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MODELLING GAMMA COEFFICIENT IN THE GOMPERTZ CURVE TO DETERMINE THE VARIABILITY OF VEHICLE OWNERSHIP SATURATION

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ABSTRACT

Vehicle ownership forecasting models based on the Gompertz curve generally employ per capita Gross Domestic Product as the primary explanatory variable. The γ coefficients in the curves specify the ultimate vehicle saturation level and α, β the profile of the S-curve shape. The γ coefficient is assumed to be the single universal value that all countries will eventually reach. The present research hypothesised that countries could have variable saturation levels and, as such, the γ coefficient would not be a universal constant. On that premise, it attempted modelling the γ coefficient as a function of several other extraneous factors that significantly influence a country's vehicle ownership and ultimately its saturation level, thus resulting in a country-specific γ; the application of the Gompertz model and the relationship of vehicle ownership would then be influenced by the country-specific characteristics. It was found that, while vehicle ownership is influenced by the GDP as in the case of Gompertz model, the country-specific γ would also be influenced by other identifiable variables such as household size, population density, share of public transport, and percentage of female drivers in each country. The results confirm the globally observable phenomenon that high-income countries do not converge to a universal vehicle ownership saturation level. Singapore and Hong Kong are examples, usually excluded from the Gompertz model, which can now be explained by the new model.

Keywords: *Vehicle Saturation Level, Gompertz Curve, Vehicle Ownership Rate, Gamma Function, Public Transport*

1. INTRODUCTION

Motorisation plays an increasingly critical role in determining the health of our cities and our transportation systems. The growth of the fleet of vehicles is an essential parameter used by any country in planning and providing road infrastructure as well as determining transport policy interventions if, indeed, motorisation should be curtailed [1],[2]. Models to forecast vehicle ownership as a function of Gross Domestic Product (GDP) per capita have emerged in recent literature [3],[4]. The Gompertz curve has been the widely used model for forecasting vehicle ownership, and the fleet of vehicles in a country has even been used as an indication of its prosperity - a popular political indicator of development. However, a careful study of countries that have reached high-income status shows exceptions to this widely held belief.

The Gompertz model captures the saturation level as a pre-defined constant called Gamma (γ) [5],[6]. Accordingly, the Gamma value (the level of maximum vehicle ownership a country will reach as income rises) is established as a global constant of 621.9 [5],[7]. Singapore and Hong Kong, which have achieved high-income status, have not reached this pre-defined value of γ; thus violating the assumptions of the Gompertz model [5].

A country j with higher population density has a higher viability for successful public transport delivery, which will lower Vehicle Ownership Rate (γ_j) at a given GDP per capita and eventually reach a lower Vehicle Ownership Saturation Rate (γ_j). Some countries leave vehicle ownership entirely to consumer choice. Others influence it through fiscal policies that severely disincentivise owning a vehicle while improving the quality of public transport [8]. The higher use of public transport in Singapore and Hong Kong reveals the combined impact that population density, the quality of public transport and transport policy can have on γ_j in the Gompertz model.

Moreover, it is observed that several countries in the Middle East, though having reached high income, demonstrate relatively lower vehicle saturation levels. This was found in this research to be due to a lower proportion of female drivers which in turn lowers vehicle ownership.

The use of γ in the Gompertz curve as a near-constant value for vehicle saturation in literature, therefore, cannot be considered appropriate. This use of a universal γ should instead be replaced by a function that represents a country-specific γ_j [7]. This result will provide a more accurate representation of the vehicle ownership in the application of the Gompertz Model allowing for more complex policy variable interactions.

Rota et al. [5] have described vehicle ownership as having both a micro and macro perspective, explaining that household income, in the long run, affects a household's demand for a vehicle at the micro-level, while the national income influences the macro perspectives for the long-run national demand for motor vehicles. In this respect, they have assumed that household characteristics such as age, sex, and employment status are micro-level variables of vehicle ownership. The number of working adults in a household, age and sex of household head has also been found to influence γ_j.

The paper attempts a two-staged model formulation by establishing a mathematical relationship between the vehicle ownership rate of 34 high-income countries and the contextual social and policy variables of the particular country [9]–[12]. The development of a functional form for the model is presented in Section 3, followed by its calibration, statistical analysis and estimates summarised in Section 4, while the findings and their applicability are discussed under Section 5.

2. LITERATURE REVIEW

The Gompertz Model can be used to determine the VOR of a country j given in vehicles per 1000 persons [9]. A sigmoidal or S-shaped curve, described as the Gompertz curve, represents a relationship with an upper limit or saturation rate in the growth of a dependent variable [13]. It assumes that growth would be slow, to begin with, and would increase exponentially before eventually slowing down on reaching its saturation rate [1].

