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Research Article

Analytical study of ion-acoustic soliton in single temperature non-isothermal electron

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Abstract

In presence of warm negative ion, propagation of ion-acoustic solitary waves has been investigated analytically in a plasma consisting of warm positive ion, warm positron and single-temperature non-isothermal electron. The necessary and sufficient conditions for the existence of compressive solitary waves are discussed critically along with the calculation of phase velocity, kinetic energy (K.E) and force (F) found out in this paper by this new analytical method. Some important general observations and Theorems related with this problem are highlighted that supports the results of previous authors. The Sagdeev potential function $\psi(\phi)$ [= - $L(u_\alpha)$] against ϕ depicted graphically under the variation of different plasma parameters is the most important and interesting situations by this analytical method.

Key Words– Solitary waves, non-isothermal single temperature electron, stream velocity, kinetic energy, force, Sagdeev pseudopotential, electrostatic potential.

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1. INTRODUCTION

In a collision less, unmagnetized with non-relativistic or relativistic plasma, solitary waves in isothermal plasmas have been investigated theoretically and experimentally by many Physicists[1-12]. In case of non-Maxwellian electron distribution, the present author[13-14] also discussed compressive solitary waves in a plasma having warm positive ion and warm negative ion. Many scientists in plasma took different types of electron distribution function for studying various kinds of solitary waves pointing out some restrictions on electrostatic potential and phase velocity. Basically, the isothermality of plasma is introduced through the electron density assuming the electron phase velocity is much smaller than the electron thermal velocity for which the electron density is expressed by the Boltzmann relation. On the other hand, trapped particles when evolved in the plasma interacted strongly with solitary waves. Again when the velocities of trapped particles for ion-acoustic wave are closed to the ion-acoustic speed, the number of trapped ions is generally ignored due to its negligible account in the dynamical system. Besides this, the shift in the electron distribution is very small, the number of trapped electrons is very large and therefore the effect of trapped electrons compared to free electrons plays an important role under the consideration of ion-acoustic dynamics for non-Maxwellian electron distribution. Schamel[15-16] first considered the non-isothermal electron plasma highlighting this free and trapped electrons and took a new form for that electron distribution function in order to study the small amplitude localised solitary waves based on the reductive perturbation technique through the derivation of the K-dV equation. This non-Maxwellian electron distribution function contains high energy electrons component. Such distributions are very common in auroral zone of ionosphere. In non-isothermal electron with cold ions Nejoh[17], Das and Sen[18-19] derived the unidirectional non-linear wave equation used to highlight the features of solitary waves for their observations in relation to laboratory and space plasmas. Kalita and Bujarbarua[20] worked on higher order contribution to ion-acoustic solitary waves by KdV equation. In presence of negative ions along with positive ion and non-isothermal electrons, Tagare and Reddy[21] also studied the effect of higher order non-linearity on ion-acoustic solitary waves. Majumder et al[22] took two-types of cold positive ions and two-temperature non-isothermal electrons mentioning third order contributions to ion-acoustic solitary waves by Bogoliubov Mitropolsky method based on reductive perturbation technique. Chattopadhyay et al[23] have investigated the ion-acoustic solitary waves in a cold positive and negative ion plasma with

non-isothermal electrons by the well-known Sagdeev potential method. In presence of warm negative and positive ions, the present author[24-25] recently studied the ion-acoustic compressive solitary waves and double layers in two-temperature non-isothermal electron along with warm positron by Sagdeev pseudopotential method. On the other hand, Ghosh et al[26] used a new analytical method for studying the propagation of ion-acoustic solitary waves in isothermal electron with relativistic positive ion and obtained some necessary and sufficient conditions for the existence of solitary waves in plasma.
The present work considers the modified plasma model of Ghosh et al[26] with warm positive ion, warm negative ion, warm positron and single temperature non-isothermal electron to investigate the solitary waves by a new analytical way. Consequently positron is included in this plasma model due to the significant change of amplitude and width of solitary waves. In addition to this, the present author also studied analytically the necessary conditions for the existence of positive potential solitary waves (compressive) in this non-isothermal single temperature electron plasma that Ghosh et al[26] did not show this in their own paper.

The plan of the paper is arranged in the following manner:

In section 2, the basic set of normalised equations with their proper boundary conditions are given. A new function L(u_α) known as kinetic energy, Sagdeev potential function $\{-L(u_\alpha)\}$ and a definite force F denoted by G(u_α) are found from the system of equations by this new analytical method. Analytical study along with some theorems containing necessary and sufficient conditions is comprehensively discussed in sec.3. The condition for the existence of solitary wave solution and some important observations are discussed in this section for real and bounded solution of the energy equation that supports the results of Ghosh et al[26] . Section 4 gives the comprehensive discussion of the present problem with graphical representation of the Sagdeev potential function $\psi(\phi)$ [= - L(u_α)] against ϕ under the variation of the different concerned plasma parameters that are shown by the figures 1 to 8. Finally concluding remarks are given in section 5.

2. FORMULATION

Considering an unmagnetized, collisionless and non-relativistic plasma consisting of warm positron, warm negative ion and warm positive ion with streaming motion in presence of single-temperature non-isothermal electron, the basic set of fluid equations in normalized form for such a plasma in unidirectional propagation are given in the following way:

Continuity equation: $\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0$ (1)

Momentum equation: $\frac{\partial u_\alpha}{\partial t}+ u_\alpha \frac{\partial u_\alpha}{\partial x}+\frac{\sigma_\alpha}{Q_\alpha n_\alpha}\frac{\partial p_\alpha}{\partial x} = - \frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x}$ (2)

Pressure equation: $\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_\alpha}{\partial x} = 0$ (3)

Poissons's equation: $\frac{\partial^2 \phi}{\partial x^2} = n_e - \sum Z_\alpha n_\alpha - n_p$ (4)

Where $n_p = \chi\, e^{-\sigma_p \phi}$ (5)

Here $\sigma_p=\frac{T_e}{T_p}$, $\sigma_\alpha = \frac{T_\alpha}{T_e}$, $Q_\alpha = \frac{m_\alpha}{m_i}$[$T_e$ = temperature of electron, T_p = temperature of positron,T_α = temperatureof positive (negative) ion,m_α = mass of positive (negative) ion, α = i for positive ion and α = j for negative ion]

In this paper n_α ,u_α , Q_α , σ_α , p_α , ϕ , n_e , n_p , σ_p , χ ,t and x are represented by the number density, velocity , mass ratio of negative to positive ion, temperature of ions, pressure, electrostatic potential, concentration of non-isothermal electrons, density of positron, temperature ratio of electron and positron, concentration of positron at ϕ = 0, time and distance of the respective ions . Here Z_α = 1, Q_α = 1for positive ion (α = i) and Z_α = - Z, Q_α = Q for negative ion (α = j).

For solitary wave solution, we assume the two dependent variables x and t on a single independent variable η defined by η = x – Vt where V is the velocity of the solitary waves. We also use the following boundary conditions:

$u_\alpha \to u_{\alpha 0}$,$n_\alpha \to n_{\alpha 0}$,$p_\alpha \to 1$, $n_e \to 1$, $n_p \to \chi$ and $\phi \to 0$ as $|x| \to \infty$ (6)

The charge neutrality condition of the plasma is

$$\chi + \sum_{\alpha= \alpha_o} Z_\alpha n_\alpha = 1 \quad (7)$$

From equations (1) to (3) after using the transformation, (6) and (7) we get,

$$n_\alpha = \frac{Q_\alpha^{\frac{1}{2}} n_{\alpha 0}^{\frac{3}{2}}}{2\sqrt{3\sigma_\alpha}}\left[\sqrt{\left(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} - \sqrt{\left(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}}\right] \quad (8)$$

$$p_\alpha = \left(\frac{V-u_{\alpha 0}}{V-u_\alpha}\right)^3 \quad (9)$$

$$\phi(u_\alpha) = \frac{Q_\alpha}{2Z_\alpha}\left[(V - u_{\alpha 0})^2 - (V - u_\alpha)^2 - \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}\left\{\left(\frac{V-u_{\alpha 0}}{V-u_\alpha}\right)^2 - 1\right\}\right] \quad (10)$$

For real n_α , the following restriction on ϕ is

$$\phi < \frac{Q_\alpha}{2Z_\alpha}\left[V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right]^2 \quad (11)$$

Now by usual conventional way we get from equation (4),

$$\frac{d^2\phi(u_\alpha)}{d\eta^2} = \mathrm{G}\,(u_\alpha) \tag{12}$$

Where $$\mathrm{G}\,(u_\alpha) = n_e - \sum Z_\alpha \frac{Q_\alpha^{\frac{1}{2}} n_{\alpha 0}^{\frac{3}{2}}}{2\sqrt{3\sigma_\alpha}} \left[\sqrt{\left(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} - \sqrt{\left(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha\phi}{Q_\alpha}} \right] - \chi\, e^{\sigma_p \phi} \tag{13}$$

Integrating equation (12) and simplifying we get finally

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 = \frac{1}{2}\left(\frac{d\phi}{du_\alpha}\right)^2 \left(\frac{du_\alpha}{d\eta}\right)^2 = \mathrm{L}(u_\alpha) \tag{14}$$

Where $$\mathrm{L}(u_\alpha) = \int \mathrm{G}\,(u_\alpha) \frac{d\phi(u_\alpha)}{du_\alpha}\, du_\alpha + \mathrm{K} \tag{15}$$

Equation (14) is known as energy equation and K is an arbitrary integration constant to be determined from initial condition.

