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An inconsistency between two classical models of the ocean buoyancy driven circulation

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ABSTRACT

A 2-layer version of the Stommel-Arons model of the abyssal circulation is shown to be inconsistent with the closure scheme used in Stommel's conceptual box model of the thermohaline circulation. The closure relates the strength of the exchange between 2 boxes, taken to represent the meridional overturning cell in the ocean, to the density difference between the boxes. Here, the Stommel-Arons model is used to argue that the difference in density, averaged over regions corresponding to Stommel's boxes, is not indicative of the rate of exchange between these regions. More generally, it is argued that to a good approximation, the zonally-averaged density field in the Stommel-Arons model is independent of both the sense and structure of the meridional overturning cell or cells. The reason for this is that, although the western boundary currents make an O(1) contribution to the zonally-averaged meridional transport, they have only a very small influence on the zonally averaged density field.

1. Introduction

Much of the ocean's rôle in regulating climate is thought to be linked to its thermohaline circulation. Two now classic papers form the basis of much of current thinking regarding both the meridional and horizontal structure of this circulation. In one of these (Stommel, 1961), Stommel proposes a simple box model driven by buoyancy fluxes to show that multiple steady states can exist under identical forcing. The model used two boxes to represent the high and low latitudes of a single hemisphere basin and has since been extended to allow for the possibility of pole to pole cells, as well as exchanges between different ocean basins (Welander, 1986). The multiple steady states obtained are generally interpreted as being analogous to multiple steady states of the zonally averaged meridional overturning cell in the ocean. A key element of Stommel's model is a closure scheme which relates the density difference between adjacent well mixed boxes to the strength of the exchange between the boxes. The second paper, (Stommel and Arons, 1960), takes the strength and structure of the zonally-averaged meridional cell to be specified and solves for the implied horizontal circulation. A success of this work was its prediction of deep western boundary currents, which are represented in the model as thin dissipative boundary layers. In this note, it is demonstrated that these two models of the buoyancy driven circulation are inconsistent with one another. In particular, it is shown that, given a separation of scale between the width of an ocean basin and the width of a deep western boundary layer, the structure and even the direction of the meridional cell is uncorrelated with the zonally averaged density field. Thus, for example, based on the zonally averaged density field alone, it is impossible to distinguish between two oppositely

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directed pole to pole cells or between a pole to pole cell and a meridional circulation which is symmetric about the equator.

It is helpful to begin with a brief review of the Stommel-Arons model of mass driven circulation. Consider two isopycnal layers separated by an interface, whose height above the sea floor is given by η. Density in the lower layer, ρ_2, is taken to be slightly larger than density in the upper layer, ρ_1. Deep water formation is represented in the model by one or more localized transfers of mass from the upper layer to the abyssal layer. A steady state requires that there be a return of this water to the upper layer. Following Stommel and Arons, it is assumed here that this diapycnal velocity, e, is uniformly distributed over the basin.

The model assumes the interior to be in a planetary geostrophic balance. In the interest of simplicity, the equatorial β-plane approximation is made (i.e., $f \equiv \beta y$, where f is the Coriolis parameter). Thus, the horizontal velocity field is taken to be geostrophic and the mass equation is taken to be fully nonlinear*. In the absence of wind forcing, the barotropic velocity vanishes and the horizontal velocity field in both layers is specified by the interface height field:

$$(u_1, v_1) = \frac{g'\eta}{H\beta y}(\eta_y, -\eta_x), \tag{1}$$

$$(u_2, v_2) = \frac{g'(H-\eta)}{H\beta y}(-\eta_y, \eta_x), \tag{2}$$

for the upper and lower layers, respectively. Continuity of mass within a layer then leads to an equation for η:

$$\frac{g'\eta(H-\eta)}{\beta H y^2}\eta_x = e, \tag{3}$$

where H is the (constant) depth of the ocean. Taking the model domain to be a rectangular basin, with $x=0$ corresponding to the eastern boundary, and integrating (3) leads to a cubic equation for $\eta(x,y)$:

$$\frac{g'H}{2}(\eta_{\text{int}}^2 - \eta_0^2) - \frac{g'}{3}(\eta_{\text{int}}^3 - \eta_0^3) = \beta H e x y^2, \tag{4}$$

where η_0 gives the the position of the interface height field at $x=0$ and the subscript $_{\text{int}}$ indicates

* Advection of the interface by the ageostrophic velocity field is, however, neglected.

