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A Study on Crenulate - Shaped Bays in Coastline of Sri Lanka

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Abstract: Various research studies have been reported in the literature investigating the crenulate-shaped bays that are frequently seen in coastlines all over the world and these coastal bays are recognized as the most stable naturally forming beaches under oblique wave attacks mostly due to swell dominant seas. Although many crenulated bays exist in Sri Lanka coastline, no considerable attention has been given so far to investigate their characteristics. The authors have studied the temporal and spatial variations of crenulated bays in many parts of the coastline of Sri Lanka using Google Earth images. Based on the detailed analysis carried out to investigate the shape of the bay profiles using empirical formulations, a hyperbolic tangent formula has been identified as the best fit for crenulated bays in the coastline of Sri Lanka. Considering various coastal sectors and monsoon seasons, different relationships have been derived between the coefficients a and b of the hyperbolic tangent formula. A relationship that has been developed co-relating the coefficient a with few coastal parameters is also discussed. Since many parts of the Sri Lanka coastline are subjected to severe erosion despite the construction of a variety of shore protection structures in traditional ways, a new approach is suggested for carrying out such work where the coastal structures will be aligned in predetermined directions depending on the coastal stretch so that stable crenulated bays will be naturally formed in between these structures, stabilizing the entire coastal stretch. This approach will be very useful in managing coastal erosion problem in the future.

Keywords: Crenulated bays, Bays in Sri Lanka, Beach planform

1. Introduction

Crenulated bays are often seen in the lee side of a headland or between two headlands when theshoreline is erodible and such bays can also be formed due to man-made breakwaters. Being an island, Sri Lanka coastline is surrounded by many crenulated bays formed with sandy beaches. Figure 1 depicts some of the crenulated bays found in Sri Lanka coastline. Sri Lanka has many beautiful beaches all around the country such as world-famous Browns beach, Hikkaduwabeach, Unawatuna beach, and Nilawelibeach, etc.

It is commonly accepted that the formation of crenulated-shaped bays on a sedimentary coastline is the most stable beach generated by nature [1]. Hence, crenulated bays can be identified as nature's way of protecting the beaches from erosion. Thus, it is vital to investigate their characteristics because the impacts of coastal erosion are severe on the Sri Lanka coastline resulting in loss of houses, undermining roads, and degradation of valuable lands. Therefore, if there is a possibility of converting eroding shorelines to form crenulated bays with the help of coastal structures, that would be one of the feasible and sustainable solutions for coastal erosion problem.

2. Literature Review

2.1 Use of Logarithmic Spiral Shape for Crenulated Shaped Bays

Various empirical formulations have been developed to fit the shape of crenulate-shaped beaches that are commonly seen in many parts of the world coastline [2]. The first approximation of the crenulated bay planform was the logarithmic spiral shape expressed in Eq.(1)[3, 4].

$$\frac{R_2}{R_1} = e^{\theta \cot\alpha} \qquad \ldots(1)$$

where, R_2 is the length of radius vector for a point measured from the pole and R_1 is length of radius which make a θ angle with R_2. Angle α is the characteristic constant angle between the tangent to the curve and radius at any point along the spiral.

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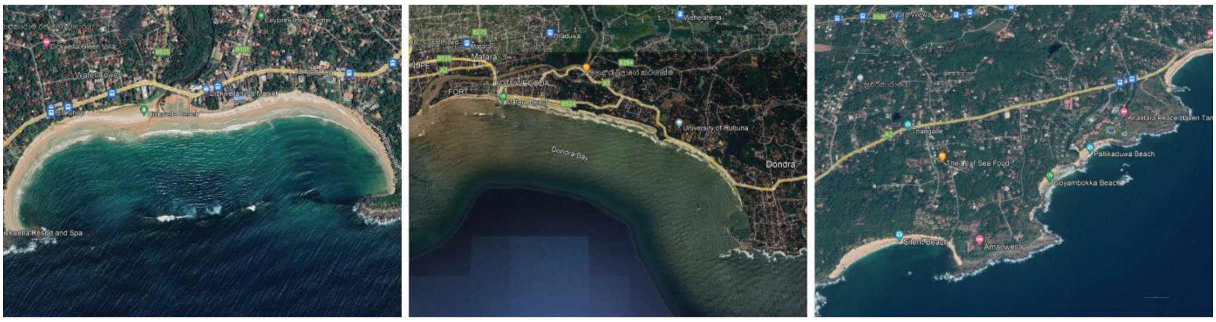

Figure 1 - Examples of Crenulated Beaches in Sri Lanka (Google Earth Images)(a) Dickwella Beach, (b) Dondra Beach (c) Series of Crenulated Bay Beaches near Tangalle

The definition sketch of the logarithmic spiral shape is depicted in Figure 2. Wave heights and periods do not appear in such empirical equations as they have only a minor effect on the final planform shape [5].

There were several limitations seen in the logarithmic spiral equations, such as origin of the spiral does not coincide with the point of diffraction and the profile shape does not describe the downcoast region of the bays realistically [6].

