TEN BASIC RULES OF INFERENCE (complete premises only)

Negation Introduction (~I – indirect proof IP)
Assume p
Get q & $\sim q$
$\vdash \sim p$

Negation Elimination (~E – version of DN)
$\sim\sim p$
$\vdash p$

Conditional Introduction (→I – conditional proof CP)
Assume p
Get q
$\vdash p \rightarrow q$

Conditional Elimination (→E – modus ponens MP)
$p \rightarrow q$
p
$\vdash q$

Conjunction Introduction (&I – conjunction CONJ)
p
q
$\vdash p$ & q

Conjunction Elimination (&E – simplification SIMP)
p & q
$\vdash p$

Disjunction Introduction (vI – addition ADD)
p
$\vdash p$ v q

Disjunction Elimination (vE – version of CD)
p v q
$p \rightarrow r$
$q \rightarrow r$
$\vdash r$

Biconditional Introduction (↔I – version of ME)
$p \rightarrow q$
$q \rightarrow p$
$\vdash p \leftrightarrow q$

Biconditional Elimination (↔E – version of ME)
$p \leftrightarrow q$
$\vdash p \rightarrow q$
or
$\vdash q \rightarrow p$

IMPORTANT DERIVED RULES OF INFERENCE (complete premises only)

Modus Tollens (MT)
$p \rightarrow q$
$\sim q$
$\vdash \sim p$

Hypothetical Syllogism (HS)
$p \rightarrow q$
$q \rightarrow r$
$\vdash p \rightarrow r$

Disjunctive Syllogism (DS)
p v q
$\sim p$
$\vdash q$

Absorption (ABS)
$p \rightarrow q$
$\vdash p \rightarrow (p$ & $q)$

Constructive Dilemma (CD)
p v q
$p \rightarrow r$
$q \rightarrow s$
$\vdash r$ v s

Repeat (RE)
p
$\vdash p$

Contradiction (CON)
p
$\sim p$
$\vdash$ Any wff

Theorem Introduction (TI)
Introduce any tautology, e.g., ~(P & ~P)

EQUIVALENCES (complete premises or sub-wffs)

De Morgan's Law (DM)
$\sim(p$ & $q) :: (\sim p$ v $\sim q)$
$\sim(p$ v $q) :: (\sim p$ & $\sim q)$

Commutation (COM)
$(p$ v $q) :: (q$ v $p)$
$(p$ & $q) :: (q$ & $p)$

Association (ASSOC)
$[p$ v $(q$ v $r)] :: [(p$ v $q)$ v $r]$
$[p$ & $(q$ & $r)] :: [(p$ & $q)$ & $r]$

Distribution (DIST)
$[p$ & $(q$ v $r)] :: [(p$ & $q)$ v $(p$ & $r)]$
$[p$ v $(q$ & $r)] :: [(p$ v $q)$ & $(p$ v $r)]$

Double Negation (DN)
$p :: \sim\sim p$

Transposition (TRANS)
$(p \rightarrow q) :: (\sim q \rightarrow \sim p)$

Material implication (MI)
$(p \rightarrow q) :: (\sim p$ v $q)$

Material Equivalence (ME)
$(p \leftrightarrow q) :: [(p$ & $q)$ v $(\sim p$ & $\sim q)]$
$(p \leftrightarrow q) :: [(p \rightarrow q)$ & $(q \rightarrow p)]$

Exportation (EXP)
$[(p$ & $q) \rightarrow r] :: [p \rightarrow (q \rightarrow r)]$

Tautology (TAUT)
$p :: (p$ & $p)$
$p :: (p$ v $p)$

Conditional-Biconditional Refutation Tree Rules
$\sim(p \rightarrow q) :: (p$ & $\sim q)$
$\sim(p \leftrightarrow q) :: [(p$ & $\sim q)$ v $(\sim p$ & $q)]$