

# Modelling of Prosumer Behaviour in Peer-to-Peer Energy Trading Environment Through Markov Chain Analysis

R.H.G. Sasikala, K.A.C. Udayakumar and Narendra de Silva

**Abstract:** The emergence of distributed generation has become significant over the last few years, bringing several challenges to traditional grid-connected electricity distribution systems. Technical limitations of integrating these new renewable distributed generation to the existing electricity grid made new avenues to change the way of energy consumption, which converted traditional consumers to prosumers. Peer-to-peer energy trading platforms enable these prosumers to trade electricity with their neighbours in microgrids where new electricity markets emerge with new technologies. This research introduces a concept of peer-to-peer energy trading for grid-connected electricity distribution networks. In its initial phase, the study models prosumer load sharing for households equipped with solar PV and battery energy storage. This modelling employs a Markov chain, where energy share states are determined by parameters in a Markov transition probability matrix. These parameters are varied during different time intervals of the day, reflecting varying cost components under a Time-of-Use (TOU) tariff structure. This approach enables prosumers to meet their load demand cost-effectively, fostering engagement in peer-to-peer energy exchanges. The proposed Markov model is simulated with 15-minute spot pricing intervals, demonstrating effective management of prosumer load demands in the peer-to-peer market environment, showcasing the potential benefits of this innovative energy trading approach.

**Keywords:** Distributed generation, Markov chain, Prosumer, Peer-to-peer energy trading, Spot pricing


## 1. Introduction

Over the last few years, there has been extensive growth in small-scale distributed energy resources. The penetration of renewable energy based distributed generations (DG) is expected to continue growing in the future. These distributed generators would range from a few kW to a few MW depending upon the availability of the resource as well as the acquisition capacity of the grid. These small-scale resources, while contributing significantly to reducing the carbon footprint of an electric grid, impose substantial integration challenges such as transient instability by intermittency and voltage rise at the distribution feeders. If managed effectively, these challenges will enable renewable energy sources (RES) to manage the energy demand more efficiently and enable a significant mix of clean energy into the grid [1]. Investigating less transient and less intermittent renewable energy sources has been one of the vibrant research areas in the energy sector [2]. Among such researches, solar PV with battery energy storage systems (BESS) plays an important role in distribution systems due to the inherent advantages of its applications. However, to achieve the maximum benefit of the energy storage system,


a mechanism needs to be devised for the customers to interactively transact energy between each storage so that a demand-based reliability market is established.

The ongoing deployment of distributed renewable energy sources, such as solar PV with battery energy storage, transform show we consume and produce electricity. This trend shows that small-scale energy technologies are becoming affordable for regular households, creating the transition from consumerism to prosumerism [2]. This development is complemented by the advancement of ICT technologies, enrolment of smart meters, and potentials from new distributed technology development (e.g. blockchains).

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Digitalization and automatization of the grid will enable closer interaction between end-users and distributed system operators (DSOs) thus introducing the possibility of a more consumer-centric power system [2].

Peer-to-peer (P2P) energy trading has been introduced as a new energy management system that is more consumer-centric, thus traditional electricity consumers become prosumers and actively engage in electricity trading. The potential of P2P energy trading comes from the diversity of the generation (equipped with DGs) and demand profiles of different customers. This diversity results in some customers needing energy at the same time as others having surplus energy that can be shared. The application of P2P makes it possible for individual consumers to become prosumers and to share their part of energy with the peers of the distribution system. From a technical point of view, Peer-to-peer trading in local electricity markets provides a new framework to operate renewable distributed generation in low-voltage systems. Such trading reduces the stress on power system operators in the main grid. Since recent past, there has been a growth in real-life pilot projects demonstrating their viability and challenges [3].

P2P Distributed Energy Trading (DET) enables everyone to exchange energy without relying on a central utility company. This energy exchange can create a competitive energy market that is not monopolized by a few utility companies, bringing profit to small-scale energy producers and consumers [4]. Moreover, distributed energy trading can also reduce power outages to prosumers by providing their own local energy sources during a power outage from the central utility provider. P2P DET also allows consumers access to a wide range of alternative energy sources according to their preferences. Furthermore, energy trading also brings various benefits to utility companies, such as increasing the overall efficiency of the grid and reducing operation costs [4].

Moreover, in most countries, the feed-in tariff for selling electricity back to the power grid is lower than the price for buying electricity from the power grid, which provides customers with economic incentives to trade with each other before trading separately with the grid [5]. From the perspective of power system operators, P2P energy trading provides a potential measure to manage high DER penetration in the future. DERs are subjected to a variety of types, features, capacities, locations,

and ownership, and are spread all over the edge of power systems. These facts make DERs impractical and costly to manage in a conventionally centralized manner. If proper P2P energy trading mechanisms are designed, DERs could autonomously facilitate a better local balance in terms of both power and energy. This could release the pressure and reduce the uncertainties for the upstream power grid [6].

However, the exchange of electricity is different from any other exchange of goods, as the consumers are connected to a complex power system. This kind of local electricity exchange structure will be directly associated with the technical constraints of the grid and the impact on the grid stability needs to be considered when designing such a P2P scheme. For example, P2P energy trading might be driven by prosumer-to-consumer overall welfare benefits and leave behind any possible challenges to grid operators [2]. Therefore, when P2P energy trading happens in a grid-connected distribution network, its impacts on grid losses, voltage variation, grid congestion, and other physical constraints on system operators must be identified [4].