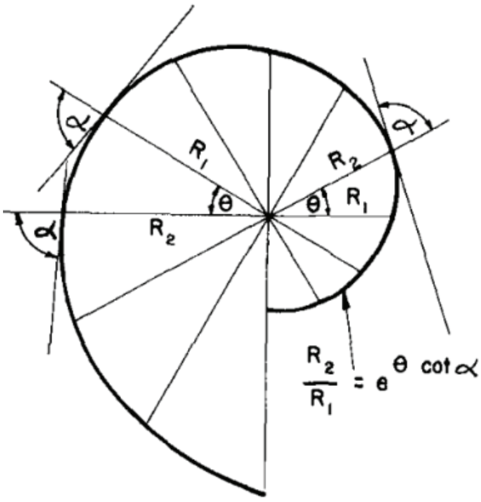

Figure 2 - Definition sketch of the logarithmic spiral shape (Silvester and Ho, 1972)

2.2. Parabolic Bay Shapes and Applications

To describe the shape of crenulated bays more accurately, Hsu and Evans (1989) proposed a parabolic bay equation, as represented in Figure 3 [5]. Radii, R can be drawn with an angle θ oriented to the wave crest line. The radius which connects the diffraction point (focus of the parabola), and the downcoast limit of the bay is R_o and it is oriented with an angle of β to the dominant wave direction. Based on model test results, R/R_o ratios are expressed in terms of β and θ angels as given in Eq. (2).

$$R/R_o = C_o + C_1\left(\beta/\theta\right) + C_2\left(\beta/\theta\right)^2 \qquad ...(2)$$

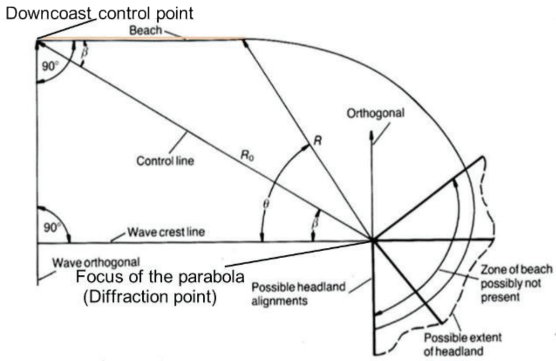

Figure 3 - Definition Sketch of Parabolic Shape (Hsu & Evans, 1989)

The coefficients C_o, C_1, C_2 vary with β. This parabolic bay shape equation has been used in coastal protection work through headland control [7]. In parabolic shape, the focus of the parabola represents the diffraction point of the waves propagating towards the coastline. As the parabolic shape is somewhat insensitive to β, uncertainties might happen in identifying the downcoast control point [8].

2.3 Hyperbolic Tangent Shape to Determine Equilibrium Shape of Crenulated Bays

Due to the limitations in the parabolic formula, a new function, called the hyperbolic-tangent profile (Eq. 3), is introduced to describe bay shapes (Figure 4), which simplifies the fitting procedure and reduces the ambiguity of shoreline shape for bays controlled by a single headland [9].

$$y = \pm \boldsymbol{a} \tanh^{\boldsymbol{m}}(\boldsymbol{b}x) \qquad ...(3)$$

where, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{m}$ are empirically determined coefficients.

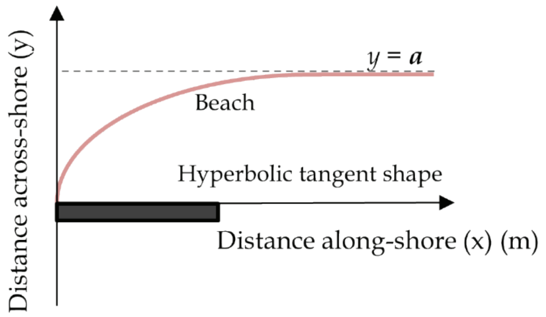

Figure 4 - Typical Sketch for Hyperbolic-Tangent Equation

The parameter ***a*** controls the magnitude of the asymptote while ***b*** is the parameter controlling the entry to the asymptotic limit. The curvature of the shape is controlled by ***m.*** For the larger values of ***m*** ($m \geq 1$) it creates more rectangular shapes which are rarely seen in natural scenarios. Meanwhile smaller ***m*** values produce more rounded shapes which are often seen in naturally formed crenulate shaped bays [8].

The focus of the parabolic equation coincides with the wave diffraction point which could represent a tip of a headland or breakwater. Hence, with the change of diffraction point, shoreline configuration represented by parabolic equation is modified accordingly to form a stable shoreline. This phenomenon could be effectively used in predicting a stable, shoreline configuration due to a change of diffraction point [10]. Logarithmic spiral and hyperbolic tangent methods cannot be used in evaluating the beach stability condition and predicting shoreline changes due to the construction of a new coastal structure since the wave diffraction point is not represented in those formulations [11].

