

## DYNAMICS OF OPTIMAL GROWTH AND OVERLAPPING GENERATIONS MODELS IN ECONOMIC CONVERGENCE AND INEQUALITY: A REVIEW

D. I. J. Samaranayake<sup>1</sup>

Department of Management Studies, Faculty of Management, University of Peradeniya, Sri Lanka<sup>1</sup>.


### ABSTRACT

*This study explains the dynamics in the Optimal Growth [OGR] and Overlapping Generations [OGE] models and their implications for (real) GDP per capita [RGDPPC] in different countries, short-run and long-run. This analysis uses RGDPPCs for ten countries consisting of five per each developed and developing category for 116 years. Sri Lanka is used as the favourite destination and calculated the RGDPPC ratios. Data on RGDPPC for ten countries explains that the relative differences between RGDPPC ratios and the baseline are varying over time. Also, it demonstrates a high variation in the short-run dynamics between countries. Both models predict a convergence of economies into a steady-state having the same amount of RGDPPCs in the long run. This prediction appears to be acceptable in the analysis. Having Sri Lanka as the benchmark, Japan exhibits low RGDPPC values at the initial stage and eventually gets closer to the RGDPPCs of developed countries. This is not a direct convergence to the steady-state but convergence to the RGDPPC level of developed countries. Also, the concept of efficient contributions by generations in the OGE model was considered and it reflects a more realistic reasoning as per the short-run dynamics, but that resulted in making differences across countries per capita GDP in the long run. The long-run dynamics revealed that growing inequality is a common issue in developing nations as a very low percentage of populations enjoys a high proportion of national output.*

**JEL Classification:** E27, E19, O47, O57

**KEYWORDS:** Convergence; Optimal growth; Overlapping generations; Real per-capita GDP

Corresponding Author: Don Isuru Samaranayake, E-mail: [dijs@mgt.pdn.ac.lk](mailto:dijs@mgt.pdn.ac.lk).

 <https://orcid.org/0000-0002-1300-4524>



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## 1. INTRODUCTION

New classical macroeconomics led economists to identify the importance of applying streamlined microeconomic models to study the aggregate economy. Thus, two landmark models in the modern macroeconomics are the Optimal Growth [OGR] and Overlapping Generations [OGE] models. These models have microeconomic foundations and are general equilibrium models (Bewley, 2007). Both models describe the behaviour of agents and focus on the fundamentals such as utility functions and initial endowments for consumers and production functions for firms. Also, agent's problems are stated, and equilibria are reviewed for utility maximization, profit maximization, and market clearance (Koopmans, 1963; John & Pecchenino, 1994; Bewley, 2007; Tvede, 2010; Kim & Spear, 2020; Gali, 2021).

Kim & Spear (2020) studied the 'Markov equilibrium' in various stochastic OGE models. Thus, it was an application of applied game theory together with dynamic strategic interactions towards macroeconomic equilibria. Gali (2021) analyzed an extension of the New Keynesian model while including the features of OGE model. Such, methodological studies emphasize the interest as well as the applicability of New Classical growth models in academia. This has been a trademark in modern economics as both OGR and OGE models let academics to think beyond Keynesian thoughts.

Both OGR and OGE models are indefinitely important in examine the optimal growth targets, strategies, and their potentials. The literature highlights various attempts that tested the applicability of these models in different economies (John & Pecchenino, 1994; Cazes et al., 1994; Mino & Shibata, 1995; Hviding & Merette, 1998; Demange & Laroque, 2000; Melkonyan & Grigorian, 2008; Watanabe, 2008; Weil, 2008; Garriga, 2017; Das et al., 2018; Khan & Lidforsky, 2019; Sun et al., 2020).

For instance, Sun et al. (2020) tested the connection between environmental performance assessments and global economic growth following the insights from efficiency and growth convergence as per OGR and OGE models and their growth targets.

Consequently, the foremost objective of the study is to examine the dynamics in the two models and their implications for Real GDP Per Capita (RGDPPC) across the developed and developing countries both in the short run and in the long run. Initially, the study elaborates technical differences between OGR and OGE models followed by an empirical analysis using RGDPPCs for ten countries consisting of five per each developed and developing categories for 116 years using data from the Maddison Project (Bolt et al., 2018).

In the analysis, Sri Lanka is considered as the favourite destination and figures are used with time on the first axis and the ratios of GDP per capita of the non-favourite countries and GDP per capita of Sri Lanka on the second axis. Thereafter, the study discusses the dynamics of two models and reflections on the short-run, long-run differences following the analysis.