Additional key challenges need to be addressed to support P2P DET. As P2P DET is based on a two-way communication network, this might expose the system to various types of security and privacy threats, which can harm the system's confidentiality, integrity, and reliability [4].

In P2P trading, prosumers are expected to trade their energy with peers having a very low (or not any) influence from a central controller. This makes P2P platforms a trustless system. Hence, it is a challenging task to encourage prosumers to cooperate in such a trustless environment.

To identify the realization of P2P energy trading, the designer should consider aspects such as demand response optimization, power routing, public energy market, money transaction mechanisms, and efficient communication networks [4].

Finally, several stakeholders in the grid may request prosumers' P2P services with different objectives in their minds. Thus, innovations are needed in the pricing scheme to prioritize these requests to deliver a non-congested service throughout the entire network while keeping the network loss minimal [7].

Creating a standardized P2P electrical energy trading model is complex due to the diverse technologies and infrastructures involved. In such an environment, household users are

primarily self-interested and driven by incentives. To promote efficient energy trading among peers, there is a need for an economic model that motivates prosumers to engage in socially beneficial trading. Particularly, the economic model for energy trading within solar PV integrated systems is a novel concept that requires further development and identification.

Though there were several trading models developed worldwide, the realization of those models is a complicated and challenging task. A simple peer-to-peer economic model that can be effectively used in any grid-connected system would benefit grid-connected prosumers.

To address the challenges of P2P energy trading and develop a model for grid-connected distribution networks, this research study proposes a simple P2P trading mechanism based on the transitional analysis of each energy trading agent. This model focuses on analyzing the dynamics of each participant in energy trading, particularly grid-connected households functioning as prosumers with rooftop solar PV and battery energy storage systems.

The first phase of this research study focuses on modelling a single household as a case study, providing insights into individual prosumer load sharing. The subsequent phase involves a peer market model, offering a comprehensive view of the dynamics within a multiple prosumer energy management system with peer energy trading. Through these results, we aim to gain a deeper understanding of the implications and performance of the proposed energy-sharing and trading mechanisms. This will enable household users to engage in energy sharing in a socially efficient manner while getting maximum benefits. The proposed model assesses the variation of house owners' energy consumption with distributed energy sources and benefits the owner in engaging peer-to-peer energy exchange.

In this manuscript, only single prosumer load sharing is elaborated through the Markov chain modelling.

## 2. Literature Review

The literature relating to the P2P trading in the electricity distribution sector can be divided into several sections.

### 2.1 P2P trading models/Architecture

Several researchers [8] discuss various aspects of P2P DET. The authors provide an overview of the whole P2P system but do not present the recent research problems associated with it.

Research done by Olamide et al. [9] compared five P2P DET projects based on their business models, including recently commercialized and pilot services. The study identified the potential development and future challenges based on the business model characteristics of each case. Chankook & Yong [10] and Wayes et al. [11] discussed the aspects of P2P energy trading without the intervention of the main grid controller or utility company.

Defining a standardized model for P2P electrical energy trading is quite challenging. Different types of P2P DET architectures have been devised by various researchers [6,12–19]. The studies [6,12] suggested a hierarchical P2P DET model. Power systems are hierarchically arranged so that microgrids consisting of several end customers are connected to a distribution system, and multiple distribution systems are connected to a transmission network. Because of this hierarchical nature, P2P DET is expected to be carried out in three hierarchical levels [6,12]. Existing industrial projects on P2P energy trading at the microgrid level are discussed in detail by Zang [3].

Zhang et al. [13] and Zhang [14] proposed a four-layer model of P2P DET to explain the design and interoperability aspects of the components of P2P DET. The hierarchical process of P2P DET is categorized into four interoperability layers: the business layer, control layer, Information and Communication Technologies (ICT) layer, and power grid layer. Ari et al. [15] proposed a distributed business model that gives solutions to P2P electricity trading, P2P grid control, and distributed ICT. The model divides the power system into three planes (trading plane, control plane, and market plane) that provide electricity trading, grid control, and wireless communication services, enabling the proposed P2P operation.

Wayes et al. [17] provided more details on P2P DET models. The authors provided an overview of the use of game theoretic approaches for P2P energy trading as a feasible and effective means of energy management. Reihani et al. [18] introduced a new model for P2P trading at the distribution level.

### 2.2 Demand Response Optimization Models

In traditional power systems, load scheduling is managed by a single centralized entity. Such scheduling becomes challenging in DG integrated distribution system. This is because power is generated and injected into the system in a decentralized manner. Moreover, power generation from distributed renewable energy

sources such as wind and solar is highly unpredictable. Demand response optimization is more difficult for P2P DET than the existing centralized system. Understanding this gap, researchers have tried to develop better energy scheduling methods and price optimization algorithms such as game theoretic-based, collaborative-based, incentive-based, and centrally controlled models.

