RESEARCH ARTICLE

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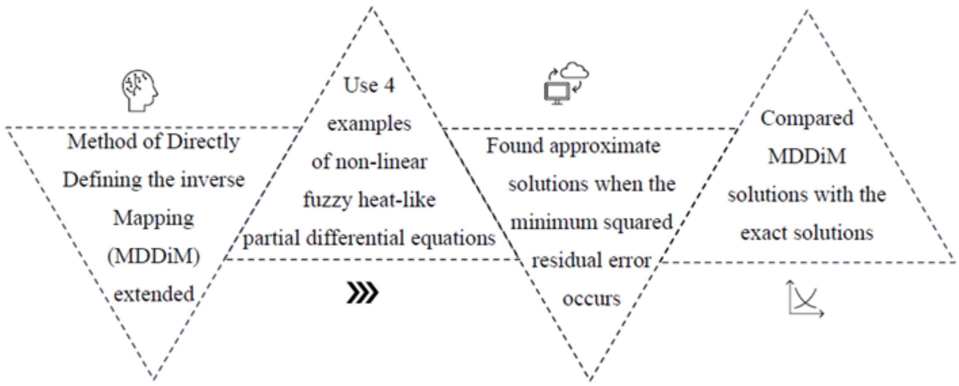

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Method of Directly Defining the Inverse Mapping for Nonlinear Fuzzy Heat-like Partial Differential Equations

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Abstract: Natural phenomena or physical systems can be described using Partial Differential Equations (PDEs), such as wave equations, heat equations, Poisson's equation, and so on. Consequently, investigations of PDEs have become one of the key areas of modern mathematical analyses, attracting a lot of attention. Many authors have recently expressed an interest in researching the theoretical framework of fuzzy Initial Value Problems (IVPs). The Method of Directly Defining the inverse Mapping (MDDiM) was effectively employed in this research to obtain the second-order approximate fuzzy solution of heat-like equations in one and two dimensions, and the results were compared with exact solutions. In each illustrated example, all the results achieved using Maple 16 were graphically depicted. This is the first time MDDiM was utilized to solve nonlinear Fuzzy Partial Differential Equations (FPDEs).

Keywords: Fuzzy numbers; Heat-like equations; Initial value problem; Method of directly defining the inverse mapping; Nonlinear fuzzy partial differential equations

INTRODUCTION

FPDEs are a branch of mathematics that extends classical partial differential equations by incorporating uncertainty and ambiguity. In FPDEs, the coefficients, boundary conditions, and even the solutions themselves are described using fuzzy logic, which allows for a more flexible representation of complex and imprecise systems. These equations are particularly useful in modeling phenomena where precise information is lacking or when dealing with data that possesses inherent fuzziness or vagueness. By embracing uncertainty, FPDEs provide a powerful framework for analyzing and solving problems in a wide range of fields, including physics, engineering, finance, and environmental science.

Zadeh introduced fuzzy sets for the first time (Zadeh, 1965) and hundreds of examples have been shown in which the nature of the uncertainty in the behavior of certain system processes is fuzzy rather than stochastic. Dubosi and Prade presented the extension principle, (Dubois & Prade, 1982) and concept of fuzzy derivative was introduced by Chang and Zadeh (Chang & Zadeh, 1996). Recently, Osman et al. applied the reduced differential transform method (RDTM) to solve fuzzy nonlinear PDEs by considering solutions as infinite series expansions which converge rapidly to the solutions (Osman et al., 2021).

MDDiM refers to a technique used in mathematics to determine the inverse of a given function without explicitly finding the formula for the inverse. Instead of solving for the inverse function symbolically, this method involves defining the inverse mapping through a set of equations or conditions. Let us consider a function $f : X \rightarrow Y$, where X and Y are sets. The inverse of f, denoted as f^{-1}, is a function that maps elements from Y back to X such that $f(f^{-1}(y)) = y$ for every y in Y. The method of directly defining the inverse mapping involves defining the inverse function f^{-1} through a set of equations or conditions that relate the input y to the output x. This approach is particularly useful when finding an explicit formula for the inverse function is difficult or impractical. By following this method, we can define the inverse function f^{-1} directly without explicitly finding the formula through algebraic manipulation.