We are now discussing the non-isothermal single-temperature electron plasmas for studying the behaviour of the particle under solitary wave motion.

2.1 NON- ISOTHERMAL SINGLE TEMPERATURE ELECTRON PLASMA

In this case the concentration of non-isothermal electron (n_e) is given by

$$n_e = 1 + \phi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 \tag{16}$$

Where $b_1 = \frac{1-\beta_1}{\sqrt{\pi}}$, $b_2 = \frac{1-\beta_1^2}{\sqrt{\pi}}$, $\beta_1 = \frac{T_{ef}}{T_{et}}$

T_{ef} is the constant temperature of free electrons and T_{et} is that of trapped electrons. The term $\frac{4}{3} b_1 \phi^{\frac{3}{2}}$ in (16) introduces the contribution of the resonant electrons (in both free and trapped case) to the electron density.

As usual way we have finally obtained from (13) for non-isothermal single temperature electron plasma

$$\mathrm{G}(u_\alpha) = 1 + \phi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3$$

$$- \sum Z_\alpha \frac{Q_\alpha^{\frac{1}{2}} n_{\alpha 0}^{\frac{3}{2}}}{2\sqrt{3\sigma_\alpha}} \left[\sqrt{\left(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} - \sqrt{\left(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}} \right] - \chi e^{\sigma_p \phi} \tag{17}$$

Thus by using (17), (15) and (10) we get

$$\mathrm{L}(u_\alpha) = \phi + \frac{1}{2}\phi^2 - \frac{8}{15} b_1 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 - \frac{16}{105} b_2 \phi^{\frac{7}{2}} + \frac{1}{24}\phi^4$$

$$+ \frac{1}{6}\sum \frac{Q_\alpha^{\frac{3}{2}} n_{\alpha 0}^{\frac{3}{2}}}{\sqrt{3\sigma_\alpha}} \left[\left\{\left(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}\right\}^{\frac{3}{2}} - \left\{\left(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^2 - \frac{2Z_\alpha \phi}{Q_\alpha}\right\}^{\frac{3}{2}} - \left(V - u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^3 + \left(V - u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\right)^3 \right]$$

$$+ \frac{\chi}{\sigma_p}\left(e^{-\sigma_p \phi} - 1\right) \tag{18}$$

In absence of negative ion, we get from (18)

$$\mathrm{L}(u_i) = \phi + \frac{1}{2}\phi^2 - \frac{8}{15} b_1 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 - \frac{16}{105} b_2 \phi^{\frac{7}{2}} + \frac{1}{24}\phi^4$$

$$+ \frac{1}{6}\sqrt{\frac{n_{i0}^3}{3\sigma_i}} \left[\left\{\left(V - u_{i0} + \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left\{\left(V - u_{i0} - \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left(V - u_{i0} + \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^3 + \left(V - u_{i0} - \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^3 \right]$$

$$+ \frac{\chi}{\sigma_p}\left(e^{-\sigma_p \phi} - 1\right) \tag{19}$$

We now observe from (14) that the term $\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2$ can be regarded as the kinetic energy (K.E) of a particle of unit mass at position ϕ and time η in (ϕ,η) plane.

In this case, K.E = $\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 = \int \mathrm{G}\,(u_\alpha)\frac{d\phi(u_\alpha)}{du_\alpha}\,\mathrm{d}u_\alpha + \mathrm{K} = \mathrm{L}(u_\alpha)$

$$= \phi + \frac{1}{2}\phi^2 - \frac{8}{15} b_1 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 - \frac{16}{105} b_2 \phi^{\frac{7}{2}} + \frac{1}{24}\phi^4$$

$$+\frac{1}{6}\sum\frac{Q_\alpha^{\frac{3}{2}}n_{\alpha0}^{\frac{3}{2}}}{\sqrt{3\sigma_\alpha}}\left[\begin{array}{c}\left\{\left(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}-\left\{\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\right\}^{\frac{3}{2}}\\ -\left(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^3\\ +\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^3\end{array}\right]$$

$$+\frac{\chi}{\sigma_p}\left(e^{-\sigma_p\phi}-1\right) \tag{20}$$

The Sagdeev potential [$\psi(\phi)$] in single temperature non-isothermal electron plasma is $\psi(\phi)$ = - [L(u_α)]

The Force (F) acting on the particle of unit mass at position ϕ and time η with velocity $\frac{d\phi}{d\eta}$ is

$$F = 1 + \phi - \frac{4}{3}b_1\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15}b_2\phi^{\frac{5}{2}} + \frac{1}{6}\phi^3$$

$$-\sum Z_\alpha\frac{Q_\alpha^{\frac{1}{2}}n_{\alpha0}^{\frac{3}{2}}}{2\sqrt{3\sigma_\alpha}}\left[\begin{array}{c}\sqrt{\left(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}}\\ -\sqrt{\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}}\end{array}\right]-\chi\, e^{\sigma_p\phi} \tag{21}$$

Very often the kinetic energy (K.E) and force identify the nature of the motion of the particle and for this reason the kinetic energy (K.E) and force regarding the motion of the particle are very important.

The kinetic energy (K.E) and force in equation (20) and (21) may be always real, finite and non-real (imaginary) if and only if the following conditions are satisfied:

For real and finite K.E and force

$$\left(V-u_{\alpha0}+\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\geq 0$$

And $\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2-\frac{2Z_\alpha\phi}{Q_\alpha}\geq 0$

Kinetic energy (K.E) and force will be non-real if

$$V < u_{\alpha0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}} \pm \sqrt{\frac{2Z_\alpha\phi}{Q_\alpha}}$$

$$V < u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}} \pm \sqrt{\frac{2Z_\alpha \phi}{Q_\alpha}}$$

But we are mainly interested with real, positive and finite values of the kinetic energy (K.E) and force.

From the above discussion we get finally for real and finite kinetic energy (K.E) and force

$$V \geq u_{\alpha 0} - \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}} \pm \sqrt{\frac{2Z_\alpha \phi}{Q_\alpha}}$$

$$V \geq u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}} \pm \sqrt{\frac{2Z_\alpha \phi}{Q_\alpha}}$$

2.2 CONDITIONS FOR THE EXISTENCE OF SOLITARY WAVE SOLUTION

From energy integral $\frac{d^2\phi}{d\eta^2} = G(u_\alpha)$, one may obtain

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 = L(u_\alpha)$$

Where $L(u_\alpha) = \int G(u_\alpha) \frac{d\phi(u_\alpha)}{du_\alpha} du_\alpha + K$, K is an arbitrary integration constant to be determined from initial condition.

The force acting on the particle of unit mass at position ϕ is $G(u_\alpha)$. Now for $\left\{-L(u_\alpha)|_{\phi=0}\right\} = \left\{-\frac{\partial L}{\partial u_\alpha}\Big|_{\phi=0}\right\} = 0$, we can say that the particle is in equilibrium at ϕ = 0 because the velocity $\frac{d\phi}{d\eta}$ as well as the force $\frac{d^2\phi}{d\eta^2}$ acting on the particle at ϕ = 0 are simultaneously equal to zero. Again if ϕ = 0 can be made an unstable position of equilibrium, the energy integral can be interpreted as the motion of an oscillatory particle if $\left[-L(u_\alpha)|_{\phi=\phi_m}\right] = 0$ for some ϕ say $\phi = \phi_m \neq 0$ where the velocity is equal to zero.

Thus the conditions for the existence of positive potential (compressive) solitary wave solution of the energy integral (14) are

(i) $\left\{-L(u_\alpha)|_{\phi=0}\right\} = \left\{-\frac{\partial L}{\partial u_\alpha}\Big|_{\phi=0}\right\} = 0$, $\left\{-\frac{\partial^2 L}{\partial u_\alpha^2}\right\}\Big|_{\phi=0} < 0$ at $u_\alpha = u_{\alpha o}$ where $\phi(u_{\alpha o}) = 0$.