#### **Centrally controlled methods**

Wu et al.[20] suggested a pricing-based optimization algorithm for local smart microgrid energy trading that is controlled by a local trading manager (LTM). The model proposed two optimization algorithms. The same authors extended this idea [21] proposing a price optimization algorithm for a hybrid energy trading market comprising a utility company and a local trading market controlled by a local trading center (LTC).

#### **Incentive-Driven Models**

Incentive-driven models encourage users by incentivizing them to continue participating and contributing to the demand response optimization process. Wang et al. [22] proposed an incentive-based renewable energy sharing mechanism. The method allows energy trading among several users simultaneously. However, the benefit to the smart grid operator, who provides a platform for users to sell and buy their surplus energy, is not explicitly mentioned. Fairness is an important criterion for effective energy trading. Tabibnia & Lieberman [23] show that fairness leads to higher happiness of users and higher energy efficiency. A survey of incentive-based energy management schemes adopted for energy trading was performed by Shang et al. [24].

#### **Corporate Based Models**

Cooperative-based models involve many producers and consumers working together for their mutual benefit. Wu et al. [25] proposed a cooperative distributed energy generation and trading system that allows prosumers with energy generation and energy storage capabilities to trade energy cooperatively to minimize their total energy-provisioning cost while ensuring the local demand of each prosumer. Xu & Zhang [26] suggested a cooperative energy trading scheme that allows a base station (BS) having local renewable energy to perform energy trading with the main grid based on coordinated multi-point (CoMP) communication powered by smart grids.

#### **Game Theoretic Models**

Game theoretic approaches are the most widely employed techniques for demand response management in energy trading. The problem of energy trading is modelled as a Multileader multi-follower Stackelberg game in several studies [27–29]. Lee et al. [30] suggested a coalitional game to derive the optimal price of electricity for energy trading between small-scale energy producers and consumers. Lee et al. [31] and Yaagoubi & Mouftah [32] proposed a game theory-based distributed energy trading algorithm that allows consumers to buy energy from neighbouring producers at a lower price than utility companies. Recent work by Mousa et al.[33] proposed a retail electricity market based on game theory for the optimal operation of home microgrids within active distribution networks.

P2P Distributed Energy Trading (DET) primarily focuses on enabling direct energy trading between prosumers and consumers, allowing them to buy and sell electricity directly without needing an intermediary utility company. While this concept is well-accepted in theory, it is important to note that the physical flow of electrical power continues to be governed by the grid's physical laws. Unlike traditional power transmission, P2P DET does not physically transport power over long distances. Instead, it operates in a manner similar to electricity markets, where the real-time balance between total demand and supply is crucial for grid stability. The key distinction lies in the trading mechanism, which empowers individuals to engage in direct energy exchanges resembling marketplace transactions. Several studies conducted in this area includes the feasibility of power packet dispatching [34, 35], power routers [36–38], power routing algorithms [39, 40], and architectures [41–43].

In addition to the above-mentioned literature survey of existing work, several researchers introduced the new concept of Blockchain into overcome key challenges related to security, privacy, and mobility of P2P DET [44–45].

### **3. Methodology**

The current research proposes a model for peer-to-peer energy trading; thus, all the parameters associated with each agent involved in energy trading need to be identified. The following methodology is used for research design.

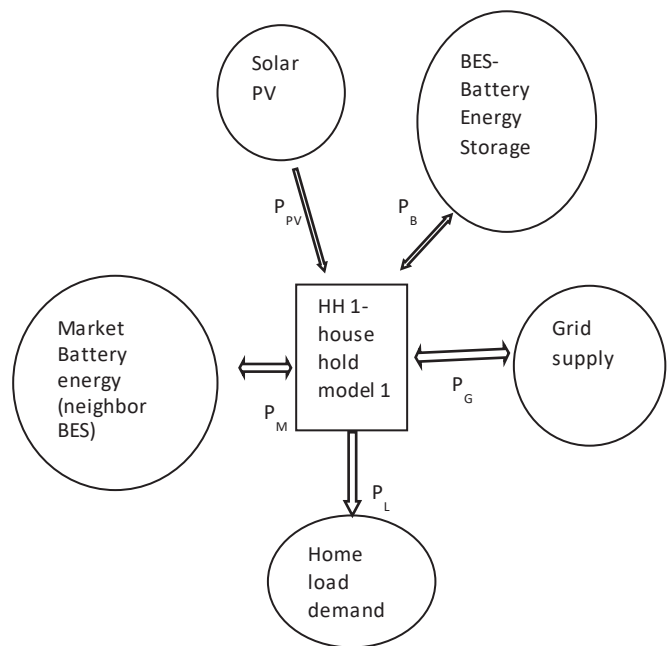
1. Select parameters of each prosumer, Solar PV panel, BESS, and network, which are

directly associated with possible energy sharing in grid-connected distribution network. Grid energy price, Peer-to-peer market energy price, generation from solar PV, battery State of Charge (SOC), Market available battery energy ( $P_{MSOC}$ ) are identified as parameters.

