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A dynamical stabilizer in the climate system: a mechanism suggested by a simple model

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ABSTRACT

A simple zonally averaged hemispheric model of the climate system is constructed, based on energy equations for two ocean basins separated at 30° latitude with the surface fluxes calculated explicitly. A combination of empirical input and theoretical calculation is used to determine an annual mean equilibrium climate for the model and to study its stability with respect to small perturbations. The insolation, the mean albedos and the equilibrium temperatures for the two model zones are prescribed from observation. The principal agent of interaction between the zones is the vertically integrated poleward transport of atmospheric angular momentum across their common boundary. This is parameterized using an empirical formula derived from a multiyear atmospheric data set. The surface winds are derived from the angular momentum transport assuming the atmosphere to be in a state of dynamic balance on the climatic timescales of interest. A further assumption that the air–sea temperature difference and low level relative humidity remain fixed at their mean observed values then allows the surface fluxes of latent and sensible heat to be calculated. Results from a radiative model, which show a positive lower tropospheric water vapour/infrared radiative feedback on SST perturbations in both zones, are used to calculate the net upward infrared radiative fluxes at the surface. In the model's equilibrium climate, the principal processes balancing the solar radiation absorbed at the surface are evaporation in the tropical zone and net infrared radiation in the extratropical zone. The stability of small perturbations about the equilibrium is studied using a linearized form of the ocean energy equations. Ice-albedo and cloud feedbacks are omitted and attention is focussed on the competing effects of the water vapour/infrared radiative feedback and the turbulent surface flux and oceanic heat transport feedbacks associated with the angular momentum cycle. The perturbation equations involve inter-zone coupling and have coefficients dependent on the values of the equilibrium fluxes and the sensitivity of the angular momentum transport. Analytical solutions for the perturbations are obtained. These provide criteria for the stability of the equilibrium climate. If the evaporative feedback on SST perturbations is omitted, the equilibrium climate is unstable due to the influence of the water vapour/infrared radiative feedback, which dominates over the effects of the sensible heat and ocean heat transport feedbacks. The inclusion of evaporation gives a negative feedback which is of sufficient strength to stabilize the system. The stabilizing mechanism involves wind and humidity factors in the evaporative fluxes that are of comparable magnitude. Both factors involve the angular momentum transport. In including angular momentum and calculating the surface fluxes explicitly, the model presented here differs from the many simple climate models based on the Budyko–Sellers formulation. In that formulation, an atmospheric energy balance equation is used to eliminate surface fluxes in favour of top-of-the-atmosphere radiative fluxes and meridional atmospheric energy transports. In the resulting models, infrared radiation appears as a stabilizing influence on SST perturbations and the dynamical stabilizing mechanism found here cannot be identified.

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1. Introduction

"I will not digress here into a discussion of the momentum budget of the coupled system but will just note that until this coupling is understood we cannot claim to have a complete theory" — R. E. Newell (1979).

A fundamental problem in climate research is that of explaining how the Earth's climate remains stable on very long time scales. Positive feedback mechanisms such as the ice-albedo feedback and the lower tropospheric water vapour/infrared radiative (WVIR) feedback on SST perturbations are known to exist which could, in principle, drive the climate system far from its observed mean state even in the absence of any external forcings. Extreme scenarios that have been envisaged are a completely ice-covered Earth (Budyko, 1969) and a runaway greenhouse such as appears to have occurred on Venus (Goody and Yung, 1989; Rennó, 1997).

There is at present no generally accepted explanation for the stability of the Earth's climate. A number of negative feedback mechanisms have been proposed, but their validity and relative importance are matters of controversy. The most obvious candidate as a possible stabilizing mechanism is the basic radiative Stefan–Boltzmann feedback (Hartmann, 1994). While this guarantees that the emission temperature of the planet will adjust so that the time-averaged outgoing longwave radiation balances the absorbed solar radiation, it places little constraint on the surface temperature. Most of the radiation emitted to space comes from the atmosphere rather than from the surface (Harries, 1996). The surface temperature depends essentially on the strength of the greenhouse effect, which is determined not only by radiation but, perhaps more importantly, by dynamics (Lindzen, 1990a, 1997). Clouds play a central role in climate and may exert a stabilizing influence. Their feedback effects, however, are extremely complicated, depending not only on cloud amount, but also on cloud type, height and microphysical properties. It has been suggested (Ramanathan and Collins, 1991) that cirrus clouds resulting from deep convection in the tropics provide a thermostat on tropical SSTs, limiting the maximum values to about 305 K. Other aspects of cloud physics have also been invoked that may permit clouds to act as a negative feedback on net insolation (Somerville and Remer, 1984; Roeckner et al., 1987; Mitchell et al., 1989; Slingo, 1990). General Circulation Model (GCM) experiments have been carried out to assess how clouds respond to an imposed uniform SST change of ± 2 K. The average result from 19 GCMs is that clouds give a slight positive feedback as their net effect (Cess et al., 1996). A number of authors have put stress on the IR radiative effect of upper tropospheric water vapour as a climate stabilizer, arguing that subsidence can influence this in such a way that it acts as a negative feedback on tropical SST perturbations (Ellsaesser, 1984; Lindzen, 1990a; Pierrehumbert, 1995). This theory has also been contested and remains controversial. A further stabilizing mechanism that has been proposed (Stone, 1984) is the feedback between dynamical heat fluxes due to baroclinic eddies and the temperature structure of the atmosphere. While this undoubtedly acts as a negative feedback on the meridional temperature gradient, it seems unlikely that it can act as a stabilizer of the global surface temperature in the presence of a positive WVIR feedback. A large number of simple climate models of the Budyko (1969) and Sellers (1969) types have been devised, which parameterize the eddy heat transport in various ways and consider its interaction with the ice-albedo feedback (North et al., 1981; Lindzen, 1990b). These models use an atmospheric energy balance equation to eliminate surface fluxes and parameterize the outgoing longwave radiation at the top of the atmosphere as $I = A + BT_s$, where T_s is the surface temperature and A and B are positive constants. A drawback of such models is that, by avoiding a direct consideration of surface fluxes, they skirt the essential processes that determine the basic climatic variable, the SST. In particular, by taking B in the above equation as a positive constant, they automatically regard IR radiation as stabilizing, whereas studies of the surface IR radiation indicate that, when the lower tropospheric water vapour feedback is taken into account, it exerts a positive feedback on SST perturbations, particularly in the tropics (Manabe and Wetherald, 1967; Ramanathan, 1981; Randall et al., 1982; Hartmann and Michelsen, 1993).

Since the dominant term balancing the absorbed solar radiation in the globally averaged surface energy balance of the Earth is evaporation (Kiehl and Trenberth, 1997), it appears reasonable to seek negative feedbacks in some of the factors determining evaporation rates. Newell (1979) has suggested that the rapid increase of evaporation due to the humidity factor linked to the Clausius–Clapeyron equation places an upper limit on tropical SSTs. Evaporation as a limiting factor on the upper values of tropical SSTs has subsequently been considered by Hoffert et al. (1983). In both of these studies, the large-scale surface wind was taken as a prescribed constant and IR radiation was regarded as an additional stabilizing factor. The stabilizing effect of evaporative fluxes on tropical SSTs has also been investigated by Hartmann and Michelsen (1993). They studied the surface energy budget of the tropical ocean and found that, for fixed surface wind and relative humidity, the stabilization by evaporative cooling was sufficiently strong to overcome the positive feedback given by their IR radiative model. They also considered the stabilizing effects of wind perturbations driven by east–west SST variations in the tropics.

In the present paper, the emphasis is again on evaporation, but the domain considered includes the extratropics as well as the tropics and the surface wind is treated as an interactive quantity determined by the global atmospheric angular momentum (AM) cycle. A simple model is constructed which includes the dependence of the evaporative fluxes on both wind strength and humidity factors. Both factors are found to be important in determining the evolution of climate perturbations and they act together to provide a global stabilizing mechanism which is of sufficient strength to overcome the destabilizing influence of the WVIR feedback for arbitrary small perturbations about the equilibrium climate. The model also includes turbulent sensible heat fluxes and poleward heat transport by ocean currents, but these are found to be of minor importance in the context of the present stabilizing mechanism. Since both the sensitivity of the zonal surface wind (which enters the evaporative wind factor) and its mean value (which enters both the evaporative wind and humidity factors) are determined by the eddy transport of AM in the atmosphere, the mechanism is referred to as a dynamical stabilizer.

2. Angular momentum transport: the empirical relationship

One of the main effects of the large-scale quasi-horizontal eddy motions in the atmosphere is to give a large poleward transport of AM. The transport can be separated into transient and stationary eddy components, the transient component being the dominant one (Peixoto and Oort, 1992). The transient eddy transport is generated by Rossby wave dispersion out of the area of baroclinic wave activity in middle latitudes (Held and Hoskins, 1985). The AM transport is of crucial importance in climate because of the role it plays in determining the surface winds and hence the turbulent surface fluxes of momentum, moisture and sensible heat. A number of attempts have been made to parameterize eddy AM transport theoretically in terms of zonal mean quantities (Charney, 1959; Green, 1970; Wiin-Nielsen and Sela, 1971; Stone and Yao, 1987). The problem is inherently difficult and the theoretical AM parameterization formulae that have been derived are rather complex, though Wiin-Nielsen (1994) has simplified the problem and attained considerable success in deriving the atmospheric zonal wind distribution, given the thermal forcing.

In the present paper, an empirical approach to AM parameterization is taken and a simple formula for the vertically integrated transport at 30° latitude is found. Latitude 30° has some special features that suggest it as a dividing latitude in a simple climate model: not only is it the half-area latitude of a hemisphere but observations show that it is the approximate latitude at which the AM transport is maximum, the mean surface winds change from easterly to westerly and the mean meridional velocity is zero (Peixoto and Oort, 1992). A data study was carried out using the observed (model assimilated) atmospheric data set compiled by Schubert et al. (1990a,b) for the period 1980–87. It was found that the seasonal mean vertically integrated poleward AM transport by transient plus stationary eddies at 30°N, denoted F_M (a list of symbol definitions and abbreviations is given in Section 9), can be simply parameterized in terms of the difference between the seasonal mean zonally averaged heights of the 500 hPa surface at 15°N ($\equiv Z_1$) and 50°N ($\equiv Z_2$). Note: 15° and 50° are approximately the half-area latitudes of the tropical and extratropical zones (taken as 0°–30° and 30°–90°) and the observations show that Z_1 and Z_2 are good approximations to the area-averaged seasonal mean heights of the 500 hPa surface in the two zones. Fig. 1 shows a plot of F_M against ΔZ_{500} ($\equiv Z_1 - Z_2$). Choosing the observed ensemble

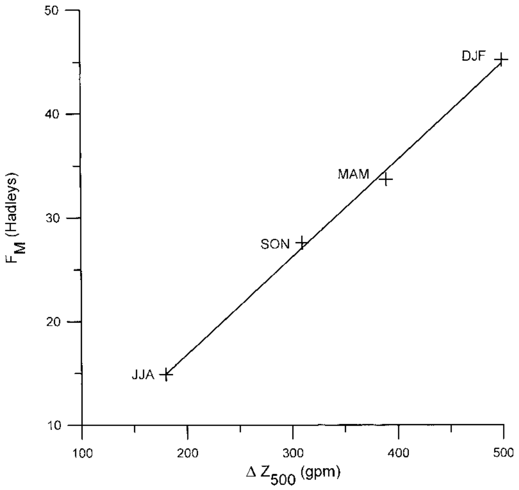

Fig. 1. Ensemble seasonal means from observations for the years 1980–87 of the vertically integrated poleward angular momentum transport by atmospheric eddies (transient plus stationary) at 30°N (F_M) versus the difference between the zonally averaged geopotential heights of the 500 hPa surface at 15°N and 50°N (ΔZ_{500}). The least squares linear fit is also shown. (DJF = December–January–February, etc. 1 Hadley = 10^{18} kg m^2 s^{-2}.)

annual mean value of ΔZ_{500} (denoted $\Delta \bar{Z}_{500}$) as a base point, it is found that the least squares linear fit equation relating F_M and ΔZ_{500} can be written

$$F_M = \bar{F}_M + a_M(\Delta Z_{500} - \Delta \bar{Z}_{500}), \tag{1}$$

where

$\Delta \bar{Z}_{500} = 345$ m, $\bar{F}_M = 30.3$ Hadleys

(1 Hadley = 10^{18} kg m^2 s^{-2}) and $a_M = 9.35 \times 10^{-2}$ Hadley m^{-1}. The closeness of the linear fit shown in Fig. 1 was quite unexpected.* A plot of F_M against the difference in observed surface temperature between 15°N and 50°N was found to show a less regular fit than that found using ΔZ_{500}. The physical basis for the close relationship between F_M and ΔZ_{500} would appear to lie in the fact that ΔZ_{500} is a gross measure of the baroclinicity of the atmosphere, and may reasonably be expected to be a determinant of the degree of mean baroclinic wave activity. It has been shown by Simmons and Hoskins (1978) that the mean AM transport over baroclinic wave life cycles cannot be represented adequately in terms of factors that refer only to the initial linearly growing stages of the waves. Since the observed transport of AM due to mean meridional motion is close to zero at 30°, the eddy transport is a close approximation to the total transport at this latitude. The steepness of the slope in Fig. 1, with a factor of three difference between the summer and winter values of F_M, implies a strong sensitivity of the AM transport to the value of ΔZ_{500}. This is important in determining the characteristics of the dynamical stabilizing mechanism.