(ii) $\left[-L(u_\alpha)|_{\phi=\phi_m}\right] = 0$ for some ϕ say $\phi = \phi_m > 0$ at $u_\alpha = u_{\alpha_1}$ where $\phi(u_{\alpha_1}) = \phi_m$ and min.$\{0, \phi_m\} < \phi <$ max.$\{0, \phi_m\}$ which satisfies inequality (11).

(iii)$\{-L(u_\alpha)\}|_\phi < 0$ for $0 < \phi < \phi_m$ at $u_\alpha = u_{\alpha_1}$ where $\phi(u_{\alpha_1}) = \phi_m$ and $\min.\{0, \phi_m\} < \phi < \max.\{0, \phi_m\}$ which satisfies inequality (11).

3. ANALYTICAL STUDY

In this paper we are now studying analytically the behaviour of the energy equation (14) together with the related important functions L(u_α) , $\phi(u_\alpha)$ and G(u_α) for single temperature non-isothermal electron plasma.

Necessary and sufficient condition

From energy equation (14), we may obtain the physically admissible solution for the ion-acoustic solitary waves in a warm negative ion, warm positive ion and warm positron with non-isothermal single temperature electron plasma. In order to study the physically admissible solution of equation (14) in non-isothermal plasma we may come to the following conclusions:

(a) $\phi(u_\alpha)$ must be positive .

(b) $\left(\frac{d\phi}{du_\alpha}\right)^2 \left(\frac{du_\alpha}{d\eta}\right)^2$ must be positive.

(c) u_α and $\frac{du_\alpha}{d\eta}$ must be bounded.

It is also found that G$(u_\alpha) \to \pm\infty$ when $u_\alpha \to$ V and this follows from $(u_\alpha)_{max} <$ V or $(u_\alpha)_{min} >$ V.

Again it is easy to establish the following observations from the expressions of the functions $\phi(u_\alpha)$, L(u_α) and G(u_α).

Observation 1. $\phi(u_\alpha) > 0$ gives finally after simplification

$$(V - u_\alpha)^4 - \left\{(V - u_{\alpha 0})^2 + \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}\right\}.(V - u_\alpha)^2 + \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}(V - u_{\alpha 0})^2 < 0$$

This is a biquadratic inequation in u_α which can have four real or imaginary values.

Observation 2. $\phi(u_\alpha)|_{u_\alpha = u_{\alpha 0}} = 0$

Observation 3. $G(u_\alpha)|_{u_\alpha = u_{\alpha 0}} = 0$

Observation 4. $L(u_\alpha)|_{u_\alpha = u_{\alpha 0}} = 0$

Observation 5. $G^{/}(u_{\alpha 0}) = \left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0$ gives the very interesting result for the existence of solitary wave solution in presence of single and two-temperature non-isothermal electron plasma when $\left.\frac{d\phi}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0$.

Now $\left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0$ gives

$$\sum \frac{n_{\alpha 0} Z_\alpha^2}{Q_\alpha (V - u_{\alpha 0})^2 - \frac{3\sigma_\alpha}{n_{\alpha 0}}} < 1 + \chi\sigma_p \text{ when } V > u_{\alpha 0} + \sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}} \text{ for } V > u_{\alpha 0} \tag{22}$$

This is the necessary condition for real and bounded solution of the energy equation (14) [i.e. the condition for the existence of solitary wave solution] in presence of single temperature non-isothermal electron plasma with warm positive ion, warm negative ion and warm positron. The above condition reduces to the following well known form containing positive (i) ion, negative (j) ion and positron,

$$\frac{Z^2 n_{j0}}{Q\left(V - u_{j0}\right)^2 - \frac{3\sigma_j}{n_{j0}}} + \frac{n_{i0}}{(V - u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}} < 1 + \chi\sigma_p \tag{23}$$

In absence of positron ($\chi = 0$), the above inequality satisfies Ref.[6,27] and also for cold ion plasma ($\sigma_i = 0$, $\sigma_j = 0$), it supports Ref.[28].Moreover the above inequality supports Ref.[29,30] in absence of negative ion and positron. The above relation is a fourth degree inequation in V giving four real or imaginary values of V. In absence of negative ion and positron, the above results exactly reduces to the result of Ref.[2].

Observation 6. (a)If $(V - u_{\alpha 0})^2 < \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}$ then $\frac{d\phi(u_\alpha)}{du_\alpha} < 0$ for $u_{\alpha 0} \le u_\alpha < V$

(b) If $(V - u_{\alpha 0})^2 > \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}$ then $\frac{d\phi(u_\alpha)}{du_\alpha} > 0$ for $u_{\alpha 0} \le u_\alpha < u_\alpha^{/}$

and (c) $\frac{d\phi(u_\alpha)}{du_\alpha} < 0$ for $u_\alpha^{/} < u_\alpha < V$ where $u_\alpha^{/}$ is determined by

$$\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_\alpha^{/}} = 0.$$

From the above condition (c) of observation 6 and by the consequence of observation 5, we get

$$\sum\left(V - u_\alpha^{/}\right)^4 = \sum \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}} (V - u_{\alpha 0})^2$$

$$\Rightarrow u_\alpha^{/} = \sum\left[V - \sqrt[4]{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\sqrt{V - u_{\alpha 0}}\right] \tag{24}$$

This result is more general form than Ref.[2] and $u_\alpha^{/}$ can be obtained for both positive and negative ion but Ghosh et al[2] only found this result for positive ion only.

Observation 7. (a) If $(V - u_{\alpha 0})^2 < \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}$ then $\frac{d\phi(u_\alpha)}{du_\alpha} > 0$ for V $< u_\alpha \leq u_{\alpha 0}$

(b) If $(V - u_{\alpha 0})^2 > \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}$ then $\frac{d\phi(u_\alpha)}{du_\alpha} < 0$ for V $< u_\alpha < u_\alpha^{/}$

(c) $\frac{d\phi(u_\alpha)}{du_\alpha} < 0$ for $u_\alpha^{/} < u_\alpha \leq$ V where $u_\alpha^{/}$ is given by $\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_\alpha^{/}} = 0$.

By the above similar arguments, we can also establish the similar result.

We now establish some important relations in the form of Theorems given below:

3.1 Theorem1. The energy equation (14) will admit a real and bounded solution for single and two-temperature non-isothermal electron plasma in presence of positron if and only if

$$\text{(a)} \sum \frac{Z_\alpha^2 n_{\alpha 0}^2}{Q_\alpha n_{\alpha 0}(V - u_{\alpha 0})^2 - 3\sigma_\alpha} < 1 + \chi\sigma_p \quad \text{when } V > u_{\alpha 0}$$

$$\text{and (b)} \sum \frac{Z_\alpha^2 n_{\alpha 0}^2}{Q_\alpha n_{\alpha 0}(V - u_{\alpha 0})^2 - 3\sigma_\alpha} > 1 + \chi\sigma_p \quad \text{when } V < u_{\alpha 0}$$

Proof: There are necessary and sufficient conditions in Theorem 1.

Part I The conditions (a) and (b) are necessary.

From equation (14), if L(u_α) > K for either $u_{\alpha 0} < u_\alpha < (u_\alpha)_{max}$ or

$(u_\alpha)_{min} < u_\alpha < u_{\alpha 0}$ is fulfilled then one can say that L(u_α)> L$(u_{\alpha 0})$ for $u_\alpha = u_{\alpha 0} + \varepsilon$, ε > 0 (ε is an arbitrary small positive number).

It follows from above that $\left.\frac{d^2 L(u_\alpha)}{du_\alpha^2}\right|_{u_\alpha = u_{\alpha 0}} > 0$

$$\Rightarrow \left.\frac{dG(u_\alpha)}{du_\alpha} \cdot \frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0 \text{ [by observation 2 and from (15)]}$$

Now two cases arise which are given below:

1st Case: $\left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0$ for $\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha = u_{\alpha 0}} > 0$

$$\Rightarrow \sum \frac{Q_\alpha}{Z_\alpha}(V - u_{\alpha 0})\left[1 - \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}} \cdot \frac{1}{(V - u_{\alpha 0})^2}\right](1 + \chi\sigma_p) - \sum \frac{Z_\alpha n_{\alpha 0}}{(V - u_{\alpha 0})} > 0 \text{ for } V > u_{\alpha 0}$$

$$\Rightarrow \sum \frac{Z_\alpha^2 n_{\alpha 0}^2}{Q_\alpha n_{\alpha 0}(V - u_{\alpha 0})^2 - 3\sigma_\alpha} < 1 + \chi\sigma_p \quad \text{when } V > u_{\alpha 0}$$

This is the condition for the existence of the solitary wave solution.