2.4 Studying the Effects of Relocating the Headland

It is evident from the previous researches that, using the empirical formulae to predict the bay profile of a crenulated bay is an effective approach in mitigating coastal erosion. Yu et al. (2017) have proposed such mitigatory measures for eroding bays in Japan and USA by applying the parabolic bay shape equation (Eq. 2). Kemp et al. (2018) have modified the hyperbolic tangent formulation to account for the profile change due to a shift in the diffraction point. A new elliptical model has been used by Li et al. (2020) to study the coastal erosion of the east coast of Laizhou Bay, China.
The authors have analysed the possibility of using the hyperbolic tangent shape formulation (Eq. 3) for bays in Sri Lanka and it is discussed in the latter part of this paper.

3. Data Collection

3.1 Bay Configuration

The images of crenulated bays around the Sri Lanka coastline were extracted using Google Earth images. From its past data viewing option (with subject to availability), the bay profiles of two monsoon seasons were collected. Although it was able to recognize more than 100 crenulated bays around the coastline of Sri Lanka, only 48 bays were selected for the analysis based on the following criteria.

i. Bays having dynamically stable configurations, indicating their constant periphery over several years. (The Google Earth Images were obtained between 2009- 2020)

ii. Bays having only sandy beaches.

iii. Bays having a unique point of diffraction (bays with islands or rocks in front of the bay were avoided).

Figure 5 shows the locations of all coastal bays selected for the study, giving numbers to these bays starting from Marawila on the west coast to Kalkudah –Thiriyai on the northeast coast. The bay configurations available in Google Earth for the past several years representing two monsoon seasons were also collected for the analysis. Several artificially formed tombolo-shaped crenulated bays due to coastal structures were selected to represent the west coast of Sri Lanka.

Also, the bays were categorized according to number of headlands associated with the bay, namely, single headland bays, two headland bays and 1.5 headland bays (bays between a headland and a partial headland).

3.2 Wave Characteristics

Sri Lanka has a tropical monsoon climate with two monsoon seasons, namely, southwest (SW) monsoon dominated by southwest winds from May-September and northeast (NE) monsoon dominated by northeast winds from December-February. Because of the small tidal range, wind and swell waves are the dominant external force generating currents in the nearshore region of the coast. The wave characteristics at few selected locations in different coastal regions were collected during this study. The summary of wave characteristics data is in Table 1. In addition, bathymetry details and grain sizes of beach material of several coastal bays were collected.

Table 1 - Summary of Wave Characteristics

Coastal Sector	Monsoon Season	Dominant Wave Direction	Wave Height
West	NE	240° - 263°	0.55 m -0.6 m
	SW	257° - 240°	0.6 m - 1.25 m
Southwest	NE	200° - 225°	0.45 m - 0.8 m
	SW	200° - 245°	0.35 m - 1.2 m
Southeast	NE	140° - 155°	0.9 m
	SW	145° - 185°	0.9 m - 1.9 m
Northeast	NE	75°	0.9 m - 1.3 m
	SW	105° - 135°	0.3 m - 0.5 m

The wave characteristics used in this study were obtained from feasibility studies carried out for various development projects in the coastal zone, such as, Port City development and rehabilitation of fishery harbours, etc. Due to lack of field data for few other coastal bays, it was assumed that the data such as wave characteristics in such bays are similar to those in the adjoining bays.

4. Analysis of Bay Configuration

4.1 Grouping of Coastal Bays According to their Locations

The bays selected during this study cover a major portion of the Sri Lanka coastlines except the North and Northwest coastlines of the island. Depending on the orientation of the shoreline, the selected coastal stretch was divided into four sectors as, west coast, southwest coast, southeast coast, and northeast coast, to study the behavior of the shoreline. The division of coastal sectors also described above is shown in Figure 5. The grouping of coastal bays to different coastal sectors with their extents are given in Table 2.

Table 2 – Grouping of Coastal Bays to different Coastal Sectors and their Extents

Coastal Sectors	Bay Number	Extent
West	Bay 1 – Bay 6	Marawila - Dikowita
Southwest	Bay 7 – Bay 24	Moratuwalla - Mirissa
Southeast	Bay 25 – Bay 38	Gandara - Okanda
Northeast	Bay 39 – Bay 48	Kalkudah - Thiriyai

4.2 Seasonal Variations

As the initial step, shoreline configurations of all coastal bays were digitized and preliminary observations were made to see whether there are any significant changes to these bay profiles on seasonal basis.

Although the dominant wave directions are almost same, the wave heights experienced by the west coastal sector during the two monsoon seasons differ. Consequently, the beach shapes developed during the two seasons appear to be somewhat different. In the southwest coast, when a river outfall is present, the beaches in the vicinity form into different crenulate shapes in the two monsoon seasons. This can be justified by the variations of sediment supply from rivers in different monsoon seasons.

Despite the difference in wave conditions in the southwest and northeast monsoons, variation of bay shape was only seen where an external sediment supply source was available in the vicinity of crenulated bays in southeast coastal sector. Similar behavior of the shoreline was noted in the north east coastal sector as well, during the two seasons, regardless of wave condition change.