With this motivation Liao and Zhao introduced MDDiM by directly defining the inverse map to Optimal Homotopy Analysis Method (OHAM) and applied it to solve nonlinear single Ordinary Differential Equations (ODEs) (Liao & Zhao, 2016). As time went by many researchers embedded this method in their projects. Nave and Elbaz successfully applied prostate cancer immunotherapy - mathematical model (Nave & Elbaz, 2018) and later, Nave introduced a new method to find the base functions for the MDDiM (Nave, 2018).

In 2018, Dewasurendra et al. extended MDDiM to solve a system of coupled nonlinear ODEs (Dewasurendra et al., 2018; 2020), and Gangadhar et al. successfully applied to obtain a series-form solution of the coupled nonlinear equations by the MDDiM and SRM (Gangadhar et al., 2022). Recently we further developed this novel technique to solve single and coupled nonlinear PDEs (Sahabandu et al., 2021; 2022). In this paper, we applied MDDiM to solve heat-like equations which describe single nonlinear FPDEs with some illustrative examples.

MATERIALS AND METHODS

Fuzzy numbers are a specific type of fuzzy set that represent numbers with uncertain or imprecise values. They are characterized by a membership function that assigns degrees of membership to different values within

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the number.

Definition 1: Let $\tilde{u}$ be a fuzzy number defined in $F(\overline{R})$. For each $r \in [0,1]$ indicated by $\tilde{u}_r$, the r-level set of $\tilde{u}$ is a crisp set that contains all the elements in R such that the membership value of $\tilde{u}$ is higher or equal to r, that is, $\tilde{u}_r = \{x \in \overline{R} \mid \tilde{u}(x) > r\}$.

When we express a fuzzy number with an r-level set, we mean that it is closed and bounded, as represented by $[\underline{u}_r, \overline{u}_r]$, where they represent the lower and upper bounds r-level set of a fuzzy number.

Definition 2: The parametric form of a fuzzy number $\tilde{u}$ is a pair $[\underline{u}_r, \overline{u}_r]$ of functions $\underline{u}_r$ and $\overline{u}_r$ for any $r \in [0,1]$, which satisfies the following requirements,

i. $\underline{u}_r$ is a bounded non-decreasing left continuous function in (0,1],

ii. $\overline{u}_r$ is a bounded non-increasing left continuous function in (0,1],

iii. $\underline{u}_r \leq \overline{u}_r$; $r \in [0,1]$.

Thus $\tilde{u}_r$ can be written as $\tilde{u}_r = [\underline{u}_r, \overline{u}_r]$.

Extend MDDiM to solve FPDEs

Consider the n^{th}- order nonlinear partial differential equation $N[\tilde{u}] = 0$. In the frame of MDDiM, the series solution of $\tilde{u}$ is given by,

$\tilde{u} = \tilde{u}_0 + \sum_{k=1}^{\infty} \tilde{u}_k$

~~S~~such that $\tilde{u}_0$ is the initial guess which satisfying the initial condition or boundary conditions and $\tilde{u}_k$ is defined as in equation (1).

$$\tilde{u}_k = \chi_k \, \tilde{u}_{k-1} + hL^{-1}[\delta_{k-1}] + \sum_{n=1}^{\mu} a_{k,n}\varphi_n \cdot \text{for } k \geq 1 \quad (1)$$

Here, L^{-1} is the inverse liner operator, N is the nonlinear operator, $a_{k,n}$ is a real constant, χ_k is the step function defined in equation (2), and h is the convergence control parameter; which should be determined (Sahabandu, 2021).

$$\chi_k = \begin{cases} 0, & \text{if } k < 1 \\ 1, & \text{if } k \geq 1 \end{cases} \quad (2)$$

By applying MDDiM to lower and upper bounds separately, we obtained ~~the~~ equations (3) and (4), respectively.

$$\underline{u}_k = \chi_k \, \underline{u}_{k-1} + hL^{-1}[\delta^k] + a_{k,0}, \text{ for } k \geq 1, \quad (3)$$

$$\overline{u}_k = \chi_k \, \overline{u}_{k-1} + hL^{-1}[\delta^k] + a_{k,0}, \text{ for } k \geq 1. \quad (4)$$

In the frame of MDDiM we have great privilege to choose an inverse linear operator with the following rules:

L^{-1} should be,

i. linear,
ii. injective,
iii. contains each base function, and
iv. finite.