A basic assumption made in this paper is that the relationship (1) that has been found between F_M and ΔZ_{500} for the case of the seasonal means (which provide the desired wide variation for a data study) will also hold for the annual means of these quantities under conditions of a perturbed climate. In the climate model with which we are concerned hereafter, seasonal variations are omitted. In that context, F_M and ΔZ_{500} will denote the time-varying annual means for the perturbed climate, while $\bar{F}_M$ and $\Delta \bar{Z}_{500}$ (with numerical values as specified above) will denote the corresponding quantities for the equilibrium climate.

* Corroboration of the parameterization (1), and of the later parameterization (12), has been provided by a recent study using the 40-year data set of the NCEP/NCAR Reanalysis (V. A. Alexeev and J. R. Bates, personal communication).

3. The simple climate model: formulation

Availing of the special features of latitude 30° noted above, and of the finding that F_M can be parameterized in terms of the first-order difference ΔZ_{500}, it is possible to construct a 1st-order model of the climate system consisting of just two zones separated at 30°. The model refers to the zonally averaged climate, assumes zero cross-equatorial transport and has tropical and extratropical ocean basins occupying zones 1 and 2 (Fig. 2). The governing equations for the model are the energy equations for the two ocean basins. The assumption is made that, on the climatic timescales of interest, the ocean energetics can be expressed in terms of the SSTs; likewise, it is assumed that horizontally averaged SSTs can be used for each zone. The basic variables of the model are thus T_{S1} and T_{S2}, the area-averaged SSTs in zones 1 and 2.

The two model zones interact primarily through

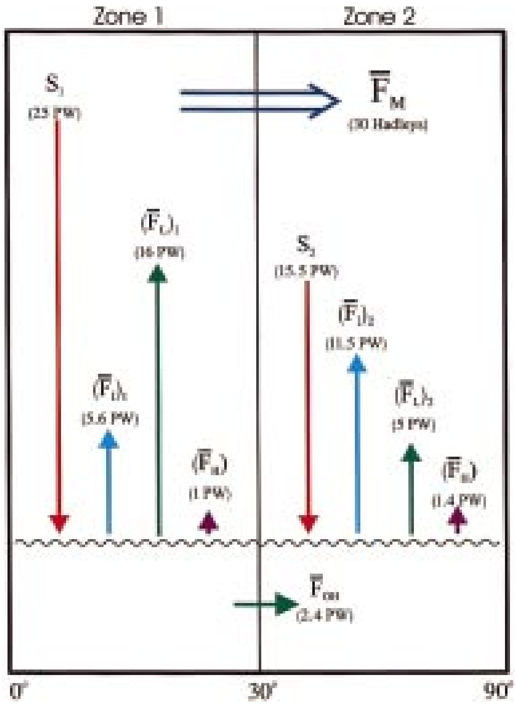

Fig. 2. Vertical cross-section of the model. The fluxes for the model's equilibrium climate are shown (see text for definitions).

the AM transport F_M and the surface fluxes of latent and sensible heat that it induces. A secondary means of interaction between the zones, also included here, is the poleward transport of heat across 30° by ocean currents (denoted F_{OH}). The model does not use any energy equations for the atmosphere, but it does include parameterizations of 500 hPa height variations in zones 1 and 2 in terms of SST variations. The extratropical 500 hPa height parameterization implicitly includes the effect of atmospheric energy transport. The model also assumes that the low level relative humidity r and air–sea temperature difference ΔT ($\equiv T_S - T_A$) are fixed, the same values being chosen in both zones. The chosen values are $r = 0.8$ and $\Delta T = 1$ K; these are suggested by the observational study of Wells and King-Hele (1990). Supporting evidence for an assumption of invariant relative humidity under conditions of a perturbed climate is available from a number of different sources: approximate invariance from summer to winter in northern hemisphere observations has been found by Manabe and Wetherald (1967); invariance in GCM experiments with prescribed SST perturbations has been found by Randall et al. (1992); invariance in GCM experiments with CO_2 doubling has been found by Bengtsson (1998). Making use of the AM parameterization, along with the parameterizations of the 500 hPa height variations, the surface fluxes of latent and sensible heat can be calculated in terms of the SSTs. The net upward infrared (IR) radiation at the sea surface is parameterized in terms of the SSTs using the results of Hartmann and Michelsen (1993).

An equilibrium climate for the model is first determined, using a combination of observed annual means and theoretically calculated quantities. The SSTs for zones 1 and 2 in the equilibrium climate are specified as $\overline{T}_{S1} = 300$ K and $\overline{T}_{S2} = 278$ K, the observed annual mean values at 15°N and 50°N, respectively (observations show that these are good approximations to the area-averaged annual mean SSTs in the two zones). The solar radiation absorbed by the earth–atmosphere system in zones 1 and 2 is derived from the results of North (1975), the fractions absorbed at the surface being chosen to give balance in the mean energy equations.

The model is then perturbed and the stability of the equilibrium climate is studied. Since attention is being focussed on a particular dynamical feedback mechanism and its interaction with the positive WVIR feedback, no ice-albedo or cloud feedbacks are included, though ice and cloud influences are implicitly included in the albedos used in determining the model's equilibrium climate. Also not included is the influence of the water vapour feedback on solar radiation absorbed at the surface. In the following subsections, details of the formulation of the various model components are given.

3.1. *The ocean energy equations*

Adopting the notation that subscripts 1 and 2 refer in all cases to zones 1 and 2, respectively, the ocean energy equations are

$$c_{O1}\frac{\mathrm{d}T_{S1}}{\mathrm{d}t} = S_1 - [(F_L)_1 + (F_H)_1 + (F_I)_1] - F_{OH}, \tag{2}$$

$$c_{O2}\frac{\mathrm{d}T_{S2}}{\mathrm{d}t} = S_2 - [(F_L)_2 + (F_H)_2 + (F_I)_2] + F_{OH}, \tag{3}$$

where c_{O1} and c_{O2} are the ocean heat capacities, S_1 and S_2 are the fluxes of solar energy absorbed at the surface, and $((F_L)_1, (F_L)_2)$, $((F_H)_1, (F_H)_2)$ and $((F_I)_1, (F_I)_2)$ are the surface energy losses due to latent heat flux, sensible heat flux and net (upward minus downward) IR radiation, respectively. The ocean heat capacities are expressed as

$$c_{O1} = \pi a^2 (c_{pw} \rho_w H_1), \tag{4}$$

$$c_{O2} = \pi a^2 (c_{pw} \rho_w H_2), \tag{5}$$

where a is the Earth's radius, c_{pw} the specific heat of seawater ($4187\ \mathrm{J\,kg^{-1}\,K^{-1}}$), ρ_w the density of seawater ($10^3\ \mathrm{kg\,m^{-3}}$), and (H_1, H_2) are the thermodynamically effective depths of the tropical and extratropical ocean basins. The latter are free parameters of the model.

3.2. *Determination of the surface zonal winds in terms of the AM transport*

It is assumed that, on the climatic timescales of interest, the atmosphere is in a state of dynamic balance so that the torque about the earth's axis exerted by the surface zonal winds in zone 1 balances the corresponding torque in zone 2. Each torque is then equal in magnitude to the total AM transport between the zones. (In the real atmosphere, the time–mean AM transport is balanced by a combination of surface frictional torques and pressure torques due to mountains. The latter constitute a minor component in an area-integrated sense (Peixoto and Oort, 1992) and are neglected here). Regarding the surface zonal wind as spatially uniform in each zone — easterly in zone 1 and westerly in zone 2 — and using the bulk aerodynamic formula (Hartmann, 1994), the surface stresses (of the Earth on the atmosphere) in zones 1 and 2 are given by

$$(\tau_1, \tau_2) = c_D \rho (u_1^2, -u_2^2), \tag{6}$$

where c_D is the drag coefficient (1.3×10^{-3}), ρ the surface air density ($1.2\ \mathrm{kg\,m^{-3}}$) and (u_1, u_2) the surface zonal winds. Regarding the eddy AM transport F_M as being a good approximation to the total AM transport at 30°, the AM balance conditions can then be written

$$2\pi a^3 \int_0^{30^\circ} \tau_1 \cos^2\phi \, \mathrm{d}\phi = F_M, \tag{7}$$

$$2\pi a^3 \int_{30^\circ}^{90^\circ} \tau_2 \cos^2\phi \, \mathrm{d}\phi = -F_M, \tag{8}$$

where ϕ is latitude. Using (6), eqs. (7) and (8) give

$$u_1 = -\left[\frac{F_M}{2\pi a^3 c_D \rho \displaystyle\int_0^{30^\circ} \cos^2\phi \, \mathrm{d}\phi}\right]^{1/2}, \tag{9}$$

$$u_2 = \left[\frac{F_M}{2\pi a^3 c_D \rho \displaystyle\int_{30^\circ}^{90^\circ} \cos^2\phi \, \mathrm{d}\phi}\right]^{1/2}. \tag{10}$$

These formulae for the zonal winds will be used in calculating the sensible and latent heat fluxes.

3.3. *Parameterization of 500 hPa height variations in terms of SST variations*

We assume that the vertical temperature profile in the tropics is determined by moist convection in the Intertropical Convergence Zone. Specifically, we determine the 1000–500 hPa thickness (D_1) for zone 1 as a function of the 1000 hPa temperature (T_{1000}) by tracing moist parcel ascent in a thermodynamic diagram, assuming a fixed relative humidity $r = 0.8$ at the 1000 hPa level. The results of this method of determining the thickness are shown in Fig. 3, where the four values $T_{1000} =$

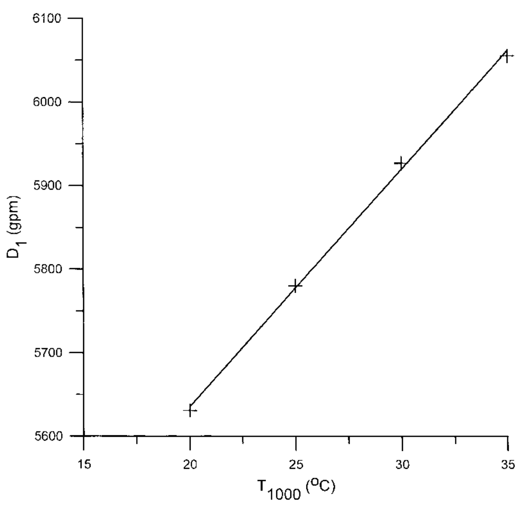

Fig. 3. Estimated thickness of the tropical 1000–500 hPa layer (D_1), derived from a thermodynamic diagram following moist parcel ascent with 80% relative humidity at 1000 hPa, versus the temperature T_{1000} at 1000 hPa. The least squares linear fit to four estimated values (corresponding to $T_{1000} = 20$, 25, 30, 35°C) is also shown.

(20, 25, 30, 35)°C have been chosen for the evaluations. The least squares linear fit equation relating the four values of D_1 and T_{1000} thus determined can be written

$$D_1 = \bar{D}_1 + a_{D1}(T_{1000} - \bar{T}_{1000}), \tag{11}$$

where $\bar{D}_1 = 5834$ m, $a_{D1} = 28.4$ m K^{-1} and $\bar{T}_{1000} = 300$ K. The closeness of the linear fit is notable.

The 1000–500 hPa thickness (D_2) in the extratropical zone is determined as a function of the surface temperature (T_{S2}) by using seasonal mean observations at 50°N, where a wide variation in these two quantities occurs. The resulting plot, derived from the NCEP/NCAR Reanalysis (Kalnay et al., 1996) for the period 1982–94, is shown in Fig. 4. The least squares linear fit equation relating the four seasonal values of D_2 and T_{2S} can be written

$$D_2 = \bar{D}_2 + a_{D2}(T_{S2} - \bar{T}_{S2}), \tag{12}$$

where $\bar{D}_2 = 5410$ m, $a_{D2} = 17.8$ m K^{-1} and $\bar{T}_{S2} = 278$ K. The closeness of the linear fit is again notable.

