JSc EUSL (2021), vol. 12, no. 2, p 61-66

RESEARCH ARTICLE

ANTI-MAGIC LIKE LABELLING OF MARIGOLD GRAPH

K. M. P. G. S. C Kapuhennayake[1*], *A. A. I. Perera*[1]

[1]*Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka*

ABSTRACT

In this paper, we present a new family of graphs called Marigold graphs and introduce a new labelling method similar to the anti-magic labelling. The Marigold graph is generated from any number of copies of fully binary trees which are going through concentric circles. All copies of trees are connected to a middle vertex and the height of the Marigold graph is increasing with concentric circles. One copy is considered as one petal in the marigold graph. A Marigold graph with n copies (petals) and height (number of concentric circles) k is denoted by M_k^n. The labelling method is defined as follows: A graph with 'm' edges and 'n' vertices is labelled as an injection from the set of edges to the integers $\{1, \dots, x\}$ such that all 'n' vertex sums are pairwise distinct, where the vertex sum is the sum of labels of all edges incident with that vertex. In our work, for edge labelling, we consider the petals one by one and denote the r^{th} edge at k^{th} level as e_r^k, and define a function to label edges of the first petal. Then define the new labelling method for other petals, such that for n^{th} petal, edge labelling is starting with $J_{n-1} + 1$ (where J_{n-1} is the summation of all edge values in $(n-1)^{\text{th}}$ petal, $\sum_{i=1}^{m} e(n-1, i) = J_{n-1}$) and continue the labelling as a monotonically increasing sequence. We discuss some illustrative examples that might be used for studying the Anti-magic like labelling of Marigold graphs.

Keywords: *Anti-magic like labelling, Marigold graph, Full binary trees, Concentric circles*

DOI. *http://doi.org/10.4038/jsc.v12i2.36*

1. INTRODUCTION

Graph labelling is one of the important areas in graph theory. Graph labelling is used to give an identity to all the vertices and edges of it. That means graph labelling is an assignment of labels, represented by integers to edges or integers to vertices of a graph G. Vertex labelling is defined as a function between the set of vertices to the set of labels and a graph with such a function is called a vertex labelled graph. Similarly, an edge labelling is a function of edges to a set of labels. In this case, the graph is called an edge

*Corresponding author: *sonalichamathka03@gmail.com*

labelled graph. On the other hand, if the domain of the mapping is both the set of vertices and edges then the labeling is called total labeling. There is a vast amount of methods to label a graph. Graceful labelling, Prime labelling, Anti-Magic labelling are some examples for special cases of graph labelling. Among these methods, for our work, we use the concept of anti-magic labelling. The idea of anti-magic labelling had come from its connection to magic labelling and magic squares. In magic labelling, we label the set of edges of a graph G using non-negative integers such that the sum of edges around any vertex in G is a constant. (Solairaju & Begam, 2012)

The notion of anti-magic labelling was introduced by Nora Hartsfield and Gerhard Ringel in 1989. After that, so many variations of anti-magic labelling have been studied by referring to their book. They speculated that all simple connected graphs except K_2 are anti-magic.

Adding an extended to the research chain of Anti-magic labelling, in 2010 Hefetz, Mütze, and Schwartz initiated the study of anti-magic labelling of digraphs. (Hefetz, Mütze, & Schwartz, 2010). During their research, they presented that “All orientation is anti-magic” is not possible for directed graphs K_1, K_2, and K_3. And in the last sections of their publication, they conclude that all connected digraphs with at least 4 vertices are antimagic. Also, they conjectured that every connected undirected graph admits an anti-magic orientation. This called “Directed and undirected anti-magicness”. Liang and Zhu proved that 3-regular graphs are anti-magic. (Liang & Zhu, 2014). In 2019, Li et.al conjectured that every graph admits an anti-magic orientation (Li, Song, Wang, Yang, & Zhang, 2019). This publication was able to strengthen the ideas of Hefetz and Mütze. K. Venkata Reddy and A. Mallikarjuna Reddy conjectured that the class of trees generated from two copies of full binary trees is anti-magic. (Reddy & Reddy, 2020)