Figure 5 - Locations of the Bays Selected for the Present Study and Coastal Sectors around Sri Lanka

4.3 Mathematical Description to bay Shapes

Describing planform shape of bays is vital in studying crenulated bays and suggesting designs of artificial headland bays. Images obtained from Google Earth software were digitized with its inbuilt facility. Then, three mathematical formulations which are presented in Section 2 for describing coastal bay shapes, namely, logarithmic spiral formulation, parabolic and hyperbolic tangent formulation, were applied for the bays in the coastline of Sri Lanka. The curve fitting tool in MATLAB was used to determine fitting coefficients and R^2 values.

Figures 6 and Figure 7 show some of the examples of the fitted profiles using three formulations.

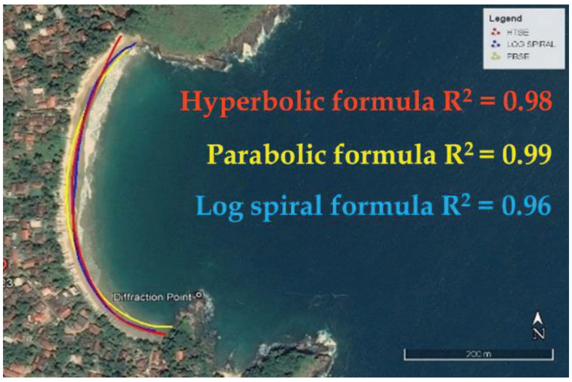

Figure 6 - Bay No. 27 at Kudawella

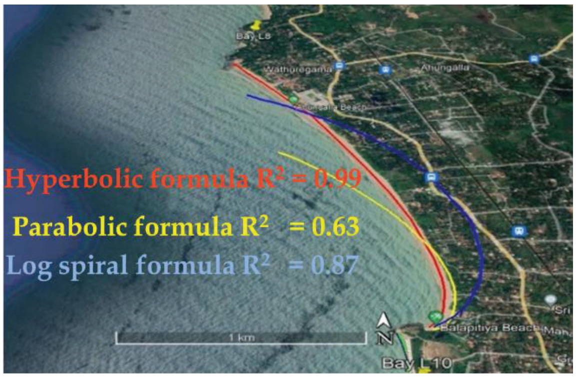

Figure 7 - Bay No. 16 at Ahungalla

Out of the three formulations, hyperbolic tangent formula appears to give much better results for all the bays, particularly single headland bays, and 1.5 headland bays around the Sri Lanka coastline. The parabolic bay formula provided acceptable results primarily for one headland bays while log-spiral formula provided acceptable results for two headland bays. Moreover, inability of hyperbolic tangent equation in verifying the stability condition of the bay was not a concern, since the stability condition was not focused in the current research. Therefore, hyperbolic tangent formulation was used in further analysis.

The authors have developed a new methodology to overcome the issue concerning the prediction of shoreline planform changes due to modification of diffraction point in the hyperbolic tangent formulation. This method-logy is briefly described in Section 4.7 which could be used for stabilizing eroding coastlines around Sri Lanka coastline.

4.4 Application of Hyperbolic Tangent Formulation to Coastal Bays

The hyperbolic tangent formulation was used to analyse all coastal bays selected during the present study. The analysis was done for these coastal bays grouping them to different coastal sectors separately. The authors have analysed the complete data set without grouping them into coastal sectors. However, the yielded results were less conclusive. Hence, they were not included in this paper. The coefficients, a, b and m of hyperbolic tangent formulation described in Section 2.3 determine the profile shape of coastal bays.

4.4.1 West Coastal Sector

In the west coastal sector, the variation of b Vs a is a 2nd order polynomial for both northeast and southwest monsoon seasons (see Figure 8). Rest of the coastal sectors seems to hold power variations between a and b coefficients. The reason for this could be attributed to the nature of the crenulated bays formation in the west coast. As stated previously, almost all the bays in this coastline are formed between series of manmade breakwaters.

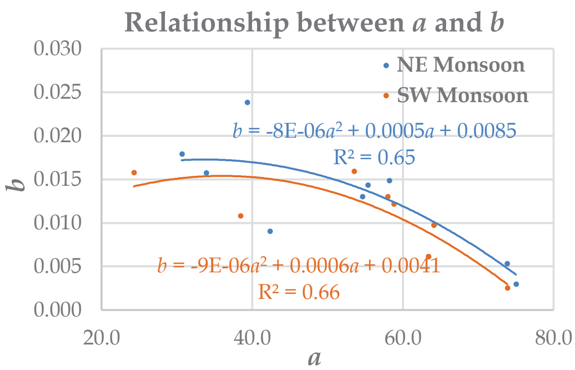

Figure 8 - Variation of b Vs a in West Coast

In addition, west coast and southwest coast are exposed to wind generated waves in both monsoon seasons [15].