Assuming that variations in the 1000–500 hPa thickness greatly outweigh variations in the height of the 1000 hPa surface in a perturbed climate (observations at 50°N, where substantial variations in the thickness occur from season to season, support such an assumption), and assuming that T_{1000} in the tropics varies as T_{S1}, eqs. (11) and (12) can be used to parameterize variations of the 500 hPa heights (Z_1 and Z_2) in terms of the SSTs; thus

$$\left[\frac{dZ_1}{dT_{S1}}, \frac{dZ_2}{dT_{S2}}\right] = [a_{D1}, a_{D2}]. \tag{13}$$

With the numerical values of (a_{D1}, a_{D2}) given above, the parameterization (13) implies a considerably greater sensitivity of the height of the 500 hPa surface in the tropical than in the extratropical zone.

The sensitivity to temperature of the 1000–500 hPa thickness in the case where the temperature is varied uniformly throughout the layer is $R_d \ln 2/g$ ($= 20.3$ m K^{-1}), where R_d is the gas constant for dry air and g the gravitational acceleration. Hence, (a_{D1}, a_{D2}) can be written in the physically meaningful form:

$$[a_{D1}, a_{D2}] = \left(\frac{R_d \ln 2}{g}\right)[(1+\varepsilon_1), (1+\varepsilon_2)], \tag{14}$$

where $(\varepsilon_1, \varepsilon_2) = (0.40, -0.12)$. If we were to set $\varepsilon_1 = \varepsilon_2 = 0$, ΔZ_{500} (and hence F_M) would vary as the difference between the SSTs at 15° and 50° latitude.

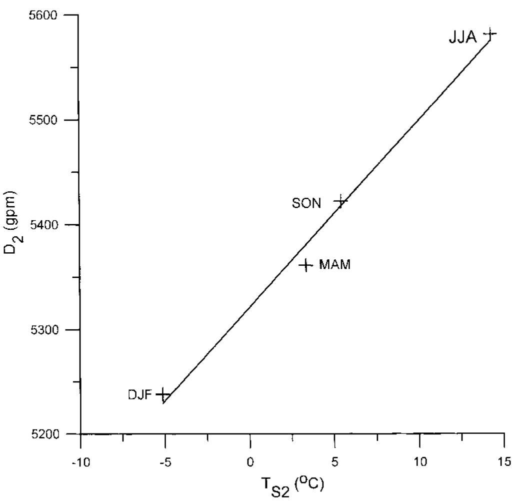

Fig. 4. Ensemble seasonal means from observations for the years 1982–94 of the zonally averaged 1000–500 hPa thickness at 50°N (D_2) versus the zonally averaged surface temperature at 50°N (T_{S2}). The least squares linear fit is also shown.

3.4. Parameterization of the turbulent surface fluxes

The latent and sensible heat fluxes are given by the bulk aerodynamic formulae (e.g., Wells and King-Hele, 1990)

$$Q_L = L\rho c_E\{q_s(T_S) - q_A\}|u|, \tag{15}$$

$$Q_H = c_p \rho c_H \Delta T|u|, \tag{16}$$

where L is the latent heat of vaporization (2.5×10^6 J kg^{-1}), c_E the turbulent exchange coefficient for water vapour (1.5×10^{-3}), $q_s(T_S)$ the saturation specific humidity at the temperature T_S of the sea surface, q_A the specific humidity of the air at 10 m above the sea surface, $|u|$ the magnitude of the time–mean zonal component of the surface wind, c_p the specific heat for dry air at constant pressure (1004 J kg^{-1} K^{-1}), and c_H the turbulent exchange coefficient for sensible heat (1.5×10^{-3}). The contribution to the turbulent fluxes by the time–mean meridional component of the surface wind is ignored.

We write (15) in an alternative form by using the approximate relationship

$$q = 0.622e/p_0, \tag{17}$$

where e is the vapour pressure and p_0 is a reference value of the surface pressure (1000 hPa); thus,

$$Q_L = \left[\frac{0.622 L\rho c_E}{p_0}\right] \Delta e|u|, \tag{18}$$

where Δe is the vapour pressure deficit, defined by

$$\Delta e = e_s(T_S) - e(T_A), \tag{19}$$

with e_s denoting saturation vapour pressure. Setting $e(T_A) = re_s(T_A)$ and $T_A = T_S - \Delta T$, Δe can be approximated by using a Taylor expansion and keeping only the first order term. Hence

$$\Delta e = e_s(T_S)\left[(1-r) + r\left(\frac{1}{e_s}\frac{de_s}{dT}\right)_{T_S} \Delta T\right]. \tag{20}$$

Using the Clausius–Clapeyron equation

$$\frac{1}{e_s}\frac{de_s}{dT} = \frac{L}{R_v T^2}, \tag{21}$$

where R_v is the gas constant for water vapour (461 J kg^{-1} K^{-1}), (20) becomes

$$\Delta e = e_s(T_S)\left[(1-r) + r\left(\frac{L}{R_v T_S^2}\right)\Delta T\right]. \tag{22}$$

Eq. (21) can be integrated to give the following approximate formula for the saturation vapour pressure as a function of temperature (Wallace and Hobbs, 1977)

$$e_s(T_S) = (6.11 \text{ hPa}) \exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{T_S}\right)\right]. \tag{23}$$

Eqs. (18) and (16), with Δe given by (22) and (23), are used to determine the turbulent surface fluxes. The use of these formulae for long-term mean flux calculations over the tropical oceans, where most of the global evaporation takes place, has been justified by Wells and King-Hele (1990).

Multiplying the unit-area fluxes (18) and (16) for zones 1 and 2 by πa^2, we have the area-integrated turbulent fluxes from the ocean surface:

$$[(F_L)_1, (F_H)_1, (F_L)_2, (F_H)_2] = \pi a^2 [(Q_L)_1, (Q_H)_1, (Q_L)_2, (Q_H)_2]. \tag{24}$$

The sensitivity to SST variations of the quantities defined in (24) are also required. These involve derivatives of Δe and $|u|$ with respect to SST in zones 1 and 2. We evaluate the relevant derivatives at this point.

Holding ΔT and r fixed and letting T_S vary, (22) gives

$$\frac{d}{dT_S}(\Delta e) = \left(\frac{de_s}{dT_S}\right)\left[(1-r) + r\left(\frac{L}{R_v T_S^2}\right)\Delta T\right] - e_s(T_S) r \left(\frac{2L}{R_v T_S^3}\right)\Delta T. \tag{25}$$

Using (21), we see that since $r < 1$ and $\Delta T/T_S \ll 1$, the second term in (25) can be neglected. Thus, (25) can be approximated as

$$\frac{d}{dT_S}(\Delta e) = \left(\frac{L}{R_v T_S^2}\right)\Delta e. \tag{26}$$

Differentiating (9) and (10) w.r.t. T_{S1} and T_{S2} and using (1) and (13), we find

$$\left[\frac{\partial|\boldsymbol{u}_1|}{\partial T_{S1}}, \frac{\partial|\boldsymbol{u}_1|}{\partial \boldsymbol{T}_{S2}}\right] = \left[\frac{\boldsymbol{a}_M|\boldsymbol{u}_1|}{2F_M}\right](\boldsymbol{a}_{D1}, -\boldsymbol{a}_{D2}), \tag{27}$$

$$\left[\frac{\partial|\boldsymbol{u}_2|}{\partial T_{S1}}, \frac{\partial|\boldsymbol{u}_2|}{\partial \boldsymbol{T}_{S2}}\right] = \left[\frac{\boldsymbol{a}_M|\boldsymbol{u}_2|}{2F_M}\right](\boldsymbol{a}_{D1}, -\boldsymbol{a}_{D2}). \tag{28}$$

The derivatives (26), (27) and (28) will be used in calculating the flux sensitivities for the perturbed climate (Subsection 5.1).

3.5. Parameterization of the surface IR radiation

The net upward IR radiation at the sea surface is parameterized using the results of Hartmann and Michelsen (1993) (hereafter HM), derived from a radiative model. HM used prescribed vertical profiles of temperature and relative humidity, with $r = 0.77$ at the surface. The relative humidity was held fixed as the SST was varied. It was found that the net upward surface IR flux decreased with increasing SST over most of the range studied, indicating a positive water vapour feedback on SST variations. The surface flux was found to be insensitive to the use of different lapse rates and relative humidity profiles, even though the outgoing longwave radiation at the top of the atmosphere was sensitive to these quantities. Using HM's Fig.1 and taking a linear fit at 300 K ($\bar{T}_{S1}$) and 278 K ($\bar{T}_{S2}$), the following formulae for the net upward IR fluxes in zones 1 and 2 of our model are found:

$$(Q_I)_1 = (\bar{Q}_I)_1 + a_{I1}(T_{S1} - \bar{T}_{S1}), \quad (29)$$

$$(Q_I)_2 = (\bar{Q}_I)_2 + a_{I2}(T_{S2} - \bar{T}_{S2}), \quad (30)$$

where $(\bar{Q}_I)_1 = 44\ \mathrm{W\ m^{-2}}$, $(\bar{Q}_I)_2 = 90\ \mathrm{W\ m^{-2}}$, $a_{I1} = -2.94\ \mathrm{W\ m^{-2}\ K^{-1}}$ and $a_{I2} = -0.30\ \mathrm{W\ m^{-2}\ K^{-1}}$. The positive water vapour feedback is reflected both in the fact that $(\bar{Q}_I)_1 < (\bar{Q}_I)_2$ and that (a_{I1}, a_{I2}) are negative. The results of HM were obtained using an assumption of clear skies. Thus, the parameterizations (29) and (30) can be expected to give accurate estimates for the tropics, where the net surface IR radiation is virtually independent of cloud cover (Webster, 1994), but only an approximate estimate for the extratropics. Since HM's results are for surface fluxes (even though a complete atmospheric column was considered), we refer to the positive feedback inherent in (29) and (30) as the lower tropospheric WVIR feedback. Evidence showing that the surface IR radiative feedback on SST perturbations is positive when the combined effects of water vapour and clouds are taken into account is provided by the GCM experiments of Randall et al. (1992).

Multiplying (29) and (30) by πa^2, we have the area-integrated IR radiative energy losses for the ocean basins in zones 1 and 2

$$[(F_I)_1, (F_I)_2] = \pi a^2 [(Q_I)_1, (Q_I)_2]. \quad (31)$$

3.6. *Parameterization of the poleward heat transport by ocean currents*

The poleward heat transport by ocean currents is an area of considerable uncertainty, with large differences being found between estimates of the total transport derived from direct oceanographic measurements and estimates derived from residual methods using atmospheric heat transport and satellite radiation measurements (Bryden et al., 1991). Further uncertainty attaches to the division of the total transport into wind-driven and thermohaline components. Direct oceanographic measurements indicate that the transport in the subtropical Atlantic is predominantly due to the thermohaline circulation (Roemmich and Wunsch, 1985) while in the subtropical Pacific it is essentially wind-driven (Bryden et al., *loc. cit.*). It appears reasonable to assume that the total transport will, to a greater or lesser degree, be correlated with the curl of the mean wind stress, which is in turn directly related to the AM transport. An observational study by Carissimo et al. (1985) using radiosonde data and satellite radiation measurements has shown a large seasonal variation in the total transport at 30°N (F_{OH}). A plot of the seasonal means of F_{OH} against ΔZ_{500} derived from their data is shown in Fig. 5. The two quantities are correlated with a coefficient of 0.82 and the least squares linear fit equation relating them can be written

$$F_{OH} = \bar{F}_{OH} + a_{OH}(\Delta Z_{500} - \Delta \bar{Z}_{500}), \quad (32)$$

where $\bar{F}_{OH} = 3.31$ PW (1 PW $= 10^{15}$ W) and $a_{OH} = 1.3 \times 10^{-2}\ \mathrm{PW\ m^{-1}}$, with $\Delta \bar{Z}_{500}$ as specified earlier. In view of (1), eq. (32) is consistent with the expectation of a relationship between ocean heat transport and atmospheric AM transport. The value of $\bar{F}_{OH}$ found above, however, is large by comparison with almost all other estimates and we adopt a value $\bar{F}_{OH} = 2.4$ PW found in a study by Keith (1995), using recent model assimilated atmospheric data and satellite radiation measurements, as more representative. The relationship (32) will otherwise be tentatively retained for the purpose of sensitivity estimation. The parameter a_{OH} is of very uncertain magnitude, but our results will not depend in any essential way on its value. As in the case of the AM transport formula (1),

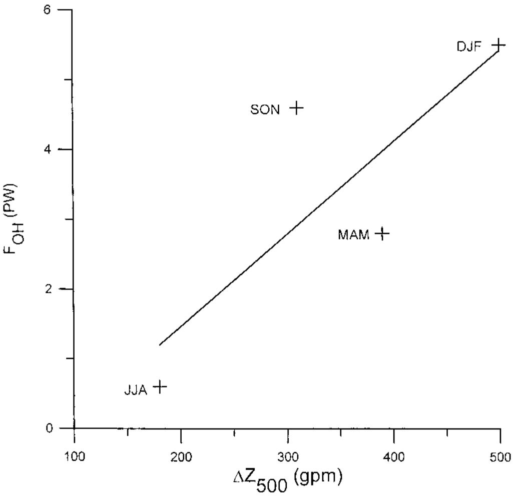

Fig. 5. Seasonal means of the poleward heat transport by ocean currents at 30°N (F_{OH}) versus the difference between the zonally averaged geopotential heights of the 500 hPa surface at 15°N and 50°N (ΔZ_{500}). The least squares linear fit is also shown. (1 PW $= 10^{15}$ W.)

we assume that the relationship found here for the seasonal mean ocean heat transport will also hold for the annual mean transport under conditions of a perturbed climate.

