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A note on the influence of breaking wind waves on the aerodynamic roughness of the sea surface as seen from below

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ABSTRACT

The purpose of this note is to incorporate the effect of wave breaking in the parametrization of the sea surface roughness as seen from below. It is shown that aerodynamically smooth conditions in the classical sense never exist below the sea surface, since effective turbulent viscosity due to shear free turbulence generated by wave breaking is much larger than molecular.

1. Introduction

When we consider the wind-induced drift current just below the sea surface, it is usually assumed that the mean velocity profile can be described by a logarithmic law. That is how the concept of the sea-surface roughness as seen from below is introduced. Until recently, it was also the only method for parametrization of the fluxes of gases across air-sea interface, whose resistance is in a water phase. The only parameter in such models, which can be still depend on the structure of the surface wave field was *again* aerodynamic roughness of the sea surface as seen from below. To determine this parameter, Zilitinkevich et al. (1991) adopted the analogy with the models for surface roughness as seen from above. In the framework of such an approach, the influence of waves was taken into account by the proper choice of the roughness parameter. This was still done by using the classical concept of aerodynamical roughness exceeding the viscous sublayer thickness (Zilitinkevich et al., 1991).

The purpose of this note is to incorporate the effect of wave breaking in the parametrization of the sea-surface roughness as seen from below. This will permit us to eliminate the contradictions created by using the analogy with rigid surfaces in interpretation of the experimental data on wind-induced drift currents.

We repeat here a sentence from our 1984 paper (Kitaigorodskii, 1984): "In real wind-wave generation conditions, even with only small-scale wave breaking, the transformation of wave energy into turbulence (due to wave breaking) can dramatically change the character of turbulence just below the wavy surface. Because of this, the whole concept of classical momentum viscous sublayer, based solely on the analogy with flow above a rigid surface, can be quite inappropriate for a description of the damping effect of the wavy free surface on the dynamics of turbulence generated by breaking waves."

2. Velocity profiles below the wavy sea surface and a parametrization of sea surface roughness as seen from below

When considering the drift current below the sea surface, it is usually assumed that the velocity profile can be described as

$$U_s - U(z) = \frac{u_*^w}{\kappa} \ln \frac{Z}{Z_{od}}, \qquad (1)$$

where U_s is the velocity of the surface drift u_*^w is the friction velocity in water ($u_*^w = \frac{1}{30} u_*^a$) and Z_{od} is the surface roughness as seen from below. In practical calculations, as a rule, an empirical relationship between U_s and friction velocity in the air, u_*^a, is used. Philipps and Banner (1974) suggest

$$\frac{U_s}{u_*^a} \approx 0.55. \tag{2}$$

The basic contribution to U_s is due to the wind-induced drift current; the rest, associated with Stokes drift U_{sd}, can be small (Wu, 1975). According to Wu (1975), $U_s/u_*^a \approx 0.53$. In strong wind conditions in open ocean and large fetches, this is not necessarily true. Laboratory experiments however (Wu, 1975; Kreiman and Karlin, 1979) show that eq. (1) is valid, especially under light wind conditions. For the existence of a logarithmic portion in the winddrift velocity profile, a range of depths should exist where neither wave breaking nor rotation and density stratification effects are important. Since the Ekman depth for drift current, u_*^w/Ω, is almost always much larger than the amplitude of breakers, the logarithmic sublayer should exist in most hydrometeorological situations. Zilitinkevich et al. (1991), suggested that Z_{od} be described by the similar formulae as in the case of roughness as seen from above, i.e.,

$$Z_{od} = m_2 \frac{v^w}{u_*^w}, \tag{3}$$

$$Z_{od} = m_3 \frac{(u_*^w)^2}{g}. \tag{4}$$

The straightforward analogy with the roughness length as seen from above gives the following values of the constants: $m_2 = 0.135$; $m_3 = 0.014 = 0.035$. The value of $m_3 = 0.034$ from the concept of moving roughness elements (Kitaigorodskii, 1973) leads to $Z_o/h_s = \frac{1}{30}$, where h_s is sand roughness, which is in good agreement with the classical results of Nikuradse (1933). In contrast, the data from the laboratory experiments performed by Kreiman and Karlin (1979), Zilitinkevich et al. (1991) and Kränenburg (1984) together with field data in lakes lead to the following estimates:

$$m_2 = 33; \qquad m_3 = 3 \cdot 10^3. \tag{5}$$

Actually, Zilitinkevich adopts the same model for Z_{od} as in Kitaigorodskii (1977) for roughness length as seen from above. He presents the analogy to Fig. (1.18) of Kitaigorodskii (1973) for the dependence of $Z_{od} u_*^w / v^w$ on the non-dimensional roughness parameter $\mathrm{Re}_s = (u_*^w)^3/g v^w$. This *analogy* however leads to *unreasonably high values* of the constants m_2 and m_3 in eq. (5). To explain this, we first start with the analysis of "rough" condititions eq. (4). The first point is that the scale $(u_*^w)^2/g$ is even smaller than Kolmogoroff's microscale. According to the latest measurements (Drennan et al., 1991), dissipation in the presence of breaking wind waves close to the sea surface is in the range $\varepsilon^w = 10$–$0.5\ \mathrm{cm}^2/\mathrm{s}^3$.

This leads to

$$l_v = \left(\frac{v^3}{\varepsilon^w}\right)^{1/4} \gg \left(\frac{u_*^w}{g}\right)^2.$$

If the breaking is important for introducing the roughness from below, then it still must be related to the amplitude of breaking waves, which is proportional to $(u_*^a)^2/g$ rather than $(u_*^w)^2/g$.

Eq. (4) can therefore be rewritten as

$$Z_{od} = m_3' \frac{(u_*^a)^2}{g} \qquad \text{with} \quad m_3' = 3. \tag{6}$$

The coefficient $m_3' = 3$, however, is still about $100\times$ larger than the same coefficient in the expression for the aerodynamic roughness length of the sea surface as seen from above, (where $Z_o \simeq 0.014 (u_*^a)^2/g$).

The explanation of this can be given by considering wave breaking as a mechanism similar to generation of shear free turbulence by the oscillating grid. The amplitude of the oscillations can be related to the amplitude of breaking waves. Then it becomes not longer impossible that the roughness of the sea surface as seen from below is much larger than from above. For the latter roughness parameter, the important length scale is related to the amplitude of short waves responsible for flow separation behind their crests. For roughness below the sea surface, the important length scales are associated with amplitudes of breakers which can be larger than the amplitude of short waves responsible for flow separation in the air. This becomes more evident if one remembers that for surface as seen from below, the effects of flow separation is highly unlikely (because of *relatively low values* of U_s compared with the phase velocities of short gravity waves). Thus, the length scale $(u_*^a)^2/g$ has different meanings for roughness as seen from above and as seen from below. This is possibly the main reason of observed differences in the values of Charnock's constant in the cases considered.

To explain the high value of the constant m_{2v} in eq. (3), we emphasize that v/u_*^w is not a proper scale of the roughness length in this case. We recall that the value of $m_2 = 33$ cited in Zilitinkevich et al. (1991) is about 300× larger than the value of a similar constant for aerodynamically smooth conditions above solid surfaces. Indeed, even for light wind conditions, microscale wave breaking, which is difficult to observe directly, can produce the diffusion of turbulence downward, close to the surface, as observed in Drennan et al. (1991) and Terray and Bliven (1985). This diffusion, according to the theory of shear free turbulence, can be better characterized by constant eddy viscosity K (Long, 1978). The value of K is as we will show below, at least 10^2 times larger than v^w. Thus, we can choose for Z_{od}, instead of eq. (3), another expression, which also follows from dimensional considerations

$$Z_{od} \approx \frac{K}{u_*^w}. \tag{7}$$

Eq. (7) just implies that in the presence of momentum flux in water and drift current, the shear free turbulence approximation for K *is more appropriate* than, e.g., the concept of the eddy viscosity associated with shear instability (which is z dependent). Thus, the constant of proportionality in (7) will be of the order of unity, and since $K/v^w \approx 10^2$, that explains the high value of m_2 in (3).

