The Statistical Distribution of Annual Maximum Rainfall in Colombo District

T. Mayooran and A. Laheetharan*

Department of Mathematics and Statistics, University of Jaffna,
Jaffna, Sri Lanka.
Corresponding Author: alahee@jfn.ac.lk

ABSTRACT

The modeling of extreme rainfall events is a fundamental part of flood hazard estimation. Establishing a probability distribution to represent the precipitation depth at various durations has long been a topic of interest in hydrology, meteorology and others. The daily rainfall data of 110 years (1900-2009) have been collected from the Meteorology station, Colombo, Sri Lanka. The data were then analyzed to identify the maximum rainfall received on any one day (24 hours duration), in during any monsoon season (4 seasons) and in a year (365 days period). The objective of this paper is to identify the best fit probability distribution of annual maximum rainfall in Colombo district for each period of study. Distribution parameters were estimated by using the maximum likelihood method. Three statistical goodness of fit test were carried out in order to find the best fitting probability distribution among 45 probability distributions for annual maximum rainfall and maximum rainfall for 4 seasons separately. After finding three best fitting distributions from the respective tests, the parameters of the selected probability distributions are used to generate random numbers for actual and estimated maximum daily rainfall for each period of study. The best fit probability distribution was identified based on minimum absolute deviation between actual and estimated values. Based on this fitting distribution, rainfall magnitudes for different return periods were calculated. The log-Pearson 3 and Burr (4P) were found as the best fit probability model for the annual and first inter monsoon season period of study, respectively. Generalized extreme value distribution was observed in remaining period of monsoon seasons. Further, the distribution reveals that the 216 mm or more of annual maximum daily rainfall return period is ten years. Similarly, the relevant estimates of return levels are listed against the return periods for extreme rainfall events during the four seasons of a year.

1. Introduction

In the advent of global warming, there are increased concerns regarding extreme weather events. As elsewhere across the globe, South Asian countries have been observing an increase in occurrence of extreme climate events in recent decades. Many researchers have found evidences of increasing extreme weather events such as heat waves, cold waves, floods, droughts and severe cyclones over the past few decades. Extreme rainfall events can have severe impacts on people's life. An investigation of extreme rainfalls by the scientific community in any country serves several purposes such as: the estimation of extreme rainfalls for design purposes; the assessment of the rarity of observed rainfalls; and, comparison of methods to estimate design rainfalls. This study also focuses the last purpose and a detailed regionalized study is practically useful for the planners and other users.

Moreover, information of spatial and temporal variability of extreme rainfall events is very useful for the design and construction of certain projects, such as dams and urban drainage systems, the management of water resources, and the prevention of flood damage as they require an adequate knowledge of extreme events of high return periods. In most cases, the return periods of interest exceed usually the periods of available records and could not be extracted directly from the recorded data. Therefore, in current engineering practice, the estimation of extreme rainfalls or flood peak discharges is accomplished based on statistical frequency analysis of maximum precipitation or maximum stream flow records where available sample data could be used to calculate the parameters of a selected frequency distribution. The fitted distribution is then used to estimate event magnitudes corresponding to return periods greater than or less than those of the recorded events do. Accurate estimation of extreme rainfall could help alleviating the damage caused by storms and floods and it can help achieving more efficient design of hydraulic structures. Several probability models have been developed to describe the distribution of annual extreme rainfalls at a single site. However, the choice of a suitable model is still one of the major problems in engineering practice since there is no general agreement as to which distribution, or distributions, that should be used for the frequency analysis of extreme rainfalls. The selection of an appropriate model depends mainly on these evaluations that yield very different conclusions than that of previous researches on this subject. Applications of probability distributions to rainfall data have

been investigated by several researchers from different regions of the world. Hirose (1994) have found that the Weibull distribution is the best fit for the annual maximum of daily rainfall in Japan. Nadarajah and Withers (2001) and Nadarajah (2005) provided the application of extreme value distributions to rainfall data over sixteen locations spread throughout New Zealand and fourteen locations in West Central Florida, respectively. Further, Nadarajah and Choi (2007) have studied annual maxima of daily rainfall for the years 1961–2001 for five locations in South Korea, and the generalized extreme value distribution is fitted to data from each location to describe the extremes of rainfall and to predict its future behaviour. They suggested that the Gumbel distribution provides the most reasonable model for four of the five locations considered. Chu et al. (2009) have applied the generalized extreme value distribution for extreme rainfall events in Hawaii Islands using three different methods based on the mean annual number of days on which 24-h accumulation exceeds a given daily rainfall amount, the value associated with a specific daily rainfall percentile, and the annual maximum daily rainfall values associated with a specific return period. For estimating the statistics of return periods, the three-parameter generalized extreme value distribution is fitted using the method of L-moments, and the spatial patterns of heavy and very heavy rainfall events across the islands are mapped separately based on the above three methods for annual maximum of daily rainfall data. Hanson and Vogel (2008) have studied the probability distribution of daily rainfall in the United States to represent the precipitation depth at various durations which has long been a focus of interest in hydrology. Sharma and Singh (2010) analyzed the daily maximum rainfall data of Pantnagar, India for a period of 37 years for annually, seasonally, monthly and weekly, and the best fitted probability distribution is identified using the least square method among the 16 compared distributions. Deka and Borah (2009) have derived the best fitted distribution to describe the annual series of maximum rainfall data for the period 1966 to 2007 of nine distantly located stations in north east India, and they considered only five extreme value distributions. In Sri Lanka, Baheerathan and Shaw (1978) have analysed data from 8 to 24 years in different stations by fitting Gumbel distribution. Dharmasena and Premasiri (1990) have used 25 years of data related to five regions in Sri Lanka, and fitted Gumbel distributions. Varathan et al. (2010) have used 110 years data in Colombo district to analyze the annual maximums of rainfall, and found that the Gumbel distribution is the best fitting distribution. Therefore, a regionalized study on the probability modeling of extreme rainfall is very much essential as the probability model may vary according to the

geographical locations of the area considered. In this study an attempt has been made to study the annual maximum daily rainfall data in Colombo, Sri Lanka, and the findings of the same along with the methodology adopted are presented. Further the best fitted probability distribution model is determined by considering more generalized forty five probability models. When considering the problem of selecting a probability distribution model to describe a maximum rainfall data, there are number of goodness-of-fit tests. Rather than selecting one of these tests, the present paper looks at how triple tests can be combined to make the final selection.