The variation of wave parameters in the two monsoon seasons results in the dissimilarity of the obtained relationships. In the northeast monsoon, the coefficient ***a*** varies from 31 to 75 and coefficient ***b*** varies from 0.003 to 0.024, whereas in the southwest monsoon, the coefficient ***a*** varies from 24 to 76 and coefficient ***b*** varies from 0.003 to 0.027.

4.4.2 Southwest Coastal Sector

The variation of ***b*** Vs ***a*** in both seasons appeared to fit well with a power function where smaller values of ***a*** gave higher values of ***b*** and vice versa.

The regression coefficients (R^2 values) obtained for these relationships were higher than those obtained for the west coastal sector. The results shown in Figure 9 do not show any significant difference to the relationships derived between the coefficients ***a*** and ***b*** for the two monsoon seasons.

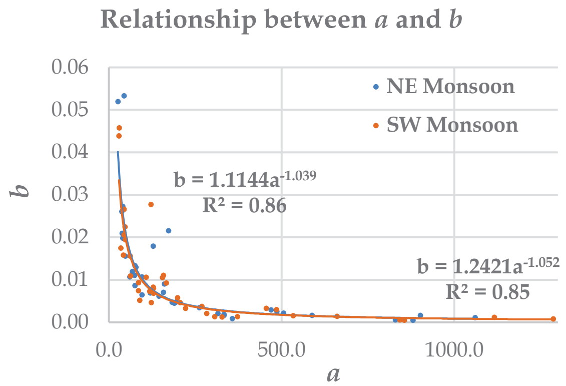

Figure 9 - Variation of *b* Vs *a* in Southwest Coast

4.4.3 Southeast Coastal Sector

Figure 10 shows the variation of coefficient ***b*** with ***a*** for coastal bays in the southeast coast during northeast and southwest monsoons. Power relationships provided a better agreement between ***a*** and ***b*** for both monsoon seasons as similar to the analysis performed for the coastal bays in the southwest coast. However, the regression coefficients obtained for the relationships derived for the southeast coast were higher than those obtained for southwest coast, indicating a much better agreement with the relationships derived for southeast coast.

Similar to coastal bays in southwest coastal sector, there was no significant difference of the relationships obtained between the coefficients ***a*** and ***b*** for the two monsoon seasons. Same as the southwest coast, the bays formed in the southeast coast appeared to shift landward direction away from the headlands for higher values of ***a***.

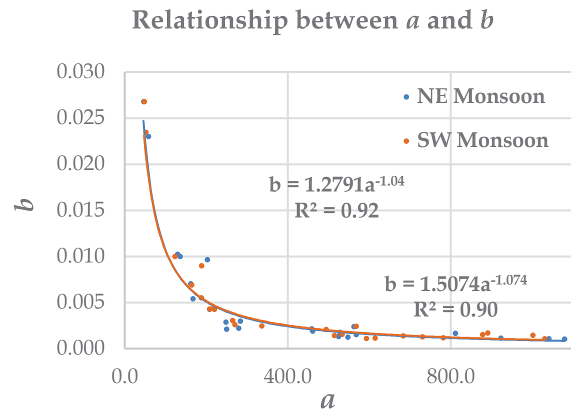

Figure 10 - Variation of *b* Vs *a* in Southeast Coast

4.4.4 Northeast Coastal Sector

The coastal bay profiles in northeast coastal sector were also be fitted well with the hyperbolic tangent formulation as shown in Figure 11. Like previous coastal sectors, there was no significant difference to the relationship between ***a*** and ***b*** coefficients for both the northeast and southwest monsoons.

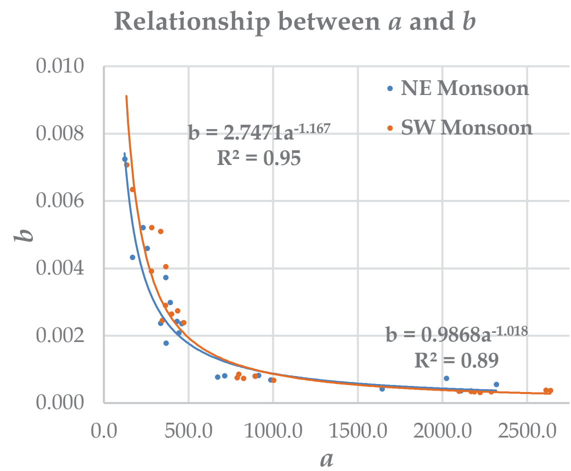

Figure 11 - Variation of *b* Vs *a* in Northeast Coast

4.4.5 Summary of the Relationships Obtained between the Coefficients *a* and *b*

Table 3 shows the relationships obtained between the coefficients ***a*** and ***b*** for different coastal sectors and the ranges of these coefficients are summarized in Table 4.