## 2. OGR AND OGE MODELS

The OGR and OGE models are two pillars in new classical macroeconomics. They are dynamic models with time extending to infinity, but in OGR economies consumers live forever and in OGE economies consumers live for finitely many years (Koopmans, 1963; John & Pecchenino, 1994; Bewley, 2007; Tvede, 2010). The dynamics of the OGR model are described by a comparison of combinations of household consumption and capital over time with respect to the household's optimization problem. Higher the consumption [ $c_t$ ] in the next year indicates a higher income level next year.

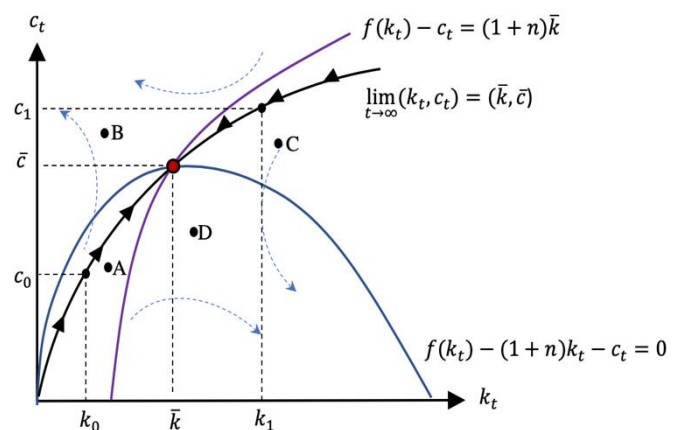
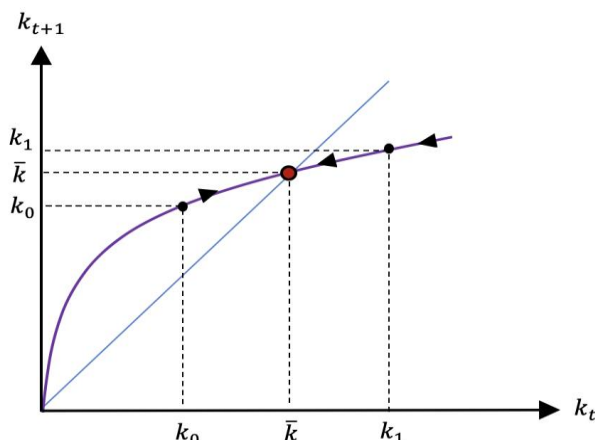


Figure 1. Steady State - OGR Model

Figure 1 reflects all the dynamics of the OGR model. Any economy with  $c_t, k_t$  combination within the area “A” experiences a situation with high growth in both capital and consumption in the short run, and it increases its per capita GDP and eventually moves to the  $\bar{k}, \bar{c}$  combination in the long run. Any country with  $c_t, k_t$  combination within the area “C” eventually moves back to the  $\bar{k}, \bar{c}$  in the long run. The area “B” represents higher levels of consumption but with low capital, and it reflects the problem with feasibility. Also, area “D” represents higher levels of capital with low consumption, and it reflects the problem with transversality.

Ultimately, this model predicts that all economies eventually reach the steady-state  $(\bar{k}, \bar{c})$  and be equally rich in the long run regardless of the initial conditions of its capital strengths.



**Figure 2. Steady State - OGE Model**

The OGE model requires each generation to actively participate in economic activities as the equilibrium changes over time with new generations. Here we consider the combinations of per capita capital ( $k_t$  and  $k_{t+1}$ ) across generations.

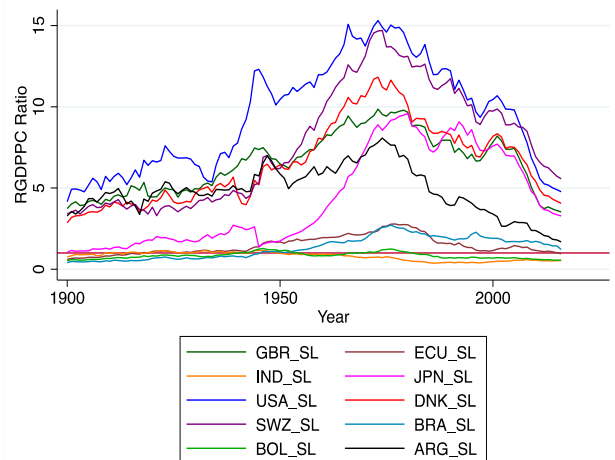
Figure 2 demonstrates how the steady-state is globally stable according to the OGE model. Here the equilibria are determinate and path independent. The equilibria are determinate as there is a unique sequence of capital for all initial endowment of savings. Also, it is path independent as the initial conditions of the economy do not matter for the long-run convergence. Thus, a country with  $k_0$  level of capital has its own unique combination across generations and reaches higher RGDPPC in the short-

run till it reaches the steady-state with a constant  $\bar{k}$ .