We considered inverse linear operator given in equation (5) for each example explore in this study for different values of A.

$$L^{-1}[t^k] = \frac{t^{k+1}}{Ak+1}; A \text{ is an arbitrary constant} \quad (6)$$

Only adding first n terms, the n^{th} - term approximate solution can be written as $\tilde{u} = \tilde{u}_0 + \sum_{k=1}^{n} \tilde{u}_k$ for the nonlinear FPDE. Since this is not the exact solution, $N[\tilde{u}] \neq 0$. Hence, $N[\tilde{u}](r)$ for r in the domain (D) of the problem gives the residual error. Now, taking the square of L^2-norm, we defined square residual error function

$$E(h) = \int_D (N[\tilde{u}])^2(r)\, dr.$$

Example 1: Consider the following one-dimensional IVP describing fuzzy heat-like equation which is a single nonlinear FPDE.

$$\frac{\partial}{\partial t}\tilde{u}(x,t) = (x^2 \oplus 5) \odot \frac{\partial^2}{\partial x^2}\tilde{u}(x,t); 0 < x < 1, t > 0 \quad (6)$$

subject to the initial condition

$\tilde{u}(x,0) = [r^n, (2-r)^n] \odot (x^2+x)$, where $0 < r < 1$, $n = 1,2,3, \ldots$.

The parametric form of (6) is given by

$$\frac{\partial}{\partial t}\underline{u}(x,t;r) = (x^2+5)\frac{\partial^2}{\partial x^2}\underline{u}(x,t;r); 0 < x < 1, t > 0, \quad (7)$$

and

$$\frac{\partial}{\partial t}\overline{u}(x,t;r) = (x^2+5)\frac{\partial^2}{\partial x^2}\overline{u}(x,t;r); 0 < x < 1, t > 0, \quad (8)$$

for $r \in [0,1]$ where $\underline{u}(x,t;r)$ and $\overline{u}(x,t;r)$ are the lower bound and the upper bound solutions respectively, subject to the initial conditions,

$$\underline{u}(x,0) = r^n(x^2+x), \quad (9)$$

and

$$\overline{u}(x,0) = (2-r)^n(x^2+x). \quad (10)$$

Here $\tilde{u}(x,t;r) = [\underline{u}(x,t;r), \overline{u}(x,t;r)]$.

By considering,

$$N[\underline{u}(x,t;r)] = \frac{\partial}{\partial t}\underline{u}(x,t;r) - (x^2+5)\frac{\partial^2}{\partial x^2}\underline{u}(x,t;r) = 0$$

and,

$$N[\overline{u}(x,t;r)] = \frac{\partial}{\partial t}\overline{u}(x,t;r) - (x^2+5)\frac{\partial^2}{\partial x^2}\overline{u}(x,t;r) = 0$$

with initial conditions in equations (9) and (10), we obtained initial guesses as,

$\underline{u}_0(x;r) = r^n(x^2+x)$, and
$\overline{u}_0(x;r) = (2-r)^n(x^2+x)$.

Now, the three term solution can be written as, $\tilde{u}(x,t;r) = \tilde{u}_0(x;r) + \tilde{u}_1(x,t;r) + \tilde{u}_2(x,t;r)$ by considering $\underline{u}(x,t;r) = \underline{u}_0(x;r) + \underline{u}_1(x,t;r) + \underline{u}_2(x,t;r)$ and $\overline{u}(x,t;r) = \overline{u}_0(x;r) + \overline{u}_1(x,t;r) + \overline{u}_2(x,t;r)$ which are lower bound and upper bound solutions, respectively. We obtained the following three-term approximate solution using Maple 16 for $r \in [0,1]$.

$$\tilde{u}(x,t;r) = \begin{cases} r^n\left[(x^2+x) - 2h(x^2+5)t - 2h\left(1+h-\frac{2h}{A+1}t\right)(x^2+5)t\right] \\ (2-r)^n\left[(x^2+x) - 2h(x^2+5)t - 2h\left(1+h-\frac{2h}{A+1}t\right)(x^2+5)t\right] \end{cases}$$