3.7. *Determination of the solar radiation absorbed at the surface*

The amounts of solar energy absorbed by the earth–atmosphere system in zones 1 and 2, denoted EA_1 and EA_2, respectively, are given by

$$EA_1 = 2\pi a^2 \int_0^{30^\circ} (1-\alpha^*) Q \cos\phi \, d\phi, \tag{33}$$

$$EA_2 = 2\pi a^2 \int_{30^\circ}^{90^\circ} (1-\alpha^*) Q \cos\phi \, d\phi, \tag{34}$$

where Q is the time-averaged flux of solar radiation at the top of the atmosphere and α^* is the albedo. The following approximate formula for Q has been given by North (1975):

$$Q = \frac{Q_0}{4}\left[1 + \frac{Q_2}{2}(3\sin^2\phi - 1)\right], \tag{35}$$

where Q_0 is the solar constant (taken as 1365 W m^{-2}) and $Q_2 = -0.482$. We assume α^* is constant in each zone and adopt the mean values given by Nakamura et al. (1994) for their tropical and extratropical zones, viz., $(\alpha_1^*, \alpha_2^*) = (0.25, 0.4)$. Using (35), eqs. (33) and (34) are then evaluated, giving $EA_1 = 38.5$ PW (area average 302 W m^{-2}) and $EA_2 = 21.4$ PW (area average 168 W m^{-2}). It is interesting to note that the mean of EA_1 and EA_2 as calculated here gives a value identical to that obtained by Kiehl and Trenberth (1997) for the global mean solar energy absorbed by the earth–atmosphere system (235 W m^{-2}).

The fractions of EA_1 and EA_2 absorbed at the surface are denoted f_1 and f_2. Thus, the amounts of solar energy absorbed by the ocean basins in zones 1 and 2 are given by

$$[S_1, S_2] = [f_1(EA)_1, f_2(EA)_2]. \tag{36}$$

The quantities f_1 and f_2 are taken as adjustable model parameters. Their values are chosen to obtain energy balance in the equilibrium climate (Section 4).

4. The model's equilibrium climate

The latent, sensible and IR radiative heat fluxes corresponding to the model's equilibrium climate are now calculated. The equilibrium value of the ocean heat transport is taken from observational estimates as $\bar{F}_{OH} = 2.4$ PW (see discussion in Subsection 3.6) and the adjustable parameters f_1 and f_2 are determined from the requirement that the equilibrium climate satisfy the steady-state version of eqs. (2) and (3), i.e.,

$$(\bar{F}_L)_1 + (\bar{F}_H)_1 + (\bar{F}_I)_1 + \bar{F}_{OH} = S_1, \tag{37}$$

$$(\bar{F}_L)_2 + (\bar{F}_H)_2 + (\bar{F}_I)_2 - \bar{F}_{OH} = S_2, \tag{38}$$

where the overbars denote equilibrium quantities. As already indicated, the following quantities are prescribed from observation: $\Delta\bar{Z}_{500} = 345$ m, $\bar{T}_{S1} = 300$ K, $\bar{T}_{S2} = 278$ K, $\Delta T = 1$ K, $r = 0.8$ (Note: a slightly different value, $r = 0.77$, was used in the radiative calculations of HM, from which we derive our parameterization of surface IR radiation. This slight inconsistency is assumed to have no significant consequence.)

The surface winds, which are used to calculate the latent and sensible heat fluxes, are calculated from the AM balance. Setting $\Delta Z_{500} = \Delta\bar{Z}_{500}$ in (1) gives $F_M = \bar{F}_M$ (i.e., 30.3 Hadleys). Substituting this into (9) and (10) gives $\bar{u}_1 = -5$ m s^{-1}, $\bar{u}_2 = 6.25$ m s^{-1}. These values are in good agreement with observation, vindicating the use of AM considerations to derive the mean (and perturbation) surface winds.

Eq. (23) gives $e_s(\bar{T}_{S1}) = 36.5$ hPa, $e_s(\bar{T}_{S2}) = 8.7$ hPa, whence eq. (22) gives $(\Delta\bar{e})_1 = 9.06$ hPa, $(\Delta\bar{e})_2 = 2.23$ hPa. Hence we find from (24) that $(\bar{F}_L)_1 = 16$ PW (area average 127 W m^{-2}), $(\bar{F}_L)_2 = 5$ PW (area average 39 W m^{-2}), $(\bar{F}_H)_1 = 1$ PW (area average 9 W m^{-2}) and $(\bar{F}_H)_2 = 1.4$ PW (area average 11.3 W m^{-2}). The mean of the above values of the latent heat fluxes (area average 83 W m^{-2}) is in reasonable agreement with Kiehl and Trenberth's global mean value (78 W m^{-2}). The distribution between the zones is also in reasonable agreement with the latitudinal distributions presented by Zhang (1996).

The surface energy losses by IR radiation for the equilibrium climate are calculated from (31) using the mean values $(\bar{Q}_I)_1$ and $(\bar{Q}_I)_2$ given in (29) and (30). Hence, $(\bar{F}_I)_1 = 5.6$ PW (area average 44 W m^{-2}) and $(\bar{F}_I)_2 = 11.5$ PW (area average

90 W m^{-2}). The mean of these (area average 67 W m^{-2}) is in excellent agreement with Kiehl and Trenberth's (1997) estimate of the global mean net surface IR flux (66 W m^{-2}).

All terms in (37) and (38) except (S_1, S_2) are now determined, leaving these terms to be calculated from the energy balance requirements. Balance is achieved if we choose $f_1 = 0.650$ and $f_2 = 0.724$ in (36), giving $S_1 = 25$ PW (area average 196 W m^{-2}) and $S_2 = 15.5$ PW (area average 122 W m^{-2}). Kiehl and Trenberth's global estimate of the fraction of the solar energy absorbed by the earth–atmosphere system that is absorbed by the surface (i.e., 0.71) lies between the above values of f_1 and f_2.

The equilibrium climate fluxes of the model are displayed in Fig. 2. It is seen that the dominant term balancing the solar radiation absorbed at the surface in zone 1 is evaporation, while in zone 2 it is net IR radiation. Both of these terms are much larger than the oceanic heat flux $\bar{F}_{\mathrm{OH}}$. The good agreement of our calculated equilibrium fluxes with the best available estimates obtained using observations provides support for the premise that the model represents a reasonable 1st-order model of the climate system.

If we define a surface greenhouse parameter $G_{\mathrm{S}} = (Q_{\mathrm{I}})^{-}/(Q_{\mathrm{I}})^{+}$, where $(Q_{\mathrm{I}})^{+}$ and $(Q_{\mathrm{I}})^{-}$ are the upward and downward components of the IR flux at the surface, and calculate the upward flux from the Stefan–Boltzmann equation, we find using the mean temperatures above and the mean fluxes from (29) and (30) that $(G_{\mathrm{S}})_1 = 0.90$ and $(G_{\mathrm{S}})_2 = 0.73$. This provides a measure of the extent to which the tropical zone of the model can be described as a strong greenhouse and the extra-tropical zone a weak one. The global value of G_{S} given by the surface IR flux components of Kiehl and Trenberth is 0.83, which is close to the mean of the above values.

5. Stability of the equilibrium climate

Having determined the model's equilibrium climate, we study its stability with respect to small perturbations. The governing equations for the perturbations are the linearized perturbation form of the ocean energy eqs. (2) and (3). Setting $T_{\mathrm{S1}} = \bar{T}_{\mathrm{S1}} + T'_{\mathrm{S1}}(t)$, etc., and regarding (S_1, S_2) as fixed at their equilibrium values, we have the perturbation equations:

$$c_{\mathrm{O1}} \frac{\mathrm{d}T'_{\mathrm{S1}}}{\mathrm{d}t} = -[(F_{\mathrm{L}})'_1 + (F_{\mathrm{H}})'_1 + (F_{\mathrm{I}})'_1] - F'_{\mathrm{OH}}, \quad (39)$$

$$c_{\mathrm{O2}} \frac{\mathrm{d}T'_{\mathrm{S2}}}{\mathrm{d}t} = -[(F_{\mathrm{L}})'_2 + (F_{\mathrm{H}})'_2 + (F_{\mathrm{I}})'_2] + F'_{\mathrm{OH}}, \quad (40)$$

where the flux quantities on the r.h.s. are calculated as linearized perturbations about the model's equilibrium climate. They involve the sensitivities of each flux quantity to variations in T_{S1} and T_{S2}, which are now determined.

5.1. *Evaluation of the sensitivities*

The sensitivities (γ_{L11}, etc.)of the various fluxes are defined in Table 1. (All are evaluated at the climate equilibrium). The latent heat flux sensitivities are evaluated from (24) using (18), (26), (27) and (28). Hence we find:

$$(\gamma_{\mathrm{L11}}, \gamma_{\mathrm{L12}}) = (\bar{F}_{\mathrm{L}})_1 \left[\frac{L}{R_{\mathrm{v}} \bar{T}^2_{\mathrm{S1}}} (1, 0) + \frac{a_{\mathrm{M}}}{2\bar{F}_{\mathrm{M}}} (a_{\mathrm{D1}}, -a_{\mathrm{D2}}) \right], \quad (41)$$

$$(\gamma_{\mathrm{L21}}, \gamma_{\mathrm{L22}}) = (\bar{F}_{\mathrm{L}})_2 \left[\frac{L}{R_{\mathrm{v}} \bar{T}^2_{\mathrm{S2}}} (0, 1) + \frac{a_{\mathrm{M}}}{2\bar{F}_{\mathrm{M}}} (a_{\mathrm{D1}}, -a_{\mathrm{D2}}) \right]. \quad (42)$$

In the above, the terms within the square brackets that involve L arise from the sensitivity of the vapour pressure deficit (we refer to them as the Δe-sensitivity terms), while the terms which

Table 1. *Sensitivities of the latent, sensible, radiative and oceanic heat fluxes*

Symbol	Definition	Value (PW K^{-1})
γ_{L11}	$\partial(F_{\mathrm{L}})_1/\partial T_{\mathrm{S1}}$	1.66
γ_{L12}	$\partial(F_{\mathrm{L}})_1/\partial T_{\mathrm{S2}}$	−0.44
γ_{L21}	$\partial(F_{\mathrm{L}})_2/\partial T_{\mathrm{S1}}$	0.22
γ_{L22}	$\partial(F_{\mathrm{L}})_2/\partial T_{\mathrm{S2}}$	0.21
γ_{H11}	$\partial(F_{\mathrm{H}})_1/\partial T_{\mathrm{S1}}$	0.04
γ_{H12}	$\partial(F_{\mathrm{H}})_1/\partial T_{\mathrm{S2}}$	−0.03
γ_{H21}	$\partial(F_{\mathrm{H}})_2/\partial T_{\mathrm{S1}}$	0.06
γ_{H22}	$\partial(F_{\mathrm{H}})_2/\partial T_{\mathrm{S2}}$	−0.04
γ_{I1}	$\partial(F_{\mathrm{I}})_1/\partial T_{\mathrm{S1}}$	−0.37
γ_{I2}	$\partial(F_{\mathrm{I}})_2/\partial T_{\mathrm{S2}}$	−0.04
γ_{O1}	$\partial(F_{\mathrm{OH}})/\partial T_{\mathrm{S1}}$	0.37
γ_{O2}	$\partial(F_{\mathrm{OH}})/\partial T_{\mathrm{S2}}$	−0.23

involve a_M arise from the sensitivity of the wind field (we refer to them as the u-sensitivity terms).