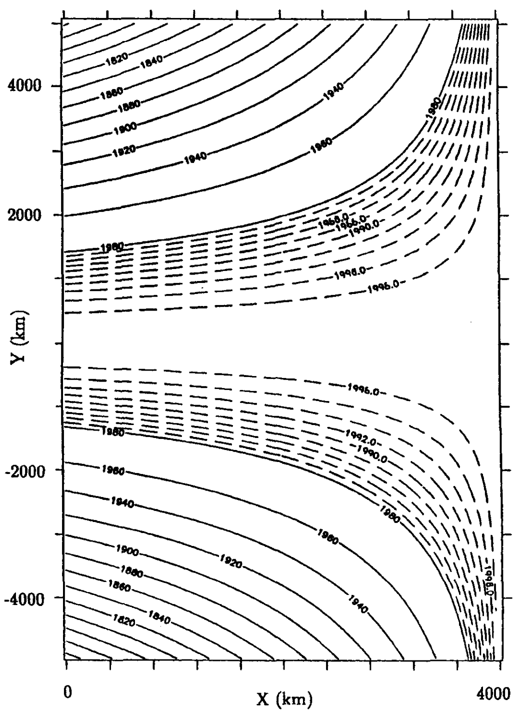

Fig. 1. η contours for the case $g' = 5 \times 10^{-3}$, $\eta_0 = 2000$ m, $H = 4000$ m, $\beta = 2e-11$ m^{-1} s^{-1} and $e = 6.7 \times 10^{-7}$ m/s. The meridional length scale (equator to pole) is 5000 km and the zonal length scale 4000 km. Note that for this choice of parameters, the variations in η are small compared to η_0 so that the linerizied verion of (4), $\eta_{\text{int}} \approx \beta H x y^2 e / g' \eta_0 (H - \eta_0)$ would have yeilded essentilly the same result. Contour intervals are 20 m for solid curves and 2 m for dashed curves.

that the solution applies to the basin's interior. Note that η_0 is taken to be specified in the problem**. Contours of the one physical root (Fig. 1) serve as streamlines for the interior horizontal flow field. The sense of the flow is such that fluid columns move poleward and eastward in the abyssal layer and equatorward and westward in the upper layer.

To meet the no normal flow boundary condition, dissipative boundary layers are appended to the interior solution along the poleward and western walls. The width of these boundary layers is

** Geostrophy, together with the no normal flow boundary condition, implies that η_0 is independent of y along the eastern boundary. Here, it is further assumed that the value of η_0 is known.

related to the strength of the dissipation. Common choices include assuming either a linear drag law or an eddy diffusion of momentum on the component of the velocity parallel to the boundary. For example, in a western boundary layer, the v-momentum equation becomes

$$\beta y u_i = -\frac{1}{\rho_0}\frac{\partial P_i}{\partial y} - r v_i + \nu \frac{\partial^2}{\partial x^2} v_i, \qquad (5)$$

where $\partial P_i/\partial y$ gives the meridional pressure gradient* in layer i and L_{bndy}, the width of the western boundary layer, is taken to be small compared to the y length scale of the problem. L_{bndy} is then given by the larger of r/β and $(\nu/\beta)^{1/3}$, depending on whether the linear drag or the eddy diffusion of momentum is dominant. For the model to be consistent with a geostrophic interior flow, r and ν must be sufficiently small so as to ensure that $\varepsilon \equiv L_{\mathrm{bndy}}/L_{\mathrm{basin}} \ll 1$. It should be noted, however, that the neglect of nonlinearities in the momentum equations (and the neglect of advection of the interface height field by the ageostrophic component of the velocity in the mass equation) prohibits taking ε to be arbitrarily small. As a rule of thumb, one expects the solution to remain consistent provided that advection of relative vorticity in the boundary layer potential vorticity equation can be neglected. Typically, it is required that $\beta v \gg u\zeta_x$, where ζ is relative vorticity, to obtain the condition $L_{\mathrm{bndy}} \gg (u/\beta)^{1/2}$. Alternatively, one could compare βv to $v\zeta_y$ to obtain the condition that $L_{\mathrm{bndy}} \gg (T/\beta L_y d)^{1/2}$, where T is the boundary layer transport, d is the thickness of the layer and L_y is the y length scale of the problem. Taking T to be 20 Sverdrups ($1\ \mathrm{Sv} = 10^6\ \mathrm{m^3/s}$), $\beta L_y \approx 10^{-5}\ \mathrm{s^{-1}}$ and $d \approx 10^3$ m gives that advection of relative vorticity can be neglected provided the boundary layer width is large compared to 45 km. For boundary layers which are thick enough such that inertial effects may be safely neglected, yet which are thin compared to the basin width, the details of the dynamics in the boundary layer (e.g., the relative importance of r and ν) do not affect the interior solution. It is this range of parameters, for which boundary layers can be treated as both passive and thin, that is considered here.