Exact solution for this example is (Osman et al., 2021),

$$\tilde{u}(x,t;r) = [r^n, (2-r)^n] \odot \left((x^2+5)e^{2t} + x - 5\right), \ 0 \leq r \leq 1$$

Example 2: Consider the following two-dimensional IVP describing fuzzy heat-like equation which is a single nonlinear FPDE.

$$\frac{\partial}{\partial t}\tilde{u}(x,y,t) = \frac{1}{2}\left(x^2 \odot \frac{\partial^2}{\partial x^2}\tilde{u}(x,y,t) \oplus y^2 \odot \frac{\partial^2}{\partial y^2}\tilde{u}(x,y,t)\right);$$

$$x > 0, y < 1, t > 0 \quad (11)$$

subject to the initial condition

$$\tilde{u}(x,y,0) = [(0.2+0.2r)^n, (0.6-0.2r)^n] \oplus (x^2+y^2),$$

where $0 < r < 1$, $n = 1,2,3, ...$

The parametric form of (11) is given by,

$$\frac{\partial}{\partial t}\underline{u}(x,y,t;r) = \frac{1}{2}\left(x^2 \frac{\partial^2}{\partial x^2}\underline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\underline{u}(x,y,t;r)\right);$$

$$x > 0, y < 1, t > 0, \quad (12)$$

and

$$\frac{\partial}{\partial t}\overline{u}(x,y,t;r) = \frac{1}{2}\left(x^2 \frac{\partial^2}{\partial x^2}\overline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\overline{u}(x,y,t;r)\right);$$

$$x > 0, y < 1, t > 0, \quad (13)$$

for $r \in [0,1]$ where $\underline{u}(x, y, t; r)$ and $\overline{u}(x, y, t; r)$ are the lower bound and the upper bound solutions, respectively, subject to the initial conditions,

$$\underline{u}(x,y,0) = (0.2+0.2r)^n + (x^2+y^2), \quad (14)$$

and

$$\overline{u}(x,y,0) = (0.6-0.2r)^n + (x^2+y^2). \quad (15)$$

Here, $\tilde{u}(x,y,t;r) = [\underline{u}(x,y,t;r), \overline{u}(x,y,t;r)]$.

By considering,

$$N[\underline{u}(x,y,t;r)] = \frac{\partial}{\partial t}\underline{u}(x,y,t;r) - \frac{1}{2}\left(x^2 \frac{\partial^2}{\partial x^2}\underline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\underline{u}(x,y,t;r)\right) = 0$$

and,

$$N[\overline{u}(x,y,t;r)] = \frac{\partial}{\partial t}\overline{u}(x,y,t;r) - \frac{1}{2}\left(x^2 \frac{\partial^2}{\partial x^2}\overline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\overline{u}(x,y,t;r)\right) = 0$$

with initial conditions in equations (14) and (15), we obtained initial guesses as,

$\underline{u}_0(x,y;r) = (0.2+0.2r)^n + (x^2+y^2)$, and

$\overline{u}_0(x,y;r) = (0.6-0.2r)^n + (x^2+y^2)$.

Now, the three term solution can be written as, $\tilde{u}(x,y,t;r) = \tilde{u}_0(x,y;r) + \tilde{u}_1(x,y,t;r) + \tilde{u}_2(x,y,t;r)$ by considering $\underline{u}(x,y,t;r) = \underline{u}_0(x,y;r) + \underline{u}_1(x,y,t;r) + \underline{u}_2(x,y,t;r)$ and $\overline{u}(x,y,t;r) = \overline{u}_0(x,y;r) + \overline{u}_1(x,y,t;r) + \overline{u}_2(x,y,t;r)$ which are lower bound and upper bound solutions respectively. We obtained the following three-term approximate solution using Maple 16 for $r \in [0,1]$.

$$\tilde{u}(x,y,t;r) = \begin{cases} \left((0.2+0.2r)^n + (x^2+y^2)\right) - h(x^2+y^2)t \\ -h\left(1+h-\frac{h}{A+1}t\right)(x^2+y^2)t \\ \left((0.6-0.2r)^n + (x^2+y^2)\right) - h(x^2+y^2)t \\ -h\left(1+h-\frac{h}{A+1}t\right)(x^2+y^2)t \end{cases}$$

Exact solution for this example is (Osman et al., 2021),

$$\tilde{u}(x,y,t;r) = [(0.2+0.2r)^n, (0.6-0.2r)^n] \oplus \left((x^2+y^2)e^t\right),$$

$0 \le r \le 1$.