The sensible heat flux sensitivities are evaluated from (24) using (16), (27) and (28). Hence we find

$$(\gamma_{H11}, \gamma_{H12}) = (\bar{F}_H)_1 \left[\frac{a_M}{2\bar{F}_M}\right](a_{D1}, -a_{D2}), \tag{43}$$

$$(\gamma_{H21}, \gamma_{H22}) = (\bar{F}_H)_2 \left[\frac{a_M}{2\bar{F}_M}\right](a_{D1}, -a_{D2}). \tag{44}$$

Here, all terms depend on the sensitivity of the wind field.

The net IR radiative flux sensitivities are evaluated from (31) using (29) and (30). Hence we find $(\gamma_{I1}, \gamma_{I2}) = \pi a^2(a_{I1}, a_{I2})$. Finally, the oceanic heat flux sensitivities are evaluated from (32) using (13), giving $(\gamma_{O1}, \gamma_{O2}) = a_{OH}(a_{D1}, -a_{D2})$.

The numerical values of the sensitivities, calculated from the above formulae using the values of all parameters as specified previously, are given in Table 1.

5.2. *Analytical solution for the perturbations*

The perturbation flux quantities, when linearized about the equilibrium climate, give the following

$$\begin{aligned}&[(F_L)'_1, (F_L)'_2, (F_H)'_1, (F_H)'_2, (F_I)'_1, (F_I)'_2, (F_{OH})']\\ &\quad = [\gamma_{L11}, \gamma_{L21}, \gamma_{H11}, \gamma_{H21}, \gamma_{I1}, 0, \gamma_{O1}]T'_{S1}\\ &\qquad + [\gamma_{L12}, \gamma_{L22}, \gamma_{H12}, \gamma_{H22}, 0, \gamma_{I2}, \gamma_{O2}]T'_{S2}.\end{aligned} \tag{45}$$

Substituting these into (39) and (40) gives

$$\frac{dT'_{S1}}{dt} = -\beta_1 T'_{S1} - \beta_2 T'_{S2}, \tag{46}$$

$$\frac{dT'_{S2}}{dt} = -\beta_3 T'_{S1} - \beta_4 T'_{S2}, \tag{47}$$

where

$$(\beta_1, \beta_2, \beta_3, \beta_4) = (\alpha_1/c_{O1}, \alpha_2/c_{O1}, \alpha_3/c_{O2}, \alpha_4/c_{O2}),$$

with

$$\alpha_1 = \gamma_{L11} + \gamma_{H11} + \gamma_{O1} + \gamma_{I1}, \tag{48}$$

$$\alpha_2 = \gamma_{L12} + \gamma_{H12} + \gamma_{O2}, \tag{49}$$

$$\alpha_3 = \gamma_{L21} + \gamma_{H21} - \gamma_{O1}, \tag{50}$$

$$\alpha_4 = \gamma_{L22} + \gamma_{H22} - \gamma_{O2} + \gamma_{I2}. \tag{51}$$

The solution of the coupled eqs. (46) and (47) is composed of two normal modes. These are obtained by seeking a solution of the form

$$(T'_{S1}, T'_{S2}) = (A, B)\exp(-\mu t). \tag{52}$$

Substitution into (46) and (47) gives

$$(\mu - \beta_1)A - \beta_2 B = 0, \tag{53}$$

$$\beta_3 A - (\mu - \beta_4)B = 0. \tag{54}$$

The discriminant gives

$$\mu = \left(\frac{\beta_1 + \beta_4}{2}\right) \pm \eta, \tag{55}$$

where $\eta^2 = \xi^2 + \beta_2\beta_3$ and $\xi = (\beta_1 - \beta_4)/2$.

The first normal mode, obtained by taking the + sign in (55) and making use of (54), can be written

$$(T'_{S1}, T'_{S2}) = A\left(1, \frac{\beta_3}{\eta + \xi}\right) \times \exp\left[-\left(\frac{\beta_1 + \beta_4}{2} + \eta\right)t\right], \tag{56}$$

where A is an arbitrary constant. The second normal mode, obtained by taking the − sign in (55) and making use of (53), can be written

$$(T'_{S1}, T'_{S2}) = B\left(-\frac{\beta_2}{\eta + \xi}, 1\right) \times \exp\left[-\left(\frac{\beta_1 + \beta_4}{2} - \eta\right)t\right], \tag{57}$$

where B is another arbitrary constant.

From the solution as written above, it is clear that if $\beta_3 \to 0$ the first mode reduces to the uncoupled solution for zone 1, while if $\beta_2 \to 0$ the second mode reduces to the uncoupled solution for zone 2.

The general solution is obtained by adding the above expressions; thus

$$T'_{S1} = A\exp\left[-\left(\frac{\beta_1 + \beta_4}{2} + \eta\right)t\right] - \left(\frac{\beta_2}{\eta + \xi}\right) \times B\exp\left[-\left(\frac{\beta_1 + \beta_4}{2} - \eta\right)t\right], \tag{58}$$

$$T'_{S2} = \left(\frac{\beta_3}{\eta + \xi}\right)A\exp\left[-\left(\frac{\beta_1 + \beta_4}{2} + \eta\right)t\right] + B\exp\left[-\left(\frac{\beta_1 + \beta_4}{2} - \eta\right)t\right]. \tag{59}$$

If we choose the constants of integration so as to satisfy the initial conditions $(T'_{S1}, T'_{S2}) = (\tau_1, \tau_2)$ at $t = 0$, the solutions (58) and (59) can be written in the form:

$$T'_{S1} = \left[\tau_1 \cosh \eta t - (\tau_1 \xi + \tau_2 \beta_2) \frac{\sinh \eta t}{\eta} \right] \times \exp\left[-\left(\frac{\beta_1 + \beta_4}{2} \right) t \right], \tag{60}$$

$$T'_{S2} = \left[\tau_2 \cosh \eta t - (\tau_1 \beta_3 - \tau_2 \xi) \frac{\sinh \eta t}{\eta} \right] \times \exp\left[-\left(\frac{\beta_1 + \beta_4}{2} \right) t \right]. \tag{61}$$

Hence we see that, for all values of the parameters, arbitrary initial conditions can be fitted by a linear combination of the normal modes without incurring any singularity in the solution.

The conditions for stability (such that both normal modes are exponentially decaying in amplitude) are seen from (56) and (57) to be as follows:

(a) If $\eta^2 > 0$, it is necessary and sufficient that both of the following conditions hold

$$\frac{\beta_1 + \beta_2}{2} + \eta > 0, \tag{62}$$

$$\frac{\beta_1 + \beta_2}{2} - \eta > 0. \tag{63}$$

These conditions can be written more conveniently as

$$\beta_1 + \beta_4 > 0, \tag{64}$$

$$\beta_1 \beta_4 - \beta_2 \beta_3 > 0. \tag{65}$$

When these conditions are satisfied, the first (fast) and second (slow) normal modes have characteristic decay times given by

$$TD_{f,s} = \left[\frac{\beta_1 + \beta_4}{2} \pm \eta \right]^{-1}. \tag{66}$$

(b) If $\eta^2 \leqslant 0$, (64) alone is necessary and sufficient for stability.

5.3. *Interpretation*

Even though our model of the climate system is extremely simplified and allows an analytical solution, the physical interpretation of the stabilizing mechanism is not self-evident. Here we interpret the analytical solution in terms of the stability and characteristic decay times of the normal modes, examining how these vary for different choices of the model parameters.

An alternative approach to seeking a physical interpretation would be to examine the initial growth or decay characteristics of particular perturbations. For instance, one could consider the initial perturbation $T'_{S1} = T'_{S2} = 1$ K. In such a case there would be an initial enhancement of the AM transport (assuming $a_{D1} > a_{D2}$), leading to increased evaporation and a tendency for initial decay of the perturbation in both zones due to both the wind and humidity factors. In this case, the role of the AM transport as a stabilizing influence appears to be immediately apparent. However, if one considers an initial perturbation $T'_{S1} = 0$, $T'_{S2} = -1$ K, the initial enhancement of the AM transport tends to amplify the perturbation in zone 2 while leading to a non-zero perturbation in zone 1. In this case, the initial tendency approach provides no insight into what role the AM transport might eventually play in stabilizing the system. The initial tendency approach is made complicated by the fact that the system can be stable and yet allow perturbations which are initially growing, and vice versa. The approach of examining the stability of the system through the stability criteria of the normal modes is free from such ambiguity.

We note that the ocean heat capacities (c_{O1}, c_{O2}) do not enter into the solution for the equilibrium climate, but are important in determining the characteristics of the perturbations. Their main influence is in determining the time constants of the perturbation modes. (If $c_{O1} = c_{O2}$, this is their only influence; if $c_{O1} \neq c_{O2}$, however, they could even, through criterion (64), determine whether the system is stable or unstable.) We define $R \equiv c_{O1}/c_{O2}$ and assume that $R \leqslant 1$ for all cases of interest.

We consider a series of special cases leading up to the general case.

(i) If only the IR radiative sensitivities are considered we have $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\gamma_{I1}, 0, 0, \gamma_{I2})$. It this case $\eta^2 > 0$ and, taking the values $(\gamma_{I1}, \gamma_{I2})$ given in Table 1, it is seen that criterion (64) is violated. The perturbations in zones 1 and 2 are uncoupled and both zones are individually unstable, the perturbations having e-folding times of $c_{O1}/|\gamma_{I1}|$ and $c_{O2}/|\gamma_{I2}|$ in the respective zones. If we take $H_1 = 100$ m and $H_2 = 500$ m, we have 4.6 years and 212 years as the respective e-folding times. This case, reflecting the destabilizing influence of the WVIR feedback, provides the reference against which the influence of the other perturbation factors are to be measured.

(ii) If only the radiative and oceanic heat transport sensitivities are considered, we have

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\gamma_{I1} + \gamma_{O1}, \gamma_{O2}, -\gamma_{O1}, \gamma_{I2} - \gamma_{O2}).$$

The zones are now coupled. Since γ_{O1} and γ_{O2} are of opposite sign it follows that $\beta_2\beta_3 > 0$ and hence $\eta^2 > 0$. Using the values given in Table 1, we see that condition (64) is satisfied but condition (65) is violated, i.e., the solution is unstable. This result holds more generally; it can be shown (see Section 10) that at least one of (64) and (65) is violated (and the solution is therefore unstable) as long as $\gamma_{I1} < 0, \gamma_{I2} < |\gamma_{I1}|(a_{D2}/a_{D1})$ and $R < a_{D1}/a_{D2}$. Thus, for all reasonable values of the radiative parameters and ocean heat capacities, the ocean heat transport sensitivities alone, regardless of the value of a_{OH}, are incapable of overcoming the destabilizing effect of the WVIR feedback.

(iii) If only the radiative and sensible heat flux sensitivities are considered we have

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\gamma_{I1} + \gamma_{H11}, \gamma_{H12}, \gamma_{H21}, \gamma_{I2} + \gamma_{H22}).$$

Using the values given in Table 1, we see that criterion (64) is violated, i.e., the sensible heat flux sensitivities alone are also unable to overcome the WVIR feedback. (They would have to be multiplied by a factor of almost 12 to allow them to act as a stabilizer).

(iv) If only the radiative and latent heat flux sensitivities are considered we have

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\gamma_{I1} + \gamma_{L11}, \gamma_{L21}, \gamma_{L21}, \gamma_{I2} + \gamma_{L22}).$$

Using the values in Table 1, we see that $\eta^2 > 0$ for $R \leqslant 1$ and criteria (64) and (65) are satisfied for all R, i.e., the latent heat flux sensitivities alone are able to overcome the WVIR feedback and stabilize the system. Taking the values of (H_1, H_2) given above, we find that the characteristic decay times of the fast and slow normal modes are 1.33 years and 34.2 years, respectively.

This case provides the central result of the present paper: a dynamical stabilizer on SST perturbations acting through the interlinked effects of AM transport and evaporation exists in the 1st-order climate model and is of sufficient strength to overcome the WVIR feedback.

There are several points to be noted about the stabilizing mechanism. Firstly, the Δe-sensitivity and u-sensitivity components of the evaporative sensitivities defined by (41) and (42) are of comparable magnitude. The Δe-sensitivity terms involve only the mean AM transport $\bar{F}_M$, while the u-sensitivity terms involve both the mean transport and the gradient of the transport w.r.t. ΔZ_{500}. With the parameters set at their standard values used in Table 1, it turns out that both the Δe-sensitivity terms on their own and the u-sensitivity terms on their own are capable of countering the WVIR feedback (the fast and slow normal modes have characteristic decay times of 2.9 years and 27.3 years in the former case, and 6.4 years and 58.6 years the latter case, using the values of H_1 and H_2 given above).

