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Some results related to new type of continuity, compactness, and connectedness in topological spaces

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Abstract Several properties of *F*-continuous functions, *F*-compact spaces, and *F*-connected spaces in the setting of topological spaces were investigated in this paper. The relationships among these concepts and their classical counterparts were established, and their preservation properties under mappings were studied. Characterizations of *F*-compactness using the finite intersection property were obtained, and several equivalent conditions for *F*-connected spaces were presented. Furthermore, The behavior of *F*-connectedness under surjective *F*-continuous mappings and *F*-homeomorphisms were examined. The results extend classical topological notions within the framework of *F*-open sets and provide a foundation for further research in generalized topology.

Keywords: *F*-connectedness, *F*-continuous, *F*-compact, *F*-open, topological spaces

1 Introduction

In general topology, numerous researchers have explored extensions of open sets to obtain a more profound comprehension of the structural characteristics of topological spaces. Key ideas like continuity, compactness, and connectedness have been examined in the literature (Engelking 1977). Throughout time, various generalizations of open sets have emerged, such as semi-open sets (Levine 1963), pre-open sets (Mashhour *et al.* 1982), α-open sets (Singal and Singal 1979), semi-preopen sets (Dontchev 1995), and other associated concepts (Levine 1970, Dontchev and Maki 1999, Császár 2005). These broad forms of openness inherently lead to related extensions of continuity, compactness, and connectedness thus offering different perspectives for examining traditional topological characteristics.

In addition to these advancements, several researchers (Dontchev and Maki 1999, Császár 2005, Levine 1970, Arya and Nour 1990, Caldas and Jafari 2006) have explored associated ideas like generalized closed sets, θ-generalized closed sets, and contra-continuous functions. These studies demonstrate that relaxing the conventional

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definition of openness frequently results in significant generalizations of topological structures and yields new theoretical understandings.

Alqahtani (2023) introduced the *F*-open sets, which provide a novel contribution to this line of research. A set is defined as *F*-open if its closure contains only finitely many points outside the set. Unlike classical or other generalized open sets, this finiteness condition serves primarily as a formal, combinatorial constraint rather than arising from natural or real-world considerations. Nevertheless, it captures scenarios in which a set is "almost open," except for a small, finite boundary, making it particularly useful for studying spaces with isolated points or finite disturbances. From this foundation, the notions of *F*-continuity, *F*-compactness, and *F*-connectedness were developed. These concepts allow classical topological properties to be examined under finite variations and provide a framework for analyzing situations where standard definitions may be overly restrictive.

Motivated by these developments, the purpose of this paper is to investigate several properties of *F*-continuous functions, *F*-compact spaces, and *F*-connected spaces and to examine their relationships with classical topological concepts.

2 Material and Methods

The useful definitions given in Alqahtani (2023) that are related to this paper together with examples are provided below.

Definition 2.1. A set A of a topological space (X, τ) is called *F*-open if $\bar{A} \backslash A$ is finite.

Example 2.2. Let $X = \mathbb{R}$ with the usual topology and consider $A = \mathbb{R} \backslash \{1,2,3\}$. Then $\bar{A} \backslash A = \{1,2,3\}$, which is finite. Hence, A is *F*-open.

Remark 2.3. The complement of an *F*-open set is called *F*-closed.

Definition 2.4. A subset *U* in a topological space (X, τ) is called *F*-cl-open, if *U* is both *F*-open and *F*-closed simultaneously.

Definition 2.5. (Alqahtani 2023) A map $k: (X, \tau) \to (Y, \sigma)$ is *F*-continuous, if $k^{-1}(U)$ is *F*-open in *X* for every open set *U* in *Y*.

Example 2.6. Let $X = \mathbb{R}$ with the usual topology, and $Y = \mathbb{R}$, and define $f(x) = x$ if $x \neq 0$ and $f(x) = 1$ if $x = 0$. Then f is *F*-continuous because the preimage of any open set in Y differs from an open set in X by at most a finite set.

Definition 2.7. (Alqahtani 2023) A map $k: (X, \tau) \to (Y, \sigma)$ is *F*-open, if $k(U)$ is *F*-open in *Y* for all open sets *U* in *X*.