2. Develop the mathematical formulation of the transaction model through transitional analysis of the agents. This analysis is supplemented with the behavioural states of selected agents in describing their socio-economic as well as technical status. This model was developed as a discrete state transition model using state transition algorithms using the Markov chain model.
  - i. All entities involved in the energy trading process (i.e. household and network) are modelled as trading agents modelled on their socio-economic status and the generation-storage states.
  - ii. Identify the internal and external transaction variables between each model.
3. Design and develop an Excel simulation model or a MATLAB/SIMULINK for the identified mathematical model.
4. Test the simulation for different scenarios based on the status of each parameter of the agents involved in energy trading.
  - i. The simulation will be carried out to verify the effectiveness of the developed model in energy sharing in real networks. This will be done by varying external parameters, which directly change the status of each selected parameter of agents in the model.
5. Model validation in LV network through a prosumer market simulation

Prosumer's load demand is supplied by each energy source attached to the household as shown in Figure 1. Prosumer load sharing is modelled through Markov chain modelling where each state of energy share is determined by the probabilities involved in the Markov transition probability matrix. External parameters, such as grid price, peer market price, and battery SOC levels, are used as parameters to develop the Markov transition probability matrix. The probabilities of the transition probability matrix are varied according to different time intervals of the day. Therefore, the segmented Markov chain is simulated with 15-minute spot pricing intervals

to evaluate the states of energy share of a prosumer.



**Figure 1 - Model for Prosumer Energy Sharing**

The first phase of this study is based on the Markov chain, and the basic concept related to the Markov chain is briefly presented below.

#### Markov chain

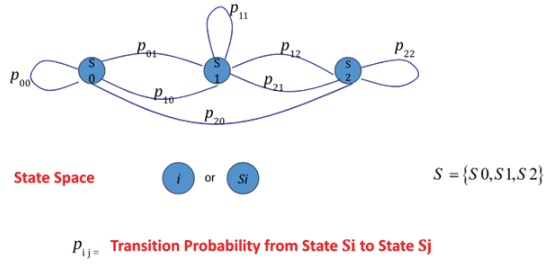
Markov chain is a stochastic model that outlines a probability associated with a sequence of events occurring based on the state in the previous event [46]. A Markov chain model depends on two key pieces of information:

- **Transition Matrix (P)** – This  $N \times N$  matrix represents the probability distribution of the state's transitions. The sum of probabilities in each row of the matrix should be one, which implies that this is a stochastic matrix.

Note: A directed, connected graph can be converted into a transition matrix. Each element in the matrix would represent a probability weight associated with an edge connecting two nodes.

- **Initial State Vector (denoted as S)** – This  $N \times 1$  vector represents the probability distribution of starting at each of the  $N$  possible states. Every element in the vector represents the probability of beginning at that state.

Given these two dependencies, as shown in Figure 2, it is possible to determine the initial state of the Markov chain by taking the product of  $P \times S$ . To predict the probability of future states occurring, you can raise your transition matrix  $P$  to the  $M$ 'th power [47].



**Figure 2 - Example of Markov chain [47]**

While Markov chain is a strong choice for this research, other methods, such as rule-based approaches, machine learning, or time series analysis, have their merits. However, Markov chain is selected for this research because it provides a structured framework to account for the stochastic and time-dependent nature of energy sharing in prosumer systems, which is crucial for accurately representing and simulating these dynamics. Markov chain is well-suited for modelling systems with distinct states that transition between each other over time. In this case, the states represent different modes of energy sharing. Markov chains inherently incorporate probabilistic elements. This is vital in modelling energy sharing, as it involves uncertainty, including variable renewable energy generation and fluctuating energy prices. Markov chains allow for the inclusion of probabilities in state transitions.

#### 4. Modelling of Prosumer through Markov Chain

A prosumer having a solar PV panel with battery energy storage is considered for the modelling of the system. A prosumer is connected to the grid while having battery storage on his premises. One household prosumer takes as an agent, and the load demand of this prosumer is met using one of the following sources depending on the energy trading rules.

1. Solar PV
2. Battery
3. Peer-to-peer market
4. Grid

The energy share between these four sources depends on the states of each agent, which again depends on some external parameters.

##### 4.1 State Variables of the Components in the Network Connected to the System

The state variables of the components in the network connected to the system represent the current status or condition of each energy mode. The Markov chain model then represents this system; the next state of energy sharing depends only on the present state and is

independent of the past data. The state variables for each component are as follows:

##### Solar PV:

**Solar Generation (Ppv):** Represents the amount of energy generated by the solar PV panel. Depending on external factors such as weather conditions, time of day, and location, it can take different values.

##### Battery Energy Storage (BES):

**State of Charge (SOC):** Represents the battery's current energy level or capacity. It is usually expressed as a percentage and ranges from 0% (fully discharged) to 100% (fully charged).

##### Peer-to-peer Market:

**Market Price (MP):** Represents the price at which energy is bought or sold in the peer-to-peer market. It can vary based on supply and demand dynamics, time of day, and other market factors.

**Total Battery Capacity in the Market (P<sub>Msoc</sub>):** This represents the total available battery capacity in the peer-to-peer market. It indicates the overall energy storage capacity that can be utilized for trading within the market.

##### Grid:

**Grid Price (GP):** This represents the price at which energy is bought from the grid. It can vary based on factors such as time of day, demand, and grid regulations.