On the other hand, a country with less contribution from the next generation reaches back to the steady state by lowering its RGDPPC in the short run. Also, OGE economies can end up in multiple equilibria solutions but it is not emphasized in this study. Eventually, the OGE model also implies that the countries have dynamics in the short run but converge into a steady-state  $\bar{k}$  irrespective of the initial condition of the country. Thus, both models predict a convergence of economies into a steady-state having same amount of RGDPPCs in the long-run but have clear differences in the way they describe the scenario.

### 3. COMPARISON OF RGDPPC RATIOS

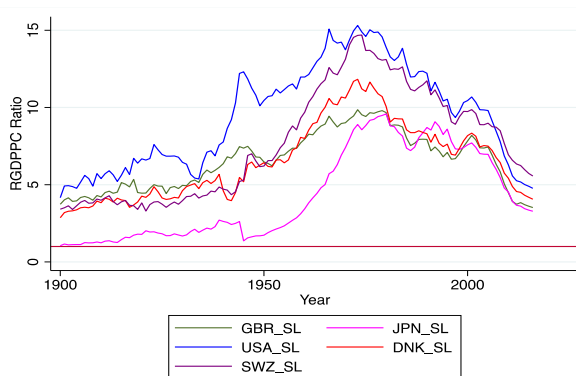
This analysis uses RGDPPCs for ten countries consisting of five per each developed and developing categories for 116 years. Sri Lanka is used as the favourite destination and calculated RGDPPC ratios.



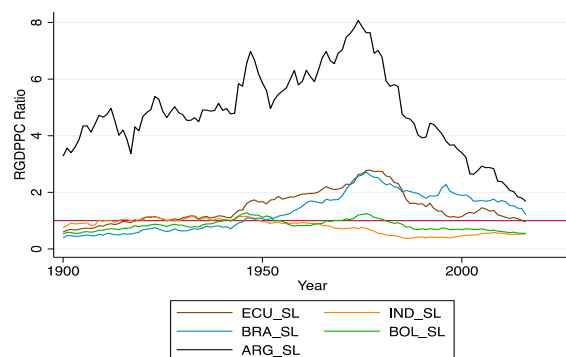
**Figure 3. RGDPPC Ratios of all Countries**

Figure 3 illustrates the RGDPPC Ratio of 10 Countries over time. The red horizontal line indicates the RGDPPC ratio ‘1’ as the baseline for the comparison. The ratio is 1 when the RGDPPC of Sri Lanka equals to the RGDPPC of other economies. We can observe that there is an increase of RGDPPC in majority of economies on average till the 1970s-1980s compared to the baseline. Thereafter, the countries demonstrate a declining trend in the ratio. In addition to that, the highest RGDPPC ratio of 15.315 is recorded by the United States and the

lowest of 0.373 by India in 1986. Also, this figure shows us that two clusters of countries depend on the differences between the ratios of developed and developing countries. The ratios for the developed countries are demonstrated in figure 4. The developed countries are more likely to represent a separate cluster. The RGDPPC of the Japanese Economy was comparatively lower at initial stages of the comparison, but it has ended up in a similar track after the 1970s up to 2016. Thus, the developed economies are 3-6 times higher than the Sri Lankan RGDPPC by 2016



**Figure 4. RGDPPC Ratio of Developed Countries**



**Figure 5. RGDPPC Ratio of Developing Countries**

Then figure 5 represents the second cluster that consists of developing countries, Ecuador, Brazil, Argentina, India and Bolivia. The figure demonstrates a clear difference between the dynamics of RGDPPC of Argentina from other developing countries. However, eventually, the developing countries are 0.3-1.8 times higher than the baseline by 2016. Also, we can observe that the Japanese RGDPPC was similar to the baseline in the

1900s, and it has ended up in the cluster for developed economies over time. In contrast, Argentina had high RGDPPC at initial times but has ended up in the cluster for developing economies.