Definition 2.8. (Alqahtani 2023) A bijective function $k: (X, \tau) \rightarrow (Y, \sigma)$ is *F*-homeomorphism if and only if k and k^{-1} are *F*-continuous.

Definition 2.9. (Alqahtani 2023) A topological space (X, τ) is *F*-compact if every open cover of X admits a finite subcover consisting of *F*-open sets.

Example 2.10. Let $X = \mathbb{Z}$ with the cofinite topology. Then X is *F*-compact: any open cover by cofinite sets has a finite subcover.

Definition 2.11. A collection I of subsets of a topological space X is said to have finite intersection property if $\bigcap_{i=1}^{n} U_i$ is nonempty for every subcollection $\{U_1, U_2, \dots, U_n\}$ of I.

Definition 2.12. A topological space (X, τ) is called *F*-connected, if it cannot be written as the union of two disjoint non-empty *F*-open sets. Note that a topological space is connected if it cannot be written as the union of two disjoint open sets.

Example 2.13. Let $X = \mathbb{Z}$ with the cofinite topology. Then X is *F*-connected because there are no two disjoint non-empty *F*-open sets whose union equals X.

3 Results and Discussion

Key findings related to *F*-continuity, *F*-compactness, and *F*-connectedness are given below. Their preservation properties were investigated through mappings, *F*-compact spaces were defined by the finite intersection property, and multiple equivalent conditions along with stability results were derived for *F*-connected spaces.

Theorem 3.1. Let $k: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous function. Then k is *F*-continuous.

Proof. Let U be any open subset of Y. Since k is continuous, $k^{-1}(U)$ is open in X. Every open set is *F*-open, because if A is open, then $\bar{A} \backslash A$ is an empty set, which is finite. Hence, $k^{-1}(U)$ is *F*-open in X. Therefore, k is *F*-continuous.

Theorem 3.2. Let $k: (X, \tau) \rightarrow (Y, \sigma)$ be *F*- continuous function and $s: (Y, \sigma) \rightarrow (Z, \eta)$ be a continuous function. Then the composite map $s \circ k: (X, \tau) \rightarrow (Z, \eta)$ is *F*-continuous.

Proof. Let V be any open set in Z. Then $s^{-1}(V)$ is an open set in Y as s is continuous. This implies that $k^{-1}(s^{-1}(V))$ is *F*-open in X as k is *F*-continuous. But $k^{-1}\big(s^{-1}(V)\big) = (s \circ k)^{-1}(V)$. Hence, $(s \circ k)^{-1}(V)$ is *F*-open in X. Thus, the map $(s \circ k)$ is *F*-continuous.

Theorem 3.3. Let $k: (X,\tau) \to (Y,\sigma)$ be F-continuous function. Then for each x in X and for each neighborhood V of $k(x)$, there exists an F-neighborhood U of x such that $k(U) \subseteq V$.

Proof. Assume that k is F-continuous. Let C be any closed set in Y. Then $Y\backslash C$ is open in Y. This implies that $k^{-1}(Y\backslash C)$ is F-open in X as k is F-continuous. Since $k^{-1}(Y\backslash C) = k^{-1}(Y)\backslash k^{-1}(C) = X\backslash k^{-1}(C)$, we have $X\backslash k^{-1}(C)$ is F-open in X. This implies that $k^{-1}(C)$ is F-closed in X. Now, let $k(x) \in V$, where V is a neighborhood of $k(x)$. So, $Y\backslash V$ is closed in Y. Then $X\backslash k^{-1}(V) = k^{-1}(Y\backslash V)$ is F -closed in X. That is, $k^{-1}(V)$ is F-open in X. Now, pick an F-open set $U = k^{-1}(V)$. Also, $k(x) \in V$. Then $x \in k^{-1}(V) = U$. Therefore, $k(U) = k\left(k^{-1}(V)\right) \subseteq V$.

Theorem 3.4. Let a bijective function $k: (X,\tau) \to (Y,\sigma)$ be F-homeomorphism. Then k is an F-closed map.