In the Markov chain model, each state represents one of the energy modes (solar, battery, market, grid), and the transition from the present state to the next state only depends on the present state of energy sharing, as defined by the probabilities in the transitional probability matrix. The state variables for each component influence these probabilities and can affect the energy-sharing decisions within the system.

##### 4.2 Markov Chain Modelling of Prosumer

The following formula represents a Markov model for a peer-to-peer energy trading system representing one household entity.

$$X_{n+1} = P X_n$$

$X_n$ - represents the present state

$X_{n+1}$ - represents the next state

$n$ - no of steps (time)

$$\begin{pmatrix} \text{Solar} \\ \text{Battery} \\ \text{Market} \\ \text{Grid} \end{pmatrix}_{n+1} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \begin{pmatrix} \text{Solar} \\ \text{Battery} \\ \text{Market} \\ \text{Grid} \end{pmatrix}_n \quad \dots(1)$$

where, Off-diagonal elements ( $P_{ij}$ ) of the transitional probability matrix elements represent the probabilities of shifting from one energy mode (solar, battery, market, and grid) to another energy mode over the time.

Diagonal elements ( $P_{ii}$ ) of the P represent the probability of remaining in the same energy mode over time.

Examples for defining some of the Off-diagonal elements ( $P_{ij}$ ) and Diagonal elements ( $P_{ii}$ ) of P matrix are as follows.

$P_{11}$ = Transition probability remaining in the solar mode

$P_{12}$ = Transition probability from solar mode to battery mode

$P_{23}$ = Transition probability from battery mode to market mode

$P_{33}$ = Transition probability remaining in the market mode

### 4.3 Define transitional probability matrix with multiple segment Markov chain and define each probability using variables of internal and external parameters

Prosumer household load-sharing options from solar PV, BES, peer market, and the grid are given in Figure3 as a flow diagram. It is assumed that the prosumer prefers to prioritize using energy from his own BES due to the lower cost than obtaining energy from the market or grid when his solar generation is insufficient to cater to the load demand.

Based on the decision of switching between each energy source, as depicted in Figure3, the transitional probability matrix P can be defined to represent the probabilities of transitioning from one energy mode to another.

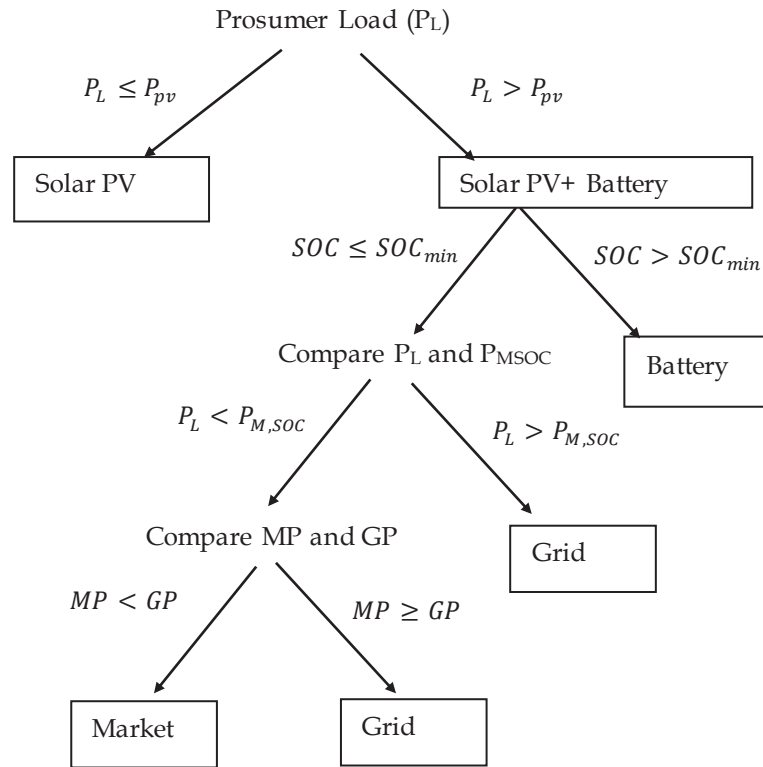


Figure 3 - Flow Diagram for Prosumer Load Sharing

Each probability can be defined using internal ( $X_1, \dots, X_n$ ) and external parameters, such as the solar generation ( $P_{pv}$ ), the SOC of the battery, the market price (MP), and the grid price (GP), for the corresponding time segment. Equations (2) to (17) define the elements of above-mentioned transition probability matrix P. The values of the probabilities can be updated for each time segment based on the values of these parameters at each time segment.