The paper is organized as follows. In section 2 description of the data is given. The methodology for fitting forty five probability models, and identifying best fitted procedures are described in section 3. Finally, the results of the best fitted probability distribution models and their implications are discussed in section 4.

2. Study Area

Sri Lanka is an island situated in the Indian Ocean, north of equator and off the lying farthest towards south tips of India. It lies between latitudes 5° 55` and 9° 50` N and longitudes 79° 42 and 81° 53 E. The surface area of the island is 65,635 sq.km and its greatest length from north south is 430 km. Sri Lanka consists of a central sloping on all sides from the Piduruthalagala peak 2528 meters to the sea. Based on the altitude the island is divided to Low, mid and Up country. The Low country is demarcated as land below 300 meters elevation, the Mid country as land between 300 meters and 1000 meters elevation, and Upcountry 1000 meters and above. The climate of Sri Lanka is strongly affected by the topographical features such as ridges, peaks, plateaus, basins, valleys and escarpments of the country, and it is classified as tropical monsoon, a wet and dry climate but with only a brief dry season, according to Koppen's classification of climates (Boucher, 1975).The climate experienced during 12 months period in Sri Lanka can be characterized in to 4 climatic seasons, First Inter monsoon Season (March- April), Southwest monsoon season (May – September), Second Inter monsoon season (October – November), Northeast Monsoon season (December – February). The data consists of daily rainfall for the years from 1900 to 2009 for the Colombo meteorological station. Fortunately, there are no missing values during the period. The data obtained from the Department of Meteorology, Colombo, Sri Lanka which lists the daily rainfalls in millimeters. The extreme values selected from the tabulated daily data. The heavy rainfall and the associated floods and landslides affect many areas. In particular, Colombo, the capital of the country faces serious flooding

problems in low-lying areas due to extreme rainfalls. Due to these extreme rainfall events, water levels will rise, and in turn will affect the coastal economy and more land will be covered with flood. The quality of the water is also be degraded due to these reasons.

3. Methodology

The present study is based on time series data related to maximum daily rainfall annually, and seasonally. The randomness of the data set were checked by using the autocorrelation plots. The general advantage of the regional approach analysis is that more data are available for the probability modeling, and parameter estimates become more reliable and spatially more coherent. In the frequency analysis problem, standard statistical techniques are used to model the extreme rainfall events from daily rainfall data. Then several standard probability distributions are used to identify the best fitted model.

Annual one-day extreme rainfall is usually defined as the maximum daily rainfall within each year, and one would have as many extreme values as the total number of years. The annual rainfall of this area is over 2000 *mm*, which is subjected to large variation. The data were then processed to identify the maximum rainfall received on any one day (24 hours duration) in a monsoon season and in a year (365 days period).

If $X_1, X_2, \ldots, X_{365}$ are daily rainfall values, then the data selection point (extreme point) value is Max$\{X_1, X_2, \ldots, X_{365}\}$; where X_i is the daily rainfall in *mm* of any particular year, for $i = 1, 2, 3 \ldots 365$. The best fitted probability distribution was evaluated by using the following systematic procedures.

3.1 Checking data homogeneity based on autocorrelation function (ACF)

Autocorrelation plots (Box and Jenkins, pp. 28-32) are a commonly used tool for checking randomness in a data set. This randomness is ascertained by computing autocorrelations for data values at varying time lags. If random, such autocorrelations should be near zero for all time-lag separations. If non-random, then one or more of the autocorrelations will be significantly non-zero. Autocorrelation plots are formed by vertical axis autocorrelation coefficient(R_h) versus horizontal axis time lag h ($h = 1, 2, \ldots$), where

$$R_h = \frac{C_h}{C_0}, -1 \le R_h \le +1 \tag{1}$$

with the auto-covariance function C_h

$$C_h = \frac{1}{n} \sum_{t=1}^{n-h} (Y_t - \bar{Y})(Y_{t+h} - \bar{Y}), \tag{2}$$

and the variance function C_0

$$C_0 = \frac{1}{n} \sum_{t=1}^{n} (Y_t - \bar{Y})^2. \tag{3}$$

If the autocorrelation plot is being used to test for randomness (i.e., there is no time dependence in the data), the following formula is recommended:

$$\pm Z_{\alpha/2}/\sqrt{n}, \tag{4}$$

where n is the sample size, Z is the percent point function of the standard normal distribution and α is the significance level. In this case, the confidence bands have fixed width that depend on the sample size. This formula is used to generate the confidence bands in the ACF plot.

3.2 Fitting the probability distributions

The generalized extreme value, gamma, Gumbel max, inverse Gaussian, log normal, normal, Pearson, Weibull were used probability models for evaluating the best fitted probability distribution for rainfall. In addition, these different forms of distributions and various other standard statistical distributions were also employed, and the following 45 probability distributions were applied: beta, Burr (3P, 4P), Cauchy, chi-square (1P, 2P), Dagum (4P), Erlang (2P, 3P), error, error function, exponential (1P, 2P), fatigue life (3P), Frechet(2P), gamma (2P, 3P), generalized gamma (3P, 4P), generalized extreme value, Gumbel max, hyperbolic secant, inverse Gaussian (2P, 3P), Kumaraswamy, Laplace, Levy, log–gamma, logistic, log-logistic (2P, 3P), lognormal (2P, 3P), log-Pearson 3 (3P), Nakagami, normal, Pareto(first kind), Pareto(second kind), Pearson type 5 (2P, 3P), Pearson type 6 (3P, 4P), Student's t, Weibull (2P, 3P). The description of various probability distribution models viz. density function, range and the parameter involved are exhibited in Appendix 1.

3.3 Testing the goodness of fit and identifying the best fitted probability distribution

The goodness-of-fit test measures the compatibility of random sample with the theoretical probability distribution function. The Anderson-Darling test (Stephens, 1974) is used to test if a sample of data came from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov (K-S)

test and gives more weight to the tails than does the K-S test. The K-S test is distribution free in the sense that the critical values do not depend on the specific distribution being tested. The Anderson-Darling test makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution. The Anderson-Darling test is an alternative to the chi-square and Kolmogorov-Smirnov goodness-of-fit tests.