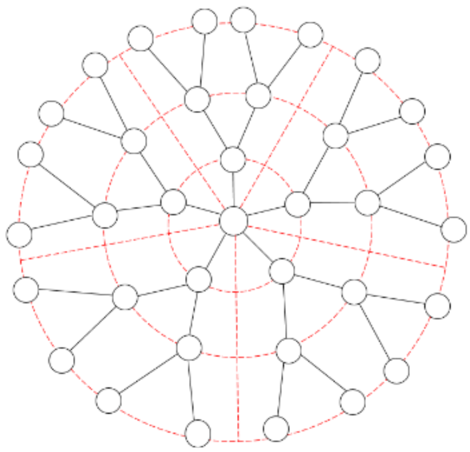

Figure 1: Marigold Graph M_3^5

Furthermore, in this paper, we introduced a new graph named a Marigold graph which is considered as the shape of a Marigold flower. This graph is generated from any number of copies of full binary trees which are going through concentric circles. All copies of trees are connected to a middle vertex. One such copy of the graph represents one petal.

Moreover, we introduced a new labelling method using the idea of the anti-magic labelling, which is the final result is same as the result of anti-magic labelling. The new labelling is given to this graph, using the result obtained for anti-magic labelling of the full binary tree. The Figure 1 illustrates a Marigold graph with 5 petals and each petal contains full binary tree with height 3.

2. MATERIAL AND METHODS

Some important definitions we used to get the result of this paper are stated below.

Definition 2.1 (Anti-Magic labelling): Let G be a simple graph with V vertices and E edges. Anti-magic labelling of graph G is a 'one-to-one' correspondence between $E(G)$ and $\{ 1,2, \dots, |E| \}$ such that the vertex sum for distinct vertices are different. Where the vertex sum is the sum of the labels assigned to edges incident to a vertex. (Sugeng, 2005)

Definition 2.2 (The new labelling method similar to the Anti-magic labelling): A graph G with 'm' edges and 'n' vertices, is labelled as an 'injection' from the set of edges to the set of integers $\{1, \dots, x\}$ (where x is any integer) such that all 'n' vertex sums are pairwise distinct. Where the vertex sum is the sum of labels of all edges incident with that vertex.

Definition 2.3 (Full binary tree): A full binary tree is a tree in which every node other than the leaves has two children.

Definition 2.4 (Marigold graph): Marigold graph is a tree with n components, each component, which is a full binary tree of height k, is attached to a common vertex, and all the vertices, other than the middle vertex, belong to the same level are in one circle. If a full binary tree of the Marigold graph has k levels (where $= \{1,2, \dots, k\}$), then there are k concentric circles in the Marigold graph. We denote the Marigold graph as M_k^n; where k is the height of the full binary tree (or the number of concentric circles) and n is the number of copies of full binary trees.

In this section, we prove that the full binary trees are anti-magic.

Theorem 2.1: Full binary trees are anti-magic.

Proof: Let B_k denote the full binary tree of height k.

The maximum number of vertices in $B_k = \sum_{i=0}^{k-1} 2^i = 2k - 1$

Maximum number of edges in $B_k = \sum_{i=1}^{k-1} 2^i = 2k - 2$

Now, let us define a function to label the edges of B_k as follows;

$f\ (e_r^{k-1}) = r$; Where $r = \{1,2,\dots,2k-2\}$ and e_r^k is the r^{th} edge at k^{th} level.

Labelling the edges as a monotonically increasing sequence starting from the bottom to top approach level wise, and approaching the edges connecting the levels k and $k+1$ in the left to right order. That is the function f is a bijection from $E(B_k)$ to $\{1,2 \dots\ 2k-2\}$. Label the vertices as the sum of all edges incident with the corresponding vertex. It can be observed that the vertex labelling is a monotonically increasing sequence from bottom to top approach level wise and approaching the vertices in each level in the left to right. That means the vertex sum for distinct vertices are different. Hence, f is anti-magic labelling for a full binary tree. Therefore, full binary trees are anti-magic.