2nd Case: $\left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_{\alpha0}} < 0$ for $\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_{\alpha0}} < 0$

The above inequality gives finally after using proper boundary condition and simplification,

$$\sum \frac{Z_\alpha^2 n_{\alpha0}^2}{Q_\alpha n_{\alpha0}(V-u_{\alpha0})^2 - 3\,\sigma_\alpha} > 1 + \chi\sigma_p \quad \text{for } V < u_{\alpha0}$$

Part II Sufficient Condition

The conditions (a) and (b) together are sufficient. We now assume that the conditions (a) and (b) of this theorem are to be satisfied. For that reason we obtain from conditions (a) & (b),

$$\left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_{\alpha0}} > 0 \quad \text{for } V > u_{\alpha0} \tag{25}$$

$$\text{And } \left.\frac{dG(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_{\alpha0}} < 0 \quad \text{for } V < u_{\alpha0} \tag{26}$$

From (25) and (26) after using the observations 2,4 and 5 we obtain

$$\left.\frac{d^2\mathrm{L}(u_\alpha)}{du_\alpha^2}\right|_{u_\alpha=u_{\alpha0}} = \left.\frac{dL^{/}(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_{\alpha0}} > 0 \tag{27}$$

From (27) and the fact that $L^{/}(u_{\alpha0}) = 0$, it follows that

$\mathrm{L}(u_{\alpha0}+\varepsilon) > \mathrm{L}(u_{\alpha0})$ for ε (>0) is an arbitrary small number.

Therefore $\mathrm{L}(u_\alpha) > \mathrm{L}(u_{\alpha0})$ where $u_\alpha = u_{\alpha0} + \varepsilon$ (28)

Thus Theorem 1 is established for necessary and sufficient conditions.

3.2 Phase Velocity: From the condition for the existence of solitary wave solution in presence of positron we can write

$$\sum \frac{n_{\alpha0} Z_\alpha^2}{Q_\alpha (V-u_{\alpha0})^2 - \frac{3\sigma_\alpha}{n_{\alpha0}}} < 1 + \chi\sigma_p$$

$$\Rightarrow \quad \frac{n_{i0}}{(V-u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{Q\left(V-u_{j0}\right)^2 - \frac{3\sigma_j}{n_{j0}}} < 1 + \chi\sigma_p$$

The phase velocity (V_p) is obtained from the equation given below:

$$\frac{n_{i0}}{\left(V_p-u_{i0}\right)^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{Q\left(V_p-u_{j0}\right)^2 - \frac{3\sigma_j}{n_{j0}}} = 1 + \chi\sigma_p \tag{29}$$

3.3 Theorem2. For single-temperature non-isothermal electron plasma, the energy equation (14) will admit real and bounded solution if and only if the sufficient condition L$\left(u_\alpha^/\right)$ - L$(u_{\alpha 0})$< 0 holds for $u_{\alpha 0}$<u_α<$u_\alpha^/$< V where $u_\alpha^/$ is given by the equation $\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_\alpha^/}$ = 0.

Proof: For a physically admissible solution of equation (14), it is obvious that $L^/(u_\alpha) \geq 0$ for u_α = $u_{\alpha 0}$ + ε, where ε > 0 is an arbitrary small number. It immediately follows from above that L$(u_{\alpha 0} + \varepsilon)$> L$(u_{\alpha 0})$.

Also from the given condition L$\left(u_\alpha^/\right)$< L$(u_{\alpha 0})$ for $u_{\alpha 0}$<u_α<$u_\alpha^/$< V.

We thus get, L$\left(u_\alpha^/\right)$< L$(u_{\alpha 0})$< L$(u_{\alpha 0} + \varepsilon)$.

Therefore, there exists a point $(u_\alpha)_{max}$ (say) or $(u_\alpha)_{min}$ (say) between $u_{\alpha 0}$ and $u_\alpha^/$ such that L$\{(u_\alpha)_{max}\}$ = L$(u_{\alpha 0})$ for $u_{\alpha 0}$<u_α<$(u_\alpha)_{max}$ (30)

Or L$\{(u_\alpha)_{min}\}$ = L$(u_{\alpha 0})$ for $(u_\alpha)_{min}$<u_α<$u_{\alpha 0}$ (31)

Consequently, there exists u_α^*between $u_{\alpha 0}$and $(u_\alpha)_{max}$ or $(u_\alpha)_{min}$ and $u_{\alpha 0}$

such that $L^/(u_\alpha^*)$ = 0 (32)

Again from (15), $L^/(u_\alpha)$ = G$(u_\alpha)\frac{d\phi(u_\alpha)}{du_\alpha}$ (33)

Then from conditions (30) and (31) we may write

$\frac{d\phi(u_\alpha)}{du_\alpha}$> 0 for V >$u_{\alpha 0}$ and $\frac{d\phi(u_\alpha)}{du_\alpha}$< 0 for V<$u_{\alpha 0}$ (34)

By (32), (33) and (34) we write $\left.\frac{d\phi(u_\alpha)}{du_\alpha}\right|_{u_\alpha=u_\alpha^/}$ = 0 where $u_{\alpha 0}$<u_α<$u_\alpha^/$< V.

$$\Rightarrow 1 - \frac{3\sigma_\alpha}{Q_\alpha n_\alpha}\frac{(V-u_{\alpha 0})^2}{\left(V- u_\alpha^/\right)^4} = 0$$

$$\Rightarrow\left(V - u_\alpha^/\right)^4 = \frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}(V - u_{\alpha 0})^2$$

$$\Rightarrow u_\alpha^/ = V - \sqrt[4]{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha 0}}}\sqrt{V - u_{\alpha 0}} \text{ (by proper boundary condition).}$$

From this Theorem some important observations are noticed.

Observation(a). From Theorem2, the sufficient condition L$\left(u_\alpha^/\right)$< L$(u_{\alpha 0})$ gives the important result given below for single temperature non-isothermal electron plasma.

$$\sum \frac{Q_\alpha}{2Z_\alpha}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2+\sum \frac{Q_\alpha^2}{8Z_\alpha^2}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^4$$

$$-\frac{8}{15}b_1\sum\left(\frac{Q_\alpha}{2Z_\alpha}\right)^{\frac{5}{2}}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^5+\sum \frac{Q_\alpha^3}{48Z_\alpha^3}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^6$$

$$-\frac{16}{105}b_2\sum\left(\frac{Q_\alpha}{2Z_\alpha}\right)^{\frac{7}{2}}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^7$$

$$+\sum n_{\alpha0}\,Q_\alpha(u_{\alpha0}-V)\left(V-\sqrt[4]{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\sqrt{V-\ u_{\alpha0}}\right)+\sum\frac{\sigma_\alpha^{\frac{1}{4}}}{3^{\frac{3}{4}}}(Q_\alpha n_{\alpha0})^{\frac{3}{4}}(V-u_{\alpha0})^{\frac{3}{2}}$$

$$+\sum \frac{Q_\alpha^4}{384Z_\alpha^4}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^8<\sum\sigma_\alpha-\sum n_{\alpha0}Q_\alpha\,(V-u_{\alpha0})u_{\alpha0}$$

$$+\frac{\chi}{\sigma_p}\left[1-\ e^{-\sigma_p\Sigma\frac{Q_\alpha}{2Z_\alpha}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2}\right] \qquad (35)$$

Where $u_\alpha^{/}=\sum\left[V-\sqrt[4]{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\sqrt{V-\ u_{\alpha0}}\right]$

And $\phi\left(u_\alpha^{/}\right)=\sum\frac{Q_\alpha}{2Z_\alpha}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2$

Under some restrictions, the above sufficient condition gives a number of special results.

Special case for single temperature non-isothermal electron plasma:

(i) When b_1 = 0 = b_2 (i.e. β_1 = 1), the above inequality (35) reduces to the isothermal warm ion plasma.

$$\sum \frac{Q_\alpha}{2Z_\alpha}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2+\sum \frac{Q_\alpha^2}{8Z_\alpha^2}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^4$$

$$+\sum \frac{Q_\alpha^3}{48Z_\alpha^3}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^6+\sum \frac{Q_\alpha^4}{384Z_\alpha^4}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^8$$

$$-\sum n_{\alpha0}\,Q_\alpha(V-u_{\alpha0})\left(V-\sqrt[4]{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\sqrt{V-u_{\alpha0}}\right)$$

$$+\sum 3^{\frac{-3}{4}}\sigma_\alpha^{\frac{1}{4}}(V-u_{\alpha0})^{\frac{3}{2}}(Q_\alpha n_{\alpha0})^{\frac{3}{4}}<\sum\sigma_\alpha-\sum n_{\alpha0}\,Q_\alpha u_{\alpha0}(V-u_{\alpha0})$$

$$+\frac{\chi}{\sigma_p}\left[1-e^{-\sigma_p\Sigma\frac{Q_\alpha}{2Z_\alpha}\left(V-u_{\alpha0}-\sqrt{\frac{3\sigma_\alpha}{Q_\alpha n_{\alpha0}}}\right)^2}\right] \qquad (36)$$