The goodness of fit tests are used for testing the following null hypothesis:
H_0: The maximum daily rainfall data follow the specified distribution
H_a: The maximum daily rainfall data does not follow the specified distribution.

Kolmogorov-Smirnov and Anderson-Darling tests are used along with the chi-square test at $\alpha(0.01)$ level of significance to select the best fitted probability distribution model. Consequently the ranking of different probability distributions were marked from 1 to 45 based on minimum test statistic value. The probability distribution having the first rank was selected for all the three tests independently. Thus the procedure for obtaining the best fitted probability distribution model on the basis of all the three identified probability distributions is explained in the next section.

3.3.1. Generating random numbers

First the parameters of the three probability distributions were used to generate the random numbers. A commonly used technique is called the *inverse transform* technique. Let U be a uniform random number in the range $(0, 1)$. If $X = F^{-1}(U)$, then X is a random variable with CDF, $F_X(X) = F$, and

$$F_X(a) = P(X \leq a) = P(F^{-1}(U) \leq a) = P(U \leq F(a)) = F(a), \qquad (5)$$

where $a \in \mathbb{R}$.

This can be used as a random number generator to generate numbers according to the uniform distribution, and this method can be used as a technique of generating the any random variable with a known probability distribution.

3.3.2. Minimum absolute deviation method

The generated random numbers for the selected distributions are treated as estimated values for actual rainfall values. These estimated values are then compared with actual rainfall values to compute the absolute deviations (*AD*) for each period:

$$AD = \left| \sum_{i=1}^{n} (Y_i - \hat{Y}_i) \right|,\qquad(6)$$

where, Y_i is the actual rainfall value,

$\hat{Y}_i$ is the estimated value $(i = 1, 2, \ldots, n)$.

Among the selected three distributions, the best fitted probability distribution was then identified based on the minimum absolute deviation between actual and estimated values.

3.4 Return period

Return period (T): Once the best probability model for the data has been determined, the interest is in deriving the return levels of rainfall. The T year return level, say x_T, is the level exceeded on average only once in T years. For example, the 2-year return level is the median of the distribution of the annual maximum daily rainfall.

Probability of occurrence (p) is expressed as the probability that an event of the specified magnitude will be equaled or exceeded during a one year period. If n is the total number of values and m is the rank of a value in a list ordered descending magnitude ($x_1 > x_2 > x_3 \ldots > x_m$), the exceeding probability of the mth largest value, x_m, is

$$P(X \geq x_m) = \frac{m}{n}.\qquad(7)$$

(See Ramachandra Rao and Hamed, page 6-7). A given return level x_T with a return period T may be exceeded once in T years. Therefore,

$$P(X \geq x_T) = \frac{1}{T}.\qquad(8)$$

If the probability model with CDF, F is assumed then on inverting

$$F(x_T) = P(X \leq x_T) = 1 - P(X \geq x_T) = 1 - \frac{1}{T},\qquad(9)$$

and get the general expression

$$\inf\left\{ x_T : F(x_T) \geq 1 - \frac{1}{T} \right\} = F^{-1}\left(1 - \frac{1}{T}\right)\qquad(10)$$

4. Results and Discussion

The methodology presented above was applied to the 110 years observational data in which lists the maximum rainfall in millimeters (*mm*) were taken from department of Meteorology, Colombo. Accordingly the data was classified into five data sets as mentioned the study period in Table 1. These five data sets were classified as 1 annual and 4 seasons to study the probability distribution pattern at different levels.

The ACF (for lags 1 to 110) for the annual maximums of daily rainfall data are within the 95% confidence limits (±0.186879, ±0.187734, ±0.186879, ±0.186879, ±0.186879). This indicates that all the five different data sets have no autocorrelation between time lags. The method of Maximum likelihood was used throughout the study periods. The test statistics for Kolmogorov-Smirnov test (D), Anderson-Darling test (A^2), and Chi-Squared test (χ^2) for each data

set were computed for 45 probability distributions. The probability distribution having the first rank along with their test statistic is presented in Table 1. It has been observed that log-Pearson 3 (3P) using Kolmogorov Smirnov test, Pearson 5 (3P) using Anderson Darling test and log-logistic (3P) using Chi-square test obtained the first rank for maximum daily annual rainfall. Thus the three probability distributions are identified as the best fit based on these three tests independently.

Table 1: First ranked probability distribution using goodness of fit tests

| Study Period | Test ranking first position | | | | | |
| | Kolmogorov-Smirnov | | Anderson-Darling | | Chi-square | |
	Distribution	Statistic	Distribution	Statistic	Distribution	Statistic
Annual	Log-Pearson 3(3P)	0.03593	Pearson 5	0.11208	Log-logistic (3P)	0.23504
North-East Monsoon	Gen. Gamma	0.04256	Gen. Extreme Value	0.25687	Log- logistic (2P)	1.2523
First-Inter Monsoon	Burr(4P)	0.05042	Burr(4P)	0.34521	Burr(3P)	5.735
South-West Monsoon	Gen. Extreme Value	0.03222	Gen. Extreme Value	0.27369	Log-logistic (3P)	1.5995
Second-Inter Monsoon	Gen. Extreme Value	0.03514	Gen. Extreme Value	0.24044	Inv. Gaussian (2P)	0.8835

The identified distributions are listed in Table 2 with the estimated parameters

for each data set. It is also observed that some of the probability distributions have the first rank in both Kolmogorov Smirnov and Anderson Darling tests.