Secondly, when the Δe-sensitivities and the u-sensitivities are both included, the perturbation evaporative fluxes $(F_L)'_1$ and $(F_L)'_2$ can be dominated by the contribution from either sensitivity component. This can be seen by examining the fluxes corresponding to the individual normal modes (see Section 11).

Thirdly, even though our standard model parameters are such $a_{D1} > a_{D2}$ (i.e., the sensitivity of the 500 hPa height to changes in SST is greater in zone 1 than in zone 2), the stability of the model's equilibrium climate does not depend on this being the case. This is easily seen by defining $A_1 \equiv (a_M a_{D1})/(2\bar{F}_M)$, $\lambda \equiv a_{D2}/a_{D1}$ and setting all the parameters except the u-sensitivities equal to their standard values. Hence, adopting the values for (H_1, H_2) used earlier, we have

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.59 + 16A_1, -16\lambda A_1, 5A_1, 0.31 - 5\lambda A_1). \tag{67}$$

Then we find that

$$\eta^2 = \left(\frac{1}{2c_{O1}}\right)^2 \times [0.28 + 1.06(16 + \lambda)A_1 + (16 - \lambda)^2 A_1^2], \tag{68}$$

$$\beta_1 + \beta_4 = \frac{1}{c_{O1}}[0.65 + (16 - \lambda)A_1], \tag{69}$$

$$\beta_1\beta_4 - \beta_2\beta_3 = \frac{1}{c_{O1}c_{O2}}[0.18 + 4.96(1 - 0.59\lambda)A_1]. \tag{70}$$

From the above, it is clear that $\eta^2 > 0$ and criteria (64) and (65) are easily satisfied for λ lying between zero and values in excess of three, i.e., the stabilizing mechanism is quite robust with respect to variation in the ratio a_{D2}/a_{D1}.

(v) Finally, we consider the general case where all the sensitivities are included, their values being taken from Table 1. In this case $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (1.70, -0.70, -0.09, 0.36)$ PW K^{-1}. It is found that $\eta^2 > 0$, and the criteria (64) and (65) are easily satisfied (regardless of the ratio of H_1 to H_2), so that the system is stable. If H_1 and H_2 are assigned the same values as above, it is found that the characteristic decay times of the fast and slow normal modes are 1.0 years and 26.3 years, respectively. Thus, the inclusion of the ocean heat transport sensitivity and the surface sensible heat flux sensitivity makes the system slightly more stable than in case (iv), where only the evaporative sensitivities counter the WVIR feedback, but no qualitative change is introduced.

If the evaporative sensitivities are omitted while all other sensitivities are included with their values taken from Table 1, the system is unstable for all R. Thus, the evaporative sensitivities are essential for stability. It is found that the evaporative sensitivities can be reduced to about one quarter of their Table 1 values before instability sets in. The dynamical stabilizer is thus seen to be robust, lending confidence to the assumption that the omission of mountain torques in the AM balance, for instance, is not a critical factor as far as the validity of the stabilizing mechanism is concerned.

If the radiative sensitivities are omitted while all other sensitivities are included with their values taken from Table 1, the system is stable for all R. If H_1 and H_2 are assigned the same values as above, it is found that the characteristic decay times of the fast and slow modes are 0.82 year and 23 years, respectively. Thus, the combined turbulent surface flux and ocean heat transport sensitivities act as a stabilizer. The decay times are, as expected, shorter than in the general case where the radiative sensitivities are included.

6. Consequences of changing the model to a Budyko–Sellers type formulation

In assigning such a central importance to the AM transport and the induced surface fluxes both in describing the equilibrium climate and in studying its stability in the face of the positive WVIR feedback, our model takes a different 1st-order view of the climate system than do the many models based on the Budyko–Sellers formulation. In the latter formulation, the surface fluxes are eliminated by making use of an atmospheric energy balance equation. The ocean energy equations are then expressed in terms of top-of-the-atmosphere radiative fluxes and poleward transports of energy in the atmosphere. The consequences of changing our two-zone model to a Budyko–Sellers formulation are explored in this section.

Letting $F^{\uparrow}_{\mathrm{SFC}}$ denote the area-integrated net upward energy flux at the surface in a given zone, we have

$$(F^{\uparrow}_{\mathrm{SFC}})_1 = (F_L)_1 + (F_H)_1 + (F_I)_1 - S_1, \tag{71}$$

$$(F^{\uparrow}_{\mathrm{SFC}})_2 = (F_L)_2 + (F_H)_2 + (F_I)_2 - S_2. \tag{72}$$

Thus, our ocean energy eqs. (2) and (3) can be re-written as

$$c_{O1}\frac{\mathrm{d}T_{S1}}{\mathrm{d}t} = -(F^{\uparrow}_{\mathrm{SFC}})_1 - F_{OH}, \tag{73}$$

$$c_{O2}\frac{\mathrm{d}T_{S2}}{\mathrm{d}t} = -(F^{\uparrow}_{\mathrm{SFC}})_2 + F_{OH}. \tag{74}$$

Assuming the atmosphere to be in a state of balance, the atmospheric energy equations for zones 1 and 2 are

$$(F^{\downarrow}_{\mathrm{TOA}})_1 + (F^{\uparrow}_{\mathrm{SFC}})_1 = F_E, \tag{75}$$

$$(F^{\downarrow}_{\mathrm{TOA}})_2 + (F^{\uparrow}_{\mathrm{SFC}})_2 = -F_E, \tag{76}$$

where $F^{\downarrow}_{\mathrm{TOA}}$ denotes the area-integrated net downward radiative energy flux at the top of the atmosphere in a given zone and F_E denotes the vertically integrated poleward transport of moist static energy in the atmosphere at 30°. Using (75) and (76), we eliminate $F^{\uparrow}_{\mathrm{SFC}}$ from (73) and (74) to give

$$c_{O1}\frac{\mathrm{d}T_{S1}}{\mathrm{d}t} = (F^{\downarrow}_{\mathrm{TOA}})_1 - (F_E + F_{OH}), \tag{77}$$

$$c_{O2}\frac{\mathrm{d}T_{S2}}{\mathrm{d}t} = (F^{\downarrow}_{\mathrm{TOA}})_2 + (F_E + F_{OH}). \tag{78}$$

$F^{\downarrow}_{\uparrow\text{TOA}}$ is the difference between the solar energy absorbed by the earth–atmosphere system and the outgoing long-wave radiation (OLR) in a given zone. In the Budyko–Sellers formulation, the OLR per unit area is parameterized as $A+BT_{\text{S}}$, where T_{S} is the SST expressed in degrees Celsius and typical values for the parameters (Nakamura et al., 1994) are $A=212\ \text{W m}^{-2}$ and $B=1.7\ \text{W m}^{-2}$ $(^{\circ}\text{C})^{-1}$. Thus,

$$(F^{\downarrow}_{\uparrow\text{TOA}})_1 = EA_1 - \pi a^2(A+BT_{\text{S1}}), \tag{79}$$

$$(F^{\downarrow}_{\uparrow\text{TOA}})_2 = EA_2 - \pi a^2(A+BT_{\text{S2}}). \tag{80}$$

Using the representation of $F^{\downarrow}_{\uparrow\text{TOA}}$ given by (79) and (80), eqs. (77) and (78) become

$$c_{\text{O1}}\frac{\text{d}T_{\text{S1}}}{\text{d}t} = EA_1 - \pi a^2(A+BT_{\text{S1}}) - (F_{\text{E}}+F_{\text{OH}}), \tag{81}$$

$$c_{\text{O2}}\frac{\text{d}T_{\text{S2}}}{\text{d}t} = EA_2 - \pi a^2(A+BT_{\text{S2}}) + (F_{\text{E}}+F_{\text{OH}}). \tag{82}$$

We refer to (81) and (82) as the ocean energy equations in Budyko–Sellers form.

An examination of the same atmospheric data set as was used in deriving our AM parameterization formula (1) shows that F_{E} can be similarly parameterized as

$$F_{\text{E}} = \bar{F}_{\text{E}} + a_{\text{E}}(\Delta Z_{500} - \Delta\bar{Z}_{500}), \tag{83}$$

where $\bar{F}_{\text{E}}=2.73$ PW and $a_{\text{E}}=0.6\times10^{-2}$ PW m^{-1}. This parameterization is derived using the seasonal mean values of the total (transient plus stationary) eddy transport of moist static energy at 30°N. The four seasonal points give a linear fit comparable to that shown for F_{M} in Fig. 1. The variation from summer to winter is not as great for F_{E} as for F_{M} (the former varies by a factor of approx. 2, while the latter varies by a factor of 3).

Using the parameterization (83), the ocean energy eqs. (81) and (82) can be used, just as were eqs. (2) and (3), to examine the mean and perturbation climate of the two-zone model. The equilibrium climate, obtained by setting $\text{d}/\text{d}t=0$ in (81) and (82), and taking the values of $(A, B, \bar{F}_{\text{E}})$ given above along with the values of $(\bar{T}_{\text{S1}}, \bar{T}_{\text{S2}}, \bar{F}_{\text{OH}})$ used earlier, give $EA_1=38$ PW and $EA_2=23$ PW. These are in good agreement with the values $EA_1=38.5$ PW and $EA_2=21.4$ PW given in Subsection 3.7.

The perturbation ocean energy equations given by (81) and (82), with EA_1 and EA_2 held fixed, are

$$c_{\text{O1}}\frac{\text{d}T'_{\text{S1}}}{\text{d}t} = -\pi a^2BT'_{\text{S1}} - (F'_{\text{E}}+F'_{\text{OH}}), \tag{84}$$

$$c_{\text{O2}}\frac{\text{d}T'_{\text{S2}}}{\text{d}t} = -\pi a^2BT'_{\text{S2}} + (F'_{\text{E}}+F'_{\text{OH}}). \tag{85}$$

Using (83) and (13), we expand F_{E} linearly about the mean climate to give

$$F'_{\text{E}} = \gamma_{\text{E1}}T'_{\text{S1}} + \gamma_{\text{E2}}T'_{\text{S2}}, \tag{86}$$

where $(\gamma_{\text{E1}}, \gamma_{\text{E2}}) = a_{\text{E}}(a_{\text{D1}}, -a_{\text{D2}})$. Using (86) along with the oceanic heat transport sensitivities given in Subsection 5.1, i.e., $(\gamma_{\text{O1}}, \gamma_{\text{O2}}) = \text{a}_{\text{OH}}(a_{\text{D1}}, -a_{\text{D2}})$, eqs. (84) and (85) can be written in the form (46) and (47), with $(\beta_1, \beta_2, \beta_3, \beta_4) = (\alpha_1/c_{\text{O1}}, \alpha_2/c_{\text{O1}}, \alpha_3/c_{\text{O2}}, \alpha_4/c_{\text{O2}})$, and:

$$\alpha_1 = \pi a^2B + a_{\text{D1}}(a_{\text{E}}+a_{\text{OH}}), \tag{87}$$

$$\alpha_2 = -a_{\text{D2}}(a_{\text{E}}+a_{\text{OH}}), \tag{88}$$

$$\alpha_3 = -a_{\text{D1}}(a_{\text{E}}+a_{\text{OH}}), \tag{89}$$

$$\alpha_4 = \pi a^2B + a_{\text{D2}}(a_{\text{E}}+a_{\text{OH}}). \tag{90}$$

The solution is again composed of two normal modes which can be written in the form (56), (57). When the initial conditions are specified, the solution can be written in the form (60), (61). Since $\alpha_2\alpha_3>0$, it is clear that $\eta^2>0$, so that the stability criteria are (64) and (65). Using (87)–(90), we see that

$$\beta_1+\beta_4 = \frac{1}{c_{\text{O1}}} \times [\pi a^2B(1+R) + (a_{\text{D1}}+Ra_{\text{D2}})(a_{\text{E}}+a_{\text{OH}})], \tag{91}$$

$$\beta_1\beta_4-\beta_2\beta_3 = \frac{\pi a^2B}{c_{\text{O1}}c_{\text{O2}}} \times [\pi a^2B + (a_{\text{D1}}+a_{\text{D2}})(a_{\text{E}}+a_{\text{OH}})]. \tag{92}$$

For $B>0$, it is clear from (91) and (92) that criteria (64) and (65) are satisfied for all R, i.e., the model is always stable when the Budyko–Sellers formulation is used. Specifying B as a positive quantity implies that IR radiation always acts as a negative feedback on SST perturbations, in the manner of a Stefan–Boltzmann feedback, and that it always combines with the atmospheric and oceanic heat transports to stabilize the system.