Its application in determining the Vehicle Ownership Rate (VOR) is represented in the Gompertz Model illustrated in Equation 1 where Gamma (γ) is to be interpreted as the universal saturation of VOR and also measured in (vehicles owned per 1000 persons) while the other two coefficients α and β, describe the shape of the curve [4],[5],[9].

$$\boldsymbol{VOR_j = \gamma e^{\alpha e^{\beta(GDP_j)}}} \quad \text{..} \quad (1)$$

Figure 1 illustrates how the shape of the curves varies with the variation of the parameters (α and β) with GDP per capita (in $000's). The Gompertz-style levelling-off in vehicle ownership indicates that all countries will experience similar patterns of GDP growth and reach a universally-established γ_s [3],[6]. The Vehicle Ownership Saturation Rate (VOSR)--defined as the maximum number of vehicles per 1000 persons that may be expected in a given country is generally determined by long-run demand factors, such as the cost of vehicle ownership and the convenience of car travel.

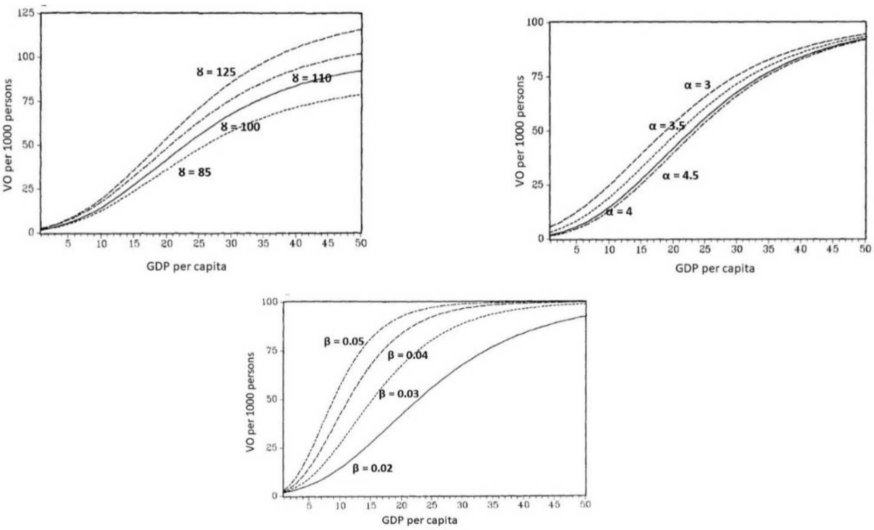

Figure 1: Projections of estimated vehicle Gompertz functions (source: [14])

Dargay [15] assumed that only the coefficient β was country-specific, while γ and α were universal, i.e., same for all countries. The differences between countries were reflected only in the curvature parameters β and not the final VOSR, which was determined as 850 vehicles per 1000 people, of which 620 being cars. It should be noted that this VOSR has not been reached in most developed countries. For instance, though it is 800 for cars and light-duty trucks in the U.S [7], it is only 450–600 in European countries, and 440 in Japan [16].

The authors have taken a more rigorous look at this observed variability of VOSR and have relaxed the assumption of a typical universal level denoted by Gamma (γ) in the Gompertz Model. Instead, they postulate that the VOSR would be country-specific and would depend on its socio-economic characteristics and specific transport policies.

$$\boldsymbol{VOR_j = \gamma_j e^{\alpha e^{\beta(GDP_j)}}} \qquad \text{...} (2)$$

Where VOR_j would represent the VOR of country j and new γ_j ("country specific asymptote") in Gompertz form would be it's VSOR. Thus, a two-stepped model was developed to derive a function for Gamma (γ) in Gompertz curve.

3. METHODOLOGY

3.1. Categorisation of Data

A simple country-based categorical analysis was employed to understand the impact of income on vehicle ownership, as shown in Table 1. The composition of income level is measured using Gross National Income (GNI) per capita and divided into four income groupings [17],[18], namely, low, lower-middle, upper-middle, and high-income categories.

It was found that, the VOR spread between 77 (Hong Kong) and 797 (United States) for HI countries confirms the hypothesis in the research that VOR would not depend on GDP alone. In fact, Dargay et al. [6], noted that the inclusion of non-OECD countries in the sample's population would introduce a high degree of variation in vehicle ownership. In order to avoid the large variations that could be observed in very small countries, only the 34 HI countries with a population of 4 million were included in the regression.