Proof. Assume that the map $k: (X,\tau) \to (Y,\sigma)$ is F-homeomorphism. Then the maps k and k^{-1} are F-continuous. Let U be an open set in X. Then $(k^{-1})^{-1}(U) = k(U)$ is F-open in Y. Thus, k is F-open and F-continuous. Let V be any closed set in X. Then $v^c = X\backslash V$ is open in X. Then $k(X\backslash V)$ is F-open in Y. That is, $k(X\backslash V) = k(X)\backslash k(V) = Y\backslash k(V)$ is F-open in Y. Thus, $k\,(V)$ is F-closed in Y. Therefore, the map k is F-closed.

Theorem 3.5. Every compact topological space is F-compact.

Proof. Let (X,τ) be compact and let $\{O_\alpha : \alpha \in I\}$ be an open cover of X. Since X is compact, there exist a finite subcover $\{O_{\alpha_i} : i = 1,2,\dots,n\}$. Every open set is F-open, hence this finite collection is a finite subcover consisting of F-open sets. Therefore X is F-compact.

Theorem 3.6. Let A, B be two F-compact subsets of X. Then $A \cup B$ is F-compact.

Proof. Let $\{O_\alpha : \alpha \in I\}$ be collection of F -open sets and $A \cup B \subseteq \cup_{\propto \in I} O_\propto$. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, we have $A \subseteq \cup_{\propto \in I} O_\propto$ and $B \subseteq \cup_{\propto \in I} O_\propto$. Since A, B are F-compact, $A \subseteq \cup_{i=1}^{m} O_{\alpha_i}$ and $B \subseteq \cup_{j=1}^{n} O_{\alpha_j}$. Then $A \cup B \subseteq \left(\cup_{i=1}^{m} O_{\alpha_i}\right) \cup \left(\cup_{j=1}^{n} O_{\alpha_j}\right)$. Thus, $A \cup B$ is F-compact.

Remark. The result of Theorem 3.6 can be extended to any finite union of F-compact subsets. Specifically, by induction on the number of subsets, one can show that if $\{A_1, A_2, \dots, A_n\}$ are F-compact, then $\cup_{i=1}^{n} A_i$ is F-compact. The base case $n = 2$ is given in Theorem 3.6. For the inductive step, assume the union of n F-compact sets is F-compact; then adding an $(n+1)th$ F-compact set preserves F-compactness by applying Theorem 3.6 to the union of the first n sets and the $(n+1)$th set.

Theorem 3.7. Let f: $(X,\tau) \to (Y,\sigma)$ be F-continuous and $K \subseteq X$ be F-compact. Then $f(K)$ is F-compact in Y.

Proof. Let $\{V_\alpha : \alpha \in I\}$ be an open cover of $f(K)$. Then $K \subseteq \bigcup_{\alpha\in I} f^{-1}(V_\alpha)$. Since f is F-continuous, each $f^{-1}(V_\alpha)$ is F-open. Since K is F-compact, there exists a finite subcollection $\{f^{-1}(V_{\alpha_i}) : i = 1,2,\dots,n\}$ covering K. Hence, $f(K) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$. Therefore, $f(K)$ is F-compact.

Theorem 3.8. Let (X,τ) be a topological space. Then X is F-compact if and only if any collection Γ of F-closed subsets of X having the finite intersection property $\cap_{C\in\Gamma} C \neq \phi$.

