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Short Communication

A model for thundercloud charge separation

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Abstract

A theoretical model of thundercloud charge separation is presented. The model assume that moisture in the updraft moving as a stream with uniform velocity condenses into particles of graupel in reaching cooler regions of the atmosphere. Falling graupels acquire a negative charge by shedding the inductive positive charge which is carried away by the updraft. Charged graupels moves downward subjected to gravity, force due to the collective electric field and air resistance. Non-linear equations governing the dynamics of the system demonstrate existence of a stable equilibrium electric field of the order of magnitude needed for dielectric breakdown of air. Model also shows that thundercloud electric field could undergo pulsations at infrasonic frequencies and synchronously emit acoustic waves at the same frequency. Preferential positive inductive charging of graupel is explained as a consequence of the proton conductivity of ice.

Keywords: thundercloud electrification, thundercloud charge separation, lightning infrasonic pulses, graupel, ice proton conductivity

1. INTRODUCTION

The fascinating phenomenon of lightning and thunder has intrigued man from time immemorial[1-2]. Mythological connections and speculations on origin of lightning and its consequences are quite common in all cultures. The fear for thunder bolt inflicted damage to life and property and the experience of electric shock seems to be the basis of most of these speculations. Relating lightning to familiar frictional electrostatics was the

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first step in scientific understanding of lightning. With the development of electromagnetic theory, explanations were sought within that framework incorporating other physical processes in the atmosphere[3-7]. Although literature is rich in models of thundercloud charge separation (TCS), and a number of quite convincing ideas put forward, many problems of TCS remain unresolved. Theories of TCS fall into two main categories, depending on whether frictional or inductive mode of charging is assumed[8-16]. Involvement of friction as well as induction seems to be a more plausible scenario. Although a seed electric field needs to be present to initiate electrification, inductive charging could lead to exponential build-up of separated charges, reaching saturation in a relatively short time. Rapid electrification enables charge accumulation against inevitable slow recombination. Furthermore, intense electric field needed for initiation of the lightning discharge is unlikely to originate entirely from a frictional process. The rate of collisions of ice particles necessary for rapid frictional electrification decreases in the final stages of TCS.

The convectional updraft of moist air and condensation of water vapor in the cooler regions of the atmosphere leads to formation of charged clouds. Most models of TCS invoke condensed phases of water hail and graupel in the proposed charging mechanism. Charge separation is a non-equilibrium process governed by electromagnetic and gravitational forces. Models of TCS should demonstrate that the non-equilibrium process finally produce a system of separated positive negative charges remaining stable until the initiation of the lightning discharge. In this note a non-linear dynamical model of TCS that demonstrate formation of stable separated charges is presented.

2. MODEL

The updraft of moist air is assumed to move as a stream of constant velocity V relative to the surface of earth. In reaching cooler regions of the atmosphere moisture condenses forming pellets of hail/graupel moving downwards with a velocity v relative to the stream (air in the updraft). For the purpose of the model pellets of graupel are assumed to be spheres of radius *R*. The charge distribution $\sigma(\theta)$ on the surface of a conducting sphere placed in a uniform electric field *E* is given by[17] (Fig.1),

$$\sigma(\theta) = 3\varepsilon_0 E \cos\theta . \qquad (1)$$

For a dielectric sphere, the expression (1) is replaced by[17]

$$\sigma(\theta) = 3\varepsilon_0 E \left[\frac{\varepsilon - 1}{\varepsilon + 1}\right] \cos\theta , \qquad (2)$$

where ε is the dielectric constant and when $\varepsilon >> 1$, Eq. (2) approximates to Eq. (1). Ice is a proton conductor of high dielectric constant and charge distribution on a sphere ice in an electric field is quite well described by Eq. (1). Integrating the Eq. (1) we obtain the charge Q on one hemisphere as:

$$Q = 3\varepsilon_0 \pi R^2 E \,. \tag{3}$$

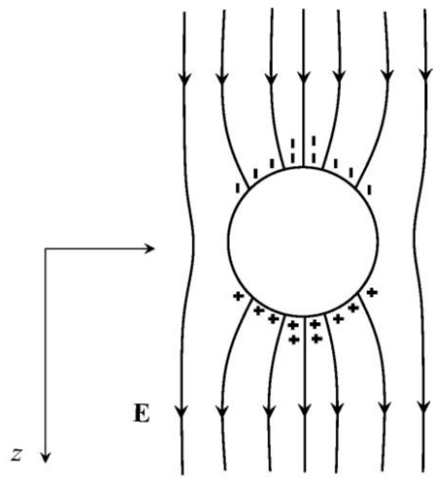

Figure 1: Charge distribution on a conducting sphere in a uniform electric field.