Table 1: Breakdown of Countries by GNI per capita

Category of GNI per capita	GNI per capita Range USD	# of Countries	Examples
High Income (HI)	12,746 < I	79	UK, USA, UAE, Australia, Hong Kong
Upper Medium Income (UMI)	4,125 < I < 12,746	60	Maldives, Jordan, Brazil, Mexico, China
Lower Medium Income (LMI)	1,045 < I < 4,125	47	Pakistan, Myanmar, Kenya, Nigeria, India, Sri Lanka
Low Income (LI)	1,045 < I	31	Somalia, Ethiopia, Haiti, Uganda, Nepal, North Korea

Source: [19]

3.2. Vehicle Ownership and Population Density

Table 2: Breakdown of the selected countries by population density

Category	Mean VO/1000 HH (VOR)	Std. Dev. of VO/1000 HH	Countries
Low population density: 0-50 persons/ km^2	1488	444	Australia, Canada, Oman, Norway, Saudi Arabia, New Zealand, Finland, Sweden, Chile, United States
Medium population density: 50-6000 persons/ km^2	1272	232	Ireland, Croatia, Greece, Spain, Austria, Hungary, Slovakia, Portugal, France, Poland, United Arab Emirates, Denmark, Czech Republic, Italy, Switzerland, Germany, United Kingdom, Japan, Belgium, Israel, Netherlands, South Korea
High population density: more than 6000 persons/ km^2	366	212	Hong Kong, Singapore

The impact of population density has been investigated in estimating the VOR in HI countries [6]. When divided into three groups by population density, these HI countries were found to have significantly different mean VORs (Table 2). Generally, it is observed that countries with low population densities have a high VOR, while those with medium population densities have moderate VOR, and those with higher population densities have lower VORs.

Figure 2 depicts the GDP and VOR of the countries by the three population density categories. Moreover, Figure 2 shows that most countries in the low population density category have a higher VOR except Chile, Oman, and Saudi Arabia.

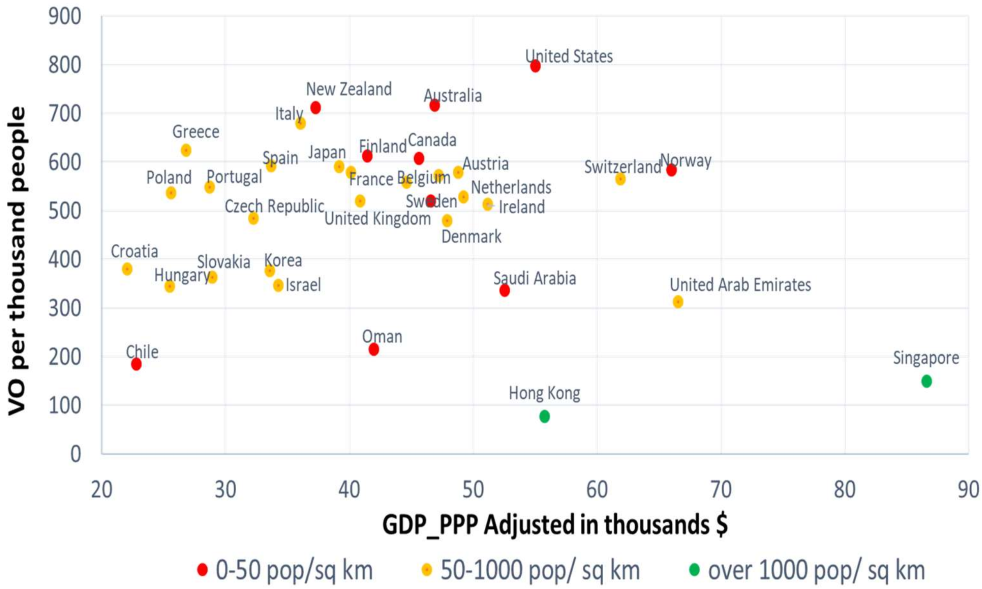

Figure 2: Vehicle ownership vs GDP vs population density

It was observed that the United Arab Emirates deviates from the general pattern of other medium population density countries while the two countries with high population density seem to have maintained relatively lower VORs. These relationships and the exceptions prompted a more detailed study of population density and VOR.

3.3. Vehicle Ownership and Household Size

One of the observations was that vehicle ownership is influenced by household size. Some countries such as Oman, Saudi Arabia, and United Arab Emirates in the HI sample have an average of 6-7 persons per household, significantly higher than around 4 per household in most other countries. Figure 3 depicts the conversion of VO per 1000 persons to VO per 1000 households (VOR), reducing the scatter of the graph and the deviations referred to above.