Example 3: Consider the following two-dimensional IVP describing fuzzy heat-like equation which is a single nonlinear FPDE.

$$\frac{\partial}{\partial t}\tilde{u}(x,y,t) = \tilde{v}(x,y,t) \oplus \frac{1}{4}\left(x^2 \odot \frac{\partial^2}{\partial x^2}\tilde{u}(x,y,t) \oplus y^2 \odot \frac{\partial^2}{\partial y^2}\tilde{u}(x,y,t)\right);$$

$$x > 0, y < 1, t > 0 \quad (16)$$

subject to the initial condition $\tilde{u}(x, y, 0) = \tilde{0}$, where

$$\tilde{v}(x,y,t;r) = (-1,0,1) \odot (x^2y^2)$$
$$= [(r-1)^n, (1-r)^n] \odot (x^2y^2),$$

where $0 < r < 1$, $n = 1,2,3, ...$

The parametric form of (16) is given by,

$$\frac{\partial}{\partial t}\underline{u}(x,y,t;r) = \underline{v}(x,y,t;r) \oplus \frac{1}{4}\left(x^2 \frac{\partial^2}{\partial x^2}\underline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\underline{u}(x,y,t;r)\right);$$

$$x > 0, y < 1, t > 0, \quad (17)$$

and

$$\frac{\partial}{\partial t}\overline{u}(x,y,t;r) = \overline{v}(x,y,t;r) \oplus \frac{1}{4}\left(x^2 \frac{\partial^2}{\partial x^2}\overline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\overline{u}(x,y,t;r)\right);$$

$$x > 0, y < 1, t > 0, \quad (18)$$

for $r \in [0,1]$, where $\underline{u}(x, y, t; r)$ and $\overline{u}(x, y, t; r)$ are the lower bound and the upper bound solutions, respectively, subject to the initial conditions,

$$\underline{u}(x,y,0) = \tilde{0}, \quad (19)$$

and

$$\overline{u}(x,y,0) = \tilde{0}. \quad (20)$$

Here, $\tilde{u}(x,y,t;r) = [\underline{u}(x,y,t;r), \overline{u}(x,y,t;r)]$.

By considering,

$$N[\underline{u}(x,y,t;r)] = \frac{\partial}{\partial t}\underline{u}(x,y,t;r) - (r-1)^n x^2y^2 - \frac{1}{4}\left(x^2 \frac{\partial^2}{\partial x^2}\underline{u}(x,y,t;r) + y^2 \frac{\partial^2}{\partial y^2}\underline{u}(x,y,t;r)\right) = 0$$

and,

$$N\,[\,\overline{u}(x,y,t;r)] = \frac{\partial}{\partial t}\overline{u}(x,y,t;r) - (1-r)^n\, x^2y^2 - \frac{1}{4}\left(x^2\,\frac{\partial^2}{\partial x^2}\overline{u}(x,y,t;r) + y^2\,\frac{\partial^2}{\partial y^2}\overline{u}(x,y,t;r)\right) = 0,$$

with initial conditions in equations (19) and (20), we obtained initial guesses as,

$\underline{u}_0(x,y;r) = \tilde{0}$, and $\overline{u}_0(x,y;r) = \tilde{0}$.