Table 3 – The Relationships Derived between the Coefficients *a* and *b*

Coastal Sector	Northeast Monsoon	Southwest Monsoon
West	31 <*a* <75, 0.003<*b* <0.024	24<*a* < 76, 0.003<*b*<0.027
Southwest	26<*a* < 1061, 0.001<*b*<0.053	29<*a* <1287, 0.001<*b*<0.046
Southeast	46<*a* <1079, 0.001<*b*<0.027	47<*a*<1030, 0.001<*b*<0.027
Northeast	122<*a* <2317, 0.0004<*b*<0.007	133 <*a*<2637, 0.0003<*b*<0.007

Table 4 – The Ranges of the Coefficients *a* and *b* in Northeast and Southwest Monsoon

Coastal Sector	Northeast Monsoon	Southwest Monsoon
West	b = -8E 06a^2+ 0.0005a+0.0085 (Eq.4)	b = -9E-06a^2 + 0.0006a + 0.0041 (Eq.5)
Southwest	$b = 1.2421a^{-1.052}$ (Eq.6)	$b = 1.1144a^{-1.039}$ (Eq.7)
Southeast	$b = 1.5074a^{-1.074}$ (Eq.8)	$b = 1.2791a^{-1.04}$ (Eq.9)
Northeast	$b = 0.9868a^{-1.018}$ (Eq.10)	$b = 2.7471a^{-1.167}$ (Eq.11)

These findings which are in connection with the hyperbolic tangent formulation fitted to the existing coastal bays in Sri Lanka coastline could be very useful in the future when trying to develop coastal bays artificially using coastal structures for mitigating shoreline erosion.

4.5 Variation of the Coefficient *m* in the Hyperbolic Tangent Formulation

The variation of *m* for all the 48 bays used in the analysis with the coastal bay numbers is shown in Figure 12.

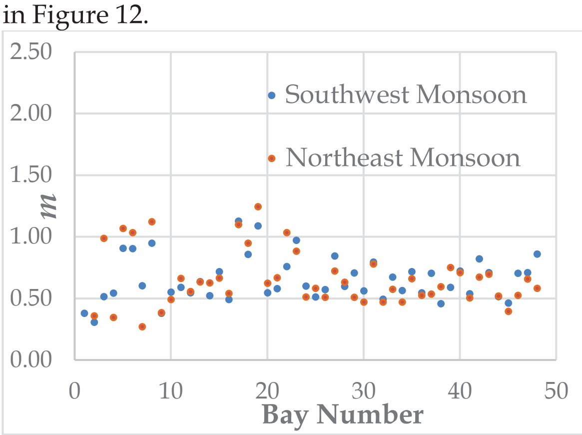

Figure 12 - Variation of *m* for 48 Coastal Bays

The values of the coefficient *m* appear to have a wider range for the coastal bays in west coast for both monsoon seasons compared to other coastal sectors, except for few bays in the southwest coast. This can be partly due to the direct exposure of waves with higher wave height for the bays on the west coast and some parts of the southwest coast during the southwest monsoon. The mean values of *m* obtained for the northeast monsoon and the southwest monsoon were 0.66 and 0.68, respectively, and which are almost similar.

A sensitivity analysis of the coefficient *m* in the hyperbolic tangent formulation in predicting the beach shape was also carried out. The coastal bay No. 24 was selected for this analysis and bay shapes were developed for the *m* values of 0.54, 0.60 and 0.66 while keeping all other parameters constants. Figure 13 shows the expected bay profiles for the different values of *m*. In this analysis, it was noticed that the curvature of the profile slightly increases when reducing the value of *m* while keeping the both ends nearly the same. Hence, it could be concluded that the coefficient *m* does not influence much on the shape of coastal bays when designing a new coastal bay as the extent of the beach profile is not governed by *m*.

Figure 13 - Sensitivity of the Coefficient *m* on Hyperbolic-Tangent Shape (a = 485.77 m; b = 0.0030 m^{-1})

With the use of lower values of *m*, bay profiles move landward and vice versa. Hence, when designing new beaches, it is safer to use lower *m* values, preferably a value between 0.50 and 0.60 as it will ensure a relatively stable beach periphery throughout the year, covering both southwest and northeast monsoon seasons. This finding is in good agreement with Moreno and Kraus (1999) who suggested the value of 0.50 as a reasonable value for *m* to be used when developing stable beach profiles and this was also confirmed by Martino et al. (2003) after carrying out extensive research work on crenulated bay profiles. Accordingly, it is suggested to use a value between 0.50 and 0.55 for the coefficient *m* when designing new coastal bay shapes and also for stabilizing existing bay profiles which are not fully developed.

4.6 Analysis of Wave Parameters and Sediment Data

Dimensional analysis was carried outto find any correlation among the model parameters, such as, wave height (H), wave direction from north (W^0), headland length (L), headland orientation (θ^0), wave period (T), density of sediment (ρ_s), density of seawater (ρ), and sediment diameter (d). Then, multiple regression analysis was performed using the data analysis option in Microsoft Excel, to derive equations among the parameters such as, wave, sediment and bay characteristics with the coefficients of the hyperbolic tangent formulation.