$$P_{11} = \begin{cases} 1, & \frac{P_{pv}}{P_L} \geq 1 \\ \frac{P_{pv}}{P_L}, & \frac{P_{pv}}{P_L} < 1 \end{cases} \quad \dots(2)$$

$$P_{12} = \begin{cases} 1 - \frac{P_{pv}}{P_L}, & \frac{P_{pv}}{P_L} < 1 \text{ and } SOC > SOC_{min} \\ 0, & \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \end{cases} \quad \dots(3)$$

$$P_{13} = \begin{cases} 0.95 - \frac{P_{pv}}{P_L}, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \leq GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP > GP \\ 0, \text{ else} \end{cases} \quad (4)$$

$$P_{14} = \begin{cases} 0.95 - \frac{P_{pv}}{P_L}, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \left\{ \left( \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP > GP \right) \text{ or } \frac{P_L}{P_{M,SOC}} \geq 1 \right\} \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } (MP < GP) \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \\ 0, \text{ else} \end{cases} \quad (5)$$

$$P_{21} = \begin{cases} 1, \frac{P_{pv}}{P_L} \geq 1 \\ 0, \text{ else} \end{cases} \quad (6)$$

$$P_{22} = \begin{cases} SOC, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC > SOC_{min} \\ 0, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \end{cases} \quad (7)$$

$$P_{23} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \geq GP \\ 1 - SOC, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC > SOC_{min} \\ 0, \text{ else} \end{cases} \quad (8)$$

$$P_{24} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP > GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 1, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} \geq 1 \\ 0, \text{ else} \end{cases} \quad (9)$$

$$P_{31} = \begin{cases} 1, \frac{P_{pv}}{P_L} \geq 1 \text{ and } \frac{P_L}{P_{M,SOC}} > 1 \\ 0, \text{ else} \end{cases} \quad (10)$$

$$P_{32} = \begin{cases} 1, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC > SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} \geq 1 \\ 0, \text{ else} \end{cases} \quad (11)$$

$$P_{33} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \geq GP \\ 0, \text{ else} \end{cases} \quad (12)$$

$$P_{34} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \geq GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 1, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} \geq 1 \\ 0, \text{ else} \end{cases} \quad (13)$$

$$P_{41} = \begin{cases} 1, \frac{P_{pv}}{P_L} \geq 1 \\ 0, \text{ else} \end{cases} \quad (14)$$

$$P_{42} = \begin{cases} 1, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC > SOC_{min} \\ 0, \text{ else} \end{cases} \quad (15)$$

$$P_{43} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \geq GP \\ 0, \text{ else} \end{cases} \quad (16)$$

$$P_{44} = \begin{cases} 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP \geq GP \\ 0.05, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} < 1 \text{ and } MP < GP \\ 0.95, \frac{P_{pv}}{P_L} < 1 \text{ and } SOC \leq SOC_{min} \text{ and } \frac{P_L}{P_{M,SOC}} \geq 1 \\ 0, \text{ else} \end{cases} \quad (17)$$

$P_{22}$ , an element within the transition probability matrix, plays a crucial role in representing the probability of remaining in battery mode during two consecutive time steps. Specifically, it reflects the likelihood of the system continuing to rely on the battery as the source of power. This situation arises when solar generation alone cannot fully meet the current load demand, and the remaining portion of the load is supplied by the battery, as illustrated in Figure 2.

The probability assigned to  $P_{22}$  varies proportionally with the state of charge (SOC) of the battery at that moment. In simpler terms, when the battery's SOC is high ( $SOC > SOC_{min}$ ), the probability of remaining in battery mode (i.e., supplying power from the battery) is elevated, signifying a greater reliance on the battery. Conversely, when the SOC is lower, the probability decreases, indicating a reduced preference for battery power. Importantly, the probability becomes zero for all SOC values falling below the minimum SOC threshold ( $SOC_{min}$ ), indicating the battery's inability to provide power when its charge is insufficient.

In summary,  $P_{22}$ 's value is contingent on the battery's state of charge, and it quantifies the probability of continued battery usage when solar generation falls short of meeting the load demand, effectively capturing the dynamic nature of the system's energy management.

To assign values for the transitional probability matrix with multiple segment Markov chain, we first need to identify the different segments of the day where the values of the transition probabilities will vary. For this system, a multiple segment Markov chain model can be used, where the probabilities are defined for each time segment of the day. In this way, the transitional probability matrix can be adjusted based on the time of energy consumption.

Transition probabilities of switching between the peer market and the grid are taken as 0.95 due to the reason that the customer may willing to

reduce his load instead of taking energy at cost either from the market or grid. Hence consumer willingness to move to one of the sources may be not exactly fully agreeable with transition probability as one, so there is a 0.05 probability margin of interest that he is not willing to buy energy[48].

## 5. Results and Discussion

### 5.1 Case Study for Prosumer Household Load Contribution

A typical household having a solar PV of 2.5 kW and BES of 200 Ah has been considered for the case study. A particular day, comprising 24 hours, is partitioned into distinct time segments. For each of these segments, transition probability matrices are derived by considering the states of the selected parameter. Equations (2) to (17) are utilized in the derivation process. The specific values corresponding to the parameters for each time segment is provided in Table 1.

For example, two transition probability matrices for different time segments (1745-1830 hrs and 0630-0715 hrs) which were defined according to Equations (2) to (17) are presented in Figure 4 and Figure 5 respectively. To calculate the values for each element in two matrices P9 and P4, the data expressed in Table 1 is used. All other transition probability matrices for other time segments are denoted in Table 1 are calculated in the same manner.

Figure 4 represents the values for the transition probability matrix (P9) for 1745-1830 hrs time segment with its graphical representation of state transition.