(ii) The inequality (35) will be reduced to isothermal cold ion plasma when

β_1 = 1, b_1 = 0 = b_2 and σ_α = 0 which gives

$$1 + \sum \frac{Q_\alpha}{4Z_\alpha}(V-u_{\alpha 0})^2 + \sum \frac{Q_\alpha^2}{24Z_\alpha^2}(V-u_{\alpha 0})^4 + \sum \frac{Q_\alpha^3}{192Z_\alpha^3}(V-u_{\alpha 0})^6$$

$$< 2 + \boldsymbol{\Sigma}\frac{2Z_\alpha\chi}{\sigma_p Q_\alpha (V-u_{\alpha 0})^2}\left[1 - e^{-\sigma_p \Sigma \frac{Q_\alpha (V-u_{\alpha 0})^2}{2Z_\alpha}}\right] \tag{37}$$

(iii) Inequality (35) reduces to non-drifting cold isothermal plasma when

β_1 = 1, b_1 = 0, b_2 = 0, σ_α = 0, u_α = 0. This gives the result

$$V^6 + \sum \frac{8Z_\alpha}{Q_\alpha}V^4 + \sum \frac{48Z_\alpha^2}{Q_\alpha^2}V^2 < 192\,\Sigma\left(\frac{Z_\alpha}{Q_\alpha}\right)^3\left[1 + \Sigma \frac{2\chi Z_\alpha}{\sigma_p Q_\alpha V^2}\left\{1 - e^{-\sigma_p \Sigma \frac{Q_\alpha V^2}{2Z_\alpha}}\right\}\right] \tag{38}$$

(iv) From inequality (35) the sufficient condition is reduced to non-isothermal cold ion plasma when $b_1 \neq 0$, $b_2 \neq 0$, σ_α = 0.

Thus we get, $\frac{1}{2} + \sum \frac{Q_\alpha}{8Z_\alpha}(V-u_{\alpha 0})^2 - \frac{\sqrt{2}}{15} b_1 \sum \left(\frac{Q_\alpha}{Z_\alpha}\right)^{\frac{3}{2}}(V-u_{\alpha 0})^3$

$$+\frac{1}{48}\sum\left(\frac{Q_\alpha}{Z_\alpha}\right)^2 (V-u_{\alpha 0})^4 - \frac{\sqrt{2}}{105} b_2 \sum \left(\frac{Q_\alpha}{Z_\alpha}\right)^{\frac{5}{2}}(V-u_{\alpha 0})^5 + \sum \frac{Q_\alpha^3}{384Z_\alpha^3}(V-u_{\alpha 0})^6$$

$$< 1 + \boldsymbol{\Sigma}\frac{Z_\alpha\chi}{\sigma_p Q_\alpha (V-u_{\alpha 0})^2}\left[1 - e^{\Sigma \frac{-\sigma_p Q_\alpha (V-u_{\alpha 0})^2}{2Z_\alpha}}\right] \tag{39}$$

4. DISCUSSION

A rigorous analytical technique is presented in this manuscript for finding the conditions of the existence of solitary wave solutions in warm positive ions, warm negative ions and warm positron together with single temperature non-isothermal electrons taking without any approximation. By applying such an exact technique some important results and Theorems are established which supports the works of previous authors.

In the case of warm negative ion together with warm positive ion, warm positron and non-isothermal single temperature electron, this exact method is done first time because no effort has been done previously with negative ion in this field. Ghosh et al[1,2,26] applied the new analytical method to find the condition for the existence of solitary wave solutions in a relativistic plasma consisting of cold positive ion and isothermal electron only. In all cases they took only warm positive ion with two-temperature isothermal electron plasma. Using this analytical method, the present author has found out the general conditions for solitary wave solutions with warm positive ion, warm negative ion, warm positron and non-isothermal single temperature electron. The result of Ghosh et al[1,2,26] can be easily found out from the result of the present author after putting n_{j0} = 0, σ_j = 0, b_1 = 0 , b_2 = 0 , χ = 0 in the respective equations .

The critical values of the velocity of solitary waves (i.e. Phase velocity V_p) are obtained from the equation (29). In absence of negative ion and positron, the four values of phase velocities V_p exactly supports the results of previous authors in this field. By this new analytical way, the Sagdeev potential function $\psi(\phi)$ for single temperature non-isothermal electron plasma can easily be found out. Moreover in this problem, the kinetic energy [K.E = $L(u_\alpha)$] and force (F) acting on a particle of unit mass in the solitary wave motion at position ϕ and time η is determined exactly.

We are now discussing the profiles of Sagdeev pseudopotential function $\psi(\phi)[= -L(u_\alpha)]$ against ϕ under the variation of different plasma parameters for the pseudoparticle moving in the potential well shown in the corresponding figs. 1 to 7.

Fig.1 shows the Sagdeev potential function $\psi(\phi)$ [= - $L(u_\alpha)$] against the electrostatic potential(ϕ) for non-isothermal single temperature electron plasma with the variation of the trapping parameters b_1 and b_2 for a fixed temperature of positive and negative ions in the plasma (Ar^+,SF_6^-) with Q = 1.9. It is evident from this figure that as $b_1$and b_2 increases for our chosen set of parameters the value of the Sagdeev potential function decreases and finally reduces to zero. The respective curves so generated in this case are represented by l_1 (b_1 = 0.15, b_2 = 0.24), l_2 (b_1 = 0.20, b_2 = 0.28) and l_3 (b_1 = 0.25, b_2 = 0.35). Also the generated curves l_1 , l_2 and l_3 are such that for every case the electrostatic potential ϕ lies in -0.072418 < ϕ < 0.309794.

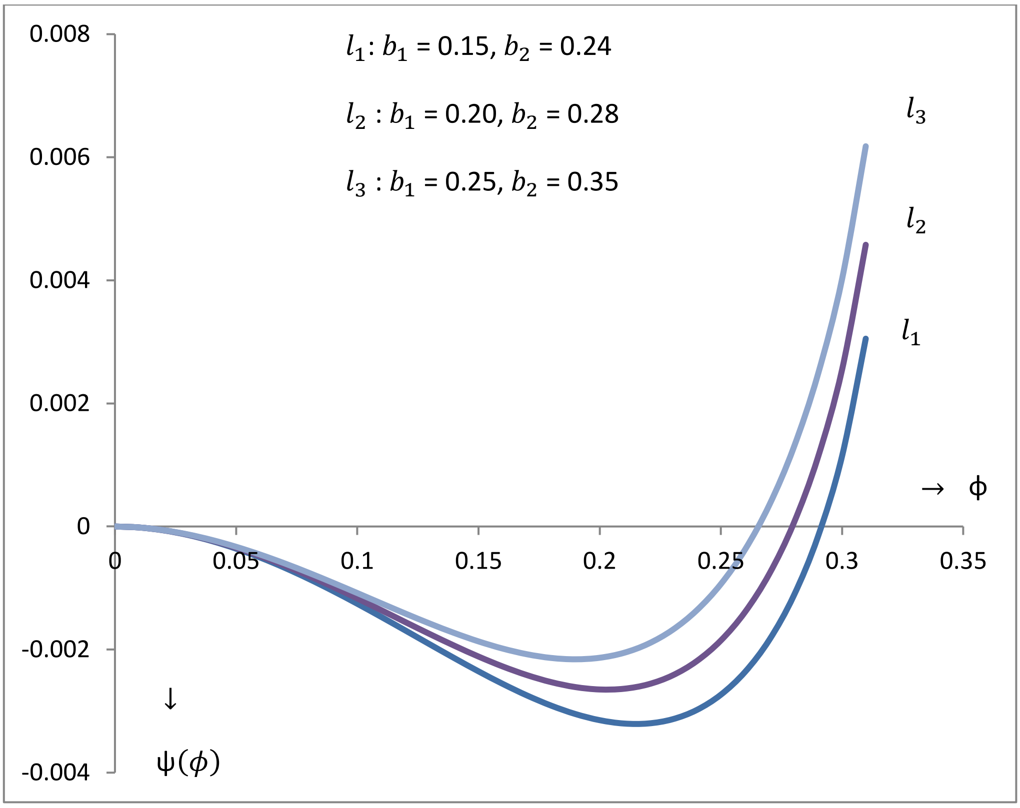

Fig.1 Profiles of Sagdeev potential function ψ(φ) against φ for the variation of the trapping parameters b_1 and b_2 in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, n_{jo} = 0.05, n_{io} = 0.88, χ = 0.17 , σ_p = 0.41, Q = 1.9 , **b_1 = 0.15, b_2 = 0.24; b_1 = 0.20, b_2 = 0.28; b_1 = 0.25, b_2 = 0.35**

Fig.2 shows the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against the electrostatic potential ϕ for non-isothermal single - temperature electron plasma with the variation of the temperature of positive (σ_i) and negative (σ_j) ions for a fixed stream velocities of positive (u_i) and negative (u_j) ions in the plasma (Ar^+, SF_6-) with Q = 1.9. As ion temperatures are an important parameter for Sagdeev potential function ψ(ϕ), so we are highlighting the Sagdeev potential function for three different situations of positive (σ_i) and negative (σ_j) ion temperatures. The Sagdeev potential function ψ(ϕ) is denoted by r_1 for $\sigma_i < \sigma_j$, by r_2 for $\sigma_i > \sigma_j$ and finally by r_3 and for $\sigma_i = \sigma_j$. The Sagdeev potential function ψ(ϕ) cuts the ϕ axis at larger distance for cold ions ($\sigma_i = \sigma_j$ = 0) while it cuts the ϕ axis at lesser distance for $\sigma_i = \frac{1}{20} > \sigma_i = \frac{1}{25}$ compared to $\sigma_i = \frac{1}{25} < \sigma_i = \frac{1}{20}$ which is really an interesting phenomena.