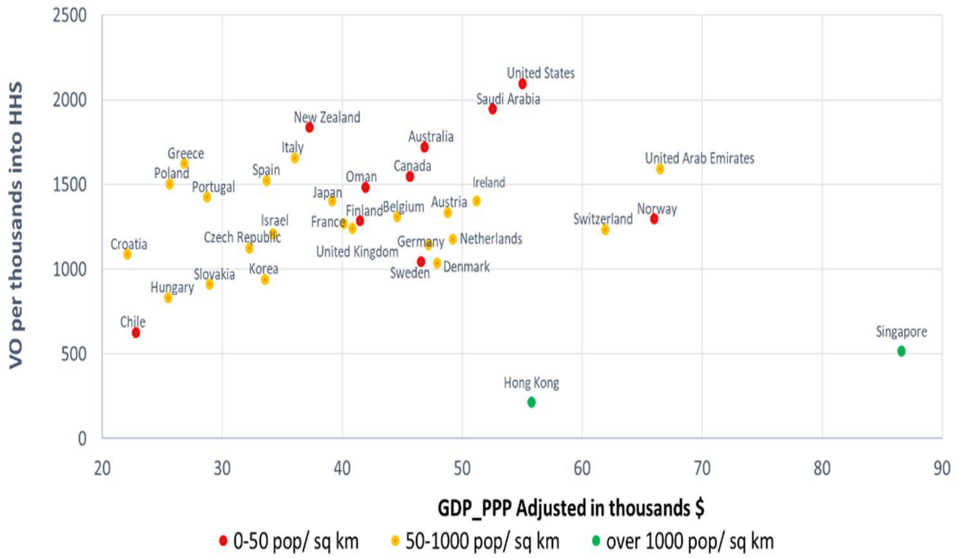

Figure 3: VOR into HHS vs GDP

3.4. Vehicle Ownership and Female Drivers

Another noteworthy observation, as shown in Figure 4, was that not all countries had an equal percentage of female drivers. For example, the three countries referred to above, namely Oman, Saudi Arabia, and the United Arab Emirates, had a significantly lower percentage of female drivers. This is another social factor influencing the country's VOR.

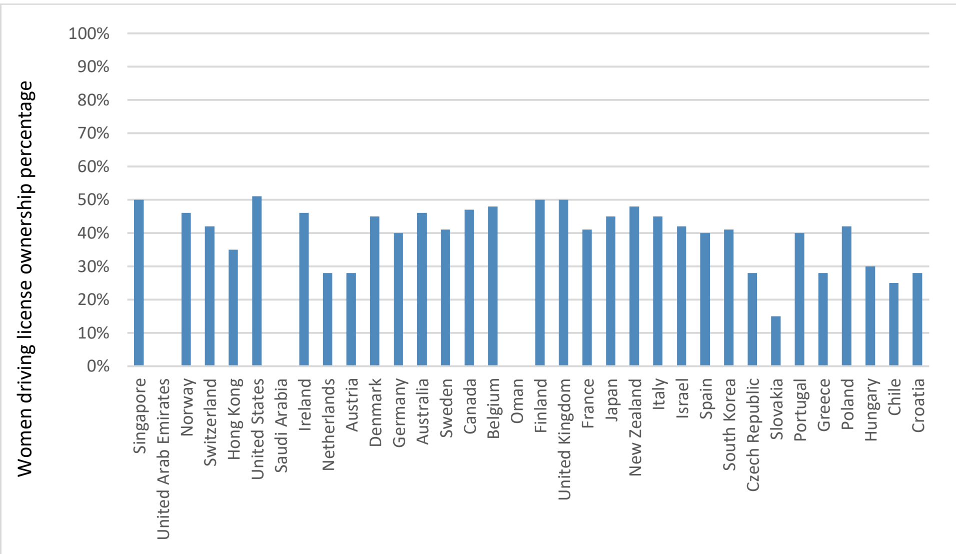

Figure 4: HHS and breakdown of women driving license ownership

3.5. Vehicle Ownership and Public Transport

As was noted earlier, Chile demonstrated a relatively lower VOR compared to other countries with low population density. It was observed that 87.3% of the total population was urban. Chile had harnessed this demographic feature to improve its public transport share to over 50%, while many lower-density countries had not [5]. Figure 5 shows a clear relationship between VO/1000 HHs and the share of public transport for all countries.