	0.5	0.5	0.0	0.0
P9	0	0.7	0.3	0
	0	1	0	0
	0.0	1.0	0.0	0.0

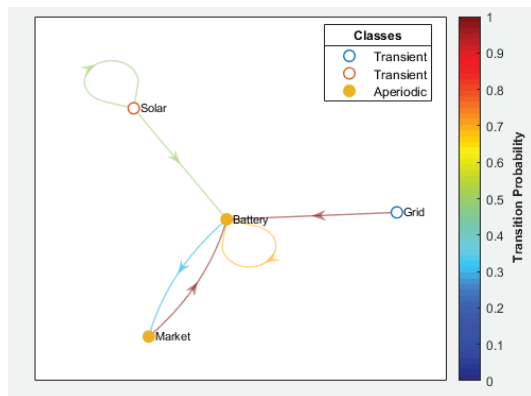


Figure 4 - Transition Probability Diagram for 1745-1830 hrs

Figure 5 represents the values for the transition probability matrix (P4) for 0630-0715 hrs time segment with its graphical representation of state transition.

	0.89	0.11	0.00	0.00
P4	0.00	0.30	0.70	0.00
	0.00	1.00	0.00	0.00
	0.00	1.00	0.00	0.00

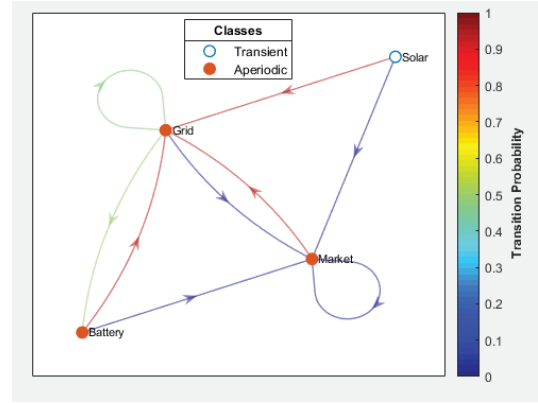


Figure 5 - Transition Probability Diagram for 0630-0715 hrs

In the context of a selected prosumer with a daily load demand of around 12 kWh, a solar PV array capacity of 2.5 kW was chosen to fulfil maximum of 10 kWh during daytime. It is assumed that all prosumers and consumers connected in the peer market trade their energy through bilateral trading bids, with detailed money transactions falling outside the scope of the research.

The selected prosumer is metered at the Time-of-Use (TOU) tariff category given by utility provider, Ceylon Electricity Board (CEB). Initially, it is assumed that all prosumers in the neighbourhood sell energy at a constant market price which is denoted by MP(i.e., the market is represented only by market price, and the price is considered as pool price).

The multiple-segment Markov process is simulated using 0.25-hour (15-minute time steps) time intervals, as it is assumed that the spot pricing for the entire market changes every 15 minutes.

Figure 6 displays the daily load contribution from each energy source connected to the prosumer's household. The load curve data and all the parameter values to determine transition probabilities are tabulated in Table 1. Equation (1) is used to obtain energy share by different sources for selected load demand at each time step as shown in Figure 6.

## 5.2 Discussion

Figure 6 illustrates how a prosumer household optimizes his energy consumption from four different energy modes, considering electricity tariffs, market prices, and resource limitations of his battery storage. The observed patterns show

that during the early morning period (steps 1 to 18), the majority of the load is supplied by the grid, with minimal contributions from the peer market, influenced by the prevailing price values.