Table 2: Parameters of the best fitted distributions

Study Period	Distribution	Parameters (MLE)
Annual	Log-Pearson 3 (3P)	α=22.459 β=0.07851 γ=3.1235
	Pearson 5	α=7.8044 β=968.61
	Log-Logistic (3P)	α=3.414 β=93.893 γ=35.394
North-East Monsoon	Gen. Gamma	k=0.98388 α=5.5839 β=11.135
	Gen. Extreme Value	k=-0.09106 σ=23.526 μ=52.481
	Log logistic (2P)	α=3.6632 β=57.79
First-Inter Monsoon	Burr(4P)	k=0.28344 α=32.325 β=284.33 γ=-240.51
	Burr(4P)	k =0.28344 α =32.325 β =284.33 γ =-240.51
	Burr(3P)	k=1.0149 α=3.5263 β=66.476
South-West Monsoon	Gen. Extreme Value	k=0.17184 σ=34.924 μ=81.845
	Gen. Extreme Value	k=0.17184 σ=34.924 μ=81.845
	Log-logistic (3P)	α= 4.6493 β= 118.81 γ= -21.45
Second-Inter Monsoon	Gen. Extreme Value	k=0.07231 σ=35.929 μ=85.264
	Gen. Extreme Value	k=0.07231 σ =35.929 μ=85.264
	Inv. Gaussian (2P)	λ=580.74 μ=109.76

The estimated parameters were used to generate random numbers for each data set and the least square method was used for the rainfall analysis. The random numbers were generated for actual and estimated observations for all the 110 years, and the residuals were computed for each data set. Sum of these deviation were obtained for all identified distributions. The probability distribution having minimum deviation was treated as the best selected probability distribution for the individual data set. The best selected probability distribution for each data set is presented in Table 3.

Table 3: Best fitted probability distribution

Study Period	Best fit distribution
Annual	Log-Pearson 3 (3P)
North-East Monsoon	Gen. Extreme Value
First-Inter Monsoon	Burr(4P)
South-West Monsoon	Gen. Extreme Value
Second-Inter Monsoon	Gen. Extreme Value

The above probability distributions were used to estimate the return levels x_T for each data set based on the return periods-T (or probability of occurrence) such as 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 150 and 200 years. The computed return levels for each data set are listed in Table 4. It has been predicted that the 2 year return period's return level is approximately 129 *mm*, which means rainfall of 129 *mm* or more, should occur at that location on the average only once every two year. In other words, the average 216*mm* or more daily extreme rainfall event occur for the period of every ten-year with the occurrence probability 0.1000. Among the monsoon seasons considered, the south-west monsoon appears to be associated with the highest return levels. As notice that, the north-east monsoon has the lowest return levels.

Table 4: Study period wise estimated return levels based on fixed return periods

Probability of Occurrence	Return Period (*T* in years)	Return Levelx_TEstimates (in *mm*) for Study Period				
		Annual	North-East Monsoon	First-Inter Monsoon	South-West Monsoon	Second-Inter Monsoon
0.5000	2	129.1089	60.9613	65.3059	95.0568	98.6085
0.2000	5	179.4439	85.4648	98.3850	141.5993	142.1865
0.1000	10	216.4185	100.3510	125.0535	177.7965	173.0677
0.0500	20	254.6869	113.7055	153.7858	217.1907	204.3066
0.0333	30	278.0240	121.0005	171.6271	242.1616	223.0282
0.0250	40	295.0924	125.9809	184.7731	260.8660	236.5685
0.0200	50	308.6500	129.7412	195.2579	275.9822	247.2334
0.0167	60	319.9457	132.7505	204.0163	288.7483	256.0586
0.0143	70	329.6556	135.2521	211.5585	299.8451	263.6014
0.0125	80	338.1887	137.3884	218.1953	309.6894	270.1971
0.0111	90	345.8119	139.2498	224.1302	318.5563	276.0637
0.0100	100	352.7094	140.8968	229.5041	326.6373	281.3509
0.0067	150	379.9690	147.0819	250.7713	359.1054	302.0553
0.0050	200	400.0298	151.3282	266.4417	383.5285	317.1008

5. Conclusion

This paper has demonstrated a probability modeling of maximum daily rainfall in Colombo, Sri Lanka using more variety of distributions. The results of rainfall study for identifying best fitted probability distributions revealed that the best probability distributions for the maximum daily rainfall for different data set are

different. The log-Pearson 3 (3P) and Burr (4P) distribution were found as the best fitted probability distribution model for the annual and first-inter monsoon season period of study, respectively. Generalized extreme value was observed in balancing three monsoons season period as best fitted probability distribution model. The scientific results clearly established that the analytical procedure devised and tested in this study may be suitably applied for the identification of the best fitted probability distribution of weather parameters.

Average values of the maximum daily rainfall amounts corresponding to return periods of 2- to 200-yrs, are derived together with their uncertainties.Despite the encouraging results of our analysis, the estimates of the extreme return values may have a limited validity. It is shown that the regional approach leads to superior results, and a similar framework may be useful in fitting probability model for maximum rainfall in other parts of the region.

References

1. Baheerathan, V.R. and Shaw, E.M. (1978). Rainfall Depth-Duration-Frequency Studies for Sri Lanka. Journal of Hydrology 37, 223-239, DOI: 10.1016/0022-1694(78)90018-5

2. Boucher, K., 1975: Global Climate. Halsted Press, New York, 326 pp.

3. Box, G. E. P., and Jenkins, G. (1976), Time Series Analysis: Forecasting and Control, Holden-Day.

4. Chu, P. S, Zhao. X, Ruan. Y and Grubbs. M, (2009). "Extreme Rainfall Events in the Hawaiian Islands" journal of applied meteorology and climatology volume 48, American Meteorological Society.

5. Dharmasena, G.T. and Premasiri, S.M, (1990). Rainfall Intensity Studies for Sri Lanka. Engineer, December.

6. Deka .S and Borah .M, (2009). Distribution of Annual Maximum Rainfall Series of North East India, European Water Publications 27/28: 3-14.

7. Hanson, L. and Vogel, R. (2008).The Probability Distribution of Daily Rainfall in the United States. World Environmental and Water Resources Congress 2008: pp. 1-10.

8. Hirose, H. (1994). Parameter Estimation in the Extreme-Value distributions using the Continuation Method, Information Processing Society of Japan Vol 35, 9.

9. Nadarajah. S and Choi. D, (2007). Maximum daily rainfall in South Korea J. Earth Syst. Sci. 116, No. 4, pp. 311–320,DOI:10.1007/s12040-007-0028-0

10. Nadarajah S., and Withers, C. S. (2001). Evidence of trend in return levels for daily rainfall in New Zealand, Journal of Hydrology New Zealand, Volume 39, p.155-166.