As charge separation in the cloud progresses, E increases and we assume that the induced positive charge over the bottom region of the pellet is removed rapidly by streaming discharge and moved upwards along with the updraft. Smaller particles of ice or clusters of water molecules that have high affinity for protons, probably act as carriers of positive charges. Positive ion chemistry in the atmosphere initiated mainly by $H_3O^+(H_2O)_n$ species formed when water molecules accept protons[18-19] .There is also experimental evidence that indicate sublimating ice acquires a negative charge[5]. The inductive positive charge on ice would naturally enhance this process. Forces acting on the ice pellet are weight Mg (downwards) , force due to the electric field QE (upwards) and the Stoke viscous drag $6\pi\eta R v$ (η is□ viscosity of air and v is graupel velocity relative to the updraft moving) upwards (see Fig. 2(b)) .The equation of motion can be written as ,

$$M\frac{dv}{dt} = Mg - QE - 6\pi\eta R v \,. \tag{4}$$

Using Eq. (3) and setting $M = \dfrac{4\pi R^3 \rho}{3}$ in Eq. (4), we obtain,

$$\frac{dv}{dt} = g - \left(\frac{9\varepsilon_0}{4\rho R}\right)E^2 - \left(\frac{9\eta}{2\rho R^2}\right)v, \tag{5}$$

where, ρ is the density of graupel.

Charge carried by falling graupel per unit time per unit area is NQv, where N is the number of graupel particles per unit volume. Assuming that the field E generated is similar to a parallel plate capacitor we obtain,

$$\frac{dE}{dt} = \frac{NQv}{\varepsilon_0} . \qquad (6)$$

Using Eqs. (3), Eq. (6) can be expressed in the form,

$$\frac{dE}{dt} = (3\pi R^2 N)Ev . \qquad (7)$$

The nonlinear Eqs. (5) and (6) has an equilibrium at

$$v = 0 \text{ and } E = E_0 = \frac{2}{3}\left(\frac{\rho g R}{\varepsilon_0}\right)^{1/2} . \qquad (8)$$

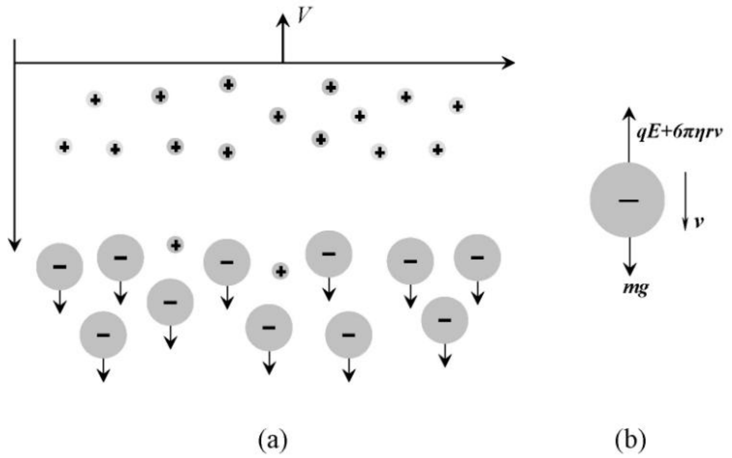

Figure 2: (a) Schematic diagram illustrating motion of graupel with a velocity v relative to the updraft moving with a velocity V relative to the earth (b) forces acting on a particle of graupel.

The saturation electric field E_o depends of the radius of graupel pellets. Setting $\rho = 500$ kg m^{-3} and R = 80 μm, typical values for graupel density and diameter, we obtain $E_0 = 1.4\times10^5$ V m^{-1}. An electric field of same order of magnitude as the strength needed for ionization of air and initiation of the lightning discharge. Setting $v = dz/dt$ (z is the distance of charge separation at time t) in Eq. (5) and Eq. (7), the distance of charge separation at the equilibrium (see Fig. 2(a) can be determined and related to the parameters of the theory. Integrating Eq. (7) we obtain,

$$E_0 = E_s \text{ Exp } [kz_0], \qquad (9)$$

where $k = 3\pi R^2 N$ and E_s is the seed electric field that existed at $t = 0, z = 0$ and z_o is the equilibrium charge separation distance . The z_o for a thundercloud is ~ 10 km and E_s= 100 V m^{-1} (fair weather electric field) and using Eq. (9), we get N ~ 10^4 m^{-3} , when R = 80 μm. Thus the value we obtain for the number density of graupels, corresponds to a water content of ~ 2 kg m^{-3} in the thundercloud, the same order of magnitude as the observed value.