If we allow $B<0$ (i.e., a positive radiative feedback) the stability criteria (64) and (65) cannot be satisfied simultaneously. For, using (91) and (92), the criteria then become

$$\frac{a_{D1}+Ra_{D2}}{1+R}>\frac{\pi a^2|B|}{a_E+a_{OH}}, \tag{93}$$

$$a_{D1}+a_{D2}<\frac{\pi a^2|B|}{a_E+a_{OH}}. \tag{94}$$

These can only be satisfied simultaneously if

$$a_{D1}+a_{D2}<\frac{a_{D1}+Ra_{D2}}{1+R}. \tag{95}$$

It is easily seen that this is impossible, since (a_{D1}, a_{D2}, R) are all positive quantities. Thus, the combined atmospheric and oceanic heat transport sensitivities cannot overcome a positive radiative feedback on SST perturbations in the Budyko–Sellers formulation. This is in marked contrast to the results presented in Section 5, where the combined surface flux and oceanic heat transport sensitivities can overcome such a positive feedback.

If we take $B=0$ (i.e., no radiative feedback), it is seen using (92) that criterion (65) cannot be satisfied. Thus, the combined atmospheric and oceanic heat flux sensitivities on their own cannot act as a stabilizer.

7. Conclusions and discussion

A simple two-zone model of the climate system based on ocean energy equations and the direct calculation of surface energy fluxes has been developed. The principal empirical inputs to the model are the solar constant, mean values of the albedo for the two model zones, and parameterization formulae for the poleward atmospheric transport of AM at 30°, the poleward oceanic transport of heat at 30° and the variations with SST of the extratropical 500 hPa height. The bulk aerodynamic formulae for the turbulent surface fluxes of momentum, moisture and sensible heat are used, with coefficients determined from observation. The equilibrium SSTs in the two model zones are prescribed from observation, along with the mean values of the low level air–sea temperature difference and relative humidity. Theoretical inputs are the parameterization of the net surface IR radiation (taken from the results of a radiative model) and of the variation with SST of the tropical 500 hPa height (based on moist adiabatic ascent).

The major model assumptions are: (a) the empirical relationship found between AM transport and 500 hPa height difference using observed seasonal means also holds for the annual means under conditions of a perturbed climate; (b) the atmosphere is in a state of dynamic balance on climatic timescales, so that the mean zonal surface winds can be determined from the AM transport (mountain torques are ignored and the mean zonal surface winds are taken as uniform in each zone); (c) the low-level air–sea temperature difference and relative humidity remain invariant when the climate is perturbed. The surface zonal winds determined from the AM balance are used to calculate the turbulent surface fluxes of latent and sensible heat. For the purpose of formulating the ocean energy equations and of calculating the surface fluxes, the planet is assumed ocean covered.

The only tuning parameters in the model are the fractions of the total absorbed solar radiation in each zone that are absorbed at the surface. These are tuned so as to give energy balance in the equilibrium ocean energy equations. It turns out that the tuned values of these parameters, and the calculated values of all the equilibrium fluxes, are in satisfactory agreement with estimates derived using satellite observations. This lends confidence to the assumption that the model represents a reasonable 1st-order model of the climate system.

The picture of the equilibrium climate portrayed by the model is that of a strong greenhouse in the tropics interacting with a weaker greenhouse in the extratropics, the principal means of interaction between the two zones being atmospheric AM transport. The model zones also interact through poleward heat transport by ocean currents, but this is of secondary importance in the present context. The solar radiation absorbed by the ocean in the tropical zone is balanced mainly by evaporation while in the extratropical zone it is balanced mainly by net infrared radiation.

Having determined the model's equilibrium climate, a linear perturbation analysis is performed to study its stability. The perturbation equations again use the parameterization formulae that have been employed in calculating the equilibrium climate quantities and make use of the mean turbulent fluxes thus determined. Cloud feedbacks, the feedback of water vapour on the solar radiation absorbed at the surface, and the ice-albedo feedback are omitted in the perturbation analysis, and attention is focussed on the competing effects of the WVIR feedback and the dynamical feedback associated with the AM cycle. The WVIR feedback tends to destabilize each zone individually but on its own leads to no interaction between the zones. The dynamical feedback, acting through the turbulent surface fluxes of latent and sensible heat induced by the AM transport, involves a coupling of the zones. It is found that the dynamical feedback is stabilizing, and of sufficient strength to overcome the WVIR feedback. It acts principally through the evaporative fluxes, and involves wind sensitivity and humidity sensitivity factors that are of comparable importance. The timescale of the perturbations depends on the thermodynamically effective depths of the two ocean basins and is measured in years to decades. Within the limitations of the simple model the dynamical stabilizing mechanism is robust, in the sense that stability continues to obtain even if the parameters determining the rates of evaporation are reduced by large fractional amounts. It is found that the ocean heat transport sensitivity on its own is incapable of overcoming the WVIR feedback and plays a minor role in determining the overall stability characteristics of the model.

In including AM transport and calculating the surface fluxes explicitly, the model presented here differs from the many simple climate models based on the Budyko–Sellers formulation. Those models eliminate the surface fluxes in favour of top-of-the-atmosphere radiative fluxes and meridional atmospheric energy transports through the use of an atmospheric energy balance equation. The outgoing long-wave radiation is parameterized in terms of the local surface temperature. The consequences of such a procedure are investigated in the context of the present 2-zone model. It is shown that the Budyko-Sellers approach gives a different picture of the equilibrium climate and leads to quite different conclusions about the maintenance of its stability. The equilibrium climate picture, though compatible with that derived using the surface flux approach, provides no insight into the importance of the AM cycle. It is found when using the Budyko–Sellers approach that the combined atmospheric and oceanic heat fluxes do not stabilize the equilibrium climate and that stability can be achieved only by having IR radiation act as a negative feedback on surface temperature perturbations. This is in contrast to the results obtained when the surface fluxes are calculated explicitly, in which case the combined surface flux and ocean heat transport sensitivities act as a stabilizer even in the presence of a positive WVIR feedback. The dynamical stabilizing mechanism that emerges when the surface fluxes are treated explicitly cannot be identified using the Budyko–Sellers formulation.

If the dynamical stabilizing mechanism found here is indeed an important feedback in the real climate system, it is essential to reproduce it correctly in coupled climate models. In many of the atmospheric GCMs currently used in coupled simulations, the resolution may not be sufficient to ensure that the AM cycle has converged. Boville (1991) has found that AM transport can increase by a factor of more than two in going from T21 to T63 resolution in a spectral GCM, with the T63 results giving favourable comparison with observation. Inadequate representation of the AM cycle through low resolution may cause coupled models to have difficulty in reaching a proper climate equilibrium and may cause them to underestimate climate stability. The necessity of using flux corrections as an artificial stabilization technique in many current coupled models may be related to this inadequacy.

The model presented here does not consider anthropogenic perturbations to the climate system. It suggests, however, that in considering the effects of increased CO_2, some attention should be directed to studying the resulting changes in the atmospheric AM cycle and its effect on the surface winds and evaporation. Existing studies in this area, dating from the early paper of Manabe and Wetherald (1975), have noted that an increase in CO_2 leads to an increase in global evaporation. In explaining this increase, however, attention has been focussed on the humidity factor in evaporation and the wind factor has received little consideration.

It is likely that drastic cooling of the climate such as occurred during ice ages would inhibit the effectiveness of the present stabilizing mechanism, both through the reduction of the mean evaporative fluxes resulting from the decreased temperature and through the mechanical insulation of the ocean by ice cover. Under such circumstances, a weakening of the dynamical stabilizer could be a factor leading to a more unstable climate such as appears from ice-core data to have occurred during the last glaciation (Dansgaard et al., 1993; Ditlevsen et al., 1996).

The AM cycle of a planetary atmosphere is intimately related to the planet's rate of rotation. GCM experiments with varying rates of rotation (e.g., Williams and Holloway, 1982) have shown that the AM cycle and surface winds become very weak as the rate of rotation is reduced to zero. The present model results, which suggest that the stability of the Earth's climate is essentially dependent on the AM cycle, thus lead one to the view that the fast rotation rate of the Earth may be an important factor maintaining the stability of its climate. If the Earth were a slowly rotating planet such as Venus its present surface energy balance could not be maintained. With a reduced AM cycle and weaker surface winds, the surface temperature would rise. The stabilizing effect of evaporation would be inhibited, since even the humidity factor in evaporation requires the existence of a mean surface wind. One is led to speculate that with the WVIR feedback becoming increasingly positive with rising temperature (as predicted by radiative models) the ultimate consequence of slow rotation could be instability leading to a runaway greenhouse.

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9. Appendix A

Symbols and abbreviations

$\bar{\psi}$ = value of any quantity ψ for the model's equilibrium climate (unless otherwise defined below).
$\psi' = \psi - \bar{\psi}$
a = earth's radius (6370 km).
a_{D1} = sensitivity of D_1 to T_{1000} (28.4 m K^{-1}); adopted as the sensitivity of Z_1 to T_{S1}.
a_{D2} = sensitivity of D_2 to T_{S2} (17.8 m K^{-1}); adopted as the sensitivity of Z_2 to T_{S2}.
a_E = sensitivity of F_E to ΔZ_{500} (0.6×10^{-2} PW m^{-1}).
a_{I1} = sensitivity of net IR flux to SST in zone 1 (-2.94 W m^{-2} K^{-1}).
a_{I2} = sensitivity of net IR flux to SST in zone 2 (-0.30 W m^{-2} K^{-1}).
a_M = sensitivity of F_M to ΔZ_{500} (9.35×10^{-2} Hadley m^{-1}).
a_{OH} = sensitivity of F_{OH} to ΔZ_{500} (1.3×10^{-2} PW m^{-1}).
AM = angular momentum.
c_D = drag coefficient (1.3×10^{-3}).
c_E = turbulent exchange coefficient for water vapour (1.5×10^{-3}).
c_H = turbulent exchange coefficient for sensible heat (1.5×10^{-3}).
c_{O1} = ocean heat capacity in zone 1.
c_{O2} = ocean heat capacity in zone 2.
c_p = specific heat of dry air at constant pressure (1004 J kg^{-1} K^{-1}).
c_{pw} = specific heat of sea water (4187 J kg^{-1} K^{-1}).
D_1 = thickness of the 1000–500 hPa layer in zone 1.
D_2 = as D_1, but for zone 2.
e = vapour pressure.
e_s = saturation vapour pressure.
EA_1 = solar energy absorbed by the earth–atmosphere system in zone 1.
EA_2 = as EA_1, but for zone 2.
f_1 = fraction of EA_1 absorbed at the surface.
f_2 = fraction of EA_2 absorbed at the surface.
F_E = vertically integrated poleward transport of moist static energy by transient plus stationary eddies at 30°N.
$\bar{F}_E$ = base value of F_E defined by the linear fit eq. (83) (2.73 PW).
$(F_H)_1$ = area-integrated sensible heat flux from the ocean surface in zone 1.
$(F_H)_2$ = as $(F_H)_1$, but for zone 2
$(F_I)_1$ = area-integrated net IR radiative heat flux from the ocean surface in zone 1.
$(F_I)_2$ = as $(F_I)_1$, but for zone 2.
$(F_L)_1$ = area-integrated latent heat flux from the ocean surface in zone 1.
$(F_L)_2$ = as $(F_L)_1$, but for zone 2.

