacryon. Below will be seen the golden relationship from the periform aperture to the distance to lateral articulare. A Φ^2 relationship is seen at the jugal process and a Φ^3 relationship is seen at the lateral articulare. The height of the nasal cavity to its width forms, essentially, a golden rectangle.

In the midface the distance between the widest points on the piriform aperture (at the anterior rim of the nasal cavity) formed a congruent area between the width of the mandible at a level which was called lateral articulare (Lar), at the crossing of the condyle neck with the outline of the zygoma. Thus, a Φ relationship was found from the nose to each lateral part of the face (Fig. 25).

In turn, the maxilla between J points was Φ^2 or 2.618 to the piriform aperture, which made the interarticulare distance Φ^3 or 4.236 times the nasal width. A golden rectangle was observed from the maximum width of the nose (NC) as related to the floor of the nose to the zygomaticofrontal suture line which approximates dacryon and is close to nasion.

In the lower face the width between the buccal surface of the crowns of the lower first molar was found golden to the mandibular width at the antegonial points (Fig.26).

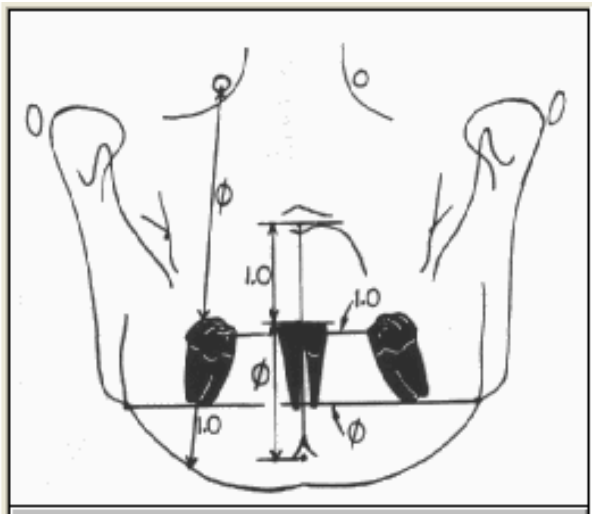


Fig. 26: The molar width is golden to the AG in this adult composite. Note also the same vertical relations seen in the lateral head film. Note also that from foramen rotundum through the lower first molar to the border of the mandible a golden relationship is seen in the oblique direction.

Vertically, the golden relation from anterior nasal spine to the lower incisor and then to the top of the trigonum mentali or protuberance menti was also reconfirmed. Laying the golden dividers on the tracing on foramen rotundum and the first molar, a Φ relationship was found to the mandibular border in that oblique line.[2]

Summary -Why Divine Proportion Works

There is a proportion that the mind registers at the subconscious level that provides beauty, comfort and pleasure to the senses. This mathematical relation is 1.0 to 1.618 and is called the golden section. Because the teeth, jaws and face are geometric structures, the more closely they are alied to this proportion the more pleasing are the sensations conveyed to the beholder esthetically. It is therefore preferable to call it the divine proportion and the two parts related are said to be golden to each other.

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2. Ricketts – Biologic significance on the divine proportion and Fibbonacci series; Am J Orthod Dentofacial Orthop, 1982 May; 351 – 370.
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