To determine the behavior of the system near the equilibrium we put $v = 0 + \delta v, E = E_0 + \delta E$ in Eq. (5) and Eq. (6), we obtain the linearized equations

$$\frac{d(\delta v)}{dt} = -3\left(\frac{g\varepsilon_0}{\rho R}\right)^{1/2}\delta E - \left(\frac{9\eta}{2\rho R^2}\right)\delta v, \tag{10}$$

$$\frac{d(\delta E)}{dt} = 2\pi N\left(\frac{g\rho R^5}{\varepsilon_0}\right)^{1/2}\delta v, \tag{11}$$

Eqs. (10) and (11) have a solution

$$E = A\,\mathrm{Exp}[-\lambda t]\sin(\omega t)\,, \tag{12}$$

where $\lambda = \dfrac{9\eta}{4\rho R^2},\ \omega = \dfrac{1}{2}\left[24\pi R^2 Ng - \left(\dfrac{9\eta}{2\rho R^2}\right)^2\right]^{1/2}$ and $\left(\dfrac{4\pi R^3\rho}{3}\right)^2 > \dfrac{3\pi\eta^2}{2gN}$. (13)

3. RESULTS AND DISCUSSION

Inequality in Eq. (13) is satisfied in all physically meaningful situations. Setting N=10^4 m^{-3} and viscosity of air, $\eta = 1.9\times10^{-5}$ kg m^{-1} s^{-1} at 0^0 C[19], R = 80 μm, we obtain $\lambda = 0.086\,\mathrm{s}^{-1}$ and $\omega = 13.6\,\mathrm{s}^{-1}$ ($\nu = 2.2\,\mathrm{s}^{-1}$). The negative sign in the exponent of Eq. (12) indicates that equilibrium is stable and any perturbations are damped with a time constant $\lambda^{-1} = 11.7$ s. Result also indicates that the displaced charges undergo damped oscillations with a frequency in the infrasonic region. As expected and seen from the expression for λ in Eq. (13), damping of oscillations is caused by viscosity of air. Thunderstorm conditions in the atmosphere are known to generate infrasonic electromagnetic and acoustic pulses[20]. It is easy to understand how the electric field oscillation in Eq. (12) will lead to acoustic waves. In the present model, thundercloud is like a parallel plate capacitor. Imagine that the distance between the plates is increased by dz, when the field energy per unit cross-section changes by amount $(\varepsilon_0 E^2/2)dz$. If the pressure inside and outside the capacitor are P_i and P_0, the external work done is $(P_0 - P_i)\,dz$. Equating the two expressions we obtain,

$$P_i = P_0 - \frac{\varepsilon_0 E^2}{2}\,. \tag{14}$$

Thus variations of *E*, will be accompanied by pressure changes. Infrasonic pulsations associated with lightning have been explained on basis of expansion of the hot channel and collapse of the electrostatic field within the thundercloud[20-21]. In the latter model, when the electric field relieves by discharge, pulsations initiate with a rarefaction. According to the present model, the separated charges possess an intrinsic oscillatory attribute associated with its stability, and oscillation could be entrained by sudden changes in the electric field, either rapid build-up of the field during charging or decay of the field when the charge is relieved.

A question that needs to answer in the context of the present model is mechanism of rapid removal of induced positive charge on the pellets of hail. Pellets of ice placed in a strong electric field develop positive corona streams. The preference for generation positive instead of negative coronas could be attributed to proton conductivity of ice. The proton conductivity of ice[22] $\sigma_p \sim 10^{-6}$ S m^{-1} and the electronic conductivity σ_e is at least six orders of magnitude smaller. On application of an electric field, the charge carriers in a conducting material relaxes with a characteristic time constant ε / σ. Thus respective relaxation times for positive and negative charges in ice are of the order 10^{-4} s and 10^{2} s respectively. As it is easier to draw positive charges into surface of ice than negative charges, ice develops positive coronas in preference to negative. The result is that particles of ice suspended in a strong electric shed its positive induced component of the charge acquiring a negative charge. There is experimental evidence to the effect that ice preferentially develops positive coronas[23].

4. CONCLUSIONS

The model presented shows that inductive negative charging of falling hail and transport of the positive charge with the updraft could lead to a stable configuration of separated charges. The model is dependent on two input variable parameters, the average radius *R* of the graupels and their number density *N*. Stable is intended to mean growth of the electric field to saturation sufficient for lightning discharge. Saturation electric field predicted by the model is of the same order of magnitude as the observed optimum electric field in thundercloud and near the ionization breakdown potential gradient for air. The model also accounts for association of infrasonic acoustic and pulsation with thunderstorm activity. A limitation of the model is the assumption that the separated charges create a field as in a parallel plate capacitor of infinite extent. Consequently, the model cannot determine the size of the cloud or any constraints to the size. Again model in its present form cannot account for tripolar or multipolar structure of thunderclouds.

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