* P and η are related as follows: In the upper layer,

$$\frac{1}{\rho_0}\nabla P_1 = \frac{-g'\eta\nabla\eta}{H} \qquad (6)$$

and, in the lower layer,

$$\frac{1}{\rho_0}\nabla P_2 = \frac{g'(H-\eta)\nabla\eta}{H}. \qquad (7)$$

2. The Inconsistency

As mentioned, Stommel's box model takes the strength of the exchange between the two boxes to be proportional to the density difference between the two (well-mixed) boxes. One would like to test for consistency between the two pictures of the thermohaline circulation. The zonally and vertically averaged density field, $\bar{\rho}(y)$, of the Stommel and Arons model is determined by the zonally averaged position of the interface height field:

$$\bar{\rho}(y) = \rho_1 + \frac{\bar{\eta}}{H}(\rho_2 - \rho_1). \qquad (8)$$

The zonally averaged value of η can be decomposed into contributions from the interior and boundary layer solutions:

$$\bar{\eta}(y) = \overline{\eta_{\mathrm{int}}}(y) + \varepsilon\overline{\eta_{\mathrm{bndy}}}(y). \qquad (9)$$

Thus, provided $\overline{\eta_{\mathrm{bndy}}}$ is the same order of magnitude as $\overline{\eta_{\mathrm{int}}}$, the zonally averaged density field is determined to $O(\varepsilon)$ by the solution to (3). The $O(1)$ contribution to $\bar{\rho}(y)$ is shown for various values of e in Fig. 2. Note that, somewhat counter-intuitively, the model predicts an abundance of dense water at low latitude. If one were to subdivide the domain into 3 regions (corresponding to two polar boxes and one equatorial box), and apply the closure used by Stommel to model fluxes between the various boxes, the solution would correspond to an exchange between the boxes which is symmetric about the equator. If this exchange is interpreted as a meridional cell in the usual way, then a symmetric cell with equatorial sinking is obtained.

Yet thus far, nothing has been said as to the placement of the localized sinking region or regions. The central point of this note is simply that $\bar{\rho}(y)$ is essentially specified independently of the position of the localized sinking. The sense and structure of the meridional circulation, on the other hand, is crucially dependent on the placement of the sinking. Placement of a single localized

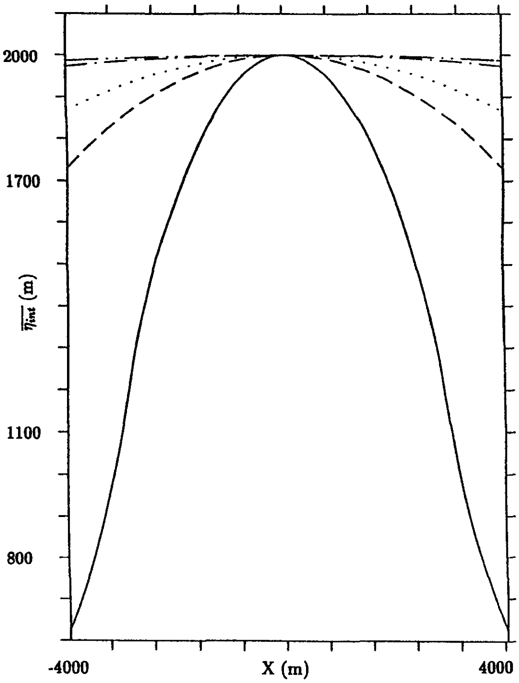

Fig. 2. Zonally averaged interface height field ($\bar{\eta}_{int}$) for various choices of e, ranging from $e = 6.7 \times 10^{-6}$ (bottom curve) to $e = 6.7 \times 10^{-8}$ (top curve). These correspond to residence times ranging from 3×10^8 s to 3×10^{10} s. Note that the signal is relatively weak for large residence times and that, curiously, there is a relative abundance of dense water in the low latitudes. For the shortest of these residence times, $\eta \to 0$ in the poleward and western corners of the domain. In this case (for which the residence time is on the order of 10 years), one might expect the boundary layer contribution to become non-negligible.