The other important aspect of using eq. (7) implies that before the logarithmic velocity profile, the first profile to develop (both in space and time) close to the surface must be a *linear* mean velocity profile based on constancy with depth of both momentum flux τ_w in water, and K. Careful laboratory measurements of wind-induced current below wind waves indicate the existence of a thin "slab" above the logarithmic sublayer (Terray and Bliven, 1985). The linear velocity profile inside such a slab can with some reservations be considered to be similar to the linear velocity distribution inside the viscous sublayer, because in both cases, the total momentum flux is constant with depth. If such conditions really hold, we will be permitted to use for the non-dimensional ratio, $Z_{od} u_*^w / K$, an empirically well founded value of 0.11 (as in the aerodynamically smooth case).

3. Aerodynamic classification of the sea surface roughness conditons as seen from below in the presence of wave breaking

Following Kitaigorodskii (1984), we assume that the wave breaking leads to generation of the shear free turbulence layer close to the sea surface. Then, instead of v^w, we must use a z-constant eddy viscosity (Long, 1978). To estimate K in the shear free turbulence model, we express it in the form:

$$K \cong a\omega L, \tag{8}$$

where a is the amplitude of the grid oscilations (which can be related to the amplitude of breaking waves), ω is the frequency of the oscillation (which can be related to the periodicity of breaking events). $\omega = 2\pi/10T_{br}$ where T_{br} is a typical period of a breaking wave, and L is a length scale which can be proportional either to a, which will lead to the expression

$$K \simeq a^2\omega, \tag{9a}$$

or to some geometrical characteristics of the grid (mesh), (which can be related to the distance between breakers). In the latter case,

$$K = a^2\omega f\left(\frac{a}{L}\right). \tag{9b}$$

For example, according to some laboratory experiments $f \approx (a/L)^{-1/2}$. Below we will use for K the simple expression (9a). Then with $v^w \approx 10^{-2}$, $a = 1$ cm and $T_{br} \approx 1$ s, we get $K/v^w > 10^2$. Now, according to our new interpretation of sea surface roughness Z_{od}, we replace the non-dimensional parameter $(u_*^w)^3/gv$ used by Zilitinkevich et al. (1991) with the ratio K/v^w which represents the roughness length Reynolds number Re_s^K for the turbulence regime below the sea surface in the presence of breaking wind waves. Using eq. (9a) with

$$\omega = \frac{2\pi}{10T_{br}} \approx \frac{1}{T_{br}} \approx \left(\frac{2\pi\lambda_{br}}{g}\right)^{-1/2}$$

and

$$a \simeq a_{br} \approx \frac{(u_*^a)^2}{g}$$

(in laboratory and light wind conditions), we have

$$\mathrm{Re}_s^K \cong \left(\frac{a_{br}}{\lambda_{br}}\right)^{1/2} \frac{a_{br}^{3/2}\, g^{1/2}}{\nu^w} \left(\frac{1}{2\pi}\right)^{1/2}$$

$$\cong \frac{(u_*^a)^3}{g\nu^w} \left(\frac{1}{2\pi}\frac{a_{br}}{\lambda_{br}}\right)^{1/2}. \tag{10}$$

Here

$$\omega = \frac{2\pi}{10T_{br}} \approx \frac{1}{T_{br}}$$

represents the periodicity of breaking events (Kitaigorodskii, 1984), *but not* the period of breaking waves T_{br}. Here, for simplicity, we just accept that each one of the 10 waves brakes. More refined definition of ω and a_{br} requires the introduction of characteristics of the so-called dissipation subrange in wind-wave spectra (Kitaigorodskii, 1986, 1991). Thus, with a constant value $(a_{br}/\lambda_{br}) \approx \frac{1}{10}$, the new Reynolds roughness *number* for the sea surface as seen from below Re_s^K is similar with that for the sea surface as seen from above. Instead of ν^a in the air, however, we have to substitute viscosity in the water. With

$$\left(\frac{a_{br}}{\lambda_{br}}\frac{1}{2\pi}\right)^{1/2} \approx 0.1,$$

this leads to the values of $\mathrm{Re}_s^K \gg \mathrm{Re}_s$ especially in moderate wind contitions when $a_{br}\, g/(u_*^a)^2 \gg 1$.