11. Nadarajah, S. (2005), The exponentiatedGumbel distribution with climate application, Environmetrics, 17(1), 13-23, DOI:10.1002/env.739

12. Pao-Shin Chu, Xin Zhao, Ying Ruan, and Melodie Grubbs, (2009): Extreme Rainfall Events in the Hawaiian Islands. *J. Appl. Meteor. Climatol.*, 48(3), 502–516, DOI:10.1175/2008JAMC1829.1

13. RamachandraRao. A andHamed. K. H, (2000). Flood Frequency Analysis, CRC Press, Boca Raton, Florida, USA.

14. Stephens, M. A, (1974). EDF Statistics for Goodness of Fit and SomeComparisons, Journal of the American Statistical Association, 69, pp. 730-737, DOI:10.1080/01621459.1974.10480196

15. Sharma, M.A. and Singh, J.B. (2010). Use of Probability Distribution in Rainfall Analysis, New York Science Journal, 40-49.

16. Snedecor, George W. and Cochran, William G. (1989), Statistical Methods, Eighth Edition, Iowa State University Press.

17. Tao.D.Q and Nguyen.V.T.V, (2002).On Selection of Probability Distributions For Representing Extreme Precipitations in Southern Quebec, Annual Conference of the Canadian Society for Civil Engineering , June 5-8.

18. Varathan, N, Perera, K and Nalin, (2010). Statistical modeling of extreme daily rainfall in Colombo, M.Sc thesis, Board of Study in Statistics and Computer Science of the postgraduate institute of science, university of Peradeniya, Sri Lanka.

Appendix 1

Table 5: Description of various probability distribution functions.

No	Distribution	Probability density function	Range	Parameters
01	Beta	$f(x) = \dfrac{1}{B(\alpha_1, \alpha_2)} \dfrac{(x-a)^{\alpha_1-1}(b-x)^{\alpha_2-1}}{(b-a)^{\alpha_1+\alpha_2-1}}$ where B is the beta function	$a \leq x \leq b$	α_1 continuous shape parameter $(\alpha_1 > 0)$ α_2 continuous shape parameter $(\alpha_2 > 0)$ a, b continuous boundary parameters $(a < b)$
02	Burr (4P)	$f(x) = \dfrac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}$	$\gamma \leq x < +\propto$	k continuous shape parameter α continuous shape parameter β continuous scale parameter γ continuous location parameter $(\gamma = 0$ yields the three-parameter Burr distribution)
03	Burr (3P)	$f(x) = \dfrac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x}{\beta}\right)^{\alpha}\right)^{k+1}}$		
04	Cauchy	$f(x) = \left(\pi\sigma(1+\left(\frac{x-\mu}{\sigma}\right)^2)\right)^{-1}$	$-\propto < x < +\propto$	σ continuous scale parameter $(\sigma > 0)$ μ continuous location parameter
05	Chi-Square (2P)	$f(x) = \dfrac{(x-\gamma)^{\frac{v}{2}-1}\exp(\frac{-(x-\gamma)}{2})}{(2)^{\frac{v}{2}}\Gamma(\frac{v}{2})}$		v degrees of freedom (positive integer) γ continuous location parameter
06	Chi-Square (1P)	$f(x) = \dfrac{(x-\gamma)^{\frac{v}{2}-1}\exp(\frac{-(x)}{2})}{(2)^{\frac{v}{2}}\Gamma(\frac{v}{2})}$	$\gamma \leq x < +\propto$	$(\gamma = 0$ yields the one-parameter Chi-Squared distribution)

07	Dagum (4P)	$$f(x) = \dfrac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha k - 1}}{\beta \left(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}$$	$\gamma \leq x < +\propto$	k continuous shape parameter α continuous shape parameter β continuous scale parameter γ continuous location parameter ($\gamma = 0$ yields the three-parameter Dagum distribution)
08	Erlang (3P)	$$f(x) = \dfrac{(x - \gamma)^{m-1}}{\beta^m \Gamma(m)} exp\left(-\left(^{x - \gamma}/_{\beta}\right)\right)$$	$\gamma \leq x < +\propto$	m shape parameter (positive integer) β continuous scale parameter ($\beta > 0$) γ continuous location parameter ($\gamma = 0$ yields the two-parameter Erlang distribution)
09	Erlang (2P)	$$f(x) = \dfrac{(x)^{m-1}}{\beta^m \Gamma(m)} exp\left(-\left(^{x}/_{\beta}\right)\right)$$		
10	Error	$$f(x) = C_1 \sigma^{-1} exp(-|C_0 Z|^k)$$ where $C_0 = \left(\dfrac{\Gamma(^3/_k)}{\Gamma(^1/_k)}\right)^{0.5}$ $C_1 = \dfrac{k C_0}{2\Gamma(^1/_k)}$,	$-\propto < x < +\propto$	k continuous shape parameter σ continuous scale parameter($\sigma > 0$) μ continuous location parameter
11	Error function	$$f(x) = \dfrac{h}{\sqrt{\pi}} exp(-(hx)^2)$$	$-\propto < x < +\propto$	h continuous inverse scale parameter($h > 0$)
12	Exponential (2P)	$$f(x) = \lambda exp(-\lambda(x - \gamma))$$	$\gamma \leq x < +\propto$	λ continuous inverse scale parameter γ continuous location parameter ($\gamma = 0$ yields the one-parameter Exponential distribution)
13	Exponential (1P)	$$f(x) = \lambda exp(-\lambda x)$$		

14	Fatigue Life (3P) (Birnbaum-Saunders)	$$f(x) = \frac{\sqrt{\frac{x-\gamma}{\beta}} + \sqrt{\frac{\beta}{x-\gamma}}}{2\alpha(x-\gamma)} \times$$ $$\varphi\left(\frac{1}{\alpha}\left(\sqrt{\frac{x-\gamma}{\beta}} - \sqrt{\frac{\beta}{x-\gamma}}\right)\right)$$ where φ is the PDF of the standard normal distribution.	$\gamma \le x < +\infty$	α continuous shape parameter ($\alpha > 0$) β continuous scale parameter ($\beta > 0$) γ continuous location parameter ($\gamma = 0$ yields the two-parameter Fatigue Life distribution)
15	Frechet (2P) (Maximum extreme value type 2)	$$f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1} \exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right)$$	$0 \le x < +\infty$	α continuous shape parameter ($\alpha > 0$) β continuous scale parameter ($\beta > 0$)
16	Gamma (3P)	$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} exp\left(-\left(^{x-\gamma}/_{\beta}\right)\right)$$	$\gamma \le x < +\infty$	α continuous shape parameter ($\alpha > 0$), β continuous scale parameter ($\beta > 0$) γ continuous location parameter ($\gamma = 0$ yields the two-parameter Gamma distribution)
17	Gamma (2P)	$$f(x) = \frac{(x)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} exp\left(-\left(^{x}/_{\beta}\right)\right)$$		
18	Generalized Gamma (4P)	$$f(x) = \frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha}\Gamma(\alpha)} exp\left(-\left(\left(\frac{x-\gamma}{\beta}\right)^{k}\right)\right)$$	$\gamma \le x < +\infty$	k continuous shape parameter α continuous shape parameter β continuous scale parameter γ continuous location parameter ($\gamma = 0$ yields the three-parameter Generalized Gamma distribution)
19	Generalized Gamma (3P)	$$f(x) = \frac{k(x)^{k\alpha-1}}{\beta^{k\alpha}\Gamma(\alpha)} exp\left(-\left(\left(\frac{x}{\beta}\right)^{k}\right)\right)$$		