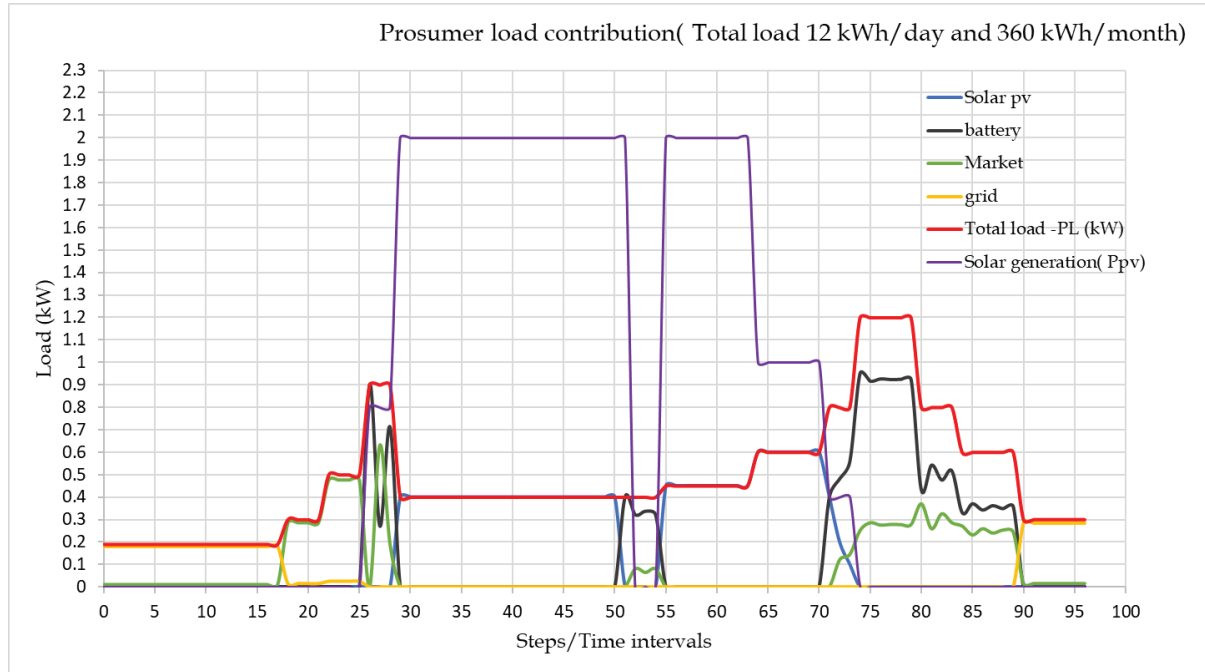


Figure 6 - Prosumer Daily Load Demand Contribution from different Energy Sources

Table 1 - Simulation Data for 24 hours

	Time steps	Time segment	Transition probability matrix	House load (P <sub>L</sub> )kW	Solar generation (P <sub>PV</sub> )kW	Market price(LKR) (MP)	Grid price (LKR) (GP)	State of charge (%) (SOC)
off-peak	0-17	2400-0430	P1	0.19	0.0	15	13	0.1
	18-21	0430-0530	P2	0.3	0.0	20	25	0.1
day	22-25	0530-0630	P3	0.5	0.0	22	25	0.1
	26-28	0630-0715	P4	0.9	0.8	26	25	0.3
	29-51	0715-1300	P5	0.4	2.0	20	25	0.8
	52-54	1300-1330	P6	0.4	0.0	20	25	0.8
	55-63	1330-1600	P7	0.45	2.0	20	25	0.7
	64-70	1600-1745	P8	0.6	1.0	20	25	0.7
	71-73	1745-1830	P9	0.8	0.4	20	25	0.7
	74-79	1830-2000	P10	1.2	0.0	40	54	0.7
peak	80-83	2000-2100	P11	0.8	0.0	55	54	0.4
	84-89	2100-2230	P12	0.6	0.0	50	54	0.3
off-peak	90-96	2230-2400	P13	0.3	0.0	15	13	0.1
		P <sub>MSOC</sub>	4 kW					
		SOC <sub>min</sub>	0.2					

At 7:30 am (step 30), where solar PV generation already started and reaches to its maximum generation, the entire load demand is fulfilled by solar PV at that time. Any excess solar PV generation can be directed towards charging the battery, selling to the peers or to the grid depending on market demand and battery limitations. These decisions are driven by the

associated costs and the battery's charging capacity.

Around 1:00-1:30 pm (steps 52 to 54), solar PV generation diminishes due to cloud conditions, but the prosumer load is consistently met by switching to battery storage. During peak demand hours in the evening, typically between 6:30-10:00 pm (steps 74 to 89), the load

demand is met by both the battery storage and the peer market, as grid energy prices reach highest levels. Figure 6 illustrates the efficient management of the prosumer's load. It begins by utilizing solar PV generation when it is available, and when solar energy production is inadequate it seamlessly transitions to relying on the battery storage system to meet the energy demand. This dynamic approach optimizes energy utilization in response to varying conditions.

During the nighttime period following 10:30 pm (as indicated in step 90), the capacity of the homeowner's battery storage system gradually decreases. Consequently, during this period, the primary source of energy shifts to the grid, which now fulfills the majority of the household's energy demand. This transition takes into account factors such as grid availability and prevailing electricity prices in the market.

## 6. Conclusion

This research presents a novel model for peer-to-peer energy trading within a grid-connected distribution network. The model focuses on a case study involving a prosumer equipped with both a solar PV system and a battery energy storage system. Through the implementation of a segmented Markov chain model within a peer energy trading environment, the prosumer's load demand contribution from different energy sources at his/her premises is demonstrated.

To capture the system's dynamics comprehensively, distinct Markov transition probability matrices were established for each time segment. These matrices were computed using a predefined set of equations, taking into account a range of internal and external factors pertinent to energy trading. The simulation of the multi-segment Markov process was executed with 15-minute time intervals, effectively accommodating the daily energy demand, which amounts to 12 kWh. This approach allows for a fine-grained analysis of energy transitions throughout the day.

The results of the study indicate that the proposed model effectively manages and represents the prosumer's load demand. By optimizing the utilization of available energy sources, the model yields maximum cost benefits for the prosumer where energy mode transition happens based on the least electricity

costs at each time step along with other parameters associated with battery storage limitations. Overall, these findings highlight the potential of the model in facilitating efficient and cost-effective energy trading within peer-to-peer networks.

## 7. Future Work

In the future, this research will be expanded to create a comprehensive market rule base that encompasses a carefully chosen group of prosumers. These rules will be designed to optimize battery capacities for each house, aiming to minimize energy costs while maximizing benefits for consumers. Importantly, these rules will take into account the unique constraints and limitations of each individual agent operating within the peer energy trading environment. The utilization of genetic algorithms holds significant promise in achieving these objectives efficiently and effectively.

By implementing these advancements, the research will contribute to the development of efficient market mechanisms and strategies in the context of peer-to-peer energy trading. It has the potential to offer valuable insights for the design and operation of future energy systems, promoting sustainable and economically beneficial practices for both prosumers and the wider community.

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