Now, the three term solution can be written as, $\tilde{u}(x,y,t;r) = \tilde{u}_0(x,y;r) + \tilde{u}_1(x,y,t;r) + \tilde{u}_2(x,y,t;r)$ by considering $\underline{u}(x,y,t;r) = \underline{u}_0(x,y;r) + \underline{u}_1(x,y,t;r) + \underline{u}_2(x,y,t;r)$ and $\overline{u}(x,y,t;r) = \overline{u}_0(x,y;r) + \overline{u}_1(x,y,t;r) + \overline{u}_2(x,y,t;r)$ which are lower bound and upper bound solutions respectively. We obtained the following three-term approximate solution using Maple 16 for $r \in [0,1]$.

$$\tilde{u}(x,y,t;r) = \begin{cases} -h(r-1)^n(x^2y^2) - h(r-1)^n\left(1+h-\dfrac{h}{A+1}\,t\right)(x^2y^2)t \\ -h(1-r)^n(x^2y^2) - h(1-r)^n\left(1+h-\dfrac{h}{A+1}\,t\right)(x^2y^2)t \end{cases}$$

Exact solution for this example is (Osman et al., 2021),

$$\tilde{u}(x,y,t;r) = [(r-1)^n,(1-r)^n] \odot \left((x^2y^2)e^t\right),\ 0 \le r \le 1.$$

Example 4: Consider the following IVP describing fuzzy heat-like equation which is a single nonlinear FPDE.

$$\frac{\partial}{\partial t}\tilde{u}(x,t) = \frac{\partial^2}{\partial x^2}\tilde{u}(x,t) \ominus \tilde{u}^2(x,t);\ 0 < x < 1,\ t > 0, \quad (21)$$

subject to the initial condition

$$\tilde{u}(x,0) = [(0.5+0.5r)^n,(1.5-0.5r)^n] \oplus 1,$$

where $0 < r < 1$, $n = 1,2,3, \ldots$.

The parametric form of (21) is given by,

$$\frac{\partial}{\partial t}\underline{u}(x,t;r) = \frac{\partial^2}{\partial x^2}\underline{u}(x,t;r) - \underline{u}^2(x,t;r);$$

$$0 < x < 1,\ t > 0, \quad (22)$$

and

$$\frac{\partial}{\partial t}\overline{u}(x,t;r) = \frac{\partial^2}{\partial x^2}\overline{u}(x,t;r) - \overline{u}^2(x,t;r);$$

$$0 < x < 1,\ t > 0, \quad (23)$$

For $r \in [0,1]$ where $\underline{u}(x,y,t;r)$ and $\overline{u}(x,y,t;r)$ are the lower bound and the upper bound solutions, respectively, subject to the initial conditions,

$$\underline{u}(x,0) = (0.5+0.5r)^n + 1, \quad (24)$$

and

$$\overline{u}(x,0) = (1.5-0.5r)^n + 1. \quad (25)$$

Here, $\tilde{u}(x,t;r) = [\underline{u}(x,t;r), \overline{u}(x,t;r)]$.

By considering,

$$N\,[\,\underline{u}(x,t;r)] = \frac{\partial}{\partial t}\underline{u}(x,t;r) - \frac{\partial^2}{\partial x^2}\underline{u}(x,t;r) + \underline{u}^2(x,t;r) = 0$$

and,

$$N\,[\,\overline{u}(x,t;r)] = \frac{\partial}{\partial t}\overline{u}(x,t;r) - \frac{\partial^2}{\partial x^2}\overline{u}(x,t;r) + \overline{u}^2(x,t;r) = 0,$$

with initial conditions in equations (24) and (25), we obtained initial guesses as, $\underline{u}_0(x;r) = (0.5+0.5r)^n + 1$, and $\overline{u}_0(x;r) = (1.5-0.5r)^n + 1$.

Now, the three term solution can be written as, $\tilde{u}(x,t;r) = \tilde{u}_0(x;r) + \tilde{u}_1(x,t;r) + \tilde{u}_2(x,t;r)$ by considering $\underline{u}(x,t;r) = \underline{u}_0(x;r) + \underline{u}_1(x,t;r) + \underline{u}_2(x,t;r)$ and $\overline{u}(x,t;r) = \overline{u}_0(x;r) + \overline{u}_1(x,t;r) + \overline{u}_2(x,t;r)$ which are lower bound and upper bound solutions, respectively. We obtained the following three-term approximate solution using Maple 16 for $r \in [0,1]$.