20	Generalized Extreme Value	$f(x) = \frac{1}{\sigma} exp\big(-(1+kz)^{-1/k}\big)(1+kz)^{-(k+1)/k}$ if $k \neq 0$	where $z = \frac{x-\mu}{\sigma}$. $1 + k\left(\frac{x-\mu}{\sigma}\right) > 0$ for $k \neq 0$, $-\infty < x < +\infty$	k continuous shape parameter σ continuous scale parameter ($\sigma > 0$) μ continuous location parameter
21	Gumbel Max (Maximum extreme value type 1)	$f(x) = \frac{1}{\sigma} exp(-Z - exp(-Z))$ where $Z = \frac{x-\mu}{\sigma}$	$-\infty < x < +\infty$	σ continuous scale parameter($\sigma > 0$), μ continuous location parameter
22	Hyperbolic Secant	$f(x) = \frac{Sech\left(\frac{\pi(x-\mu)}{\sigma}\right)}{2\sigma}$	$-\infty < x < +\infty$	σ continuous scale parameter($\sigma > 0$)μ continuous location parameter
23	inverse Gaussian (3P)	$f(x) = \sqrt{\frac{\lambda}{2\pi(x-\gamma)^3}}\, exp\left(\frac{-\lambda(x-\gamma-\mu)^2}{2\mu^2(x-\gamma)}\right)$	$\gamma \leq x < +\infty$	λ continuous parameter ($\lambda > 0$) μ continuous parameter ($\mu > 0$) γ continuous location parameter ($\gamma = 0$ yields the two-parameter Inverse Gaussian distribution)
24	inverse Gaussian (2P)	$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}}\, exp\left(\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right)$		
25	Kumaraswamy	$f(x) = \frac{\alpha_1\alpha_2 Z^{(\alpha_1-1)}(1-Z^{\alpha_1})^{(\alpha_2-1)}}{(b-a)}$	$a \leq x \leq b$	α_1 continuous shape parameter ($\alpha_1 > 0$) α_2 continuous shape parameter ($\alpha_2 > 0$) a, b continuous boundary parameters ($a < b$)
26	Laplace (Double Exponential)	$f(x) = \frac{\lambda}{2} exp(-\lambda|x-\mu|)$	$-\infty < x < +\infty$	λ continuous inverse scale parameter $\lambda > 0$ μ continuous location parameter

27	Levy	$f(x) = \sqrt{\dfrac{\sigma}{2\pi}}\dfrac{\exp\left(\frac{-0.5\sigma}{x}\right)}{(x)^{3/2}}$	$0 < x < +\infty$	σ continuous scale parameter$(\sigma > 0)$
28	Log-Gamma	$f(x) = \dfrac{(\ln(x))^{\alpha-1}}{x\beta^{\alpha}\Gamma(\alpha)}exp\left(-\ln(x)/\beta\right)$	$0 < x < +\infty$	α continuous parameter $(\alpha > 0)$ β continuous parameter $(\beta > 0)$
29	Logistic	$f(x) = \dfrac{\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)}{\sigma\left(1 + \exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)^{2}}$	$-\infty < x < +\infty$	σ continuous scale parameter$(\sigma > 0)$ μ continuous location parameter
30	Log-Logistic (3P)	$f(x) = \dfrac{\alpha}{\beta}\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha-1}\left(1 + \left(\dfrac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-2}$	$\gamma \le x < +\infty$	α continuous shape parameter $(\alpha > 0)$ β continuous scale parameter $(\beta > 0)$ γ continuous location parameter $(\gamma = 0$ yields the two-parameter Log-Logistic distribution)
31	Log-Logistic (2P)	$f(x) = \dfrac{\alpha}{\beta}\left(\dfrac{x}{\beta}\right)^{\alpha-1}\left(1 + \left(\dfrac{x}{\beta}\right)^{\alpha}\right)^{-2}$		
32	Lognormal (3P)	$f(x) = \dfrac{exp\left(-\frac{1}{2}\left(\frac{\ln(x-\gamma)-\mu}{\sigma}\right)^{2}\right)}{(x-\gamma)\sigma\sqrt{2\pi}}$	$\gamma \le x < +\infty$	σ continuous parameter$(\sigma > 0)$ μ continuous parameter γ continuous location parameter $(\gamma = 0$ yields the two-parameter Lognormal distribution)
33	Lognormal (2P)	$f(x) = \dfrac{exp\left(-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^{2}\right)}{(x)\sigma\sqrt{2\pi}}$		
34	Log-Pearson 3 (3P)	$f(x) = \dfrac{1}{x\lvert\beta\rvert\Gamma(\alpha)}\left(\dfrac{\ln(x)-\gamma}{\beta}\right)^{\alpha-1}$ $* \exp\left(-\dfrac{\ln(x)-\gamma}{\beta}\right)$	$0 < x \le e^{\gamma}$ for $\beta < 0$ and $e^{\gamma} \le x < +\infty$ for $\beta > 0$	α continuous parameter $(\alpha > 0)$ β continuous parameter $(\beta \ne 0)$ γ continuous parameter