#### 4. SHORT-RUN DYNAMICS

According to both OGR and OGE models, the short-run dynamics lead an economy towards a steady state in the long-run. When we compare the data on RGDPPC for ten countries, we can see that the relative differences between RGDPPC ratios and the baseline are varying over time. Also, it demonstrates a high variation between the short-run dynamics between countries. We know from the OGR model that the income level of an individual depends on the consumption, and it encourages buying more capital. This is indicated by the economies such as the USA, Switzerland, and Japan by short-run boosts in their RGDPPC values in different time periods and what should lie in the area “A” of figure 1. However, the steady-state in this example is the baseline where all economies should converge in the long run but not feasible in the short-run dynamics for these countries. Also, the OGR model is not clear with India and Bolivia. Since India and Bolivia are very similar to the baseline ratio until the 1950s and then indicates a different behaviour with a decline of their RGDPPCs. This model does not have a specific explanation for such dynamic behaviours across countries.

Then we can consider the short-run dynamics of the OGE model. It seems that the OGE model has a fairly realistic explanation for such data sets because it focuses on the importance of effective combinations of  $k_t$  across generations. Each combination has its uniqueness as countries may have different RGDPPC values. For instance, India experiences a declining trend in their RGDPPC as the contribution of their new generations are ineffective or low in the short run.

Moreover, in OGR economies, the consumer does not put enough weight into the future. However, the data on different countries show how they reach different income levels across generations. According to the OGE model, the concept of efficient contributions by

generations was concerned and it reflects more realistic reasoning. We know that both models predict a convergence of economies into a steady-state in the long run and no matter what the initial conditions are. This prediction appears to be acceptable to some extent according to the data. For instance, Japan exhibits low RGDPPC values at the initial stage and eventually it get closer to the RGDPPCs of developed countries. This is not a direct convergence to the steady-state but a convergence to the RGDPPC level of developed countries. Also, we can see how the rapid adjustments in Argentina reach the RGDPPC range of developing countries. Also, figure 3 encourages us to expect convergence to the baseline usage in the distant future as there is a severe trend shown in data though it was not there in 2016.

## 5. LONG-RUN DIFFERENCES

The creation of the economic output is more complex than most of the economic models describe. As a result, we can observe a long-run difference in RGDPPCs. If we consider the Japanese economy as an example, they had the worst experience during the World War II and later reflects significant improvements in their RGDPPC. The Indian economy suffers from a growing population and inequality as their RGDPPC decreases over time. Therefore, inequality, unemployment, corruptions, high birth and mortality rates, policy changes, and technology are some reasons for such long-run differences. When we try to identify these reflections in relation to the two models, OGR model exhibits some unrealistic features as it does not focus some structural contexts in the countries. According to the model, a consumer lives forever, and demographic structure seems unrealistic. The issues in mortality, ageing population and population growth are not considered, and even the issue of unemployment is not considered due to this limitation of the model. Anyway, the OGE model appears to be more realistic as it considers that consumers live for finitely many days.

Growing inequality is a common issue in developing nations as a very low percentage of populations enjoys a high proportion of national output. Therefore, this generates instability in the explanation given by OGE model regarding the

effective contribution of individuals from all generations for production. For instance, Indian economy struggles from both high population growth and inequality which creates issues such as less contribution from the economically active population and high unemployment. Also, both models do not focus on factors such as technology and market structures apart from the spot market, which can create major differences. Especially, the advanced technology can architect differences in income levels over time. Therefore, such issues can create significant differences to make long-run differences in RGDPPCs against the predictions by the models on long-run convergence to the steady-state.

## 6. CONCLUSION

New classical macroeconomics led economists to identify the importance of applying streamlined microeconomic models to study the aggregate economy. Two landmark models in the modern macroeconomics are the OGR and OGE models. Both models predict a convergence of economies into a steady-state having the same amount of RGDPPCs in the long-run but with clear differences in the approaches. The analysis on RGDPPC for ten countries reflect relative differences between RGDPPC ratios and the baseline and are varying over time. It exhibits high variation between the short-run dynamics between countries. Further, we observe long-run difference in RGDPPCs. Inequality, unemployment, corruptions, high birth, mortality rates, policy changes, and technology are some fundamental reasons for such long-run differences. Consequently, the OGR model exhibits unrealistic features as it does not focus on some of the key structural contexts highlighted above. A consumer lives forever, and the demographic structure seems unrealistic in the OGR. In contrast, the OGE model appears to be more realistic as it considers that consumers live for finitely many days. Moreover, in optimal growth economies, the consumer does not put enough weight into the future. However, the data on different countries show how they reach different output levels depending on their plans into future and the degree of weight creates differences across the per-capita GDP between countries. According to the OGE model, the concept of efficient contributions by

generations such as baby boomers, X, Y, Z and Alpha was concerned, and it reflects more realistic reasoning. Yet, in contrast to the predictions made by OGE model, data show differences in per-capita GDPs between developed and developing countries in the long-run due to prolonged structural differences.

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