F_M = vertically integrated poleward AM transport by transient plus stationary eddies at 30°N (denotes seasonal mean values in the data study and annual mean values in the model).
$\bar{F}_M$ = base value of F_M defined by the linear fit eq. (1) (30.3 Hadleys); adopted as the value for the model's equilibrium climate.
F_{OH} = poleward transport of heat across 30° by ocean currents.
$\bar{F}_{OH}$ = estimated value of F_{OH} for the current annual mean climate (2.4 PW); adopted as the value for the model's equilibrium climate.
g = gravitational acceleration (9.81 m s^{-2}).
GCM = general circulation model.
gpm = geopotential metre.
H_1 = thermodynamically effective depth of the ocean in zone 1.
H_2 = as H_1, but for zone 2.
IR = infrared.
K = Kelvin (degrees).
L = latent heat of vaporization of water (2.5×10^6 J kg^{-1}).
p_0 = reference value of surface pressure (1000 hPa).
PW = petawatt (10^{15} watt).
q = specific humidity.
q_A = specific humidity of the air at the 10 m level.
q_s = saturation specific humidity.
Q_0 = solar constant (1365 W m^{-2}).
$(Q_H)_1$ = sensible heat flux per unit area in zone 1
$(Q_H)_2$ = as $(Q_H)_1$, but for zone 2.
$(Q_I)_1$ = net (upward minus downward) IR radiative flux per unit area in zone 1.
$(Q_I)_2$ = as $(Q_I)_1$, but for zone 2.
$(Q_L)_1$ = latent heat flux per unit area in zone 1.
$(Q_L)_2$ = as $(Q_L)_1$, but for zone 2.
r = relative humidity at 10 m above the sea surface.
$R = c_{O1}/c_{O2}$.
R_d = gas constant for dry air (287 J kg^{-1} K^{-1}).
R_v = gas constant for water vapour (461 J kg^{-1} K^{-1}).
SST = sea surface temperature.
S_1 = area-integrated net solar energy flux into the ocean surface in zone 1.
S_2 = as S_1, but for zone 2.
t = time.
T = temperature.
T_A = air temperature at 10 m above the sea surface.
TD_f = characteristic decay time of the fast normal mode.
TD_s = characteristic decay time of the slow normal mode.
T_S = sea surface temperature.
T_{S1} = area-averaged SST in zone 1.
$\bar{T}_{S1}$ = value of T_{S1} for the current annual mean climate and for the model's equilibrium climate; approximated by the observed value at 15°N (300 K).
T_{S2} = area-averaged SST in zone 2.
$\bar{T}_{S2}$ = value of T_{S2} for the current annual mean climate and for the model's equilibrium climate; approximated by the observed value at 50°N (278 K).
T_{1000} = air temperature at 1000 hPa.
u = surface zonal wind.
u_1 = surface zonal wind in zone 1 (assumed spatially uniform).
u_2 = surface zonal wind in zone 2 (assumed spatially uniform).
WVIR = water vapour/infrared radiative (feedback on SST perturbations).
Z_1 = geopotential height of the 500 hPa surface at 15°N (taken as representative of zone 1).
Z_2 = geopotential height of the 500 hPa surface at 50°N (taken as representative of zone 2).

α^* = albedo.
α_1^* = mean albedo for zone 1 (0.25).
α_2^* = mean albedo for zone 2 (0.4).
Δe = vapour pressure deficit.
$\Delta T = T_S - T_A$ (value fixed at 1 K).
$\Delta Z_{500} = Z_1 - Z_2$.
$\Delta\bar{Z}_{500}$ = value of ΔZ_{500} for the current annual mean climate (345 m); adopted as the value for the model's equilibrium climate.
ϕ = latitude.
ρ = surface air density (1.2 kg m^{-3}).
ρ_w = density of sea water (10^3 kg m^{-3}).

10. Appendix B

Instability with radiative and oceanic heat flux sensitivities only

We assume zone 1 to be radiatively unstable, i.e., $\gamma_{I1} < 0$, while allowing zone 2 to be either radiatively stable or unstable (taking account of the fact that the value of γ_{I2} in Table 1 is only weakly negative). We also allow a_{OH} to be of either sign, while assuming that a_{D1} and a_{D2} are positive. We consider two cases separately.

(a) Zone 2 radiatively unstable ($\gamma_{I2} < 0$).
In this case we can write

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = -[(|\gamma_{I1}| - a_{OH}a_{D1}), a_{OH}a_{D2}, a_{OH}a_{D1}, (|\gamma_{I2}| - a_{OH}a_{D2})]. \tag{B.1}$$

Since $\alpha_2\alpha_3 > 0$, it is clear that $\eta^2 > 0$. Thus, both of the criteria (64) and (65) must be satisfied for stability to obtain. We can write

$$\beta_1 + \beta_4 = -\frac{1}{c_{O1}}[(|\gamma_{I1}| - a_{OH}a_{D1}) + R(|\gamma_{I2}| - a_{OH}a_{D2}), \tag{B.2}$$

$$\beta_1\beta_4 - \beta_2\beta_3 = \frac{1}{c_{O1}c_{O2}} [(|\gamma_{I1}| - a_{OH}a_{D1})(|\gamma_{I2}| - a_{OH}a_{D2}) - a_{OH}^2 a_{D1}a_{D2}]. \tag{B.3}$$

If $a_{OH} \leqslant 0$, it is seen from (B.2) that criterion (64) is violated. If $a_{OH} > 0$, there are three possibilities to consider. To examine these, we hold $(|\gamma_{I1}|, |\gamma_{I2}|)$ fixed and let a_{OH} increase from a small (but non-zero) positive value:

(i) If $(|\gamma_{I1}| - a_{OH}a_{D1})$ and $(|\gamma_{I2}| - a_{OH}a_{D2})$ are both positive, (B.2) shows that criterion (64) is violated for all R.

(ii) If one of these quantities is positive while the other is $\leqslant 0$, (B.3) shows that criterion (65) is violated.

(iii) If both of these quantities are $\leqslant 0$, the first product in (B.3) becomes $\geqslant 0$, but clearly it is numerically less than the second product, i.e., criterion (65) is again violated.

Thus, in all the above cases, we have instability.

(b) Zone 2 radiatively stable ($\gamma_{I2} > 0$).
In this case we can write

$$\alpha_4 = \gamma_{I2} + a_{OH}a_{D2},$$

while $(\alpha_2, \alpha_3, \alpha_4)$ are written as before. We again have $\eta^2 > 0$ and we now write

$$\beta_1 + \beta_4 = -\frac{1}{c_{O1}}[(|\gamma_{I1}| - R\gamma_{I2}) - a_{OH}(a_{D1} + Ra_{D2})], \tag{B.4}$$

$$\beta_1\beta_4 - \beta_2\beta_3 = -\frac{1}{c_{O1}c_{O2}}[|\gamma_{I1}|\gamma_{I2} + a_{OH}(|\gamma_{I1}|a_{D2} - \gamma_{I2}a_{D1})]. \tag{B.5}$$

If $a_{OH} \leqslant 0$, we see from (B.4) that a sufficient condition for criterion (64) to be violated is

$$\gamma_{I2} < \frac{1}{R}|\gamma_{I1}|. \tag{B.6}$$

If $a_{OH} > 0$, we see from (B.5) that a sufficient condition for criterion (65) to be violated is

$$\gamma_{I2} < \frac{a_{D2}}{a_{D1}}|\gamma_{I1}|. \tag{B.7}$$

Condition (B.7) is easily satisfied for all realistic values of the parameters and, making the easily satisfied assumption that $R < a_{D1}/a_{D2}$, condition (B.6) is satisfied when (B.7) is satisfied. Thus, we again have instability in all cases.

11. Appendix C

Perturbation evaporative fluxes for the individual normal modes in the case where only the radiative and latent heat flux sensitivities are included

We examine the contributions of the Δe-sensitivity and the u-sensitivity to the perturbation evaporative fluxes $(F_L)_1'$ and $(F_L)_2'$ for both normal modes, in the case where only the radiative and evaporative sensitivities are included.

From (45), using (41) and (42), we can write the ratio of the perturbation to the mean evaporative fluxes as

$$\frac{(F_L)_1'}{(\bar{F}_L)_1} = \left[\frac{L}{R_v\bar{T}_{S1}^2} + \frac{a_M a_{D1}}{2\bar{F}_M}\right]T_{S1}' - \left[\frac{a_M a_{D2}}{2\bar{F}_M}\right]T_{S2}', \tag{C.1}$$

$$\frac{(F_L)_2'}{(\bar{F}_L)_2} = \left[\frac{a_M a_{D1}}{2\bar{F}_M}\right]T_{S1}' + \left[\frac{L}{R_v\bar{T}_{S2}^2} - \frac{a_M a_{D2}}{2\bar{F}_M}\right]T_{S2}'. \tag{C.2}$$

We assume that $\eta^2 > 0$ and that the stability criteria (64) and (65) are satisfied, so that mode 1 given by (56) is the fast mode and mode 2 given by (57) is the slow mode. We define

$$E_{\mathrm{f}} = \exp\left[-\left(\frac{\beta_1+\beta_2}{2}+\eta\right)t\right], \tag{C.3}$$

$$E_{\mathrm{s}} = \exp\left[-\left(\frac{\beta_1+\beta_2}{2}-\eta\right)t\right]. \tag{C.4}$$

Then substituting from (56) into (C.1) and (C.2) we have the following relationships for the fast mode:

$$\frac{1}{AE_{\mathrm{f}}}\frac{(F_{\mathrm{L}})'_1}{(\bar{F}_{\mathrm{L}})_1} = \left[\left(\frac{\mathrm{L}}{\mathrm{R_v}\bar{\mathrm{T}}^2_{\mathrm{S1}}}+\frac{\mathrm{a_M a_{D1}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)-\left(\frac{\beta_3}{\eta+\xi}\right)\left(\frac{\mathrm{a_M a_{D2}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)\right] = \left[(0.06+0.044)-\left(\frac{\beta_3}{\eta+\xi}\right)(0.027)\right]\mathrm{K}^{-1}, \tag{C.5}$$

$$\frac{1}{AE_{\mathrm{f}}}\frac{(F_{\mathrm{L}})'_2}{(\bar{F}_{\mathrm{L}})_2} = \left[\left(\frac{\mathrm{a_M a_{D1}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)+\left(\frac{\beta_3}{\eta+\xi}\right)\left(\frac{\mathrm{L}}{\mathrm{R_v}\bar{\mathrm{T}}^2_{\mathrm{S2}}}-\frac{\mathrm{a_M a_{D2}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)\right] = \left[(0.044)+\left(\frac{\beta_3}{\eta+\xi}\right)(0.07-0.027)\right]\mathrm{K}^{-1}. \tag{C.6}$$

Similarly, substituting from (57) into (C.1) and (C.2) we have the following relationships for the slow mode:

$$\frac{1}{BE_{\mathrm{s}}}\frac{(F_{\mathrm{L}})'_1}{(\bar{F}_{\mathrm{L}})_1} = -\left[\left(\frac{\beta_2}{\eta+\xi}\right)\left(\frac{\mathrm{L}}{\mathrm{R_v}\bar{\mathrm{T}}^2_{\mathrm{S1}}}+\frac{\mathrm{a_M a_{D1}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)+\left(\frac{\mathrm{a_M a_{D2}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)\right] = -\left[\left(\frac{\beta_2}{\eta+\xi}\right)(0.06+0.044)+(0.027)\right]\mathrm{K}^{-1}, \tag{C.7}$$

$$\frac{1}{BE_{\mathrm{s}}}\frac{(F_{\mathrm{L}})'_2}{(\bar{F}_{\mathrm{L}})_2} = \left[-\left(\frac{\beta_2}{\eta+\xi}\right)\left(\frac{\mathrm{a_M a_{D1}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)+\left(\frac{\mathrm{L}}{\mathrm{R_v}\bar{\mathrm{T}}^2_{\mathrm{S2}}}-\frac{\mathrm{a_M a_{D2}}}{2\bar{\mathrm{F}}_{\mathrm{M}}}\right)\right] = \left[-\left(\frac{\beta_2}{\eta+\xi}\right)(0.044)+(0.07-0.027)\right]\mathrm{K}^{-1}, \tag{C.8}$$

where the Δe-sensitivity and u-sensitivity terms have been evaluated numerically using our standard model values.

From (C.5) and (C.6) it is clear that, depending on the magnitude of the coupling factor $\beta_3/(\eta+\xi)$, the u-sensitivty terms can dominate over the Δe-sensitivity terms in one or other of the zones for the fast mode. Likewise, from (C.7) and (C.8), it is clear that, depending on the magnitude of the coupling factor $\beta_2/(\eta+\xi)$, a similar situation can hold for the slow mode.

Considering our special case where the radiative sensitivities are included in addition to the evaporative sensitivities (but all other sensitivities are neglected), we find using Table 1 that $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (1.29, -0.44, 0.22, 0.17)$ PW K^{-1}. Hence, choosing $H_1 = 100$ m and $H_2 = 500$ m, we find that $\beta_3/(\eta+\xi) = 0.035$ and $\beta_2/(\eta+\xi) = -0.35$. Using these values, (C.5) and (C.6) show that, for the fast mode, the Δe-sensitivity is slightly dominant (by a factor of 1.4) in determining $(F_{\mathrm{L}})'_1$, but the u-sensitivity is overwhelmingly dominant in determining $(F_{\mathrm{L}})'_2$. On the other hand, (C.7) and (C.8) show that, for the slow mode, the Δe-sensitivity is dominant (by a factor of 1.8) in determining $(F_{\mathrm{L}})'_1$ and more markedly dominant in determining $(F_{\mathrm{L}})'_2$.

In general, it is clearly important to include both types of sensitivity in examining the stability properties of the system.

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