35	Nakagami	$f(x) = \dfrac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} exp\left(-\dfrac{m}{\Omega}x^2\right)$	$0 < x < +\infty$	m continuous parameter $m \geq 0.5$, Ωcontinuous parameter $(\Omega > 0)$
36	Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} exp(-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2)$	$-\infty < x < +\infty$	σ continuous scale parameter$(\sigma > 0)$ μ continuous location parameter
37	Pareto (first kind)	$f(x) = \dfrac{\alpha\beta^\alpha}{x^{\alpha+1}}$	$\beta < x < +\infty$	α continuous parameter $(\alpha > 0)$ β continuous parameter $(\beta > 0)$
38	Pareto (second kind)	$f(x) = \dfrac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}$	$0 < x < +\infty$	α continuous parameter $(\alpha > 0)$ β continuous parameter $(\beta > 0)$
39	Pearson type 5 (3P)	$f(x) = \dfrac{exp(-\beta/(x-\gamma))}{\beta\Gamma(\alpha)((x-\gamma)/\beta)^{\alpha+1}}$	$\gamma < x < +\infty$	α continuous shape parameter $(\alpha > 0)$ β continuous scale parameter $(\beta > 0)$ γ continuous location parameter $(\gamma = 0$ yields the two-parameter Pearson 5 distribution)
40	Pearson type 5 (2P)	$f(x) = \dfrac{exp(-\beta/x)}{\beta\Gamma(\alpha)(x/\beta)^{\alpha+1}}$		
41	Pearson type 6 (4P)	$f(x) = \dfrac{((x-\gamma)/\beta)^{\alpha_1-1}}{\beta B(\alpha_1,\alpha_2)(1+(x-\gamma)/\beta)^{\alpha_1+\alpha_2}}$	$\gamma \leq x < +\infty$	α_1 continuous shape parameter $(\alpha_1 > 0)$ α_2 continuous shape parameter $(\alpha_2 > 0)$ βcontinuous scale parameter $(\beta > 0)$ γcontinuous location parameter $(\gamma = 0$ yields the two-parameter Pearson 5 distribution)
42	Pearson type 6 (3P)	$f(x) = \dfrac{(x/\beta)^{\alpha_1-1}}{\beta B(\alpha_1,\alpha_2)(1+x/\beta)^{\alpha_1+\alpha_2}}$		

43	Student's t	$f(x) = \dfrac{1}{\sqrt{\pi v}} \dfrac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \left(\dfrac{v}{v+x^2}\right)^{\left(\frac{v+1}{2}\right)}$	$-\infty < x < +\infty$	$v$degrees of freedom (positive integer)
44	Weibull (3P)	$f(x) = \dfrac{\alpha}{\beta}\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha-1} exp\left(-\left(\dfrac{x-\gamma}{\beta}\right)^{\alpha}\right)$	$\gamma \le x < +\infty$	αcontinuous shape parameter ($\alpha > 0$) βcontinuous scale parameter ($\beta > 0$) γcontinuous location parameter ($\gamma \equiv 0$ yields the two-parameter Weibull distribution)
45	Weibull (2P)	$f(x) = \dfrac{\alpha}{\beta}\left(\dfrac{x}{\beta}\right)^{\alpha-1} exp\left(-\left(\dfrac{x}{\beta}\right)^{\alpha}\right)$		

Appendix 2

Table 6: Estimates of the parameters for each distribution -using Maximum Likelihood method.

No	Distribution	Parameters	Estimated Value for Study Period				
			Annual	North-East Monsoon	First –Inter Monsoon	South-West Monsoon	Second-Inter Monsoon
01	Beta	α_1	1.577	1.7204	1.7511	2.2646	4.2085
		α_2	6.1855	2.2627	5.3776	8.1868	422.01
		a	46.35	9.68	5.68	$8.2523 * 10^{-11}$	-4.4411
		b	493.7	132.5	284.6	493.7	11812.0
02	Burr (4P)	k	1.4388	10.187	0.2834	8338.9	3396.5
		α	2.7971	2.4006	32.325	1.6575	1.8298
		β	103.89	165.96	284.33	27769.0	10052.0
		γ	42.421	7.0969	-240.51	$-3.9041 * 10^{-7}$	$-1.7292 * 10^{-7}$
03	Burr (3P)	k	0.6581	7.8196	1.0149	17.765	1182.6
		α	5.6733	2.7775	3.5263	1.6564	1.8169

		β	115.7	146.24	66.476	651.12	5768.2
04	Cauchy	σ	28.124	15.735	18.53	25.477	24.335
		μ	125.3	59.057	64.126	91.845	94.488
05	Chi-Square (2P)	ν	1663	340	769	99	101
		γ	-1521.1	-276.15	-694.73	$7.6417 * 10^{-11}$	$7.930 * 10^{-10}$
06	Chi-Square (1P)	ν	142	64	75	109	108
07	Dagum (4P)	k	243.89	218.19	291.53	2.0363	0.37039
		α	15.271	7.0093	4.7989	0.34556	5.2018
		β	345.61	73.502	18.658	0.92158	131.72
		γ	-498.84	-163.92	-34.183	$1.000 * 10^{-10}$	$3.6500 * 10^{-5}$
08	Erlang (3P)	m	3	9	4	5	8
		β	31.597	8.9502	19.146	22.719	16.639
		γ	45.143	-16.261	-0.1379	-13.972	-27.9
09	Erlang (2P)	m	5	5	3	3	4
		β	25.882	11.135	23.323	34.19	22.584
10	Error	k	1.0	2.3714	1.0	1.0	1.3634
		σ	60.726	26.717	41.898	61.071	49.56
		μ	142.48	64.103	75.265	109.09	108.76
11	Error function	h	0.0116	0.0265	0.0169	0.01158	0.01427
12	Exponential (2P)	λ	0.0109	0.0192	0.0147	0.00917	0.00919
		γ	51.5	12.1	7.1	$9.999 * 10^{-11}$	$9.9999 * 10^{-10}$
13	Exponential (1P)	λ	0.0070	0.0156	0.0133	0.00917	0.00919
14	Fatigue Life (3P)	α	0.4698	0.2282	0.4072	28378.0	26046.0
		β	106.79	113.96	85.919	$1.5264 * 10^{-7}$	$1.6111 * 10^{-7}$
		γ	23.913	-52.827	-17.765	$1.000 * 10^{-10}$	$1.0000 * 10^{-9}$
15	Frechet (2P) (Maximum extreme value type 2)	α	3.3922	2.4397	2.2223	0.14387	0.16087
		β	110.96	45.918	50.51	1.5312	2.4113
16	Gamma (3P)	α	3.0805	8.979	3.9383	5.4166	8.2132