The following dimensionless groups were formed among the parameters governing the problem using *Buckingham Pi Theorem*;

$\pi_2 = L/H$, $\pi_3 = Sin(\theta)/Cos(W)$, $\pi_4 = gT^2/H$

$\pi_5 = \rho_s/\rho$, $\pi_6 = d/H$

To reflect the influence of the coefficient $\boldsymbol{a}$ with other parameters, Eq. (12) can be written.

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \quad ...(12)$$

where, $\pi_1 = \boldsymbol{a}/H$

In the same manner, to reflect the influence of the coefficients $\boldsymbol{b}$ and $\boldsymbol{m}$, Eq. (13) and Eq. (14) can be written.

$$\pi_1' = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \quad ...(13)$$

where, $\pi_1' = \boldsymbol{b} \times H$ and

$$\pi_1'' = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \quad ...(14)$$

where, $\pi_1'' = \boldsymbol{m}$

The term $\pi_5 = \rho_s/\rho$ was neglected since this term is uniform in the entire coastline.

The terms $\pi_4 = gT^2/H$ and $\pi_6 = d/H$ were also not considered in the analysis due to limited data available for wave period and sediment diameter.
According to the model tests done by Vichetpan (1969) and Ho (1971), the effects of wave period would mostly influence the total duration to form the final equilibrium shape.

Accordingly, the relationship between the coefficients in the hyperbolic tangent equation and other coastal parameters can be expressed as in Eq. (15), Eq.(16), and Eq.(17).

$$\frac{\boldsymbol{a}}{H} = f\left({}^{L}/_{H}, \frac{Sin(\theta)}{Cos(W)}\right) \quad ...(15)$$

$$\boldsymbol{b} \times H = f\left({}^{L}/_{H}, Sin(\theta)/Cos(W)\right) \quad ...(16)$$

$$\boldsymbol{m} = f({}^{L}/_{H}, Sin(\theta)/\ Cos(W),) \quad ...17)$$

The results of the multiple regression analysis performed for the above equations are described below.

4.6.1 Variation of *a* with the Coastal Parameters

The parameter $\boldsymbol{a}$ is noted to be the most governing parameter of the hyperbolic tangent shape which gives the position of the down-drift shoreline beyond the influence of the headland. The effect of wave height, wave direction, headland length, and headland orientation to $\boldsymbol{a}$ was found.

Eq. (18) and Eq.(19) were obtained using multi regression analysis as a solution to Eq.(15) for northeast monsoon and southwest monsoon, respectively. For this analysis, only the bays with wave parameters available were used. However, few bays where past profiles were available in Google Earth were also used. The statistical parameters describing the degree of best fit among the non-dimensional terms given in these equations for northeast monsoon and southwest monsoon are tabulated in Table 5 and Table 6, respectively.

$$\frac{\boldsymbol{a}}{H} = 67.39 + 1.14(L/H) - 2.13 \quad ...(18)$$

$$\frac{\boldsymbol{a}}{H} = 6.92 + 1.48(L/H) - 31.61 \quad ...(19)$$

Table 5 – Statistical Parameters of Eq. (18) for Northeast Monsoon

Parameter	Value
Multiple R	0.82
R^2 value	0.67
Adjusted R^2	0.66
Standard Error	193.15
No. of observations	56

Table 6 - Statistical Parameters of Eq. (19) for Southwest Monsoon

Parameter	Value
Multiple R	0.86
R^2 value	0.74
Adjusted R^2	0.73
Standard Error	248.20
No. of observations	61

From the above equations, a reasonable value for the parameter a can be estimated for any coastline, knowing wave height (H), wave direction ($Cos\,W$), and assuming headland orientation ($Sin\,\theta$) and headland length (L). However, this method might give a certain error on the parameter predicted due to the omission of two other parameters, such as wave period and sediment diameter, in the present analysis. This approach can be used to allow crenulated bays to form when manmade headlands, such as different coastal structures are provided as appropriate.

Similarly, regression analysis was carried out to derive relationships for the coefficients b and m with other coastal parameters. However, those relationships gave low regression coefficients indicating a poor correlation among those parameters. Hence, they were not considered further.

4.7 Stabilization of Eroding Coastlines around Sri Lanka using Crenulated Shaped Bays

Based on the relationships obtained in this research, the authors suggest the following approach in aligning coastal structures as headlands to facilitate the formation of stable, acceptable beachfront between the structures as a feasible method to stabilize eroding coastlines around Sri Lanka.

The methodology outlined here needs an iterative process, as the estimation of both orientation and length of artificial headlands to be constructed are unknown. Any given set of values for these two parameters will eventually develop a unique crenulated shaped beachfront as per the hyperbolic tangent formulation. However, the beach shape so developed would be acceptable to that particular coastline without making any undue erosion to damage existing properties and also to make sure that adequate beach width is formed.

The orientation of artificial headlands to be constructed usually depends on the coastal sector where the structure is to be built. As a guide for choosing a suitable orientation of the proposed structure, the range of orientations of the existing headlands and coastal structures in different parts of Sri Lanka coastline is presented in Table 7.