$$\tilde{u}(x,t;r) = \begin{cases} ((0.5+0.5r)^n+1) + h((0.5+0.5r)^n+1)^2t \\ +\,h((0.5+0.5r)^n+1)^2\left(1+h+\dfrac{2((0.5+0.5r)^n+1)h}{A+1}t\right)t \\ ((1.5-0.5r)^n+1) + h((1.5-0.5r)^n+1)^2t \\ +\,h((1.5-0.5r)^n+1)^2\left(1+h+\dfrac{2((1.5-0.5r)^n+1)h}{A+1}t\right)t \end{cases}$$

Exact solution for this example is (Osman et al., 2021),

$$\tilde{u}(x,t;r) = [(0.5+0.5r)^n,(1.5-0.5r)^n] \oplus \left[\frac{1}{1+t}\right],$$

$0 \le r \le 1$.

RESULTS AND DISCUSSION

The following graphs obtained for three-term approximate solutions of MDDiM and exact solutions for each example using maple 16.

Now, taking squared residual error for above examples as; $E(h) = \int_0^1 (N[\tilde{u}])^2(r)\,dr$, we obtained the following results for the converge control parameter h when minimum error occurs.

Table 1: Values of corresponding squared residual errors $E(h)$ and values of convergence control parameter h.

Example	$\underline{u}(x,t)$		$\overline{u}(x,t)$	
	h	$E(h)$	h	$E(h)$
1	-1.21320	2.0146×10^{-9}	-1.21320	4.4022×10^{-9}
2	-2.90514	9.9354×10^{-18}	-2.90514	9.9354×10^{-18}
3	-2.45000	8.8415×10^{-43}	-2.4500	8.8415×10^{-43}
4	-0.00100	2.0805×10^{-3}	-0.00100	2.0805×10^{-3}

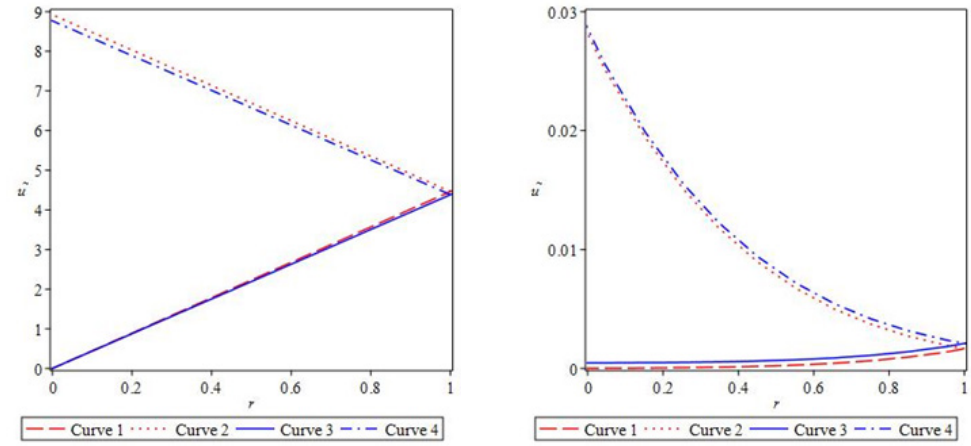

Figure 1: Example 1: $A = 1$, $x = 0.2$, $t = 0.3$, $n = 1$ (left), Example 2: $A = 19$, $x = 0.0004$, $y = 0.0005$, $t = 7$, $n = 7$ (right); In each example, curves 1, 2 and curves 3, 4 represent MDDiM and the exact solutions, respectively. Furthermore, the curves 1, 3 and curves 2, 4 represent the lower bound and the upper bound solutions, respectively.