Proof. Assume that X is F -compact and Γ be any collection of F closed subsets of X having the finite intersection property. If $\cap_{i\in I} C_i = \phi$ where $C_i \in \Gamma, i \in I$, then $\bigcup_{i\in I} C_i^c = \emptyset^c = X$. Thus, $\{C_i^c : i \in I\}$ forms an open cover for X with collection of F-open ses. Since X is F −compact, we have $X \subseteq \bigcup_{j=1}^{m} C_{i_j}^c$. Then $X^c = \emptyset \supseteq \bigcap_{j=1}^{m} C_{i_j}$. That is, $\bigcap_{j=1}^{m} C_{i_j} = \emptyset$ which contradicts Γ having finite intersection property. Thus, $\cap_{C\in\Gamma} C \neq \phi$. Conversely assume that any collection Γ of F -closed subsets of X having the finite intersection property, $\cap_{C\in\Gamma} C \neq \phi$. Let $X \subseteq \cup_{\propto\in I} O_\propto$, where each O_α is F -open. Let $\{O_\alpha^c : \alpha \in I\}$ be a collection of F-closed sets. If X is not F-compact, then X cannot be covered by any finite subcollection of F-open sets $\{O_\alpha : \alpha \in I\}$. If $\{O_{\alpha_i}\}_{i=1}^{m}$ is any finite subcollection of $\{O_\alpha : \alpha \in I\}$, then $X \nsubseteq \bigcup_{i=1}^{m} O_{\alpha_i}$. This implies that $\bigcap_{i=1}^{m} O_{\alpha_i}^c \neq \emptyset$. Thus, $\{O_\alpha^c : \alpha \in I\}$ has the finite intersection property. By assumption, $\bigcap_{\alpha\in I} O_\alpha^c \neq \emptyset$. This contradicts to $X \subseteq \cup_{\propto\in I} O_\propto$. Thus, X is F -compact.

Theorem 3.9. Let X be a topological space. Then the following statements are equivalent.

1) the space X is F-connected,
2) the only F-cl-open subsets of X are $\emptyset$ and X itself,
3) any F-continuous map f from space X to any discrete space D is a constant map.

Proof. First, check (1) ⇒ (2): Assume that the topological space X is F-connected. Suppose that there exists an F-cl-open set A of X and $A \neq \emptyset$, $A \neq X$. So, $X = A \cup A^c$ and A, A^c both are F -open. This contradicts that the space X is F-connected. Thus, the supposition is wrong. Hence, (1) ⇒ (2).

Next, check (2) ⇒ (3): Assume that the only F -cl-open sets of X are $\emptyset$ and X itself. Suppose there is a F-continuous map f: $X \to D$, where D is a discrete space. Pick $x_0 \in X$. Then $f(x_0) = C_0 \in D$. Now, $f^{-1}(C_0) = \{x \in X: f(x) = C_0\}$. Since f is F-continuous and D is discrete, we have $f^{-1}(C_0)$ is F-cl-open. Also, $f^{-1}(C_0) \neq \phi$ as $x_0 \in f^{-1}(C_0)$. Thus, $f^{-1}(C_0) = X$. That is, $f(x) = C_0$, for each $x \in X$. In other words, f is a constant map. Hence, (2) ⇒ (3).

Finally, check (3) ⇒ (1): Assume that any F-continuous map f from the space X to any discrete space D is a constant map. Suppose X is not F- connected. Then there exist two F -open sets in X such that $X = U \cup V$. Define the function $\phi: X \longrightarrow \{-1,1\}$ by $\phi(x) = 1$ for $x \in U$ and $\phi(x) = -1$ for $x \in V$. Then $\phi^{-1}(1) = U$ and $\phi^{-1}(-1) = V$. Also, $\phi^{-1}\{-1,1\} = X$ and $\phi^{-1}(\Phi) = \Phi$. Thus, ϕ is F -continuous as the inverse image of each open set is F-open. Hence, (3) ⇒ (1).

Theorem 3.10. A surjective F-continuous image of an F-connected space is connected.

Proof. Let (X, τ) and (Y, σ) be two topological spaces and k: $(X, \tau) \rightarrow (Y, \sigma)$ is a surjective F-continuous map with an F-connected space X. Assume that Y is not connected. Then there exist two disjoint open sets U, V in Y such that $Y = k(X) = U \cup V$. This implies that $X = k^{-1}(U \cup V) = k^{-1}(U) \cup k^{-1}(V)$. Since k is F-continuous and U, V are open sets in Y, $k^{-1}(U)$ and $k^{-1}(V)$ are F-open sets in X, and $k^{-1}(U) \cap k^{-1}(V) = k^{-1}(U \cap V) = k^{-1}(\emptyset) = \emptyset$. That is, there exists two disjoint F-open sets $k^{-1}(U)$ and $k^{-1}(V)$ in X such that $X = k^{-1}(U) \cup k^{-1}(V)$. Thus, X is not F-connected which contradicts to the fact that X is F -connected. Therefore, the assumption is wrong. Hence Y is connected.