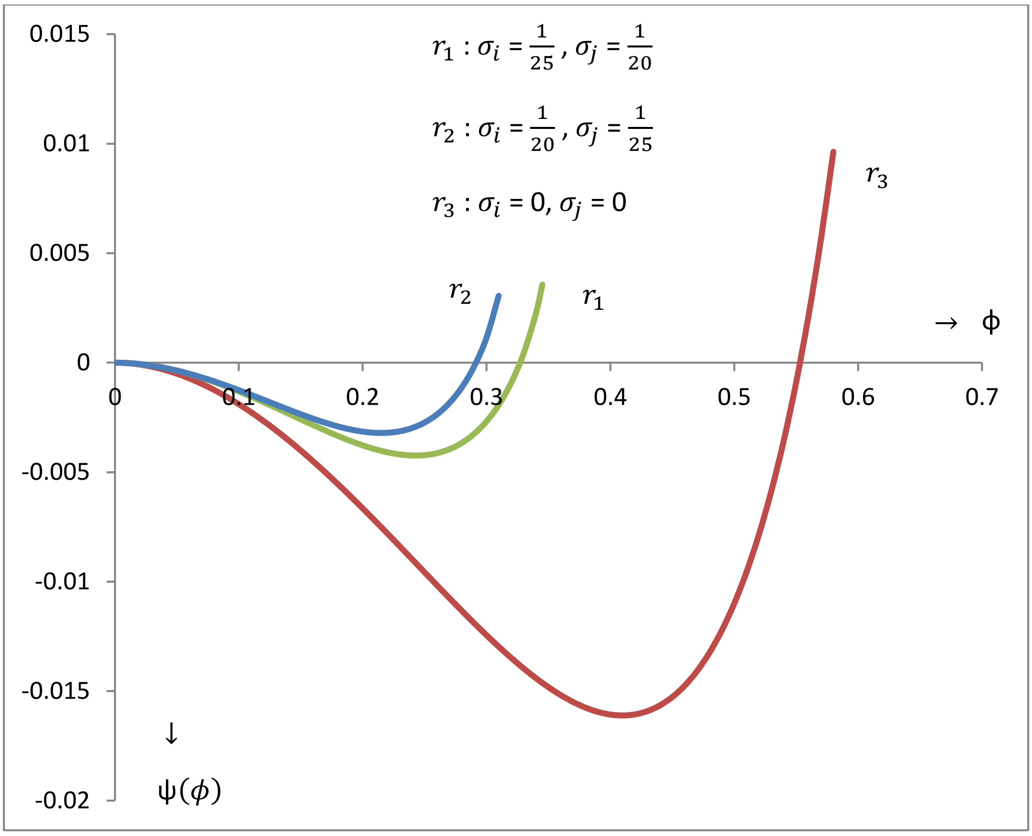

Fig.2 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of the temperature of positive (σ_i) and negative (σ_j) ions in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\boldsymbol{\sigma_i = \frac{1}{20}, \sigma_j = \frac{1}{25}}$; $\boldsymbol{\sigma_i = \frac{1}{25}, \sigma_j = \frac{1}{20}}$; $\boldsymbol{\sigma_i = 0, \sigma_j = 0}$; n_{jo} = 0.05, n_{io} = 0.88, χ = 0.17 , σ_p = 0.41, Q = 1.9 , b_1 = 0.15, b_2 = 0.24.

In **Fig.3**, the Sagdeev potential function $\psi(\phi)$ [= - L(u_α)] against the electrostatic potential ϕ is plotted for single-temperature non-isothermal electron plasma with the variation of the concentration of negative ions (n_{j0}) for (Ar^+,SF_6^-) plasma with Q = 1.9.When negative ion concentration (n_{j0}) increases, the Sagdeev potential function $\psi(\phi)$ cuts the ϕ axis at lesser value than the higher values of the concentration of negative ions. For n_{j0} = 0, the Sagdeev potential function $\psi(\phi)$ does not cut the ϕ axis and so no solitary wave is formed for this value of negative ion concentration. The Sagdeev potential function $\psi(\phi)$ is denoted by h_1 for n_{jo} = 0, by h_2 for n_{j0} = 0.05 and finally by h_3 for n_{j0} = 0.09.

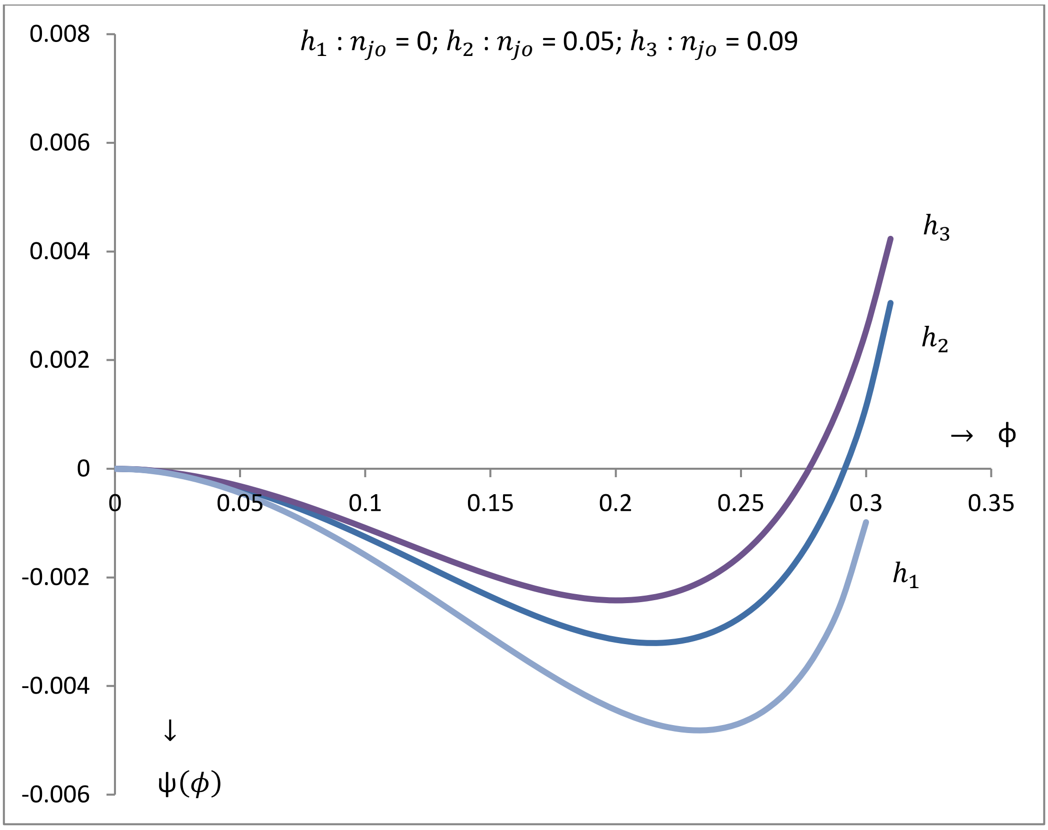

Fig.3 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of negative ion concentration (n_{jo}) in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, $\boldsymbol{n_{jo}}$ **= 0.05, 0.09, 0;** n_{io} = 0.88,0.92, 0.83; χ = 0.17 , σ_p = 0.41, Q = 1.9 , b_1 = 0.15, b_2 = 0.24.