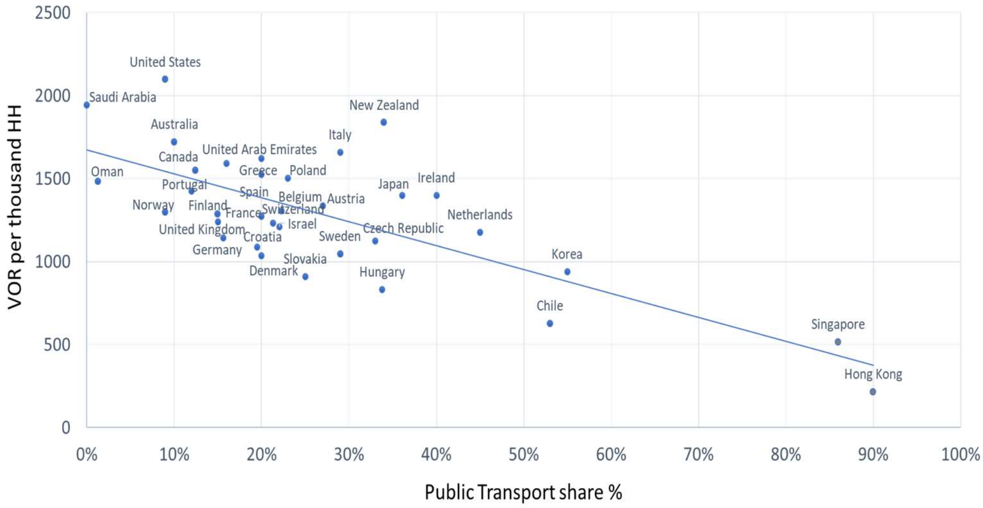

Figure 5: VOR vs Public Transport Share

It also clearly shows that the high population density countries, namely Singapore and Hong Kong, both are city-states, have the highest (more than 80%) public transport share, resulting in a low VOR. Both these countries follow public transport priority policies and have effectively curtailed the rise of VOR by even using measures to control private vehicle ownership and resort to such policies explicitly.

3.6. Study Procedure

The methodology adopted in this research was to (a) progressively study the patterns, trends, and influential factors for vehicle ownership levels in 34 High Income (HI) countries, (b) identify parameters that appeared likely to contribute to the country-specific VOR reached, and thereafter, to (c) calibrate a mathematical model for Gamma (γ) in Gompertz curve based on the identified country-specific variables. Thus, this proposed model formulation consists of two stages; first, modelling VOR using the traditional Gompertz curve mentioned under Equation 1, and thereafter, deriving a function for country-specific Gamma (γ), denoted for the purposes of this research as Gamma γ_j, "country specific asymptote" in Gompertz form would be derived as depicted by the Equation 2.

4. CALIBRATION OF THE VEHICLE OWNERSHIP MODEL

4.1. Stage 01: Estimation of Generalised Gamma for the Model Development

The model described in Equation (1) was estimated using Stata Statistical software for the pooled cross-country data on vehicle ownership for the 34 countries. The resulting estimates are shown in Table 3.

Table 3: Parameter estimates for Equation 1

Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
$\boldsymbol{\gamma}$	505.19	32.44	439.02	571.36
$\boldsymbol{\alpha}$	-13056.93	170737.89	-361279.15	335165.29
$\boldsymbol{\beta}$	-45.24	57.68	-162.88	72.40
Adjusted R^2: 84%				

All coefficients were found statistically significant, and the estimated coefficients were with expected signs: Gamma (γ) being positive and α, β negative. The results of the regression returned a common saturation level of 505.19 vehicles per 1000 people.

On the basis of assumptions mentioned under Gompertz curve, the model (Equation 3) projects Vehicle Ownership Rate (VOR) for each country.

$$\boldsymbol{\widehat{VOR}_j = \gamma e^{\alpha e^{\beta (GDP_j)}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

As the model results indicate (see Appendix A), the estimated country-specific VORs ($\widehat{VOR}_j$) demonstrated a considerable deviation from actual VOR data observed. This fact could be explained by the relevance of unobservable factors in the long run demand of automobiles. These unobservable variables could not be attributed to economic variables, like GDP per capita, since the differences in the estimated VOR were observed among countries with similar levels of economic development.

These patterns observed further substantiated the importance of deriving a country specific Gamma coefficient to be used in the Gompertz curve, including more information at country level. This was attempted in the country-level case study during the second phase of this research.

4.2. Stage 02: Definition of a Country Specific Gamma

In order to derive a function that would replace the common Gamma coefficient (γ) in the Gompertz curve, a residual adjustment mechanism was used as indicated below:

$$\widehat{\boldsymbol{Y_j}} = \ln(\boldsymbol{VOR_j}) - \ln(\widehat{\boldsymbol{VOR_j}}) \quad \text{.....} (4)$$

Here, VOR_j is the actual vehicle ownership and $\widehat{VOR_j}$ is the predicted vehicle ownership rate estimated using the Equation 1. The difference in log form ($\widehat{Y_j}$) therefore could be recognized as reflecting the disparity between actual and predicted values as per Gompertz curve and capturing factors other than the GDP per capita, including public transport mode share; household size; female driver ratio; and population density. The examination in Section 3 revealed the significant relationships between VOR and other determinants, namely, population density (PD), public transport mode share (PTS), household size (HHS) and female driver ratio (FDS) which yielded substantial justification for developing an improved vehicle ownership model. The present study therefore attempted to regress this unexplained variability of the log form of VOR, based on country-specific characteristics in HI countries as indicated in Equation (5).

$$\boldsymbol{Deviation\ of\ ln(VOR)} = \widehat{\boldsymbol{Y_j}} = \boldsymbol{f(PD_j, PTS_j, HHS_j, FDS_j)} \quad \text{.....} (5)$$

A stepwise regression was employed to determine the significance of each exploratory variable when determining a particular country's VOR. The logarithmic transformations respond to more substantial influences from the large values, mostly from larger countries. The linear and log formulations were tested. The logarithmic transformation became not significant ($p > 0.05$). Table 4 summarises the result of the linear regression.