		β	31.597	8.9502	19.146	22.719	16.639
		γ	45.143	-16.261	-0.1379	-13.973	-27.9
17	Gamma (2P)	α	5.5048	5.7569	3.227	3.1906	4.8157
		β	25.882	11.135	23.323	34.19	22.584
18	Generalized Gamma (4P)	k	0.5372	2.1924	0.5677	0.8687	1.3071
		α	11.957	1.0594	13.588	7.5743	2.7871
		β	0.9512	62.301	0.7665	11.98	51.505
		γ	39.019	7.0816	-4.6883	-17.964	-0.2665
19	Generalized Gamma (3P)	k	1.0575	0.9839	1.0528	0.82657	0.75181
		α	6.1156	5.5839	3.471	2.4346	3.067
		β	25.882	11.135	23.323	34.19	22.584
20	Generalized Extreme Value	k	0.1269	-0.0911	0.1676	0.17184	0.07231
		σ	38.694	23.526	25.304	34.924	35.929
		μ	114.63	52.481	55.68	81.845	85.264
21	Gumbel Max (Maximum extreme value type 1)	σ	47.348	20.831	32.668	47.617	38.642
		μ	115.15	52.079	56.408	81.6	86.453
22	Hyperbolic Secant	σ	60.726	26.717	41.898	61.071	49.56
		μ	142.48	64.103	75.265	109.09	108.76
23	Inverse Gaussian (3P)	λ	530.79	2284.1	549.45	504.38	580.74
		μ	119.92	118.04	93.556	110.09	580.74
		γ	22.562	-53.934	-18.291	$8.9074 * 10^{-11}$	$9.1109 * 10^{-10}$
24	Inverse Gaussian (2P)	λ	784.31	369.03	242.88	348.04	523.74
		μ	142.48	64.103	75.265	109.09	108.76
25	Kumaraswamy	α_1	1.6387	1.0	1.7769	1.6313	0.99998
		α_2	3111.1	1.2	916.07	6121.6	1.2
		a	50.117	12.1	5.7799	$-1.7374 * 10^{-8}$	$1.0000 * 10^{-9}$
		b	14058	132.5	3657.0	24663.0	270.1
26	Laplace (Double	λ	0.0233	0.0529	0.0337	0.02316	0.02854

	Exponential)	μ	142.48	64.103	75.265	109.09	108.76
27	Levy	σ	124.11	51.649	56.024	90.363	92.309
28	Log-Gamma	α	172.51	76.787	62.538	No fit	No fit
		β	0.0283	0.0529	0.0669		
29	Logistic	σ	33.48	14.73	23.099	33.67	27.324
		μ	142.48	64.103	75.265	109.09	108.76
30	Log-Logistic (3P)	α	3.414	6.3343	3.9491	4.6493	5.2461
		β	93.893	94.614	72.985	118.81	130.97
		γ	35.394	-33.896	-6.6198	-21.45	-31.169
31	Log-Logistic (2P)	α	4.8603	3.6632	3.2736	0.26318	0.29312
		β	130.91	57.79	65.018	75.654	78.902
32	Lognormal (3P)	σ	0.4726	0.2243	0.4164	0.35907	0.28413
		μ	4.6325	4.7472	4.4015	4.8943	5.0723
		γ	27.313	-54.089	-13.974	-33.91	-57.453
33	Lognormal (2P)	σ	0.3704	0.4617	0.5272	2.6582	2.44
		μ	4.8866	4.0644	4.1881	4.3432	4.3794
34	Log-Pearson 3 (3P)	α	22.459	7.3684	15.219	0.03961	0.03979
		β	0.0785	-0.1709	-0.1357	-13.418	-12.288
		γ	3.1235	5.3235	6.2541	4.8747	4.8683
35	Nakagami	m	0.7883	1.5556	0.5156	No fit	1.0745
		Ω	23954	4816.4	7404.2		14262.0
36	Normal	σ	60.726	26.717	41.898	61.071	49.56
		μ	142.48	64.103	75.265	109.09	108.76
37	Pareto (first kind)	α	1.0582	0.6364	0.4488	0.03654	0.03984
		β	51.5	12.1	7.1	$1.0000 * 10^{-10}$	$1.0000 * 10^{-9}$
38	Pareto (second kind)	α	113.81	109.17	84.464	83.01	201.72
		β	16968	5651.5	6182.5	9237.4	19969.0
39	Pearson type 5 (3P)	α	7.8532	36.713	9.5689	5.6877	6.0756

		β	978.24	5652.2	930.67	513.96	560.84
		γ	-0.4039	-94.154	-33.605	$9.4621 * 10^{-11}$	$9.2305 * 10^{-10}$
40	Pearson type 5 (2P)	α	7.8044	4.3281	3.2378	5.6877	6.0756
		β	968.61	223.54	181.4	513.96	560.84
41	Pearson type 6 (4P)	α_1	11.278	305.45	56.655	0.91225	3.6571
		α_2	8.5333	36.845	9.6282	0.2401	$4.3086 * 10^{+9}$
		β	77.246	17.603	15.466	1.9312	$1.419 * 10^{+11}$
		γ	26.332	-85.889	-26.608	$1.0001 * 10^{-10}$	-6.2079
42	Pearson type 6 (3P)	α_1	81.756	5.8576	5.8732	1.5751	1.7652
		α_2	8.5887	$7.3473 * 10^{+7}$	12.253	$3.2850 * 10^{+8}$	229.32
		β	13.203	$7.9743 * 10^{+8}$	143.98	$2.2772 * 10^{+10}$	14073.0
43	Student's t	υ	2	2	2	2	2
44	Weibull (3P)	α	1.6393	2.2497	1.7757	0.31977	0.25362
		β	103.52	63.887	78.307	$2.5157 * 10^{-5}$	$2.1831 * 10^{-5}$
		γ	50.158	7.5097	5.7621	$9.0667 * 10^{-11}$	$9.20 * 10^{-10}$
45	Weibull (2P)	α	3.3485	2.6347	2.3011	0.21209	0.23585
		β	154.8	71.503	82.972	1066.1	851.73