In **Fig.4**, the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against the electrostatic potential ϕ is shown in single-temperature non-isothermal electron plasma with the variation of the concentration (χ) of positron for the plasma (Ar^+,SF_6^-) with Q = 1.9. As χ increases from χ = 0 to χ = 0.17, the electrostatic potential (ϕ) takes larger values where Sagdeev potential function ψ(ϕ) cuts the ϕ axis. The Sagdeev potential function ψ(ϕ) [= - L(u_α)] against ϕ is represented by the curve g_1 for χ = 0, by g_2 for χ = 0.08 and finally by g_3 for χ = 0.17. The maximum value of ϕ will be obtained for χ = 0.17 and the minimum value of ϕ is found for χ = 0 in our chosen set of parameter values.

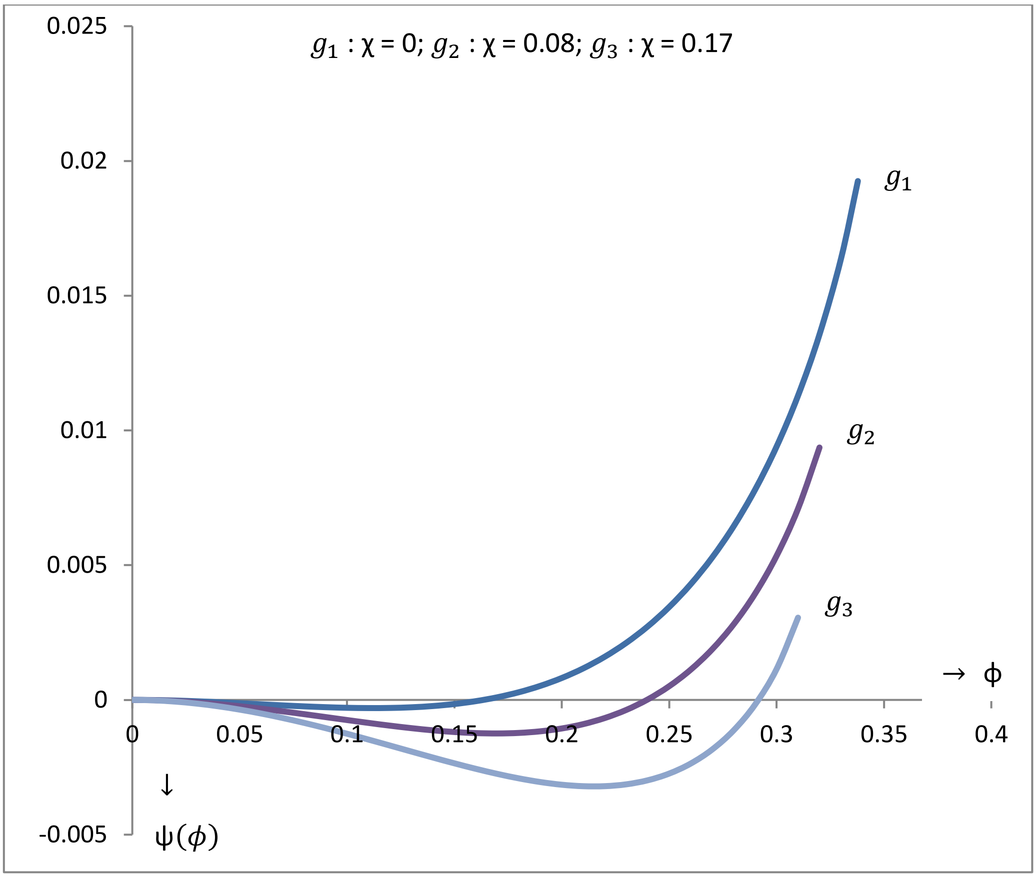

Fig.4 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of the concentration (χ) of positron in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, n_{jo} = 0.05, n_{io} = 1.05, 0.97, 0.88; **χ = 0, 0.08, 0.17 ;** σ_p = 0.41, Q = 1.9 , b_1 = 0.15, b_2 = 0.24.

Fig.5 shows the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ in single-temperature non-isothermal electron plasma with the variation of the temperature ratios (σ_p) of electron (T_e) and positron (T_p) for (Ar^+,SF_6^-) plasma with Q = 1.9. When the temperature ratios (σ_p) of electron (T_e) and positron (T_p) are increasing from σ_p = 0.12 to σ_p= 1.1, the corresponding values of the electrostatic potential (ϕ) are increasing where it is observed that the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ cuts the ϕ axis at larger values for soliton formation except at σ_p= 1.1 where no soliton is formed. The Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ is denoted by the curve w_1 for σ_p = 0.12, by the curve w_2 for σ_p = 0.30, by the curve w_3 for σ_p = 0.41 and finally by the curve w_4 for σ_p = 1.1.

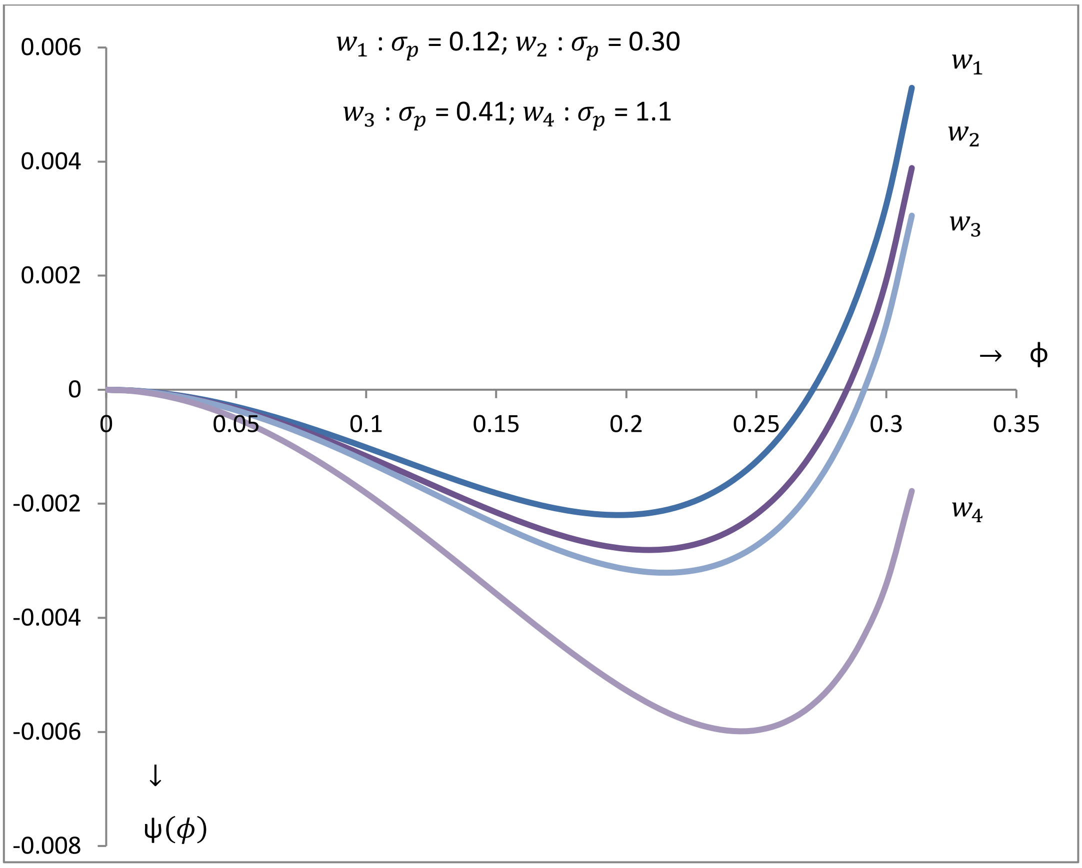

Fig.5 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of the temperature ratios (σ_p) of electron (T_e) and positron (T_p) in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, n_{jo} = 0.05, n_{io} = 0.88; χ = 0.17 ; $\boldsymbol{\sigma_p}$ **= 0.12, 0.30, 0.41, 1.1; Q = 1.9 ,** b_1 = 0.15, b_2 = 0.24.

In **Fig.6**, the profiles of the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ in single-temperature non-isothermal electron plasma with the variation of the mass ratios $(Q)$of negative ion (m_j) to positive ion (m_i) are shown for the plasmas (Ar^+,SF_6^-) with Q = 1.9, (He^+,O^-) plasma with Q = 4 and (H^+,Cl^-) plasma with Q = 35.5 .When Q increases from 1.9 to 35.5, the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ cuts the ϕ axis at larger values than the previous value of the mass ratios Q. The curves so generated by the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against electrostatic potential ϕ are denoted by s_1 for Q = 1.9, by s_2 for Q = 4 and finally by s_3 for Q = 35.5. The maximum value of ϕ will be obtained for Q = 35.5 and the least value will be found for Q = 1.9 in our chosen set of parameters.