The anti-magic orientation of the full binary trees can be represented by Figure 2.

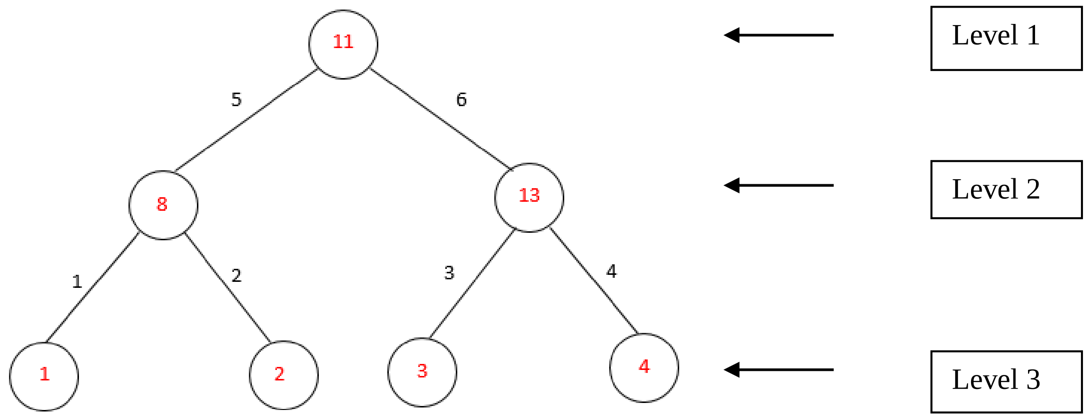

Figure 2: Anti-magic orientation of the full binary tree

Now, consider the Marigold graph.

Let's start labelling with the first copy. Label the edges of that as a monotonically increasing sequence starting from bottom to top approach level wise, and approaching the edges at the same level from left to right. Then it will be the new labelling method similar to the anti-magic labelling according to Theorem 2.1.

Assume the summation of edges in the first copy as;

$$\sum_{i=1}^{m} e(1,i) = J_1,$$

where m is the number of edges in each full binary tree.

Then label the edges in the second copy starting with $(J_1 + 1)$ and as a monotonically increasing sequence.

Take the summation of edges of the second copy as J_2. Then

$$\sum_{i=1}^{m} e(2, \mathrm{i}) = J_2.$$

Similarly, label the edges in the third copy starting with $(J_2 + 1)$ and as a monotonically increasing sequence. Repeat the procedure for every copy in the Marigold Graph.

Then in the n^{th} copy, edge labelling should start with $(J_{n-1} + 1)$ and the summation would be

$$J_n ; \ \sum_{i=1}^{m} e(\mathrm{n}, \mathrm{i}) = J_n.$$

Now, label the edges which are connected those copies to the middle vertex as a monotonically increasing sequence starting with $(J_n + 1)$. Note that we should label those edges starting with the edge which connects the first copy to the main vertex.

As usual, the vertex sum is the sum of labels of all edges incident with that vertex.

When we move on to each copy, the labelling of edges would be increased. Therefore all the vertices in n copies of full binary trees of the Marigold graph are distinct.

Since the edges which are connected to the middle vertex are also in a monotonically increasing sequence the middle vertex is different from all the other vertices in the Marigold graph. And it is the largest vertex in the graph. Therefore all the vertices of Marigold graph have distinct values and its labelling is a 'onto' correspondence between $E(G)$ and the set of integers $\{ 1,2, \dots, x \}$ where x is any integer.