Thus, the sea surface as seen from below is more rough in the *classical sense* than as seen from above. This rather unexpected conclusion deserves to be accurately checked against observations.

The limits for the behaviour of Z_{od} are thus:

$$\text{as} \quad \mathrm{Re}_s^K \to 0 \qquad Z_{od} \to 0.1\,\frac{\nu^w}{u_*^w} \tag{11}$$

$$\text{as} \quad \mathrm{Re}_s^K \to \infty \qquad Z_{od} \to 0.1\,\frac{K}{u_*^w}. \tag{12}$$

Of course, the unknown proportionality constants in the expressions $K \approx a\omega L$ and $a_{br} \approx (u_*^a)^2/g$ make the range of variability of the Reynolds roughness number Re_s^K still not well-known.

4. The dependence of the sea surface roughness, as seen from below, on wave development

The easiest way to investigate the differences in behaviour of sea-surface roughness as seen from above and below, as dependent on the stage of wave-development, is to rewrite eq. (7) in the form

$$Z_{od} \cong \frac{K}{u_*^w} = \frac{a^2\omega}{u_*^w} = \frac{(u_*^a)^4}{g^2 u_*^w}\frac{2\pi}{10T_{br}}$$

$$\approx \frac{(u_*^a)^4}{g^2 u_*^w}\left(\frac{g}{2\pi\lambda_{br}}\right)^{1/2}, \tag{13}$$

and take

$$a \approx a_{br} \simeq \frac{(u_*^a)^2}{g},$$

$$\omega \simeq \frac{2\pi}{10T_{br}},$$

$$T_{br} = \left(\frac{2\pi\lambda_{br}}{g}\right)^{1/2}.$$

By analogy with the Charnock constant $m = (gZ_o/u_*^a)^2$ for the sea surface roughness as seen from above, we can introduce a new constant

$$m_1 = \frac{gZ_{od}}{(u_*^a)^2}, \tag{14}$$

and take from (13) the following expression for m_1:

$$m_1 = \frac{(u_*^a)}{u_*^w}\frac{(u_*^a)}{C_{br}}. \tag{15}$$

Note that u_*^a/C_{br} is necesserily proportional to u_*^a/C_p (even though the effective value of C_{br} is probably smaller than C_p, the peak phase velocity, and

$$\frac{u_*^a}{u_*^w} \approx \text{coast} \approx 30, \qquad \frac{u_*^a}{C_p} = (2 \cdot 10^{-2} - 1).$$

Thus, we come to the conclusion that the roughness of the sea surface from below is larger for young waves than for developed waves. This behaviour is different from that which was found from the data on the dependence of the m on u_*^a/C_p (see, e.g., Toba et al., 1990). This also deserves to be checked against observation, since it contradicts intuitive feeling about an increase of K with wave growth up to a very developed state.

5. Concluding remarks

We have demonstrated in this note that the physical phenomena determining the roughness conditions above and below the sea surface are different, even though the basic mechanisms in both cases are connected with wind-generated surface waves, and in particular with the process of "dissipation" of wave energy through their breaking.

It is shown that the introduction of the effective depth constant eddy viscosity, typical for shear free turbulence in the description of the roughness conditions below the sea surface can explain observed variability of the roughness parameter Z_{od}. The range of changes of new Reynolds roughness number Re_s^K, however, remains still unknown, which does not permit us to determine which aerodynamic conditions below the sea surface prevail.

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