Table 4: Statistical results of the stepwise regressor

OLS Regressor	Coefficient	Significance Level (1-tailed)
Intercept	0.2909	0.036*
Population density	-0.0001	0.000***
Share of Public Transport	-0.3175	0.024*
Household Size	-0.1393	0.018*
Share of Female Drivers	0.8515	0.039*
Adjusted R^2: 82.7%		

*** p<0.001, ** p<0.01, * p<0.05

The results (Table 4) show that, at a 95% confidence interval, the optimum regression output satisfies all the significant values ($p < 0.05$) and rejects the null hypothesis that there is no significant linear relationship between deviation of ln(VOR) estimated through Gompertz curve as the dependent variable and the four socio-demographic variables tested as independent variables. Furthermore, the adjusted R^2 value of 82.7% is considered significant in explaining the fitted model. The population density (coefficient= -0.0001) and public transport mode share (coefficient= -0.7590) contribute negatively to the VOR difference in log form, indicating that the difference of ln(VOR) estimate would decrease with a higher population density and higher public transport share. In contrast, female drivers (coefficient= 0.8515) appeared to increase the difference of ln(VOR).

Cross-sectional studies often range between very small and large values and, thus, are more likely to have heteroscedasticity effects. The developed OLS assumed that the spread of the residuals would be constant across the plot (null hypothesis). Therefore, the following OLS diagnostic statistics were obtained to check the violation of assumptions, if any. Breusch-Pagan/Cook-Weisberg test for heteroskedasticity had 6.63 Chi-squared value (Prob > Chi-squared = 0.0100). Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE), and Mean Square Error (MSE) were 1.75E-01, 2.5%, and 4.38E-02, respectively, indicating an acceptable model result (see Appendix B).

The model for $\widehat{\boldsymbol{Y_j}}$ depicted in Equation (6) could thus be derived using the regression output.

$$\boldsymbol{Deviation\ of\ ln(VOR) = \widehat{Y_j} = (0.2909 - 0.0001(PD) - 0.3175(PTS) - 0.1393(HHS) + 0.8515(FDS)} \quad \ldots\ldots (6)$$

This relationship was fitted into Gompertz formula for $\widehat{\boldsymbol{VOR_j}}$ as described in the following steps, to derive a formula for country-specific Gamma coefficient (γ_j).

Firstly, the Equation (7) could be developed by substituting the equation (6) into equation (4).

$$\boldsymbol{\ln(VOR_j) = 0.2909 - 0.0001(PD) - 0.3175(PTS) - 0.1393(HHS) + 0.8515(FDS) + \ln(\widehat{VOR_j})} \quad \ldots\ldots (7)$$

Next, the logarithmic transformation of Equation (3) was substituted for $\ln(\widehat{VOR_j})$ in Equation (7), which yielded the Equation (8):

$$\boldsymbol{\ln(VOR_j) = 0.2909 - 0.0001(PD) - 0.3175(PTS) - 0.1393(HHS) + 0.8515(FDS) + \ln\gamma + \alpha\, e^{\beta(GDP_j)}} \quad \ldots\ldots (8)$$

Third, Equation (9) could be worked out a indicated below, by taking the Antilog of Equation (8).

$$\boldsymbol{VOR_j} = \boldsymbol{e}^{[\mathbf{0.2909-0.0001}(\boldsymbol{PD})\mathbf{-0.3175}(\boldsymbol{PTS})\mathbf{-0.1393}(\boldsymbol{HHS})\mathbf{+\ 0.8515}(\boldsymbol{FDS})]}.\boldsymbol{\gamma}.\,\boldsymbol{e}^{\alpha e^{\beta(GDP_j)}} \quad ..\ (9)$$

Finally, the Vehicle Ownership Rate for country j ($\boldsymbol{VOR_j}$) could be expressed in the form of the initial Gompertz model, as depicted in the Equation (10).

$$\boldsymbol{VOR_j} = \boldsymbol{\gamma_j} \boldsymbol{e}^{\alpha e^{\beta(GDP_j)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (10)$$

Such that, $\boldsymbol{\gamma_j} = \boldsymbol{e}^{[\mathbf{0.2909-0.0001}(\boldsymbol{PD})\mathbf{-0.3175}(\boldsymbol{PTS})\mathbf{-0.1393}(\boldsymbol{HHS})\mathbf{+\ 0.8515}(\boldsymbol{FDS})]}.\boldsymbol{\gamma}]$

where "γ_j" represents the country specific asymptote in Gompertz form and γ denotes Gamma coefficient in the initial Gompertz model.