Table 7 – Range of Headland Orientation in Different Beach Segments

Beach Segment	Headland Orientation
Marawila -Wennappuwa	340^o -350^o
Daluwakotuwa - Negombo	290^o -295^o
Dikkowita - Moratuwella	250^o -255^o
Maggona - Balapitiya	270^o -310^o
Ambalangoda - Dodanduwa	255^o -280^o
Koggala - Midigama	240^o -250^o
Mirissa - Kudawella	70^o -200^o
Kudawella - Rekawa	55^o -80^o
Kalametiya - Ambalanthota	100^o -120^o
Situlpawwa - Okanda	70^o -90^o
Kalkudah - Vakarei	110^o -130^o
Vakarai - Kaddeiparichchan	40^o -70^o
Mutur - Thiriyai	05^o-20^o

Then, length of the coastal structure which is to be built as an artificial headland needs to be assumed as an initial guess. Knowing the orientation and length of the structure, nearshore wave height and wave direction, the coefficient a of hyperbolic tangent shape can be computed using Eqns. (18) and (19), considering both northeast and southwest monsoons, respectively. The coefficient b can also be obtained using the relationships developed between a and b for different coastal sectors described in Section 4.4. As mentioned in Section 4.5, the coefficient m can be chosen between 0.55-0.50 for conservative design.

The coefficients a, b, and m in the hyperbolic tangent formulation can be used to predict the shape of the coastline which is to be expected after construction of the artificial headland. Thus, the predicted bay shape can be checked with the required advancement of the beach front. If the crenulated shaped bay developed between the headlands do not meet the expected beachfront, the orientation and length of the headland can be changed and the process repeated until the best suitable bay shape is obtained. In this approach, the minimal headland length required for stabilizing eroding coastlines by forming crenulated-shaped bays can be determined. Hence, the proposed method can be effectively used in designing shore protection schemes economically.

5. Conclusions and Recommendations

The following conclusions can be drawn from this research:

- Almost all coastal bays analyzed in this study were able to represent using the hyperbolic tangent formulation. Therefore, this formulation appears to be the best among others for describing coastal bays in Sri Lanka. In particular, 1.5 headland bays and single headland crenulated bays are very well fitted with the hyperbolic tangent formulation.

- The parabolic bay formula could provide acceptable results in predicting beach shapes only for single headlands. The formulation based on logarithmic spiral was able to produce better results only for crenulated shaped bays with two headlands.

- Seasonal variations of coastal bay shapes were seems to be somewhat significant for coastal bays on the west coast, while the bays on the northeast coast showed minimum seasonal variations. However, the seasonal variations of the bay shape were generally noticeable at places where additional sediment supply sources such as, river outfalls are present in the vicinity of the bays.

- The relationship between the coefficients ***a*** and ***b*** of the hyperbolic tangent formulation applied to coastal bays on the west coast shows a 2nd order polynomial, while the bays in the rest of the coastal sectors show power functions for the relationship between ***a*** and ***b***. This difference could have been due to the fact that the coastal bays formed on the west coast are mainly artificially created bays using coastal structures and also from higher wave conditions.

- Dimensional analysis has been carried out to derive relationships among nearshore coastal parameters, orientation and length of headland and coefficients in the hyperbolic tangent formulation. Multi-regression analysis gave a reasonably acceptable correlation for the coefficient ***a*** with the rest of the model parameters. However, a similar relationship derived for the coefficient ***b*** with coastal parameters showed a low regression coefficient, indicating a poor correlation between the coefficient ***b*** with the rest of the model parameters. Hence, this relationship was not considered in the analysis.

- The average values of the coefficient ***m*** in hyperbolic tangent formulation fitted to coastal bays during both monsoon seasons seem to be nearly the same. Any value between 0.50 and 0.55 is recommended for the coefficient ***m*** in designing new bays.

- A methodology is devised for planning crenulated shaped bays using coastal structuresaligned in predetermined directions depending on the coastal stretch. Consequently, stable crenulated bays will be naturally formed in between these structures, stabilizing the entire coastal stretch. One of the key features of this method is the ability to predict the shape of beachfront that will be developed prior to the construction of the structures so that acceptability of the bay shape that will be developed could be pre-assessed. This approach will be very useful in managing coastal erosion problem in the future.

6. Limitations

The following are several limitations to the present study:

- For many of the selected crenulated bays there were only a limited number of past bay profiles. Hence, it always was not possible to obtain Google Images right at the end of the each monsoon season. Therefore, it is recommended to collect more bay profiles using satellite images to improve the results.

- The nearshore wave characteristics were available only for several bays. For others, the wave characteristics from adjoining bays were assumed with minor adjustments. Hence, coastal sectors were not considered separately when developing relationship between **a** and coastal parameters.

- The sediment sizes in beaches were not considered due to lack of data. However, it is important to consider sediment size as it is directly related to the grain movement and beach formation.

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