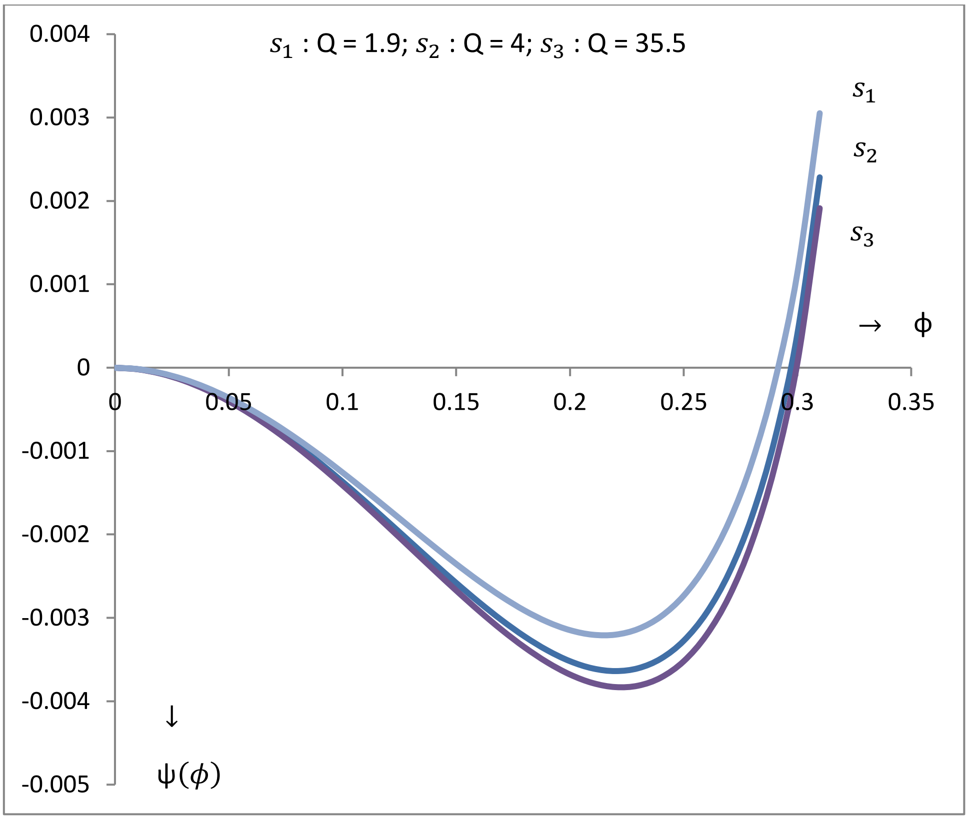

Fig.6 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of the mass ratios (Q) of negative (m_j) to positive (m_i) ions in single temperature non-isothermal electron plasma when V = 1.6, u_{io} = 0.4, u_{jo} = 0.2, $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, n_{jo} = 0.05, n_{io} = 0.88; χ = 0.17 ; σ_p = 0.41, **Q = 1.9 ,4 , 35.5;** b_1 = 0.15, b_2 = 0.24.

Fig.7 shows the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against the electrostatic potential(ϕ) for non-isothermal single temperature electron plasma with the variation of the stream velocities of positive (u_{io}) and negative (u_{jo}) ions for a fixed temperature of positive and negative ions in the plasma (Ar^+,SF_6^-) with Q = 1.9. It is observed from this figure that the Sagdeev potential function ψ(ϕ) [= - L(u_α)] against the electrostatic potential(ϕ) does not form any solitary waves for the stream velocities u_{io} = 0.3, u_{jo} = 0.1 denoted by f_1 while the other two curves f_2 for u_{io} = 0.4, u_{jo} = 0.2 and f_3 for u_{io} = 0.5, u_{jo} = 0.3 form solitary wave structures in this figure.

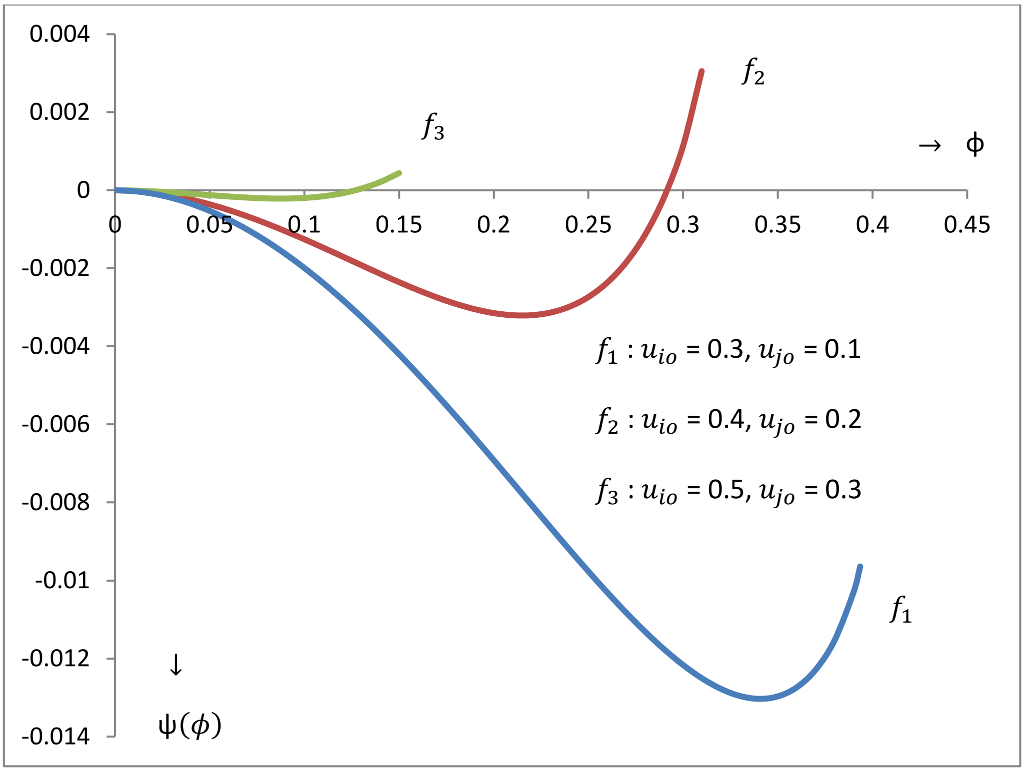

Fig.7 Profiles of Sagdeev potential function ψ(ϕ) against ϕ for the variation of the stream velocities of positive (u_{io}) and negative (u_{jo}) ions in single temperature non-isothermal electron plasma when V = 1.6, $\boldsymbol{u_{io}}$ **= 0.3,** $\boldsymbol{u_{jo}}$ **= 0.1;** $\boldsymbol{u_{io}}$ **= 0.4,** $\boldsymbol{u_{jo}}$ **= 0.2;** $\boldsymbol{u_{io}}$ **= 0.5,** $\boldsymbol{u_{jo}}$ **= 0.3;** $\sigma_i = \frac{1}{20}$, $\sigma_j = \frac{1}{25}$, n_{jo} = 0.05, n_{io} = 0.88; χ = 0.17 ; σ_p = 0.41, Q = 1.9 , b_1 = 0.15, b_2 = 0.24.

5. CONCLUDING REMARKS

The present author has investigated theoretically the compressive solitary waves in a single temperature non-isothermal electron plasma consisting of warm positive ion, warm negative ion along with warm positron by a new analytical method under the variation of different plasma parameters. It is also observed from this paper that the Sagdeev potential function ψ(ϕ), kinetic energy L(u_α), force F [= G(u_α)] and phase velocity (V_p) are derived by this new analytical method which are very important and interesting phenomena. The new analytical observations with sufficient and necessary conditions are obtained for the existence of ion-acoustic compressive solitary waves in the single temperature non-isothermal plasma which have wide applications to various physical situations in laboratory and space plasmas. In this paper, the kinetic energy L(u_α) is an important topic and as a consequence of this energy, the pseudoparticles are moving in the potential well. The Sagdeev potential function ψ(ϕ) is found from this kinetic energy L(u_α) by the relation ψ(ϕ) = - L(u_α) obtained by the new analytical method for a given set of plasma parameters in which the pseudoparticle is trapped. The graphical nature of Sagdeev potential function $\psi(\phi)$ [= - $L(u_\alpha)$] against ϕ is shown very clearly by the respective figures 1 to 7 under the variation of trapping parameters $(b_1 \& b_2)$, temperatures of positive (σ_i) and negative (σ_j) ions, concentration of negative ion (n_{jo}), concentration of positron (χ), temperature ratios (σ_p) of electron (T_e) and positron (T_p), mass ratios (Q) of negative (m_j) to positive (m_i) ions and stream velocities of positive (u_{io}) and negative (u_{jo}) ions.

In future, the present author wish to develop a time dependent energy function for relativistic warm positive ion, warm negative ion and warm positron with non-thermal electron plasma.

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