sinking region along the northern boundary, for example, corresponds to a pole to pole cell with sinking in the northern hemisphere. Placement of the sinking region in the opposite hemisphere reverses the sense of the meridional circulation, but has only an $O(\varepsilon)$ effect on the zonally (or regionally) averaged density fields. Thus, given the density fields predicted by the Stommel and Arons model for these two oppositely directed pole to pole cells, the closure used in Stommel's box model would predict essentially the same meridional circulation in each case. Furthermore, this predicted circulation is, to leading order, symmetric about the equator. The underlying reason for this discrepancy between the two models is that, although the boundary layers make an $O(1)$ contribution to the net meridional transport, they make only an $O(\varepsilon)$ contribution to $\bar{\rho}(y)$. Given this, it is then straight-forward to construct cells corresponding to all 4 steady states of a 3-box version of Stommel's box model (Welander, 1986), without significantly affecting $\bar{\rho}(y)$ or its average value over the three subregions of the domain. In this sense, these two classic models of ocean's buoyancy driven circulation are seen to contradict one another.

3. Discussion

It would thus seem that one is forced either to reject Stommel's closure (or any closure relating the strength of the meridional circulation to the zonally averaged density field), or to reject the Stommel and Arons (1960) model as giving an order 1 description of the global mass driven circulation. On the one hand, Stommel's closure scheme is based on the intuitive idea that heavy water should want to flow under light water, so that meridional density gradients should tend to drive an overturning in the y,z plane. Since the length scales of the flows of interest here are very much larger than the internal Rossby radius, however, this intuition becomes suspect. The Stommel and Arons model, on the other hand, is more physically based; but is highly unrealistic in that it does not allow for surface density gradients. Furthermore, the model is of limited use to researchers interested in issues issues relating to climate dynamics since the meridional cell is specified, not solved for, in the model.

Some effort has recently gone into finding empirical correlations between zonally averaged quantities from numerical general circulation models and the strength of the meridional circulation in these models. Wright et al. (1995) did this and found that a closure scheme relating strength of the meridional cell to integrals of zonally averaged quantities could be tuned to give a good correlation with data from a general circulation model. If these correlations prove to be robust, then the explanation must involve dynamics beyond those discussed above. However, given the coarse resolution of the general circulation models, their dynamics would appear to be essentially equivalent to those explored here (i.e., a roughly planetary geostrophic balance in the interior, with dissipative boundary layers). Several possible differences which might help to explain the correlation which they found are the following. (This is not intended to be an exhustive list of possible explanations). Firstly, ε may not be entirely negligible in numerical general circulation models. Wright et al. (1995) used an eddy diffusivity of 2.5×10^5 m^2 s^{-1}, leading to a Munk boundary layer width of roughly 500 km in a 60° wide basin. Furthermore, it is straight forward to argue (at least for mass driven flow) that the O(ε) contribution to $\bar{\rho}(y)$ is such that the boundary layer contribution to the meridional transport is directed in the sense assumed by the Stommel (1961) closure scheme. Thus, if ε becomes non-negligible, some correlation between $\bar{\rho}$ and the sense of the boundary layer transport will develop. Of course, the sense of the transport in the western boundary layer does not always correspond to the sense of the net meridional transport. Additionally, it is not clear to what extent diapycnal upwelling is spatially distributed throughout the ocean's interior, as assumed in the Stommel and Arons framework, or whether it occurs primarily in the boundary layers themselves. Indeed numerical simulations (Bryan, 1987) suggest the latter possibility. Some of this may be related to the horizontal, rather than isopycnal, eddy diffusivities used by these models.

Another possibility is that the dynamics assumed within the Stommel and Arons framework is not representative of the meridional circulation. Sakai and Peltier, for example, (Sakai and Peltier, 1995) argue that the mean flow in the interior is not in geostrophic balance. In particular, they consider the meridional component of this flow to be described by a balance between the Coriolis force and a parameterized Reynolds stress. Additionally, the limited vertical structure of the Stommel and Arons model may be insufficient to adequately describe the meridional circulation. If one imagines a many layer version of this model, then it is not clear that characteristic equations (corresponding to eq. (3) in the present paper) can be derived for each of the vertical modes. In this case, the interior solution may not be specified by knowledge of the eastern boundary condition and the forcing alone. Given this, one can not rule out the possibility that O(1) asymmetries between $\bar{\rho}$ in the sinking and upwelling hemispheres could develop.

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