With the introduction of this new parameter, authors were able to determine how the traditional Gompertz model could be used for policy variables. Accordingly, the growth of female drivers, and improvement in public transport share are the vital factors that will change the VOR. The relationship between population increases and VOR was negatively correlated, while denser countries tend to have lower VOR given the viability of public transport. In the proposed study of calibrated vehicle ownership modelling, all coefficients were found statistically significant, and the estimated coefficients were with expected signs which confirms the observation we made. Through this model, significant improvement has been made to the traditional Gompertz curve with the regression-based model justifying the unexplained variability of VOR based on country-specific characteristics in HI countries.

5. CONCLUSIONS AND POLICY IMPLICATIONS

This study was built on numerous previous research efforts to derive a function for γ_j as a country-specific value of Vehicle Ownership Saturation Rate instead of it being considered a universal constant in the Gompertz function. Our two-staged model specification exploited the similarity of response in VORs to per-capita income across countries, while allowing for cross-country variation in the instruments that could be used to direct vehicle ownership and the eventual saturated vehicle ownership.

Our study revealed that this relationship would be significantly affected by the large HHS and low prevalence of driving licenses among women in Middle Eastern countries. High population density in countries would tend to support rapid uptake of public transport. This better explains the observation that countries like Singapore and Hong Kong account for the most extensive public transport ownership. However, the effects were found to vary across high to low levels of the population density. They were further subject to the urban population percentages. Accordingly, it was observed that the sustainable level of motor vehicles would be determined by the ability of an area to accommodate a given level of vehicle ownership without imposing a burden on available land, the economy, or the environment.

Based on a stepwise regression analysis, the VOR was found to depend on the attributes of household size, population density, percentage of public transport share, percentage of female drivers in a country, in addition to the traditional understanding of its relationship to changes in GDP and per capita income. It indicated that social, demographic, and transport policy variables identified and calibrated with statistical significance would be contributory; providing a much more plausible, robust, and functional model to forecast the desirable vehicle ownership level (VOR_j) for a given country "j" at its given GDP, and its ultimate saturation level (γ_j). The application of the model shows that the household size, the share of female drivers, the population density, and the public transport service level would therefore be the factors that would determine the γ_j of a country, and eventually, its VOR.

The findings of this paper are expected to benefit policymakers, especially in middle and lower-income countries, to manipulate the transport, urban densification and other instruments that could be used to manage vehicle ownership and the eventual saturation level of vehicle ownership. It also looks to encourage future studies that explore the influence of other variables that could possibly further increase the predictability of the model.

REFERENCES

[1] P. H. Franses, "Fitting a gompertz curve," *J. Oper. Res. Soc.*, vol. 45, no. 1, pp. 109–113, 1994, doi: 10.1057/jors.1994.11.

[2] N. C. D. Silva *et al.*, "An Experimental Study on Smart Electric Bicycle with Weight Minimized Power Pack," Nov. 2021, pp. 80–86, doi: 10.1109/EECON52960.2021.9580942.

[3] H. Lu, H. Ma, Z. Sun, and J. Wang, "Analysis and Prediction on Vehicle Ownership Based on an Improved Stochastic Gompertz Diffusion Process," *J. Adv. Transp.*, vol. 2017, 2017, doi: 10.1155/2017/4013875.

[4] T. Wu, M. Zhang, and X. Ou, "Analysis of future vehicle energy demand in China based on a Gompertz function method and computable general equilibrium model," *Energies*, vol. 7, no. 11, pp. 7454–7482, Nov. 2014, doi: 10.3390/en7117454.

[5] M. Felis Rota, M. C. Julián, and J. P. García, “Dual approach for modelling demand saturation levels in the automobile market. The Gompertz curve: Macro versus micro data,” 2016.

[6] J. Dargay, D. Gately, and M. Sommer, “Vehicle Ownership and Income Growth, Worldwide: 1960-2030,” *Energy J.*, vol. Volume 28, no. Number 4, pp. 143–170, 2007, Accessed: May 13, 2021. [Online]. Available: https://ideas.repec.org/a/aen/journl/2007v28-04-a07.html.

[7] H. Huo and M. Wang, “Modeling future vehicle sales and stock in China,” *Energy Policy*, vol. 43, pp. 17–29, Apr. 2012, doi: 10.1016/j.enpol.2011.09.063.

[8] M. M. De Silva and A. S. Kumarage, “A Gap Analysis of EV Policies for promoting E-Mobility in Sri Lanka,” 2021, Accessed: Feb. 03, 2022. [Online]. Available: https://www.researchgate.net/publication/358200899_A_Gap_Analysis_of_EV_Policies_for_promoting_E-obility_in_Sri_Lanka.

[9] T. Wu, H. Zhao, and X. Ou, “Vehicle ownership analysis based on GDP per capita in China: 1963-2050,” *Sustain.*, vol. 6, no. 8, pp. 4877–4899, Aug. 2014, doi: 10.3390/su6084877.