3 RESULTS AND DISCUSSION

Figure 3 illustrate the new labelling method similar to the anti-magic labelling, for the Marigold graph with 5 copies of full binary trees and 3 concentric circles; M_3^5, where

$$J_1 = 21, J_2 = 147, J_3 = 903, J_4 = 5439, J_5 = 32655$$

When the new labelling method applied, the vertex values of the Marigold graph get increased when we go through the first petal to the last petal in order, and also they are increased when go inside to the graph, through the concentric circles. That is the vertices in the outer circle have less values in each copy of perfect binary trees. Then, the middle vertex has the highest vertex value.

CONCLUSION

In our work, we defined a new graph called the Marigold graph which is generated by any number of copies of full binary trees, and the height of the graph is increasing with concentric circles. Furthermore, we introduced a new labelling method which is similar to the anti-magic labelling for the Marigold graph. The vertex sums of the final labelling graph are pairwise distinct. As our future work, we are planning to apply this new labelling method to reduce the tendency of Gold grabbing game.

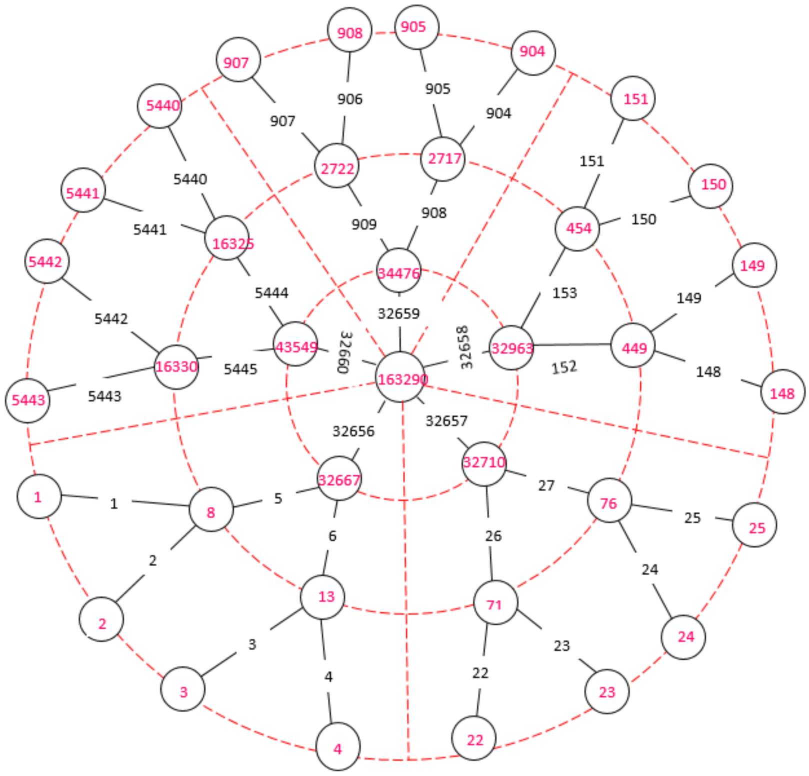

Figure 3: New labelling for M_3^5 which has a result same as the anti-magic labelling

REFERENCES

1. Hartsfield, N., & Ringel, G. (1994). *Pearls in graph theory*. Boston: Academic press, INC, Boston.

2. Hefetz, D., Mütze, T., & Schwartz, J. (2010). *On antimagic directed graphs*. Journal of Graph Theory, 219-232.

3. Li, T., Song, Z.-X., Wang, G., Yang, D., & Zhang, C.-Q. (2019). *Antimagic orientations of even regular graphs*. Journal of Graph Theory, 46-53.

4. Liang, Y.-C., & Zhu, X. (2014). *Antimagic Labeling of Cubic Graphs*. Journal of graph theory, 31-36.

5. Reddy, K. V., & Reddy, A. M. (2020). *A new class of Anti-magic Trees*. Journal of Xi'an University of Architecture & Technology, 118-123.

6. Solairaju, A., & Begam, R. R. (2012). *Edge-Magic Labeling of some Graphs.* International Journal of Fuzzy Mathematics and Systems. , 47-51.

7. Sugeng, K. A. (2005). *Magic and Antimagic Labeling of Graphs.* Victoria , Australia