Theorem 3.11. Let (X, τ) and (Y, σ) be two topological spaces. Then X is F- connected and (Y, σ) is F-homeomorphism of (X, τ) then Y is F-connected.

Proof. The contradiction method was used to prove Y is connected. Assume Y is disconnected. Then there exists two non-empty disjoint open sets A, B in Y such that $Y = A \cup B$. Since the map k is F- homeomorphism, k and k^{-1} are F-continuous and bijective. Since $k^{-1}(Y) = X = k^{-1}(A \cup B)$, we have $X = k^{-1}(A) \cup k^{-1}(B)$. Moreover, $k^{-1}(A) \cap k^{-1}(B) = k^{-1}(A \cap B) = \phi$, and $k^{-1}(A)$ and $k^{-1}(B)$ are F-open in X. This contradicts that X is F-connected. Thus, Y is F-connected.

Theorem 3.12. Let (X, τ) be a topological space and A, B be two subsets of X such that $A \subseteq B \subseteq \bar{A}$. Then if A is F-connected then B is also F-connected.

Proof. Assume that A is F-connected and $A \subseteq B \subseteq \bar{A}$. Suppose that B is not F - connected. Then there exists two non-empty disjoint F-open sets C, D in X such that $A \subseteq B = C \cup D$. This gives $A \subseteq C$ or $A \subseteq D$ as $A = (A \cap C) \cup (A \cap D)$ and both $A \cap C$ and $A \cap D$ are F-open sets in A. Since A is F-connected, $A \cap C = \emptyset$ or $A \cap D = \emptyset$. If $A \subseteq C$ then $\bar{A} \subseteq \bar{C}$. This implies that $A \cap D \subseteq C \cap D = \emptyset$. Then $A \cap D = \emptyset$. Then $B \cap D = \emptyset$. So, $D = \emptyset$ as $B = C \cup D$ and $D \subseteq B$. This is a contradiction. So, B is F-connected.

Theorem 3.13. Let $\{A_\alpha: \alpha \in I\}$ be a family of F-connected sets in the topological space X such that $\cap_{\alpha \in I} A_\alpha \neq \emptyset$. Then $\cup_{\alpha \in I} A_\alpha$ is F-connected.

Proof. Let $\{A_\alpha \, \alpha \in I\}$ be a collection of F-connected sets in the topological space X. Assume that $\cup_{\alpha \in I} A_\alpha$ is not F-connected. Then there exists two disjoint F-open sets U, V such that $\cup_{\alpha \in I} A_\alpha = C \cup D$. Pick $A_{\alpha_0} \subseteq \cup_{\alpha \in I} A_\alpha = C \cup D$. This implies that $A_{\alpha_0} \subseteq C$ or

$A_{\alpha_0} \subseteq D$ as A_{α_0} is connected. If $A_{\alpha_0} \subseteq \mathbb{C}$, then $\cap_{\propto \in I} A_{\propto} \subseteq A_{\alpha_0} \subseteq C$. This implies that $\cap_{\propto \in I} A_{\propto} \subseteq A_{\alpha_0} \subseteq C$. This gives $C \cap A_{\propto} \neq \phi$ for every $\alpha \in I$. This gives $A_{\propto} \subseteq \mathbb{C}$ for every $\alpha \in I$. $\cup_{\propto \in I} A_{\propto} \subseteq \mathbb{C}$. That is, $C \cup D \subseteq C$ which is a contradiction. So, the assumption is not correct. Hence, $\cup_{\propto \in I} A_{\propto}$ is *F*-connected.

4 Conclusions

This paper studied several properties of *F*-continuous mappings, *F*-compact spaces, and *F*-connected spaces within the framework of generalized topology. Their preservation results under composition, *F*-homeomorphisms, and surjective *F*-continuous mappings were established. Characterizations of *F*-compactness using the finite intersection property and equivalent conditions for *F*-connectedness were obtained. These results demonstrate that many classical topological properties remain stable under finite perturbations introduced by *F*-openness. Future work may include investigations of separation axioms, as well as extensions to bitopological and tritopological settings.

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