[10] J. Dargay, D. Gately, and M. Sommer, “Vehicle Ownership and Income Growth, Worldwide: 1960-2030,” *Energy J.*, vol. 28, no. 4, 2007, Accessed: 10/6/2021[Online]. Available:https://www.jstor.org/stable/41323125?seq=1.

[11] R. R. Hocking, “The Analysis and Selection of Variables in Linear Regression,” *Biometrics*, vol. 32, no. 1, p. 1, Mar. 1976, doi: 10.2307/2529336.

[12] J. Neter, M. H. Kutner, C. J. Nachtsheim, and W. Wasserman, “Applied linear statistical models,” 1996. https://www.worldcat.org/title/applied-linear-statistical-models/oclc/36222556 (accessed May 13, 2021).

[13] M. Carrillo and J. M. González, “A new approach to modelling sigmoidal curves,” *Technol. Forecast. Soc. Change*, vol. 69, no. 3, pp. 233–241, Apr. 2002, doi: 10.1016/S0040-1625(01)00150-0.

[14] P. H. Franses, “Gompertz curves with seasonality,” *Technol. Forecast. Soc. Change*, vol. 45, no. 3, pp. 287–297, Mar. 1994, doi: 10.1016/0040-1625(94)90051-5.

[15] J. M. Dargay, “The effect of income on car ownership: evidence of asymmetry,” *Transp. Res. Part A Policy Pract.*, vol. 35, no. 9, pp. 807–821, Nov. 2001, doi: 10.1016/S0965-8564(00)00018-5.

[16] S. C. Davis, S. W. Diegel, and R. G. Boundy, "Transportation Energy Data Book: Edition 29," 2010, Accessed: Feb. 24, 2022. [Online]. Available: http://www.osti.gov/contact.html.

[17] World Bank Group, "How does the World Bank classify countries?," 2016. https://datahelpdesk.worldbank.org/knowledgebase/articles/378834-how-does-the-world-bank-classify-countries (accessed May 13, 2021).

[18] R. S. Sugathadasa, M. Lakshitha, A. Thibbotuwawa, and K. A. C. P. Bandara, "Motivation factors of engineers in private sector construction industry," *J. Appl. Eng. Sci.*, vol. 19, no. 3, pp. 794–805, Apr. 2021, doi: 10.5937/JAES0-29201.

[19] D. Wadhwa, "WDI - Classifying countries by income," *The World Bank*, 2019.https://datatopics.worldbank.org/world-development-indicators/stories/the-classification-of-countries-by-income.html (accessed Jun. 10, 2021).

APPENDIX A

Country	VOR_j (Actual)	$\widehat{VOR_j}$ (Estimated)
Singapore	149	505.19
United Arab Emirates	313	505.19
Norway	584	505.19
Switzerland	566	505.19
Hong Kong	77	505.19
United States	797	505.19
Saudi Arabia	336	505.19
Ireland	513	505.19
Netherlands	528	505.19
Austria	578	505.19
Denmark	480	505.19
Germany	572	505.18
Australia	717	505.18
Sweden	520	505.18
Canada	607	505.18
Belgium	559	505.18
Oman	215	505.15
Finland	612	505.14
United Kingdom	519	505.13
France	578	505.1
Japan	591	505.06
New Zealand	712	504.87
Italy	679	504.65
Israel	346	503.97
Spain	593	503.62
South Korea	376	503.53
Czech Republic	485	502.17
Slovakia	364	491.7
Portugal	548	490.56
Greece	624	471.2
Poland	537	447.48
Hungary	345	445.13
Chile	184	326.83
Croatia	380	277.18

APPENDIX B

Country	VOR_j **(Actual)**	$\lambda_{(s)_j}$ **(New** $(\gamma_{(s)_j})$ **)**
Singapore	149	106.00
United Arab Emirates	313	288.98
Norway	584	684.02
Switzerland	566	589.80
Hong Kong	77	112.20
United States	797	672.10
Saudi Arabia	336	296.56
Ireland	513	499.56
Netherlands	528	414.87
Austria	578	498.85
Denmark	480	618.01
Germany	572	615.27
Australia	717	663.12
Sweden	520	579.05
Canada	607	645.01
Belgium	559	588.06
Oman	215	256.06
Finland	612	687.14
United Kingdom	519	636.01
France	578	595.06
Japan	591	514.77
New Zealand	712	546.78
Italy	679	548.63
Israel	346	473.62
Spain	593	561.08
South Korea	376	411.11
Czech Republic	485	470.84
Slovakia	364	429.19
Portugal	548	576.56
Greece	624	472.59
Poland	537	477.84
Hungary	345	417.76
Chile	184	224.20
Croatia	380	268.46