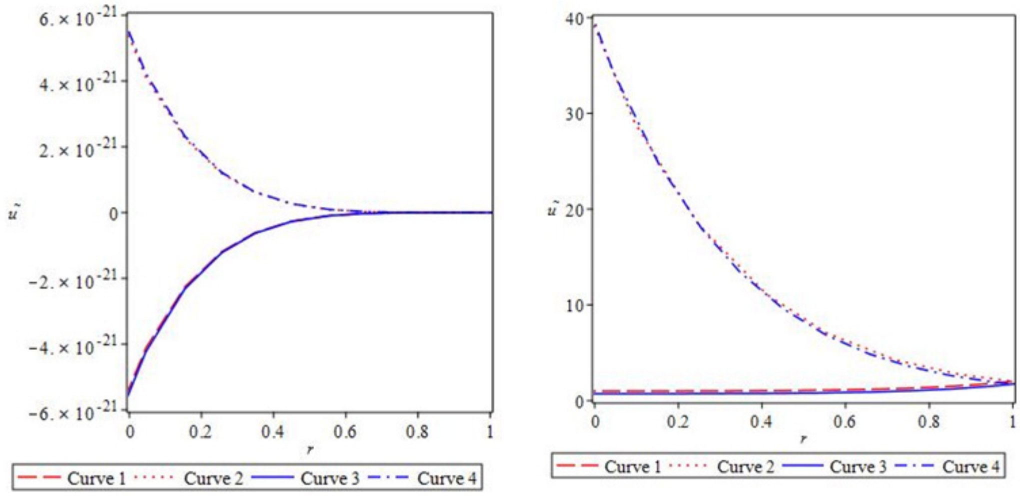

Figure 2: Example 3: $A = 0$, $x = 0.000002$, $y = 0.000003$, $t = 5$, $n = 5$ (left), Example 4: $A = 0$, $t = 0.4$, $n = 9$ (right); In each example, curves 1, 2 and curves 3, 4 represent MDDiM and the exact solutions, respectively. Furthermore, the curves 1, 3 and curves 2, 4 represent the lower bound and the upper bound solutions, respectively.

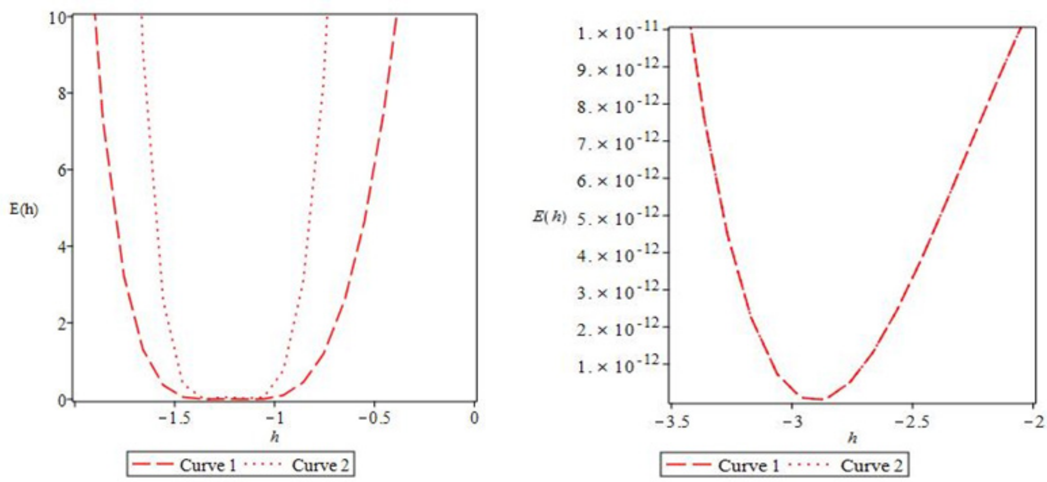

Figure 3: Corresponding error graphs of lower (curve 1) and upper (curve 2) bounds verses h for Example 1(left) and Example 2 (right) respectively.

CONCLUSION

In this paper, MDDiM has been successfully applied to solve fuzzy heat-like equations in one and two dimensions with variable coefficients. We obtained three-term approximate solutions for the nonlinear FPDEs using Maple 16. All the solutions are accurate enough with the squared residual errors as given in table 1. Figures 1 and 2 illustrate that, the lower functions of the *r*-level set of $\tilde{u}$ are always increasing functions of *r*, whereas the upper functions of the *r*-level set of $\tilde{u}$ are always decreasing functions of *r*. Additionally, we can see that the MDDiM approximate solutions converge to the exact solutions.

DECLARATION OF CONFLICT OF INTEREST

The authors declare no conflict of interest.

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